

## HW 11 Solutions

1. The first transistors were fabricated using germanium (Ge) as the semiconductor material. Ge has a much smaller bandgap energy ( $E_{g,Ge} = 0.66$  eV) than silicon ( $E_{g,Si} = 1.12$  eV). Would you expect the intrinsic carrier concentration in a Ge sample to be larger or smaller than that in a Si sample maintained at the same temperature? Explain your answer qualitatively, *i.e.* without resorting to any equations.

The lower melting point of Ge indicates that the covalent bonds are weaker in Ge. Therefore, it is easier to free an electron from a covalent bond in Ge, to generate an electron-hole pair. Thus, at a given temperature, the intrinsic carrier concentration in a Ge sample is larger than that in a Si sample.

2. Consider a Si sample maintained under thermal equilibrium conditions at  $T = 300\text{K}$ , doped with arsenic at a concentration of  $3 \times 10^{16} \text{ cm}^{-3}$ .

- a) Is this material n-type or p-type? What are the majority and minority carrier concentrations?
- b) Suppose this sample is additionally doped with phosphorus at a concentration of  $10^{16} \text{ cm}^{-3}$ . How will the carrier concentrations change? Explain qualitatively why this is the case.

### Solution:

- a) The Si sample is N-type doped. From lecture,

$$n \text{ (Majority Carrier Concentration)} \approx N_D = 3 \cdot 10^{16} \text{ cm}^{-3},$$

$$p \text{ (Minority Carrier Concentration)} \approx n_i^2 / N_D = (10^{10})^2 / 3 \cdot 10^{16} = 0.333 \cdot 10^4 \text{ cm}^{-3}.$$

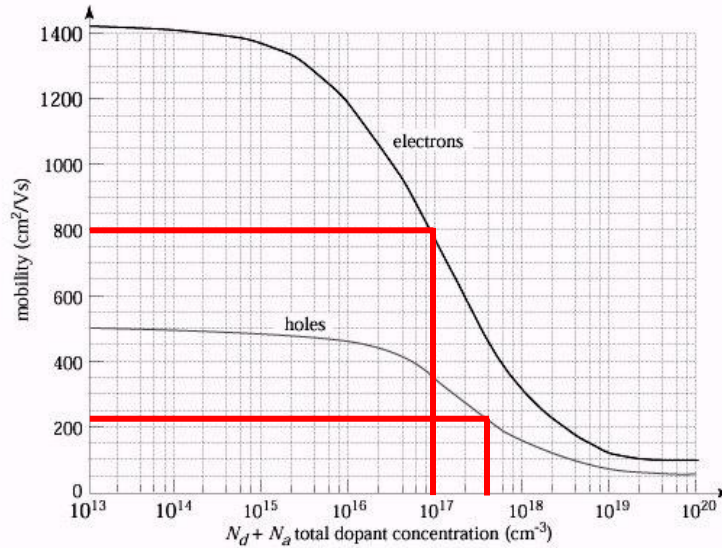
- b) Phosphorous is an n-type dopant. Therefore, adding arsenic increases the n-type doping concentration and reduces the p-type concentration. Simply add the existing n-type carrier concentration (arsenic) to the new concentration (phosphorous).

$$n \text{ (Majority Carrier Concentration)} \approx N_{D,As} + N_{D,P} = 3 \cdot 10^{16} \text{ cm}^{-3} + 10^{16} \text{ cm}^{-3} = 4 \cdot 10^{16} \text{ cm}^{-3},$$

$$p \text{ (Minority Carrier Concentration)} \approx n_i^2 / N_D = (10^{10})^2 / 4 \cdot 10^{16} = 0.25 \cdot 10^4 \text{ cm}^{-3}.$$

3. The electron and hole mobilities in Si at 300K are plotted as a function of **total** dopant concentration below. Consider a silicon sample maintained at 300K, doped with arsenic at a concentration of  $10^{17} \text{ cm}^{-3}$ .

- If an electric field equal to  $0.2 \text{ V}/\mu\text{m}$  is applied, what are the drift velocities of the electrons and holes?
- Calculate the resistivity of this sample.
- How will the resistivity of this sample change if it were to be additionally doped with boron to a concentration of  $2 \times 10^{17} \text{ cm}^{-3}$ ?



### Solution:

- The electron velocity is given by

$$\mathbf{v}_e = -\mu_n \mathbf{E} \text{ [cm/s]},$$

where  $\mu_n$  is the electron mobility in  $\text{cm}^2/\text{V}\cdot\text{s}$ , and  $\mathbf{E}$  is the applied electric field in  $\text{V}/\text{cm}$ .

In addition, the hole velocity is given by

$$\mathbf{v}_h = \mu_p \mathbf{E} \text{ [cm/s]},$$

where  $\mu_p$  is the hole mobility in  $\text{cm}^2/\text{V}\cdot\text{s}$ , and  $\mathbf{E}$  is the applied electric field in  $\text{V}/\text{cm}$ .

From the plot of mobility vs.  $N_D + N_A$  total dopant concentration, we can extract  $\mu_n \approx 800 \text{ cm}^2/\text{V}\cdot\text{s}$  and  $\mu_p \approx 350 \text{ cm}^2/\text{V}\cdot\text{s}$ .

With  $E = |\mathbf{E}| = 0.2 \text{ V}/\mu\text{m} = 2000 \text{ V}/\text{cm}$ , the electron mobility is calculated to be

$$v_e = |\mathbf{v}_e| = |-\mu_n \mathbf{E}| = \mu_n E = 800 [\text{cm}^2/\text{V}\cdot\text{s}] \cdot 2000 [\text{V}/\text{cm}] = 16.0 \cdot 10^5 [\text{cm/sec}].$$

In addition, given  $E = |\mathbf{E}| = 0.2 \text{ V}/\mu\text{m} = 2000 \text{ V}/\text{cm}$ , the hole mobility is calculated to be

$$v_h = |\mathbf{v}_h| = |\mu_p \mathbf{E}| = \mu_p E = 350 [\text{cm}^2/\text{V}\cdot\text{s}] \cdot 2000 [\text{V}/\text{cm}] = 7 \cdot 10^5 [\text{cm/sec}].$$

- The resistivity of a semiconductor sample is expressed as

$$\rho \equiv \frac{1}{\sigma} = \frac{1}{qp\mu_p + qn\mu_n},$$

where  $q$  is the electric charge in C,  $p$  is the hole concentration in holes/ $\text{cm}^3$ ,  $\mu_p$  is the hole mobility in  $\text{cm}^2/\text{V}\cdot\text{s}$ ,  $n$  is the electron concentration in electrons/ $\text{cm}^3$ , and  $\mu_n$  is the electron mobility in  $\text{cm}^2/\text{V}\cdot\text{s}$ .

At room temperature,

$$\begin{aligned}
q &= 1.6 \cdot 10^{-19} \text{ C}, \\
n &\approx N_D = 10^{17} \text{ cm}^{-3} \\
\mu_p &= 350 \text{ cm}^2/\text{V}\cdot\text{s} \\
p &\approx n_i^2 / N_D = (10^{10})^2 / 10^{17} = 10^3 \text{ cm}^{-3}, \\
\mu_n &= 800 \text{ cm}^2/\text{V}\cdot\text{s}.
\end{aligned}$$

In the given N-type semiconductor sample,

$$p \ll n,$$

leading to

$$qp\mu_p + qn\mu_n \approx qn\mu_n.$$

Thus, the resistivity of the sample is estimated to be

$$\rho \equiv \frac{1}{\sigma} = \frac{1}{qp\mu_p + qn\mu_n} \cong \frac{1}{qn\mu_n} = \frac{1}{1.6 \cdot 10^{-19} \cdot 10^{17} \cdot 800} \cong 0.08 \Omega\text{-cm}.$$

- c) The additional boron doping compensates the arsenic doping such that holes (rather than electrons) become the majority carriers. Although the majority carrier concentration remains the same, the mobility of the majority carriers is much lower ( $\mu_p \approx 200 \text{ cm}^2/\text{V}\cdot\text{s}$  for the compensated sample vs.  $\mu_n \approx 800 \text{ cm}^2/\text{V}\cdot\text{s}$  for the uncompensated sample), so the resistivity of the compensated sample is higher than that of the uncompensated sample.

At room temperature,

$$\begin{aligned}
q &= 1.6 \cdot 10^{-19} \text{ C}, \\
p &\approx N_A - N_D = 2 \cdot 10^{17} - 10^{17} = 10^{17} \text{ cm}^{-3} \\
\mu_p &= 200 \text{ cm}^2/\text{V}\cdot\text{s} \\
n &\approx n_i^2 / (N_A - N_D) = (10^{10})^2 / 10^{17} = 10^3 \text{ cm}^{-3}, \\
\mu_n &= 450 \text{ cm}^2/\text{V}\cdot\text{s}.
\end{aligned}$$

In the compensated P-type semiconductor sample,

$$n \ll p,$$

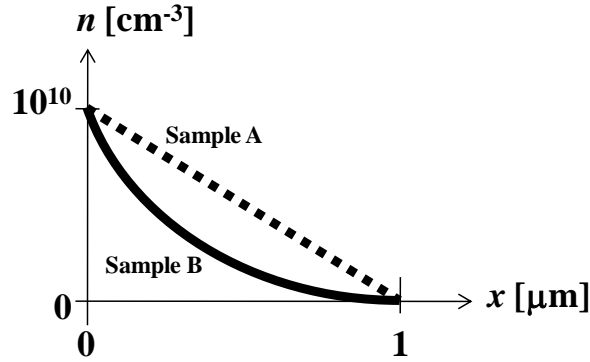
leading to

$$qp\mu_p + qn\mu_n \approx qp\mu_p.$$

Thus, the resistivity of the sample is estimated to be

$$\rho \equiv \frac{1}{\sigma} = \frac{1}{qp\mu_p + qn\mu_n} \cong \frac{1}{qp\mu_p} = \frac{1}{1.6 \cdot 10^{-19} \cdot 10^{17} \cdot 200} \cong 0.31 \Omega\text{-cm}.$$

4. Suppose electrons are continuously injected (at  $x = 0$ ) into two p-type silicon samples maintained at 300K (each with  $N_A = 10^{17} \text{ cm}^{-3}$ ), resulting in the following electron concentration profiles:



- Indicate the direction ( $+x$  or  $-x$ ) of electron diffusion and the direction of electron current flow.
- In which sample is the electron injection rate larger? Explain briefly.
- Calculate the diffusion current density (in units of  $\text{A/cm}^2$ ) in Sample A.

**Solution:**

- The electrons diffuse from a region of higher concentration to a region of lower concentration, i.e. in the  $+x$  direction. Since electrons are negatively charged, this results in negative current in the  $+x$  direction, i.e. the direction of current flow is in the  $-x$  direction.
- The electron injection rate of Sample B is larger than that of Sample A, since the concentration gradient (tangent) at  $x = 0 \text{ μm}$  is larger.
- The diffusion current density is

$$J_{n,diff} = qD_n \frac{dn}{dx} = q\mu_n \frac{kT}{q} \frac{dn}{dx} [\text{A/cm}^2],$$

where  $q$  is the electric charge in C,  $\mu_n$  is the electron mobility in  $\text{cm}^2/\text{V}\cdot\text{sec}$ ,  $k$  is Boltzmann's constant eV/K,  $T$  is the temperature in K, and  $n$  is the electron carrier concentration in electrons/ $\text{cm}^3$ .

Given

$$q = 1.6 \cdot 10^{-19} \text{ C},$$

$$k = 8.62 \cdot 10^{-5} \text{ eV/K},$$

$$\mu_n \approx 750 \text{ cm}^2/\text{V}\cdot\text{sec} \text{ (extracted from the plot of mobility vs. } N_D + N_A \text{ total dopant concentration),}$$

$$T = 300 \text{ K},$$

$$dn = 0 - 10^{10} = -10^{10} \text{ cm}^{-3},$$

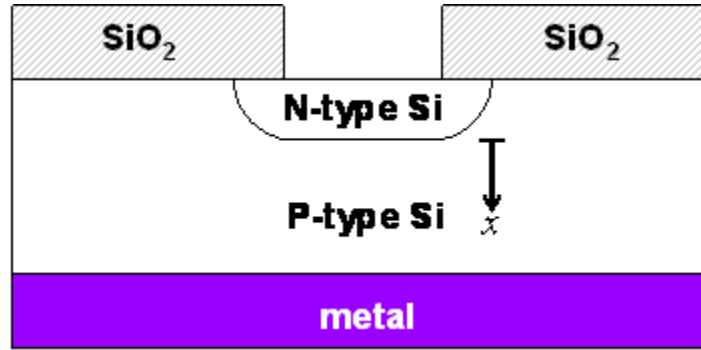
$$dx = 1 - 0 = 1 \text{ μm} = 10^{-4} \text{ cm},$$

$$kT/q = 0.026 \text{ V},$$

the diffusion current density in Sample A is calculated to be

$$J_{n,diff} = qD_n \frac{dn}{dx} = q\mu_n \frac{kT}{q} \frac{dn}{dx} = 1.6 \cdot 10^{-19} \cdot 750 \cdot 0.026 \cdot \frac{-10^{10}}{10^{-4}} \cong 0.3125 [\text{mA/cm}^2].$$

5. Suppose a PN junction is formed by locally introducing arsenic into the surface region of a silicon sample that is uniformly doped with  $3 \times 10^{17} \text{ cm}^{-3}$  boron:



The arsenic concentration is uniformly  $3 \times 10^{18} \text{ cm}^{-3}$  in the N-type region. You may use the depletion approximation.

- Calculate the built-in potential,  $V_0$ .
- Calculate the width of the depletion region,  $W_{\text{dep}}$ .
- Plot (with qualitative accuracy) the charge-density distribution  $\rho(x)$ , the electric-field distribution  $E(x)$ , and potential distribution  $V(x)$ . Indicate the charge density values on the N side and P side, the peak electric field strength, and the total potential dropped across the depletion region, respectively, in your plots. Remember that the depletion width is larger on the more lightly doped side of the PN junction.
- Calculate the junction capacitance,  $C_j$ , in units of  $\text{F/cm}^2$ .

**Solution:**

- The built-in potential is given by

$$V_0 = \frac{kT}{q} \ln \frac{N_A N_D}{n_i^2} \text{ [V]},$$

where  $k$  is Boltzmann's constant in eV/K,  $T$  is the temperature in K,  $q$  is the electric charge in C,  $N_A$  is the acceptor concentration in acceptors/ $\text{cm}^3$ ,  $N_D$  is the donor concentration in donors/ $\text{cm}^3$ , and  $n_i$  is the intrinsic carrier concentration in electrons/ $\text{cm}^3$ .

At room temperature,

$$\begin{aligned} q &= 1.6 \cdot 10^{-19} \text{ C}, \\ k &= 8.62 \cdot 10^{-5} \text{ eV/K}, \\ kT / q &\approx 0.026 \text{ V}, \\ N_A &= 3 \times 10^{17} \text{ cm}^{-3}, \\ N_D &= 3 \times 10^{18} \text{ cm}^{-3}, \\ n_i &\approx 10^{10} \text{ cm}^{-3}. \end{aligned}$$

Substituting these values into the equation for the built-in potential,

$$V_0 = \frac{kT}{q} \ln \frac{N_A N_D}{n_i^2} = \frac{kT}{q} \ln \frac{3 \times 10^{17} \cdot 3 \times 10^{18}}{(10^{10})^2} = \frac{kT}{q} \ln 9 \times 10^{15} = 0.955 \text{ [V]}.$$

$$\text{b) } W_{\text{dep}} = \sqrt{\frac{2\epsilon_{\text{si}}}{q} \left( \frac{1}{N_A} + \frac{1}{N_D} \right) (V_0 - V_D)},$$

where  $\epsilon_{\text{si}}$  is the permittivity of silicon in  $\text{F/cm}$ ,  $q$  is the electric charge in C,  $N_A$  is the acceptor concentration in acceptors/ $\text{cm}^3$ ,  $N_D$  is the donor concentration in donors/ $\text{cm}^3$ ,  $V_0$  is the built-in potential in V, and  $V_D$  is the external voltage in V applied across the diode.

Given

$$\epsilon_{Si} = 11.7\epsilon_0 = 11.7 \cdot 8.854 \cdot 10^{-14} \text{ F/cm} \approx 10^{-12} \text{ F/cm},$$

$$q = 1.6 \cdot 10^{-19} \text{ C},$$

$$N_A \approx 3 \times 10^{17} \text{ cm}^{-3},$$

$$N_D \approx 3 \times 10^{18} \text{ cm}^{-3},$$

$$V_0 = 0.955 \text{ V},$$

$$V_D = 0 \text{ V},$$

the width of the depletion region is calculated to be

$$W_{dep} = \sqrt{\frac{2\epsilon_{Si}}{q} \left( \frac{1}{N_A} + \frac{1}{N_D} \right) (V_0 - V_D)} = \sqrt{\frac{2 \cdot 11.7 \cdot 8.854 \cdot 10^{-14}}{1.6 \cdot 10^{-19}} \left( \frac{1}{3 \times 10^{17}} + \frac{1}{3 \times 10^{18}} \right) \cdot 0.955} \approx 0.0673 \mu\text{m}$$

c) From Slide 26 in Lecture 2,

$$C_j = \frac{C_{j0}}{\sqrt{1 + \frac{V_R}{V_0}}},$$

$$C_{j0} = \sqrt{\frac{\epsilon_{Si} q}{2} \frac{N_A N_D}{N_A + N_D} \frac{1}{V_0}},$$

where  $\epsilon_{Si}$  is the permittivity of silicon in F/cm,  $q$  is the electric charge in C,  $N_A$  is the acceptor concentration in acceptors/cm<sup>3</sup>,  $N_D$  is the donor concentration in donors/cm<sup>3</sup>,  $V_0$  is the built-in potential in V, and  $V_R$  is the external reverse-bias voltage in V applied across the diode.

Since

$$q = 1.6 \cdot 10^{-19} \text{ C},$$

$$N_A \approx 3 \times 10^{17} \text{ cm}^{-3},$$

$$N_D \approx 3 \times 10^{18} \text{ cm}^{-3},$$

$$V_0 = 0.955 \text{ V},$$

$$V_D = 0 \text{ V},$$

the junction capacitance is estimated to be

$$C_j = \frac{C_{j0}}{\sqrt{1 + \frac{V_R}{V_0}}} = C_{j0} = \sqrt{\frac{\epsilon_{Si} q}{2} \frac{N_A N_D}{N_A + N_D} \frac{1}{V_0}} = \sqrt{\frac{11.7 \cdot 8.854 \cdot 10^{-14} \cdot 1.6 \cdot 10^{-19}}{2} \cdot \frac{3 \times 10^{17} \cdot 3 \times 10^{18}}{3 \times 10^{17} + 3 \times 10^{18}} \cdot \frac{1}{0.955}} \approx 154 \text{ [nF/cm}^2\text{]}.$$

The typical junction area in integrated-circuit devices ranges from 0.01  $\mu\text{m}^2$  to 10  $\mu\text{m}^2$ . Based on the calculated capacitance value of 90 nF/cm<sup>2</sup>, the typical capacitance values vary from 1.54 e-17 to 1.54 fF.