Problem 5.59 Relate v_{out} to v_{i} in the circuit of Fig. P5.59. Assume $v_{\text{C}} = 0$ at $t = 0$.

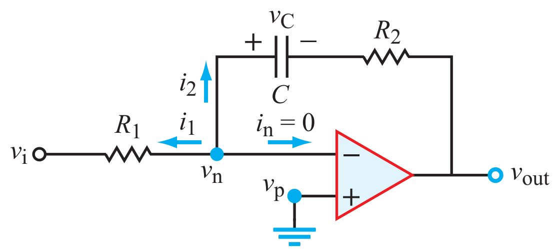


Figure P5.59: Circuit for Problem 5.59.

Solution:

$$i_1 = \frac{v_{\text{n}} - v_{\text{i}}}{R_1}$$

$$v_{\text{n}} - v_{\text{out}} = i_2 R_2 + \frac{1}{C}\int_0^t i_2 \, dt$$

But $v_{\text{n}} = v_{\text{p}} = 0$, and

$$i_2 = -i_1 = \frac{v_{\text{i}}}{R_1},$$

which leads to

$$v_{\text{out}} = -\left(\frac{R_2}{R_1} v_{\text{i}} + \frac{1}{R_1 C}\int_0^t v_{\text{i}} \, dt\right).$$

Problem 5.49 After having been in position 1 for a long time, the switch in the circuit of Fig. P5.49 was moved to position 2 at $t = 0$. Determine $i_1(t)$ and $i_2(t)$ for $t \geq 0$ given that $I_0 = 6$ mA, $R_0 = 12\ \Omega$, $R_1 = 10\ \Omega$, $R_2 = 40\ \Omega$, $L_1 = 1$ H, and $L_2 = 2$ H.

Solution:

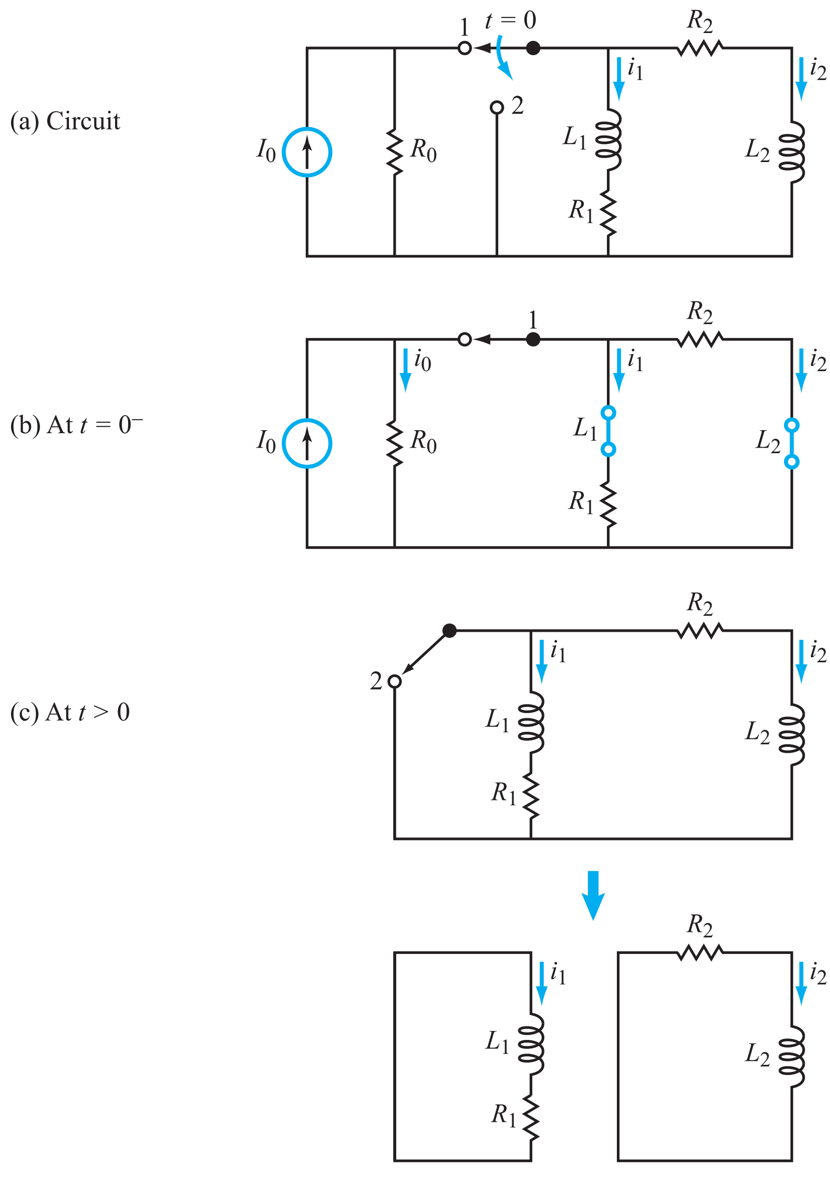


Figure P5.49

At $t = 0^-$ (Fig. P5.49(b)), I_0 will flow through the three branches such that

$$i_0 R_0 = i_1 R_1 = i_2 R_2,$$

and $I_0 = i_0 + i_1 + i_2$. Hence,

$$i_1(0^-) = \frac{R_0 R_2 I_0}{R_0 R_1 + R_0 R_2 + R_1 R_2} = 2.88 \qquad \text{(mA)},$$

$$i_2(0^-) = \frac{R_0 R_1 I_0}{R_0 R_1 + R_0 R_2 + R_1 R_2} = 0.72 \qquad \text{(mA)}.$$

At $t > 0$, we have two independent RL circuits sharing a common short circuit.

$R_1 L_1$ Circuit

$$i_1(0) = i_1(0^-) = 2.88 \qquad \text{(mA)}$$
$$i_1(\infty) = 0$$
$$\tau_1 = \frac{L_1}{R_1} = \frac{1}{10} = 0.1 \text{ s}$$
$$i_1(t) = 2.88 e^{-10t} \qquad \text{(mA)}, \qquad \text{for } t \geq 0.$$

$R_2 L_2$ Circuit

$$i_2(0) = i_2(0^-) = 0.72 \qquad \text{(mA)}$$
$$i_2(\infty) = 0$$
$$\tau_2 = \frac{L_2}{R_2} = \frac{2}{40} = 0.05 \text{ s}$$
$$i_2(t) = 0.72 e^{-20t} \qquad \text{(mA)}, \qquad \text{for } t \geq 0.$$

Chapter 7

Section 7-1: Sinusoidal Signals

Problem 7.1 Express the sinusoidal waveform

$$v(t) = -4\sin(8\pi \times 10^3 t - 45^\circ) \text{ V}$$

in standard cosine form and then determine its amplitude, frequency, period, and phase angle.

Solution:
(a)

$$\begin{aligned} v(t) &= -4\sin(8\pi \times 10^3 t - 45^\circ) \\ &= 4\cos(8\pi \times 10^3 t - 45^\circ + 90^\circ) \qquad [-\sin x = \cos(x + 90^\circ)] \\ &= 4\cos(8\pi \times 10^3 t + 45^\circ) \qquad \text{(V).} \end{aligned}$$

(b)

$$\text{amplitude} = 4 \text{ V}$$

(c)

$$f = 4 \times 10^3 \text{ Hz} = 4 \text{ kHz}$$

(d)

$$T = \frac{1}{f} = \frac{1}{4 \times 10^3} = 0.25 \text{ ms}$$

(e)

$$\phi = 45^\circ$$

Problem 5.62 The two-stage op-amp circuit in Fig. P5.62 is driven by an input step voltage given by $v_{\rm i}(t) = 10u(t)$ mV. If $V_{\rm cc} = 10$ V for both op amps and the two capacitors had no charge prior to $t = 0$, determine and plot:

(a) $v_{\rm out_1}(t)$ for $t \geq 0$

(b) $v_{\rm out_2}(t)$ for $t \geq 0$

Solution:

(a)

$$v_{\rm out_1}(t) = -\frac{1}{R_1C_1}\int_0^t v_{\rm i}\,dt$$

$$R_1C_1 = 5\times 10^3 \times 4\times 10^{-6} = 0.02.$$

Hence,

$$v_{\rm out_1}(t) = -50\int_0^t 10\times 10^{-3}\,dt = -0.5t \qquad \text{(V)}, \qquad \text{for } t \geq 0.$$

(b) For the second stage:

$$v_{\rm out_2}(t) = -\frac{1}{R_2C_2}\int_0^t v_{\rm out_1}(t)\,dt$$

$$R_2C_2 = 1\times 10^6 \times 5\times 10^{-6} = 5$$

$$v_{\rm out_2}(t) = -\frac{1}{5}\int_0^t (-0.5t)\,dt = 0.1\,\frac{t^2}{2} = 0.05t^2 \qquad \text{(V)}, \qquad \text{for } t \geq 0.$$

Plots of $v_{\rm i}(t)$, $v_{\rm out_1}(t)$, and $v_{\rm out_2}(t)$ are shown below. We note that $v_{\rm out_1}(t)$ reaches saturation at $-V_{\rm cc} = -10$ V after 20 s, and $v_{\rm out_2}(t)$ reaches saturation at $V_{\rm cc} = +10$ V at $t = 14.14$ s.

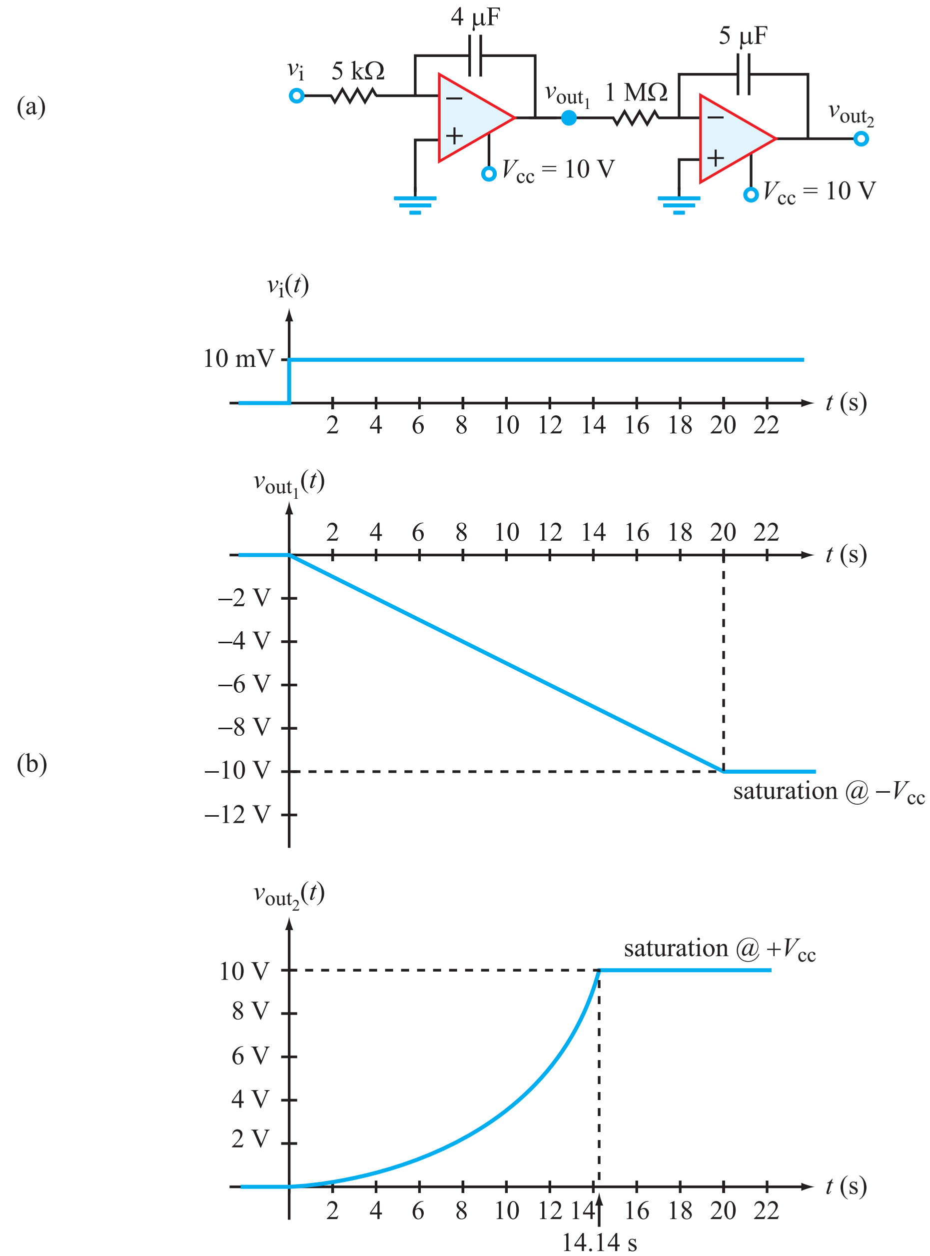

(a)
v_i
5 kΩ
4 μF
v_{out_1}
1 MΩ
5 μF
v_{out_2}
V_{cc} = 10 V
V_{cc} = 10 V
(b)
$v_i(t)$
10 mV
t (s)
2 4 6 8 10 12 14 16 18 20 22
$v_{out_1}(t)$
2 4 6 8 10 12 14 16 18 20 22
t (s)
−2 V
−4 V
−6 V
−8 V
−10 V
−12 V
saturation @ $-V_{cc}$
$v_{out_2}(t)$
saturation @ $+V_{cc}$
10 V
8 V
6 V
4 V
2 V
t (s)
2 4 6 8 10 12 14 16 18 20 22
14.14 s

Figure P5.62

Problem 7.11 Express the following complex numbers in rectangular form:

(a) $\mathbf{z}_1 = 2e^{j\pi/6}$

(b) $\mathbf{z}_2 = -3e^{-j\pi/4}$

(c) $\mathbf{z}_3 = \sqrt{3}\,e^{-j3\pi/4}$

(d) $\mathbf{z}_4 = -j^3$

(e) $\mathbf{z}_5 = -j^{-4}$

(f) $\mathbf{z}_6 = (2+j)^2$

(g) $\mathbf{z}_7 = (3-j2)^3$

Solution:

(a) $\mathbf{z}_1 = 2e^{j\pi/6} = 2e^{j30^\circ} = 2\cos 30^\circ + j2\sin 30^\circ = 1.73 + j1.$

(b) $\mathbf{z}_2 = -3e^{-j\pi/4} = -3e^{-j45^\circ} = -3[\cos(-45^\circ) + j\sin(-45^\circ)] = -2.12 + j2.12.$

(c) $\mathbf{z}_3 = \sqrt{3}\,e^{-j3\pi/4} = \sqrt{3}\,e^{-j135^\circ} = \sqrt{3}\,[\cos 135^\circ - j\sin 135^\circ] = -1.22 - j1.22.$

(d) $\mathbf{z}_4 = -j^3 = -j \cdot j^2 = j.$

(e) $\mathbf{z}_5 = -j^{-4} = \dfrac{-1}{j^4} = -1.$

(f)

$$\begin{aligned}
\mathbf{z}_6 = (2+j)^2 = \left[\sqrt[+]{2^2+1^2}\,e^{j\tan^{-1}(1/2)}\right]^2 &= \left[\sqrt{5}\,e^{j26.565^\circ}\right]^2 \\
&= 5e^{j53.13^\circ} \\
&= 5\cos 53.13^\circ + j5\sin 53.13^\circ = 3 + j4.
\end{aligned}$$

(g)

$$\begin{aligned}
\mathbf{z}_7 = (3-j2)^3 = \left[\sqrt[+]{3^2+2^2}\,e^{-j\tan^{-1}(2/3)}\right]^3 &= \left[\sqrt{13}\,e^{-j33.69^\circ}\right]^3 \\
&= 46.87e^{-j101.1^\circ} \\
&= 46.87(\cos 101.1^\circ - j\sin 101.1^\circ) \\
&= -9 - j46.
\end{aligned}$$

Problem 7.29 The circuit in Fig. P7.29 is in the phasor domain. Determine the following:

(a) The equivalent input impedance $\mathbf{Z}$ at terminals (a,b).

(b) The phasor current $\mathbf{I}$, given that $\mathbf{V}_s = 25\angle 45^\circ$ V.

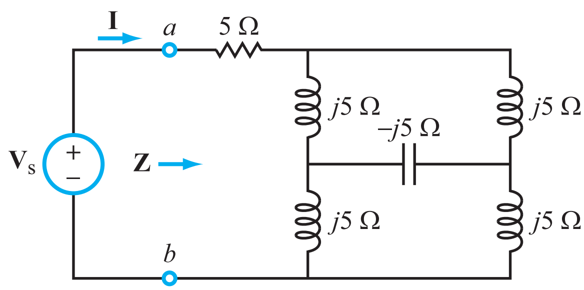
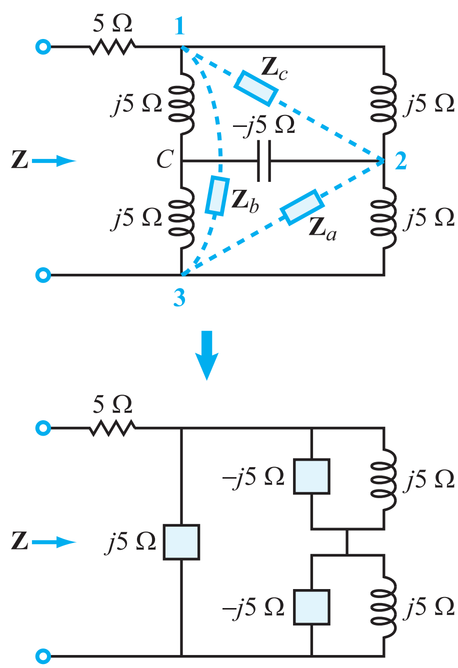


Figure P7.29: Circuit for Problem 7.29.

Solution:

(a)



$$\mathbf{Z}_a = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_1\mathbf{Z}_3}{\mathbf{Z}_1}$$

$$= \frac{(j5)(-j5) + (-j5)(j5) + (j5)^2}{j5} = -j5\ \Omega$$

$$\mathbf{Z}_b = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_1\mathbf{Z}_3}{\mathbf{Z}_2} = j5\ \Omega$$

$$\mathbf{Z}_c = \mathbf{Z}_a = -j5\ \Omega.$$

$$\mathbf{Z} = 5 + j5 \parallel 2(j5 \parallel -j5)$$

$$
\begin{aligned}
&= 5 + j5 \parallel 2\left(\frac{25}{j5 - j5}\right) \\
&= 5 + j5 \parallel \infty \\
&= (5 + j5) = 5\sqrt{2}\, e^{j45^\circ} \qquad (\Omega).
\end{aligned}
$$

(b)

$$
\mathbf{I} = \frac{\mathbf{V}_\mathrm{s}}{\mathbf{Z}} = \frac{25e^{j45^\circ}}{5\sqrt{2}\, e^{j45^\circ}} = \frac{5}{\sqrt{2}} = 3.54 \text{ V}.
$$