EECS 215 Winter 2004 Midterm 2

Name: Fred Terry

Lecture Section Solutions

Rules:

1. 6-7:30 PM Monday, March 22, 2004 and 2-3:30 PM Monday
2. Closed Book, Closed Notes, etc.
3. A formulae sheet is provided on the back of this exam and can be removed if desired. No other pages should be removed.
4. Calculators Needed and Allowed
5. Work to be done in Exam booklet.
6. **DO NOT WRITE ON THE BACK OF PAGES.**
7. **Exam given under CoE Honor Code**
8. Show your work and *briefly* explain major steps to maximize partial credit. (ex: i3=i1+i2 ,node A, KCL). **NO CREDIT WILL BE GIVEN IF NO WORK IS SHOWN.**
9. *WRITE YOUR FINAL ANSWERS IN THE AREAS PROVIDED*

This Exam Contains
4 problems over 19 pages (including workspace& formulae page).

Sign the College of Engineering Honor Code Below (NO credit will be given for the exam without a signed pledge):

I have neither given nor received aid on this examination.

Signed: ____________________

Do not write on this page below this line – Instructional Staff Use Only!

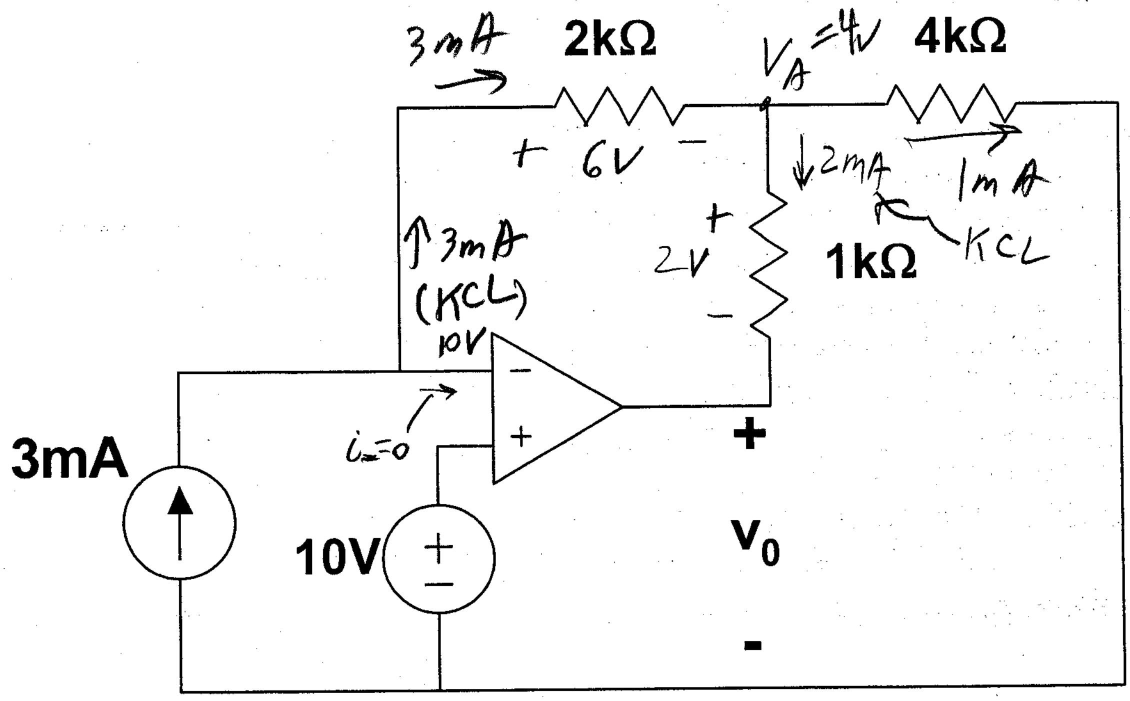
[ ] Prob 1 [ ] Prob 3

[ ] Prob 2 [ ] Prob 4

Problem 1: Op-Amps (20 points total)

Problem has parts a & b. You may draw directly on the circuits if you want, but be sure to clearly explain your reasoning to qualify for partial credit.

a) For the circuit below, what is v_0? (10 points)



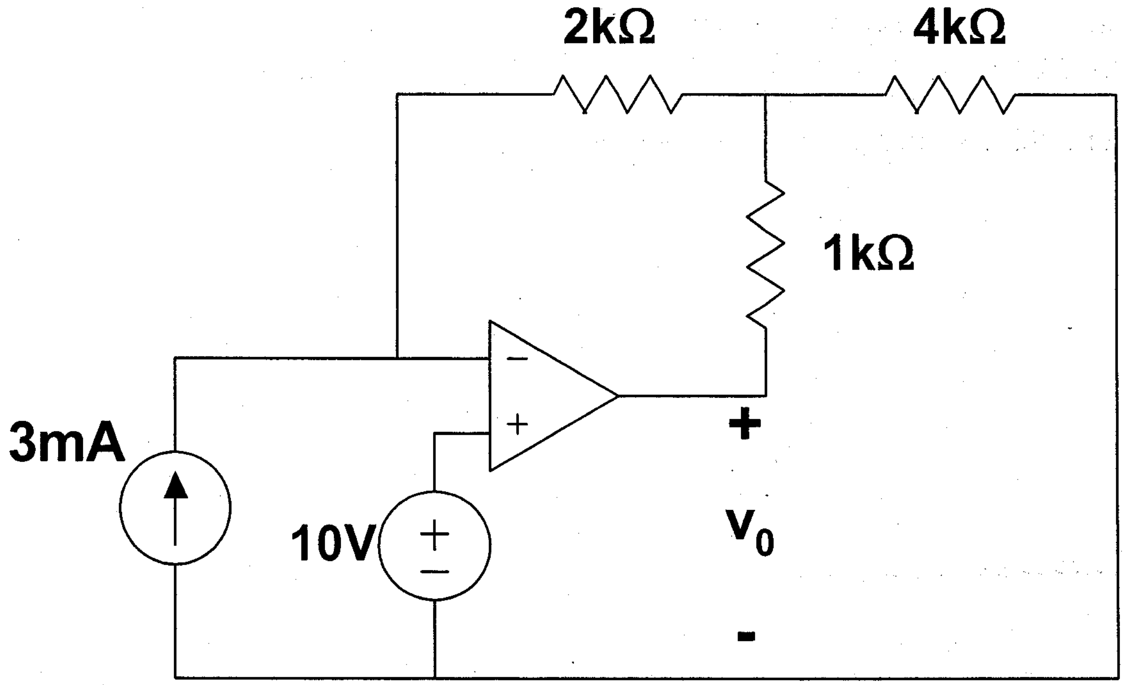
$v_0 =$ +2 V

$i_- = 0$ ideal opamp $v_- = 10V = v_+$

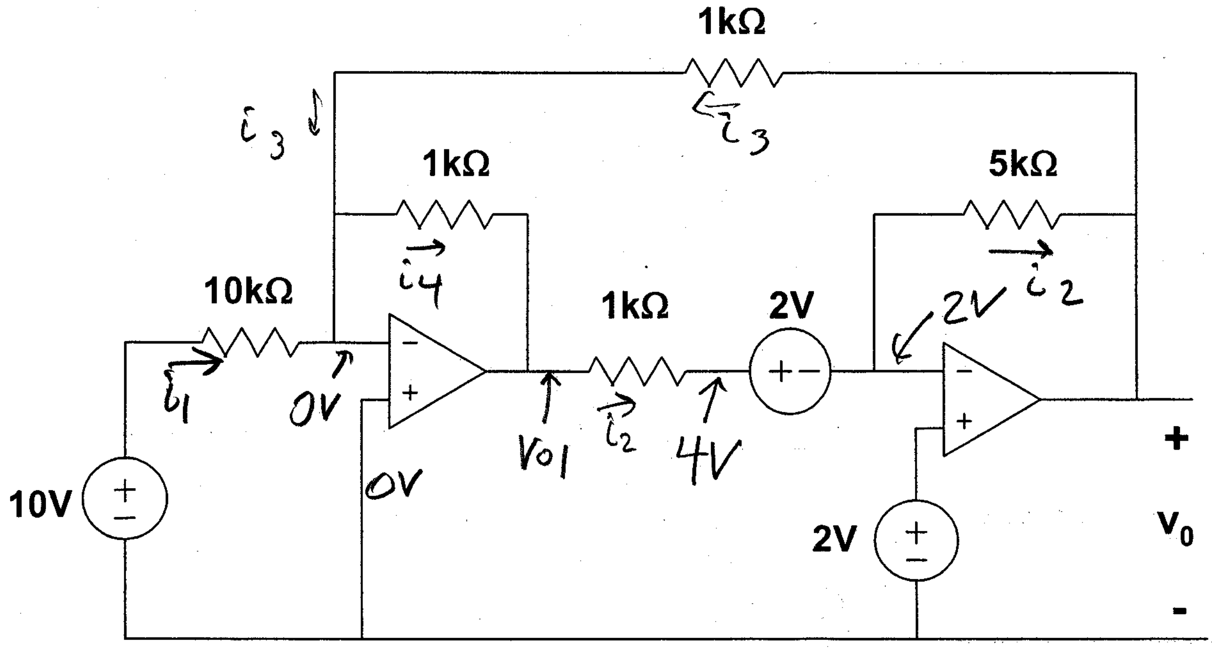
$V_A = 10V - 6V = 4V$ KVL

$V_0 = 4V - 2V = 2V$ KVL

additional space for 1(a) if needed



b) For the circuit below, what is v_0? (10 points)



$v_0 = \underline{-\frac{27}{4} = -6.75V}$ V

$i_1 = 10V/10K\Omega = 1mA$

$i_2 = \dfrac{V_{01} - 4V}{1K\Omega}$

$V_0 = 2V - (5K\Omega)\, i_2 = 2V - (5V_{01} - 20V)$

$= 22V - 5V_{01}$

$i_3 = \dfrac{V_0}{1K\Omega} \Rightarrow i_4 = \dfrac{V_0}{1K\Omega} + i_1 = \dfrac{V_0}{1K\Omega} + 1mA$

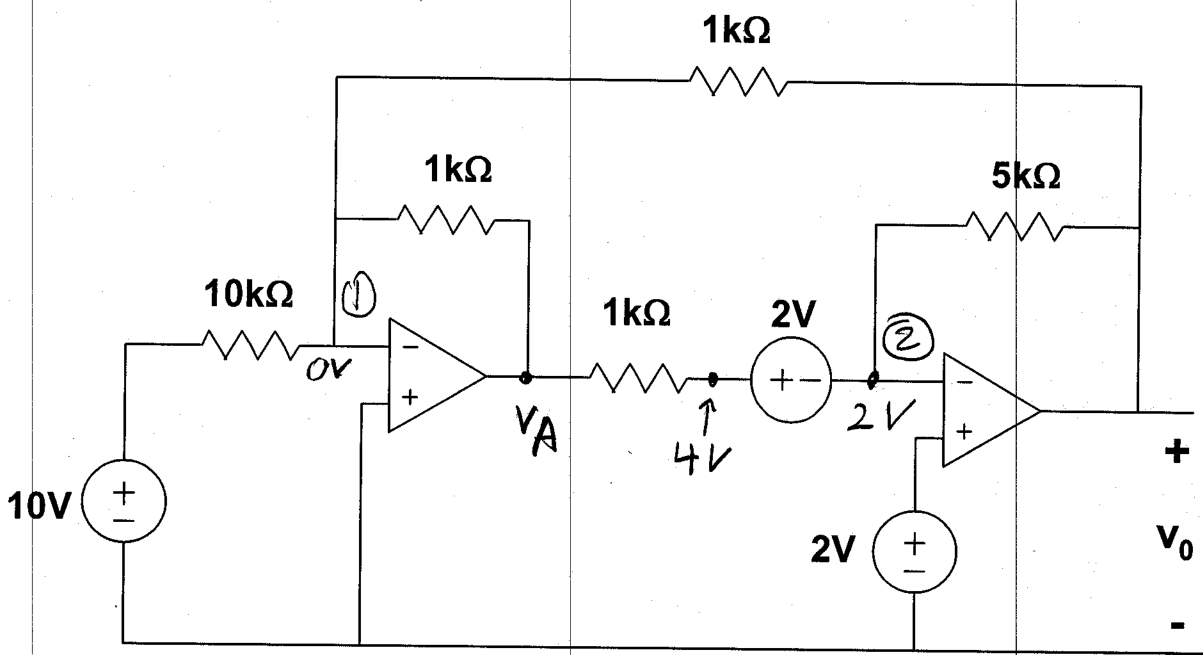
$V_{01} = (-1K\Omega)\, i_4 = -V_0 - 1V$

so $V_0 = 22V + 5V_0 + 5V$

$\Rightarrow 4V_0 = -27V \Rightarrow V_0 = -\frac{27}{4}V$

Same idea, more elegance!

additional workspace for 1(b) if needed



After using $V_+ = V_-$ on both op-amps

+ Doing the simple KVL for the 4V point, we have a

2 node KCL/nodal problem (at the 2 V_- nodes)

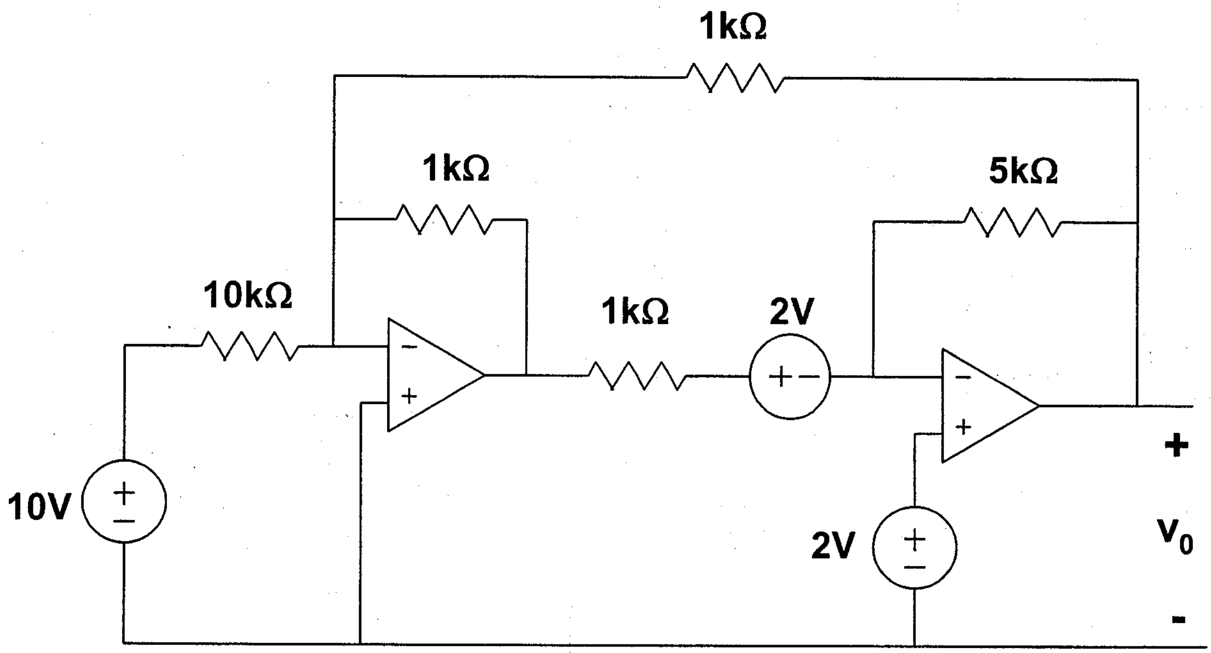
① $$\frac{10V}{10k\Omega} + \frac{V_0}{1k\Omega} + \frac{V_A}{1k\Omega} = 0 \Rightarrow V_A + V_0 = -1V$$

② $$\frac{V_A - 4V}{1k\Omega} + \frac{V_0 - 2V}{5k\Omega} = 0 \Rightarrow 5V_A + V_0 = 22V$$

$$\begin{bmatrix} 1 & 1 \\ 5 & 1 \end{bmatrix} \begin{bmatrix} V_A \\ V_0 \end{bmatrix} = \begin{bmatrix} -1 \\ 22 \end{bmatrix} \Rightarrow \begin{bmatrix} 23/4 \\ -27/4 \end{bmatrix} V = \begin{bmatrix} V_A \\ V_0 \end{bmatrix}$$

$$\Rightarrow V_0 = -27/4$$

additional workspace for 1(b) if needed



Problem 2: First Order Circuits (30 points total)

Problem has only 1 part (all quantities in the box below)

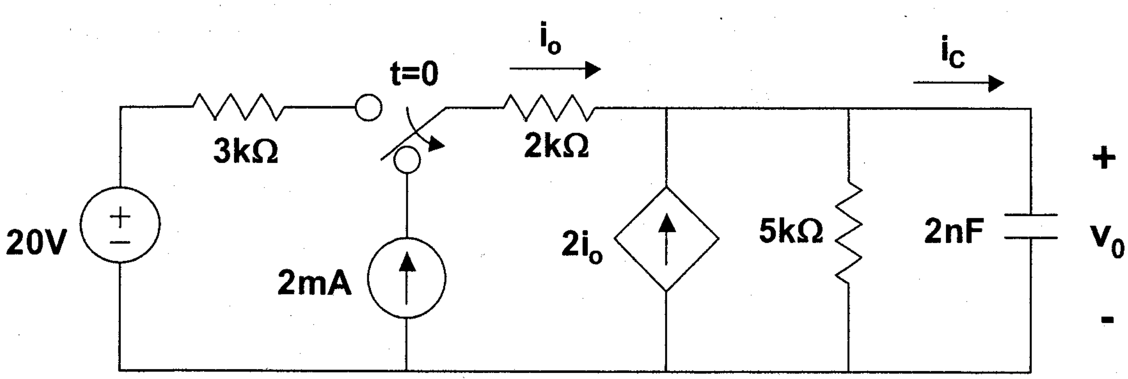
For the circuit below, find the following quantities (box below). Show your work clearly.
No credit will be given without clear supporting work.

$v_0(0^-)$= $+15V$ V
$v_0(0^+)$= $+15V$ V

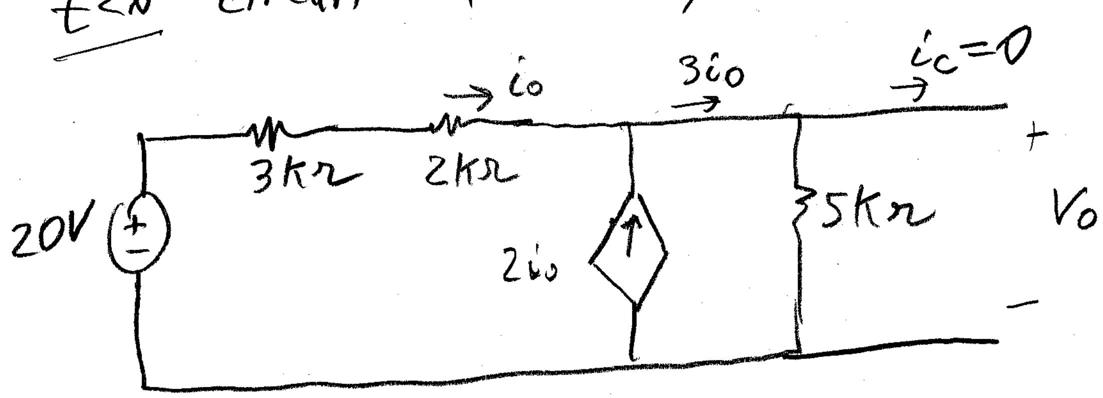
same by V-continuity

$v_{0f}=v_C(t\to\infty)$= $+30V$ V

$v_0(t)$= $\left(30 - 15e^{-t/10\mu s}\right)$ V (for t>0)

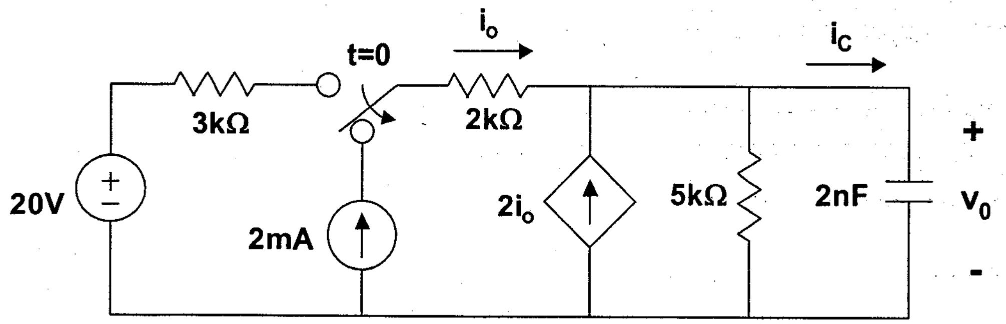


$t<0$ circuit DC steady state

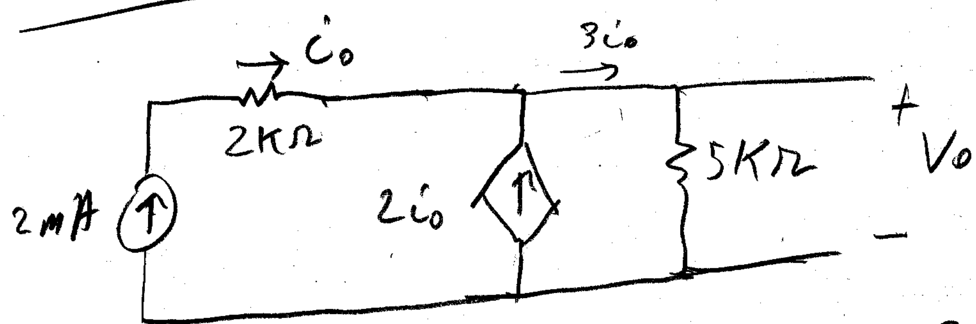


$$20V = (5k\Omega)i_o + (15k\Omega)i_o \quad \text{KVL/ohm}$$
$$= (20k\Omega)i_o$$
$$\Rightarrow i_o = 1mA \quad \& \quad v_o = (3i_o)5k\Omega = 15V$$

Additional Workspace for problem 2



$t \to \infty$ circuit D.C. steady state



$V_{of} = (5k\Omega)3i_o$ $\quad i_o = 2mA$ by KCL

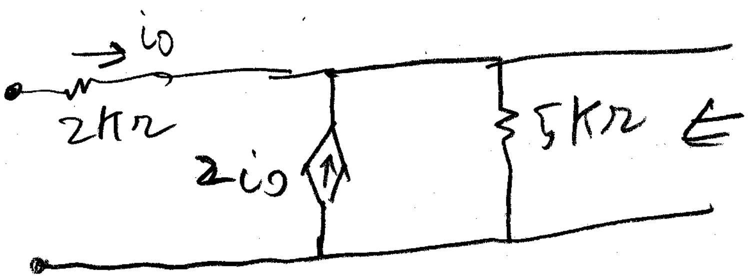
$= (5k\Omega)6mA$

$= 30\ V$

note this circuit also is the basis for find our $t > 0$ Thevenin equivalent.

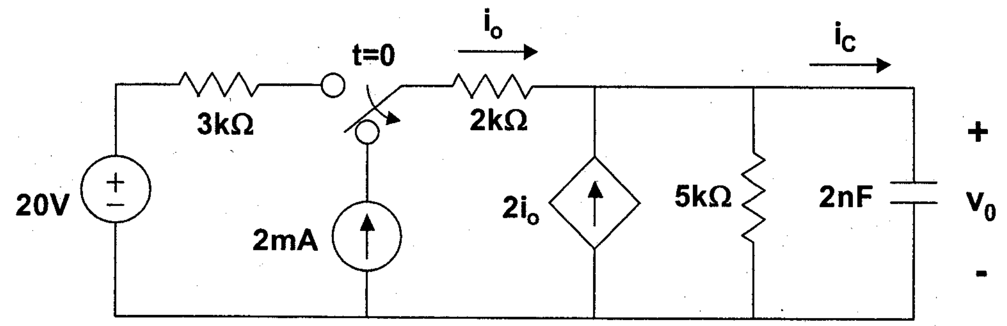
So $V_{oc} = V_{TH} = V_{of} = +30V$

R_{eq}? turn off 2mA source (open



$\leftarrow R_{eq} = R_{TH}$

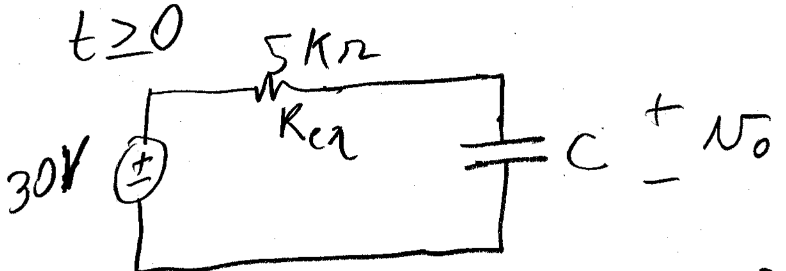
Workspace for problem 2



So i_o in this circuit $= 0$

$+ R_{eq} = 5K\Omega$

$t \geq 0$



$$\tau = R_{eq} C = (5\,K\Omega)\,2nF = 10\,\mu s$$

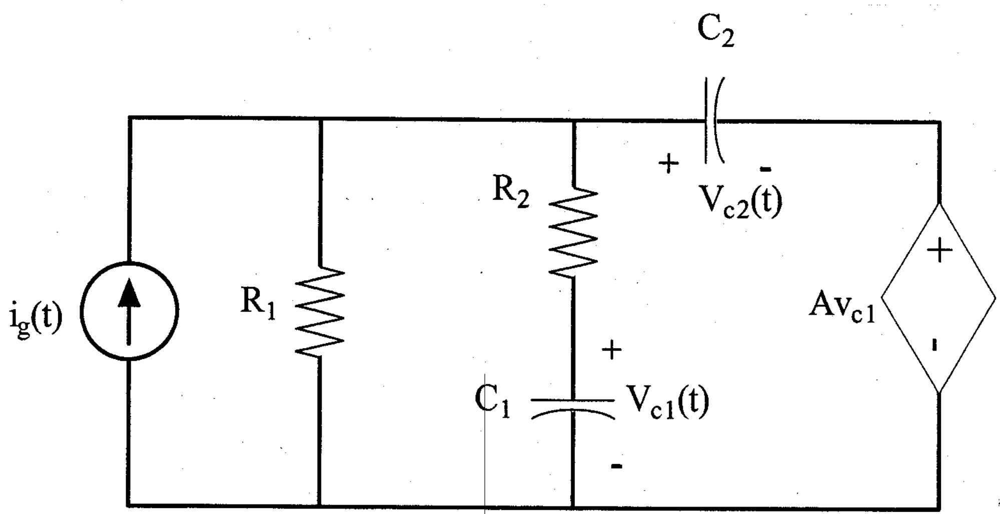
$$V_c(t) = 30V + Ae^{-t/\tau} = V_{cf} + V_{cn}$$

$$V_c(0) = 15V = 30V + A \Rightarrow A = -15V$$

$$\underline{V_c(t) = 30 - 15e^{-t/10\mu s}\ V}$$

Problem 3: Second Order Circuits (20 points total)

Problem has only 1 part



For the circuit picture above, find the differential equation that relates $V_{c1}(t)$ to $i_g(t)$. Write the equation in standard form - $\frac{d^2V_{c1}}{dt^2} + A\frac{dV_{c1}}{dt} + BV_{c1} = function(i_g)$. V_{c1} must be the only unknown (assuming $i_g(t)$ is known). You may use KVL/KCL/time domanin methods or s-domain, but you must clearly show your work to receive full or partial credit. **Warning**: *Attempts to mix time-domain and s-domain approaches are likely to result in zero credit.*

Differential Equation:

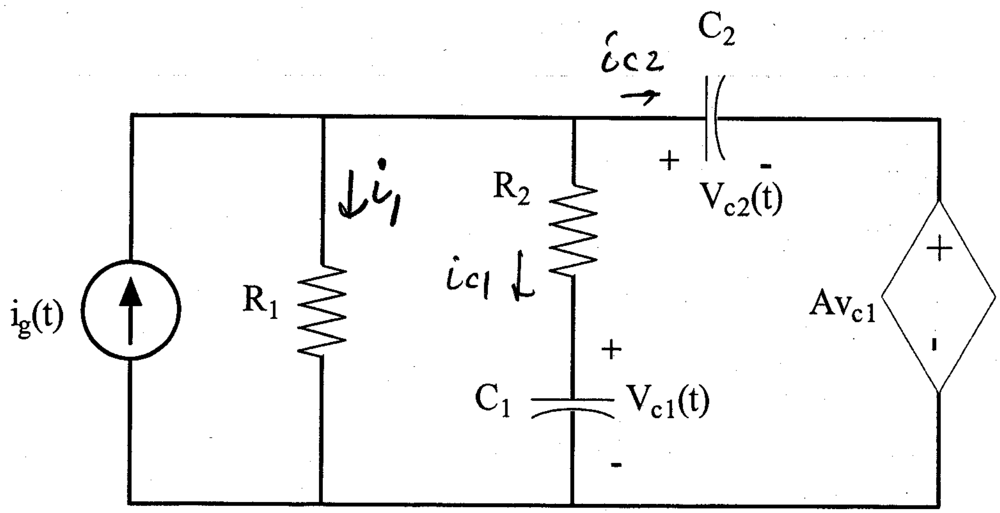
$$\left\{\left(\frac{d^2V_{c1}}{dt^2}\right) + \left[\frac{R_1+R_2}{R_1R_2C_2} + \frac{(1-A)}{R_2C_1}\right]\frac{dV_{c1}}{dt} + \left(\frac{1}{R_1R_2C_1C_2}\right)V_{c1}\right\}$$

$$= i_g/(R_2C_1C_2)$$

Note that you can check this numerically using the component values and the numerically specified differential equation in problem 4. Also note that the units all work (v/s^2 on is the unit for all terms) and that the D.C. steady state forced response (if ig=const) is Vc1=R1ig which agrees with D.C. circuit analysis.

Time Domain

Workspace for problem 3



KVL: $R_2 i_{c1} + V_{c1} = V_{c2} + A V_{c1} = i_1 R_1$

$\Rightarrow \quad V_{c2} = R_2 i_{c1} + (1-A) V_{c1}$

KCL: $i_g = i_1 + i_{c1} + i_{c2}$

$i_1 = \frac{R_2}{R_1} i_{c1} + \frac{1}{R_1} V_{c1}$ from KVL/Ohm's Law

$i_{c1} = C_1 \frac{dV_{c1}}{dt}$

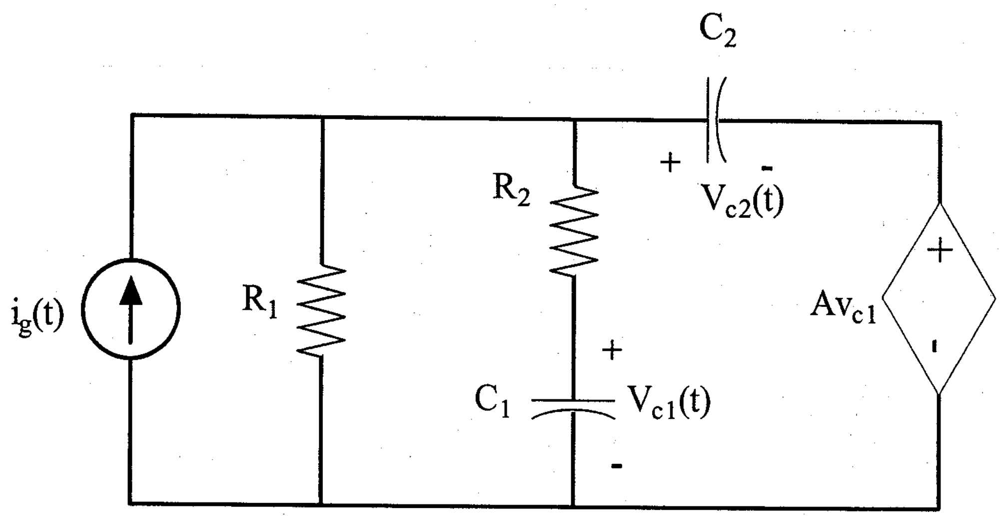
$i_{c2} = C_2 \frac{dV_{c2}}{dt}$

$= R_2 C_2 C_1 \frac{d^2 V_{c1}}{dt^2} + (1-A) C_2 \frac{dV_{c1}}{dt}$

So

$$i_g = \frac{R_2}{R_1} C_1 \frac{dV_{c1}}{dt} + \frac{V_{c1}}{R_1} + C_1 \frac{dV_{c1}}{dt} + R_2 C_1 C_2 \frac{d^2 V_{c1}}{dt^2} + (1-A) C_2 \frac{dV_{c1}}{dt}$$

additional workspace for problem 3



$$i_g = R_2 C_1 C_2 \frac{d^2 V_{c1}}{dt^2} + \left[\left(1 + \frac{R_2}{R_1}\right)C_1 + (1-A)C_2\right] \frac{dV_{c1}}{dt}$$

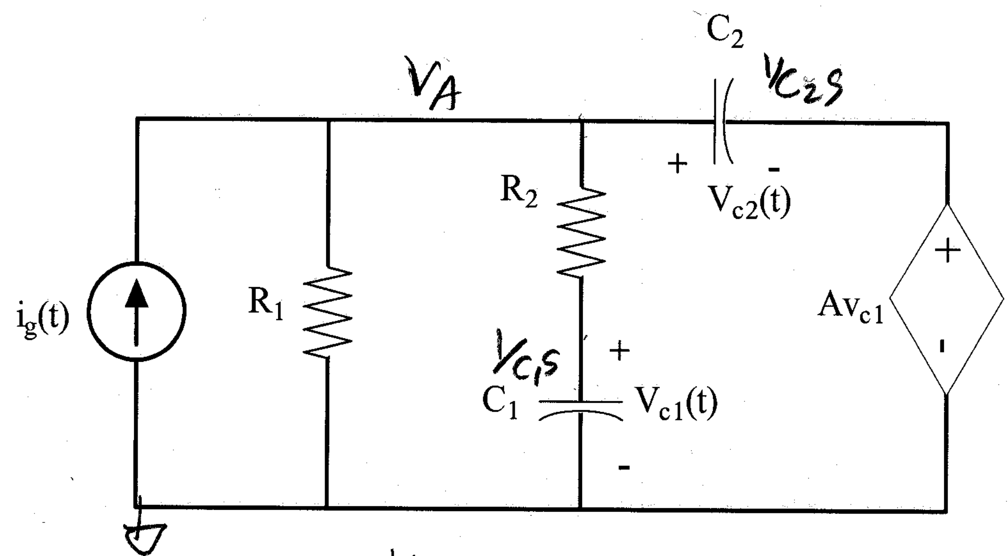
$$+ V_{c1}/R_1$$

$$\left(\frac{i_g}{R_2 C_1 C_2}\right) = \left\{ \frac{d^2 V_{c1}}{dt^2} + \left[\left(\frac{R_1 + R_2}{R_1 R_2 C_2}\right) + \frac{(1-A)}{R_2 C_1}\right] \frac{dV_{c1}}{dt}\right.$$

$$\left. + \left(\frac{V_{c1}}{R_1 R_2 C_1 C_2}\right) \right\}$$

S-Domain Nodal

Workspace for problem 3



$$V_{C1} = V_A\left[\frac{1/C_1 s}{R_2 + 1/C_1 s}\right] = \frac{V_A}{1+R_2C_1 s}$$ Voltage Divider

$$\Rightarrow V_A = (1+R_2C_1 s)V_{C1}$$

KCL A: $-V_A\left[\frac{1}{R_1} + \frac{1}{R_2+\frac{1}{C_1 s}} + sC_2\right] - AV_{C1}sC_2 = i_g$

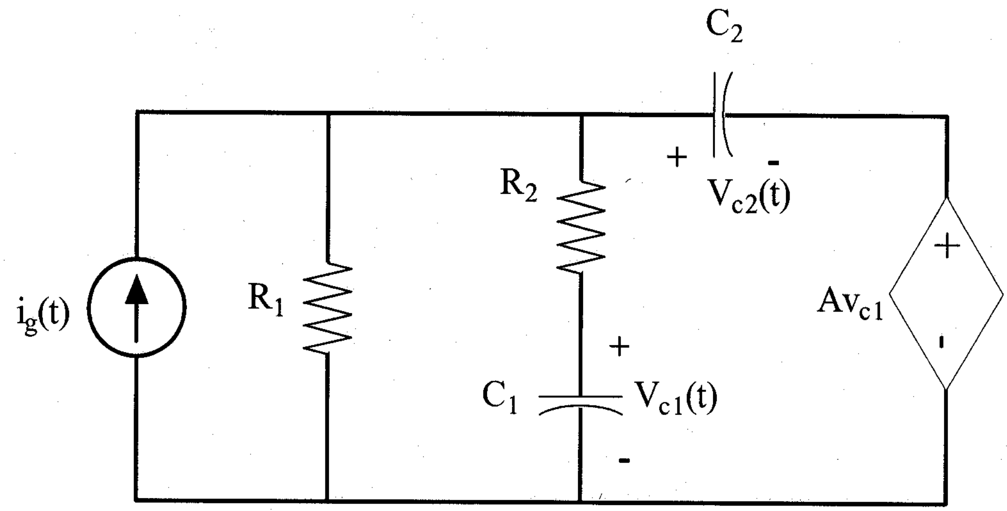
$$V_{C1}\left[(1+R_2C_1 s)\left(\frac{1}{R_1}+\frac{C_1 s}{1+R_2C_1 s}+sC_2\right) - AC_2 s\right] = i_g$$

$$V_{C1}\left[\frac{1}{R_1}+\frac{R_2}{R_1}C_1 s + C_1 s + sC_2 + R_2C_1C_2 s^2 - AC_2 s\right]=i_g$$

$$V_{C1}\left[\frac{1}{R_1} + \left[\left(1+\frac{R_2}{R_1}\right)C_1 + (1-A)C_2\right]s + R_2C_1C_2 s^2\right] = i_g$$

$$V_{C1}\left[s^2 + \left(\frac{R_1+R_2}{R_1R_2C_2} + \frac{1-A}{R_2C_1}\right)s + \frac{1}{R_1R_2C_1C_2}\right] = \frac{i_g}{R_2C_1C_2}$$

additional workspace for problem 3



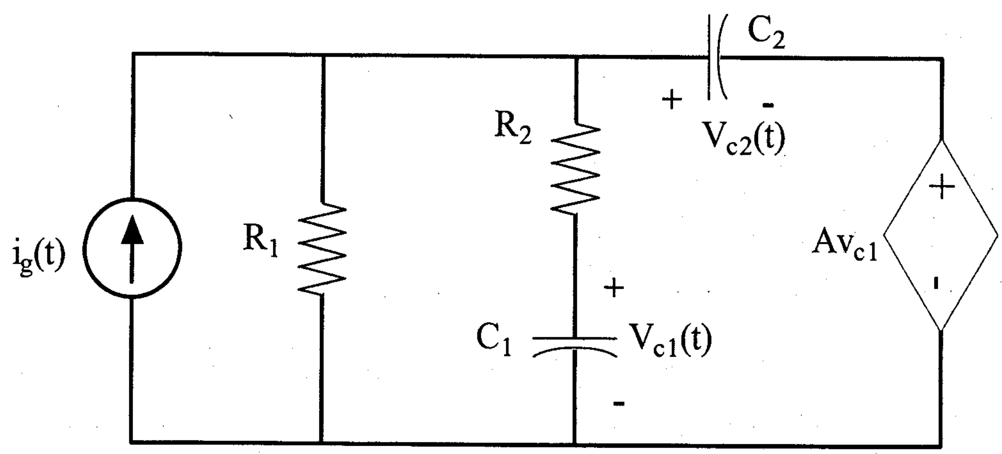
$$s \to \frac{d}{dt} \qquad s^2 \to \frac{d^2}{dt^2}$$

so we get the same result

$$\left\{ \frac{d^2 V_{c1}}{dt^2} + \left[\left(\frac{R_1 + R_2}{R_1 R_2 C_2} \right) + \frac{(1-A)}{R_2 C_1} \right] \frac{dV_{c1}}{dt} + \left(\frac{1}{R_1 R_2 C_1 C_2} \right) V_{c1} \right\} = \left(\frac{i_g}{R_2 C_1 C_2} \right)$$

Problem 4: Second Order Circuits (30 points total)

Problem has parts a, b, c, and d



Now suppose we have the circuit above with the following component values:

$\mathbf{R}_1 = 1K\Omega$ $\quad \mathbf{R}_2 = 5K\Omega$ $\quad A = 0.7$ $\quad C_1 = 1nF$ $\quad C_2 = 5nF$

and we will let $i_g(t) = [3mA]u(t)$

This results in a differential equation for this circuit (for t>0):

$$\left(2.5x10^{-11}s^2\right)\frac{d^2V_{c1}}{dt^2} + \left(7.5x10^{-6}s\right)\frac{dV_{c1}}{dt} + V_{c1} = 3V$$

where s **denotes seconds, not the Laplace differential operator**

a) Find the quantities below. Show your work on the following 2 pages (5 pts)

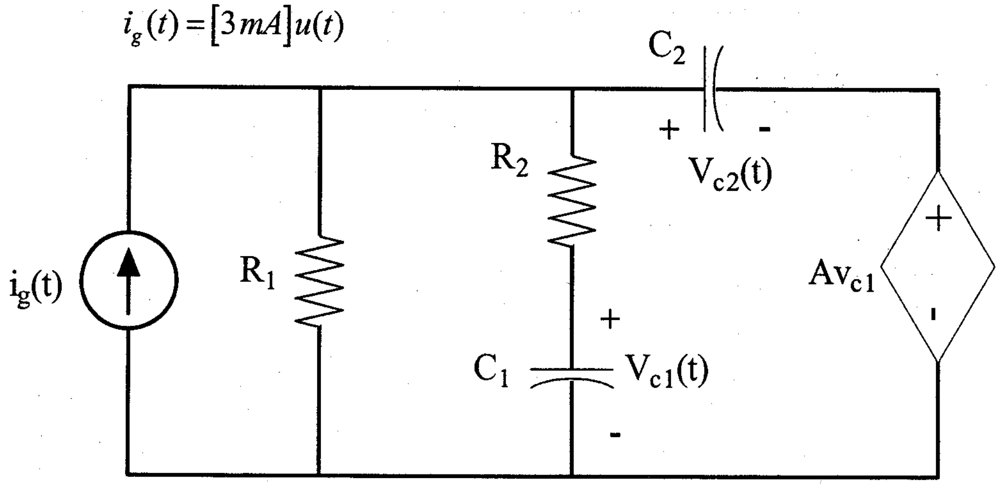
$v_{c1}(0^-)=$	0	V
$v_{c2}(0^-)=$	0	V
$i_{c1}(0^-)=$	0	A
$i_{c2}(0^-)=$	0	A

$v_{c1}(0^+)=$	0	V
$v_{c2}(0^+)=$	0	V
$i_{c1}(0^+)=$	0	A
$i_{c2}(0^+)=$	+3mA	A

Workspace for (a)

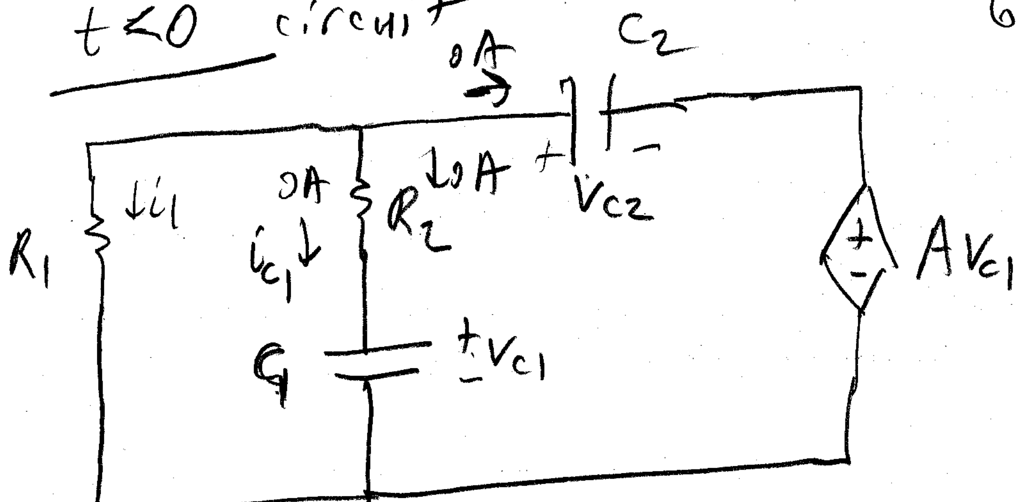
$\mathbf{R}_1 = 1K\Omega \quad \mathbf{R}_2 = 5K\Omega \quad A = 0.7 \quad C_1 = 1nF \quad C_2 = 5nF$

$i_g(t) = [3mA]u(t)$



t<0 circuit

Get $t=0^-$ quantities



DC, Steady state $i_{c1} = 0$ $\quad i_{c2} = 0$

by KCL $i_1 = 0A$

by KVL $i_1 R_1 = 0V = i_{c1} R_2 + V_{c1} \Rightarrow V_{c1} = 0V$

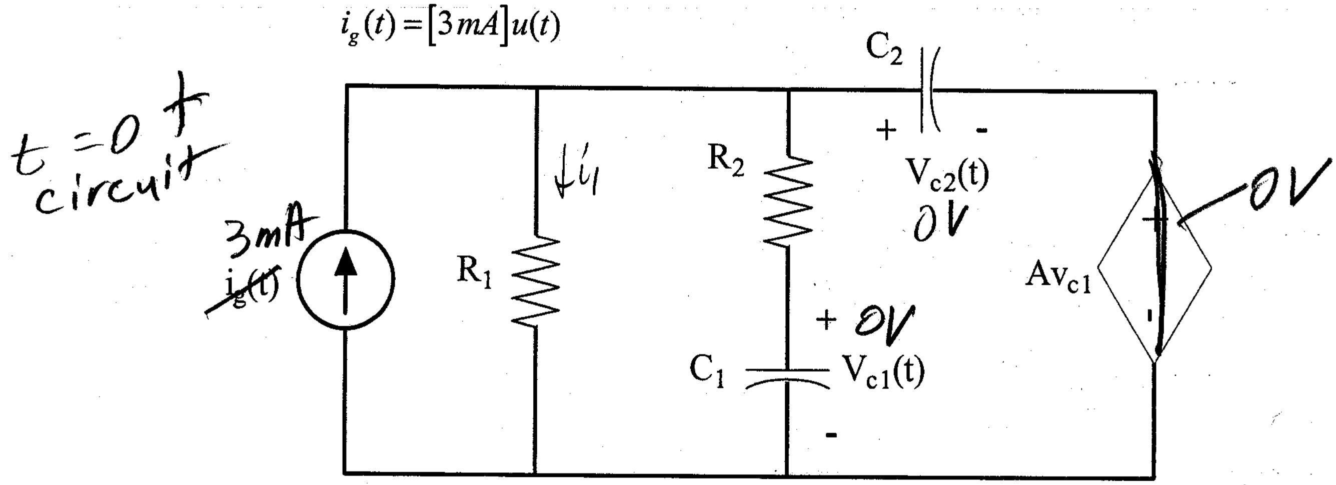
by KVL $\underbrace{i_{c1} R_2 + V_{c1}}_{0} = V_{c2} + \underbrace{A V_{c2}}_{0}$

$\Rightarrow V_{c2} = 0V$

additional workspace for (a) if needed

$\mathbf{R}_1 = 1K\Omega \quad \mathbf{R}_2 = 5K\Omega \quad A = 0.7 \quad C_1 = 1nF \quad C_2 = 5nF$

$i_g(t) = [3mA]u(t)$



$t = 0^+ \quad V_{c1} = 0 \quad V_{c2} = 0 \quad i_g = 3mA$

by KVL $R_1 i_1 = V_{c2} + AV_{c1} = 0V$

$\Rightarrow \underline{i_1 = 0}$

by KVL $i_{c1} R_2 + \underbrace{V_{c1}}_{0V \text{ by Cap. V-continuity}} = V_{c2} + AV_{c1} = 0V$

$\Rightarrow \underline{i_{c1} = 0}$

So by KCL $i_{c2} = 3mA$

b) Find $v_{c1}(\infty)$, $v_{c2}(\infty)$, $i_{c1}(\infty)$, and $i_{c2}(\infty^+)$. (10 pts)

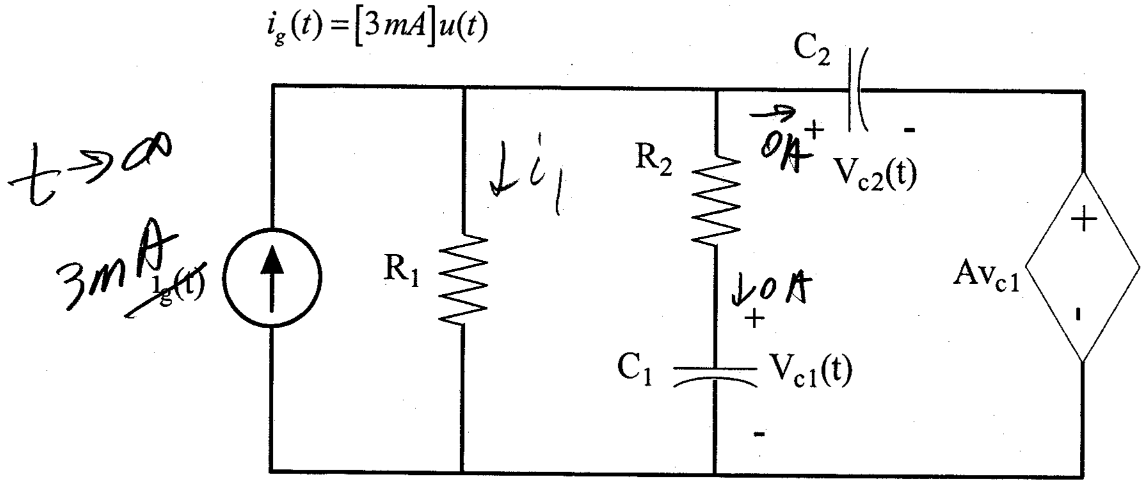
$v_{c1}(\infty)=$	$+3V$	V
$v_{c2}(\infty)=$	$+0.9V$	V
$i_{c1}(\infty)=$	0	A
$i_{c2}(\infty)=$	0	A

Workspace for (b)

$\mathbf{R_1} = 1K\Omega \quad \mathbf{R_2} = 5K\Omega \quad A = 0.7 \quad C_1 = 1nF \quad C_2 = 5nF$

$i_g(t) = [3mA]u(t)$

$t \to \infty$



$i_{c1} \to 0 \quad i_{c2} \to 0$ DC steady state

KCL $\quad i_1 = 3mA$

KVL $\quad \underbrace{R_2 i_{c1}}_{0} + V_{c1} = R_1 i_1 = 3V$

$\Rightarrow V_{c1} = 3V$

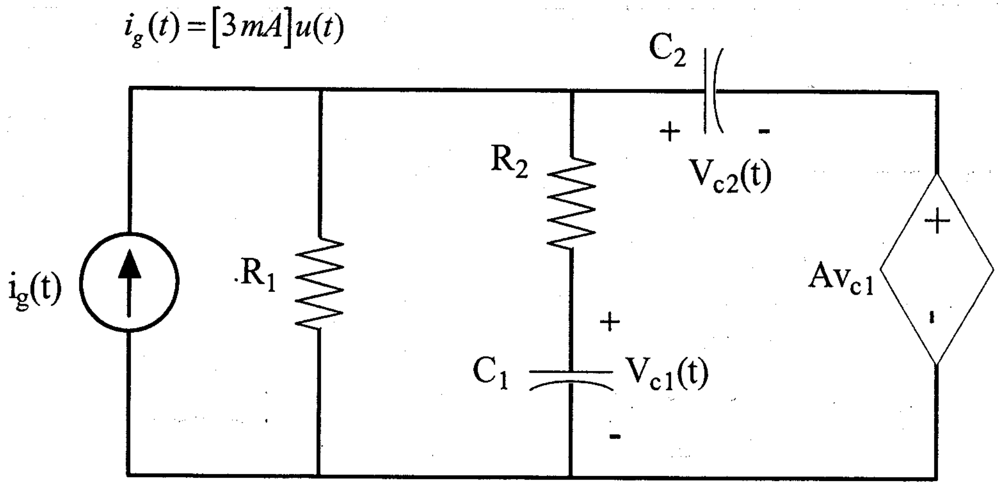
KVL $\quad \underbrace{R_2 i_{c2}}_{0} + V_{c1} = V_{c2} + A V_{c1}$

$\Rightarrow (1-A) V_{c1} = V_{c2} = (0.3)3V = 0.9V$

additional workspace for (b) if needed

$\mathbf{R}_1 = 1K\Omega \quad \mathbf{R}_2 = 5K\Omega \quad A = 0.7 \quad C_1 = 1nF \quad C_2 = 5nF$

$i_g(t) = [3\,mA]u(t)$



c) Find the natural solution for Vc1 (with 2 and only 2 unknown coefficients). (5 pts)

$V_{c1,n}(t)=$	$[B_1\cos(\omega_d t)+B_2\sin(\omega_d t)]e^{-\alpha t}$ V
(with 2 and only 2 unknown coefficients)	$\alpha=1.5\times10^5 s^{-1}$ $\omega_d\approx1.323\times10^5 s^{-1}$

workspace for (c):

$$\omega_o^2=\frac{1}{2.5\times10^{-11}s^2}=4\times10^{10}s^{-2}$$

from given diff. eqn,

$$\Rightarrow \omega_o=2\times10^5 s^{-1}$$

$$2\alpha=\frac{7.5\times10^{-6}s}{2.5\times10^{-11}s}=3\times10^5 s^{-1}$$

$$\Rightarrow \alpha=1.5\times10^5 s^{-1}$$

$\alpha<\omega_o \Rightarrow$ Underdamped response

$$\Rightarrow V_{c1,n}(t)=[B_1\cos(\omega_d t)+B_2\sin(\omega_d t)]e^{-\alpha t}$$

$$\omega_d=[\omega_o^2-\alpha^2]^{1/2}\approx[1.75\times10^{10}s^{-2}]^{1/2}$$

$$\approx1.3229\times10^5 s^{-1}$$

d) Match the initial conditions to the complete solution to find the final numerical solution for $v_{c1}(t)$ for this problem. (10 pts)

$v_{c1}(t)=$ $3 - [3\cos(\omega_d t) + 3.4017\sin(\omega_d t)]e^{-\alpha t}$ V

$\alpha = 1.5\times10^5\, s^{-1}$ $\omega_d \approx 1.323\times10^5\, s^{-1}$

workspace for (d):

$$V_{c1}(t) = 3V + [B_1\cos(\omega_d t) + B_2\sin(\omega_d t)]e^{-\alpha t}$$

$$V_{c1}(0) = 0 = 3V + B_1 \Rightarrow B_1 = -3V$$

$$i_{c1}(0) = 0 = C[B_1(-\alpha) + B_2\omega_d]$$

$$\Rightarrow B_2 = \frac{+B_1\alpha}{\omega_d} = -3V\left(\frac{\alpha}{\omega_d}\right)$$

$$\approx -3.4017\ V$$