HKN EE 40 Review

Nov 2011
Yunjae Cho

Photo credits to:
Neel Shah (Summer 2011)
Michel Maharbiz (Fall 2011)
Vivek Subramanian

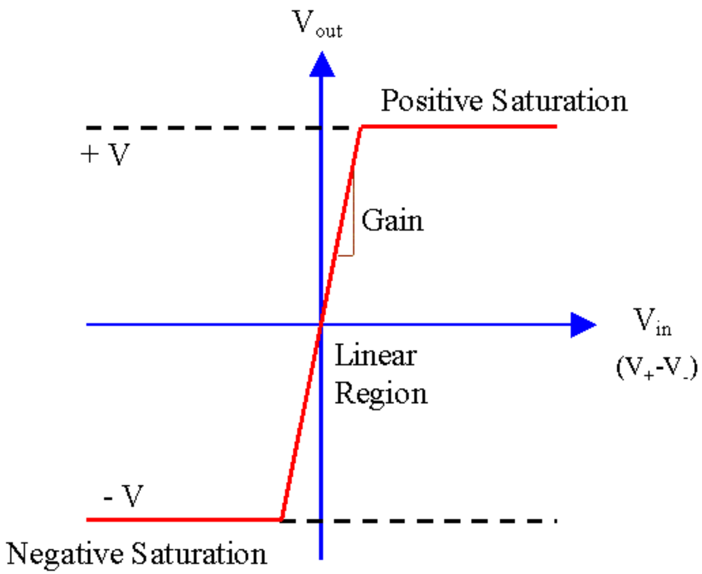
EE 40 MT 2 Review

1. Op Amps
2. RC & RL Circuits
3. RLC Circuits
4. AC Response (Phasor Methods)
5. Frequency Response (Passive & Active Filtering)

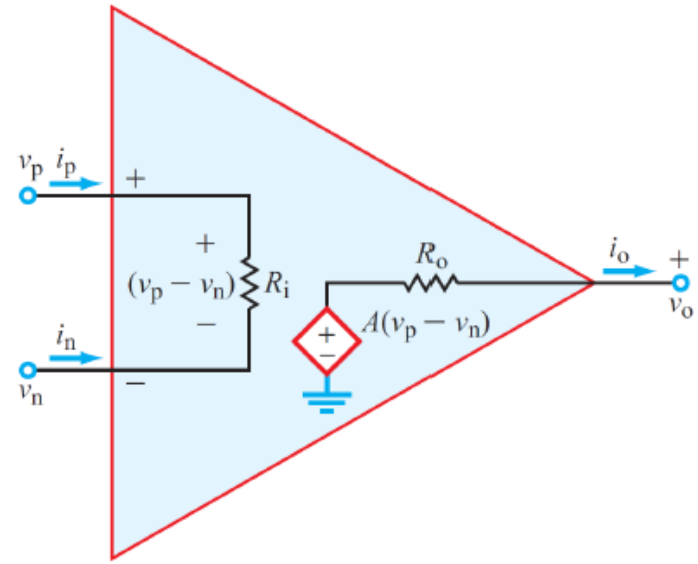
Operational Amplifiers

- Ideal vs. Non-Ideal Op Amps
- Inverting Op Amps
- Non-Inverting Op Amps
- Inverter Adder
- Differential Amplifier
- Inverter Integrater
- Inverter Differentiater
- ***General Strategies***
- Question: a "scary" Op Amp

Ideal vs Non-Ideal Op Amps



Ideal Op Amp



Non-Ideal Op Amp

Linearity of the input-output relationship

Gain = potentially Inf

R_{in}, input resistance = Inf

R_{out}, output resistance = 0

Non-Linearity of the input-output relationship

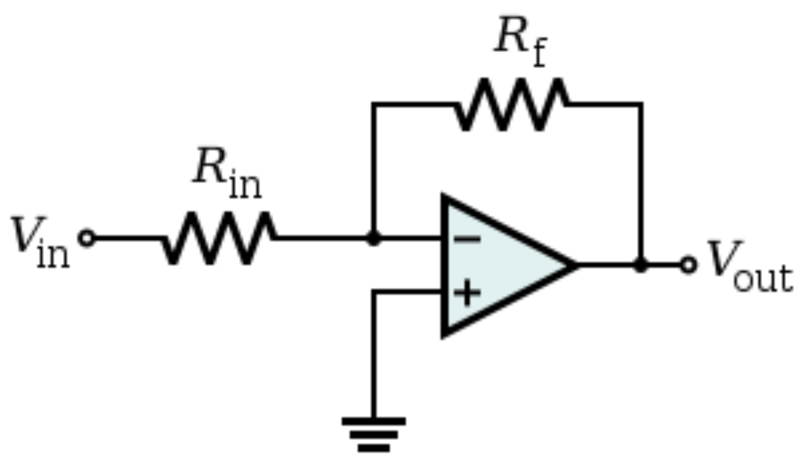
Gain = Limited (finite)

R_{in}, input resistance = high (finite)

R_{out}, output resistance = very low

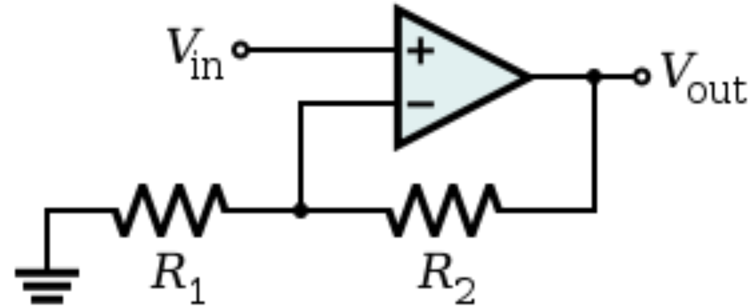
Inverting and Non-Inverting Amplifiers

Inverting Amplifier



$$V_{out} = -\frac{R_f}{R_{in}} V_{in}$$

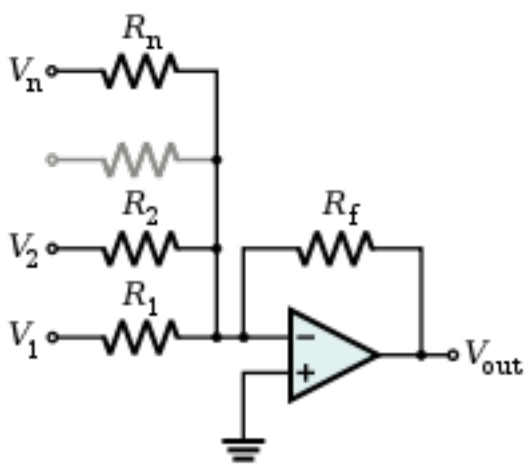
Non-Inverting Amplifer



$$V_{out} = V_{in}\left(1 + \frac{R_2}{R_1}\right)$$

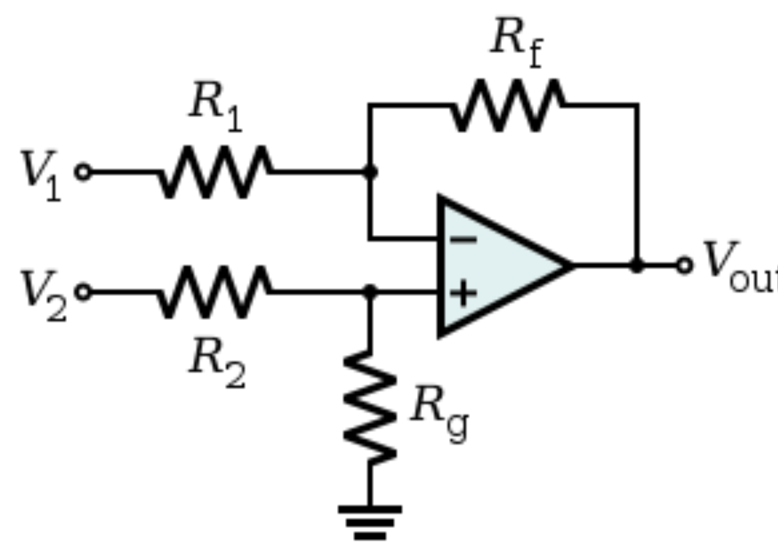
Inverter Adder and Differential Amplifier

Inverter Adder



$$V_{out} = -R_f \left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \cdots + \frac{V_n}{R_n} \right)$$

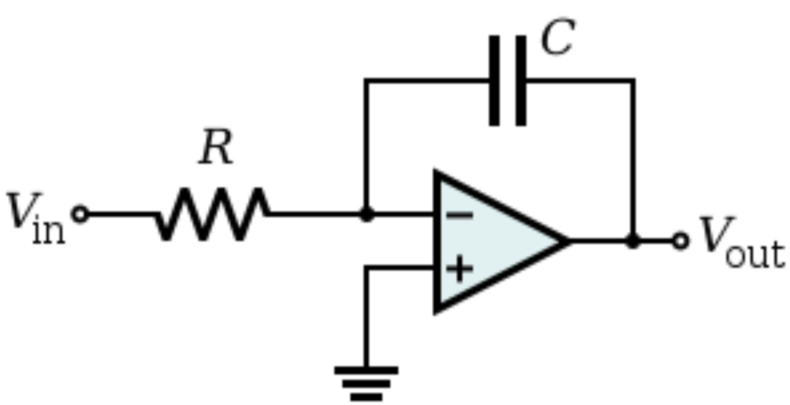
Differential Amplifier



$$V_{out} = \frac{(R_f + R_1) R_g}{(R_g + R_2) R_1} V_2 - \frac{R_f}{R_1} V_1$$

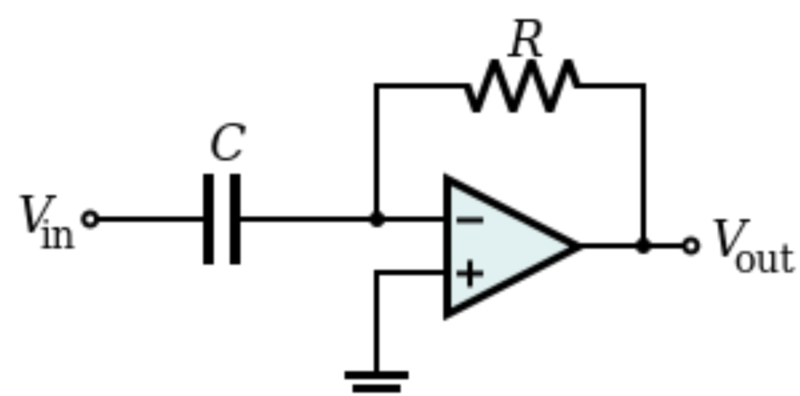
Inverter Integrater and Inverter Differentiater

Inverter Integrater



$$V_{out} = -\int_0^t \frac{V_{in}}{RC}\, dt + V_{initial}$$

Inverter Differentiater



$$V_{out} = -RC\,\frac{dV_{in}}{dt}$$ where V_{in} and V_{out} are functions of time.

General Strategies

Mark all the nodes with the same voltages
(i.e. $V^+=V^-$, $V_{GND} = 0$, etc)
And the Op Amps may or may not simply disappear :)
Set up Node Voltage Equations for each node
Solve for V_{out} in terms of V_{in}

RC, RL & RLC Circuits

1. Capacitors and Inductors (Properties)

2. Solutions to Diff Eq's

3. τ, the time constant

4. Transient Analysis of RC & RL Circuits

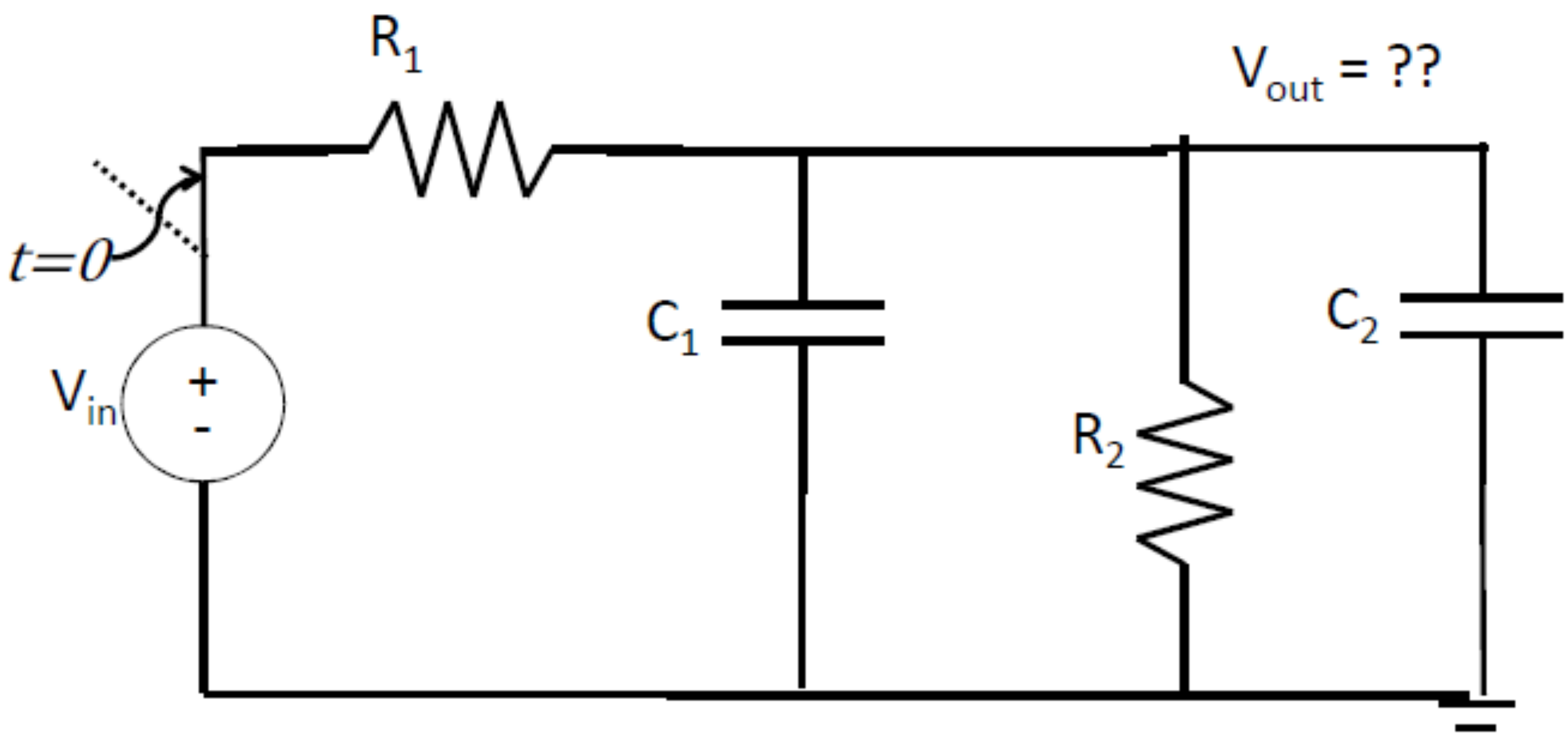
5. Solution of 2nd Order Diff Eq & Damping

Summary of properties

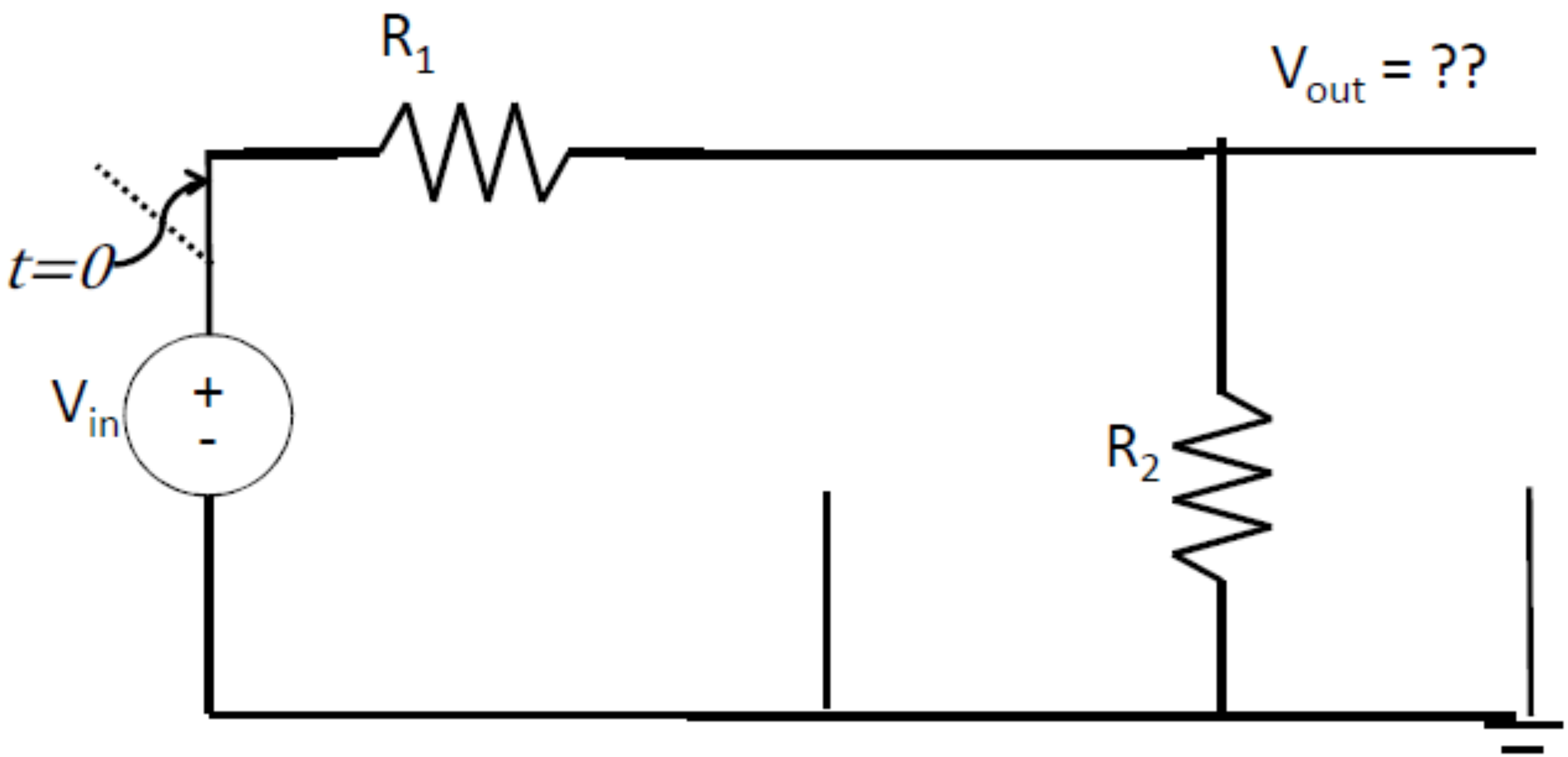
Property	R	L	C
i–v relation	$i = \frac{v}{R}$	$i = \frac{1}{L}\int_{t_0}^{t} v\,dt + i(t_0)$	$i = C\,\frac{dv}{dt}$
v-i relation	$v = iR$	$v = L\,\frac{di}{dt}$	$v = \frac{1}{C}\int_{t_0}^{t} i\,dt + v(t_0)$
p (power transfer in)	$p = i^2 R$	$p = Li\,\frac{di}{dt}$	$p = Cv\,\frac{dv}{dt}$
w (stored energy)	0	$w = \frac{1}{2}Li^2$	$w = \frac{1}{2}Cv^2$
Series combination	$R_{\text{eq}} = R_1 + R_2$	$L_{\text{eq}} = L_1 + L_2$	$C_{\text{eq}} = \frac{C_1 C_2}{C_1 + C_2}$
Parallel combination	$R_{\text{eq}} = \frac{R_1 R_2}{R_1 + R_2}$	$L_{\text{eq}} = \frac{L_1 L_2}{L_1 + L_2}$	$C_{\text{eq}} = C_1 + C_2$
dc behavior	no change	short circuit	open circuit
Can v change instantaneously?	yes	yes	no
Can i change instantaneously?	yes	no	yes

RC - Steady State (DC) Practice

The Steady State Voltage



The Steady State Voltage



$$V_{out} = V_{in} R_2/(R_1 + R_2)$$

Solution to Diff Eq's

$Y(x) = Y_i + (Y_f - Y_i)*(1-e^{-x/a})$

(You can use these equations for both V(t) and I(t))

Ex:

Charging (a compacitor or an inductor)

$V(t) = V_i + (V_f - V_i) * (1-e^{-t/\tau})$

if V_i=0: $V(t) = V_f * (1-e^{-t/\tau})$

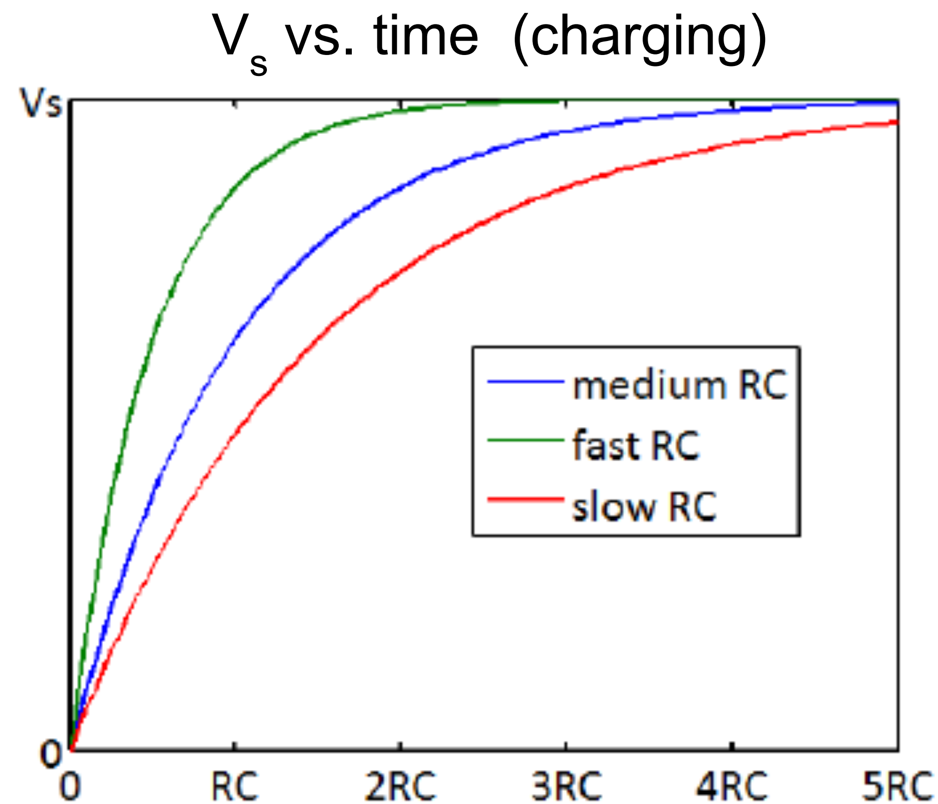
Discharging

$V(t) = V_i * (e^{-t/\tau})$

*τ is the time constant of the circuit

τ, the time constant

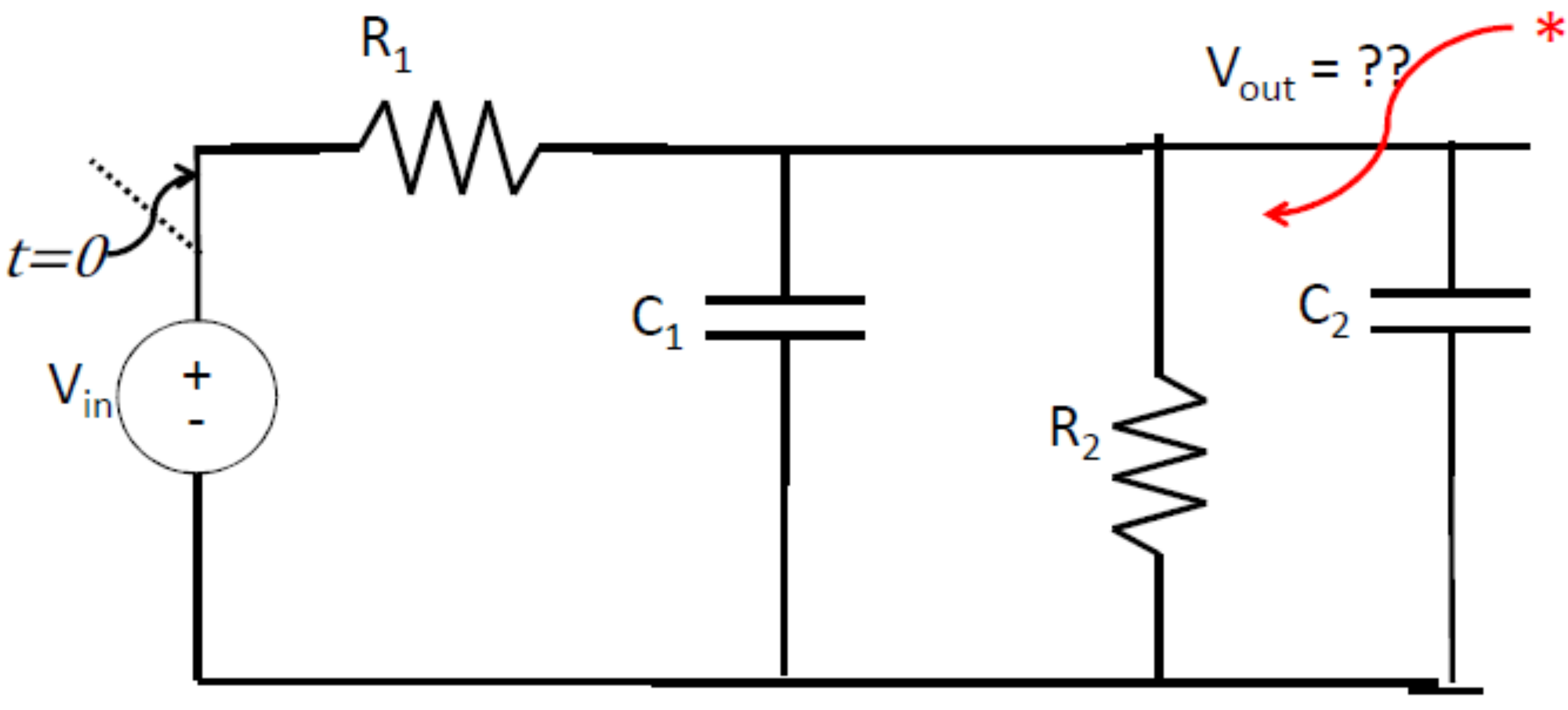
- The time constant τ is a measure of how quickly a circuit responds to a change in state
  - Large τt values respond slowly
  - Small τ values respond quickly
  - Which one is better depends on the application



τ for RC circuit: $R_{eq}*C_{eq}$

τ for RL ciruit: L_{eq}/R_{eq}

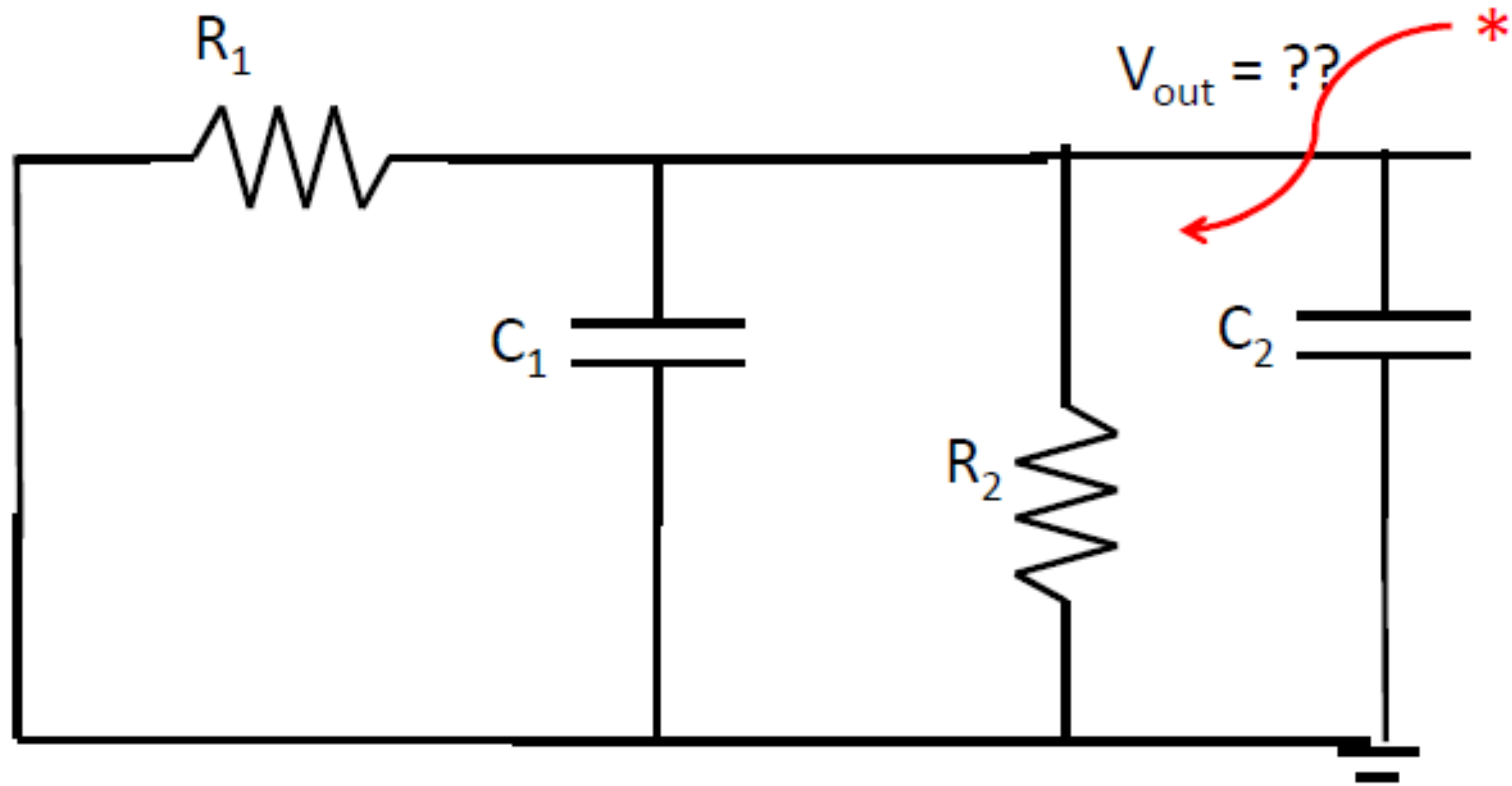
The Time Constant



Can find just by finding the Thévenin output impedance!

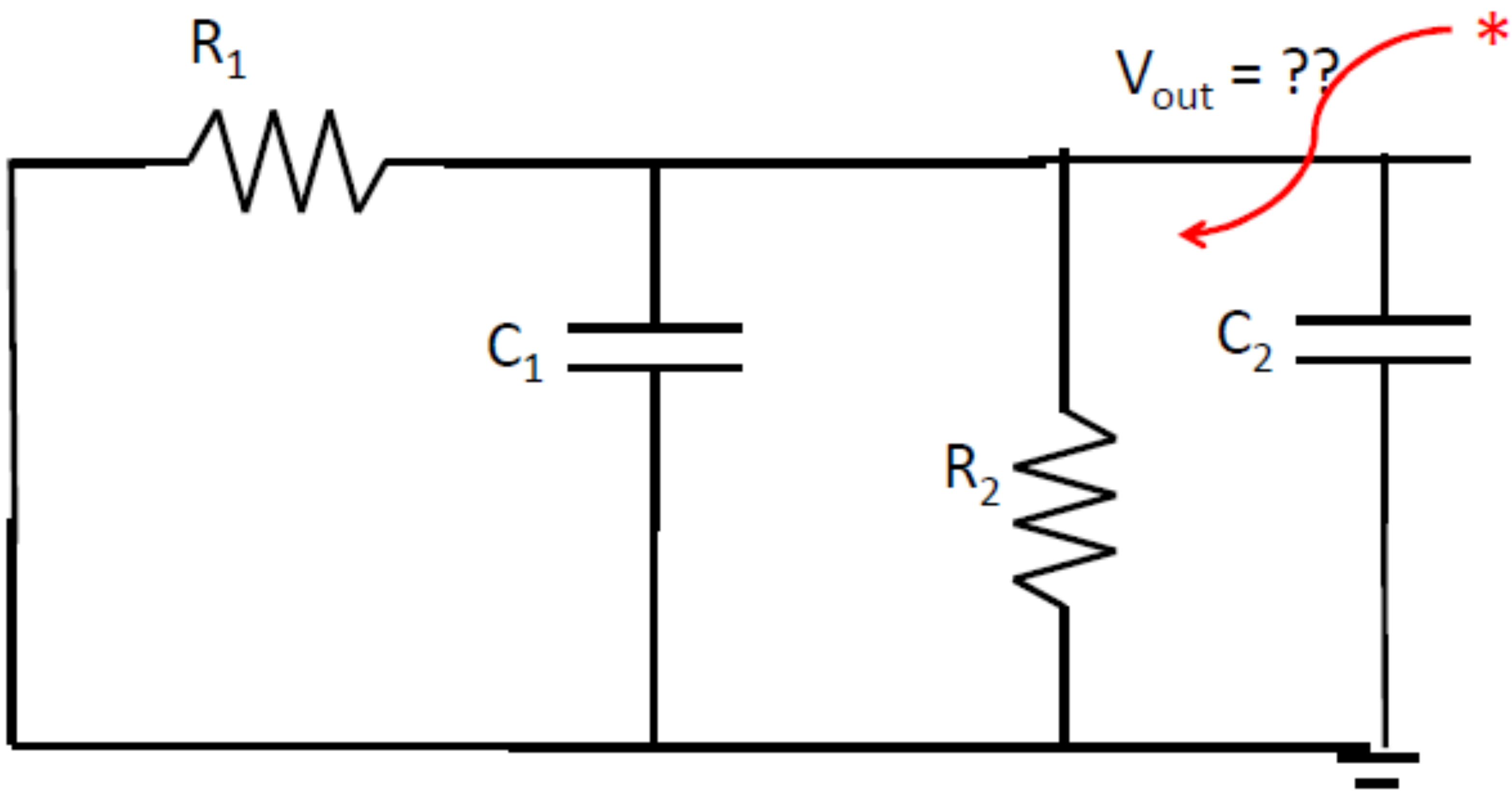
The Time Constant

- Short Vin



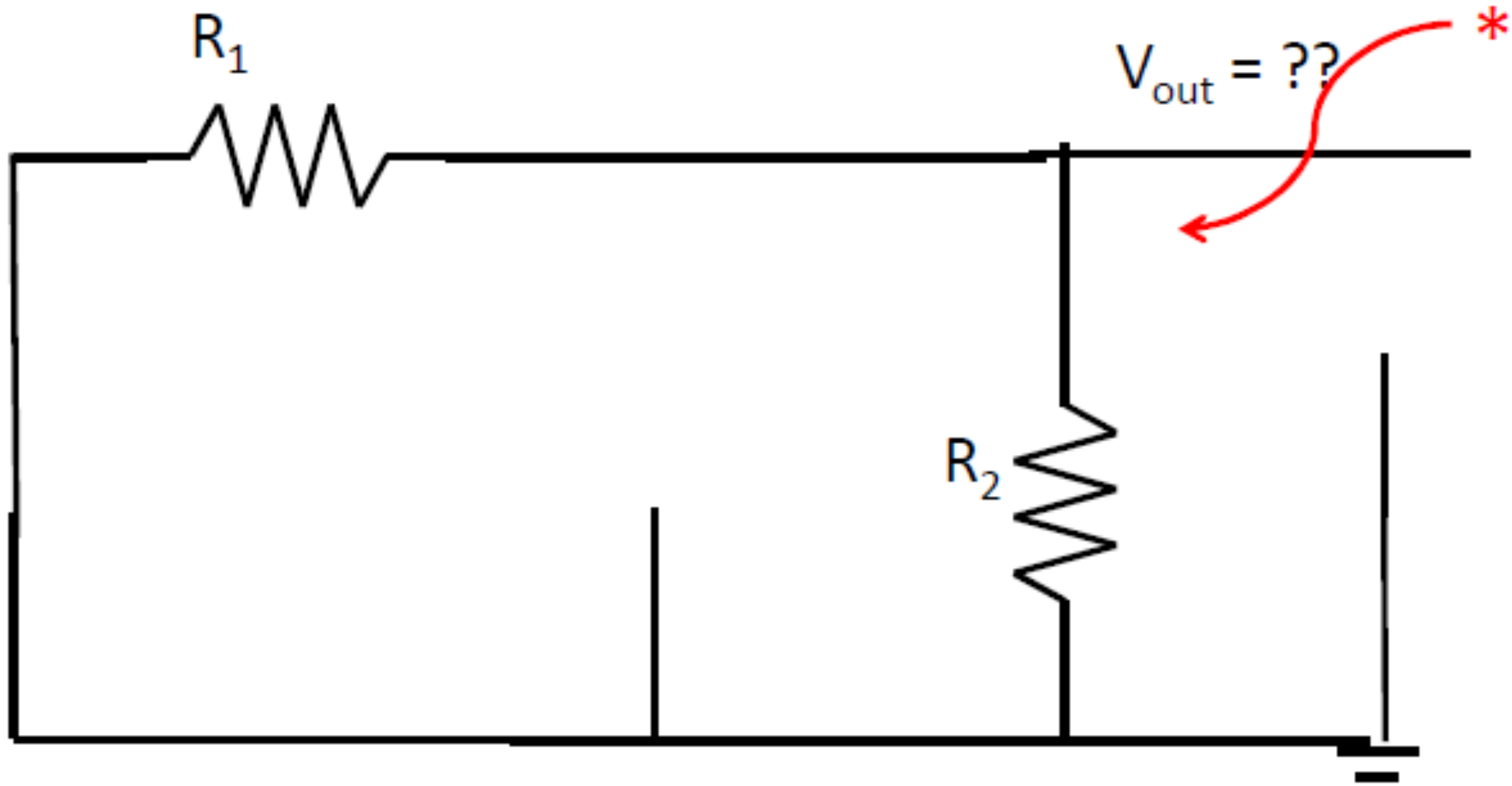
The Time Constant

- Find impedance from V_{out} to ground



The Time Constant

- Look first at just the resistors

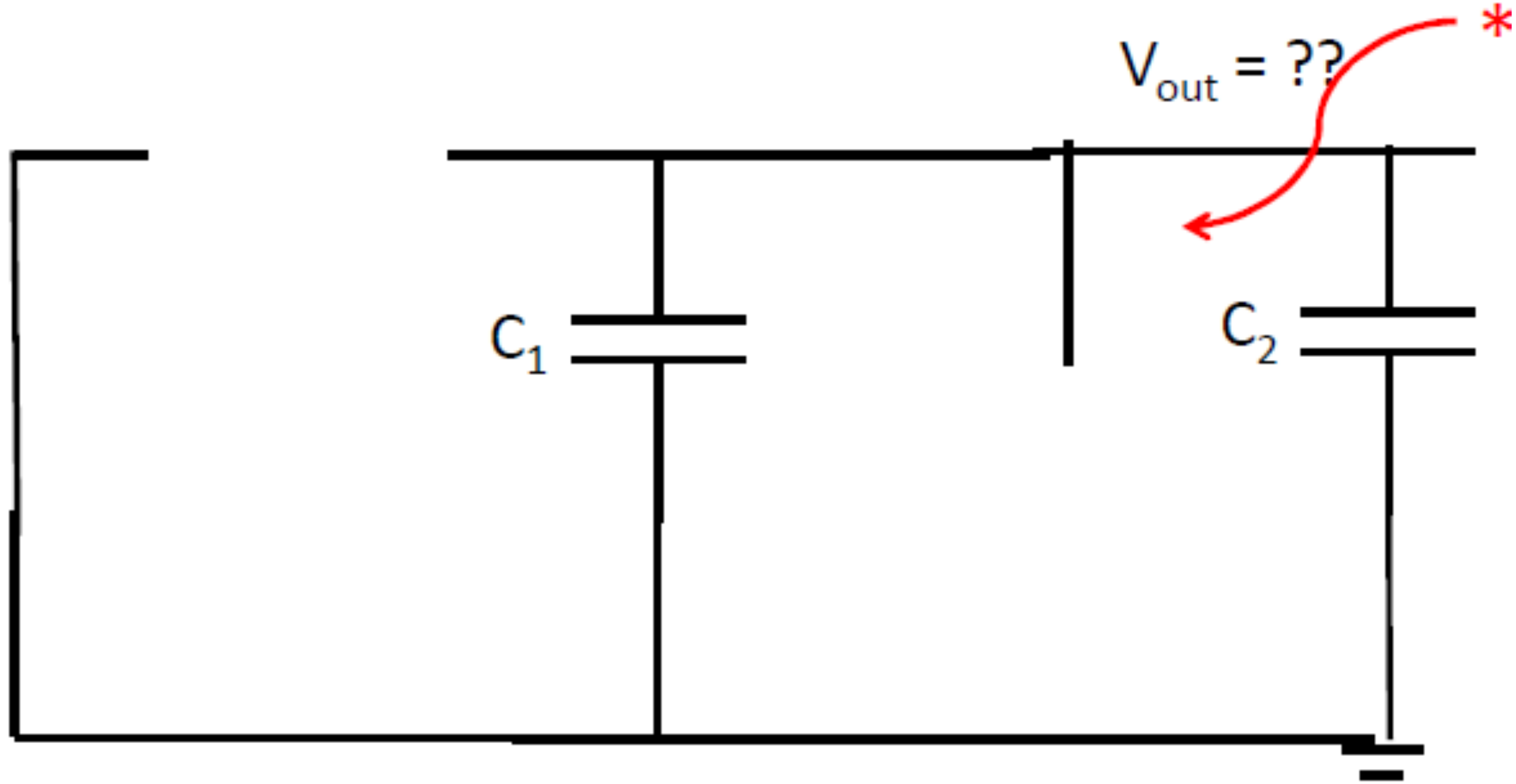


- Series or parallel?

$R_{eq} = R_1 || R_2 = R_1R_2/(R_1+R_2)$

The Time Constant

- Now look at just the capacitors

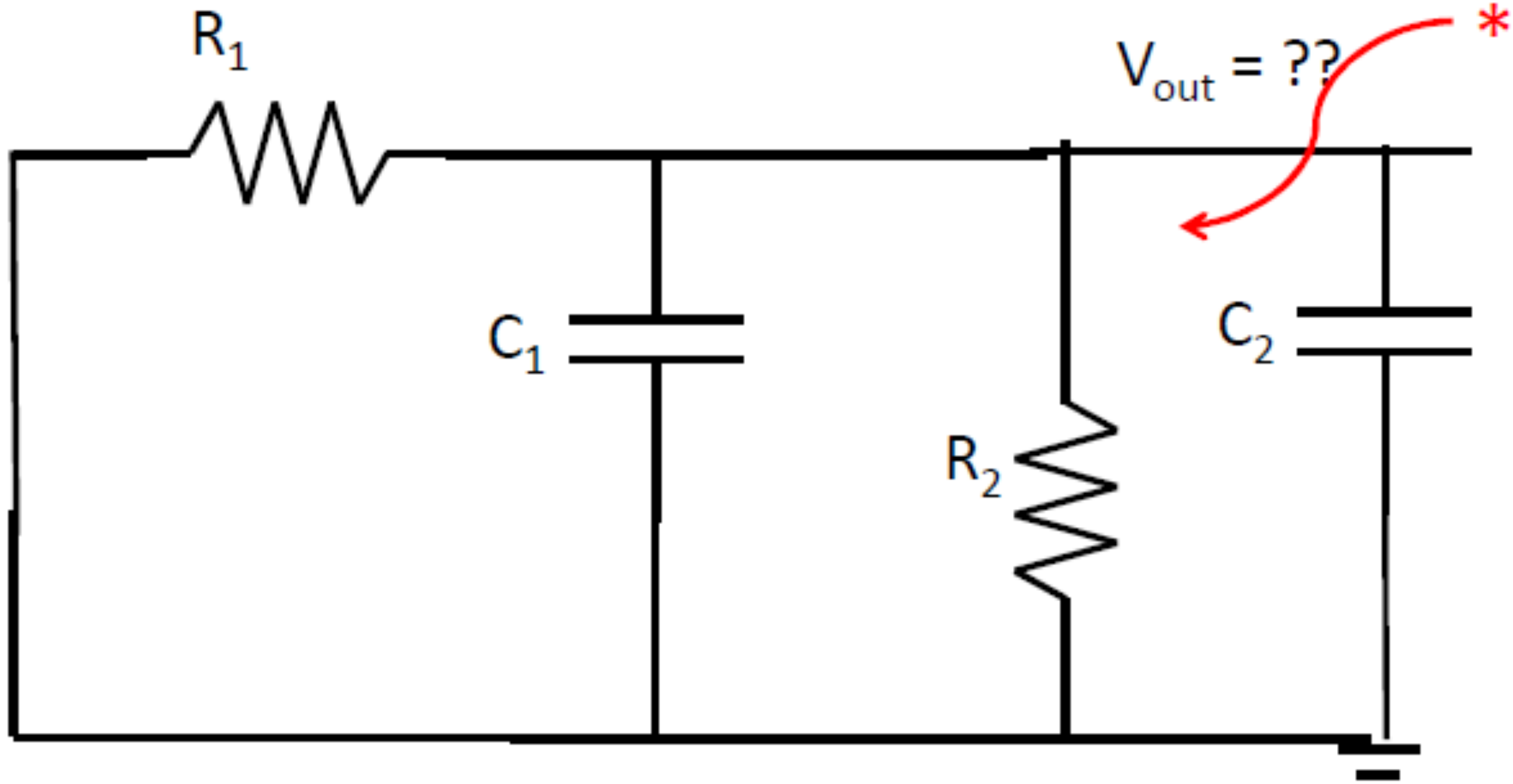


- Series or parallel?

$C_{eq} = C_1 || C_2 = C_1 + C_2$

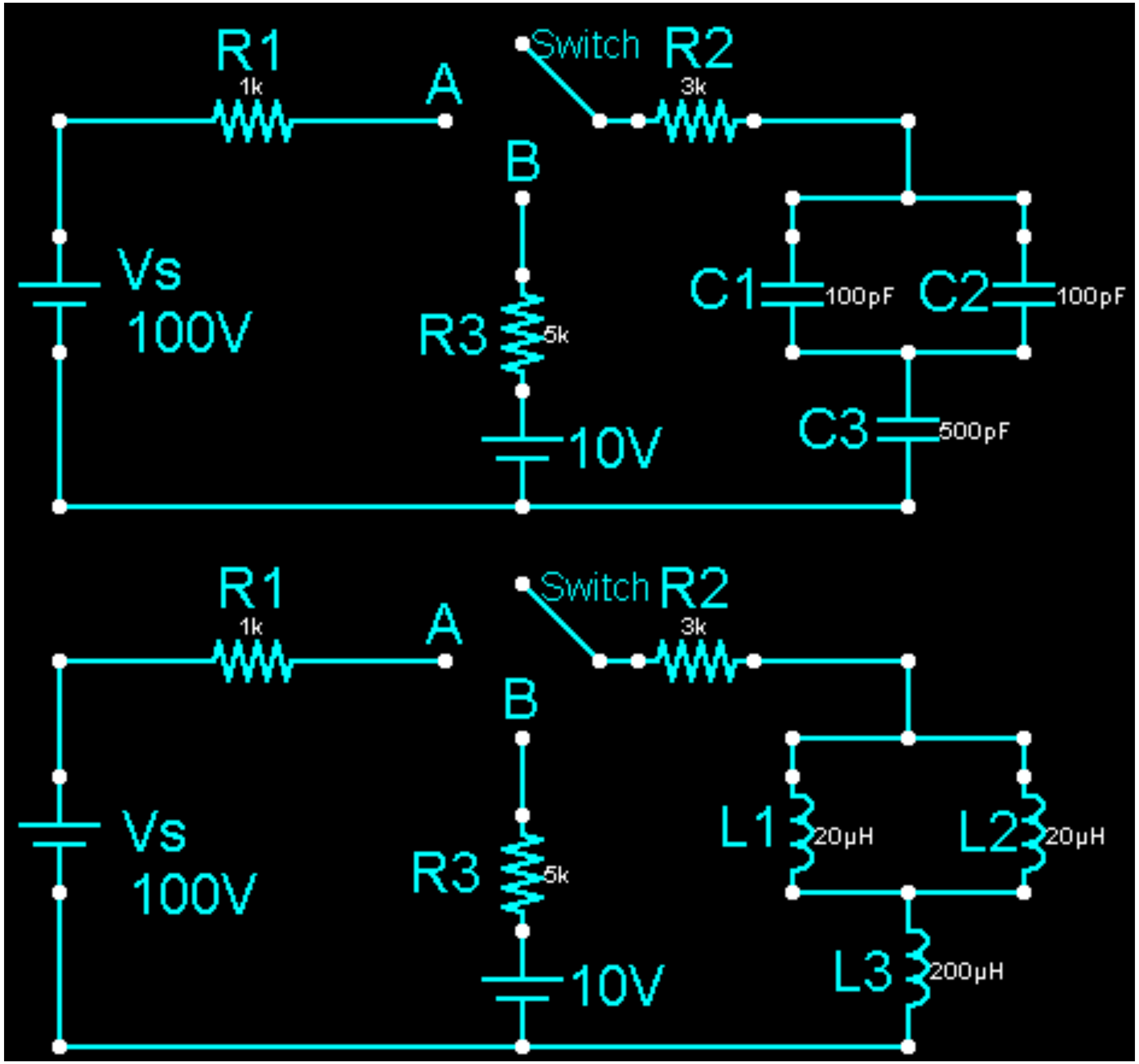
The Time Constant

- Now just put them together



$$\tau = R_{eq}C_{eq} = (R_1||R_2)(C_1 + C_2)$$

Transient Analysis Example (RC & RL)



R1=1kOhms, R2= 3kOhms, R3= 5kOhms
C1=C2=100pF, C3=500pF,
L1=L2=20uH, L3=200uH

Questions: (solve for both circuits)

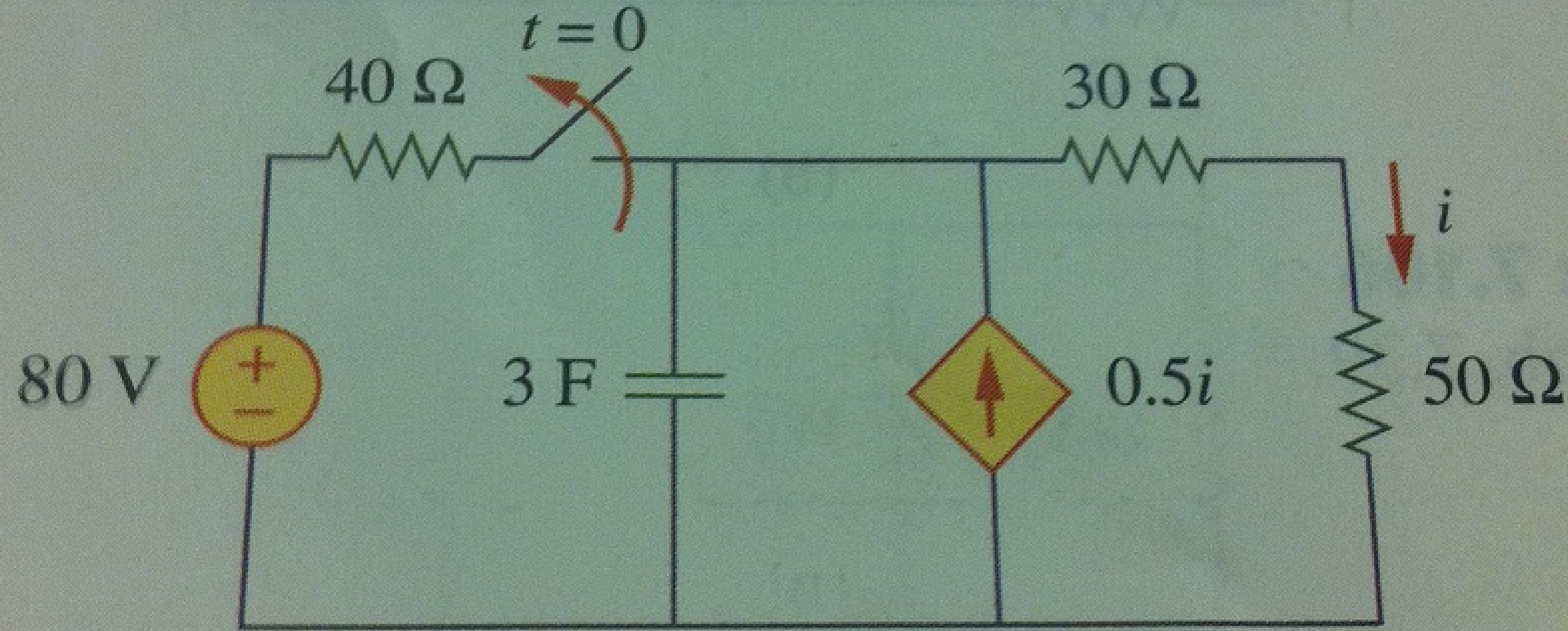
1. If both switches are connected to node A at t=0, what will be voltage across R2 after 1ms, 1s and 1000s?

2. If both switches are connected connected to node B when voltage across R2 = 50V, how long will it take (from connection to B) for voltage across R3 to reach 10V?

RC Transient Question 1

Figure 7.109
For Prob. 7.42.

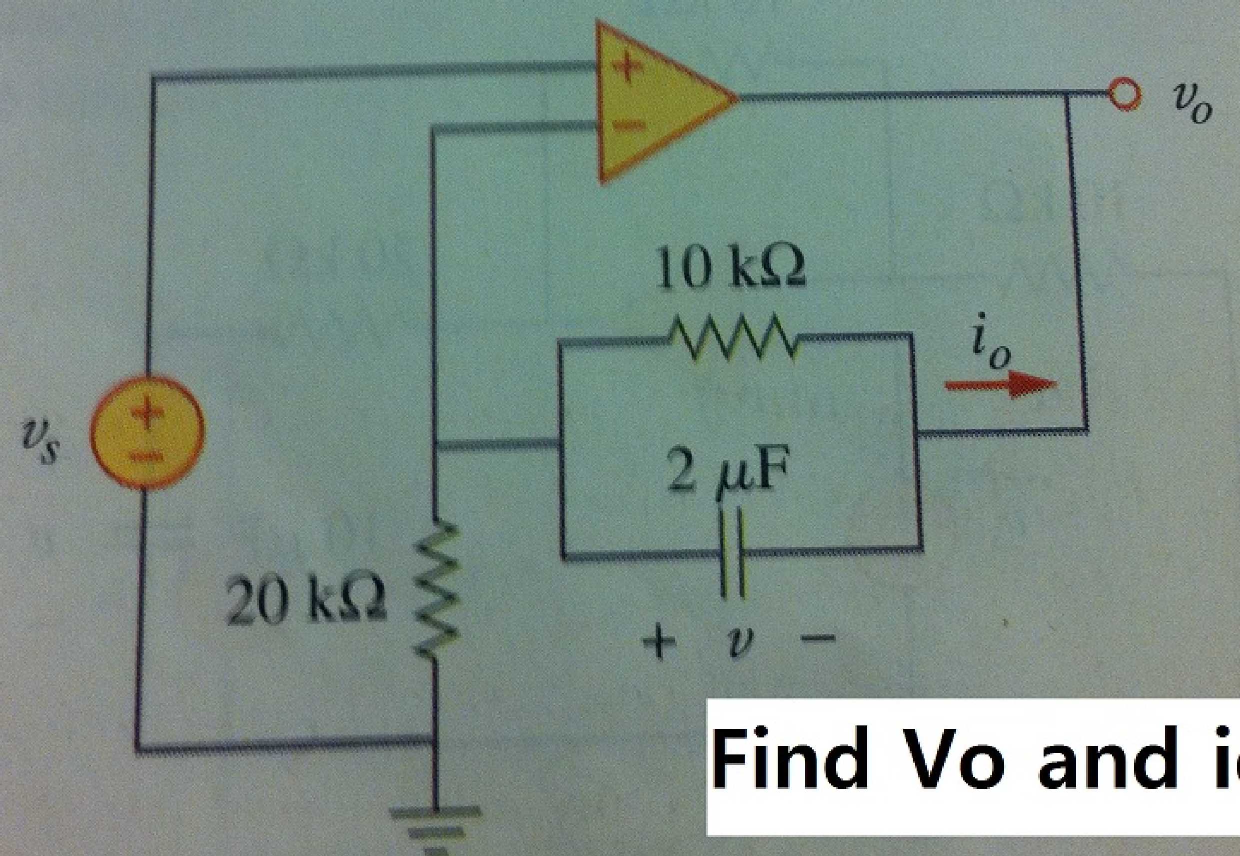
7.43 Consider the circuit in Fig. 7.110. Find $i(t)$ for $t < 0$ and $t > 0$.



RC circuit Question 2 (with an Op Amp)

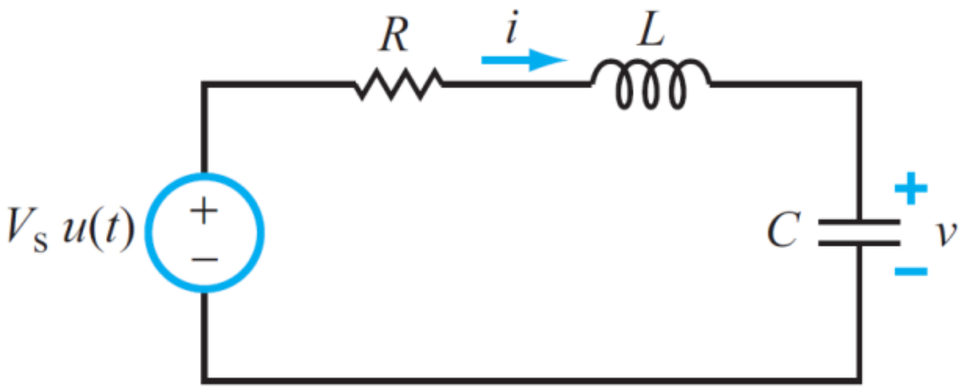
Figure 7.139
For Prob. 7.74.

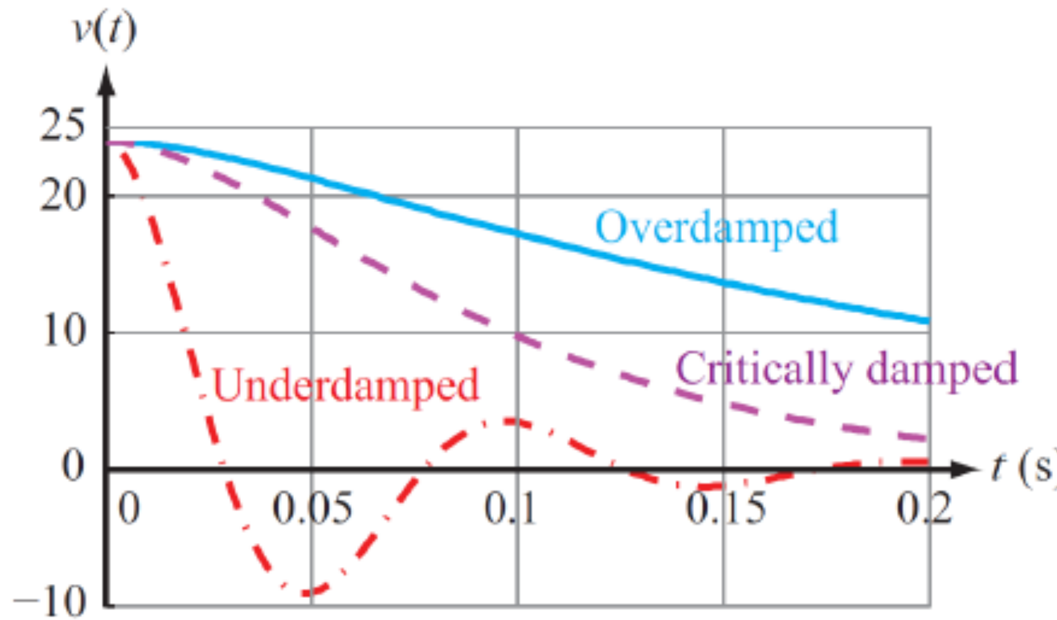
7.75 In the circuit of Fig. 7.140, find v_o and i_o, given that $v_s = 4u(t)$ V and $v(0) = 1$ V.



Find Vo and io

Solutions to 2nd Order Diff Eq's and Damping





$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^{\,2}}$$

$\alpha =$ damping factor

$\omega_0 =$ resonant frequency

$$\alpha = \frac{R}{2L} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

Overdamped ($\alpha > \omega_0$)

$$v(t) = v(\infty) + A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

Critically Damped ($\alpha = \omega_0$)

$$v(t) = v(\infty) + (B_1 + B_2 t)e^{-\alpha t}$$

Underdamped ($\alpha < \omega_0$)

$$v(t) = v(\infty) + e^{-\alpha t}(D_1 \cos \omega_d t + D_2 \sin \omega_d t)$$

AC Response

1. Complex Numbers

2. Phasor Domain

3. Impedance

4. How phasor method works

5. General Strategy

6. Question

Complex Numbers

We will find it is useful to represent sinusoids as complex numbers

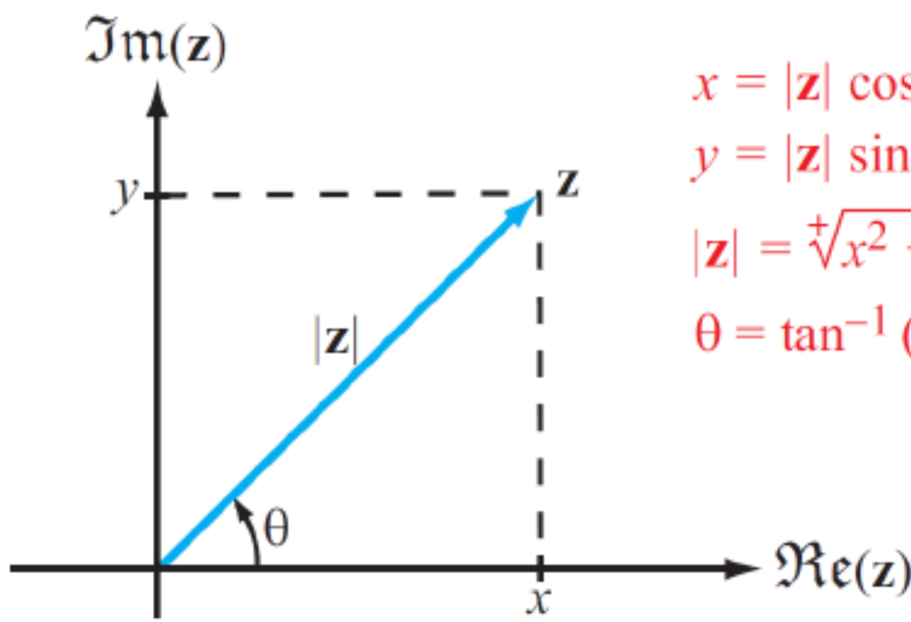
$$j = \sqrt{-1}$$

$z = x + jy$ **Rectangular coordinates**

$z = |z|\angle\theta = |z|e^{j\theta}$ **Polar coordinates**

$$\mathrm{Re}(z) = x$$

$$\mathrm{Im}(z) = y$$



$x = |\mathbf{z}| \cos \theta$

$y = |\mathbf{z}| \sin \theta$

$|\mathbf{z}| = +\sqrt{x^2 + y^2}$

$\theta = \tan^{-1}(y/x)$

Relations based on Euler's Identity

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

Complex Number Arithmetic

Do you know how to:

Express complex numbers in both rectangular and polar forms

Add complex numbers

Subtract complex numbers

Multiply complex numbers

Divide complex numbers?

I'm sure you do! :)

Phasor Domain

$$
\begin{aligned}
v(t) &= V_0 \cos(\omega t + \phi) \\
&= \mathfrak{Re}[\underbrace{V_0 e^{j\phi}}_{\text{Phasor counterpart of } v(t)} e^{j\omega t}]
\end{aligned}
$$

Time Domain		**Phasor Domain**
$v(t) = V_0 \cos \omega t$	$\leftrightarrow$	$\mathbf{V} = V_0$
$v(t) = V_0 \cos(\omega t + \phi)$	$\leftrightarrow$	$\mathbf{V} = V_0 e^{j\phi}.$

If $\phi = -\pi/2$,

$$v(t) = V_0 \cos(\omega t - \pi/2) \quad \leftrightarrow \quad \mathbf{V} = V_0 e^{-j\pi/2}.$$

Impedance

Impedance is voltage/current

$$\mathbf{Z} = R + jX$$

$R = \text{resistance} = \text{Re}(Z)$

$X = \text{reactance} = \text{Im}(Z)$

Resistor	$\mathbf{Z} = R$
Inductor	$\mathbf{Z} = j\omega L$
Capacitor	$\mathbf{Z} = 1/\,j\omega C$

From Ohm's Law,

V = I*Z

Z = V / I

Ha, That was easy! :)

How Phasor Method Works

Equivalent Circuit
(Power of Abstraction)

Find:
Input Impedance
Input voltage
Input Current

From Ohm's Law again,

$V_{in} = I_{in} * Zeq$

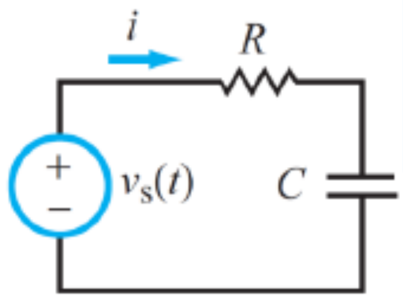
$I_{in} = V_{in} / Zeq$

$Zeq = V_{in} / I_{in}$

That's all there is... :)

General Strategies for Phasors

Step 1
Adopt Cosine Reference
(Time Domain)



$v_s(t) = 12\sin(\omega t - 45°)$ (V

Step 3
Cast Equations in
Phasor Form

$$\mathbf{I}\left(R + \frac{1}{j\omega C}\right) = \mathbf{V}_s$$

Step 4
Solve for Unknown Variable
(Phasor Domain)

$$\mathbf{I} = \frac{\mathbf{V}_s}{R + \dfrac{1}{j\omega C}}$$

Step 2
Transfer to Phasor Domain

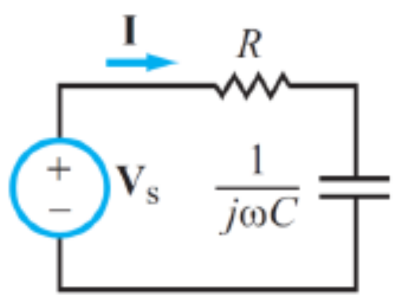
$i \longrightarrow \mathbf{I}$

$v \longrightarrow \mathbf{V}$

$R \longrightarrow \mathbf{Z}_R = R$

$L \longrightarrow \mathbf{Z}_L = j\omega L$

$C \longrightarrow \mathbf{Z}_C = 1/j\omega C$



$\mathbf{V}_s = 12e^{-j135°}$ (V)

Step 5
Transform Solution
Back to Time Domain

$$i(t) = \Re\mathfrak{e}[\mathbf{I}e^{j\omega t}]$$
$$= 6\cos(\omega t - 105°)$$
(mA)

Frequency Response

1. System H(jw)

2. What comes out of the system

3. Bode Plot

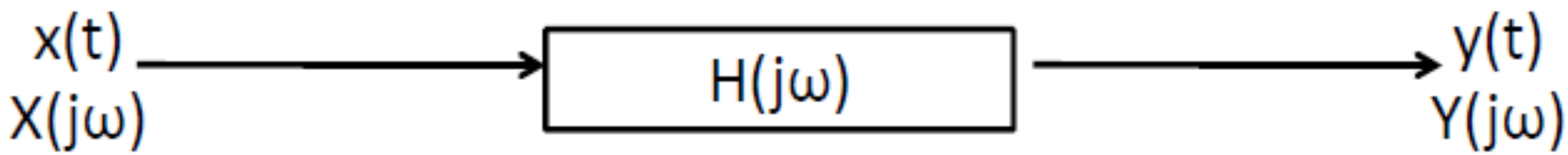
4. Summary of First Order System

5. Summary of Second Order System

6. Circuit Intuition

A Quick Review of Frequency Response

Transfer Function of a System (circuit in this case):
Scales and shifts the input



V_{in} -> $H_v(jw)$ -> V_{out}

For a voltage gain system, $H_v(jw)$, if the Input is

$$V_{in} = v_s \cos(\omega t + \phi)$$

Then the Output is (scaled and shifted version of input)

$$V_O = |H(j\omega)| v_s \cos\left(\omega t + \phi + \angle H(j\omega)\right)$$

Where

$$|H(j\omega)| = \sqrt{Re\left(H(j\omega)\right)^2 + Im\left(H(j\omega)\right)^2}$$

$$\angle H(j\omega) = \tan^{-1} Im\left(H(j\omega)\right)/Re\left(H(j\omega)\right)$$

First Order Circuit Summary

Low Pass Filter

- Canonical form
$$H(j\omega) = \frac{1}{1 + j\omega\tau}$$
- DC magnitude response
  - Short inductors, open capacitors, solve for V_{SS}
- HF magnitude rolloff
  - -20 dB/decade
- DC phase = 0°
- HF phase = -90°
- Corner frequency $\omega_c = 1/\tau$
  - Magnitude plot: -3 dB pt
  - Phase plot: -45° pt

High Pass Filter

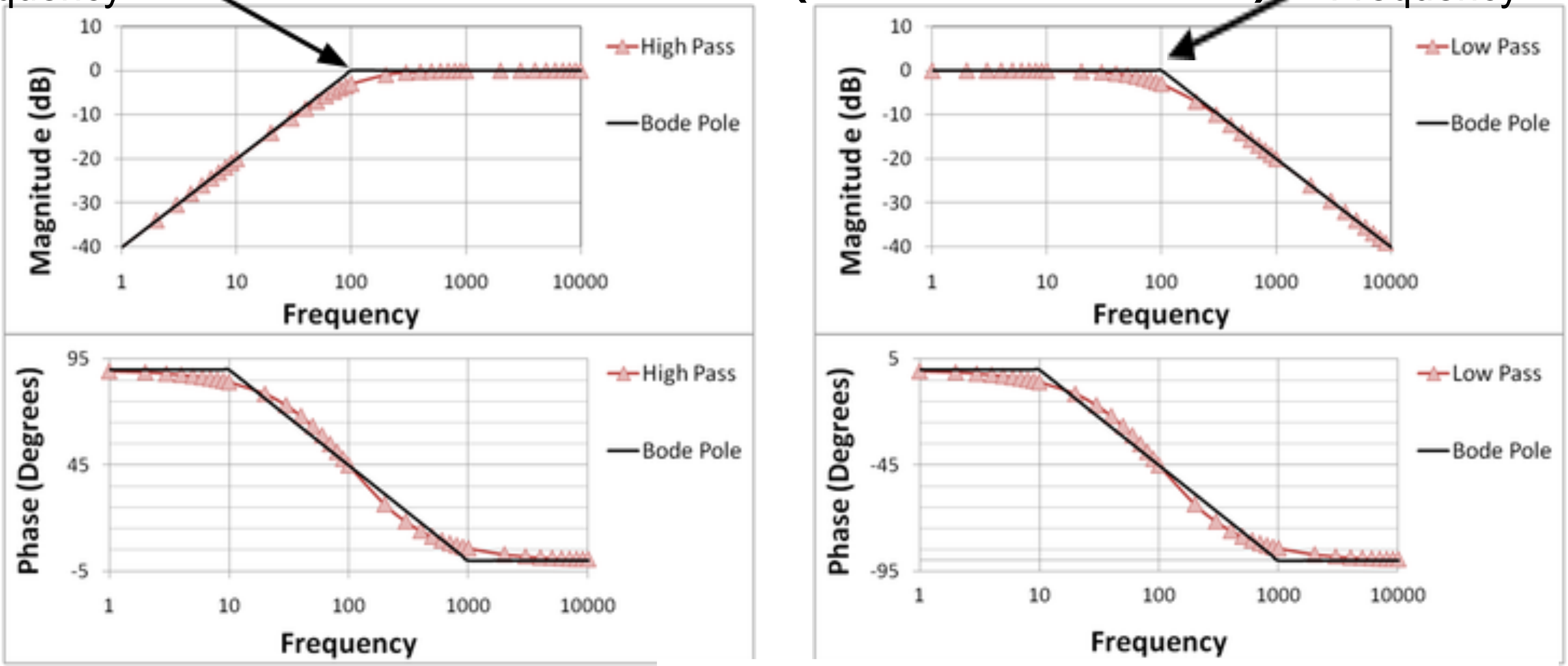
- Canonical form
$$H(j\omega) = \frac{j\omega\tau}{1 + j\omega\tau}$$
- DC magnitude rolloff
  - +20 dB/decade
- HF magnitude response
  - Open inductors, short capacitors, solve for V_{SS}
- DC phase = 90°
- HF phase = 0°
- Corner frequency $\omega_c = 1/\tau$
  - Magnitude plot: -3 dB pt
  - Phase plot: 45° pt

$\omega_c = 2\pi f_c$
$= 1/\tau$
$= 1/(R_{eq}C_{eq})$ (for an RC Circuit)

Bode Plots (First Order)

Corner Frequency

Corner Frequency



Bode plots are approximations of graphs of Magnitude and Phase vs. frequency relationship of a filter.

They tell you how the magnitude and phase shift of the transfer function change with respect to the frequency of **the input**

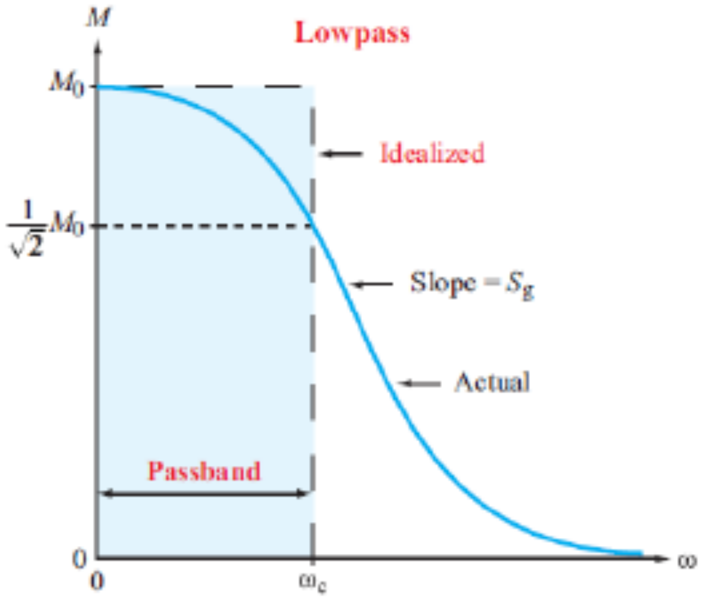
Low Pass Filter

- DC magnitude response
  - Short inductors, open capacitors, solve for V_{SS}
- HF magnitude rolloff
  - -20 dB/decade
- DC phase = 0°
- HF phase = -90°
- Corner frequency $\omega_c = 1/\tau$
  - Magnitude plot: -3 dB pt
  - Phase plot: -45° pt

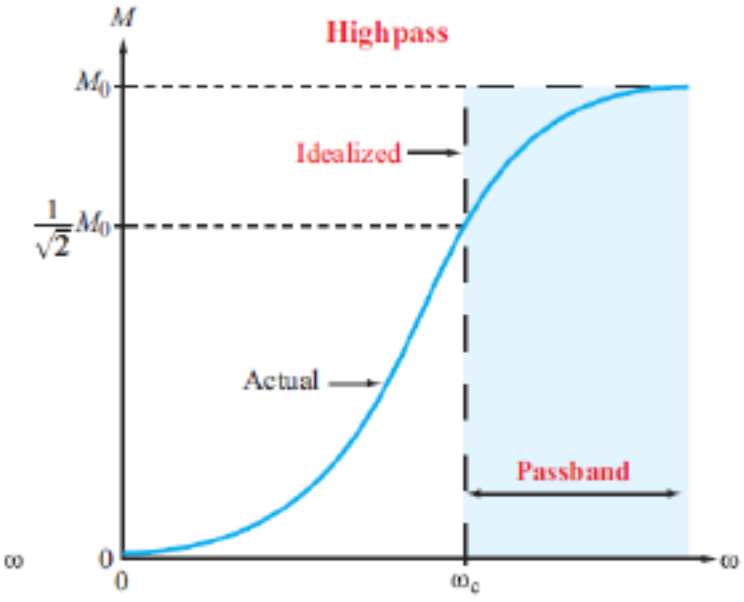
High Pass Filter

- DC magnitude rolloff
  - +20 dB/decade
- HF magnitude response
  - Open inductors, short capacitors, solve for V_{SS}
- DC phase = 90°
- HF phase = 0°
- Corner frequency $\omega_c = 1/\tau$
  - Magnitude plot: -3 dB pt
  - Phase plot: 45° pt

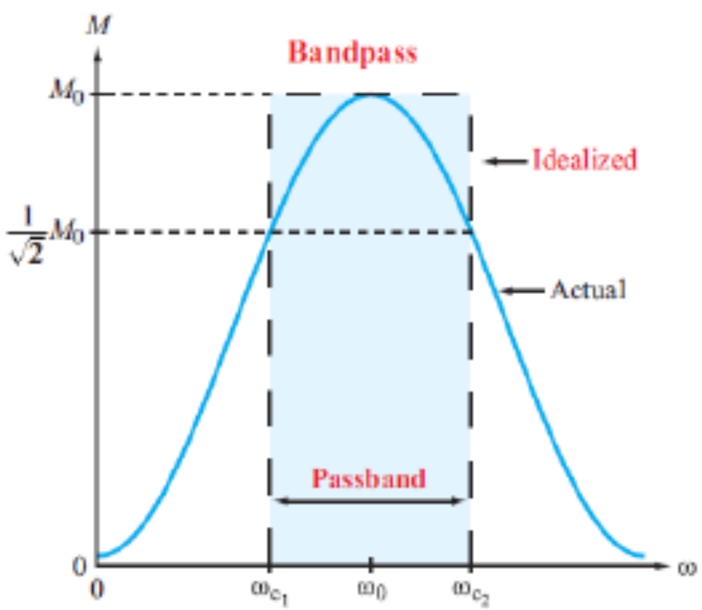
Magnitude Plot



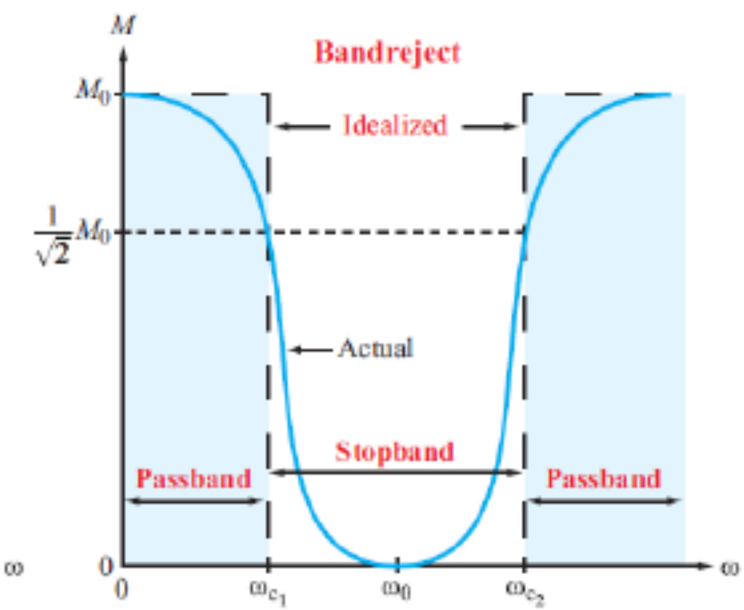
(a) Lowpass filter



(b) Highpass filter



(c) Bandpass filter



(d) Bandreject filter

Second Order Filters

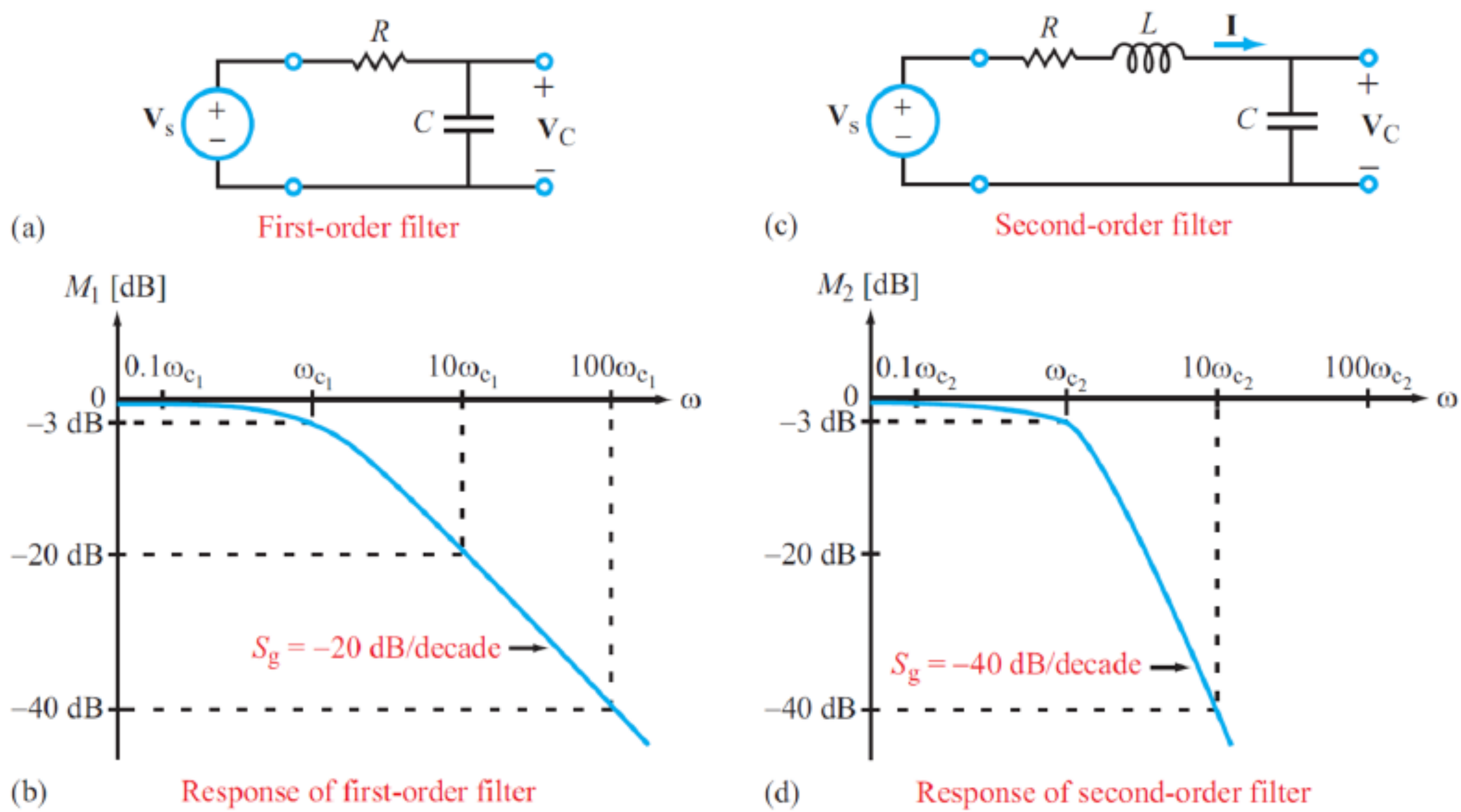
- You should be able to identify what kind of filter a second order system is
- You can easily do this by examining the DC and HF magnitude response
- There are several ways to implement the same kind of filter!

- You also need to be able to identify the resonant frequency and the quality factor
  - Resonant frequency always $\omega_0 = \frac{1}{\sqrt{LC}}$
  - Quality factor either…
    - Series $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$
    - Parallel $Q = R\sqrt{\frac{C}{L}}$

1st Order vs. 2nd Order Filters

These are low-pass 1st and 2nd order filters.
1st and 2nd order High-pass filters have
a shape of a high-pass filter (which is?)
and positive slopes, 20dB/dec and 40dB/dec,
respectively

Filter Order

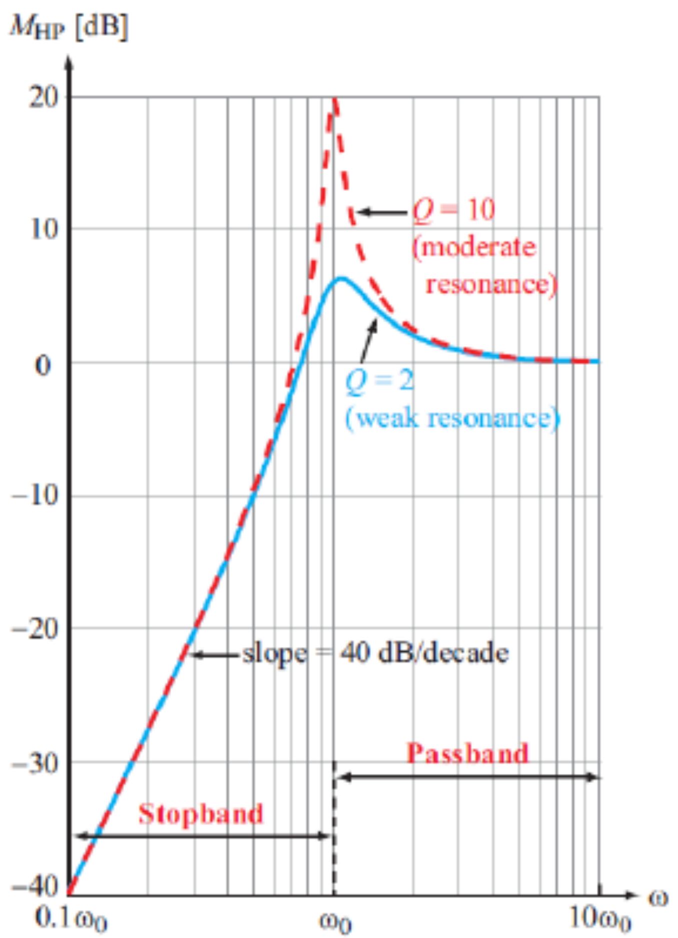


Note the difference in slope;
Second order filters have a higher slope, |40dB/dec|, than that of First order filters, |20dB/dec|

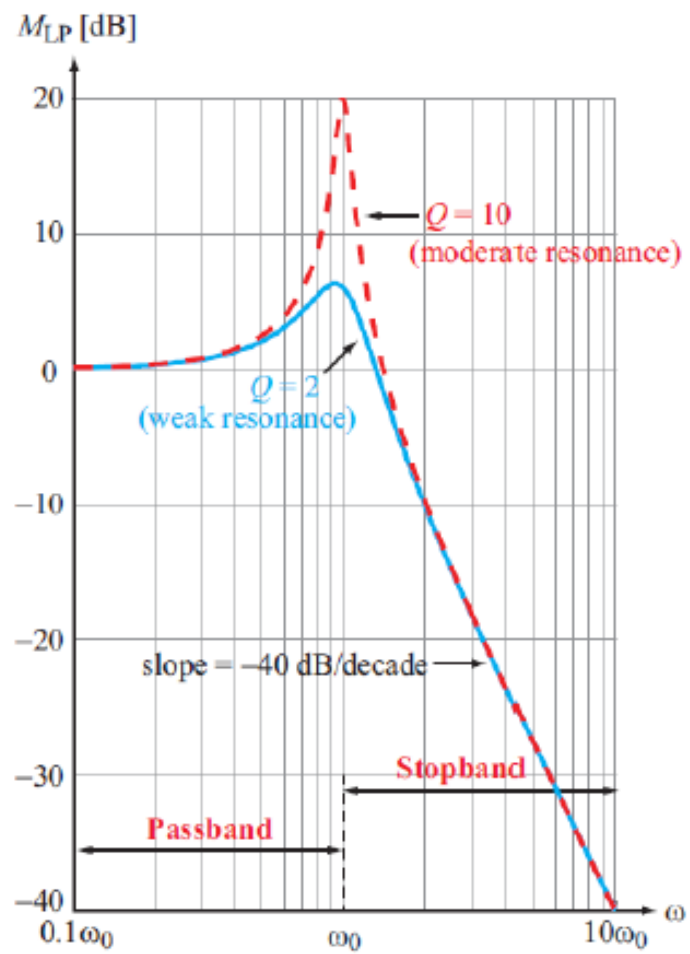
W_o (Resonant Frequency) and Q (Quality Factor) (HP & LP)

To examine the roles of W_o and Q, let us look at magnitude plots for HP & LP Filters

Highpass Filter Lowpass Filter



(b) Magnitude spectrum



(b) Magnitude spectrum

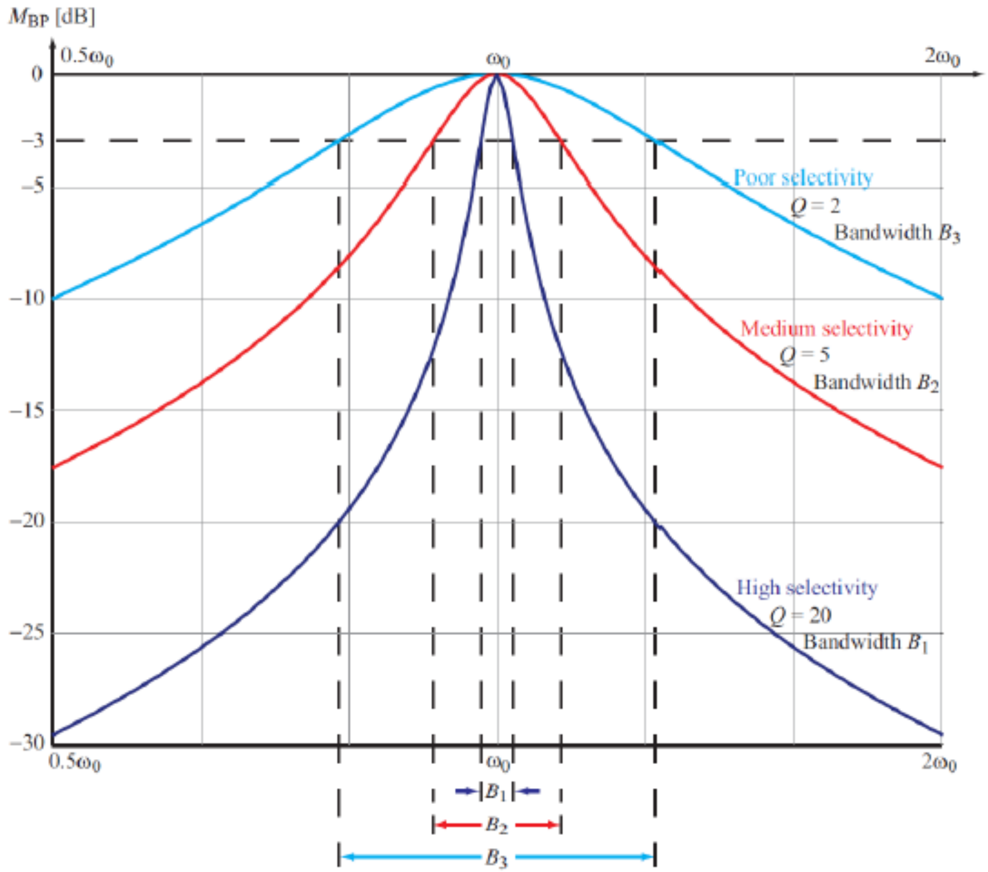
at wo,
the circuit "resonates:"
highest magnitude point

Q factor determines
the resonance, or the
max magnitude value

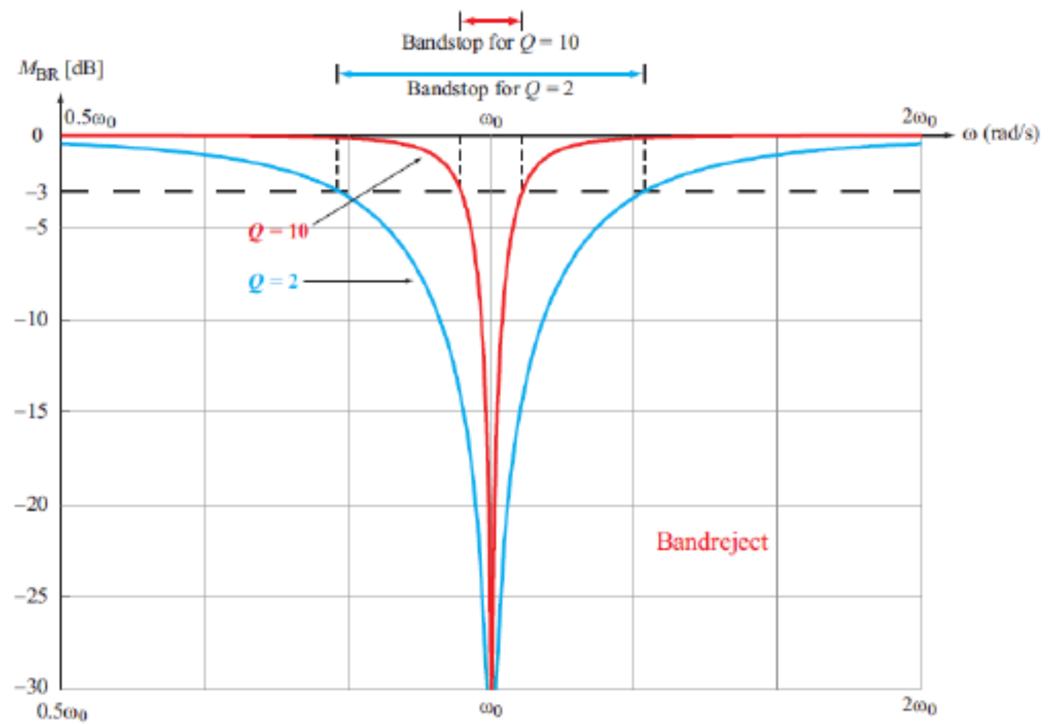
W_o (Resonant Frequency) and Q (Quality Factor) (BP & BR)

For Band-pass and Band-reject filters, it's a bit different.

Effect of Q



Bandreject Filter



For Band pass and Band reject filters
Wo determines the middle point of the band.
Q determines the bandwidth and the slope.
(Higher Q = Smaller Bandwidth and vice versa)

Circuit Intuition

Circuit? Intuitive? What??? lol

Believe it or not, circuits can be *intuitive*
if you think about it a little harder.
These exercises are designed to build
your **qualitative intuition** in identifying different
types of filters.

Just remember,

1. Capacitors pass high frequency inputs
2. Inductors pass low frequency inputs

On the other hand,

3. Capacitors block low frequency inputs
4. Inductors block high frequency inputs

<u>Try not to write down any equations</u> during this exercise.
Rather, examine the filter outputs for inputs with:
Low frequencies (0 Hz)
Middle frequencies
High frequencies (inf)

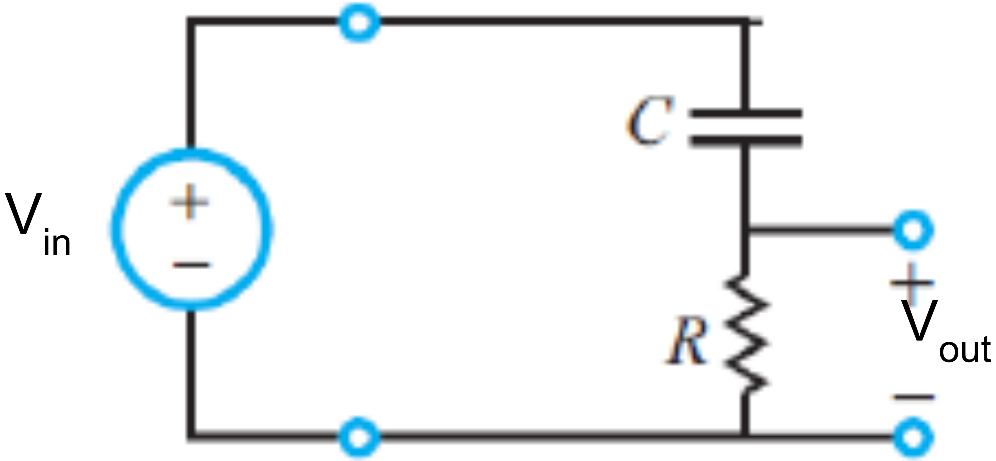
In case you're a visual person like me, :)

	Low Frequency (e.g. w=0)	High Frequency (e.g. w=1 GHz)
Capacitor	Block!	Pass!
Inductor	Pass!	Block!

Your good intuition might save you some time on exam / circuit designs

Circuit Intuition (RC)

Let's warm up with a simple one

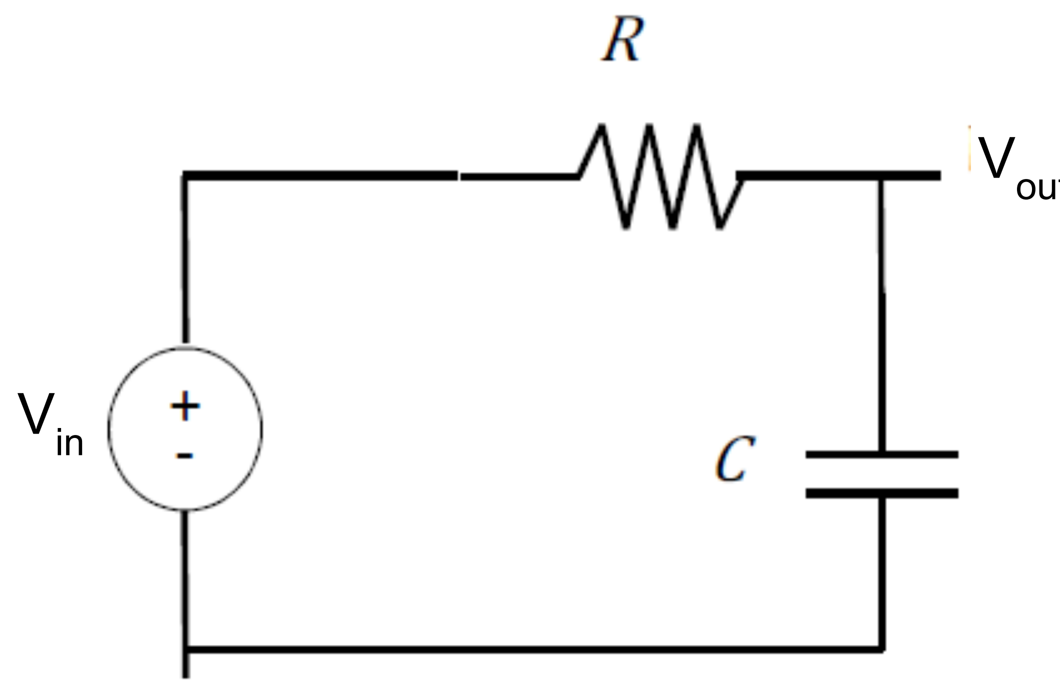


(a) RC circuit

Is this:
1. High-Pass Filter
2. Low-Pass Filter
3. Band-Pass Filter
4. Band-Reject Filter
5. I-don't-know Filter

answer: HP

mkay... what about this one?

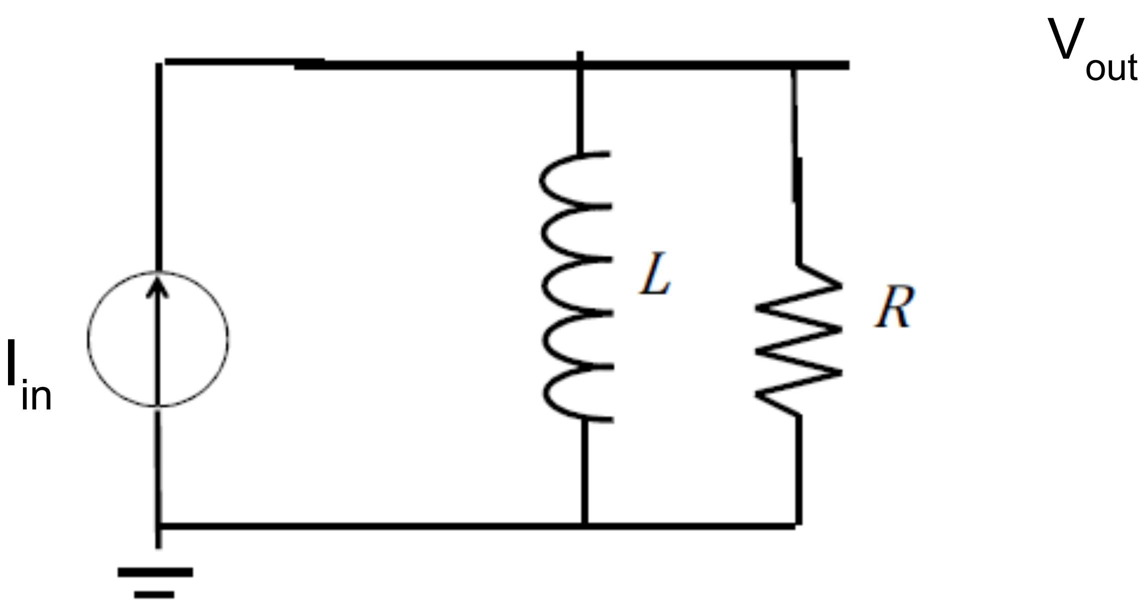


1. High-Pass Filter
2. Low-Pass Filter
3. Band-Pass Filter
4. Band-Reject Filter
5. I-don't-know Filter

answer: LP

Circuit Intuition (RL) (still 1st order)

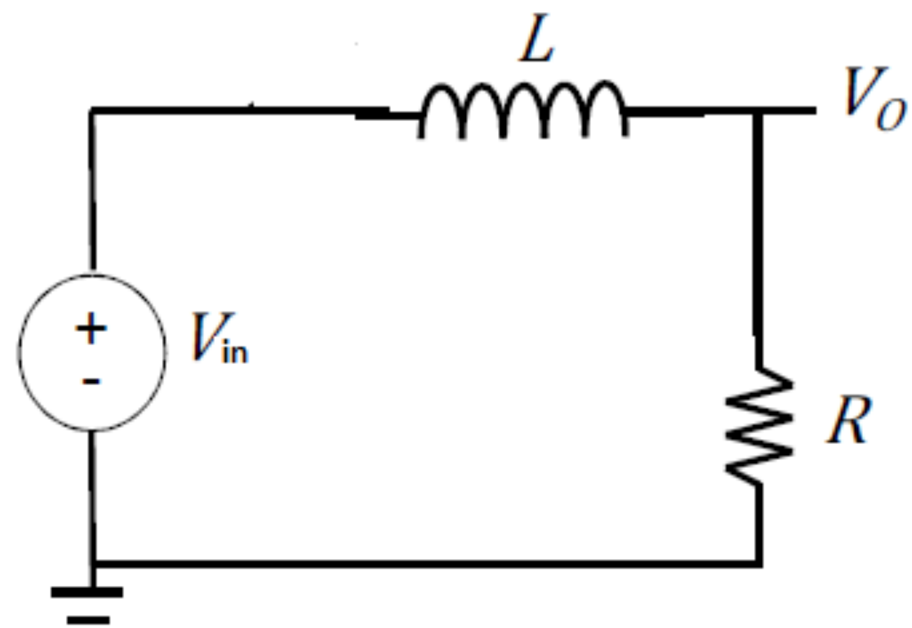
Try one with an inductor
and a current source



1. High-Pass Filter
2. Low-Pass Filter
3. Band-Pass Filter
4. Band-Reject Filter
5. I-don't-know Filter

answer: HP

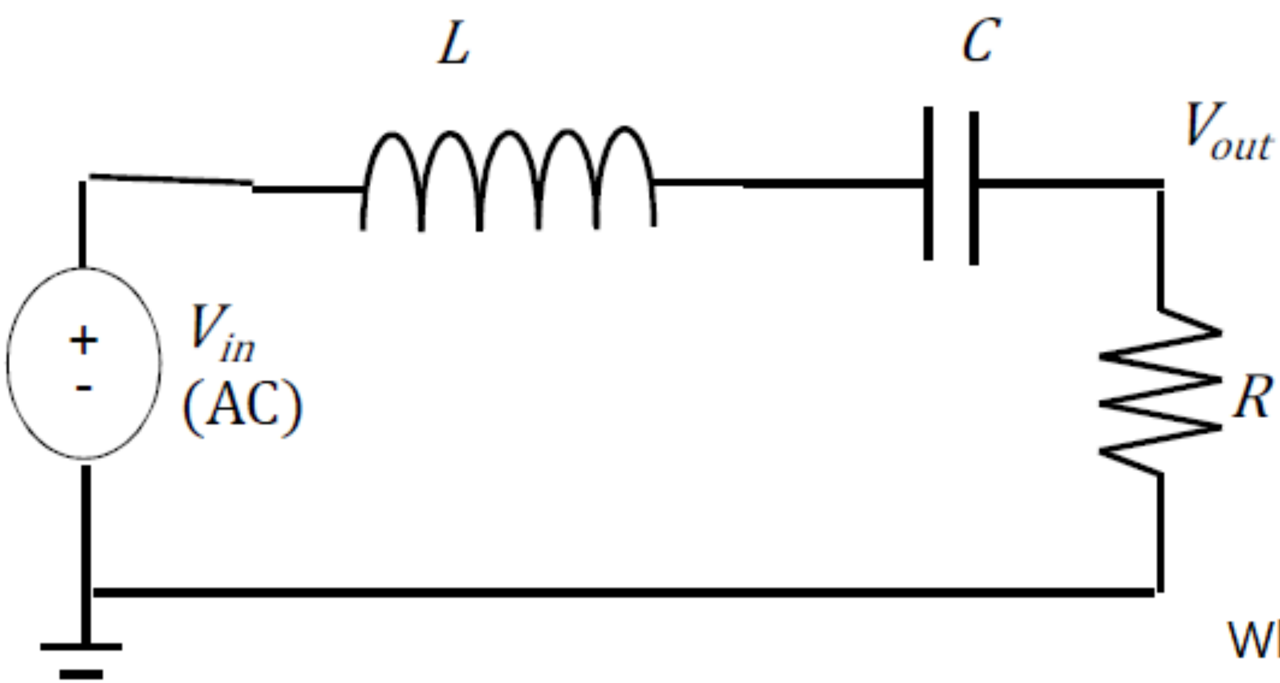
Or not...

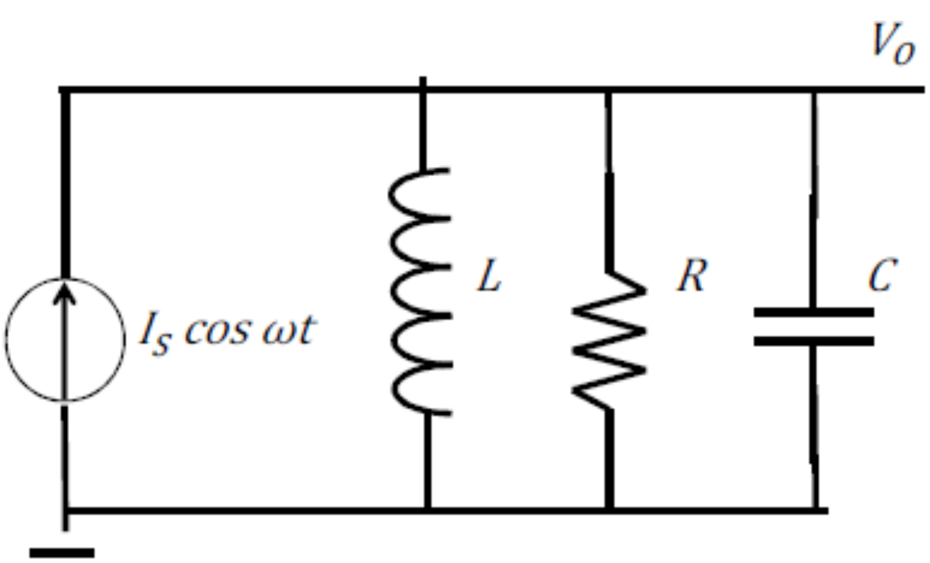


1. High-Pass Filter
2. Low-Pass Filter
3. Band-Pass Filter
4. Band-Reject Filter
5. I-don't-know Filter

answer: LP

Circuit Intuition (RLC - 2nd Order)





What Kind of Filter are they?

A = Low pass filter

B = High pass filter

C = Band pass filter

D = Band stop filter

Hint: They are the same type

answer: BP

Circuit Intuition (RLC - 2nd Order)

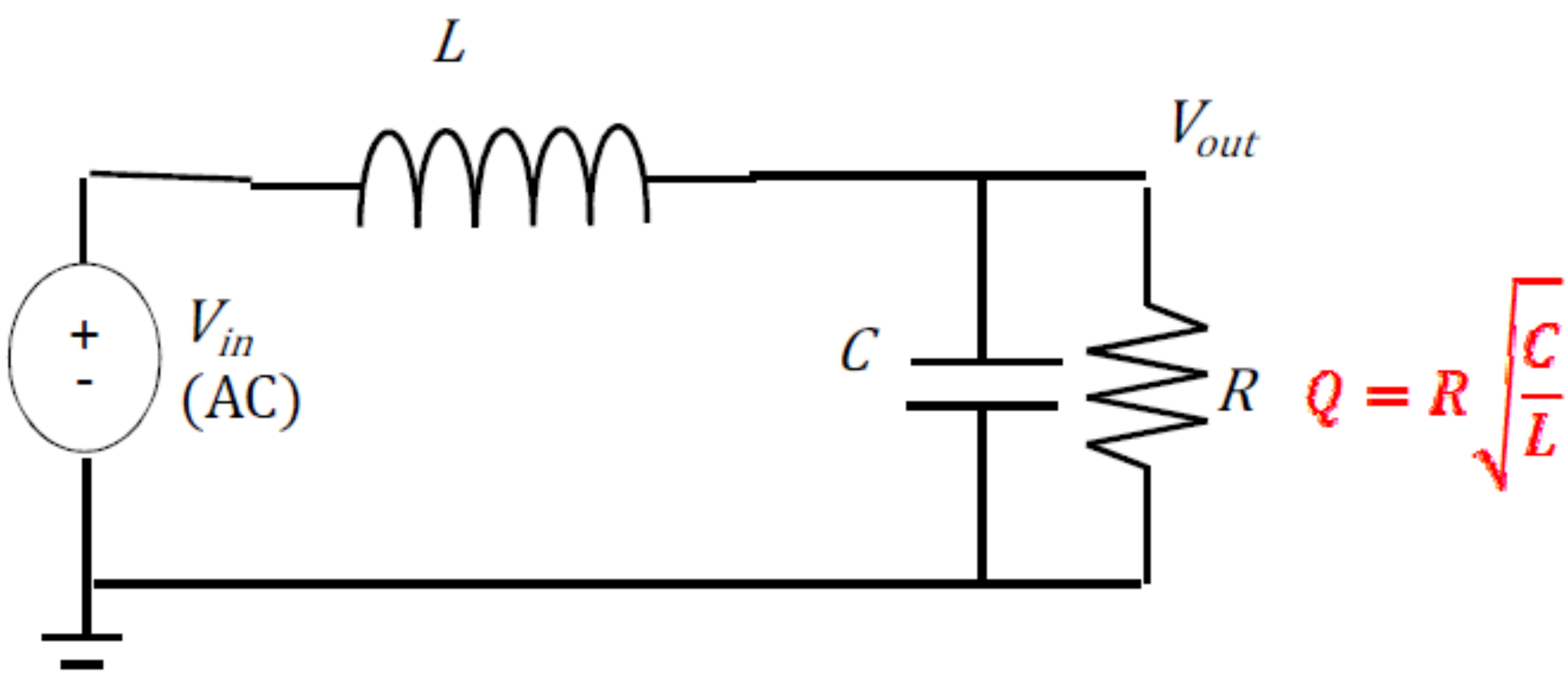
What kind of filter is this circuit?

A = Low pass filter

B = High pass filter

C = Band pass filter

D = Band stop filter



answer: LP

Circuit Intuition (RLC - 2nd Order)

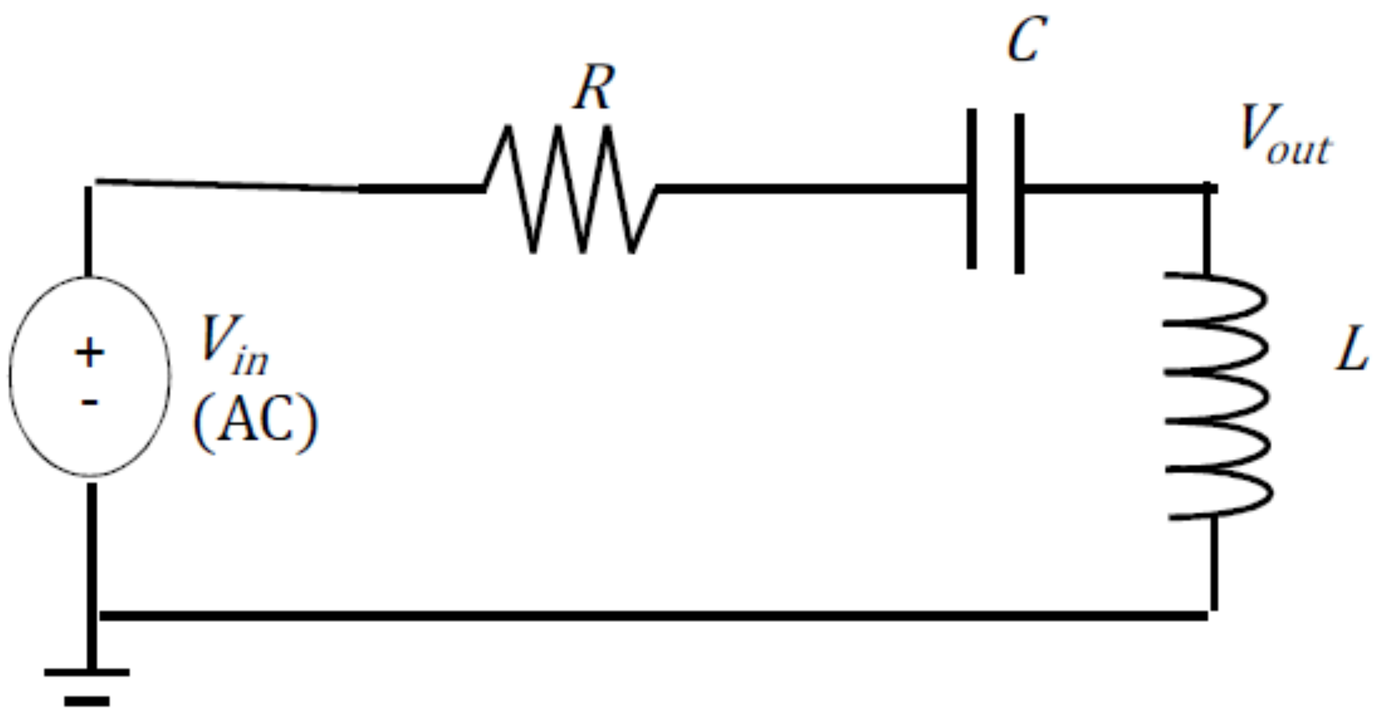
What kind of filter is this circuit?

A = Low pass filter

B = High pass filter

C = Band pass filter

D = Band stop filter

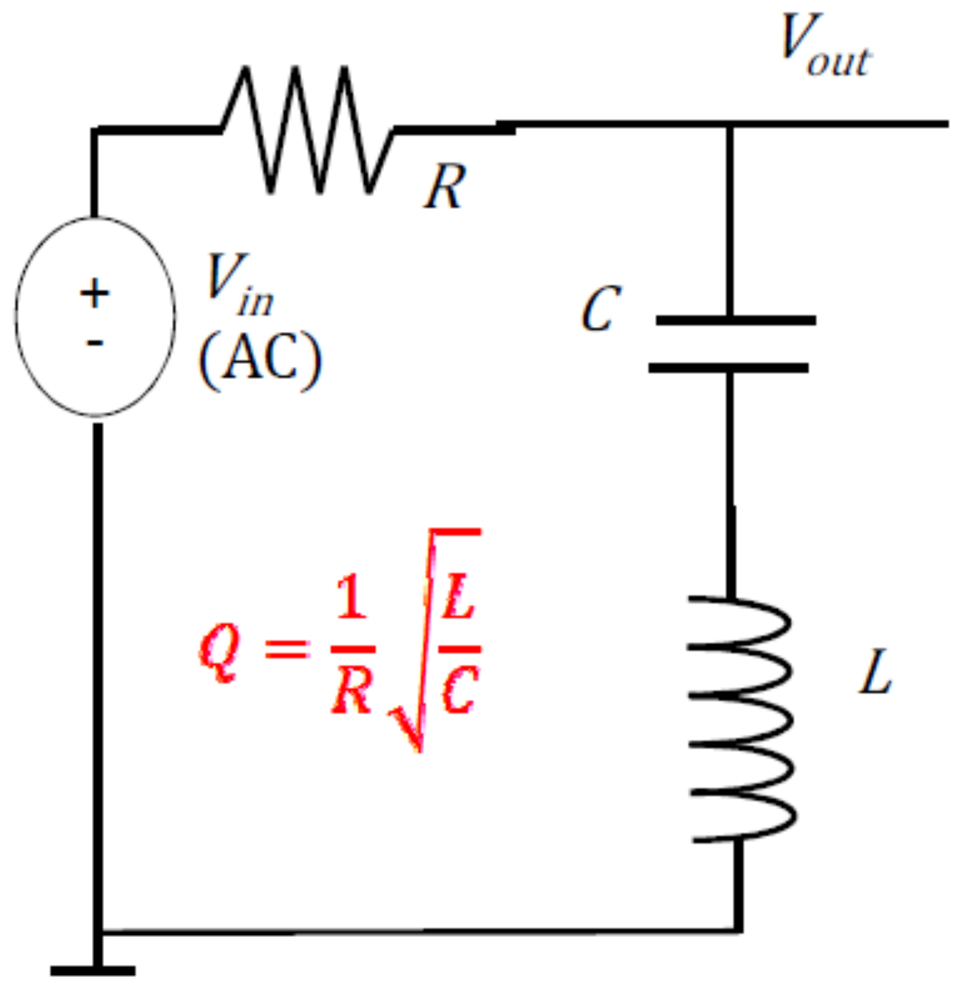


$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

answer: HP

Circuit Intuition (RLC - 2nd Order)

- What kind of filter is this circuit?
  - A = Low pass filter
  - B = High pass filter
  - C = Band pass filter
  - D = Band stop filter

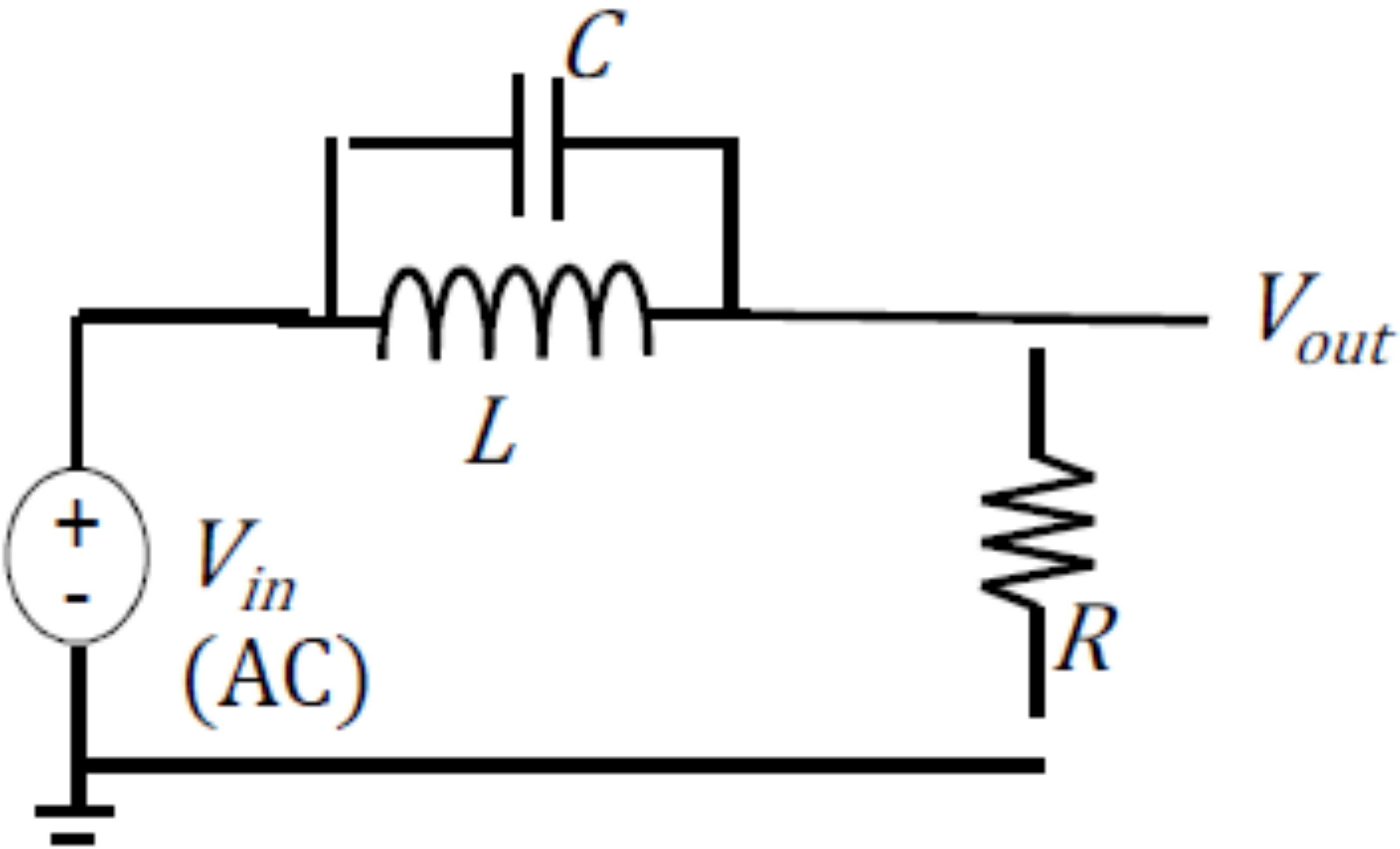


$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Darn it, I just gave it away! lol

Circuit Intuition (RLC - 2nd Order)

Now you can tell me what this one is:



Guess what, this is the same type as the last one (Band-Reject)

Circuit Intuition (cont'd)

Now, if you have time, go back to the circuits examples and

1. Determine the Transfer Function, H(w) (canonical forms?) to convince yourself that the answers are right.
2. Calculate w_c or w_o, whichever is appropriate.
3. Calculate the quality factor, Q (for 2nd order filters)

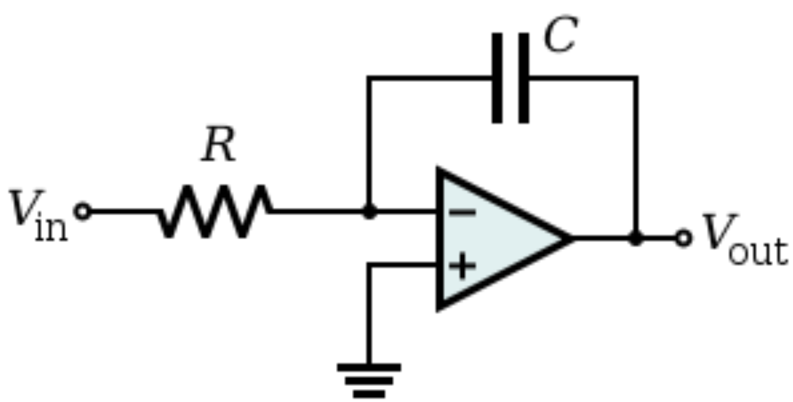
After all, I hope you're more familiar with different types of filters and their different configurations.

Again, your intuition will come in handy on the exam because you will have some idea of what you're going for.

Active Filtering

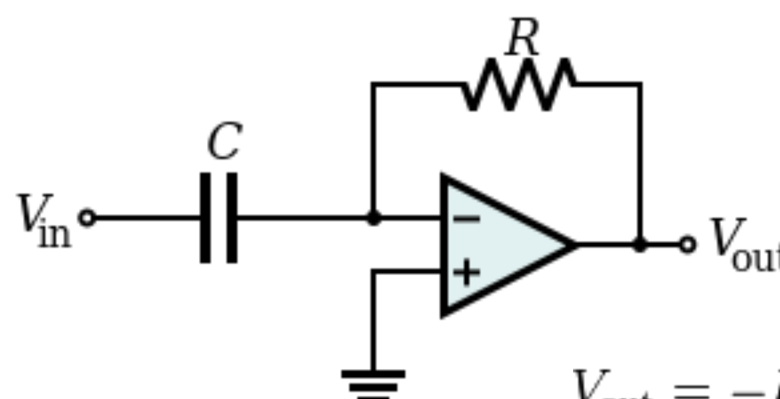
1st or 2nd Order Filters + Op Amps
Examples:

Inverter Integrater



$$V_{\text{out}} = -\int_0^t \frac{V_{\text{in}}}{RC}\,\mathrm{d}t + V_{\text{initial}}$$

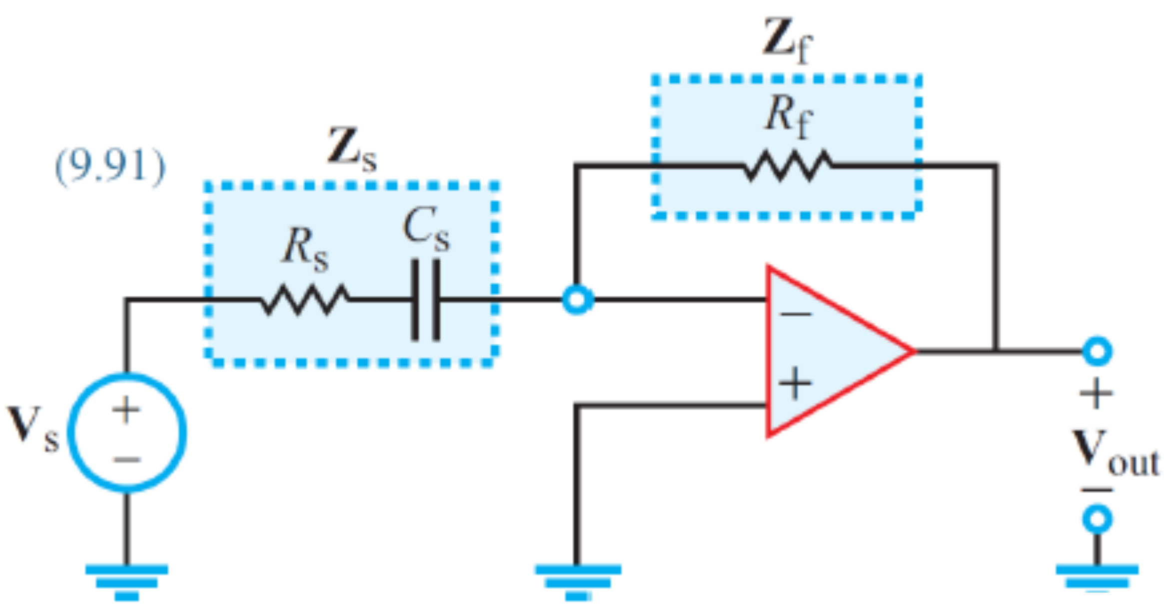
Inverter Differentiater



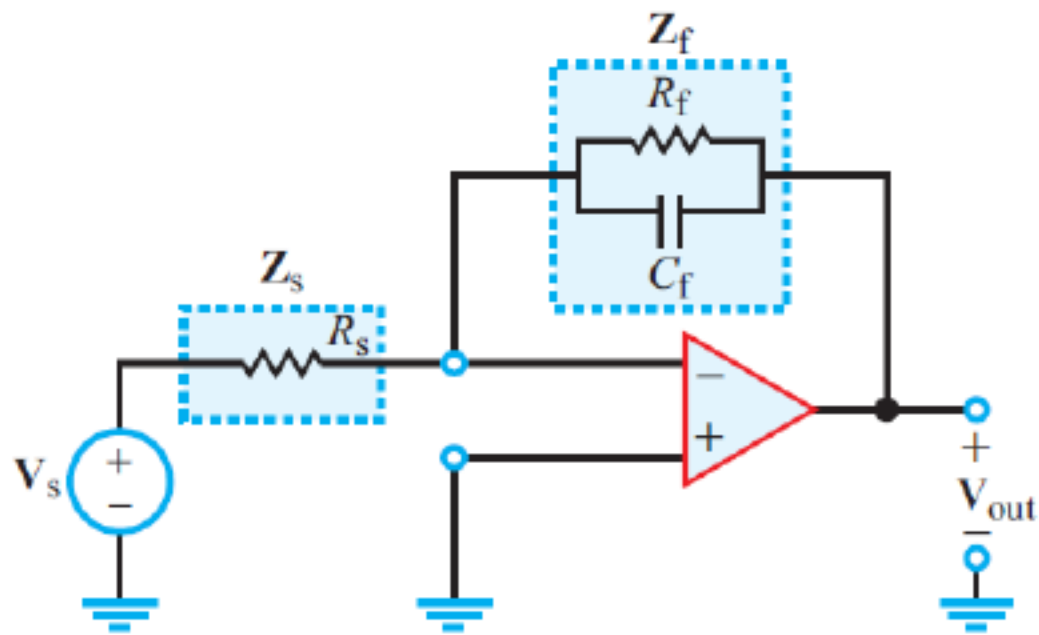
$$V_{\text{out}} = -RC\,\frac{\mathrm{d}V_{\text{in}}}{\mathrm{d}t}$$

Active Filtering (2nd Order)

(9.91)

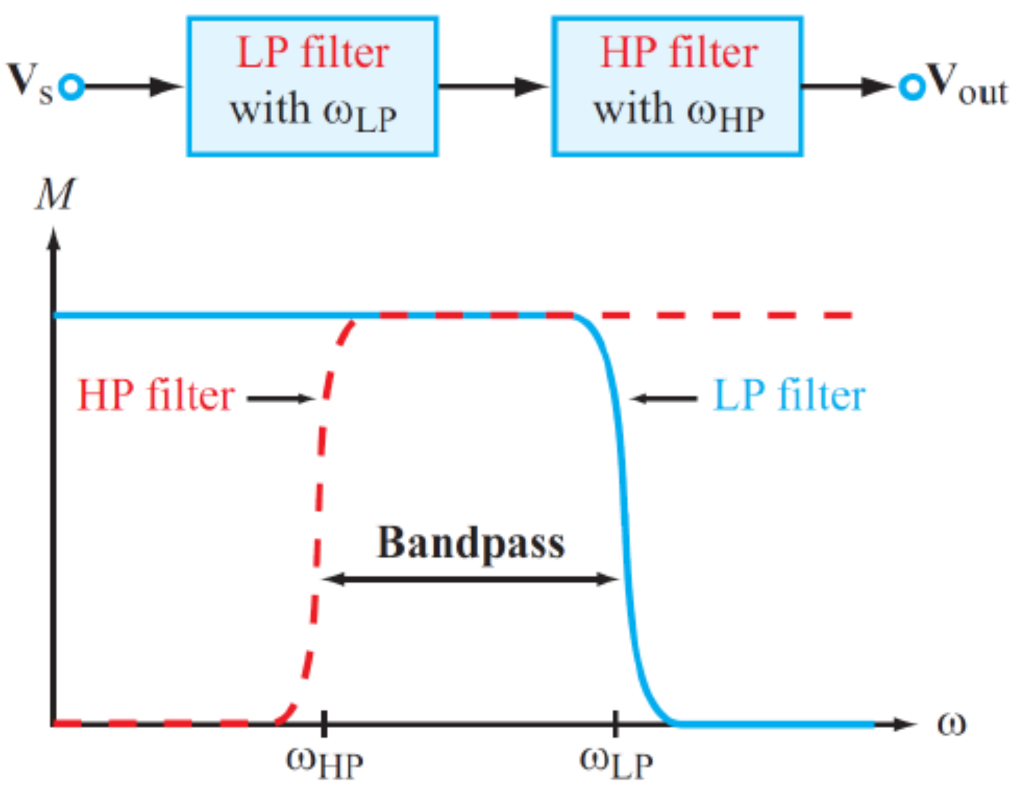


High Pass or Low Pass?

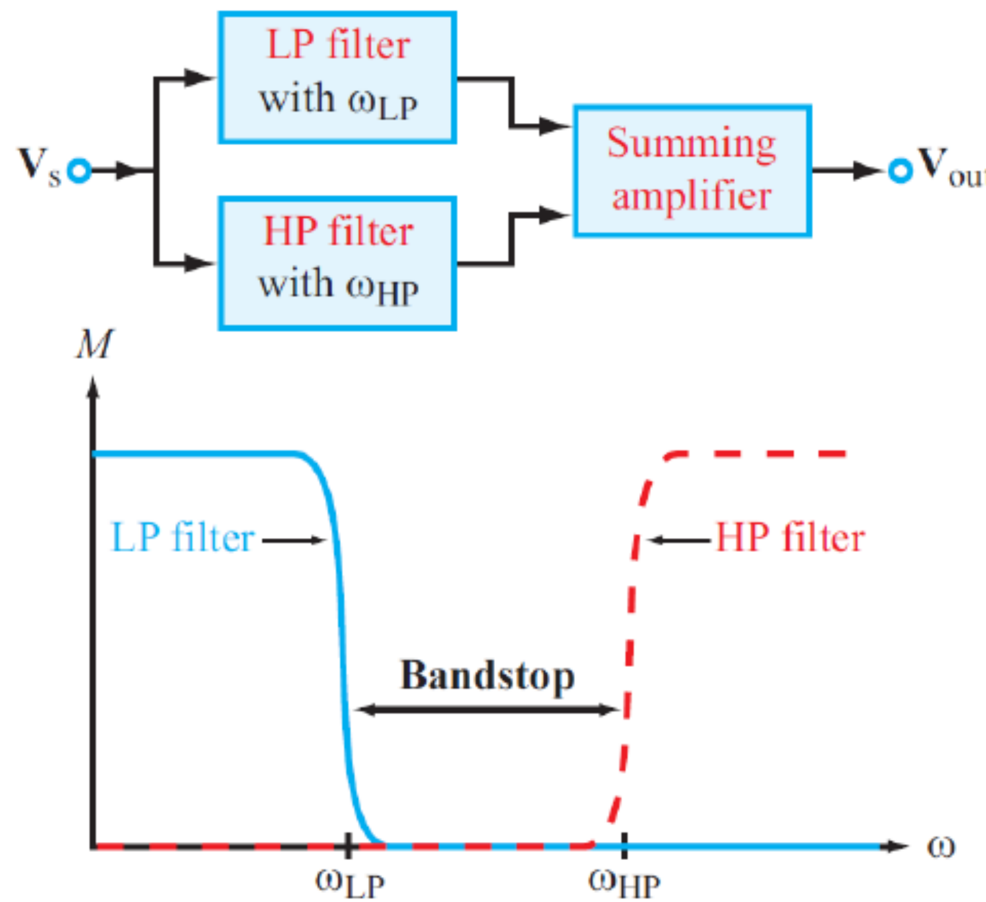


Active Filtering (Cascading)

You can't do this by combining simple circuit elements in some kind of order without any Op Amps. (why?)



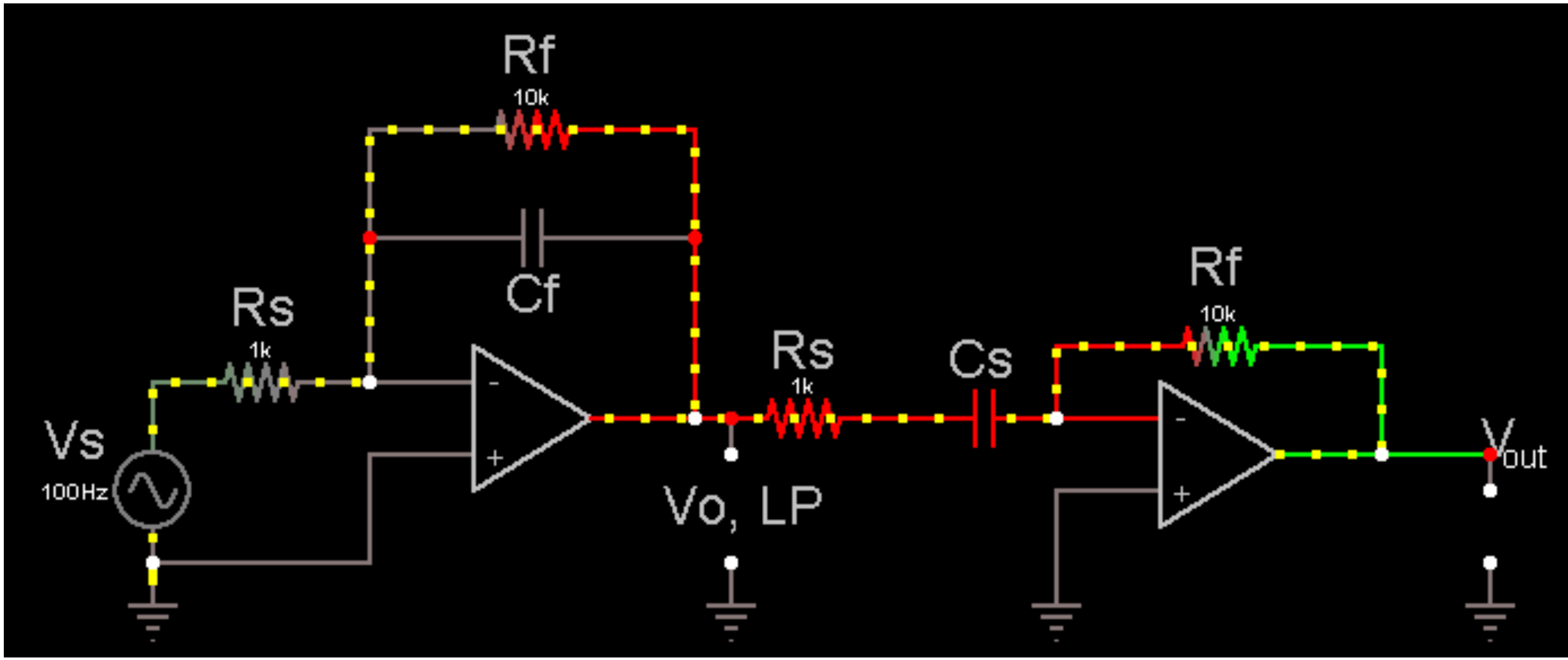
(a) Bandpass filter



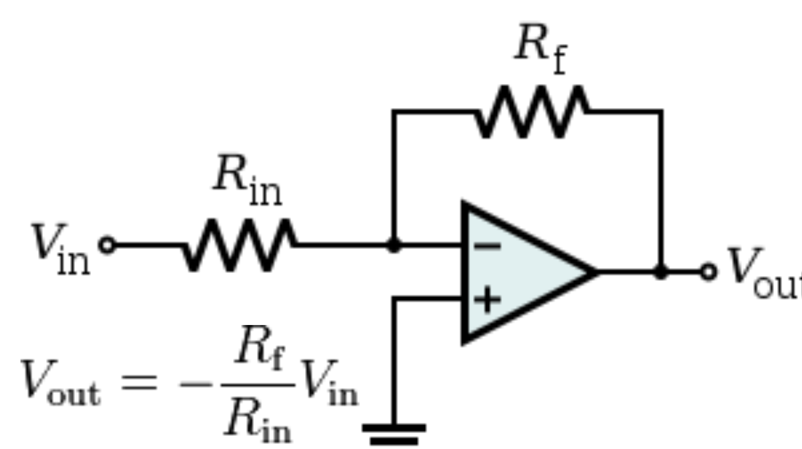
(b) Bandreject filter

Filters combined in Series : Multiply transfer functions
Filters combined in Parallel : Add transfer functions

Frequency Response Example (Noah)



Inverting Amplifier



Question:

1. Find the frequency response (transfer function) of the cascaded filters (Low-pass (+ or * ?) High-pass) in terms of R_s, R_f, C_s, C_f.
2. Determine the values of C_s and C_f required to design a hearing aid for human voice range (50 Hz ~ 20 kHz) if R_s =1 kOhms and R_f = 10 kOhms