

## EECS 215 Winter 2005 Midterm 2

Name: \_\_\_\_\_

Lecture Section (circle one): McAfee Terry

### Rules:

1. One (1) 8.5x11" note sheet allowed. No other information aids allowed.
2. A formulae sheet is provided on the back of this exam and can be removed if desired. No other pages should be removed.
3. DO NOT UNSTAPLE THE PAGES OF THIS EXAM.
4. TURN IN ALL PAGES EXCEPT THE FORMULAE SHEET..
5. Calculators Needed and Allowed
6. Work to be done in Exam booklet.
7. **DO NOT WRITE ON THE BACK OF PAGES.**
8. **Exam given under CoE Honor Code**
9. Show your work and *briefly* explain major steps to maximize partial credit. (ex:  $i_3 = i_1 + i_2$ , node A, KCL). **NO CREDIT WILL BE GIVEN IF NO WORK IS SHOWN.**
10. *WRITE YOUR FINAL ANSWERS IN THE AREAS PROVIDED*

This Exam Contains

4 problems over 15 pages (including workspace& formulae page).

**Sign the College of Engineering Honor Code Below (NO credit will be given for the exam without a signed pledge):**

**I have neither given nor received aid on this examination.**

**Signed:** \_\_\_\_\_

Do not write on this page below this line – Instructional Staff Use Only!

[      ] Prob 1

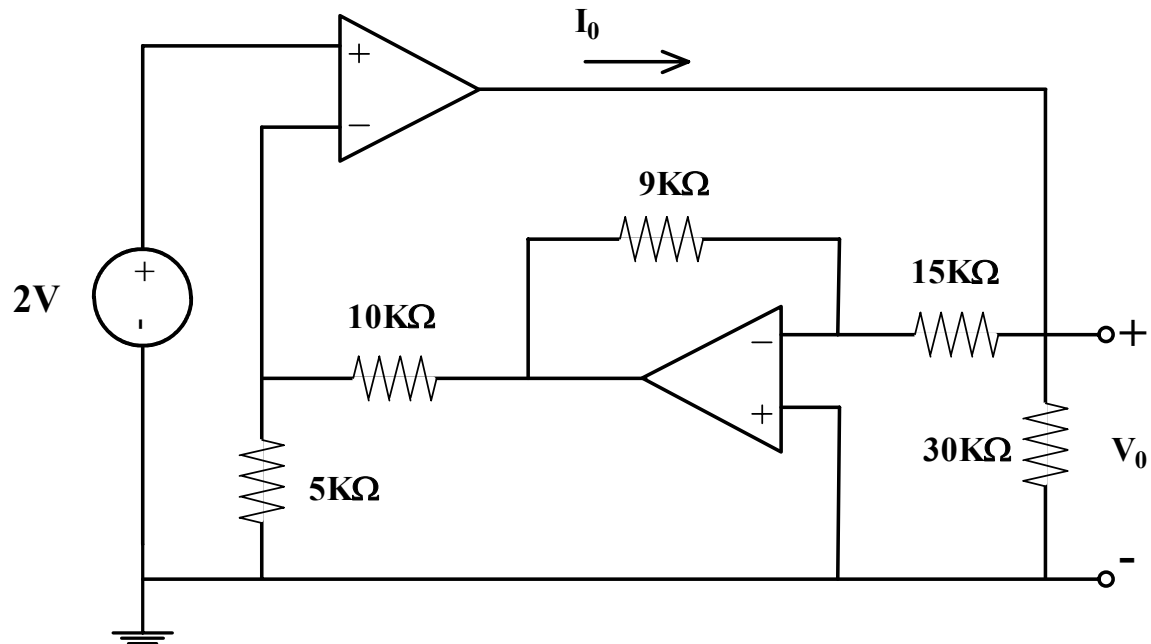
[      ] Prob 3

[      ] Prob 2

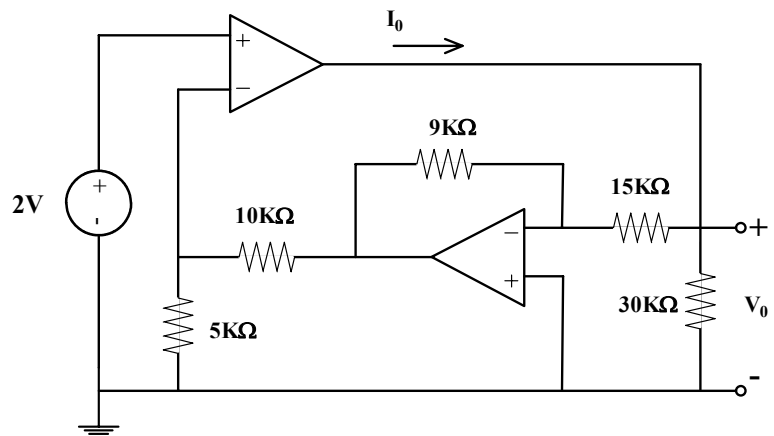
[      ] Prob 4

### **Problem 1: Op-Amps (15 points total)**

For the circuit shown below, find  $V_0$  and  $I_0$ . You may assume ideal opamps under negative feedback conditions.

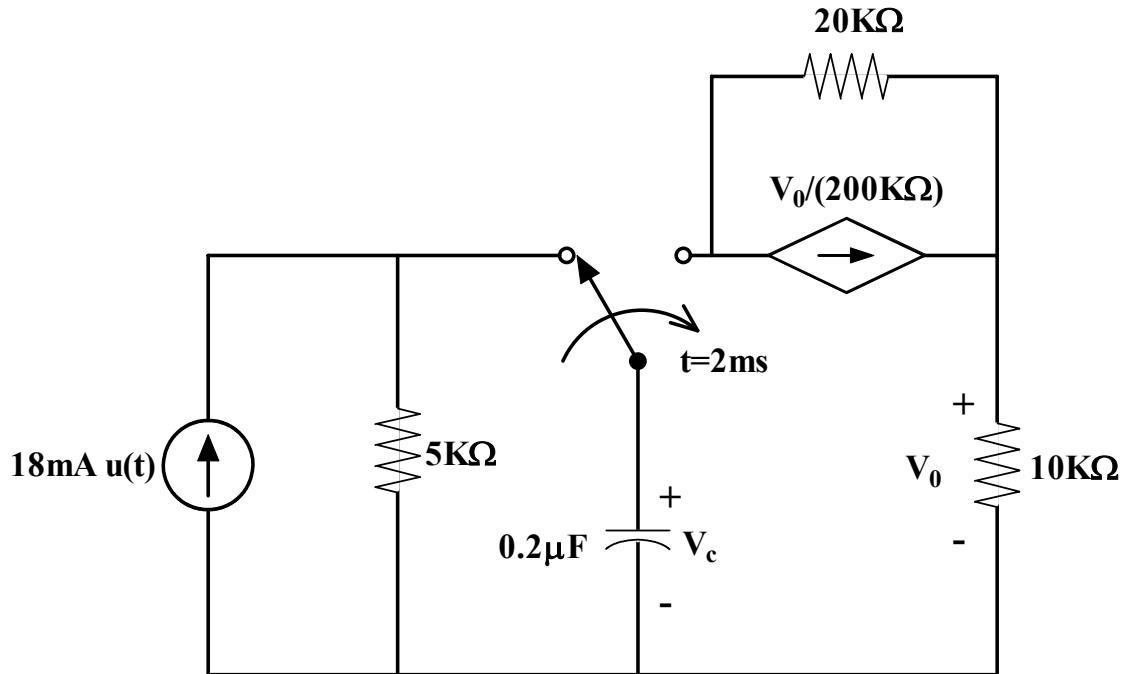


additional workspace for problem 1



## Problem 2: First Order Circuits (25 points)

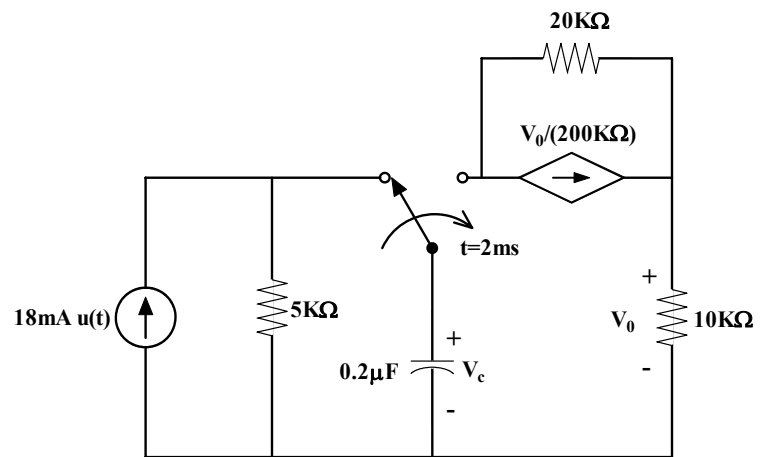
For the circuit shown below, find  $v_c(t)$  for  $t \geq 0$ . Note that the independent current source is  $(18\text{mA})u(t)$  and the switch is toggled at  $t=2\text{ms}$ .



a) Find  $V_c(t)$  for  $t=0$  to  $2\text{ms}$ .

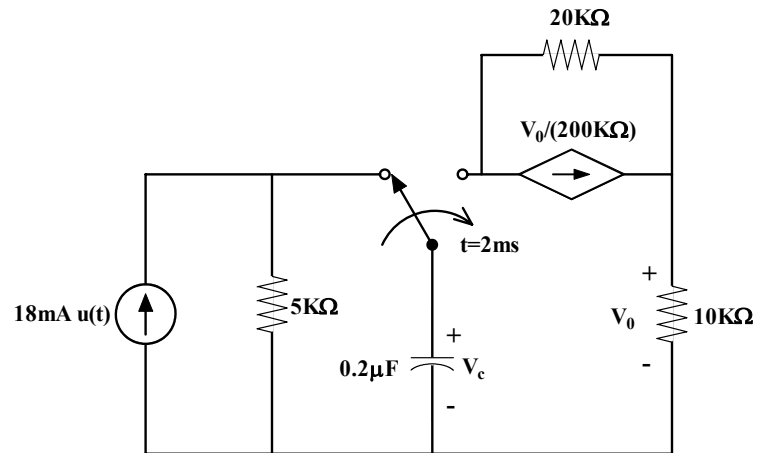
$V_c(t) =$  \_\_\_\_\_  $0 \leq t \leq 2\text{ms}$

Additional workspace for 2(a)

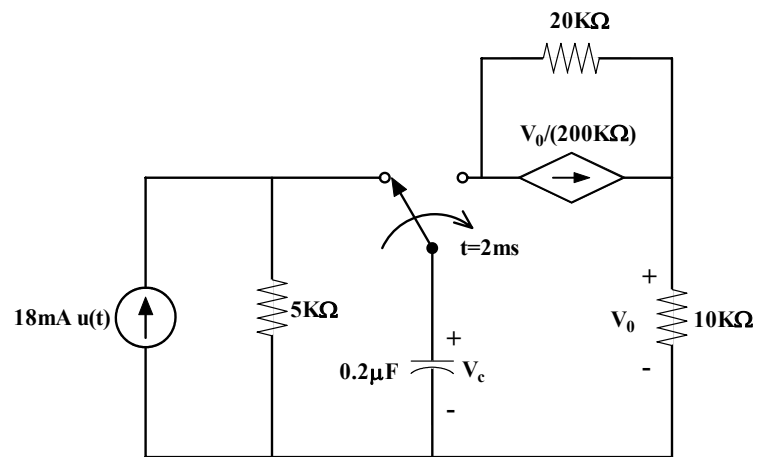


b) Find  $V_c(t)$  for  $t > 2\text{ms}$

$V_c(t) = \underline{\hspace{10cm}} \quad t \geq 2\text{ms}$

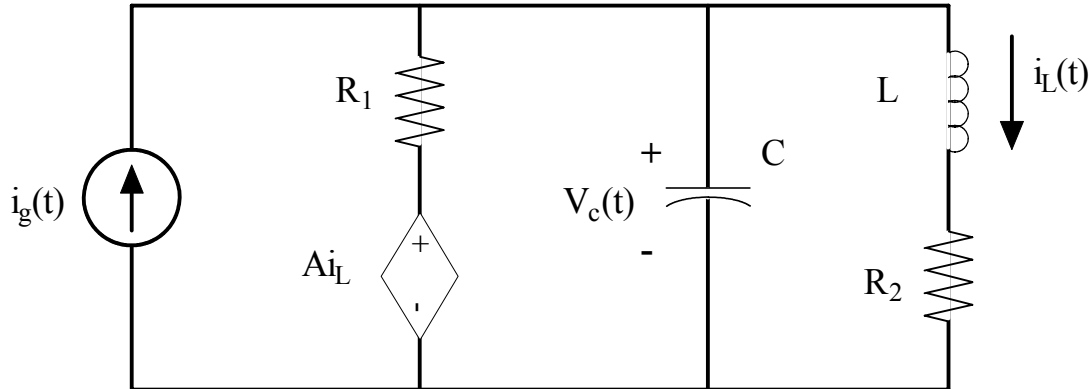


additional workspace for 2(b)



### **Problem 3: Second Order Circuits (35 points)**

Problem has parts (a) & (b). These two parts can be done independently of each other.



- a) For the circuit picture above, find the differential equation that relates  $i_L(t)$  to  $i_g(t)$ .

Write the equation in one of the standard forms -  $\frac{d^2 i_L}{dt^2} + A \frac{di_L}{dt} + B i_L = \text{function}(i_g)$  or

$D \frac{d^2 i_L}{dt^2} + F \frac{di_L}{dt} + i_L = \text{function}(i_g)$ .  $V_c$  must be the only unknown (assuming  $i_g(t)$  is

known). You may use KVL/KCL/time domain methods or s-domain, but you must clearly show your work to receive full or partial credit. **Warning:** Attempts to mix time-domain and s-domain approaches are likely to result in zero credit.

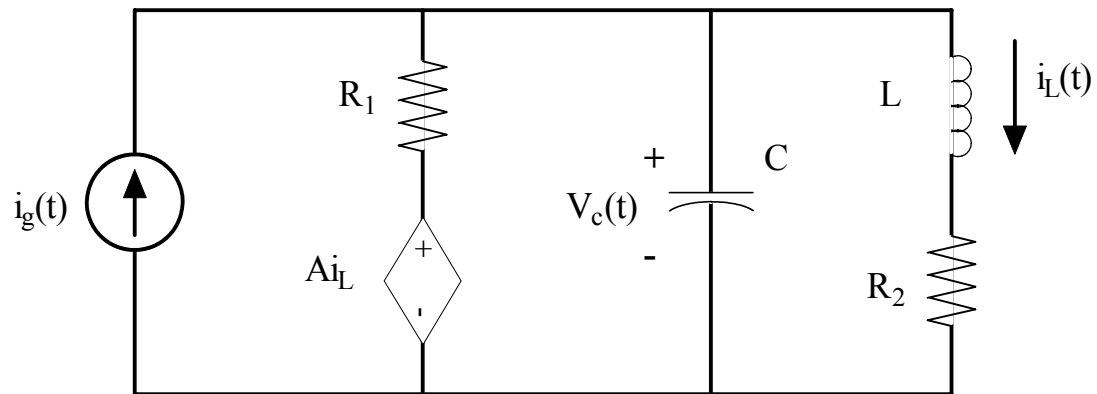
**Differential Equation:**

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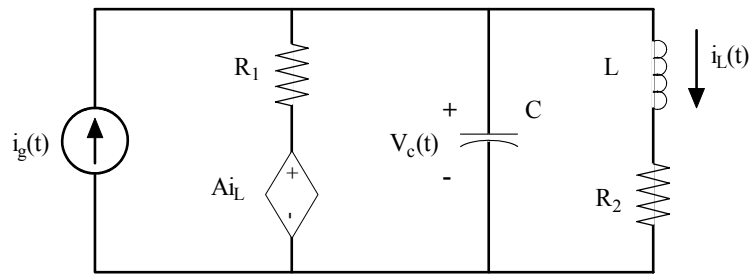
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Workspace for 3(a)



Workspace for 3(a)



- b) Assume that the source had the following behavior:  $i_g(t) = i_a + i_b u(t)$ , where  $i_a$  &  $i_b$  are constants. Find  $i_L(t)$  &  $V_L(t)$  for  $t=0^-, 0^+$ , and  $t \rightarrow \infty$ .

$i_L(0^-) =$  \_\_\_\_\_

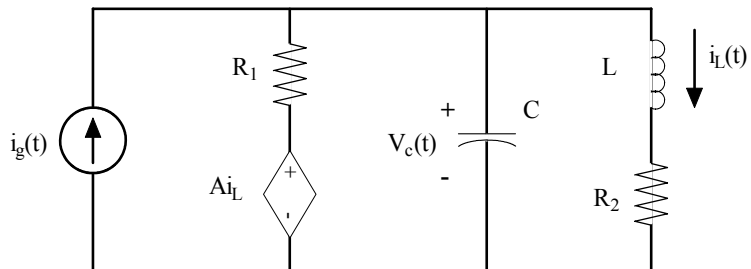
$V_L(0^-) =$  \_\_\_\_\_

$i_L(0^+) =$  \_\_\_\_\_

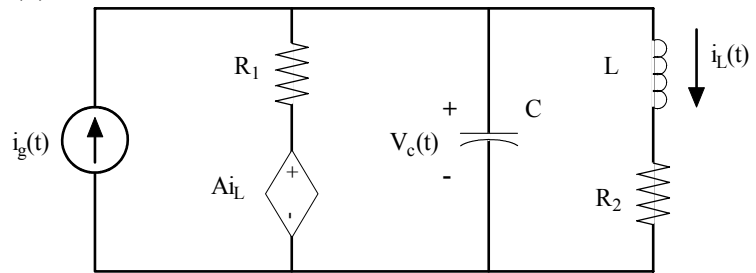
$V_L(0^+) =$  \_\_\_\_\_

$i_L(\infty) =$  \_\_\_\_\_

$V_L(\infty) =$  \_\_\_\_\_



additional workspace for 3(b)



### **Problem 4: Second Order Circuits (25 points)**

Using the same circuit as problem 3, assume we have the following component values:

$$R_1 = 1K\Omega \quad R_2 = 5\Omega \quad A = 996\Omega \quad L = 1mH \quad C = 1\mu F$$

with these values, the differential equation becomes:

$$\left(\frac{1}{9} \times 10^{-6} s\right) \frac{d^2 i_L}{dt^2} + \left(\frac{2}{3} \times 10^{-4} s\right) \frac{di_L}{dt} + i_L = i_g$$

or equivalently:

$$\frac{d^2 i_L}{dt^2} + (6 \times 10^3 s^{-1}) \frac{di_L}{dt} + (9 \times 10^6 s^{-1}) i_L = i_g$$

where  $s$  here is the unit seconds

- a) Find the natural (source-free/homogeneous) solution for this case and name the damping type.

**Damping type (circle only one)**

**Underdamped**

**Critically Damped**

**Overdamped**

**$i_{L,n}(t) =$**  \_\_\_\_\_

- b) Assuming  $i_g(t) = [9u(t) + 81]mA$ , find the complete solution (with no unknowns) for  $i_L(t)$ .

$i_L(t) =$  \_\_\_\_\_

## Formulae

*General Second Order Equation :*

$$\frac{\partial^2 y}{\partial t^2} + 2\alpha \frac{\partial y}{\partial t} + \omega_0^2 y = f(t) \quad \text{or} \quad \omega_0^{-2} \frac{\partial^2 y}{\partial t^2} + 2\alpha \omega_0^{-2} \frac{\partial y}{\partial t} + y = p(t)$$

*Natural (Source Free, Homogeneous) Part :*

$$\frac{\partial^2 y}{\partial t^2} + 2\alpha \frac{\partial y}{\partial t} + \omega_0^2 y = 0 \quad \text{or} \quad \omega_0^{-2} \frac{\partial^2 y}{\partial t^2} + 2\alpha \omega_0^{-2} \frac{\partial y}{\partial t} + y = 0$$

*Trial Solution to natural equation :*

$$y = Ae^{st}$$

*Result :*

$$s^2 + 2\alpha s + \omega_0^2 = 0 \quad \text{or} \quad \omega_0^{-2} s^2 + 2\alpha \omega_0^{-2} s + 1 = 0 \quad \text{Characteristic Equation}$$

$$s_{1,2} = -\alpha \pm [\alpha^2 - \omega_0^2]^{1/2} \quad \text{Time Constants}$$

*Three Possibilities for natural solutions :*

$\alpha > \omega_0$  Overdamped Response

$s_{1,2}$  are real numbers (negative)

$$y_n(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$\alpha < \omega_0$  Underdamped Response

$s_{1,2}$  are complex numbers

$$y_n(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$$s_{1,2} = -\alpha \pm [\alpha^2 - \omega_0^2]^{1/2} = -\alpha \pm j\omega_d$$

$$\omega_d = [\omega_0^2 - \alpha^2]^{1/2} \quad \text{damped frequency of oscillation}$$

$$y_n(t) = [B_1 \cos(\omega_d t) + B_2 \sin(\omega_d t)] e^{-\alpha t}$$

$\alpha = \omega_0$  Critically Damped Response

$s_1 = s_2 = -\alpha$  a negative real number

$$y_n(t) = A_1 e^{-\alpha t} + A_2 t e^{-\alpha t} = (A_1 + A_2 t) e^{-\alpha t}$$