Problem 5.7 After opening a certain switch at $t = 0$ in a circuit containing a capacitor, the voltage across the capacitor started decaying exponentially with time. Measurements indicate that the voltage was 7.28 V at $t = 1$ s and 0.6 V at $t = 6$ s. Determine the initial voltage at $t = 0$ and the time constant of the voltage waveform.

Solution:

$$v(t) = v_o e^{-t/\tau} \qquad \text{(V)}.$$

$$7.28 = v_o e^{-1/\tau}$$

$$0.6 = v_o e^{-6/\tau}$$

$$\frac{7.28}{0.6} = \frac{e^{-1/\tau}}{e^{-6/\tau}} = e^{5/\tau}$$

$$\ln\left(\frac{7.28}{0.6}\right) = \frac{5}{\tau}$$

$$\tau = \frac{5}{2.5} = 2.$$

$$7.28 = v_o e^{-1/2} = 0.61 v_o$$

$$v_o = \frac{7.28}{0.61} = 12 \text{ V}.$$

$$v(t) = 12e^{-0.5t} \qquad \text{(V)}.$$

Problem 5.12 The current through a 40-μF capacitor is given by a rectangular pulse as

$$i(t) = 40\text{rect}\left(\frac{t-1}{2}\right) \text{ mA}.$$

If the capacitor was initially uncharged, determine $v(t)$, $p(t)$, and $w(t)$.

Solution: For $0 \le t \le 2$,

$$\begin{aligned} v(t) &= v(0) + \frac{1}{C}\int_0^t i\,dt \\ &= 0 + \frac{1}{40\times 10^{-6}}\int_0^t 40\times 10^{-3}\,dt \\ &= 1000t \qquad \text{(V)}. \end{aligned}$$

Hence,

$$v(t) = \begin{cases} 0 & \text{for } t < 0 \\ 1000t \ \ \text{(V)} & \text{for } 0 \le t \le 2 \text{ s} \\ 2000 \ \ \text{(V)} & \text{for } t \ge 2 \text{ s} \end{cases}$$

$$\begin{aligned} p(t) &= i(t)\,v(t) \\ &= \begin{cases} 0 & \text{for } t < 0 \\ 40t \ \ \text{(A)} & \text{for } 0 \le t \le 2 \text{ s} \\ 0 & \text{for } t > 2 \text{ s} \end{cases} \end{aligned}$$

$$w(t) = \frac{1}{2}Cv^2 = \begin{cases} 0 & \text{for } t < 0 \\ 20t^2 \ \ \text{(mJ)} & \text{for } 0 \le t \le 2 \text{ s} \\ 80 \ \ \text{(mJ)} & \text{for } t \ge 2 \text{ s} \end{cases}$$

Problem 5.15 Determine voltages v_1 to v_3 in the circuit of Fig. P5.15 under dc conditions.

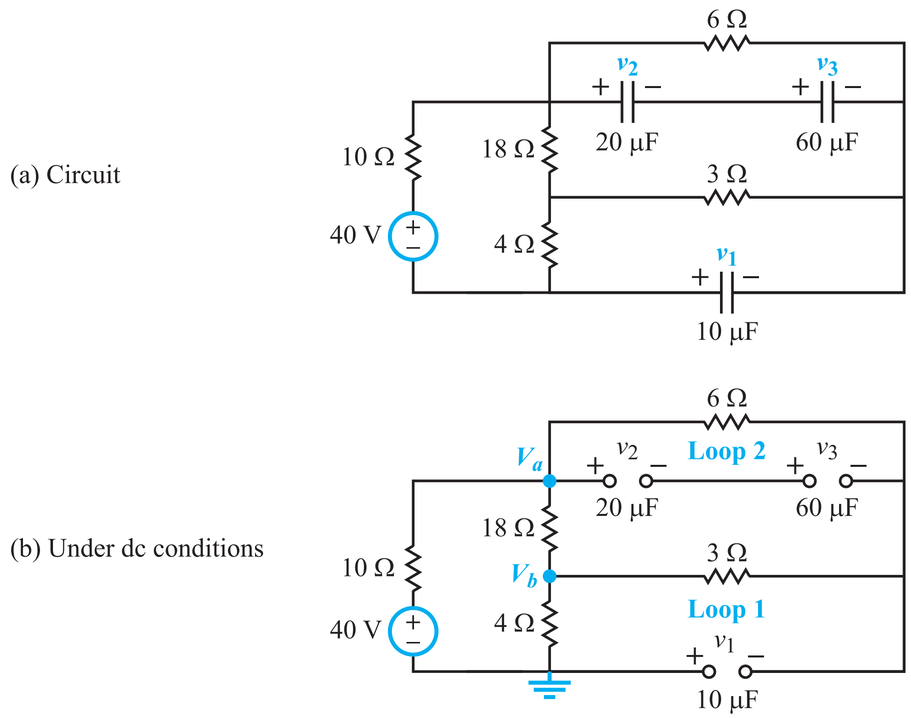


Figure P5.15: Circuit for Problem 5.15.

Solution: KCL at nodes V_a and V_b:

$$\frac{V_a - 40}{10} + \frac{V_a - V_b}{18} + \frac{V_a - V_b}{6+3} = 0$$

$$\frac{V_b}{4} + \frac{V_b - V_a}{18} + \frac{V_b - V_a}{6+3} = 0$$

Solution gives

$$V_a = 20 \text{ V}, \qquad V_b = 8 \text{ V}.$$

For Loop 1,

$$-v_1 - V_b + (V_b - V_a)\frac{3}{9} = 0,$$

which gives

$$v_1 = -12 \text{ V}.$$

For Loop 2,

$$-v_3 - v_2 + (V_a - V_b)\frac{6}{9} = 0$$

which gives

$$v_3 + v_2 = (20-8)\frac{6}{9} = 8 \text{ V}. \qquad (1)$$

For two capacitors in series, Eq. (5.47) gives

$$v_2 C_2 = v_3 C_3$$

or

$$20 v_2 = 60 v_3 \tag{2}$$

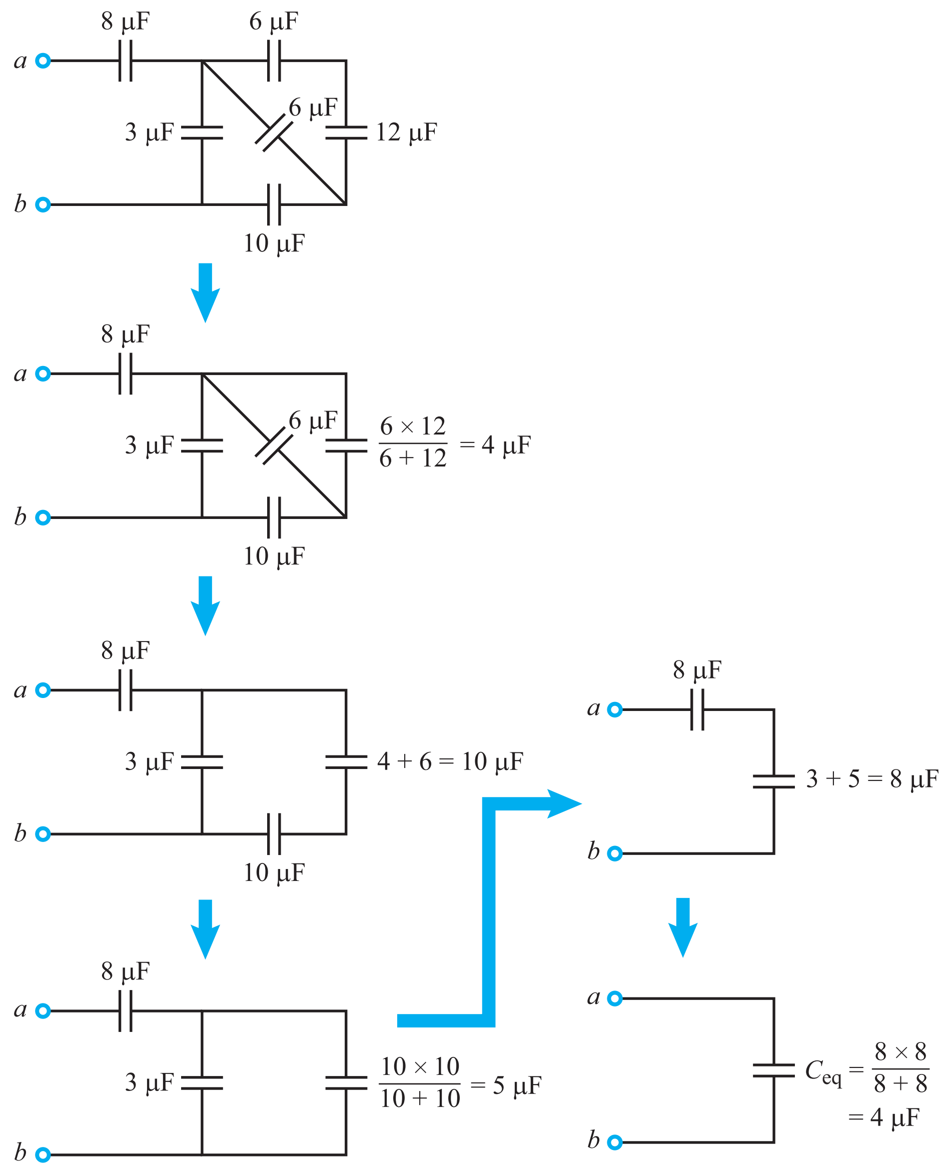
Simultaneous solution of Eqs. (1) and (2) leads to

$$v_2 = 6 \text{ V}, \qquad v_3 = 2 \text{ V}.$$

Problem 5.21 Assume that a 120-V dc source is connected at terminals (a,b) to the circuit in Fig. P5.17. Determine the voltages across all capacitors.

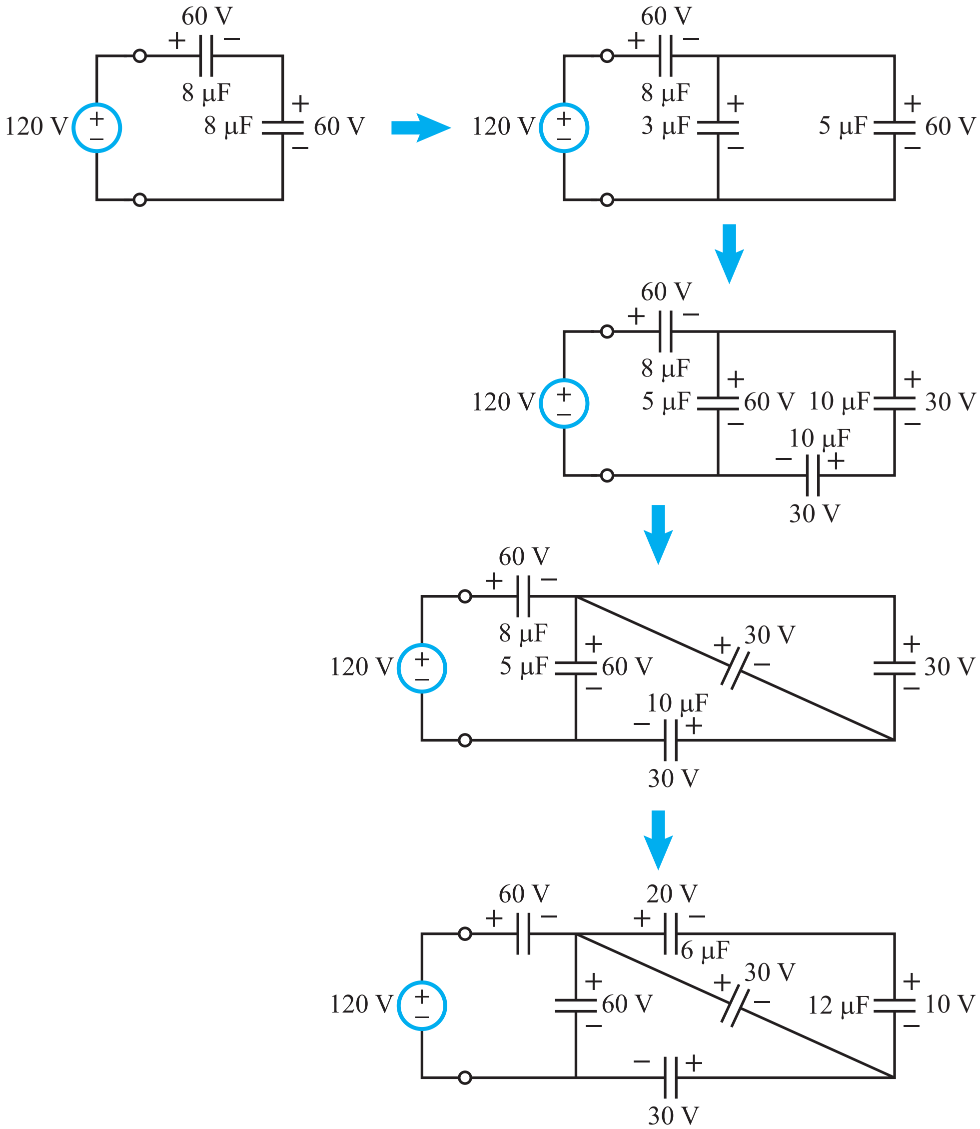
Solution: We first simplify the circuit to find C_{eq} at terminals (a,b), and then we work backwards to determine the voltages across the capacitors.

Step 1: Find C_{eq}.



Step 2: Find voltages.

- Two capacitors in parallel share the same voltage.
- For two capacitors in series: $C_1 v_1 = C_2 v_2$.


60 V
+ −
8 μF
120 V
+
−
8 μF
60 V
60 V
3 μF
5 μF
60 V
8 μF
5 μF
60 V
10 μF
30 V
10 μF
30 V
30 V
10 μF
20 V
6 μF
12 μF
10 V

Problem 5.35 The circuit in Fig. P5.35 contains two switches, both of which had been open for a long time before $t = 0$. Switch 1 closes at $t = 0$, and switch 2 follows suit at $t = 5$ s. Determine and plot $v_C(t)$ for $t \geq 0$ given that $V_0 = 24$ V, $R_1 = R_2 = 16$ kΩ, and $C = 250$ μF. Assume $v_C(0) = 0$.

Solution:

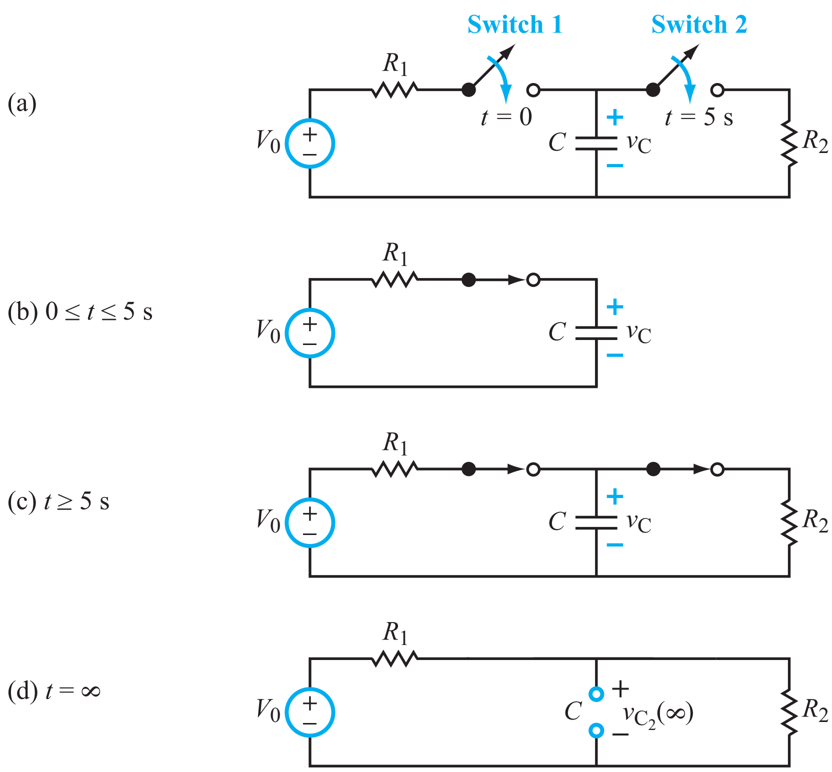


Figure P5.35

Time Segment 1: $0 \leq t \leq 5$ s

$$\tau_1 = R_1 C = 16 \times 10^3 \times 250 \times 10^{-6} = 4 \text{ s.}$$

$$\begin{aligned}
v_{C_1}(t) &= v_{C_1}(\infty) + (v_{C_1}(t) - v_{C_1}(\infty))e^{-t/\tau_1} \\
&= V_0 + (0 - V_0)e^{-0.25t} \\
&= 24(1 - e^{-0.25t}), \qquad \text{for } 0 \leq t \leq 5 \text{ s.}
\end{aligned}$$

Time Segment 2: $t \geq 5$ s

Through source transformation, it is easy to see that R_1 and R_2 should be combined in parallel. Hence:

$$\tau_2 = \left(\frac{R_1 R_2}{R_1 + R_2}\right) C = 8 \times 10^3 \times 250 \times 10^{-6} = 2 \text{ s.}$$

$$v_{C_2}(t) = v_{C_2}(\infty) + [v_{C_2}(5 \text{ s}) - v_{C_2}(\infty)]e^{-(t-5)/\tau_2}$$

$$v_{C_2}(\infty) = \frac{V_0 R_2}{R_1 + R_2} = \frac{24 \times 16}{16 + 16} = 12 \text{ V}.$$

$$v_{C_2}(5 \text{ s}) = v_{C_1}(5 \text{ s}) = 24(1 - e^{-0.25 \times 5}) = 17.12 \text{ V}$$

$$\begin{aligned} v_{C_2}(t) &= 12 + [17.12 - 12]e^{-0.5(t-5)} \\ &= 12 + 5.12e^{-0.5(t-5)}, \qquad \text{for } t \geq 5 \text{ s}. \end{aligned}$$

Plot is shown in Fig. P5.35(e).

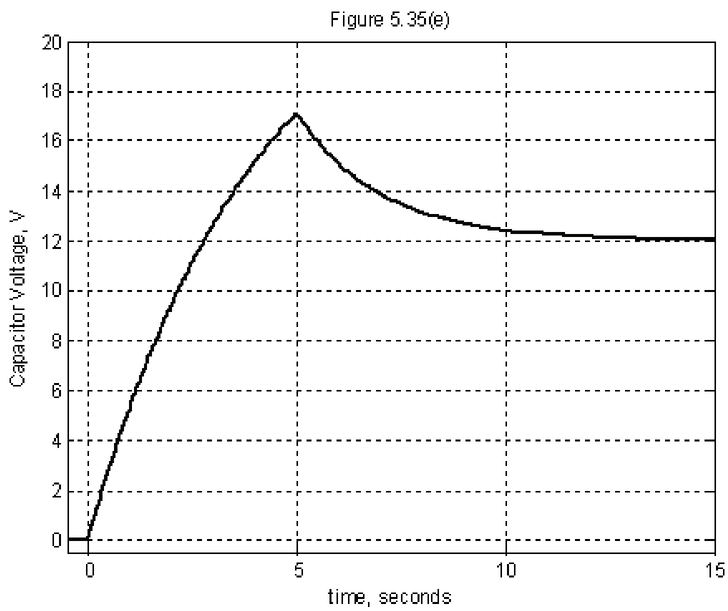


Figure P5.35(e)

Section 5-4: Response of the RC Circuit

Problem 5.33 After having been in position 1 for a long time, the switch in the circuit of Fig. P5.33 was moved to position 2 at $t = 0$. Given that $V_0 = 12$ V, $R_1 = 30$ kΩ, $R_2 = 120$ kΩ, $R_3 = 60$ kΩ, and $C = 100$ μF, determine:

(a) $i_C(0^-)$ and $v_C(0^-)$

(b) $i_C(0)$ and $v_C(0)$

(c) $i_C(\infty)$ and $v_C(\infty)$

(d) $v_C(t)$ for $t \geq 0$

(e) $i_C(t)$ for $t \geq 0$

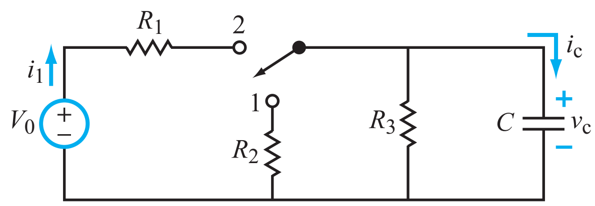


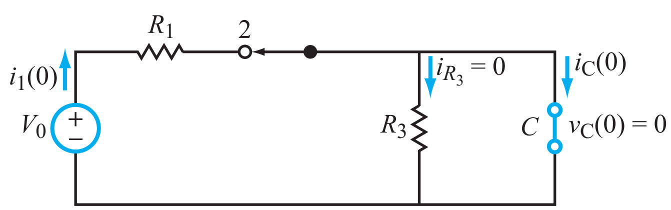
Figure P5.33: Circuit for Problem 5.33.

Solution: (a) Since the capacitor had access to resistors R_2 and R_3 prior to $t = 0$, it has dissipated any charge it may have had, long before $t = 0$. Hence,

$$i_C(0^-) = v_C(0^-) = 0.$$

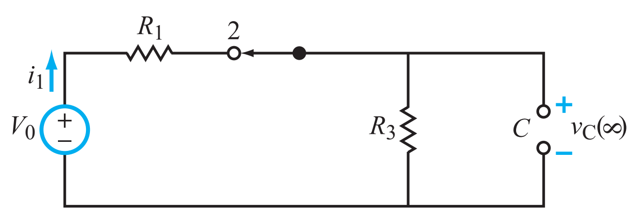
(b) At $t = 0$, the capacitor acts like a short circuit (because its voltage cannot change instantaneously). Since the voltage across R_3 is zero, no current flows through it. Hence,

$$i_1(0) = \frac{V_0}{R_1} = \frac{12}{30\text{k}} = 0.4 \text{ mA}.$$
$$v_C(0) = v_C(0^-) = 0.$$



At $t = 0$

(c) At $t = \infty$, capacitor acts like an open circuit.



At $t = \infty$

Hence,

$$i_C(\infty) = 0$$

$$v_C(\infty) = \frac{V_0 R_3}{R_1 + R_3} = \frac{12 \times 60}{30 + 60} = 8 \text{ V}.$$

(d)

$$\begin{aligned} v_C(t) &= v_C(\infty) + (v_C(0) - v_C(\infty))e^{-t/\tau} \\ &= 8 + (0 - 8)e^{-t/\tau} \\ &= 8(1 - e^{-t/\tau}) \qquad \text{(V)}, \qquad \text{for } t \geq 0, \end{aligned}$$

where

$$\tau = \left(\frac{R_1 R_3}{R_1 + R_3}\right) C = \frac{30 \times 60}{30 + 60} \times 10^3 \times 10^{-4} = 2 \text{ s}.$$

(e)

$$\begin{aligned} i_C(t) &= C\,\frac{dv_C}{dt} \\ &= 10^{-4}\frac{d}{dt}\,[8(1 - e^{-0.5t})] \\ &= 0.4e^{-0.5t} \qquad \text{(mA)} \qquad \text{for } t \geq 0. \end{aligned}$$

Problem 5.39 The switch in the circuit of Fig. P5.39 had been in position 1 for a long time until it was moved to position 2 at $t = 0$. Determine $v(t)$ for $t \geq 0$, given that $I_0 = 6$ mA, $V_0 = 18$ V, $R_1 = R_2 = 4$ kΩ, and $C = 200$ μF.

Solution:

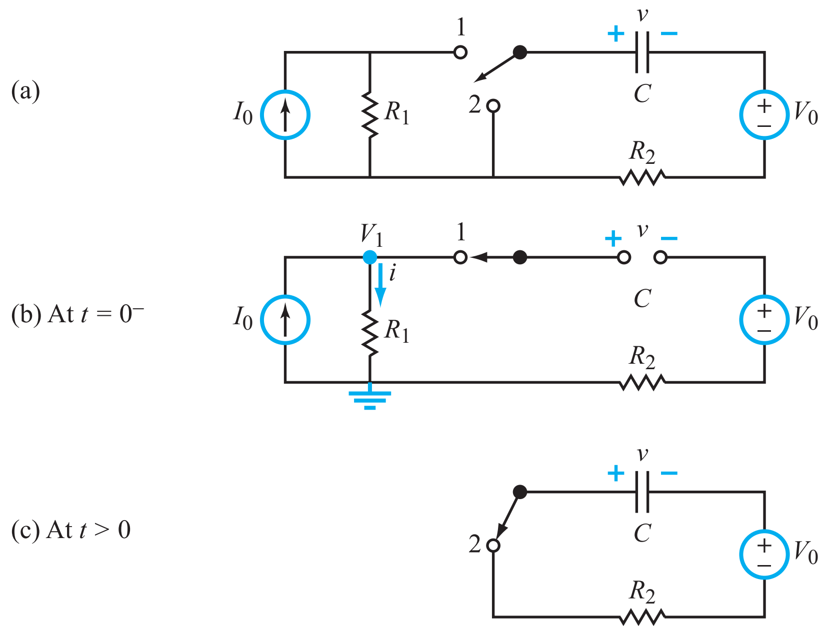


Figure P5.39

At $t = 0^-$, the circuit assumes the condition shown in Fig. 5.39(b).

$$V_1 = I_0 R_1 = 6 \times 10^{-3} \times 4 \times 10^3 = 24 \text{ V}.$$

$$v(0^-) = V_1 - V_0 = 24 - 18 = 6 \text{ V}.$$

At $t > 0$, circuit becomes as shown in Fig. P5.39(c). Now,

$$v(\infty) = -V_0 = -18 \text{ V}.$$

$$\tau = R_2 C = 4 \times 10^3 \times 2 \times 10^{-4} = 0.8 \text{ s}.$$

Hence,

$$\begin{aligned} v(t) &= [v(\infty) + [v(0) - v(\infty)]e^{-t/\tau}] \\ &= [-18 + [6 + 18]e^{-1.25t}] \\ &= [-18 + 24e^{-1.25t}] \qquad \text{(V)}, \qquad \text{for } t \geq 0. \end{aligned}$$