EECS 215 Winter 2005 Midterm 2

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Lecture Section (circle one): McAfee Terry

Rules:

1. One (1) 8.5x11" note sheet allowed. No other information aids allowed.
2. A formulae sheet is provided on the back of this exam and can be removed if desired. No other pages should be removed.
3. DO NOT UNSTAPLE THE PAGES OF THIS EXAM.
4. TURN IN ALL PAGES EXCEPT THE FORMULAE SHEET.
5. Calculators Needed and Allowed
6. Work to be done in Exam booklet.
7. **DO NOT WRITE ON THE BACK OF PAGES.**
8. **Exam given under CoE Honor Code**
9. Show your work and *briefly* explain major steps to maximize partial credit. (ex: i3=i1+i2 , node A, KCL). **NO CREDIT WILL BE GIVEN IF NO WORK IS SHOWN.**
10. *WRITE YOUR FINAL ANSWERS IN THE AREAS PROVIDED*

This Exam Contains
4 problems over 15 pages (including workspace & formulae page).

Sign the College of Engineering Honor Code Below (NO credit will be given for the exam without a signed pledge):

I have neither given nor received aid on this examination.

Signed: ____________________

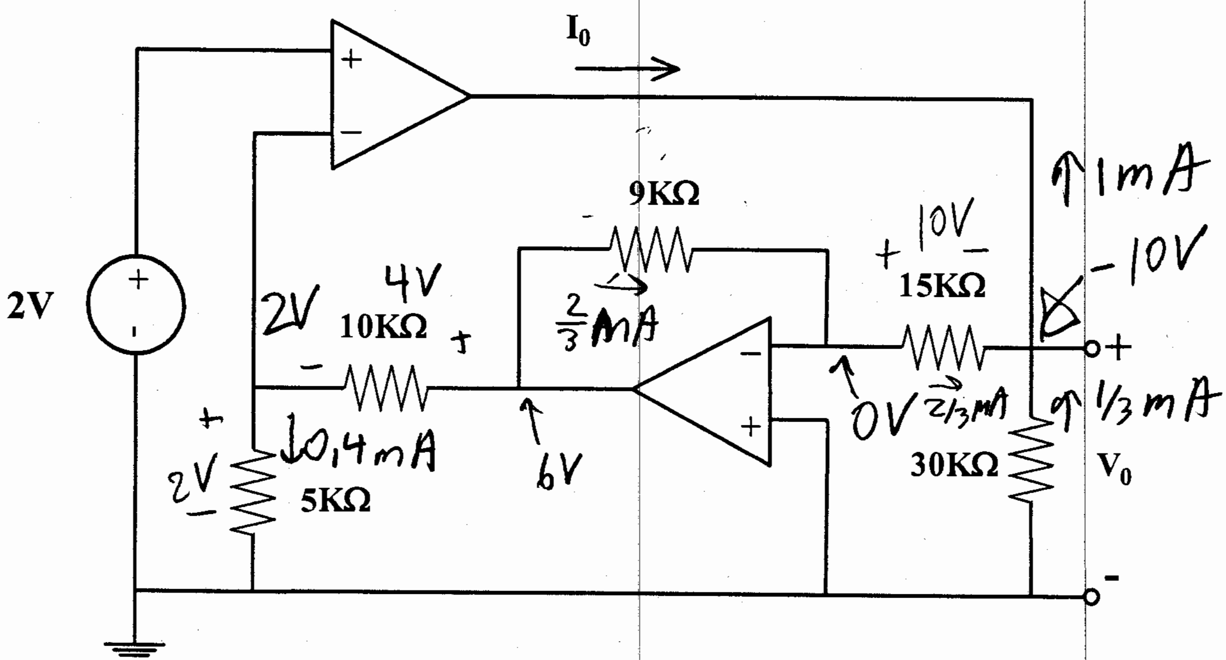
Do not write on this page below this line – Instructional Staff Use Only!

[ ] Prob 1 [ ] Prob 3

[ ] Prob 2 [ ] Prob 4

Problem 1: Op-Amps (15 points total)

For the circuit shown below, find V_0 and I_0. You may assume ideal opamps under negative feedback conditions.



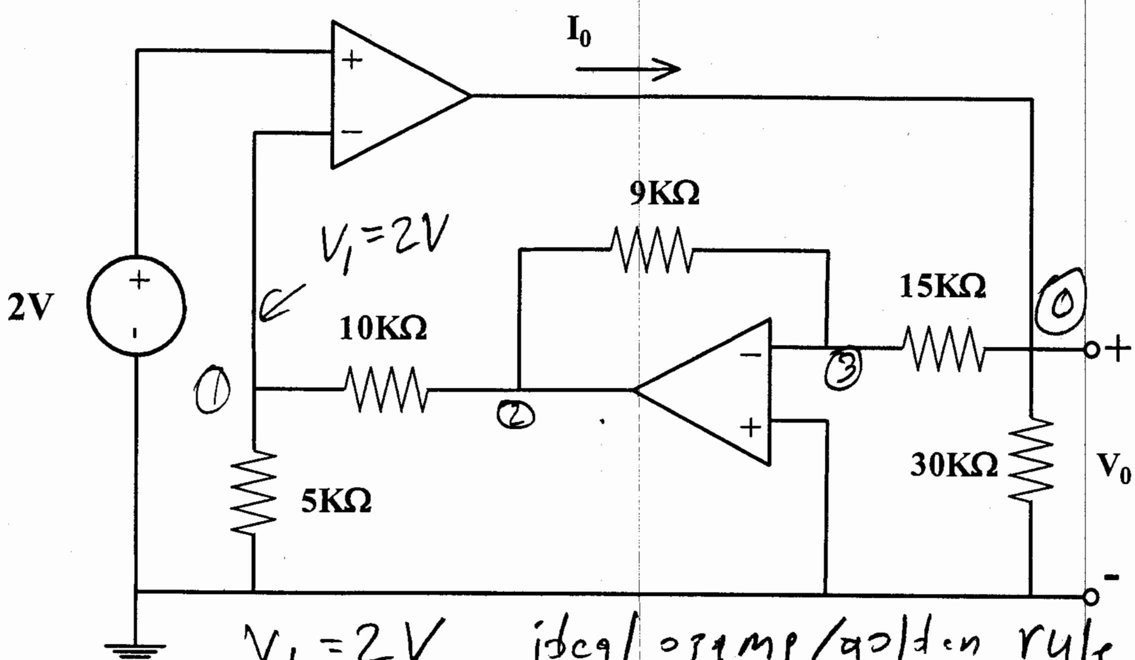
$V_0 = -10V$

$I_0 = -1mA$

Done step by step "nibbling" approach on the diagram

Problem 1: Op-Amps (15 points total)

For the circuit shown below, find V_0 and I_0. You may assume ideal opamps under negative feedback conditions.



$\underline{V_1 = 2V}$ ideal opamp/golden rule

node 1 KCL $\frac{V_1}{5K\Omega} + \frac{V_1 - V_2}{10K\Omega} = 0$

$\Rightarrow V_2 = 3V_1 = 6V$

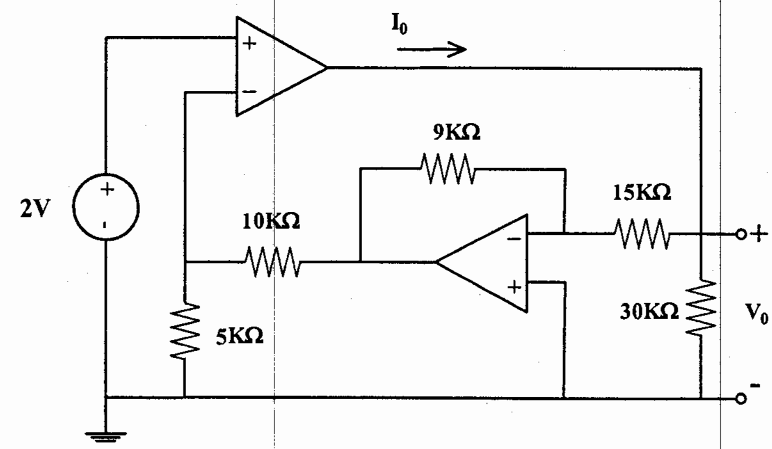
no KCL at 2 for V's since output of opamp

$V_3 = 0V$ ideal opamp/golden rule

node 3 KCL

$$\frac{V_3 - V_2}{9K\Omega} + \frac{V_3 - V_0}{15K\Omega} = 0$$

additional workspace for problem 1



$$\frac{0-6V}{9k\Omega} + \frac{0-V_0}{15k\Omega} = 0$$

$$\Rightarrow \boxed{V_0 = -6V\left(\frac{15}{9}\right) = -10V}$$

KCL node (a)

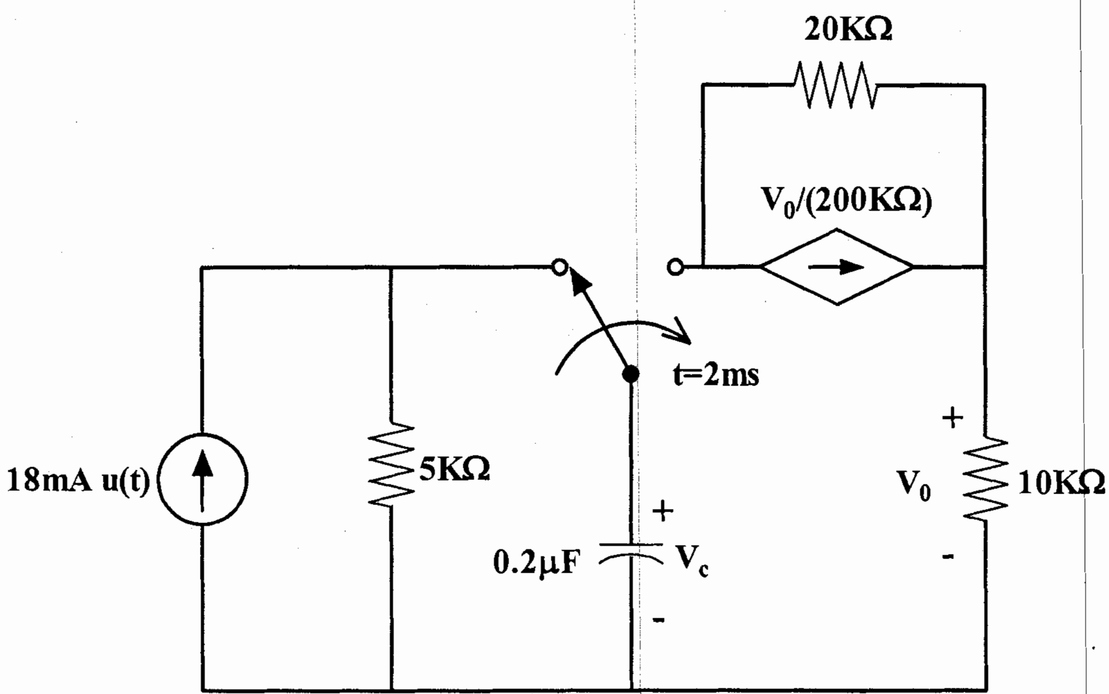
$$I_0 = \frac{V_0 - V_3}{15k\Omega} + \frac{V_0 - 0}{30k\Omega} = \quad V_0\left(\frac{1}{15k\Omega} + \frac{1}{30k\Omega}\right)$$

$$\boxed{I_0 = -1\,mA}$$

Problem 2: First Order Circuits (25 points)

For the circuit shown below, note that the independent current source is (18mA)u(t) and the switch is toggled at t=2ms.

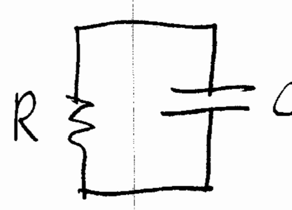
Problem 2 has part (a) and part (b).



a) Find Vc(t) for t=0 to 2ms.

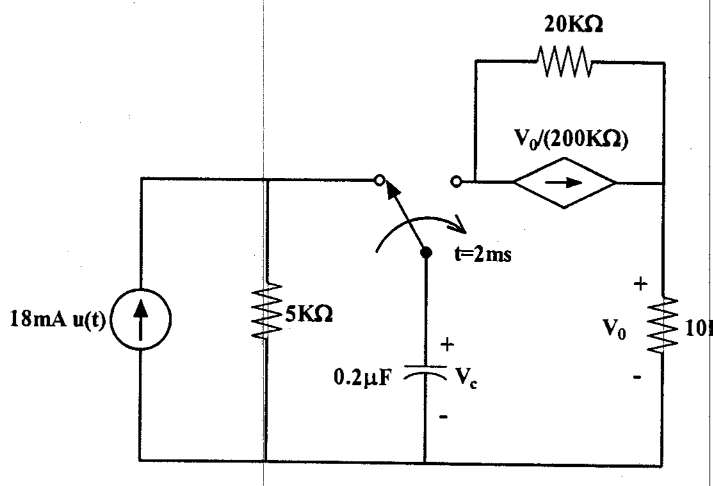
$V_c(t)=$ $90(1-e^{-t/1ms})$ V 0 ≤t ≤2ms

$t<0$

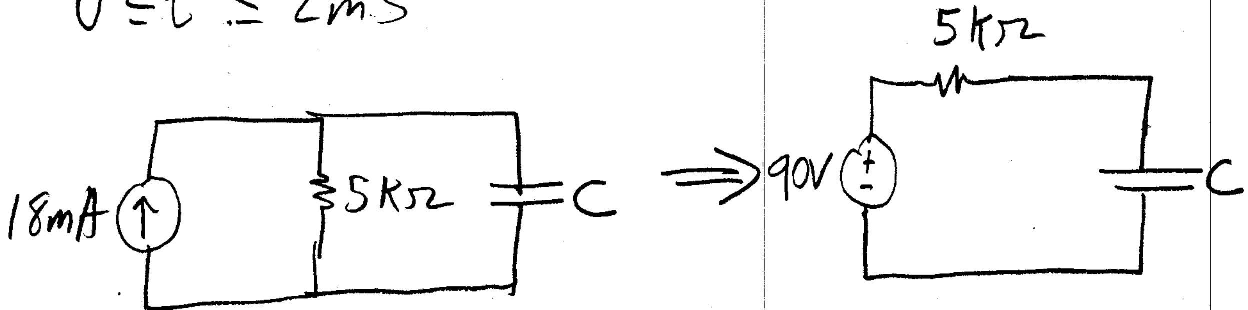


$\Rightarrow V_c = 0$

Additional workspace for 2(a)



$0 \leq t \leq 2ms$



$V_c(t) = 90V + Ae^{-t/\tau}$ $\quad$ $\tau = RC = (5K\Omega)\,0.2\mu F$
$= 1ms$

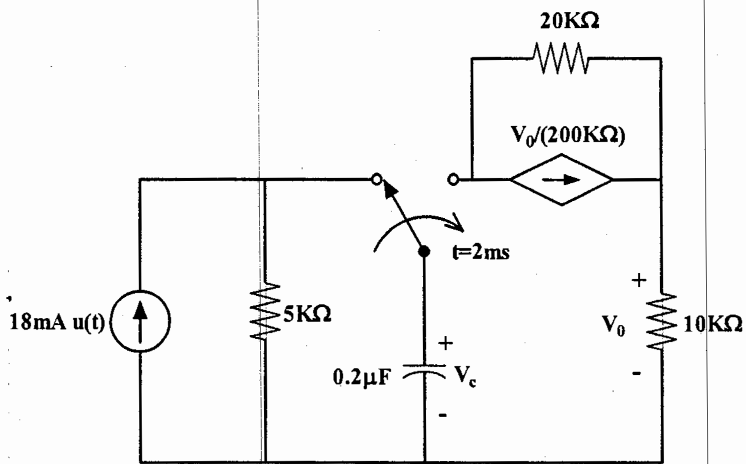
$V_c(0) = 0 = 90V + A$
$\Rightarrow A = -90V$

$V_c(t) = 90(1 - e^{-t/1ms})\ V$

at $t = 2ms$ $\quad V_c(2ms) = 90(1 - e^{-2})$
$\approx 90(0.8647)$
$\approx 77.8198V$

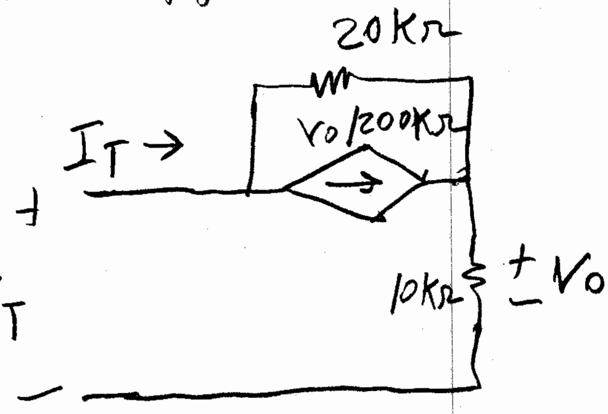
b) Find Vc(t) for t≥2ms

Vc(t)= $77.82\, e^{-(t-2ms)/5.8ms}$ V t ≥ 2ms



Now capacitor sees a circuit with no independent sources ⇒ only yields an equivalent resistance & natural decay.

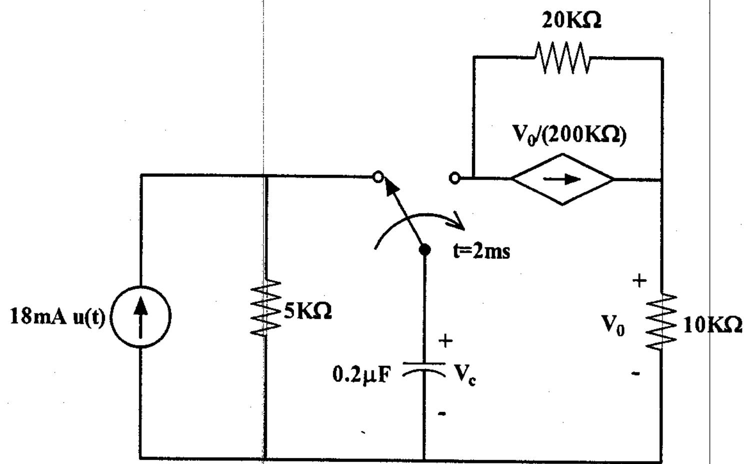
What is Reff ?



$$I_T = V_0/10K\Omega = \frac{V_0}{200K\Omega} + \frac{V_T - V_0}{20K\Omega} \quad KCL$$

$$\Rightarrow 2V_0 = \frac{V_0}{10} + V_T - V_0$$

additional workspace for 2(b)



$$V_T = 3V_0 - \frac{V_0}{10} = 2.9 V_0$$

$$R_{eff} = \frac{V_T}{I_T} = 29 K\Omega$$

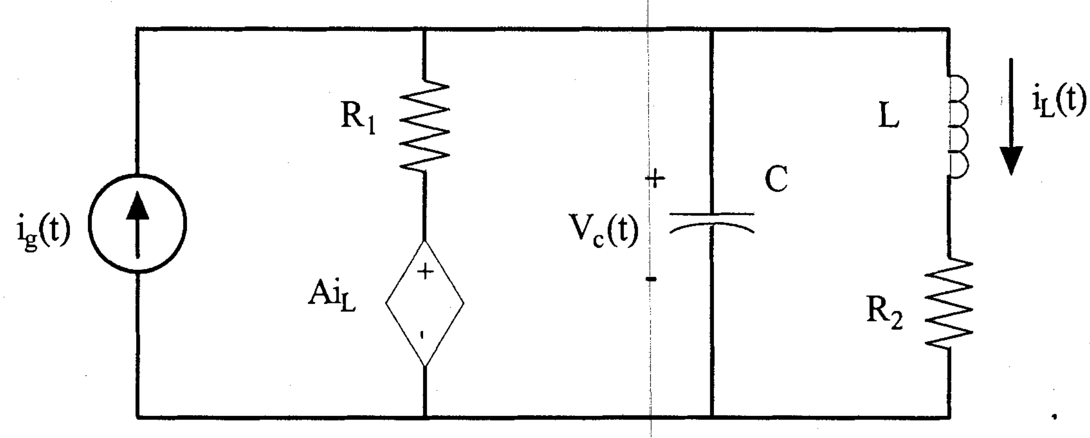
so $\tau_2 = C R_{eff} = 5.8ms$

$$V_c(t) = V_c(2ms)\, e^{-(t-2ms)/5.8ms}$$

$$\approx (77.82)\, e^{-(t-2ms)/5.8ms} \; V$$

Problem 3: Second Order Circuits (35 points)

Problem has parts (a) & (b). These two parts can be done independently of each other, or may be used to partially check the work of the alternate part.



a) For the circuit picture above, find the differential equation that relates $i_L(t)$ to $i_g(t)$. Write the equation in one of the standard forms: $\frac{d^2 i_L}{dt^2} + A\frac{di_L}{dt} + Bi_L = function(i_g)$ or $D\frac{d^2 i_L}{dt^2} + F\frac{di_L}{dt} + i_L = function(i_g)$. i_L must be the only unknown (assuming $i_g(t)$ is known). You may use KVL/KCL/time domain methods or s-domain methods, but you must clearly show your work to receive full or partial credit. **Warning**: *Attempts to mix time-domain and s-domain approaches are likely to result in zero credit.*

Differential Equation:

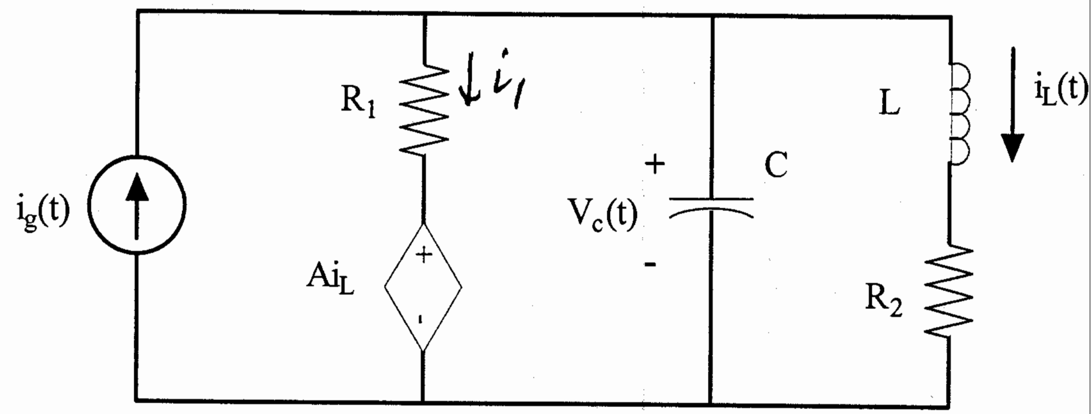
$$LC\frac{d^2 i_L}{dt^2} + \left(R_2 C + \frac{L}{R_1}\right)\frac{di_L}{dt} + \left(\frac{R_1+R_2-A}{R_1}\right) i_L = i_g$$

$$\frac{d^2 i_L}{dt^2} + \left(\frac{R_2}{L} + \frac{1}{R_1 C}\right)\frac{di_L}{dt} + \left(\frac{R_1+R_2-A}{R_1}\right)\left(\frac{1}{LC}\right) i_L = \left(\frac{1}{LC}\right) i_g$$

either form

these do check numerically with part 4

Workspace for 3(a)



KVL/KCL approach:

$$V_C = V_L + i_L R_2 = L\frac{di_L}{dt} + R_2 i_L$$ have V_C in terms of i_L!

$$i_1 = \frac{V_C - Ai_L}{R_1}$$ KVL & Ohm's Law

$$= \frac{L}{R_1}\frac{di_L}{dt} + \left(\frac{R_2 - A}{R_1}\right) i_L$$

$$i_C = C\frac{dV_C}{dt} = LC\frac{d^2 i_L}{dt^2} + R_2 C\frac{di_L}{dt}$$

$$i_g = i_1 + i_C + i_L$$ KCL

$$i_g = LC\frac{d^2 i_L}{dt^2} + \left(R_2 C + \frac{L}{R_1}\right)\frac{di_L}{dt} + \left(1 + \frac{R_2 - A}{R_1}\right) i_L$$

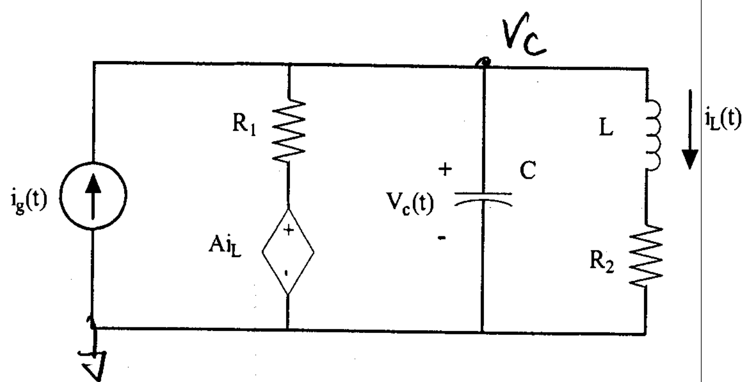
$$i_g = LC\frac{d^2 i_L}{dt^2} + \left(R_2 C + \frac{L}{R_1}\right)\frac{di_L}{dt} + \left(\frac{R_1 + R_2 - A}{R_1}\right) i_L$$

or $$\frac{i_g}{LC} = \frac{d^2 i_L}{dt^2} + \left(\frac{R_2}{L} + \frac{1}{R_1 C}\right)\frac{di_L}{dt} + \left(\frac{R_1 + R_2 - A}{R_1 L C}\right) i_L$$

Workspace for 3(a)

s-domain approach /nodal

KCL at top node



$$i_g = \frac{V_c - Ai_L}{R_1} + \frac{V_c}{1/Cs} + \underbrace{\frac{V_c}{Ls+R_2}}_{i_L}$$

$$i_L = \frac{V_c}{Ls+R_2} \quad \text{so} \quad V_c = i_L(Ls+R_2)$$

$$i_g = i_L\left(\frac{L}{R_1}s + \frac{R_2-A}{R_1}\right) + i_L(Ls+R_2)Cs + i_L$$

$$= LCs^2 i_L + \left(R_2C + \frac{L}{R_1}\right)s\,i_L + \left(1+\frac{R_2-A}{R_1}\right)i_L$$

$s \to d/dt \quad s^2 \to d^2/dt^2$

$$\Rightarrow i_g = LC\frac{d^2 i_L}{dt^2} + \left(R_2C + \frac{L}{R_1}\right)\frac{di_L}{dt} + \left(\frac{R_1+R_2-A}{R_1}\right)i_L$$

b) Assume that the source had the following behavior: $i_g(t) = i_a + i_b u(t)$, where i_a & i_b are constants. Find expressions for $i_L(t)$ & $V_L(t)$ for $t=0^-, 0^+$, and $t \to \infty$

$i_L(0^-)$= $i_q \left(\frac{R_1}{R_1+R_2-A}\right)$

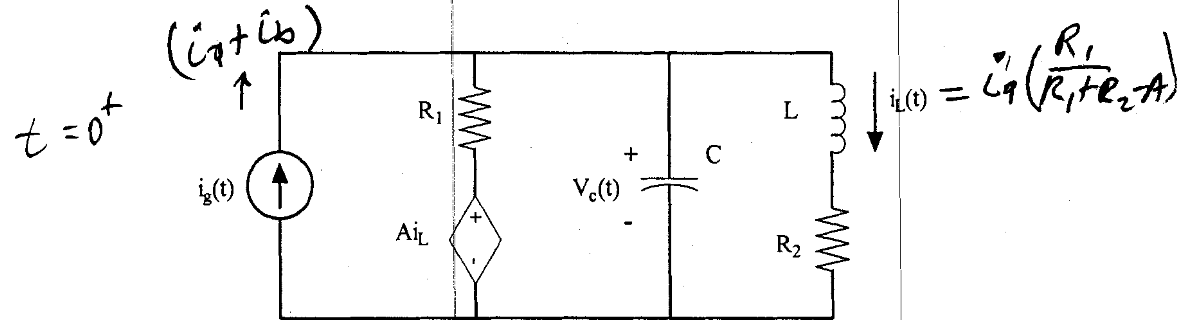
$V_L(0^-)$= 0 V

$i_L(0^+)$= $i_L(0^-)$ current continuity

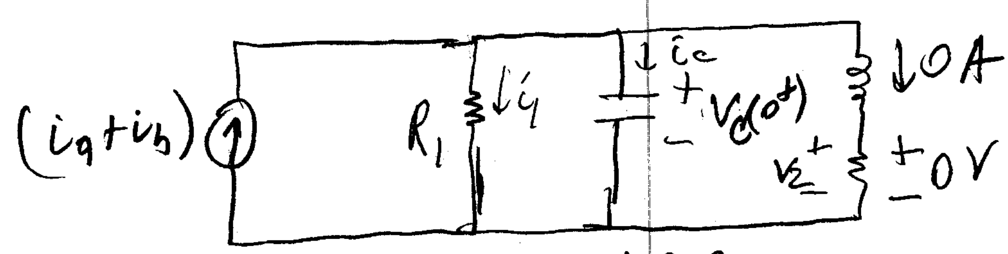
$V_L(0^+)$= 0

$i_L(\infty)$ = $(i_q + i_b)\left(\frac{R_1}{R_1+R_2-A}\right)$

$V_L(\infty)$= 0 V

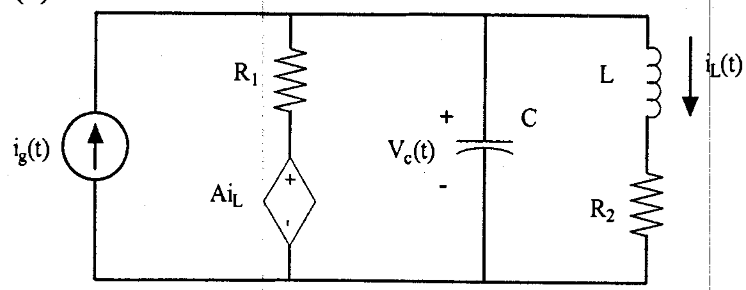


$t=0+$

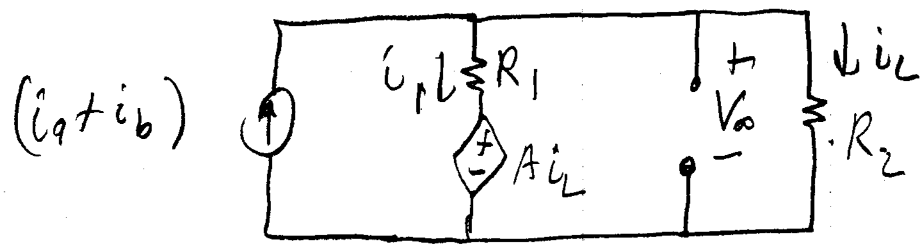


$$V_c(0^-) = V_c(0^+) = \left(\frac{R_1 R_2}{R_1+R_2-A}\right) i_q$$

$V_L(0^+) = 0$ since i_L + V_c don't change additional workspace for 3(b)



$t \to \infty$



$$V_\infty = i_{L\infty} R_2$$

$$i_1 = (V_\infty - A i_{L\infty})/R_1$$

$$(i_a + i_b) = i_1 + i_{L\infty} = i_{L\infty} \frac{R_2 - A}{R_1} + i_{L\infty}$$

$$= \left(\frac{R_1 + R_2 - A}{R_1}\right) i_{L\infty}$$

$$i_{L\infty} = (i_a + i_b)\left(\frac{R_1}{R_1 + R_2 - A}\right)$$

note this agrees with diff. eqn. result

$$V_{L\infty} = 0$$

since $\frac{di_L}{dt} \to 0$

Problem 4: Second Order Circuits (25 points)

Using the same circuit as problem 3, assume we have the following component values:

$$R_1 = 1000\Omega \quad R_2 = 5\Omega \quad A = 996\Omega \quad L = 1mH \quad C = 1\mu F$$

with these values, the differential equation becomes:

$$\left(\frac{1}{9}x10^{-6}s^2\right)\frac{d^2 i_L}{dt^2} + \left(\frac{2}{3}x10^{-3}s\right)\frac{di_L}{dt} + i_L = \left(\frac{1}{9}x10^3\right)i_g$$

or equivalently:

$$\frac{d^2 i_L}{dt^2} + \left(6x10^3 s^{-1}\right)\frac{di_L}{dt} + \left(9x10^6 s^{-2}\right)i_L = \left(1x10^9 s^{-2}\right)i_g$$

where s here is the abbreviation for the unit seconds

a) Find the natural (source-free/homogeneous) solution for this case and name the damping type.

Damping type (circle only one)

Underdamped **Critically Damped** **Overdamped**

$i_{L,n}(t)$= $(A_1 + A_2 t)e^{-\alpha t}$ $\alpha = 3000\, s^{-1}$

$\alpha = 3000\, s^{-1}$

$\omega_o^2 = (9 \times 10^6 s^{-2}) \Rightarrow \omega_o = 3000\, s^{-1}$

from eqn's given above

$\alpha = \omega_o \Rightarrow$ critically damped

$i_{Ln}(t) = (A_1 + A_2 t)e^{-\alpha t}$

b) Assume $i_g(0^+) = [9 + 81u(t)]mA$, $i_L(0^+) = 1A$, and $\left.\frac{di_L}{dt}\right|_{0^+} = 0$. Find the complete solution (with no unknowns) for $i_L(t)$.

$i_L(t) = \underline{10 - 9(1 + 3000t)e^{-3000t} \quad A}$

$$i_{Lf} = 90mA\left(\frac{R_1}{R_1 + R_2 - A}\right)$$

$$= 90mA\left(\frac{1000}{9}\right)$$

$$= 10A \quad \text{(matches with given eqn)}$$

$$i_L(t) = 10A + (A_1 + A_2 t)e^{-\alpha t}$$

$$i_L(0) = 1A = 10A + A_1 \Rightarrow A_1 = -9A$$

$$\left.\frac{di_L}{dt}\right|_0 = -\alpha A_1 + A_2 = 0$$

$$\Rightarrow A_2 = \alpha A_1 = (3000s^{-1})(-9) = -27{,}000 \text{ A/s}$$

$$i_L(t) = 10 - 9(1 + (3000s^{-1})t)e^{-3000t}$$