Problem 7.51 Apply nodal analysis to determine $\mathbf{I}_C$ in the circuit of Fig. P7.51.

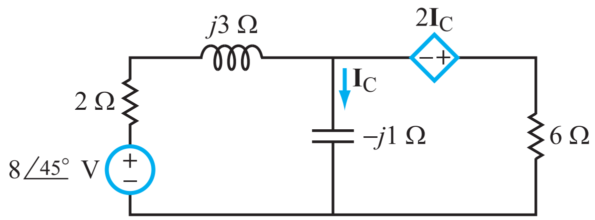
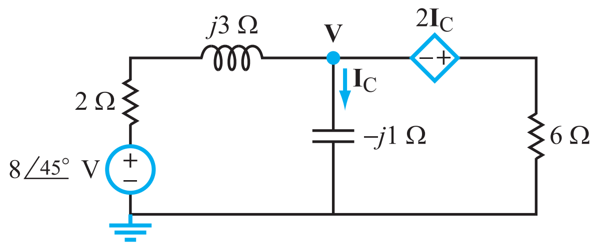


Figure P7.51: Circuits for Problems 7.51 and 7.52.

Solution:



$$\frac{\mathbf{V}-8e^{j45^\circ}}{2+j3}+\frac{\mathbf{V}}{-j1}+\frac{\mathbf{V}+2\mathbf{I}_C}{6}=0$$

Also,

$$\mathbf{I}_C=\frac{\mathbf{V}}{-j1}=j\mathbf{V}.$$

Hence,

$$\mathbf{V}\left(\frac{1}{2+j3}+j+\frac{1}{6}+\frac{j}{3}\right)=\frac{8e^{j45^\circ}}{2+j3}$$

Solution gives

$$\mathbf{V}=\frac{48e^{j45^\circ}}{-(16-j19)}=1.93e^{-j85.1^\circ}\text{ V}$$

and

$$\mathbf{I}_C=j\mathbf{V}=1.93e^{j4.9^\circ}\text{ A}.$$

Problem 7.56 Use any analysis technique of your choice to determine $i_C(t)$ in the circuit of Fig. P7-56.

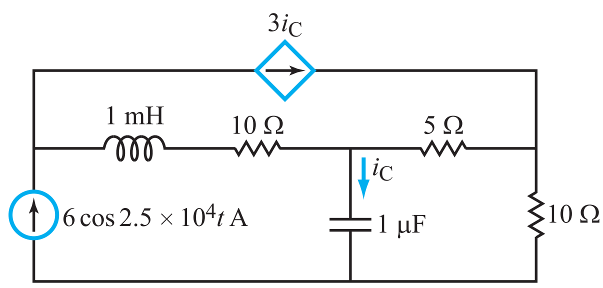
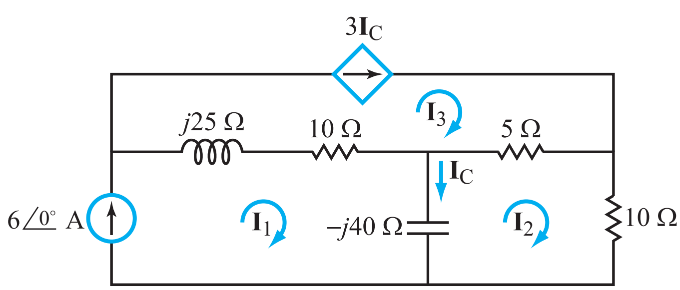


Figure P7.56: Circuit for Problem 7.56.

Solution:



$$\mathbf{Z}_L = j\omega L = j2.5 \times 10^4 \times 10^{-3} = j25\ \Omega$$

$$\mathbf{Z}_C = \frac{-j}{\omega C} = \frac{-j}{2.5 \times 10^4 \times 10^{-6}} = -j40\ \Omega$$

The mesh-current method gives:

Mesh 1:	$\mathbf{I}_1 = 6$ A
Mesh 2:	$-j40(\mathbf{I}_2 - \mathbf{I}_1) + 5(\mathbf{I}_2 - \mathbf{I}_3) + 10\mathbf{I}_2 = 0$
Mesh 3:	$\mathbf{I}_3 = 3\mathbf{I}_C$
Auxiliary:	$\mathbf{I}_C = \mathbf{I}_1 - \mathbf{I}_2$.

Simultaneous solution leads to

$$\mathbf{I}_1 = 6\text{ A}, \qquad \mathbf{I}_2 = (4.92 - j1.44)\text{ A}.$$

$$\begin{aligned} \mathbf{I}_C &= \mathbf{I}_1 - \mathbf{I}_2 \\ &= 1.8e^{j53.13^\circ}\text{ A}. \\ i_C(t) &= \mathfrak{Re}[\mathbf{I}_C e^{j\omega t}] \\ &= 1.8\cos(2.5 \times 10^4 t + 53.13^\circ) \qquad \text{(A)}. \end{aligned}$$

Problem 7.58 The input signal in the op-amp circuit of Fig. P7.58 is given by

$$v_{in}(t) = V_0 \cos \omega t.$$

Assuming the op amp is operating within its linear range, obtain an expression for $v_{out}(t)$ by applying the phasor-domain technique, and then evaluate it for $\omega RC = 1$.

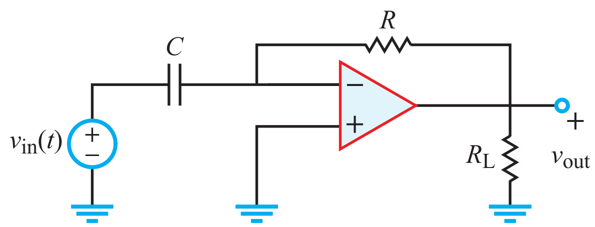
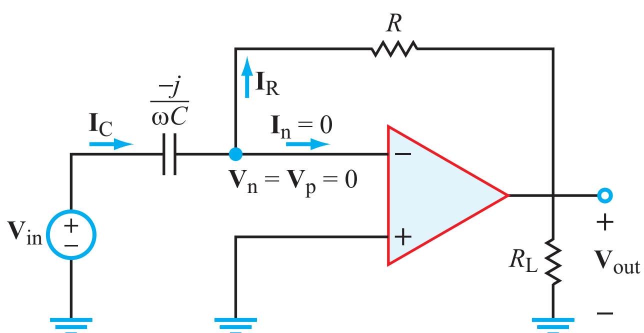


Figure P7.58: Op-amp circuit for Problem 7.58.

Solution:



$\mathbf{V}_{in} = V_0$.

Because $\mathbf{V}_n = \mathbf{V}_p = 0$,

$$\mathbf{I}_C = \frac{\mathbf{V}_{in}}{\mathbf{Z}_C} = \frac{\mathbf{V}_{in}}{-j/\omega C} = j\omega C\mathbf{V}_{in}.$$

$$\mathbf{I}_R = \frac{\mathbf{V}_n - \mathbf{V}_{out}}{R}.$$

Also, $\mathbf{V}_n = 0$ and $\mathbf{I}_n = 0$.

Hence

$$\begin{aligned}
\mathbf{V}_{out} &= -R\mathbf{I}_R \\
&= -R\mathbf{I}_C \\
&= -j\omega RC\mathbf{V}_{in} \\
&= -j\omega RCV_0 \\
&= \omega RCV_0 e^{-j90^\circ}. \\
v_{out}(t) &= \mathfrak{Re}[\mathbf{V}_{out} e^{j\omega t}] \\
&= \mathfrak{Re}[\omega RCV_0 e^{-j90^\circ} e^{j\omega t}] \\
&= \omega RCV_0 \cos(\omega t - 90^\circ)
\end{aligned}$$

$$= \omega RCV_0 \sin \omega t.$$

For $\omega RC = 1$,

$$v_{\text{out}}(t) = V_0 \sin \omega t.$$

CHAPTER 9

Section 9-1: Transfer Function

Problem 9.1 Determine the resonant frequency of the circuit shown in Fig. P9.1, given that $R = 100\ \Omega$, $L = 5$ mH, and $C = 1\ \mu$F.

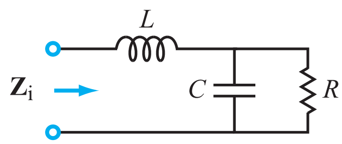


Figure P9.1: Circuit for Problem 9.1.

Solution:

$$
\begin{aligned}
\mathbf{Z}_i &= \left(R \parallel \frac{1}{j\omega C}\right) + j\omega L \\
&= \frac{\frac{R}{j\omega C}}{R + \frac{1}{j\omega C}} + j\omega L \\
&= \frac{R}{1 + j\omega RC} + j\omega L \\
&= \frac{R(1-\omega^2 LC) + j\omega L}{1 + j\omega RC} \times \frac{1 - j\omega RC}{1 - j\omega RC} \\
&= \frac{[R(1-\omega^2 LC) + \omega^2 RLC] + j[\omega L - \omega R^2 C(1-\omega^2 LC)]}{1 + \omega^2 R^2 C^2} \\
&= \frac{R + j\omega[L - R^2 C + \omega^2 R^2 LC^2]}{1 + \omega^2 R^2 C^2} \, .
\end{aligned}
$$

At resonance, the imaginary part of $\mathbf{Z}_i$ is zero. Thus,

$$
\omega[L - R^2 C + \omega^2 R^2 LC^2] = 0,
$$

which gives two solutions, a trivial resonance at $\omega = 0$, and

$$
\begin{aligned}
\omega_0 &= \sqrt{\frac{1}{LC} - \frac{1}{R^2 C^2}} \\
&= \sqrt{\frac{1}{5\times 10^{-3} \times 10^{-6}} - \frac{1}{10^4 \times 10^{-12}}} = 10^4 \text{ rad/s.}
\end{aligned}
$$

Problem 9.4 For the circuit shown in Fig. P9.4, determine (a) the transfer function $\mathbf{H} = \mathbf{V}_o/\mathbf{V}_i$, and (b) the frequency ω_o at which $\mathbf{H}$ is purely real.

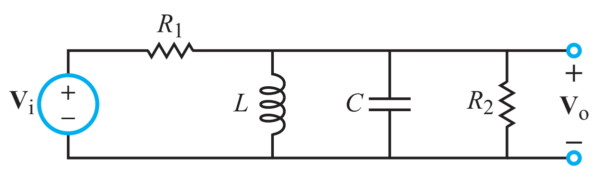


Figure P9.4: Circuit for Problem 9.4.

Solution:

(a) KCL at mode $\mathbf{V}_0$:

$$\frac{\mathbf{V}_o - \mathbf{V}_i}{R_1} + \mathbf{V}_o \left(\frac{1}{j\omega L} + j\omega C + \frac{1}{R_2} \right) = 0.$$

Solution leads to

$$\mathbf{H} = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{jR_2 \omega L}{R_1 R_2 (1 - \omega^2 LC) + j\omega L (R_1 + R_2)} .$$

(b) Rationalizing the expression for $\mathbf{H}$:

$$\begin{aligned} \mathbf{H} &= \frac{jR_2 \omega L}{R_1 R_2 (1 - \omega^2 LC) + j\omega L (R_1 + R_2)} \times \frac{R_1 R_2 (1 - \omega^2 LC) - j\omega L (R_1 + R_2)}{R_1 R_2 (1 - \omega^2 LC) - j\omega L (R_1 + R_2)} \\ &= \frac{\omega^2 R_2 L^2 (R_1 + R_2) + j\omega L R_1 R_2^2 (1 - \omega^2 LC)}{R_1^2 R_2^2 (1 - \omega^2 LC)^2 + \omega^2 L^2 (R_1 + R_2)^2} . \end{aligned}$$

The imaginary part of $\mathbf{H}$ is zero when $\omega = 0$ (trivial resonance) or when

$$\omega_0 = \frac{1}{\sqrt{LC}} .$$

Problem 9.3 For the circuit shown in Fig. P9.3, determine (a) the transfer function $\mathbf{H} = \mathbf{V}_o/\mathbf{V}_i$, and (b) the frequency ω_o at which $\mathbf{H}$ is purely real.

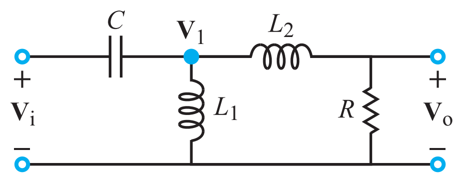


Figure P9.3: Circuit for Problem 9.3.

Solution:

(a) KCL at node $\mathbf{V}_1$ gives:

$$\frac{\mathbf{V}_1 - \mathbf{V}_i}{\mathbf{Z}_C} + \frac{\mathbf{V}_1}{\mathbf{Z}_{L_1}} + \frac{\mathbf{V}_1}{R + \mathbf{Z}_{L_2}} = 0,$$

where $\mathbf{Z}_C = 1/j\omega C$, $\mathbf{Z}_{L_1} = j\omega L_1$, and $\mathbf{Z}_{L_2} = j\omega L_2$.

Also, voltage division gives

$$\mathbf{V}_o = \frac{\mathbf{V}_1 R}{R + j\omega L_2}\,.$$

Solving for the transfer function gives

$$\mathbf{H} = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{-\omega^2 R L_1 C}{R(1-\omega^2 L_1 C) + j\omega(L_1 + L_2 - \omega^2 L_1 L_2 C)}\,.$$

(b) We need to rationalize the expression for $\mathbf{H}$:

$$\begin{aligned}
\mathbf{H} &= \frac{-\omega^2 R L_1 C}{R(1-\omega^2 L_1 C) + j\omega(L_1 + L_2 - \omega^2 L_1 L_2 C)} \\
&\quad \times \frac{R(1-\omega^2 L_1 C) - j\omega(L_1 + L_2 - \omega^2 L_1 L_2 C)}{R(1-\omega^2 L_1 C) - j\omega(L_1 + L_2 - \omega^2 L_1 L_2 C)} \\
&= \frac{-\omega^2 R^2 L_1 C(1-\omega^2 L_1 C) + j\omega^3 R L_1 C(L_1 + L_2 - \omega^2 L_1 L_2 C)}{R^2(1-\omega^2 L_1 C)^2 + \omega^2(L_1 + L_2 - \omega^2 L_1 L_2 C)^2}\,.
\end{aligned}$$

The imaginary part of $\mathbf{H}$ is zero if $\omega = 0$ (trivial solution) or if

$$\omega_0 = \sqrt{\frac{L_1 + L_2}{L_1 L_2 C}}\,.$$