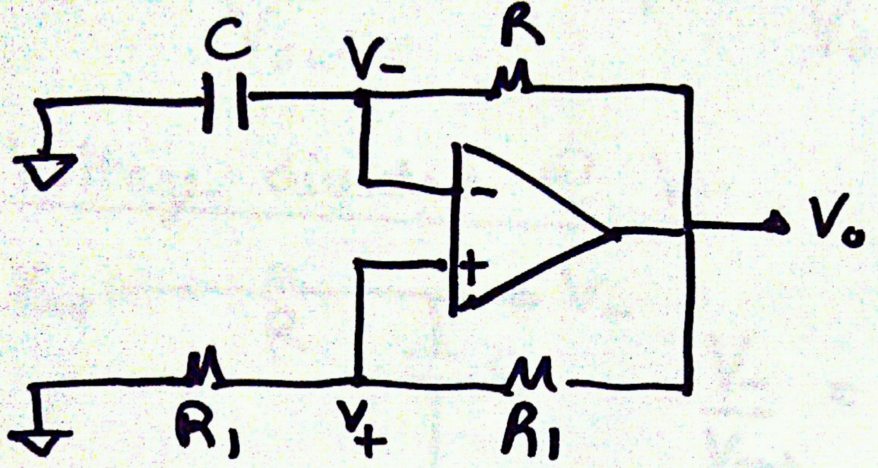
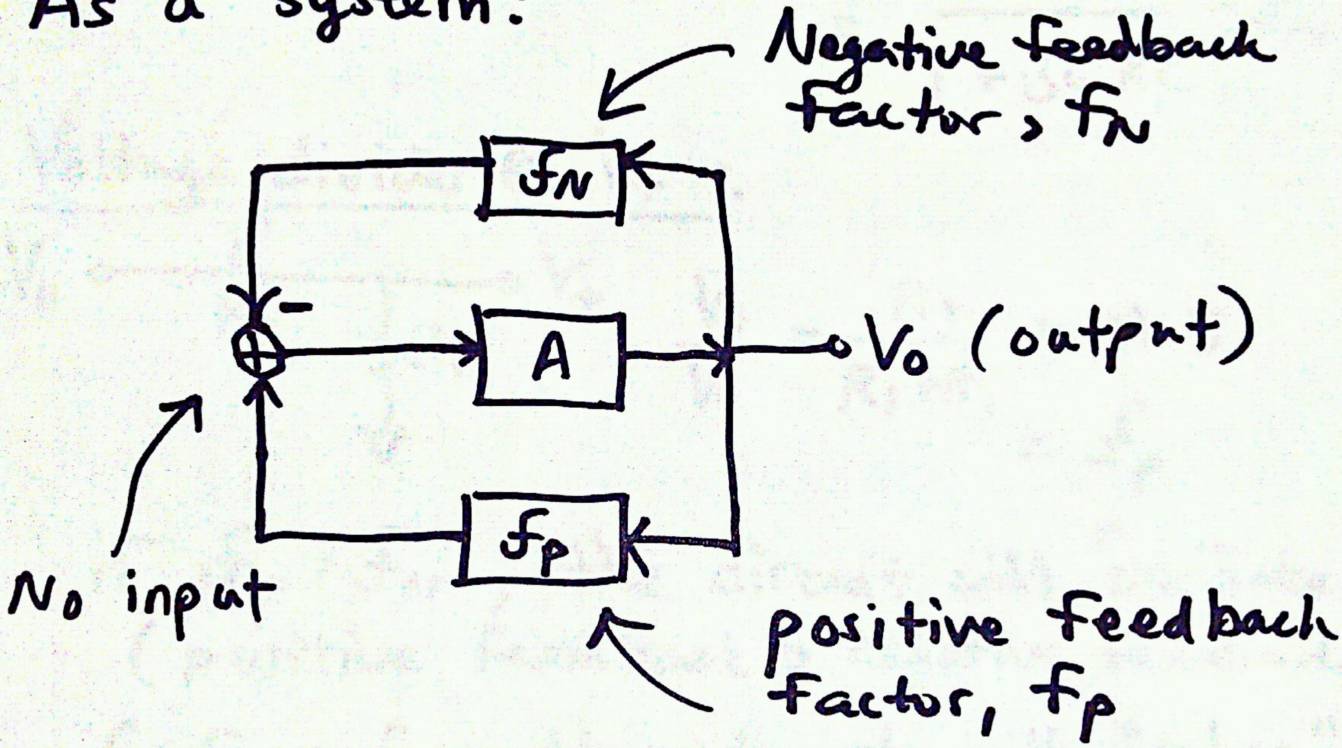
Relaxation Oscillator Demystified

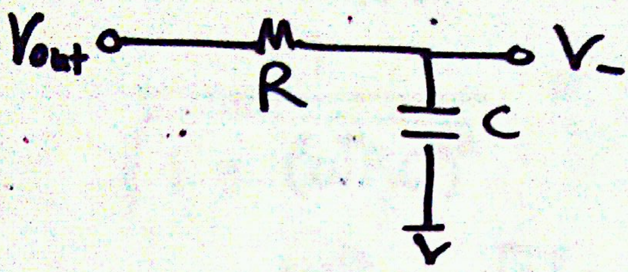


As a system:



$$f_N \triangleq \frac{V_-}{V_{out}} \quad , \quad f_P \triangleq \frac{V_+}{V_{out}}$$

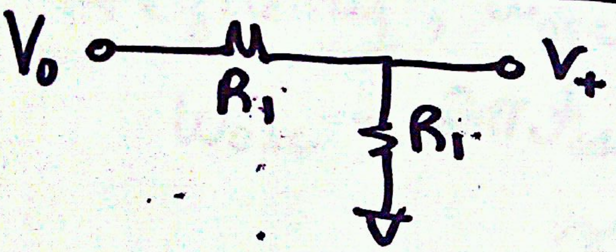
<u>Voltage divider @ V_- :</u>



$$\frac{V_-}{V_{out}} = \frac{1/j\omega C}{1/j\omega C + R}$$

$$= \frac{1}{1 + j\omega RC} = f_N$$

<u>Voltage divider @ V_+ :</u>



$$\frac{V_+}{V_0} = \frac{R_1}{R_1 + R_1} = 0.5$$

$$= f_P$$

if $f_P > f_N$, the circuit will "oscillate"
(positive feedback > negative feedback)

if $f_N > f_P$, the circuit will "relax"
(negative feedback > positive feedback)

At some frequency, ω^*, the circuit transitions from (−) FB to (+) FB.

$$|f_N| = |f_P|$$

$$\frac{1}{\sqrt{1 + (\omega^* RC)^2}} = 0.5$$

$$\omega^* = \frac{\sqrt{3}}{RC} = \sqrt{3} \cdot \omega_0 \; ; \; \omega_0 = \frac{1}{RC}$$

We are told that the oscillation frequency, ω_{osc}, is given by:

$$\omega_{osc} = 2\pi f_{osc} = 2\pi \cdot \frac{1}{2\ln(3) RC}$$

$$= \frac{\pi}{\ln(3)} \omega_0$$

Since the circuit oscillates, we expect that $\omega_{osc} > \omega^*$

$$\omega_{osc} = \frac{\pi}{\ln(3)} \omega_0 \approx 2.86\,\omega_0$$

$$\omega^* = \sqrt{3}\,\omega_0 \approx 1.73\,\omega_0$$

so $\omega_{osc} > \omega^*$ as expected ✓

IF we assume that $\frac{V_{cc}}{|A|} \approx 0$

(which is not a bad assumption)
since $A \sim 10^6$ and $V_{cc} \sim 10V$

then

$$V_{out} = \begin{cases} +V_{cc} & (V_+ - V_-) > 0 \\ -V_{cc} & (V_+ - V_-) < 0 \end{cases}$$

When $V_{out} = +V_{cc}$

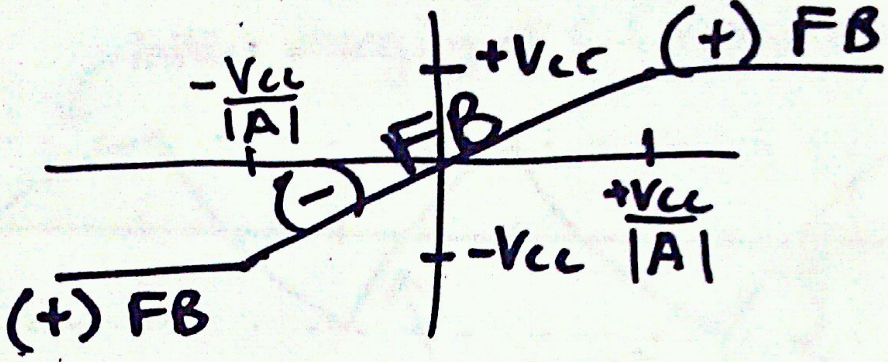
- C charges from its initial voltage to $(V_{cc} - V_{initial})$ as long as $V_{out} = +V_{cc}$
- $V_+ = 0.5 V_{out} = 0.5 V_{cc}$

When $V_{out} = -V_{cc}$

- C discharges from $V_{initial}$ towards $-V_{cc}$ as long as $V_+ - V_- < 0$

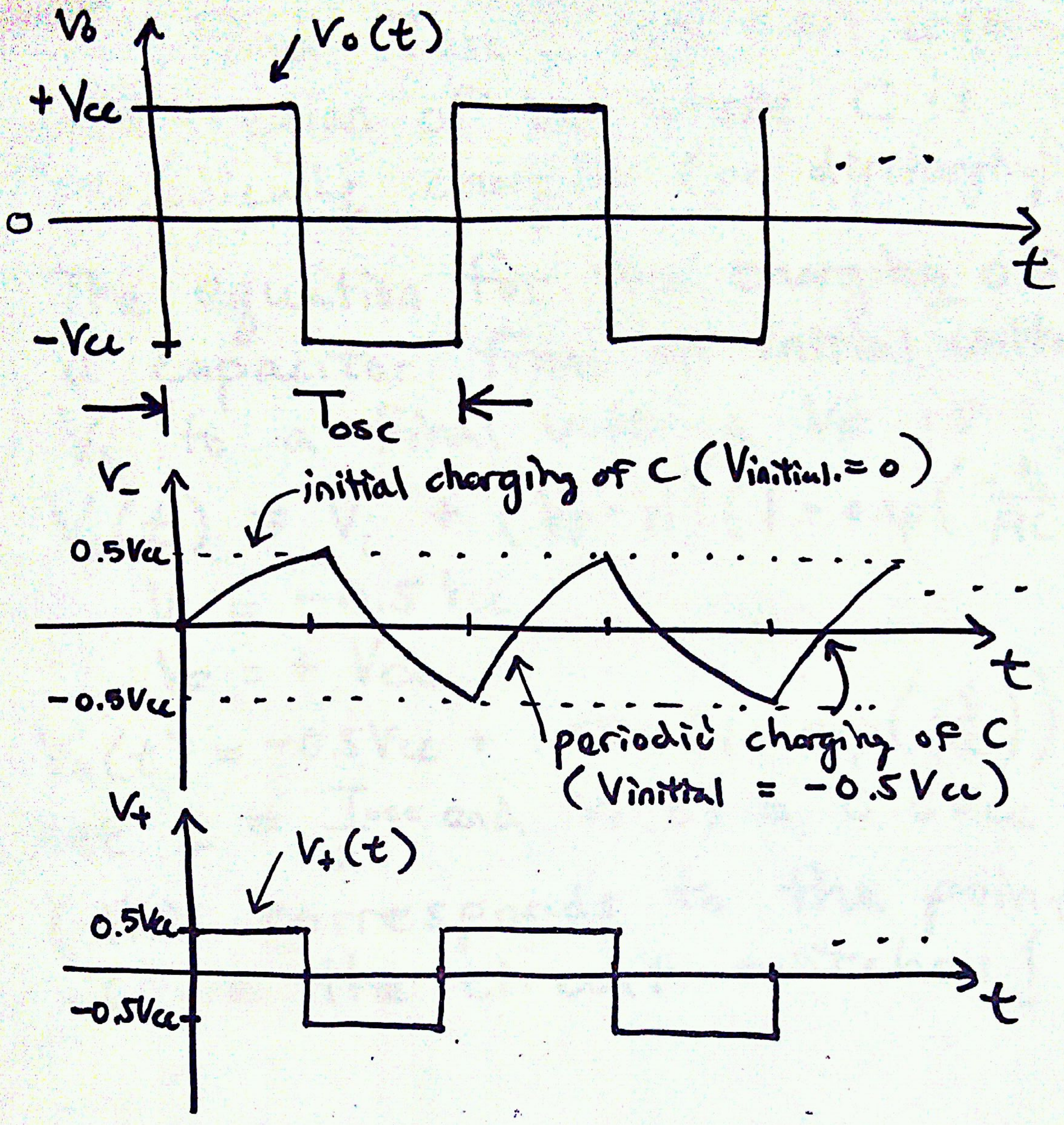
Sketching the output

Since the circuit oscillates (i.e. it is in positive FB) Vout can only take on $+V_{cc}$ or $-V_{cc}$ (assuming an ideal op amp transfer curve)



$$V_{out} = \begin{cases} +V_{cc} & |A|(V_+ - V_-) > +V_{cc} \\ -V_{cc} & |A|(V_+ - V_-) < -V_{cc} \end{cases}$$

The switching frequency (oscillation frequency) will be determined by $V_+ - V_-$



V_o
$V_o(t)$
$+V_{cc}$
0
t
$-V_{cc}$
T_{osc}
V_-
initial charging of C ($V_{initial} = 0$)
$0.5V_{cc}$
t
$-0.5V_{cc}$
periodic charging of C
($V_{initial} = -0.5V_{cc}$)
V_+
$V_+(t)$
$0.5V_{cc}$
t
$-0.5V_{cc}$

To find T_{osc}, we can use the region of V_- where C is periodically charging (or discharging)

The equation for the charging of a capacitor from an initial voltage V_i to a final voltage V_F is:

$$V_o(t) = V_i + (V_F - V_i)\left(1 - \exp\left(\frac{-t}{RC}\right)\right)$$

$$V_i = -0.5V_{cc}$$

$$V_F = +V_{cc}$$

$$V_o(t) = -0.5V_{cc} + 1.5V_{cc}\left(1 - \exp\left(\frac{-t}{RC}\right)\right)$$

set $t = \frac{T_{osc}}{2}$ and $V_o(t) = 0.5V_{cc}$

(this corresponds to the point where the circuit switches!

$$0.5\cancel{\hat{V}_{cc}} = -0.5\cancel{\hat{V}_{cc}} + 1.5\cancel{\hat{V}_{cc}}\left(1 - \exp\left(-\frac{T_{osc}}{2RC}\right)\right)$$

$$1 = 1.5\left[-\exp\left(-\frac{T_{osc}}{2RC}\right) + 1\right]$$

$$2 = 3\left[1 - \exp\left(-\frac{T_{osc}}{2RC}\right)\right]$$

$$= 3 - 3\exp\left(-\frac{T_{osc}}{2RC}\right)$$

$$3\exp\left(-\frac{T_{osc}}{2RC}\right) = 1$$

$$\exp\left(-\frac{T_{osc}}{2RC}\right) = \frac{1}{3}$$

$$\ln\left[\quad " \quad\right] = \ln\left[\quad " \quad\right]$$

$$-\frac{T_{osc}}{2RC} = \ln\left(\frac{1}{3}\right) = -\ln(3)$$

$$T_{osc} = 2\ln(3)RC$$

$$\text{so } \boxed{f_{osc} = \frac{1}{T_{osc}} = \frac{1}{2\ln(3)RC}}$$