Problem 9.12 Convert the following dB values to voltage ratios:

(a) 46 dB

(b) 0.4 dB

(c) -12 dB

(d) -66 dB

Solution:

(a) $10^{(46/20)} = 10^{2.3} = 199.5 \simeq 200.$

(b) $10^{(0.4/20)} = 10^{0.02} = 1.047.$

(c) $10^{(-12/20)} = 10^{-0.6} = 0.25.$

(d) $10^{(-66/20)} = 10^{-3.3} = 5 \times 10^{-4}.$

Problem 9.14 Generate Bode magnitude and phase plots for the following voltage transfer functions:

(a) $\mathbf{H}(\omega) = \dfrac{4\times 10^4(60+j6\omega)}{(4+j2\omega)(100+j2\omega)(400+j4\omega)}$

(b) $\mathbf{H}(\omega) = \dfrac{(1+j0.2\omega)^2(100+j2\omega)^2}{(j\omega)^3(500+j\omega)}$

(c) $\mathbf{H}(\omega) = \dfrac{8\times 10^{-2}(10+j10\omega)}{j\omega(16-\omega^2+j4\omega)}$

(d) $\mathbf{H}(\omega) = \dfrac{4\times 10^4\omega^2(100-\omega^2+j50\omega)}{(5+j5\omega)(200+j2\omega)^3}$

(e) $\mathbf{H}(\omega) = \dfrac{j5\times 10^3\omega(20+j2\omega)}{(2500-\omega^2+j20\omega)}$

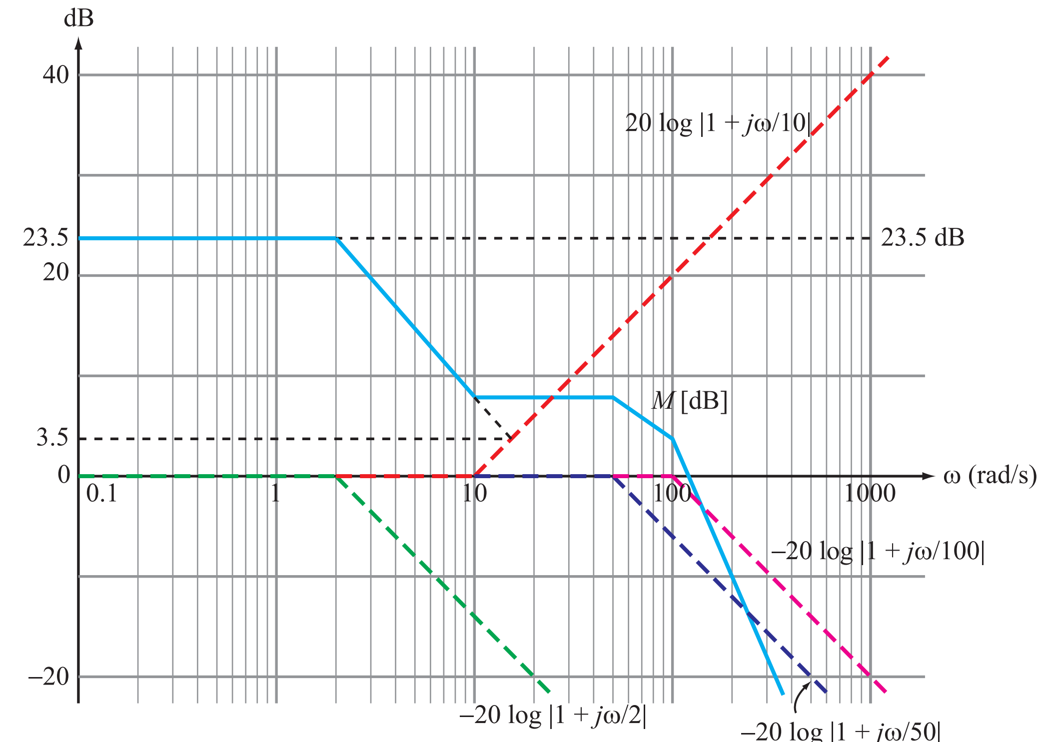
(f) $\mathbf{H}(\omega) = \dfrac{512(1+j\omega)(4+j40\omega)}{(256-\omega^2+j32\omega)^2}$

(g) $\mathbf{H}(\omega) = \dfrac{j(10+j\omega)\times 10^8}{(20+j\omega)^2(500+j\omega)(1000+j\omega)}$

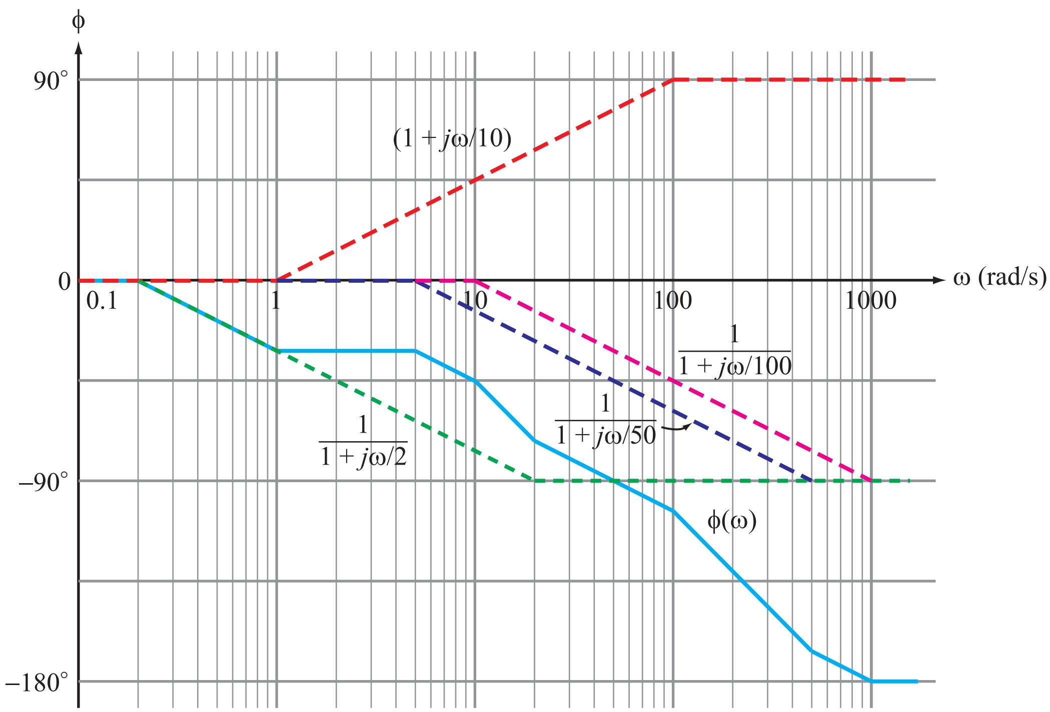
Solution:
(a)

$$\begin{aligned}
\mathbf{H}(\omega) &= \frac{4\times 10^4(60+j6\omega)}{(4+j2\omega)(100+j2\omega)(400+j4\omega)}\\
&= \frac{4\times 10^4\times 60(1+j\omega/10)}{4\times 100\times 400(1+j\omega/2)(1+j\omega/50)(1+j\omega/100)}\\
&= \frac{15(1+j\omega/10)}{(1+j\omega/2)(1+j\omega/50)(1+j\omega/100)}\ .
\end{aligned}$$

- Constant term 15 $\Longrightarrow$ 23.5 dB
- Simple pole with $\omega_c = 2$ rad/s
- Simple zero with $\omega_c = 10$ rad/s
- Simple pole with $\omega_c = 50$ rad/s
- Simple pole with $\omega_c = 100$ rad/s



Magnitude



Phase

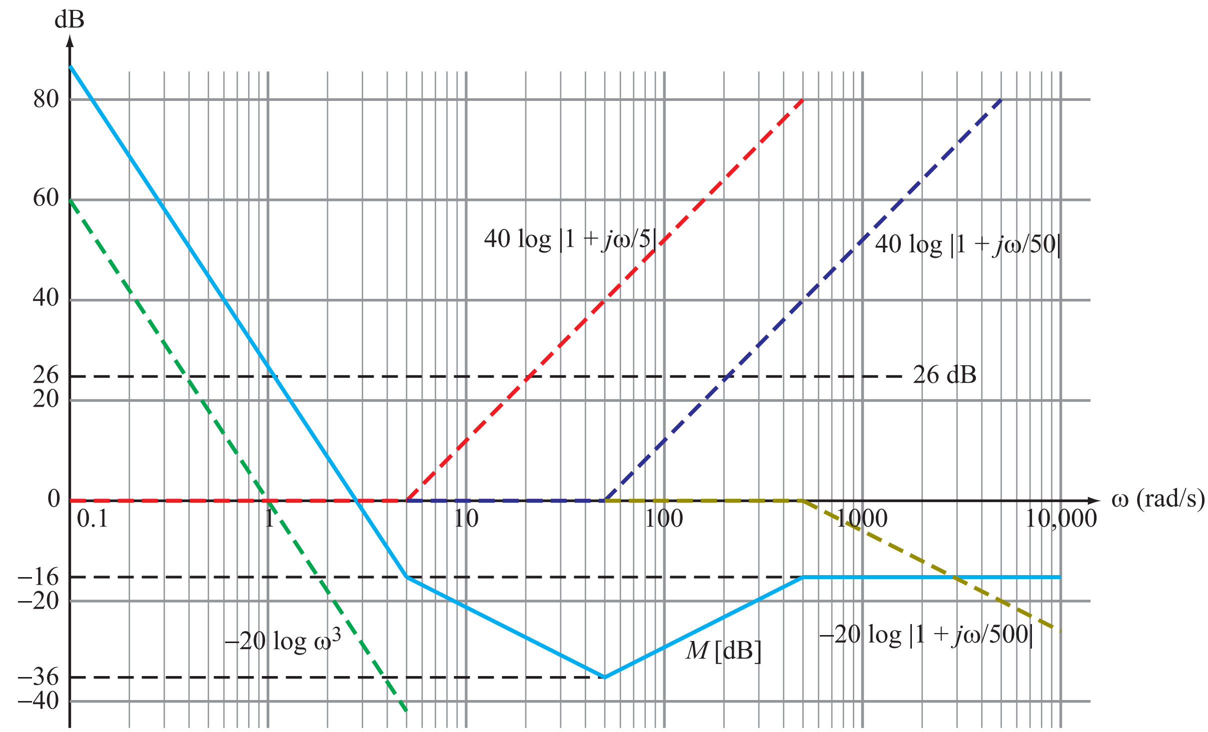
(b)

$$\mathbf{H}(\omega) = \frac{(1 + j0.2\omega)^2(100 + j2\omega)^2}{(j\omega)^3(500 + j\omega)}$$

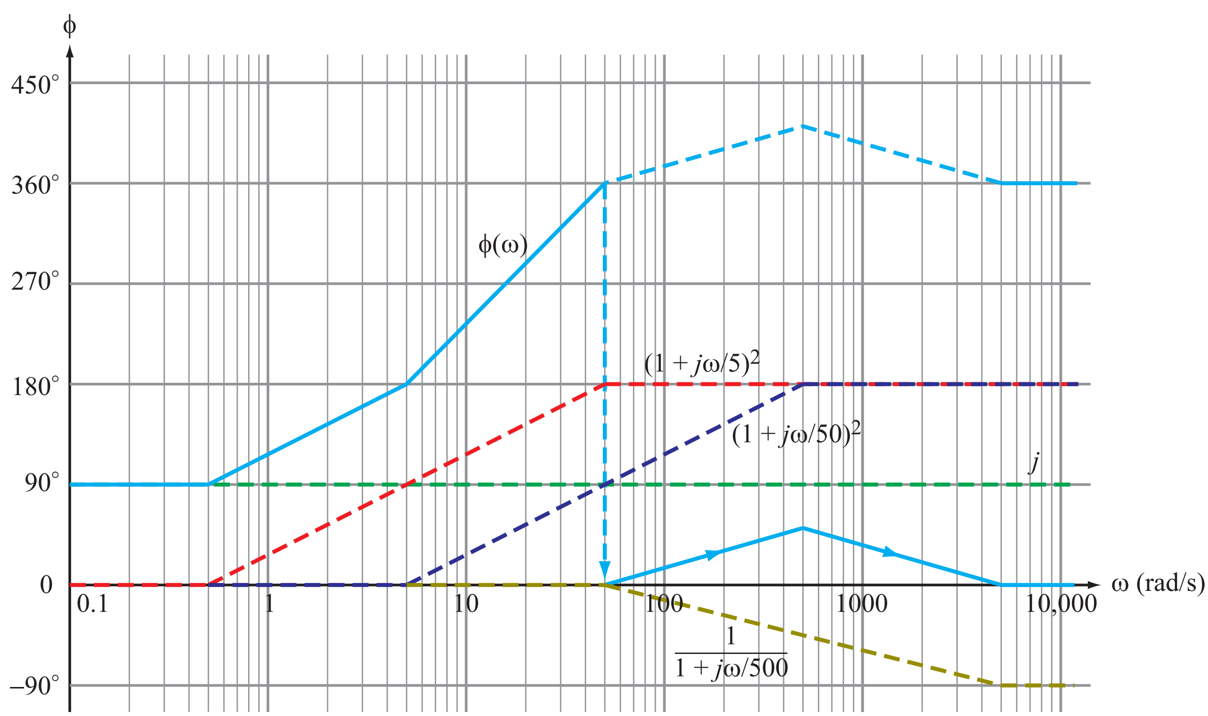
$$= \frac{j10^4(1+j\omega/5)^2(1+j\omega/50)^2}{500\omega^3(1+j\omega/500)}$$

$$= \frac{j20(1+j\omega/5)^2(1+j\omega/50)^2}{\omega^3(1+j\omega/500)}$$

- Constant term 20 $\implies$ 26 dB
- Pole of order 3 @ origin
- Simple zero with $\omega_c = 5$ rad/s, of order 2
- Simple zero of order 2 with $\omega_c = 50$ rad/s
- Simple pole with $\omega_c = 500$ rad/s



Magnitude



Phase

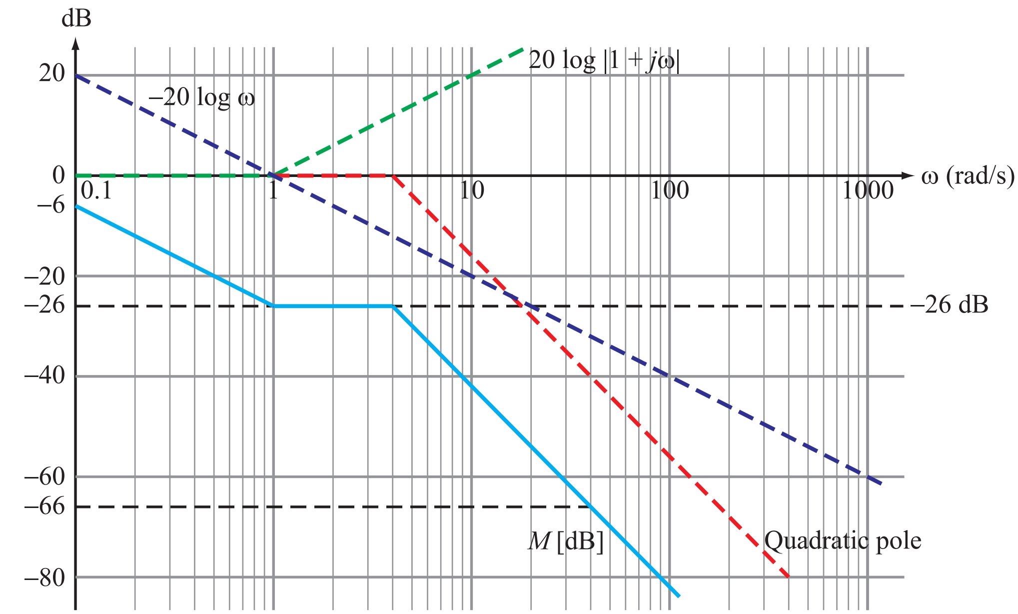
(c)

$$\mathbf{H}(\omega) = \frac{8 \times 10^{-2}(10 + j10\omega)}{j\omega(16 - \omega^2 + j4\omega)}$$

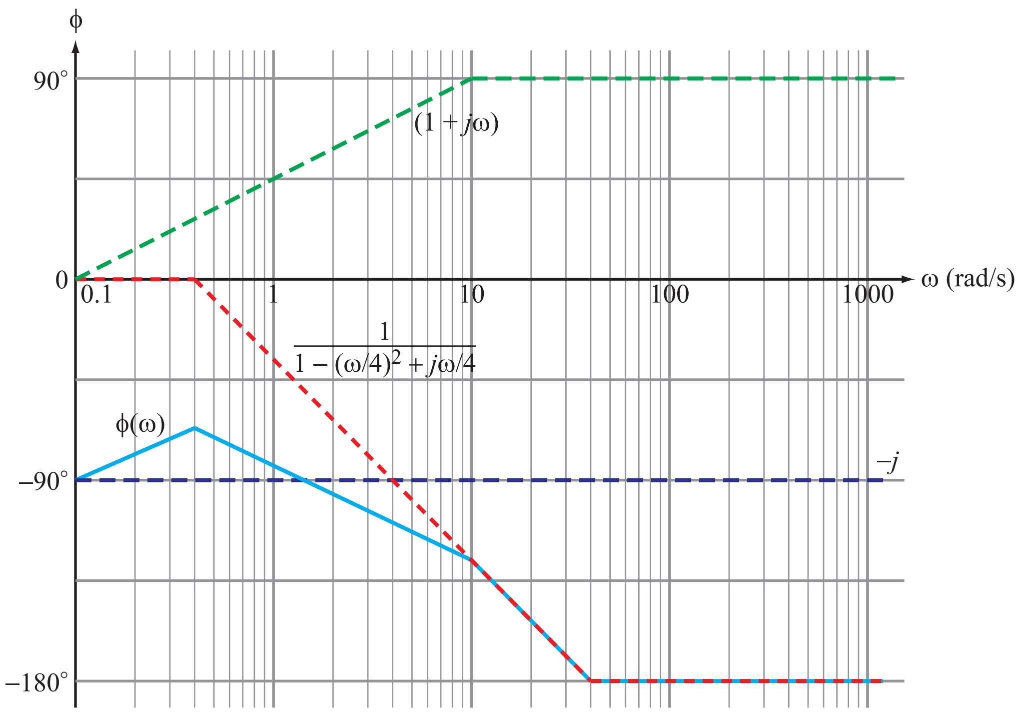
$$= \frac{-j8 \times 10^{-2} \times 10(1 + j\omega)}{16\omega[1 - (\omega/4)^2 + j\omega/4]}$$

$$= \frac{-j5 \times 10^{-2}(1 + j\omega)}{\omega[1 - (\omega/4)^2 + j\omega/4]}$$

- Constant factor $5 \times 10^{-2} \quad \Longrightarrow \quad -26$ dB
- Zero factor with $\omega_c = 1$ rad/s
- Quadratic pole with $\omega_c = 4$ rad/s
- Pole @ origin



Magnitude



Phase

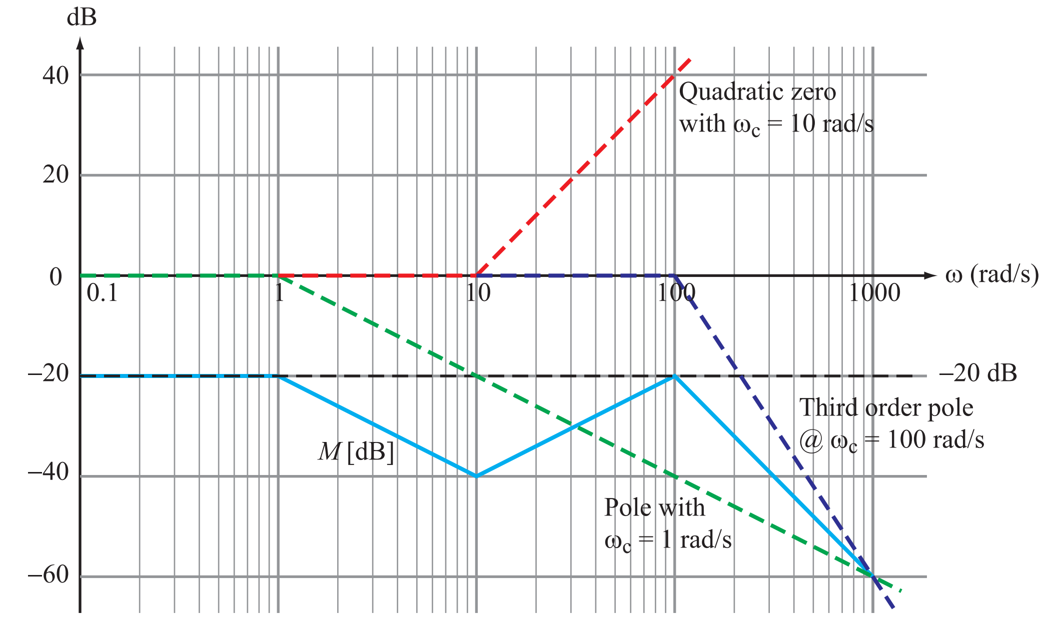
(d)

$$\mathbf{H}(\omega) = \frac{4\times 10^4 \omega^2 (100-\omega^2+j50\omega)}{(5+j5\omega)(200+j2\omega)^3}$$

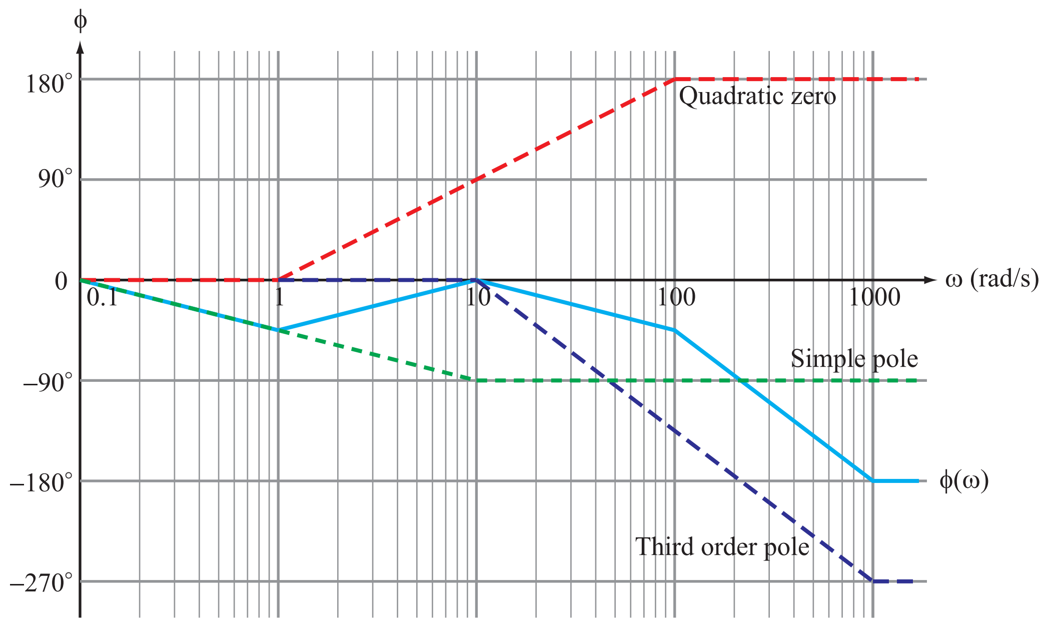
$$= \frac{4\times 10^4 \times 100\omega^2[1-(\omega/10)^2+j\omega/2]}{5\times(200)^3(1+j\omega)(1+j\omega/100)^3} = \frac{0.1\omega^2[1-(\omega/10)^2+j\omega/2]}{(1+j\omega)(1+j\omega/100)^3}$$

- Constant factor 0.1 $\Longrightarrow$ -20 dB

- Zero at origin of order 2
- Simple pole with $\omega_c = 1$ rad/s
- Quadratic zero with $\omega_c = 10$ rad/s
- Third order pole with $\omega_c = 100$ rad/s



Magnitude



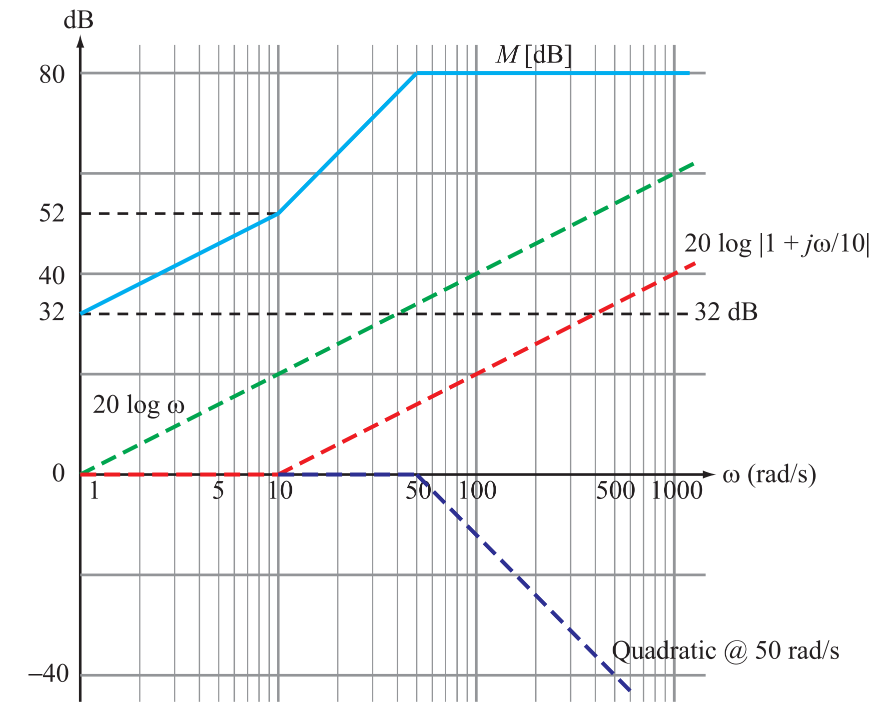
Phase

(e)

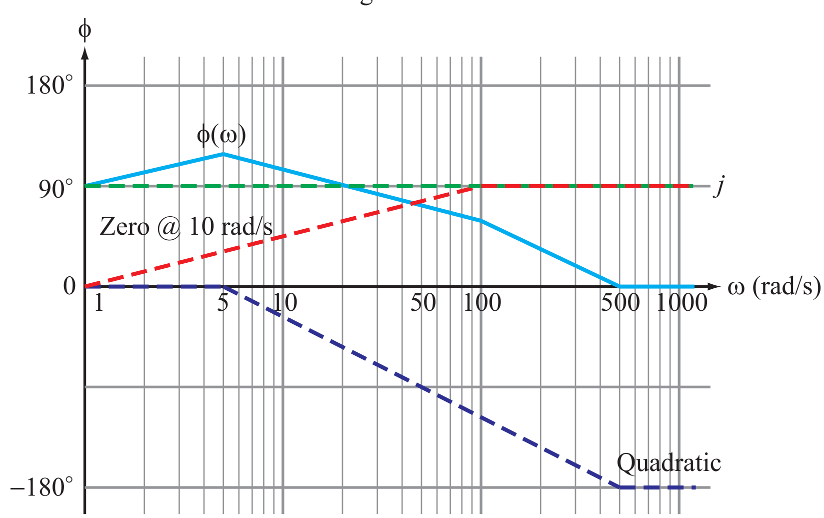
$$\mathbf{H}(\omega) = \frac{j5\times 10^3\omega(20+j2\omega)}{(2500-\omega^2+j20\omega)} = \frac{j5\times 10^3\times 20\omega(1+j\omega/10)}{2500[1-(\omega/50)^2+j\omega/125)}$$

$$= \frac{j40\omega(1 + j\omega/10)}{[1 - (\omega/50)^2 + j\omega/125)}$$

- Constant term 40 $\implies$ 32 dB
- Zero @ origin
- Simple zero with $\omega_c = 10$ rad/s
- Quadratic pole with $\omega_c = 50$ rad/s



Magnitude



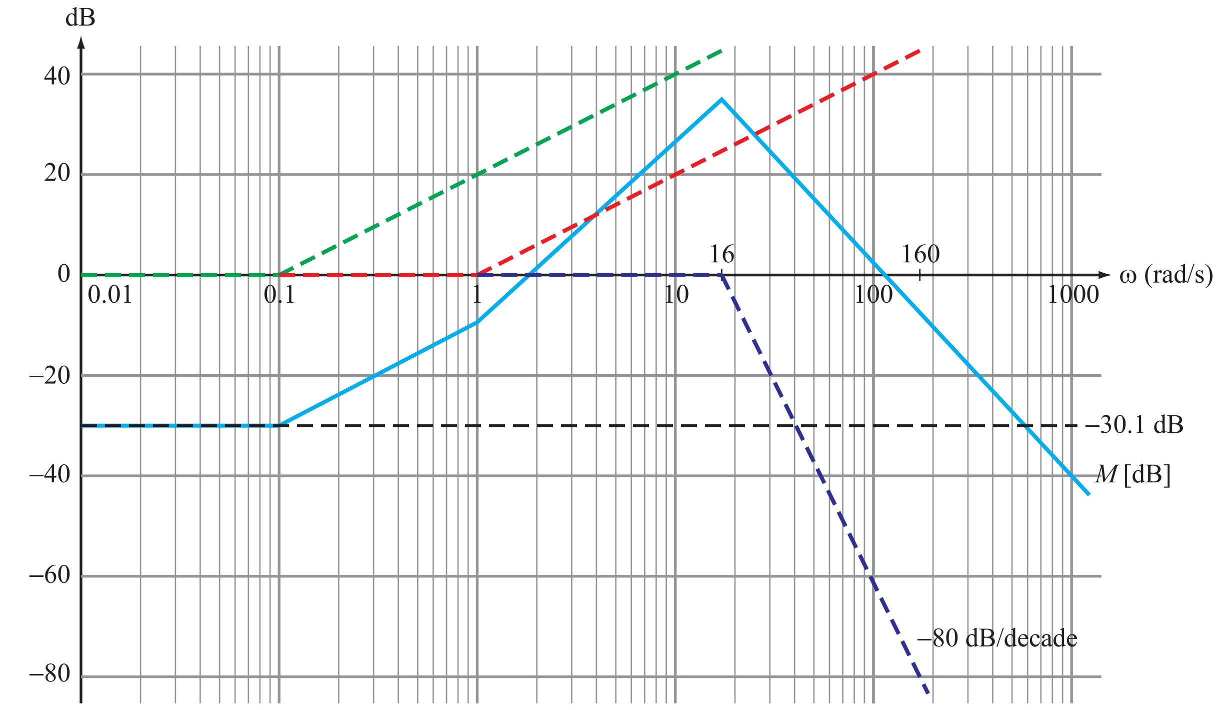
Phase

(f)

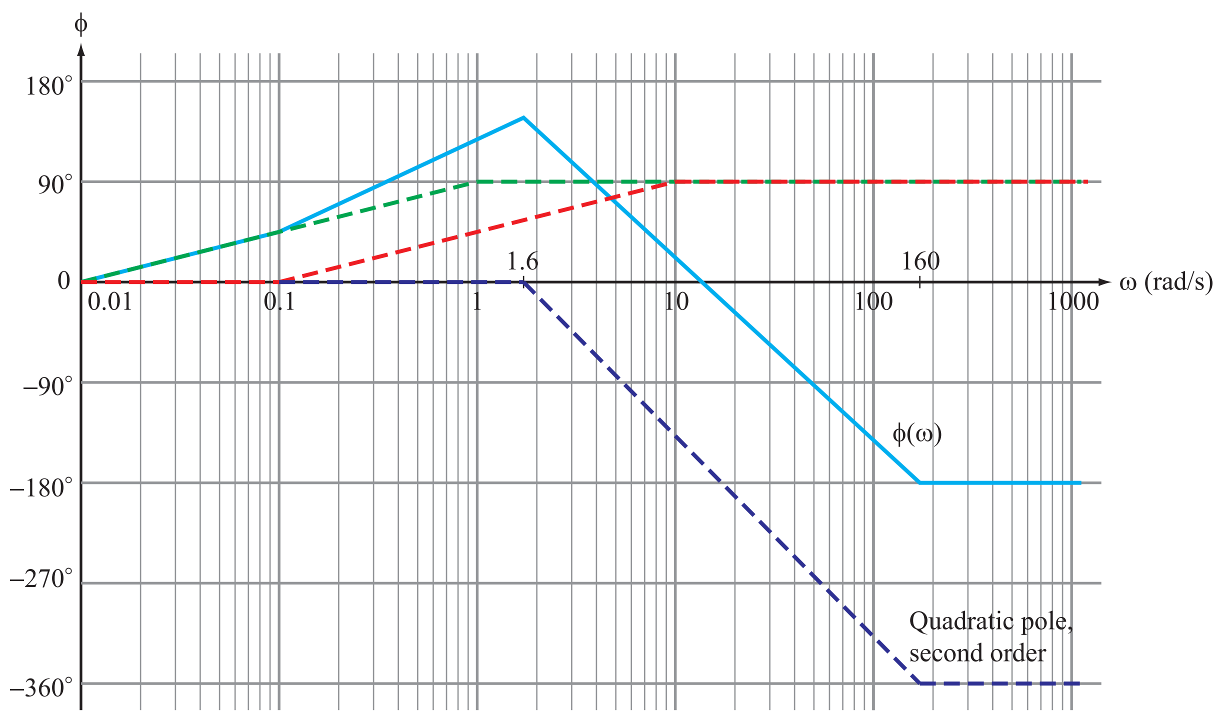
$$\mathbf{H}(\omega) = \frac{512(1 + j\omega)(4 + j40\omega)}{(256 - \omega^2 + j32\omega)^2}$$

$$
\begin{aligned}
&= \frac{512 \times 4(1 + j\omega)(1 + j\omega/0.1)}{256^2[1 - (\omega/16)^2 + j\omega/8]^2} \\
&= \frac{(1 + j\omega/0.1)(1 + j\omega)}{32[1 - (\omega/16)^2 + j\omega/8]^2}
\end{aligned}
$$

- Constant term $1/32 \quad \Longrightarrow \quad -30.1$ dB
- Simple zero with $\omega_\text{c} = 0.1$ rad/s
- Simple zero with $\omega_\text{c} = 1$ rad/s
- Quadratic pole of order 2 with $\omega_\text{c} = 16$ rad/s



Magnitude



Phase

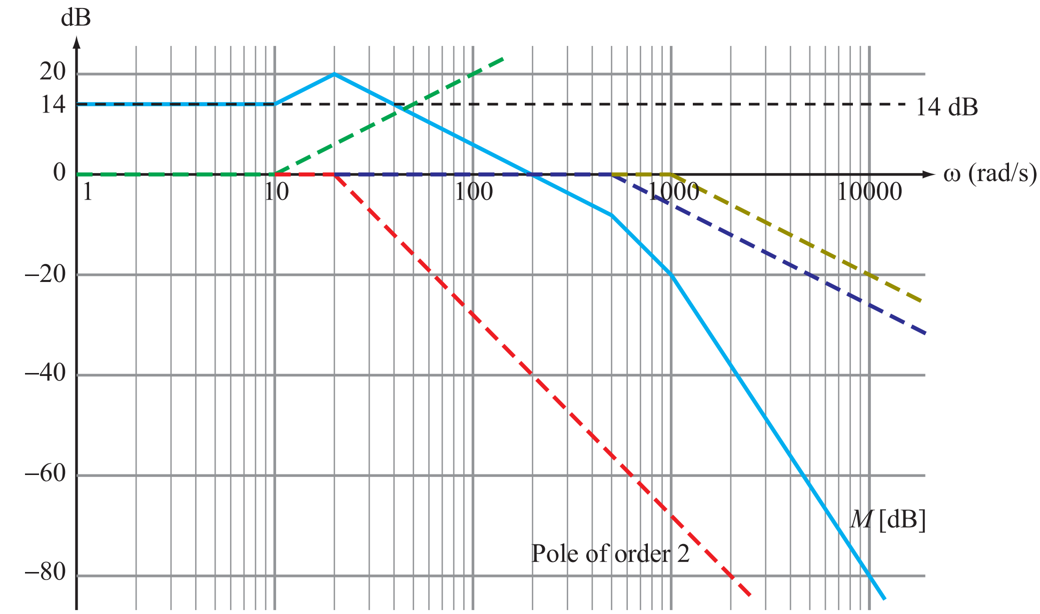
(g)

$$\mathbf{H}(\omega) = \frac{j(10 + j\omega) \times 10^8}{(20 + j\omega)^2(500 + j\omega)(1000 + j\omega)}$$

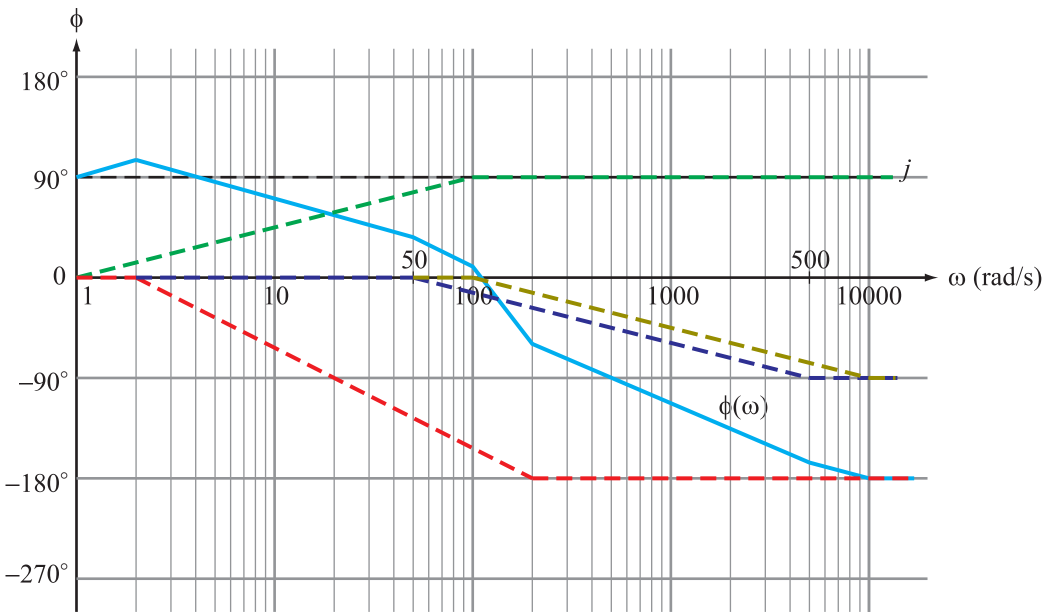
$$= \frac{j10 \times 10^8 (1 + j\omega/10)}{20^2 \times 500 \times 1000 (1 + j\omega/20)^2 (1 + j\omega/500)(1 + j\omega/1000)}$$

$$= \frac{j5(1 + j\omega/10)}{(1 + j\omega/20)^2 (1 + j\omega/500)(1 + j\omega/1000)}$$

- Constant term 5 $\implies$ 14 dB
- Simple zero with $\omega_c = 10$ rad/s
- Simple pole of order 2 with $\omega_c = 20$ rad/s
- Simple pole with $\omega_c = 500$ rad/s
- Simple pole with $\omega_c = 1000$ rad/s



Magnitude



Phase

Problem 9.16 Determine the voltage transfer function $\mathbf{H}(\omega)$ corresponding to the Bode magnitude plot shown in Fig. P9.16. The phase of $\mathbf{H}(\omega)$ is 90° at $\omega = 0$.

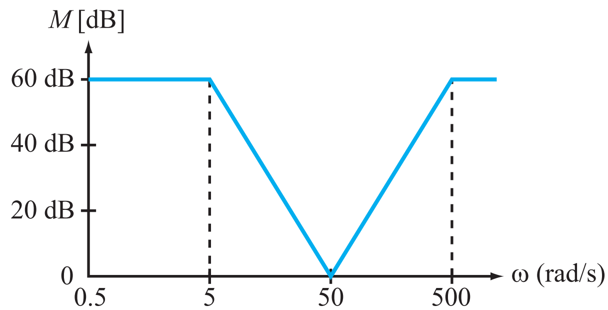


Figure P9.16: Bode magnitude plot for Problem 9.16.

Solution: $\mathbf{H}(\omega)$ consists of:

(1) A constant term K whose dB value is 60 dB, or

$$K = 10^{60/20} = 1000.$$

(2) A simple pole of order 3 with $\omega_c = 5$ rad/s (slope $= -60$ dB/decade)

(3) A simple zero of order 6 with $\omega_c = 50$ rad/s (slope reverses from -60 dB/decade to $+60$ dB/decade)

(4) A simple pole of order 3 with $\omega_c = 500$ rad/s (slope changes to 0 dB at $\omega_c = 500$ rad/s).

Hence,

$$\mathbf{H}(\omega) = \frac{(j)^N 1000(1 + j\omega/50)^6}{(1 + j\omega/5)^3(1 + j\omega/500)^3} = \frac{j1000(50 + j\omega)^6}{(5 + j\omega)^3(500 + j\omega)^3} .$$

Given that the phase of $\mathbf{H}(\omega)$ is 90° at $\omega = 0$, it follows that $N = 1$.

Problem 9.22 A series RLC circuit is driven by an ac source with a phasor voltage $\mathbf{V}_s = 10\angle 30^\circ$ V. If the circuit resonates at 10^3 rad/s and the average power absorbed by the resistor at resonance is 2.5 W, determine the values of R, L, and C, given that $Q = 5$.

Solution: At resonance,

$$\mathbf{Z}_{\text{in}} = R, \qquad P_{\text{av}} = \frac{1}{2}\,\frac{|\mathbf{V}_s|^2}{R}\,.$$

Hence,

$$2.5 = \frac{10^2}{2R} \qquad \Longrightarrow \qquad R = 20\ \Omega.$$

From

$$Q = \omega_0\,\frac{L}{R} \qquad \Longrightarrow \qquad L = \frac{RQ}{\omega_0} = \frac{20\times 5}{10^3} = 0.1\ \text{H}.$$

From

$$\omega_0^2 = \frac{1}{LC} \qquad \Longrightarrow \qquad C = \frac{1}{\omega_0^2 L} = \frac{1}{10^6\times 0.1} = 10\ \mu\text{F}.$$

Problem 9.23 The element values of a parallel RLC circuit are $R = 100\ \Omega$, $L = 10$ mH, and $C = 0.4$ mF. Determine ω_0, Q, B, ω_{c_1}, and ω_{c_2}.

Solution: From Table 9-3:

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-2} \times 0.4 \times 10^{-3}}} = 500 \text{ rad/s},$$

$$Q = \frac{R}{\omega_0 L} = \frac{100}{500 \times 10^{-2}} = 20,$$

$$B = \frac{\omega_0}{Q} = \frac{500}{20} = 25 \text{ rad/s},$$

$$\omega_{c_1} \simeq \omega_0 - \frac{B}{2} = 500 - 12.5 = 487.5 \text{ rad/s},$$

$$\omega_{c_2} \simeq \omega_0 + \frac{B}{2} = 500 + 12.5 = 512.5 \text{ rad/s}.$$

Problem 7.37 Determine the Thévenin equivalent of the circuit in Fig. P7.37 at terminals (a, b), given that

$$v_s(t) = 12\cos 2500t \text{ V},$$
$$i_s(t) = 0.5\cos(2500t - 30^\circ) \text{ A}.$$

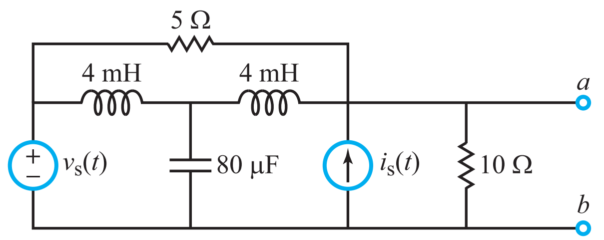
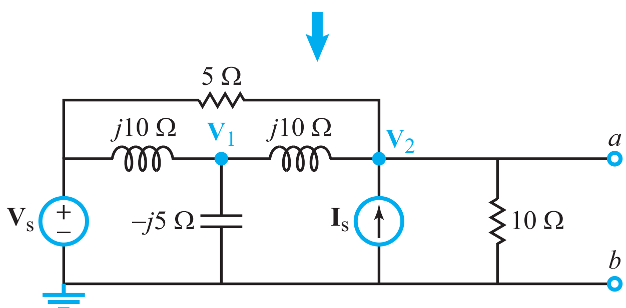


Figure P7.37: Circuit for Problem 7.37.

Solution:



(a)

At $\omega = 2500$ rad/s,

$$\mathbf{Z}_L = j\omega L = j2500 \times 4 \times 10^{-3} = j10\ \Omega$$
$$\mathbf{Z}_C = \frac{-j}{\omega C} = \frac{-j}{2500 \times 80 \times 10^{-6}} = -j5\ \Omega.$$

Also,

$$\mathbf{V}_s = 12\angle 0^\circ \text{ V},$$
$$\mathbf{I}_s = 0.5\angle -30^\circ \text{ A}.$$

To obtain the Thévenin equivalent circuit, we can either apply impedance and source transformations to simplify the circuit or apply one of the analysis techniques. We opt to apply nodal analysis to determine $\mathbf{V}_{oc}$ at terminals (a, b).

At node $\mathbf{V}_1$:

$$\frac{\mathbf{V}_1 - \mathbf{V}_s}{j10} + \frac{\mathbf{V}_1}{-j5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j10} = 0$$

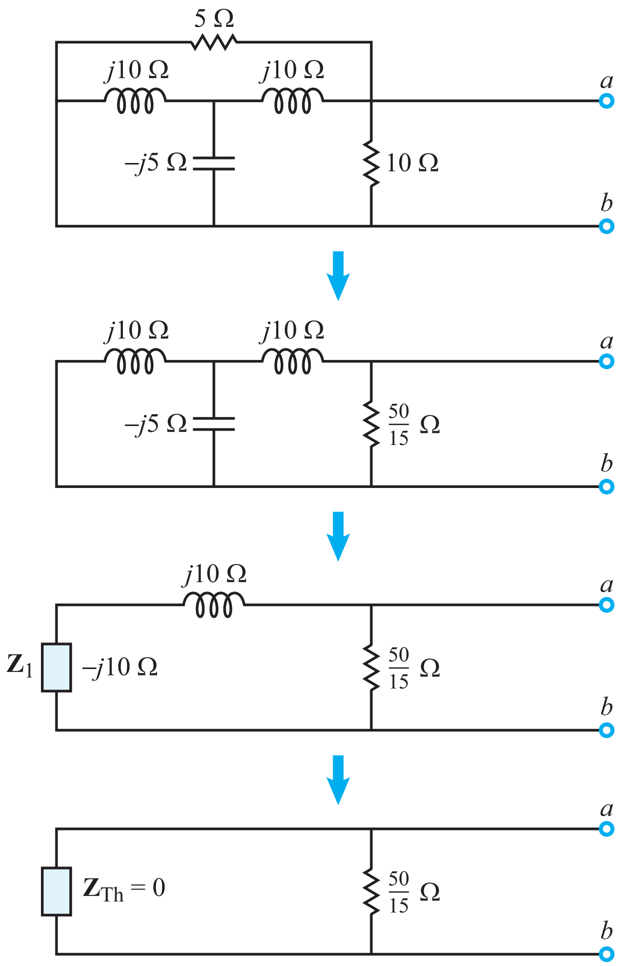
At node $\mathbf{V}_2$:

$$\frac{\mathbf{V}_2 - \mathbf{V}_1}{j10} + \frac{\mathbf{V}_2 - \mathbf{V}_s}{5} + \frac{\mathbf{V}_2}{10} - \mathbf{I}_s = 0$$

Upon inserting the values for $\mathbf{V}_s$ and $\mathbf{I}_s$ and then solving for $\mathbf{V}_1$ and $\mathbf{V}_2$, we determine that

$$\mathbf{V}_{\text{Th}} = \mathbf{V}_{\text{oc}} = \mathbf{V}_2 = -12 \text{ V}.$$

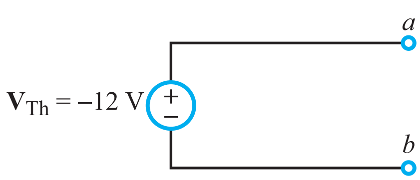
To determine $\mathbf{Z}_{\text{Th}}$, we eliminate the sources and then simplify the circuit.



$$\mathbf{Z}_1 = \frac{(j10)(-j5)}{j10 - j5} = -j10 \ \Omega$$

$$\mathbf{Z}_{\text{Th}} = (j10 - j10) \parallel \frac{50}{15} = 0.$$

Hence, the Thévenin equivalent circuit is:



Problem 7.47 Apply nodal analysis in the phasor domain to determine $i_C(t)$ in the circuit of Fig. P7.47.

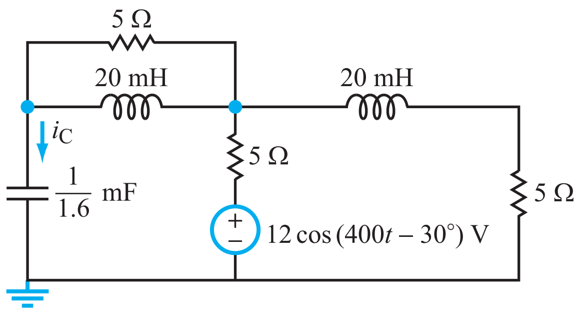
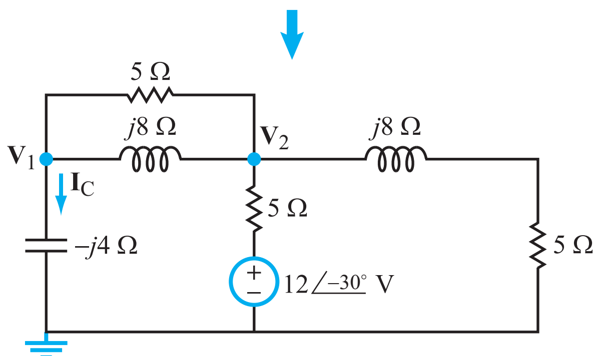


Figure P7.47: Circuit for Problem 7.47.

Solution:



$$\mathbf{Z}_L = j\omega L = j400 \times 20 \times 10^{-3} = j8\ \Omega$$

$$\mathbf{Z}_C = \frac{-j}{\omega C} = \frac{-j}{400 \times \frac{1}{1.6} \times 10^{-3}} = -j4\ \Omega$$

Node $\mathbf{V}_1$:

$$\frac{\mathbf{V}_1}{-j4} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j8} = 0$$

Node $\mathbf{V}_2$:

$$\frac{\mathbf{V}_2 - \mathbf{V}_1}{5} + \frac{\mathbf{V}_2 - \mathbf{V}_1}{j8} + \frac{\mathbf{V}_2 - 12e^{-j30^\circ}}{5} + \frac{\mathbf{V}_2}{5 + j8} = 0$$

Simultaneous solution leads to:

$$\mathbf{V}_1 = 4.986e^{-j96.352^\circ}, \qquad \mathbf{V}_2 = 4.986e^{-j32.341^\circ},$$

$$\mathbf{I}_C = \frac{\mathbf{V}_1}{-j4} = \frac{j}{4}\mathbf{V}_1 = 1.25e^{-j6.532^\circ},$$

$$i_C(t) = 1.25\cos(400t - 6.352^\circ) \qquad \text{(A)}.$$