

## EECS 215 Winter 2004 Midterm 2

Name: \_\_\_\_\_

Lecture Section \_\_\_\_\_

### Rules:

1. 6-7:30 PM Monday, March 22, 2004 and 2-3:30 PM Monday
2. Closed Book, Closed Notes, etc.
3. A formulae sheet is provided on the back of this exam and can be removed if desired. No other pages should be removed.
4. Calculators Needed and Allowed
5. Work to be done in Exam booklet.
6. **DO NOT WRITE ON THE BACK OF PAGES.**
7. **Exam given under CoE Honor Code**
8. Show your work and *briefly* explain major steps to maximize partial credit. (ex:  $i_3 = i_1 + i_2$ , node A, KCL). **NO CREDIT WILL BE GIVEN IF NO WORK IS SHOWN.**
9. *WRITE YOUR FINAL ANSWERS IN THE AREAS PROVIDED*

This Exam Contains

4 problems over 19 pages (including workspace & formulae page).

**Sign the College of Engineering Honor Code Below (NO credit will be given for the exam without a signed pledge):**

***I have neither given nor received aid on this examination.***

**Signed:** \_\_\_\_\_

Do not write on this page below this line – Instructional Staff Use Only!

[     ] Prob 1

[     ] Prob 3

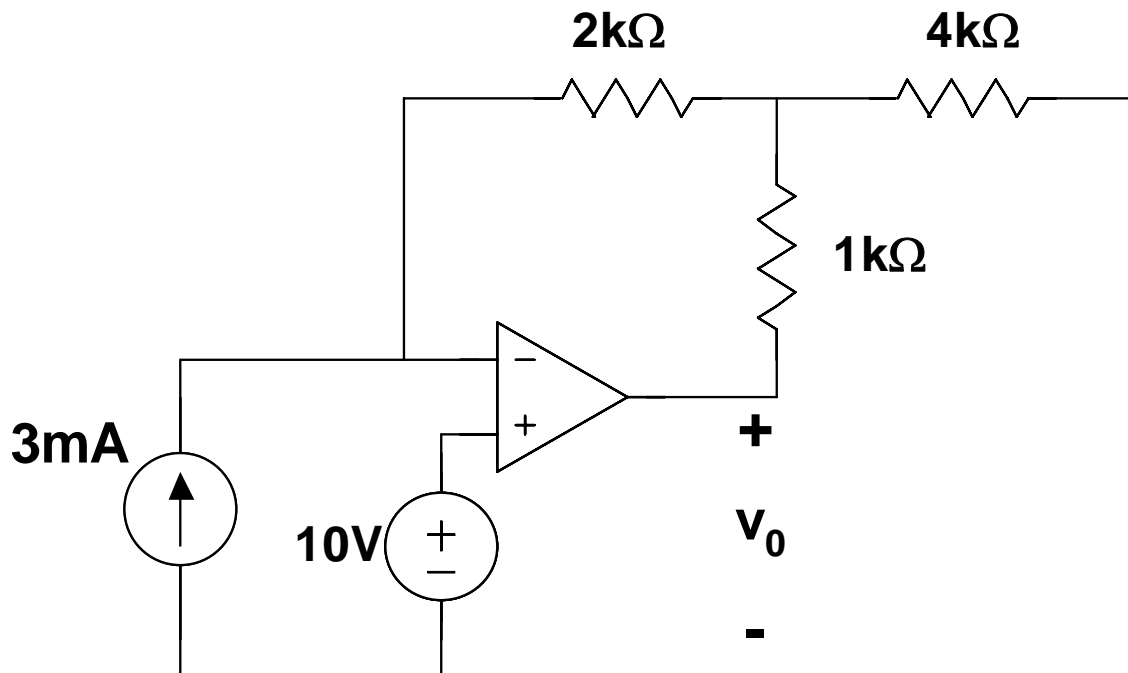
[     ] Prob 2

[     ] Prob 4

### **Problem 1: Op-Amps (20 points total)**

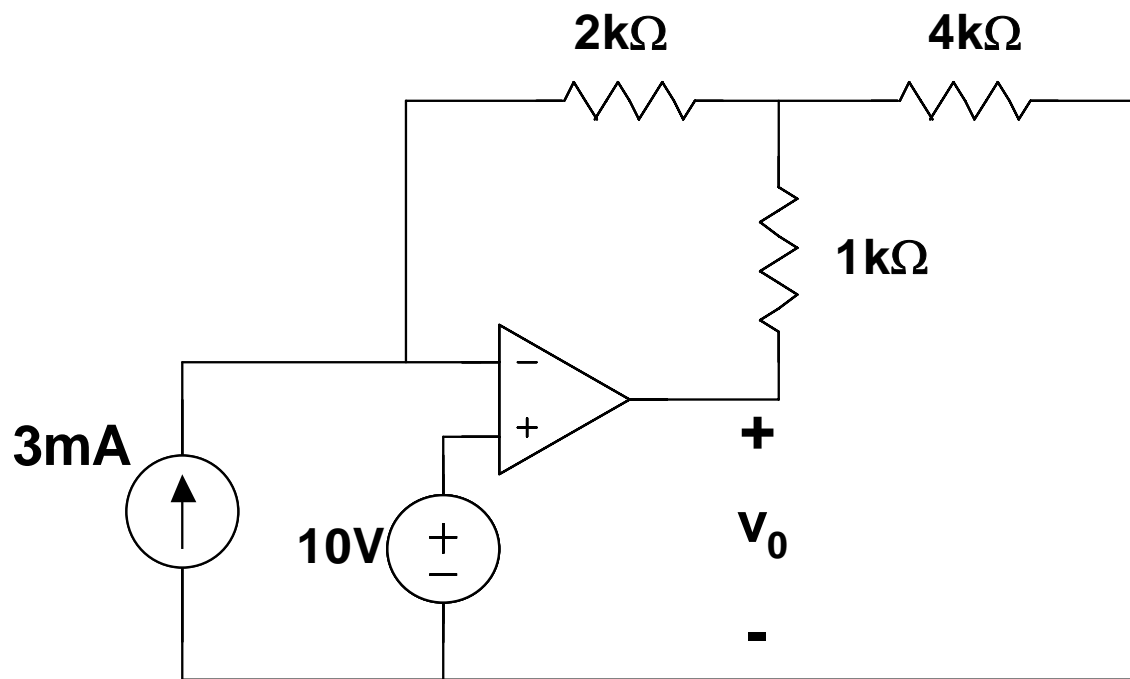
Problem has parts a & b. You may draw directly on the circuits if you want, but be sure to clearly explain your reasoning to qualify for partial credit.

a) For the circuit below, what is  $v_0$ ? (10 points)

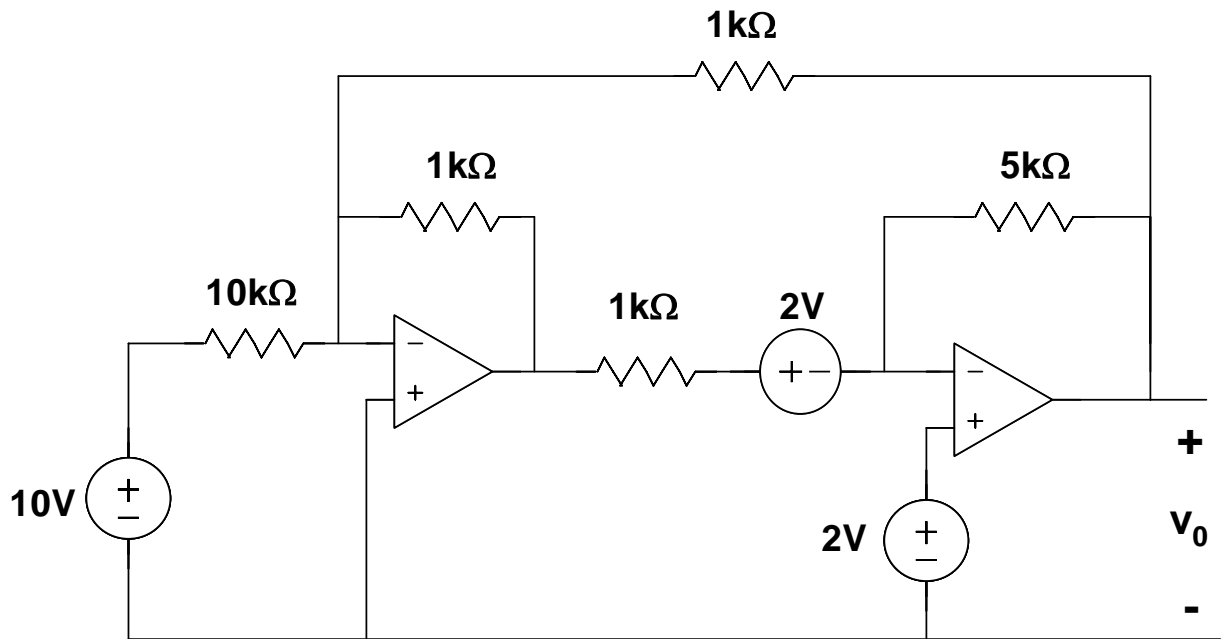


$$v_0 = \underline{\hspace{2cm}} \text{ V}$$

additional space for 1(a) if needed

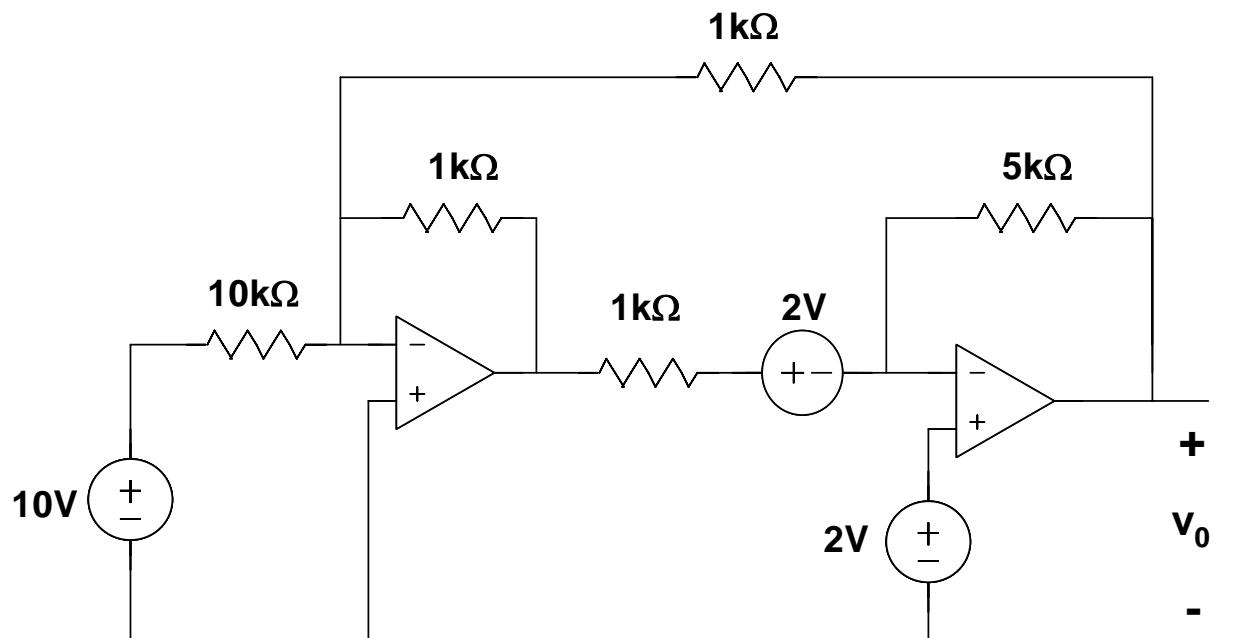


b) For the circuit below, what is  $v_0$ ? (10 points)



$v_0 = \underline{\hspace{2cm}} \text{ V}$

additional workspace for 1(b) if needed



## **Problem 2: First Order Circuits (30 points total)**

Problem has only 1 part (all quantities in the box below)

For the circuit below, find the following quantities (box below). Show your work clearly.

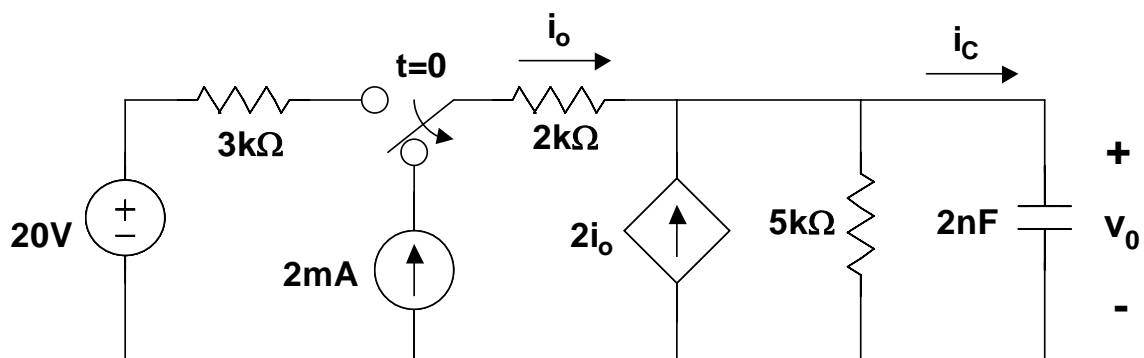
**No credit will be given without clear supporting work.**

$$v_0(0^-) = \underline{\hspace{10cm}} \text{ V}$$

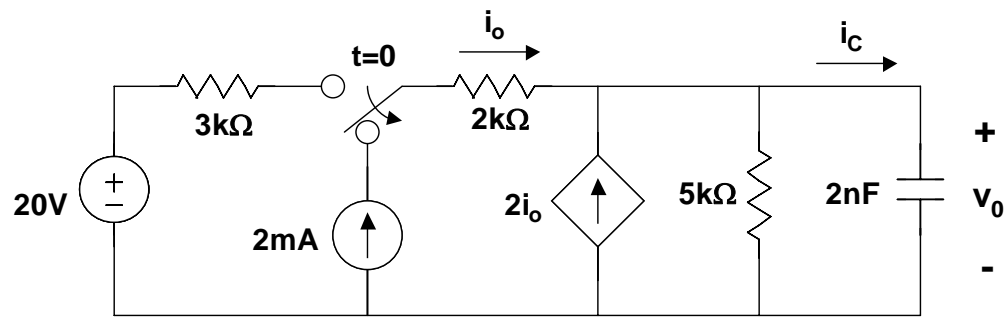
$$v_0(0^+) = \underline{\hspace{10cm}} \text{ V}$$

$$v_{0f} = v_C(t \rightarrow \infty) = \underline{\hspace{10cm}} \text{ V}$$

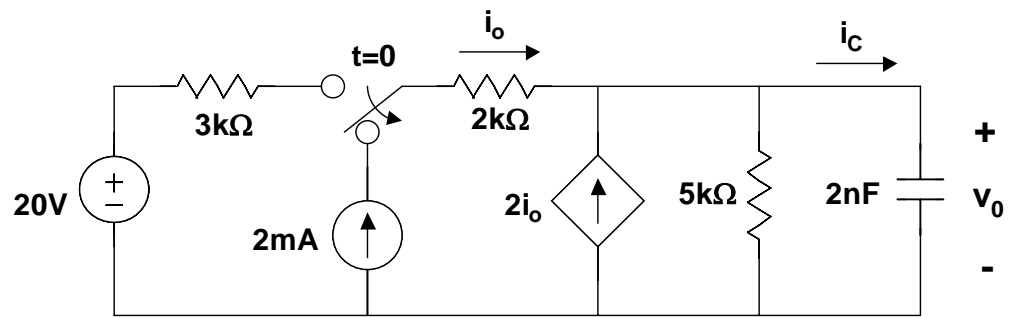
$$v_0(t) = \underline{\hspace{10cm}} \text{ A (for } t > 0)$$



Additional Workspace for problem 2



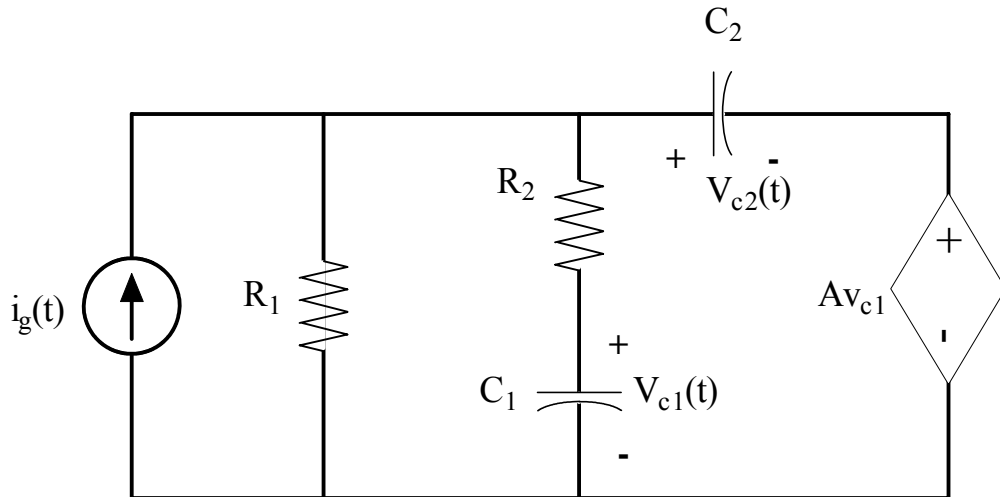
Workspace for problem 2





### **Problem 3: Second Order Circuits (20 points total)**

Problem has only 1 part



For the circuit picture above, find the differential equation that relates  $V_{c1}(t)$  to  $i_g(t)$ .

Write the equation in standard form -  $\frac{d^2 V_{c1}}{dt^2} + A \frac{dV_{c1}}{dt} + B V_{c1} = \text{function}(i_g)$ .  $V_{c1}$  must be

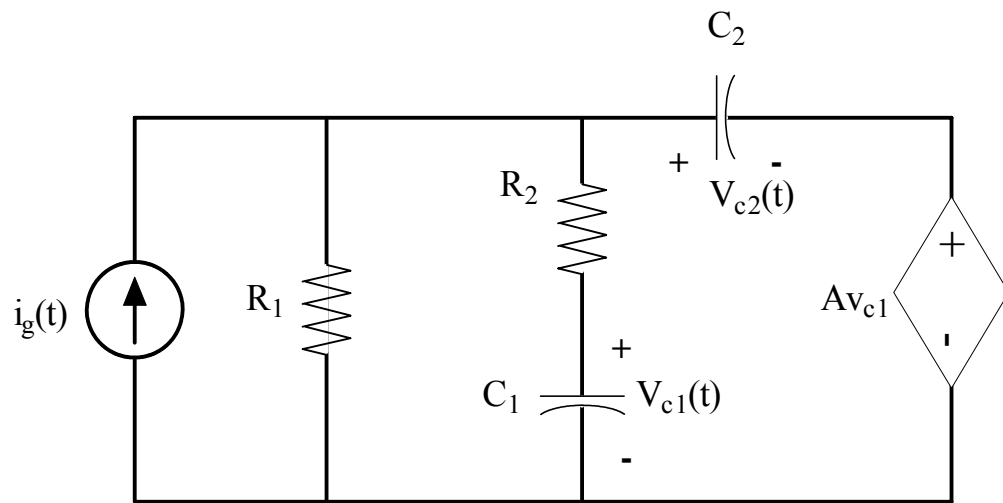
the only unknown (assuming  $i_g(t)$  is known). You may use KVL/KCL/time domain methods or s-domain, but you must clearly show your work to receive full or partial credit. **Warning:** Attempts to mix time-domain and s-domain approaches are likely to result in zero credit.

**Differential Equation:**

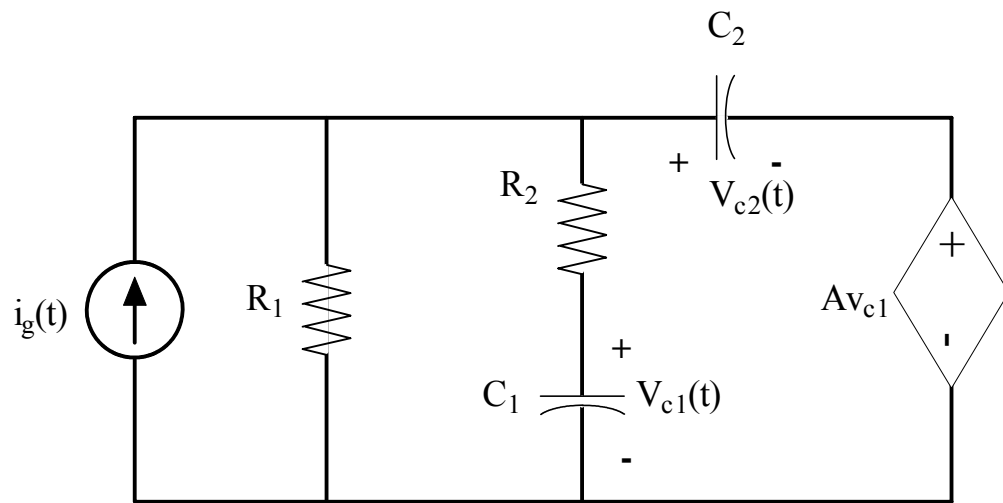
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Workspace for problem 3

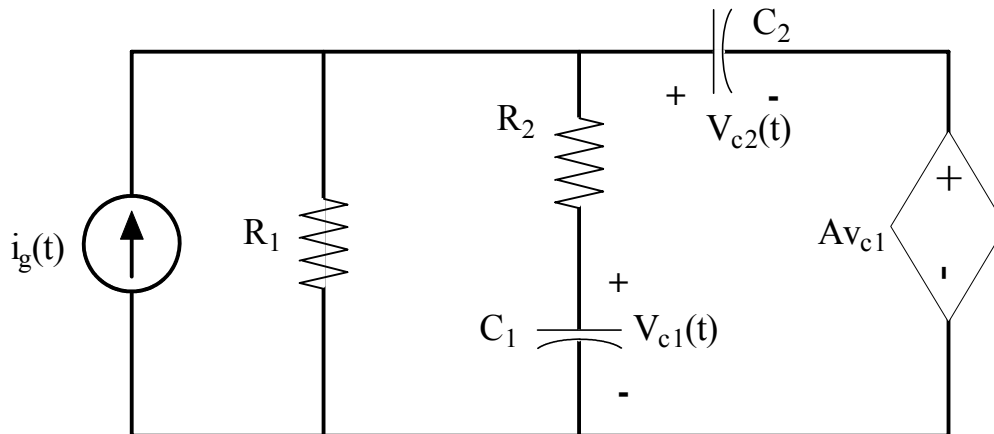


additional workspace for problem 3



## Problem 4: Second Order Circuits (30 points total)

Problem has parts a, b, c, and d



Now suppose we have the circuit above with the following component values:

$$\mathbf{R_1 = 1K\Omega \quad R_2 = 5K\Omega \quad A = 0.7 \quad C_1 = 1nF \quad C_2 = 5nF}$$

$$\text{and we will let } i_g(t) = [3mA]u(t)$$

This results in a differential equation for this circuit (for  $t > 0$ ):

$$(2.5 \times 10^{-11} s^2) \frac{d^2 V_{c1}}{dt^2} + (7.5 \times 10^{-6} s) \frac{dV_{c1}}{dt} + V_{c1} = 3V$$

where  $s$  denotes seconds, not the Laplace differential operator

a) Find the quantities below. Show your work on the following 2 pages (5 pts)

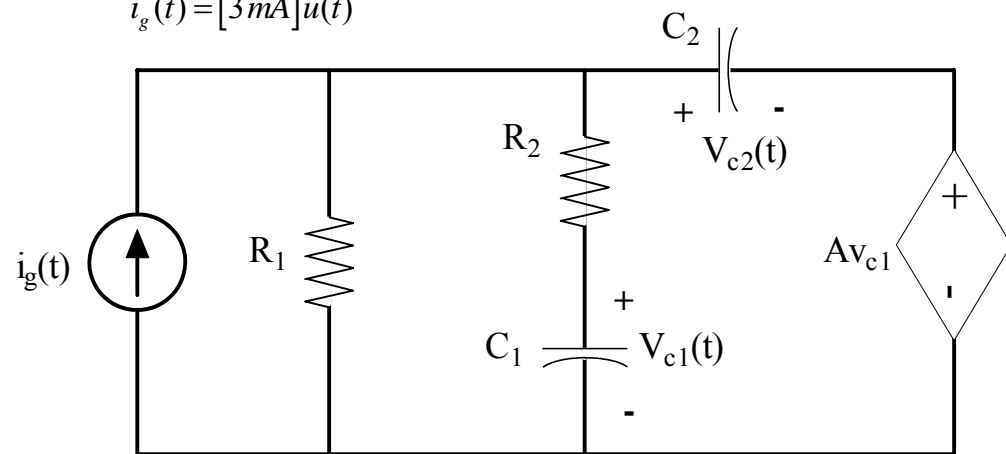
$v_{c1}(0^-) =$	_____	V
$v_{c2}(0^-) =$	_____	V
$i_{c1}(0^-) =$	_____	A
$i_{c2}(0^-) =$	_____	A

$v_{c1}(0^+) =$	_____	V
$v_{c2}(0^+) =$	_____	V
$i_{c1}(0^+) =$	_____	A
$i_{c2}(0^+) =$	_____	A

Workspace for (a)

$$\mathbf{R}_1 = 1K\Omega \quad \mathbf{R}_2 = 5K\Omega \quad A = 0.7 \quad C_1 = 1nF \quad C_2 = 5nF$$

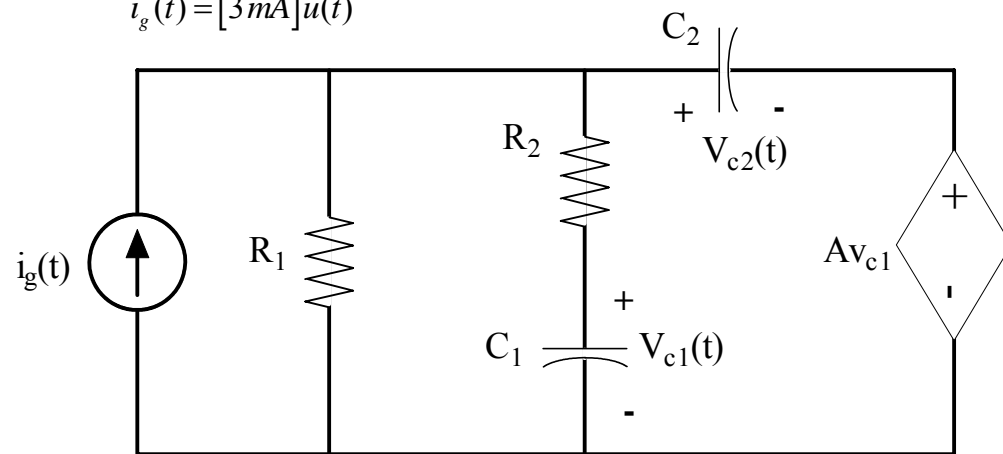
$$i_g(t) = [3mA]u(t)$$



additional workspace for (a) if needed

$$\mathbf{R_1 = 1K\Omega \quad R_2 = 5K\Omega \quad A = 0.7 \quad C_1 = 1nF \quad C_2 = 5nF}$$

$$i_g(t) = [3mA]u(t)$$



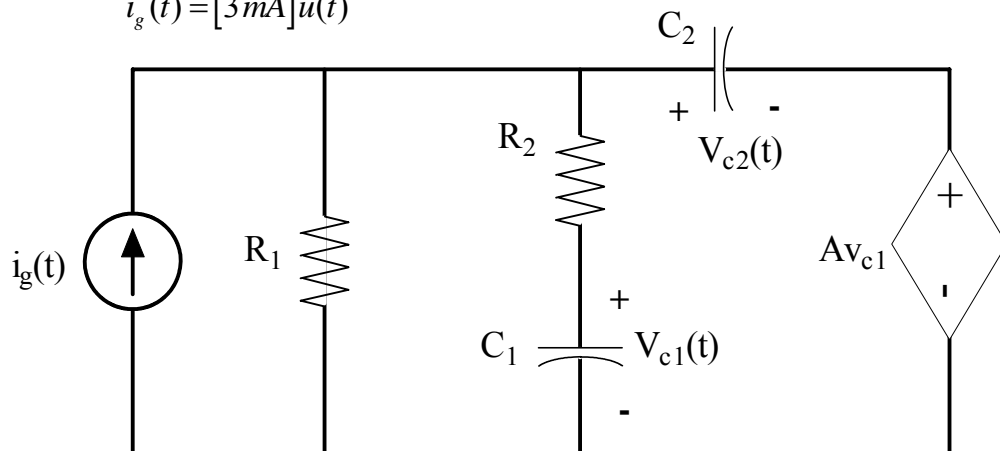
b) Find  $v_{c1}(\infty)$ ,  $v_{c2}(\infty)$ ,  $i_{c1}(\infty)$ , and  $i_{c2}(\infty^+)$ . (10 pts)

$v_{c1}(\infty) =$	_____	V
$v_{c2}(\infty) =$	_____	V
$i_{c1}(\infty) =$	_____	A
$i_{c2}(\infty) =$	_____	A

Workspace for (b)

$$\mathbf{R_1 = 1K\Omega \quad R_2 = 5K\Omega \quad A = 0.7 \quad C_1 = 1nF \quad C_2 = 5nF}$$

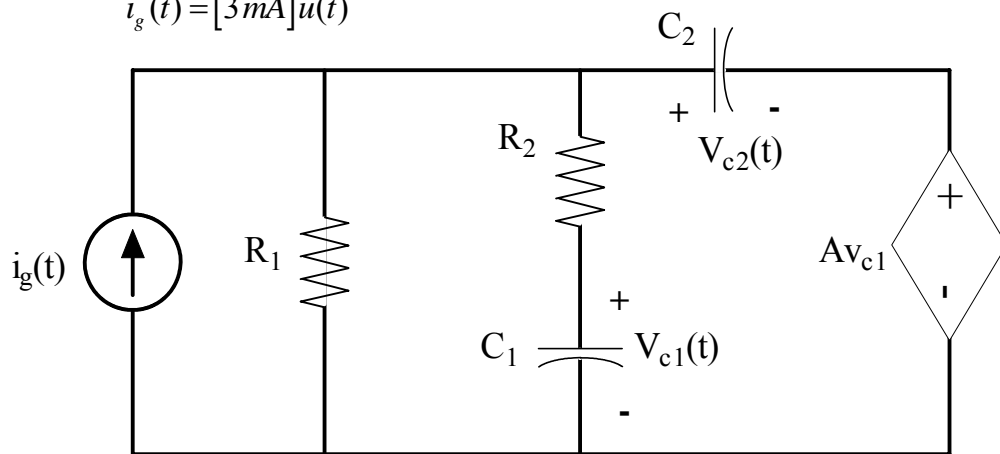
$$i_g(t) = [3mA]u(t)$$



additional workspace for (b) if needed

$$\mathbf{R}_1 = 1K\Omega \quad \mathbf{R}_2 = 5K\Omega \quad A = 0.7 \quad C_1 = 1nF \quad C_2 = 5nF$$

$$i_g(t) = [3mA]u(t)$$





- c) Find the natural solution for  $V_{c1}$  (with 2 and only 2 unknown coefficients). (5 pts)

$$v_{c1,n}(t) = \frac{V}{\dots}$$

(with 2 and only 2 unknown coefficients)

workspace for (c):

- d) Match the initial conditions to the complete solution to find the final numerical solution for  $v_{c1}(t)$  for this problem. (10 pts)

$v_{c1}(t) =$	_____	<u>V</u>
_____		

workspace for (d):

## Formulae

### *General Second Order Equation*

$$\frac{\partial^2 y}{\partial t^2} + 2\alpha \frac{\partial y}{\partial t} + \omega_0^2 y = f(t) \quad \text{or} \quad \omega_0^{-2} \frac{\partial^2 y}{\partial t^2} + 2\alpha \omega_0^{-2} \frac{\partial y}{\partial t} + y = p(t)$$

### *Natural (Source Free) Part :*

$$\frac{\partial^2 y}{\partial t^2} + 2\alpha \frac{\partial y}{\partial t} + \omega_0^2 y = 0 \quad \text{or} \quad \omega_0^{-2} \frac{\partial^2 y}{\partial t^2} + 2\alpha \omega_0^{-2} \frac{\partial y}{\partial t} + y = 0$$

### *Trial Solution :*

$$y = Ae^{st}$$

### *Result :*

$$s^2 + 2\alpha s + \omega_0^2 = 0 \quad \text{or} \quad \omega_0^{-2} s^2 + 2\alpha \omega_0^{-2} s + 1 = 0 \quad \text{Characteristic Equation}$$

$$s_{1,2} = -\alpha \pm [\alpha^2 - \omega_0^2]^{1/2} \quad \text{Time Constants}$$

### *Three Possibilities :*

#### $\alpha > \omega_0$ Overdamped Response

$s_{1,2}$  are real numbers (negative)

$$y_n(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

#### $\alpha < \omega_0$ Underdamped Response

$s_{1,2}$  are complex numbers

$$y_n(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$$s_{1,2} = -\alpha \pm [\alpha^2 - \omega_0^2]^{1/2} = -\alpha \pm j\omega_d$$

$$\omega_d = [\omega_0^2 - \alpha^2]^{1/2} \quad \text{damped frequency of oscillation}$$

$$y_n(t) = [B_1 \cos(\omega_d t) + B_2 \sin(\omega_d t)] e^{-\alpha t}$$

#### $\alpha = \omega_0$ Critically Damped Response

$s_1 = s_2 = -\alpha$  a negative real number

$$y_n(t) = A_1 e^{-\alpha t} + A_2 t e^{-\alpha t} = (A_1 + A_2 t) e^{-\alpha t}$$