40 – Resistive Circuits

Reading Material:

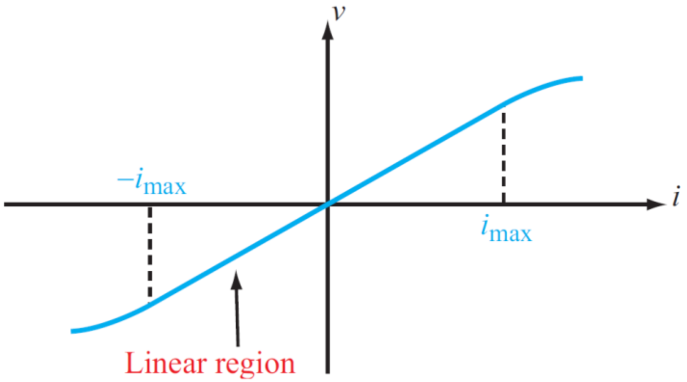
Chapter 2

Ohm's Law

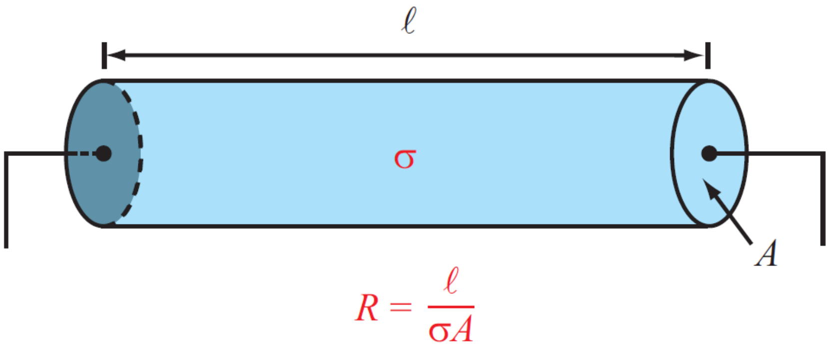
Voltage across resistor is proportional to current

$$\upsilon = iR$$

$$R = \frac{\upsilon}{i}$$



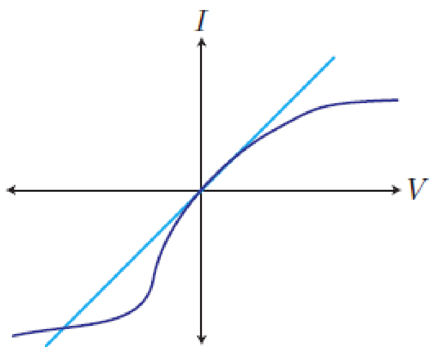
Resistance: ability to resist flow of electric current



$$R = \frac{\ell}{\sigma A} = \rho \frac{\ell}{A} \qquad (\Omega),$$

ρ = **resistivity**

Is Ohm's Law Strange?



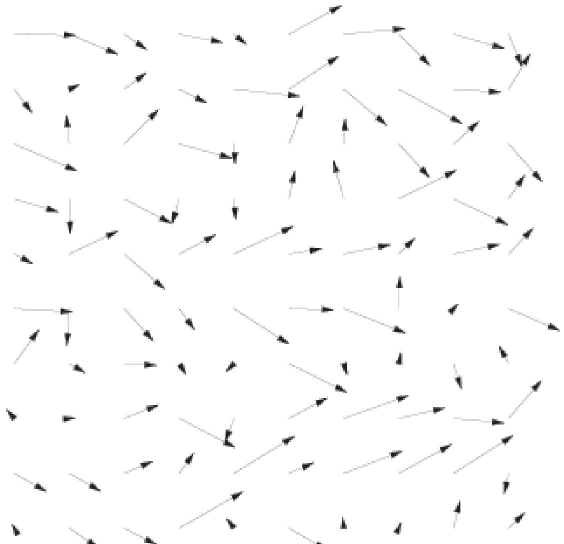
- It's also remarkable that this linear relation hold true over a wide range of voltages. In general, we can say that

$$V = f(I) = R_1 I + R_2 I^2 + \cdots$$

- For all practical purposes, R_1 is the only term that matters.
- The voltage drop across a resistor is V. Note that the V represents how much energy is gained by a unit of charge as it moves the the resistor element.
- The current I we have learned is proportional to the velocity of charge carriers (such as electrons). We would therefore expect a quadratic relation, not linear, between the current and voltage (Kinetic energy).

Is Ohm's Law Strange?

- The answer to this riddle lies in the fact that carriers do not move unimpeded through a conductor (in vacuum they would in fact have a quadratic dependence) but rather the motion is mostly random motion.
- On average the charge carriers move only a short distance before colliding with atoms (impurities) in the crystal. After the collision all "memory" of the previous path of motion is lost and the energy of the carrier is converted into heat (vibrations in the crystal). The gain in momentum is proportional to the voltage.



$$\bar{v} = \frac{1}{N}\sum_i \mathbf{v}_i \approx \hat{\mathbf{x}} v_d$$

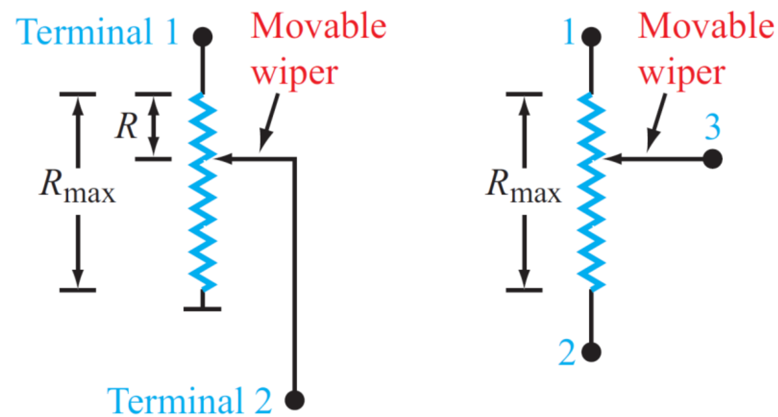
Conductivity

Table 2-1: Conductivity and resistivity of some common materials at 20°C.

Material	Conductivity σ (S/m)	Resistivity ρ (Ω-m)
Conductors		
Silver	6.17×10^7	1.62×10^{-8}
Copper	5.81×10^7	1.72×10^{-8}
Gold	4.10×10^7	2.44×10^{-8}
Aluminum	3.82×10^7	2.62×10^{-8}
Iron	1.03×10^7	9.71×10^{-8}
Mercury (liquid)	1.04×10^6	9.58×10^{-8}
Semiconductors		
Carbon (graphite)	7.14×10^4	1.40×10^{-5}
Pure germanium	2.13	0.47
Pure silicon	4.35×10^{-4}	2.30×10^3
Insulators		
Paper	$\sim 10^{-10}$	$\sim 10^{10}$
Glass	$\sim 10^{-12}$	$\sim 10^{12}$
Teflon	$\sim 3.3 \times 10^{-13}$	$\sim 3 \times 10^{12}$
Porcelain	$\sim 10^{-14}$	$\sim 10^{14}$
Mica	$\sim 10^{-15}$	$\sim 10^{15}$
Polystyrene	$\sim 10^{-16}$	$\sim 10^{16}$
Fused quartz	$\sim 10^{-17}$	$\sim 10^{17}$

Table 2-3: Common resistor terminology.

Thermistor	R sensitive to temperature
Piezoresistor	R sensitive to pressure
Rheostat	2-terminal variable resistor
Potentiometer	3-terminal variable resistor



(a) Rheostat (b) Potentiometer

Power loss in resistors

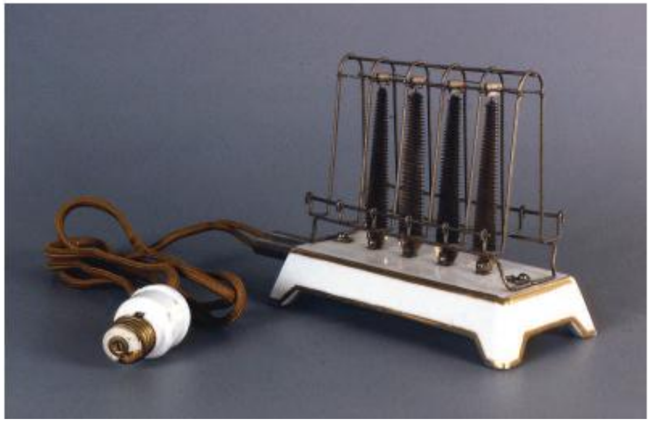
- From the equation for the power loss in a component, we have

$$P = V \cdot I = (R \cdot I) \cdot I = I^2 R$$

- Or in terms of conductance

$$P = V \cdot I = V \cdot (G \cdot V) = V^2 G$$

- This power is lost to heat or "Joule Heating". In fact, most electric ovens use resistors to heat up the oven.
- Nichrome (nickel-chromium alloy) is often used as the heating element. It has a high melting point of $1400°$C and a hight resistivity and resistance to oxidation at high temperature. It is widely used in ovens, hair dryers, and toasters.



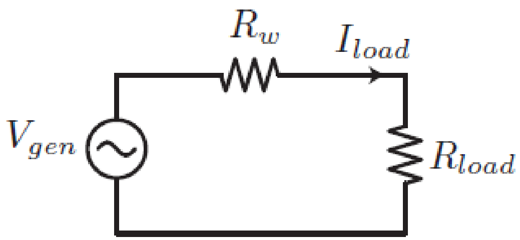
Example: Why Power is Delivered with High Voltages





- For economic and environmental reasons, power is usually generated remotely (wind farms, electric dam, nuclear power plants, etc). Many hundreds of miles of wires are required to carry the energy to the factories and homes.
- Since wires have resistance, this is energy which is completely lost without doing any useful work (except warming up the planet).
- Since the line voltage is fixed (say 120V into the home), the power draw is represented by a varying I_{load}. The equivalent circuit (next page) shows that the "load" can be represented by R_{load} and the power loss is proportional to $I_{load}^2 R_{wire}$.

100-km Energy Loss



- Say a house is dissipating $P = 1.2$kW of power. If the house operated from a 120V DC source, then that's a current draw of

$$I_{load} = P/V = 1\text{kW}/120\text{V} = 10\text{A}$$

- Assume a copper wire of 5mm radius and 100km long

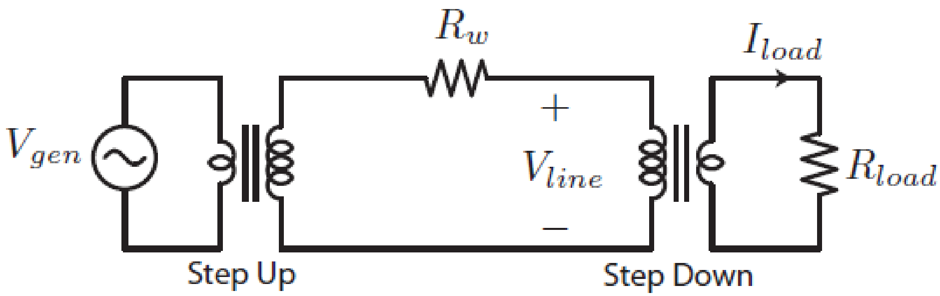
$$R_w = \rho\frac{L}{A} = \rho\frac{L}{\pi r^2} = 1.72 \times 10^{-8} \times \frac{100 \times 10^3}{\pi(5 \cdot 10^{-3})^2} = 22\Omega$$

- The power lost to heat is

$$P_{loss} = I_{load}^2 R_w = 2.2\text{kW}$$

- That's more than the power delivered! The efficiency of this system is very low.

High-Voltage Lines



- The key is to transform the voltage to a much higher value to minimize the current. One key reason why we use AC power is so that we can use transformers to easily boost the voltage on the lines, and then to drop the voltage to more reasonable values as we get near residential areas (for safety).
- In the above example, the voltage is increased to 100kV, and so the equivalent current needed to deliver 1.2kW is decreased substantially

$$I_{load,line} = P/V_{line} = 0.012\text{A}$$

- And so that the energy lost to heat is minimized

$$P_{loss} = I_{load,line}^2 R_w = 3\text{mW}$$

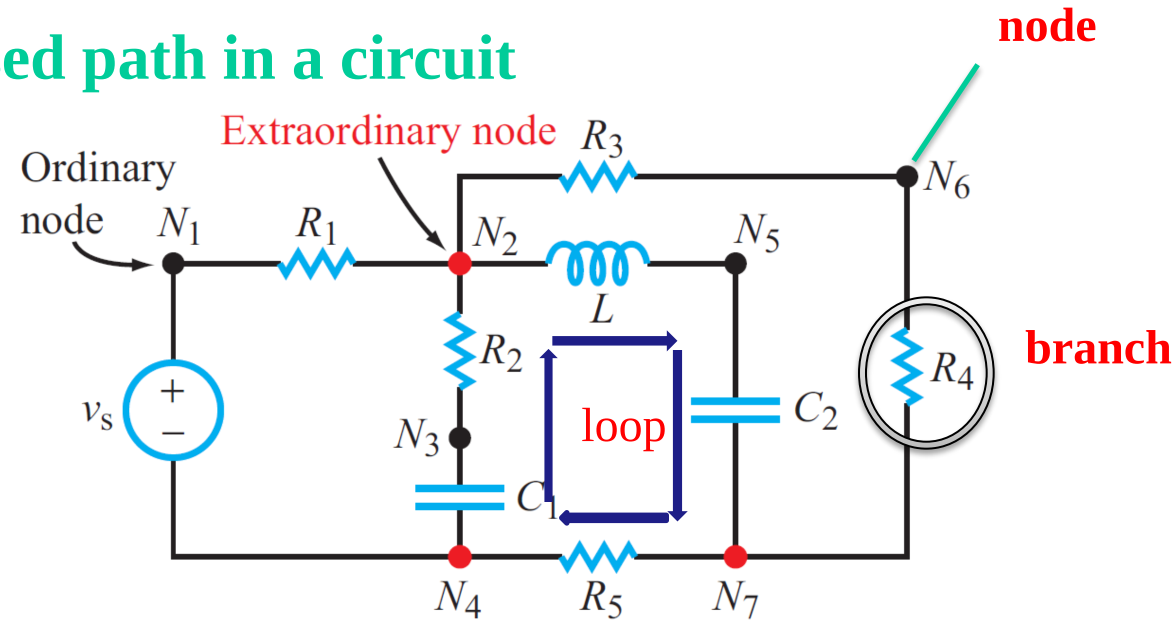
Circuit Topology

Branch: single element, such as a resistor or source

Node: connection point between two or more branches

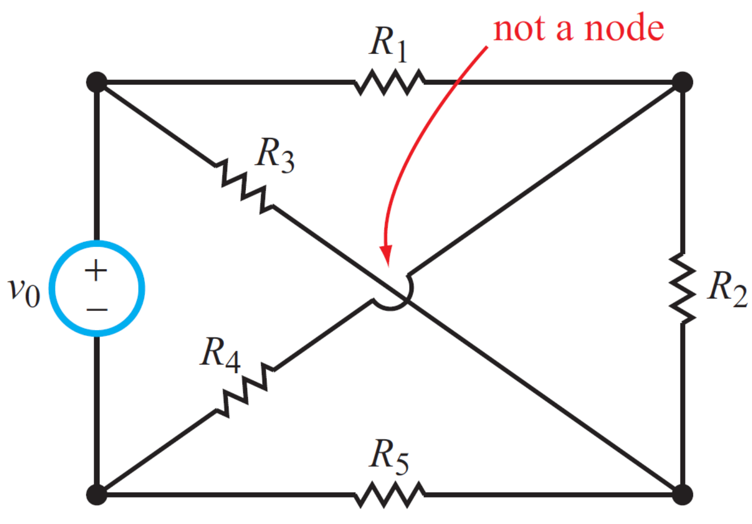
Extraordinary Node: connection point between at least 3 branches

Loop: closed path in a circuit

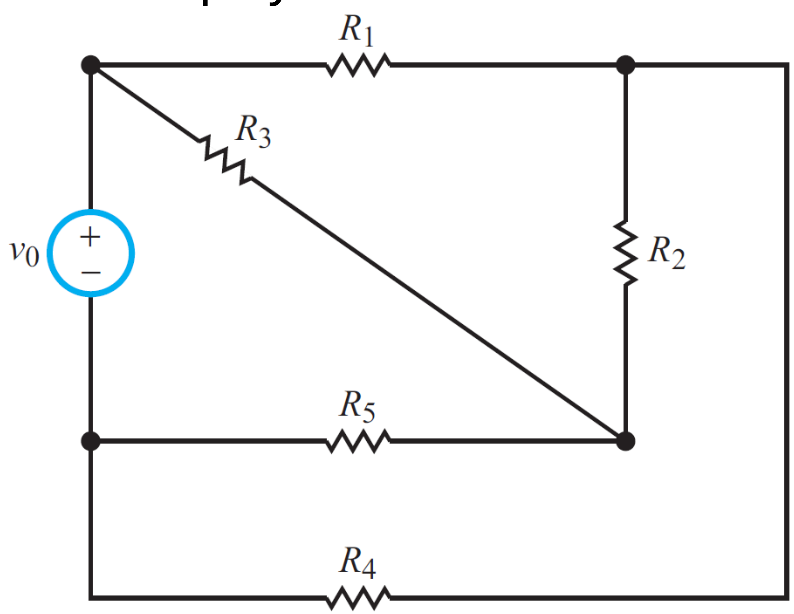


Planar Circuits

- Planar circuits: can be drawn in 2-D without branches crossing each other
- Whenever possible, re-draw circuit to simplify!



(a) Original circuit



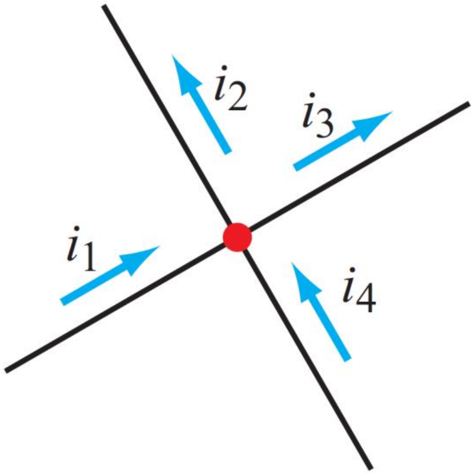
(b) Redrawn

Slide 2-11

Kirchhoff's Current Law

Sum of currents entering a node is zero

Also holds for closed boundary



$$\sum_{n=1}^{N} i_n = 0 \qquad \text{(KCL)},$$

$$i_1 - i_2 - i_3 + i_4 = 0$$

$$i_1 + i_4 = i_2 + i_3$$

Example: KCL

If V_4, the voltage across the 4–Ω resistor in Fig. 2-12, is 8 V, determine I_1 and I_2.

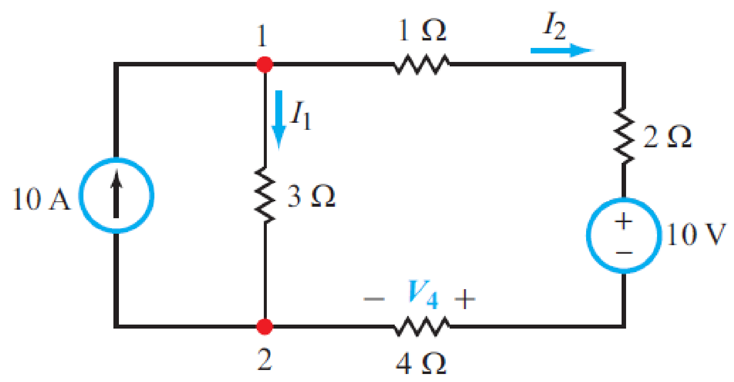


Figure 2-12: Circuit for Example 2-3.

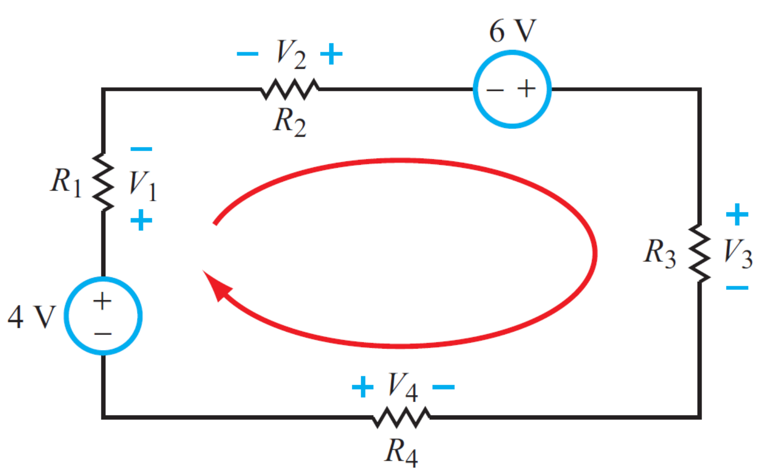
Solution: Applying Ohm's law,

Kirchhoff's Voltage Law (KVL)

Sum of voltages around a closed path is zero

Sum of voltage drops = sum of voltage rises

$$\sum_{n=1}^{N} v_n = 0 \qquad \text{(KVL)},$$

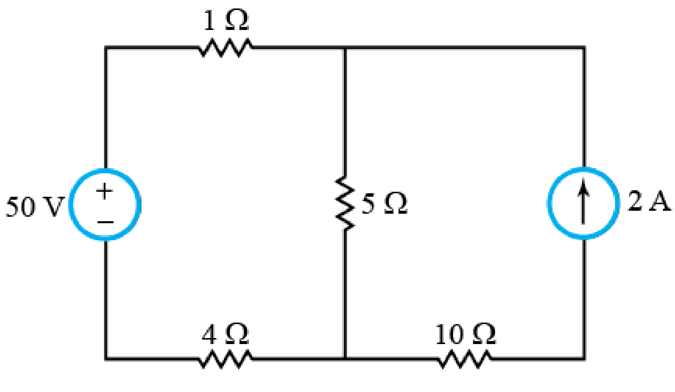


Sign Convention

- Add up the voltages in a systematic clockwise movement around the loop.
- Assign a positive sign to the voltage across an element if the (+) side of that voltage is encountered first, and assign a negative sign if the (−) side is encountered first.

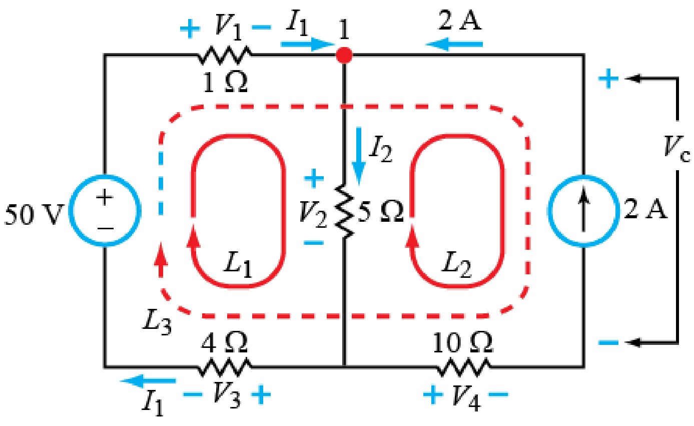
$$-4 + V_1 - V_2 - 6 + V_3 - V_4 = 0$$

Example: KCL/KVL



(a)

Solution:

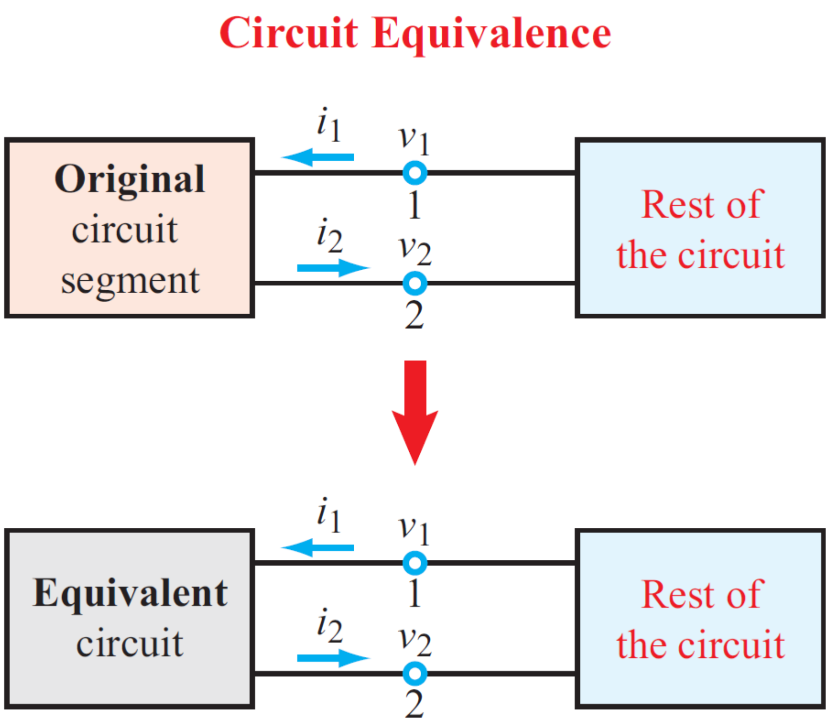


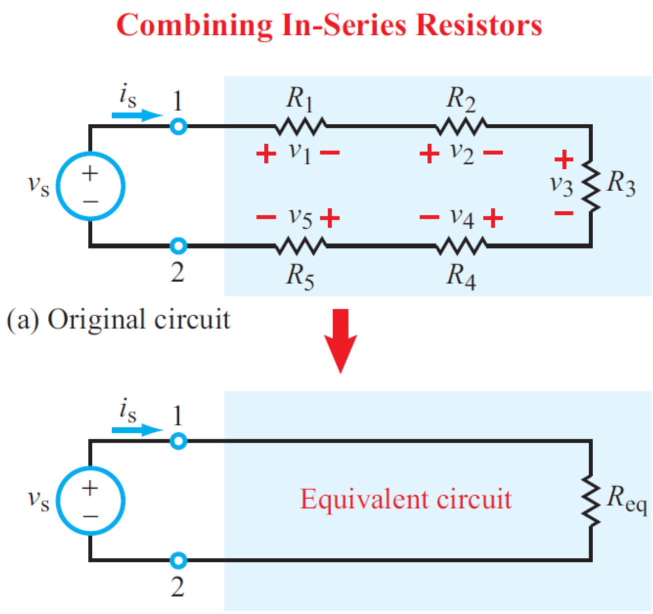
(b)

Determine all currents & voltages

Equivalent Circuits

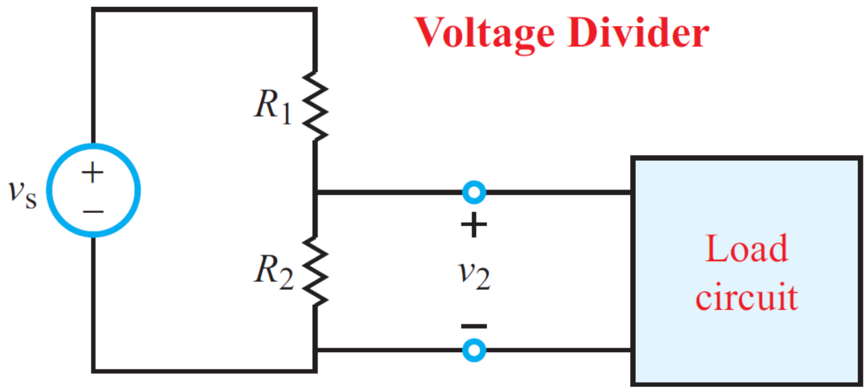
- If the current and voltage characteristics at nodes are identical, the circuits are considered "equivalent"
- Identifying equivalent circuits simplifies analysis





Resistors in Series

Equivalent resistance (series) is sum of resistances



$$v_2 = \left(\frac{R_2}{R_1 + R_2}\right) v_s$$

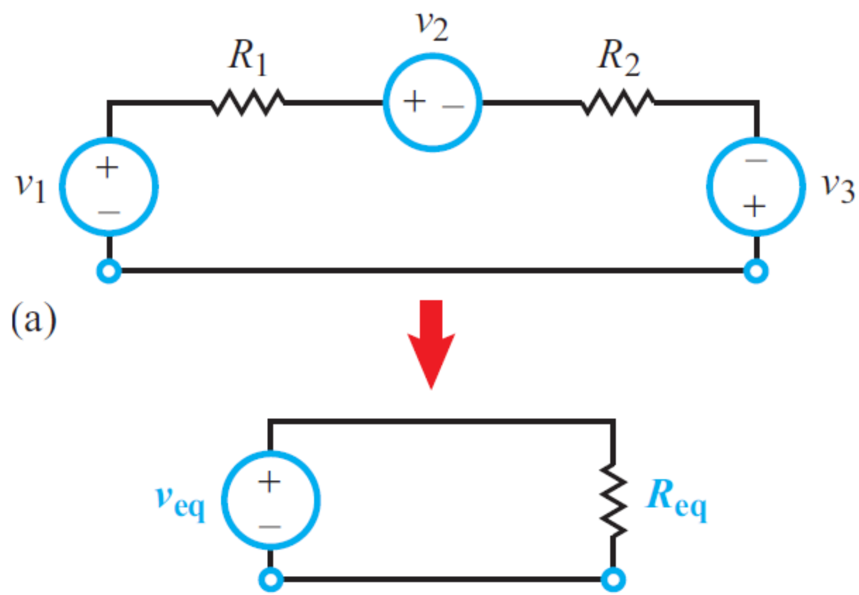
$$R_{eq} = \sum_{i=1}^{N} R_i \qquad \text{(resistors in series)},$$

$$v_i = \left(\frac{R_i}{R_{eq}}\right) v_s.$$

Voltage divided over resistors (voltage divider)

Adding Sources In Series

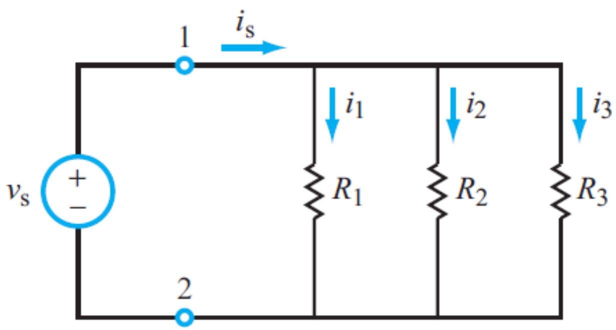
Combining voltage sources



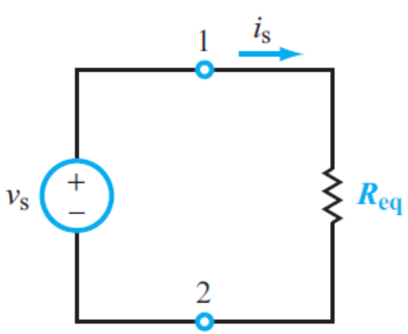
(b) $v_{eq} = v_1 - v_2 + v_3 \qquad R_{eq} = R_1 + R_2$

Resistors in Parallel

Combining In-Parallel Resistors



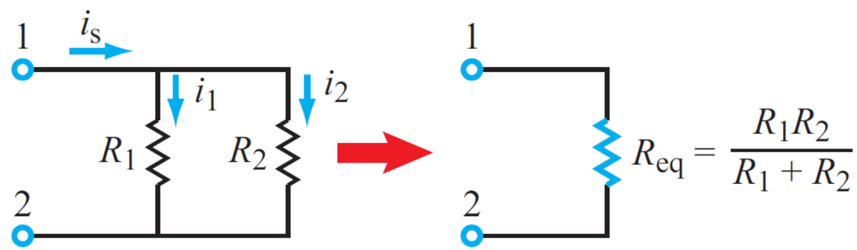
(a) Original circuit



(b) Equivalent circuit

$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)^{-1} \qquad i_2 = \left(\frac{R_{eq}}{R_2} \right) i_s$$

Current Division



$$i_1 = \left(\frac{R_2}{R_1 + R_2} \right) i_s \qquad i_2 = \left(\frac{R_1}{R_1 + R_2} \right) i_s$$

$$i_s = \frac{v_s}{R_1} + \frac{v_s}{R_2} + \frac{v_s}{R_3}$$

$$= \frac{v_s}{R_{eq}}.$$

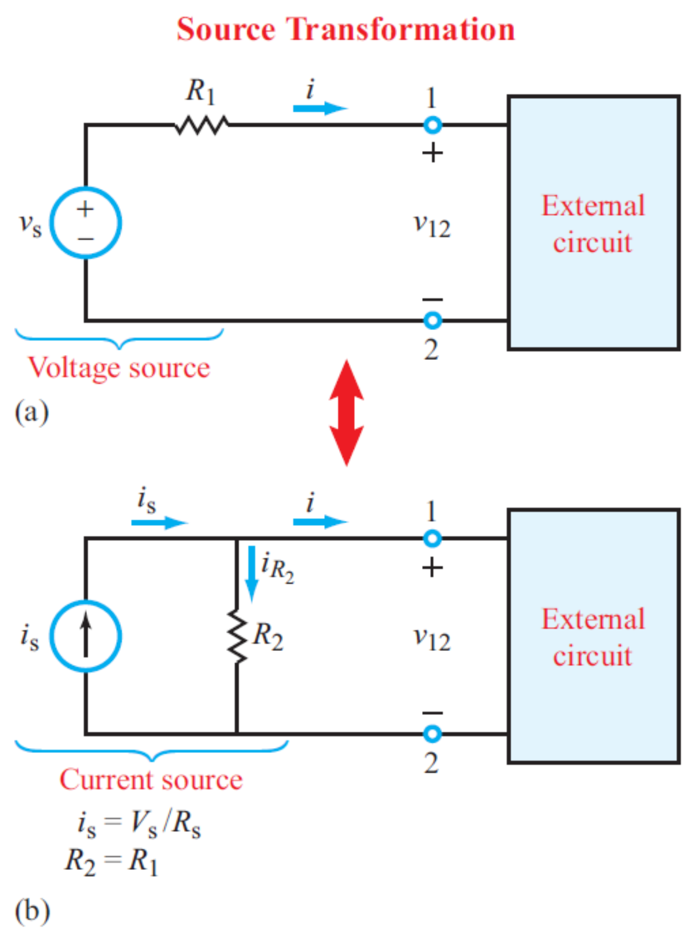
$$\frac{1}{R_{eq}} = \sum_{i=1}^{N} \frac{1}{R_i} \qquad \text{(resistors in parallel).}$$

Source Transformation

$$-v_s + iR_1 + v_{12} = 0$$

Hence, $$i = \frac{v_s}{R_1} - \frac{v_{12}}{R_1}.$$

$$i = i_s - i_{R_2}$$
$$= i_s - \frac{v_{12}}{R_2}$$

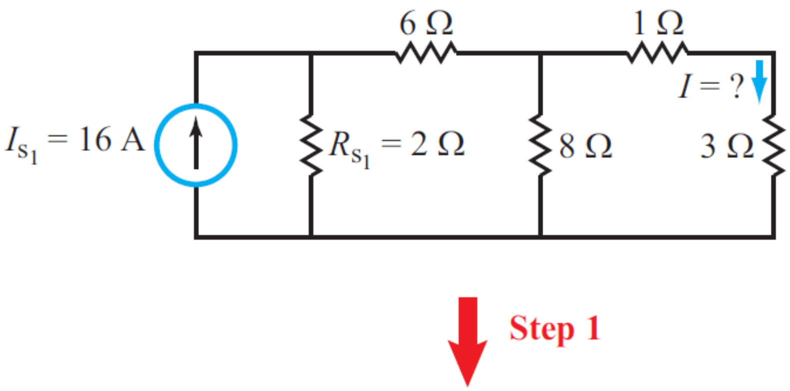


For the two circuits to be equivalent :

$$\boxed{R_1 = R_2} \qquad \boxed{i_s = \frac{v_s}{R_1}.}$$

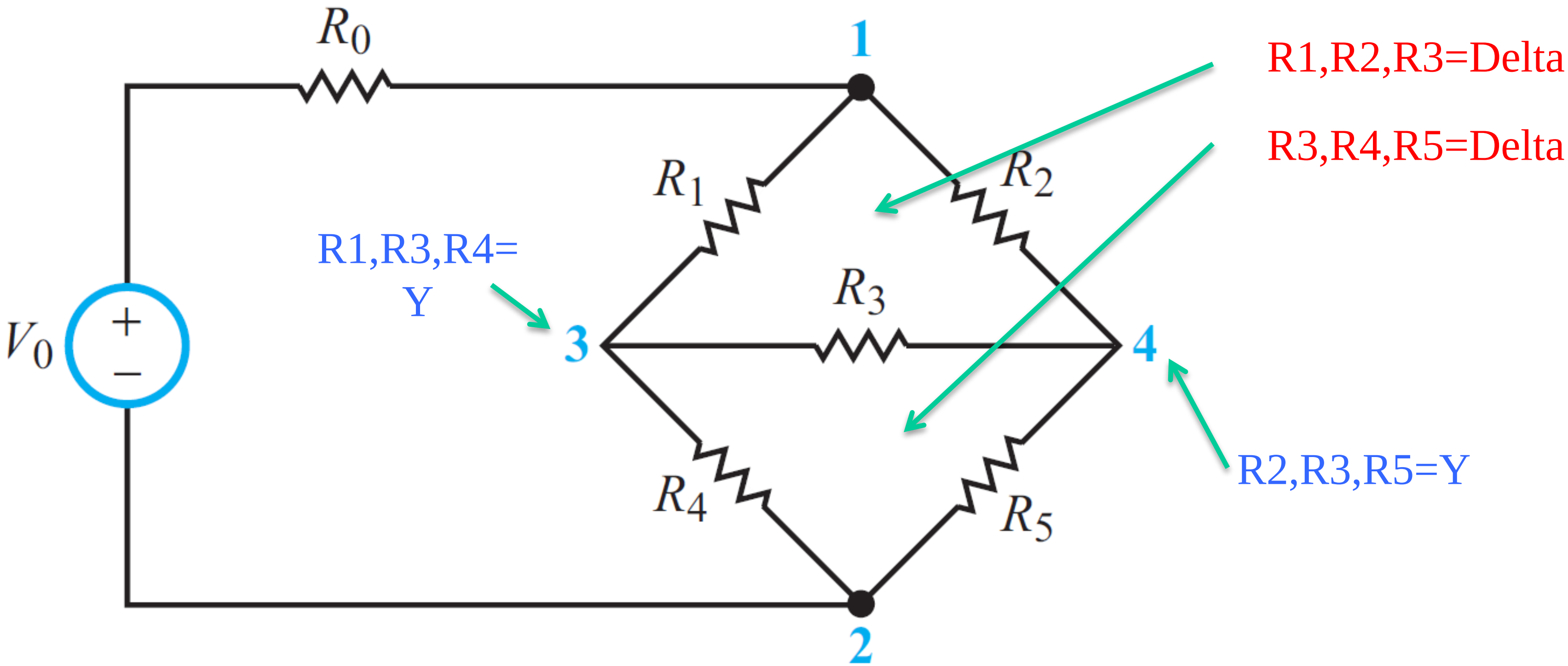
Example: *Source Transformation*

Determine the current I in the circuit of Fig. 2-26(a).



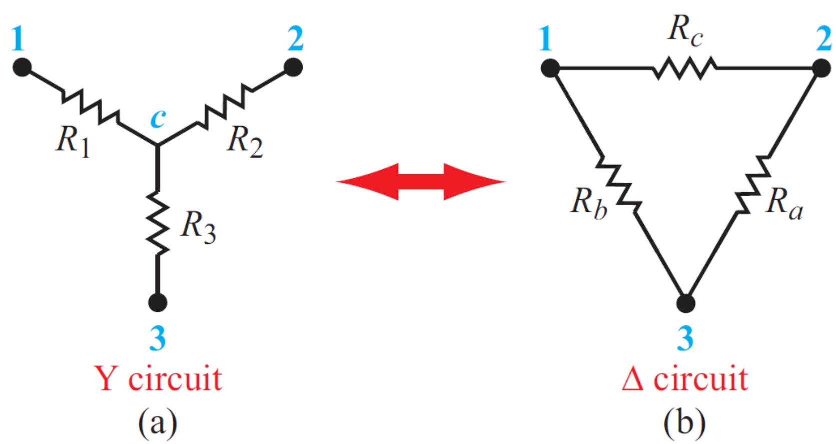
$$I = \frac{V_{s_2}}{4+1+3} = \frac{16}{8} = 2\text{ A}.$$

Wye–Delta (Y–Δ) Transformation



Circuit with no two resistors sharing the same current or same voltage

Wye–Delta (Y–Δ) Transformation



Between nodes 1 & 2:

$$R_{12} = R_1 + R_2 \qquad \text{(Y-circuit)}$$

$$R_{12} = \frac{R_c(R_a + R_b)}{R_a + R_b + R_c} \qquad (\Delta\text{-circuit})$$

(Assuming no connection at node 3)

Hence,

$$R_1 + R_2 = \frac{R_c(R_a + R_b)}{R_a + R_b + R_c}. \qquad (2.57a)$$

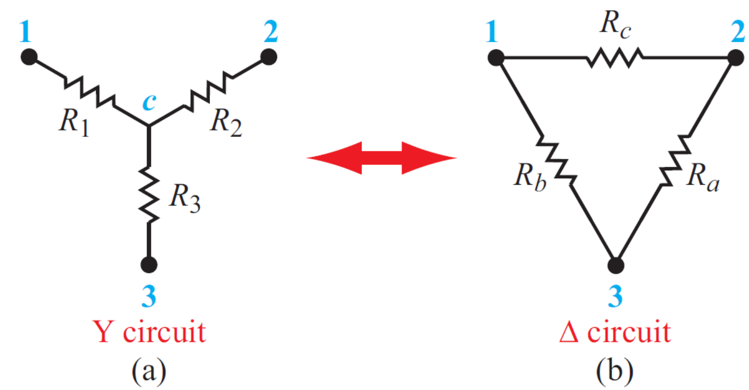
When applied to the other two combinations of nodes, the foregoing procedure leads to:

$$R_2 + R_3 = \frac{R_a(R_b + R_c)}{R_a + R_b + R_c} \qquad (2.57b)$$

and

$$R_1 + R_3 = \frac{R_b(R_a + R_c)}{R_a + R_b + R_c}. \qquad (2.57c)$$

Wye–Delta (Y–Δ) Transformation



Simultaneous solution leads to:

Δ →Y Transformation

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_a R_c}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

Y→ Δ Transformation

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_1}$$

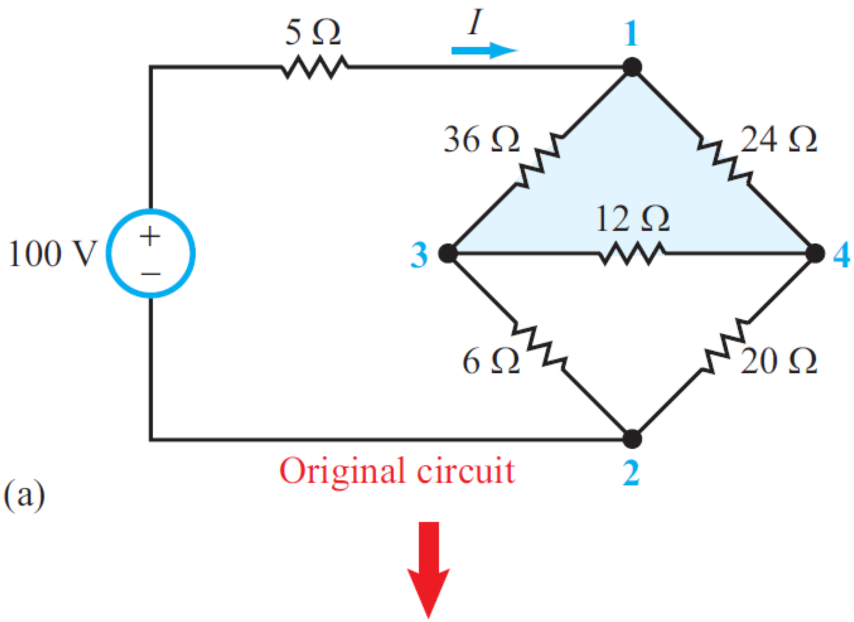
$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_2}$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_3}$$

Example: Y-Δ Circuit

Simplify the circuit in Fig. 2-30(a) by applying the Y–Δ transformation so as to determine the current I.

Solution: Noting the symmetry rules associated with the transformation, the Δ circuit connected to nodes 1, 3, and 4 can be replaced with a Y circuit, as shown in Fig. 2-30(b), with



(a)

i–v Relationships

Linear *i-v* relationships