EECS 215 Winter 2004 Midterm I

Name: Fred Terry

Room: SOLN

Lecture Section

Rules:

1. 6-7:30 PM Monday, February 16, 2004
2. Closed Book, Closed Notes, etc.
3. Calculators Needed and Allowed
4. Work to be done in Exam booklet.
5. **DO NOT WRITE ON THE BACK OF PAGES.**
6. **Exam given under CoE Honor Code**
7. Show your work and *briefly* explain major steps to maximize partial credit. (ex: i3=i1+i2 ,node A, KCL). **NO CREDIT WILL BE GIVEN IF NO WORK IS SHOWN.**
8. *WRITE YOUR FINAL ANSWERS IN THE AREAS PROVIDED*

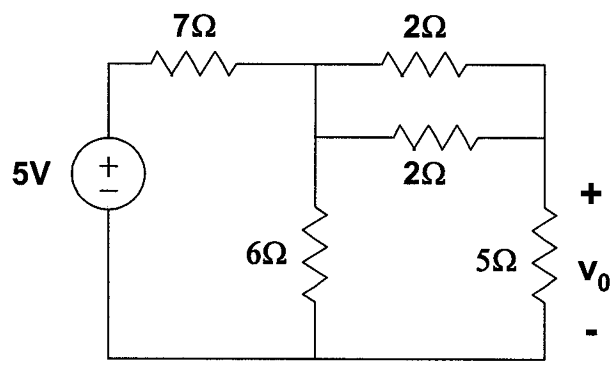
This Exam Contains
4 problems over 16 pages (including workspace).

Write and Sign the College of Engineering Honor Code Below (NO credit will be given for the exam without a signed pledge):

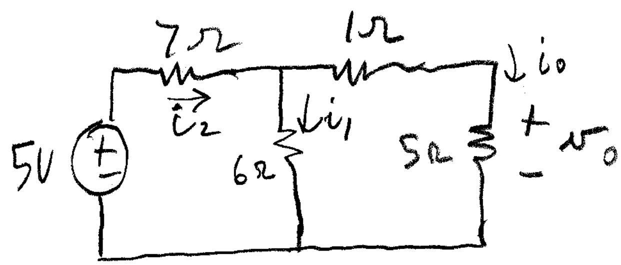
Problem 1: Short Basic Problems (20 points)

Problem has parts (a) & (b)

a) For the circuit below, what is v_0? (10 pts)



$v_0 =$ 1.25 V



$i_1 = i_o$ (both 6Ω branches)

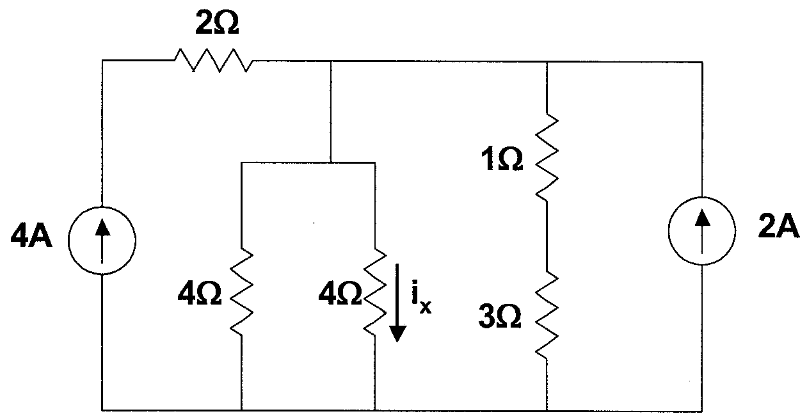
$i_2 = 2 i_o$ by KCL

$$i_2 = \frac{5V}{7\Omega + 6\|6\Omega} = \frac{5V}{10\Omega} = 0.5A$$

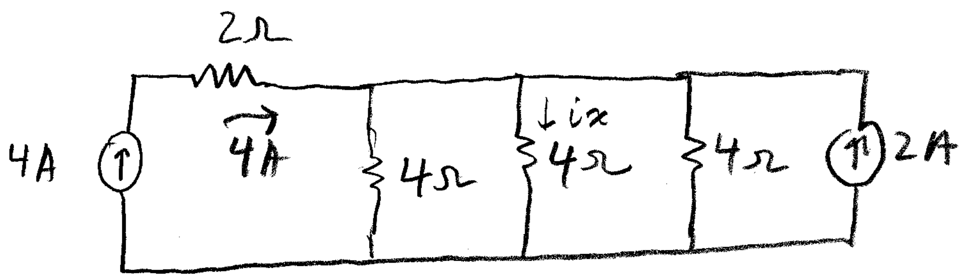
$\Rightarrow i_o = 0.25A$

$\Rightarrow V_o = i_o 5\Omega = 1.25V$

b) Find the current i_x in the circuit below (10 pts)



$i_x =$ 2 A



$$i_x = \frac{1}{3}(4A + 2A) = 2A$$

Problem 2: Nodal Analysis – Nonplanar Circuit

Problem has parts (a), (b), & (c).

Consider the circuit shown on the next page. For this circuit, there are 6 unknowns (v_1, v_2, v_3, v_4, i_a, and ib).

a) Using Ohm's Law, write the initial equations for i_a, and i_b in terms of the node voltages (v_1, v_2, v_3, v_4).

i_a= $V_2 / 40\,\Omega$

i_b= $(V_4 - 150\,V)/7.5\,\Omega$

b) By using the results of (a), we now can reduce the problem to a combination of *4 (and only 4)* KCL and/or KVL equations are needed to find the unknown node voltages (v_1, v_2, v_3, v_4). Write down these 4 equations with so that (v_1, v_2, v_3, v_4) are the *only* unknowns in the equation. The variables i_a and i_b should not appear in these equations. **For each equation Circle whether it is a *KVL* (voltage units)or *KCL* (current units) equation**. Show your intermediate work on the following pages.

1) $V_1 - V_3 = 20\,V$

KVL or KCL

2) $V_4 = -V_2 + 300\,V$

KVL or KCL

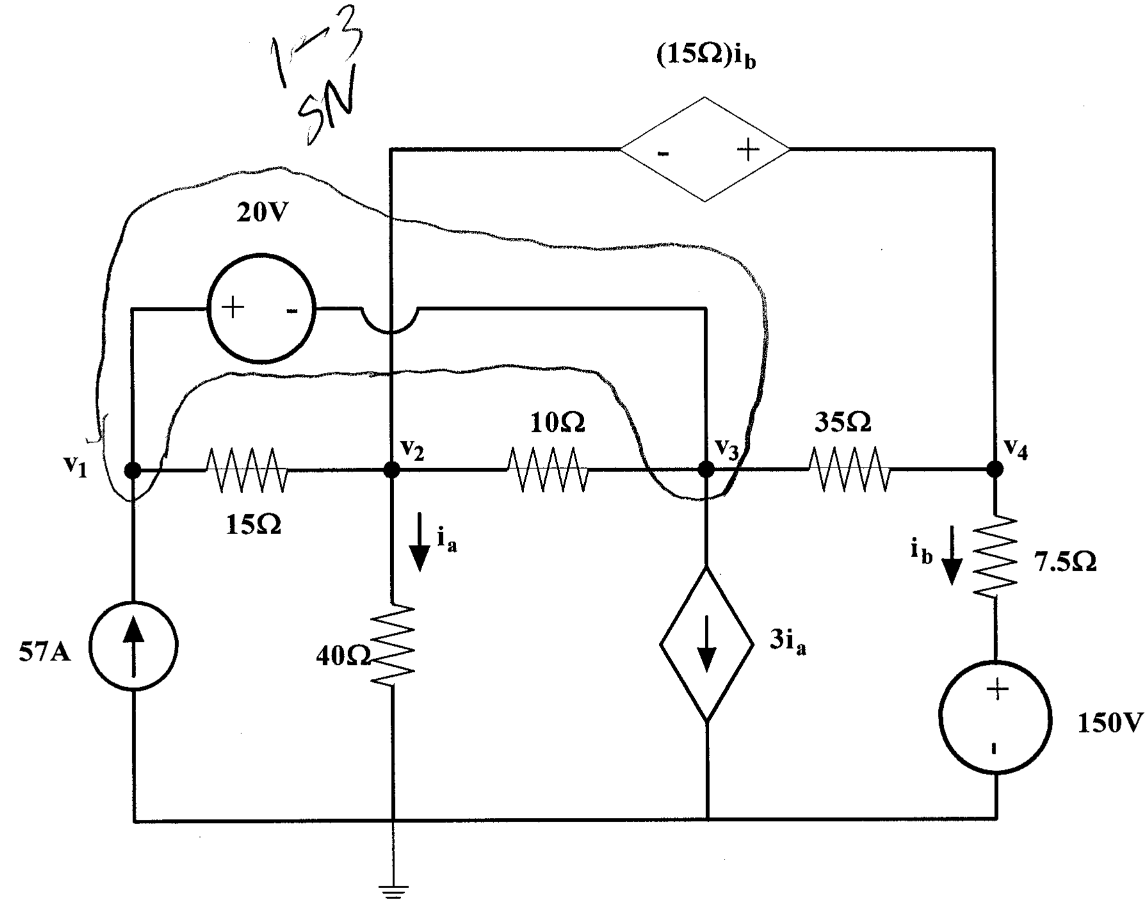
3) $V_1\left[\frac{1}{15}\right] + V_2\left[-\frac{11}{120}\right] + V_3\left[\frac{9}{70}\right] + V_4\left[-\frac{1}{35}\right] = 57A$

KVL or KCL

4) $V_1\left[-\frac{1}{15}\right] + V_2\left[\frac{23}{120}\right] + V_3\left[-\frac{9}{70}\right] + V_4\left[\frac{17}{105}\right] = 20A$

KVL or KCL

note: exact fractions are not required



SN 1-3 KCL

$$-57A + \frac{V_1 - V_2}{15\Omega} + \frac{V_3 - V_2}{10\Omega} + \frac{V_3 - V_4}{35\Omega} + 3i_q = 0$$

$$i_q = V_2/40\Omega$$

$$\boxed{V_1\left[\frac{1}{15\Omega}\right] + V_2\left[-\frac{1}{15\Omega} - \frac{1}{10\Omega} + \frac{3}{40\Omega}\right] + V_3\left[\frac{1}{10\Omega} + \frac{1}{35\Omega}\right] + V_4\left[-\frac{1}{35\Omega}\right] = 57A}$$

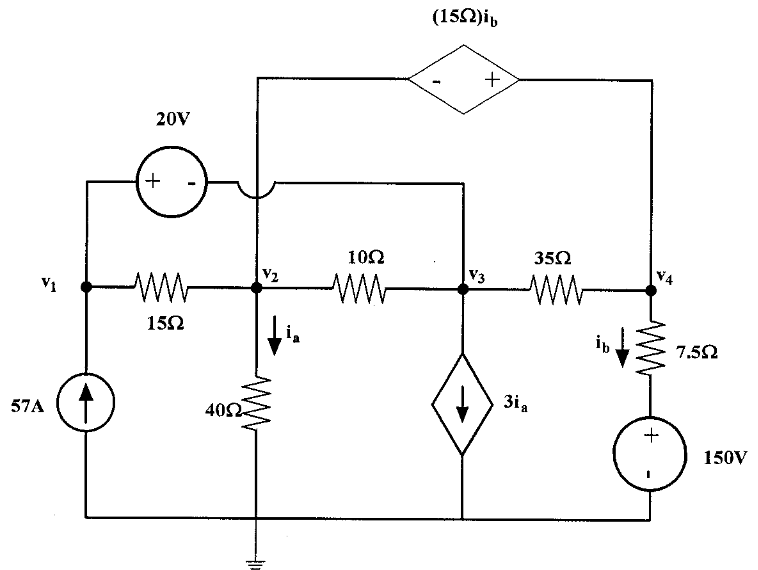
KVL: $V_1 - V_3 = 20V \Rightarrow V_3 = V_1 - 20V$

KVL; $V_4 - V_2 = 15\Omega\, i_b = 15\Omega\left(\frac{V_4 - 150V}{7.5\Omega}\right)$

$= 2V_4 - 300V$

$\Rightarrow V_4 = -V_2 + 300V$

Workspace for 3(b)



SN 1-3 KCL

$$\boxed{V_1\left[\frac{1}{15\Omega}\right] + V_2\left[-\frac{11}{120}\right] + V_3\left[\frac{9}{70}\right] + V_4\left[-\frac{1}{35}\right] = 57}$$

$(V_1 - 20V)$ $(-V_2 + 300V)$

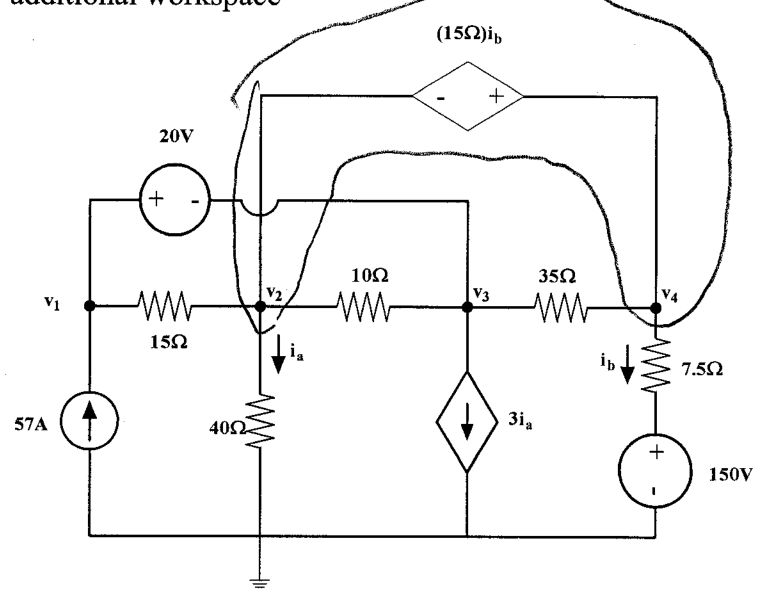
$$\boxed{V_1\left[\frac{41}{210}\right] + V_2\left[-\frac{53}{840}\right] = \frac{477}{7}}$$

Numerical form

$V_1(0.0667) + V_2(-0.0917) + V_3(0.12857) +$
$V_4(-0.02857) = 57A$

or $V_1(0.1952) + V_2(-0.0631) = 68.143\ A$

additional workspace



2-4
SN

2-4 SN KCL

$V_1 - 20V$

$$V_1\left[-\frac{1}{15\Omega}\right] + V_2\left[\frac{1}{15}+\frac{1}{40}+\frac{1}{10}\right] + V_3\left[-\frac{1}{10} + -\frac{1}{35}\right] + V_4\left[\frac{1}{35}+\frac{1}{7.5}\right] = \frac{150V}{7.5A} = 20A$$

$-V_2 + 300V$

$$V_1\left[\frac{1}{15}\right] + V_2\left[\frac{23}{120}\right] + V_3\left[-\frac{9}{70}\right] + V_4\left[\frac{17}{105}\right] = 20A$$

$$V_1(-0.0667) + V_2(0.1917) + V_3(-0.1286) + V_4(0.1619) = 20A$$

or

$$V_1\left[-\frac{41}{210}\right] + V_2\left[\frac{5}{168}\right] = -\frac{218}{7}$$

c) Using your results from (b), write down in matrix-vector form the equation you need to solve to get the node voltages. You may express the matrix equation as either a 4x4 mixed KCL-KVL form or an appropriately reduced KCL-only form ($\vec{G}\cdot\vec{V} = \vec{I}$). Solve to find the numerical values for (v_1, v_2, v_3, v_4). Round your answer to the nearest mV and be careful to specify the **sign** of the voltage.

Matrix Equation:

$$\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 \\ \left(\frac{1}{15}\right) & \left(\frac{-11}{120}\right) & \left(\frac{9}{70}\right) & \left(\frac{-1}{35}\right) \\ \left(\frac{-1}{15}\right) & \left(\frac{23}{120}\right) & \left(\frac{-9}{70}\right) & \left(\frac{17}{105}\right) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 20V \\ 300V \\ 57A \\ 20A \end{bmatrix}$$

or

$$\begin{bmatrix} \left(\frac{41}{210}\right) & \left(-\frac{53}{840}\right) \\ \left(-\frac{41}{210}\right) & \left(\frac{5}{168}\right) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} (477/7)A \\ (-218/7)A \end{bmatrix}$$

$V_1 \approx$ $-9.6\overline{95121}$ V $= -\frac{795}{82}$ V

$V_2 \approx$ -1110 V

$= -\frac{2435}{82}$ V

$V_3 \approx$ **-29.695**$\overline{121}$ V (given so you can check)

$V_4 \approx$ $+1410$ V

Problem 3: Mesh Analysis (30 points)

Problem has parts (a), (b), & (c).

a) For the circuit shown on the next page, write an equation for v_x in which the only unknowns are the mesh loop currents. (5 pts)

$v_x =$ $(4\Omega)\, i_3$

b) there are 4 mesh loops, and thus 4 equations are needed to solve for (i_1, i_2, i_3, i_4). Find these equations and write them below so that the *only* unknowns are the mesh currents (i_1, i_2, i_3, i_4). Indicate whether these equations are KVL (voltage units) or KCL (current units) equations. (20 pts)

1) $i_4 - i_1 = 3A$

KVL or KCL

2) $i_3 - i_2 = 2A$

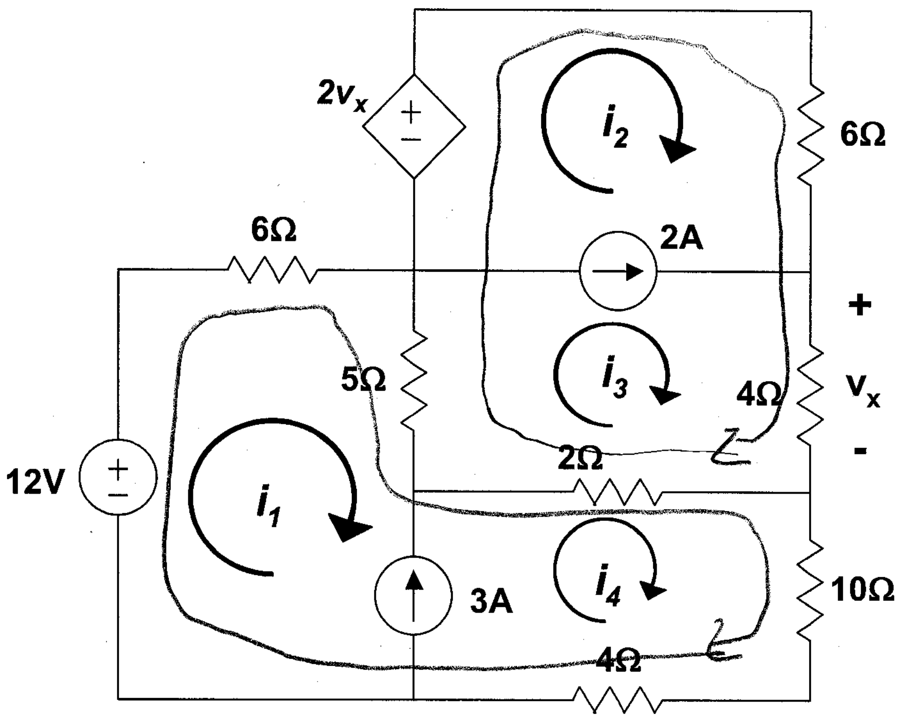
KVL or KCL

3) $i_1(11\Omega) + i_3(-7\Omega) + i_4(16\Omega) = +12V$

KVL or KCL

4) $i_1(-5\Omega) + i_2(6\Omega) + i_3(3\Omega) + i_4(-2\Omega) = 0$

KVL or KCL



2 supermesh loops

KCL: $i_4 - i_1 = 3A \Rightarrow i_4 = i_1 + 3A$ } to eliminate i_3 & i_4

KCL: $i_3 - i_2 = 2A \Rightarrow i_3 = i_2 + 2A$

<u>1-4 SM KVL:</u>

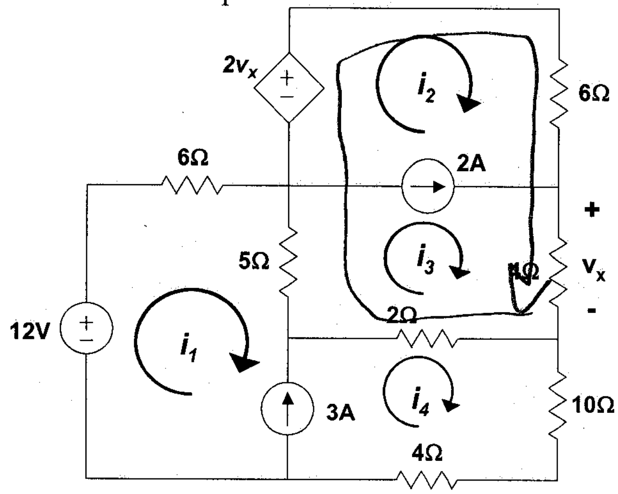
$\{ -12V + (6\Omega) i_1 + 5\Omega(i_1 - i_3) + 2\Omega(i_4 - i_3)$

$+ (10 + 4\Omega) i_4 = 0$

$$\boxed{i_1(11\Omega) + i_2(0) + \underbrace{i_3}_{i_2+2A}(-7\Omega) + \underbrace{i_4}_{i_1+3A}(16\Omega) = +12V}$$

$$\boxed{i_1(27\Omega) + i_2(-7\Omega) = 12V + 14V - 48V = -22V}$$

Additional workspace



2-3 SM KVL

$$(i_3 - i_4)(2\Omega) + (i_3 - i_1)5\Omega \underbrace{-2V_x}_{-8\Omega i_3} + i_2(6\Omega) + i_3(4\Omega) = 0$$

$$\boxed{i_1(-5\Omega) + i_2(6\Omega) + i_3(3\Omega) + i_4(-2\Omega) = 0}$$

$i_2 + 2A$ $i_1 + 3A$

$$\boxed{i_1(-7\Omega) + i_2(9\Omega) = -6V + 6V = 0}$$

Using your results from (b), write down in matrix-vector form the equation you would need solve to get numerical answers. You may express the matrix equation as either a 4x4 mixed KVL-KCL form or an appropriately reduced KVL-only form ($\vec{R} \bullet \vec{I} = \vec{V}$). Find the numerical values of the resulting mesh currents (i_1, i_2, i_3, i_4). Be sure to specify the sign of the current. (5 pts)

Matrix Equation:

$$\begin{bmatrix} -1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \\ 11 & 0 & -7 & 16 \\ -5 & 6 & 3 & -2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} 3\,A \\ 2\,A \\ 12\,V \\ 0\,V \end{bmatrix}$$

or

$$\begin{bmatrix} 27 & -7 \\ -7 & 9 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} -22\,V \\ 0\,V \end{bmatrix}$$

$i_1 =$ $-99/97 \approx -1.0206$ A

$i_2 =$ $-77/97 \approx -0.7938$ A

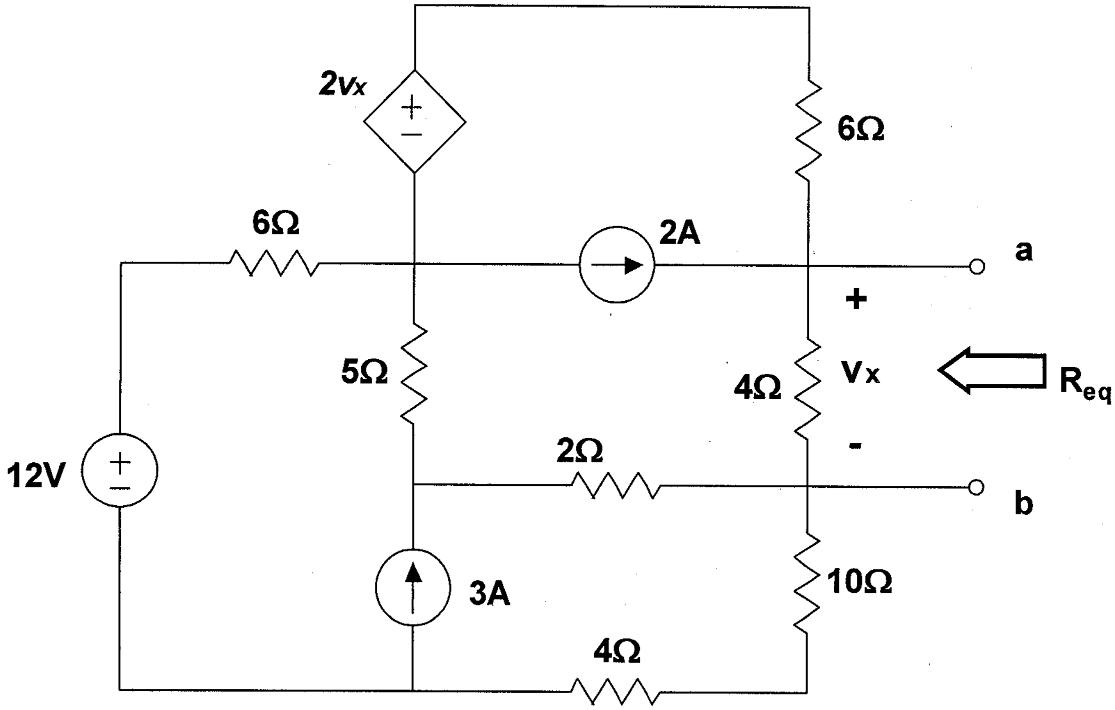
$i_3 =$ $+117/97\text{A} \approx 1.206\text{A}$ (provided as a check)

$i_4 =$ $+192/97 \approx +1.9794$ A

Problem 4: Equivalent Circuits

Problem has parts (a), & (b).

Suppose we have the circuit from the previous problem:



a) Find the equivalent resistance (the Thevenin/Norton resistance) seen between the terminals a&b {compute a number here, you may use any means you like, but show your work} (15 pts)

R_{eq}= $+604/97\,\Omega \approx 6.227\,\Omega$

Method 1: If I trust my work from #3

I note $V_{TH} = i_3\, 4\Omega = +(468/97)V$

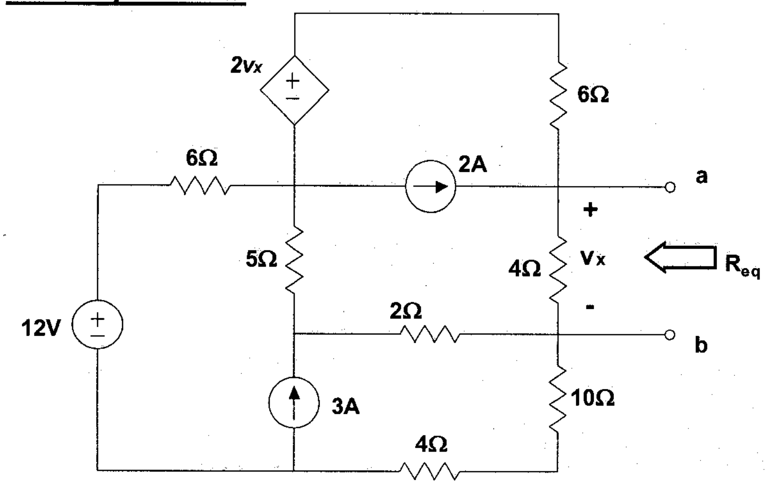
$I_N = I_{SC}$ will result if I find i_3 with a-b shorted.

The 2-3 SM equation becomes

$i_1(-5\Omega) + i_2(6\Omega) + i_3(+7\Omega) + i_4(-2\Omega) = 0$

All other eqn's are the same

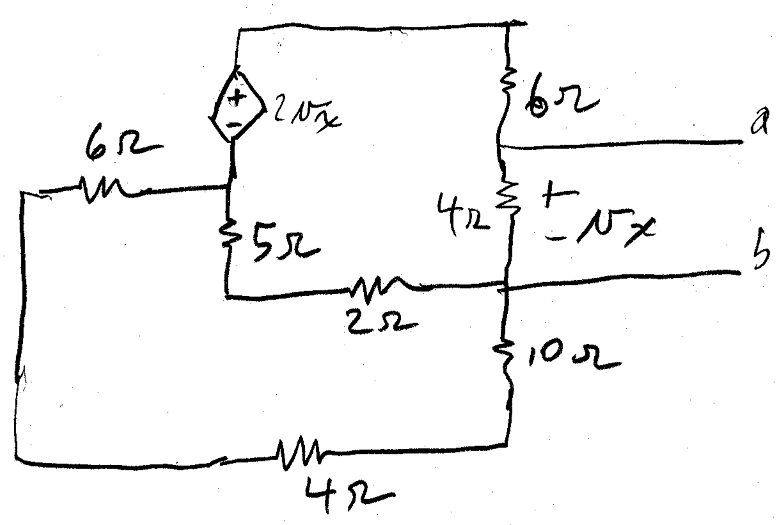
Work Space for 4



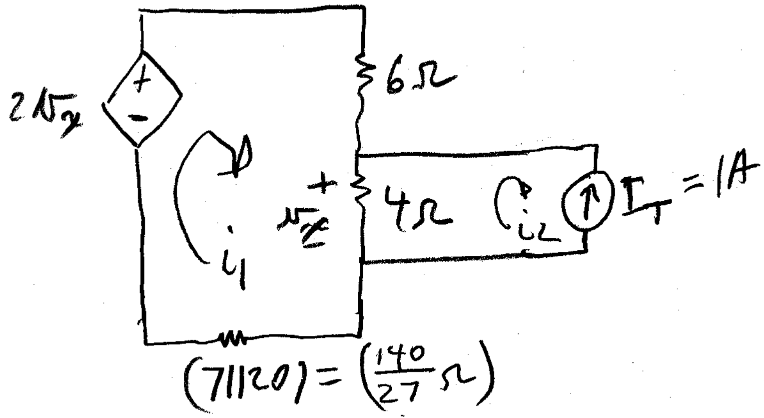
resolving yields $i_3 = +117/151$ A $= I_N = I_{SC}$

$R_{eq} = \dfrac{V_{TH}}{I_N} = \dfrac{V_{OC}}{I_{SC}} = +\dfrac{604}{97}\ \Omega \approx 6.227\ \Omega$

If I don't trust my work #3, find the equivalent R by turning off <u>indepen.</u> sources



Additional Workspace



Test current is easy here

$i_2 = -I_T = -1A$

$v_x = V_T = (i_1 - i_2)4\Omega$
$= (i_1 + 1A)4\Omega$

KVL: $-2v_x + i_1\left(6 + 4 + \frac{140}{27}\right) + i_2(-4\Omega) = 0$

$i_1\left(-8 + 6 + 4 + \frac{140}{27}\right) = +8V - 4V = 4V$

$i_1 = +54/97\ A$

$v_x = +604/97\ V \Rightarrow R_{eq} = \frac{+604}{97}\Omega$ ✓

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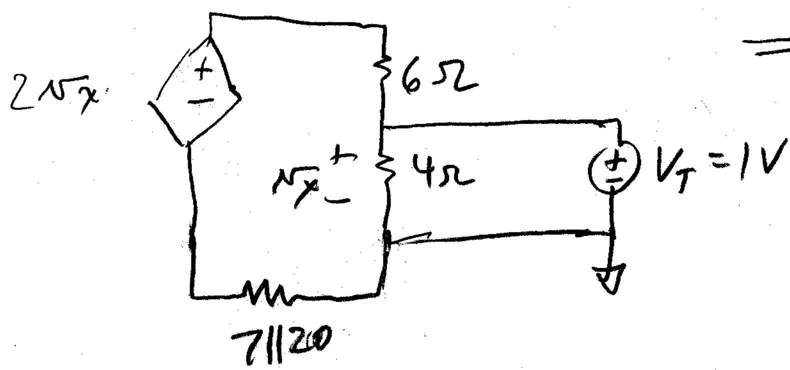
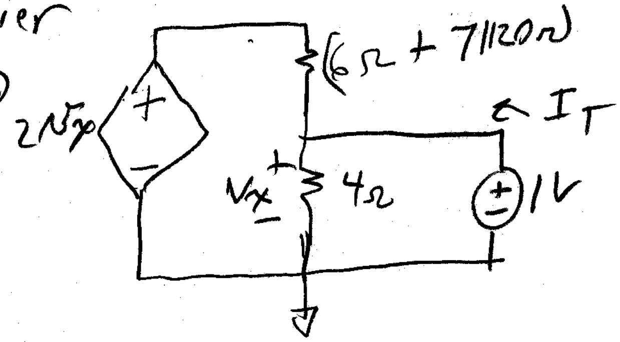
nodal works too



easier ⇒



$v_x = 1V,\ 2V_x = 2V$

$I_T = \frac{1V}{4\Omega} + \frac{1V - 2V}{6\Omega + 20\|7\Omega} = 97/604\ A$

$\Rightarrow R_{eq} = \frac{604}{97}\Omega$ ✓

b) What is Norton Equivalent Current (I_N) for this circuit? (5 pts)

$I_N =$ $+(117/151) \approx +0.7748\,A$

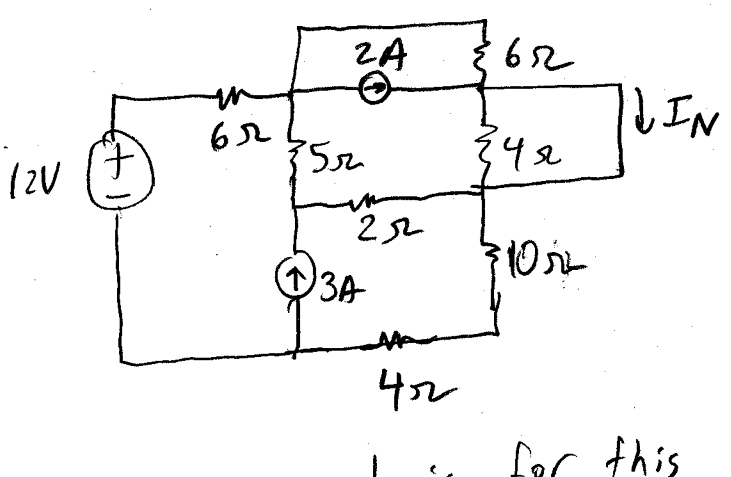
From the given result in 3 (b)

$$V_{oc} = V_{TH} = (4\Omega)(i_3) = +\frac{468}{97}\,V \approx +4.825\,V$$

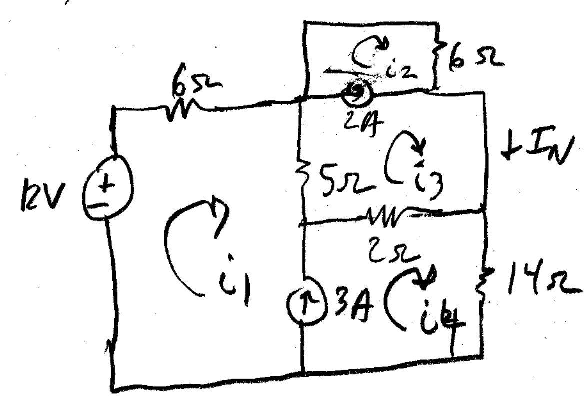
$$I_N = I_{sc} = V_{TH}/R_{eq} \approx \frac{468}{604}\,A = \frac{117}{151}\,A$$

$$\approx +0.7748\,A$$

I could Also get I_{sc} from a straight-forward Attack. With a-b shorted $v_x = 0 \Rightarrow 2v_x = 0$



⇒



Mesh analysis for this was done in the answer to part (a)