EE 40 – AC Response

Reading Material:

Chapter 7

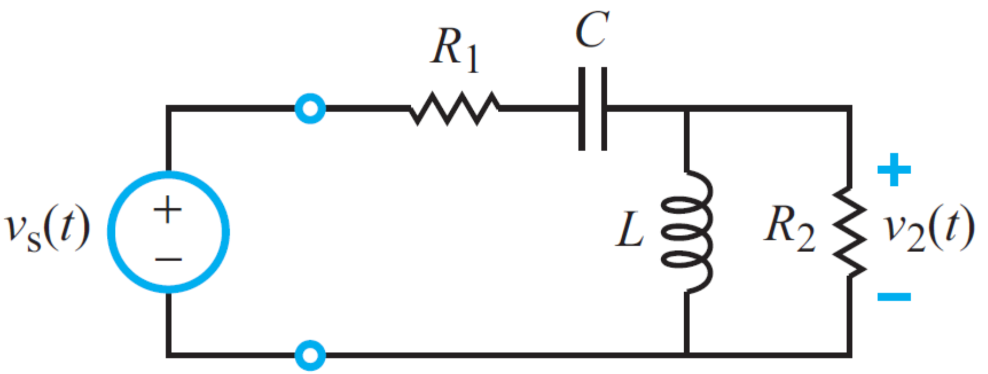
Linear Circuits at ac

Objective: To determine the steady state response of a linear circuit to ac signals

$v_s(t) = V_0 \cos(\omega t + \phi)$

angular frequency ω

ϕ is called its ***phase angle***



- Sinusoidal input is common in electronic circuits
- Any time-varying periodic signal can be represented by a series of sinusoids (Fourier Series)
- Time-domain solution method can be cumbersome

Complex Numbers

We will find it is useful to represent sinusoids as complex numbers

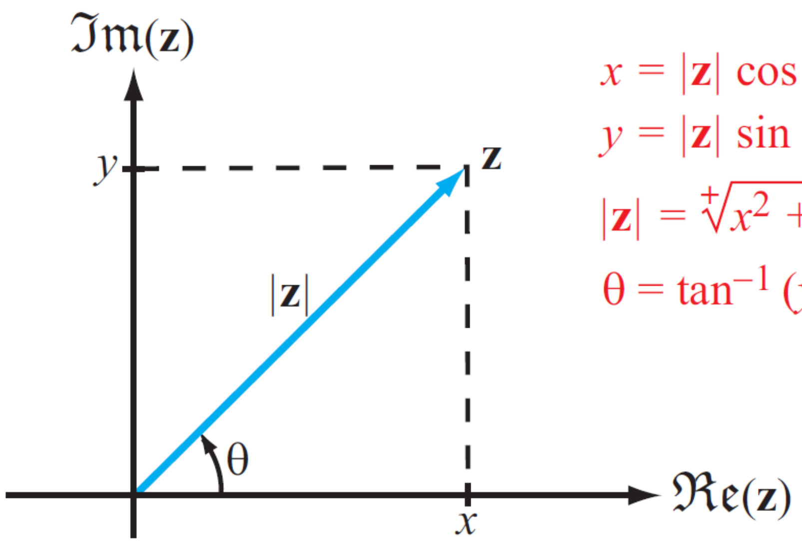
$$j = \sqrt{-1}$$

$z = x + jy$ **Rectangular coordinates**

$z = |z|\angle\theta = |z|e^{j\theta}$ **Polar coordinates**

$$\mathrm{Re}(z) = x$$

$$\mathrm{Im}(z) = y$$



$x = |\mathbf{z}| \cos\theta$

$y = |\mathbf{z}| \sin\theta$

$|\mathbf{z}| = \sqrt[+]{x^2 + y^2}$

$\theta = \tan^{-1}(y/x)$

Relations based on Euler's Identity

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

Phasor Domain

A ***domain transformation*** is a mathematical process that converts a set of variables from their domain into a corresponding set of variables defined in another domain.

1. The phasor-analysis technique transforms equations from the time domain to the phasor domain.

2. Integro-differential equations get converted into linear equations with no sinusoidal functions.

3. After solving for the desired variable--such as a particular voltage or current-- in the phasor domain, conversion back to the time domain provides the same solution that would have been obtained had the original integro-differential equations been solved entirely in the time domain.

Phasor Domain

$$v(t) = V_0 \cos(\omega t + \phi)$$
$$= \Re\mathfrak{e}[\underbrace{V_0 e^{j\phi}}_{\text{Phasor counterpart of } v(t)} e^{j\omega t}]$$

Time Domain		**Phasor Domain**
$v(t) = V_0 \cos \omega t$	$\leftrightarrow$	$\mathbf{V} = V_0$
$v(t) = V_0 \cos(\omega t + \phi)$	$\leftrightarrow$	$\mathbf{V} = V_0 e^{j\phi}.$

If $\phi = -\pi/2$,

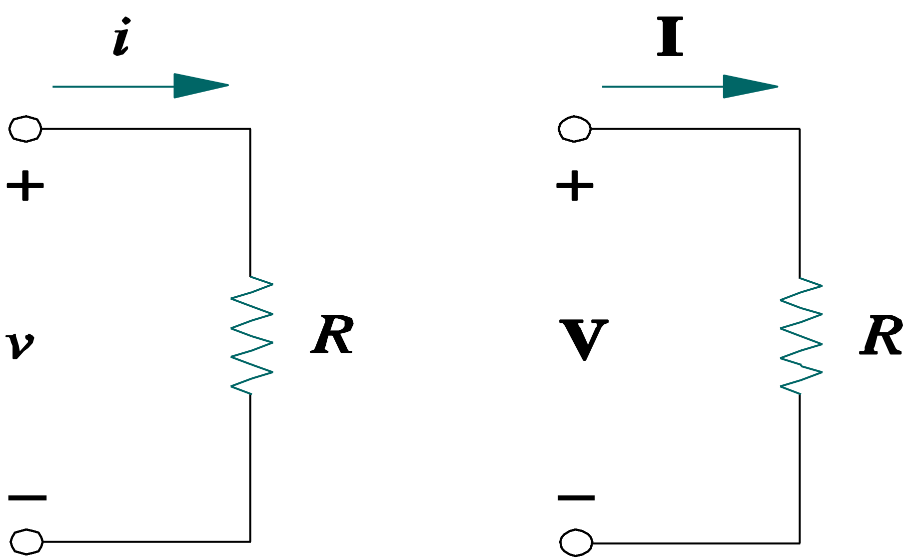
$$v(t) = V_0 \cos(\omega t - \pi/2) \leftrightarrow \mathbf{V} = V_0 e^{-j\pi/2}.$$

Phasor Relation for Resistors

Current through a resistor

Time Domain

Frequency Domain



$v = iR$

$\mathbf{V} = \mathbf{I}R$

Time domain

$$i = I_{\mathrm{m}} \cos(\omega t + \phi)$$

$$v = iR = RI_{\mathrm{m}} \cos(\omega t + \phi)$$

Phasor Domain

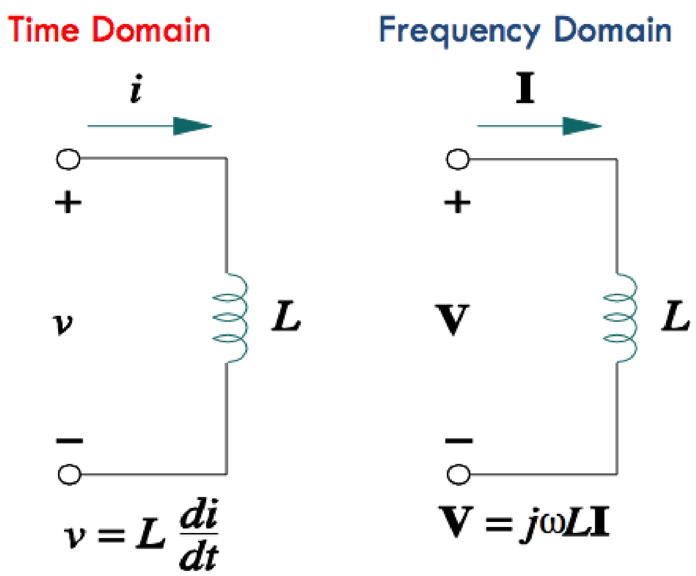
$$\mathbf{V} = R\mathbf{I}$$

$$= RI_{\mathrm{m}} \angle \phi$$

Phasor Relation for Inductors

Current through inductor in time domain

$$i = I_m \cos(\omega t + \phi)$$



Time domain $\upsilon = L\dfrac{di}{dt}$

Phasor Domain

$$\upsilon_L = \Re\mathfrak{e}[\mathbf{V}_L e^{j\omega t}]$$

and

$$i_L = \Re\mathfrak{e}[\mathbf{I}_L e^{j\omega t}].$$

Consequently,

$$\Re\mathfrak{e}[\mathbf{V}_L e^{j\omega t}] = L\,\frac{d}{dt}[\Re\mathfrak{e}(\mathbf{I}_L e^{j\omega t})]$$
$$= \Re\mathfrak{e}[j\omega L\mathbf{I}_L e^{j\omega t}],$$

which leads to

$$\mathbf{V}_L = j\omega L\mathbf{I}_L$$

and

$$\mathbf{Z}_L = \frac{\mathbf{V}_L}{\mathbf{I}_L} = j\omega L.$$

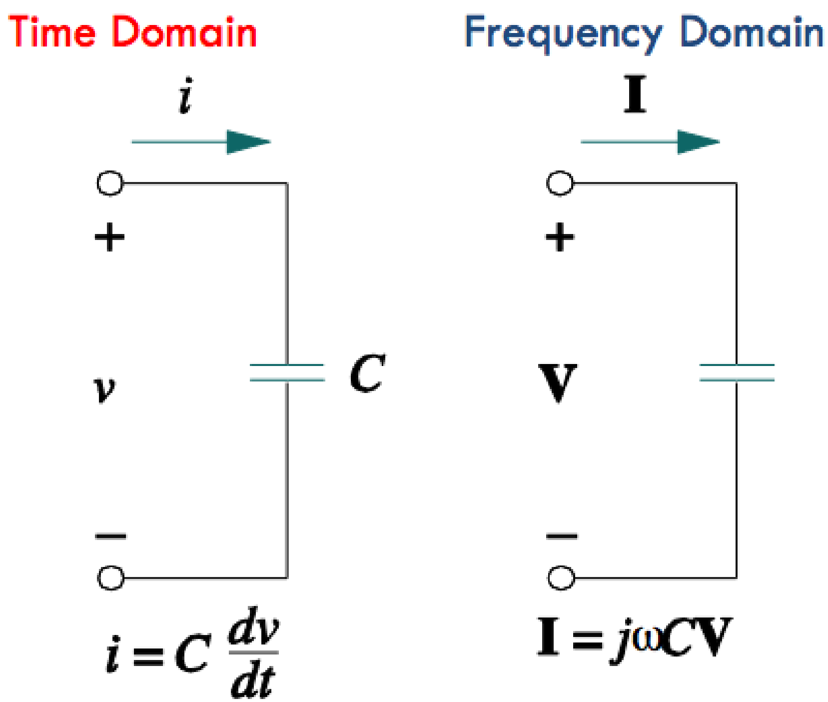
Phasor Relation for Capacitors

Voltage across capacitor in time domain is

$$v = V_{\mathrm{m}} \cos(\omega t + \phi)$$

Time domain

$$i = C\frac{dv}{dt}$$



Phasor Domain

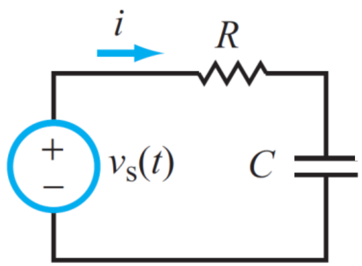
$$\mathbf{I}_{\mathrm{C}} = j\omega C\mathbf{V}_{\mathrm{C}}$$

$$\mathbf{Z}_{\mathrm{C}} = \frac{\mathbf{V}_{\mathrm{C}}}{\mathbf{I}_{\mathrm{C}}} = \frac{1}{j\omega C}.$$

ac Phasor Analysis General Procedure

Using this procedure, we can apply our techniques from dc analysis

Step 1
Adopt Cosine Reference
(Time Domain)



$v_s(t) = 12 \sin(\omega t - 45^\circ)$ (V

Step 3
Cast Equations in
Phasor Form

$$\mathbf{I}\left(R + \frac{1}{j\omega C}\right) = \mathbf{V}_s$$

Step 4
Solve for Unknown Variable
(Phasor Domain)

$$\mathbf{I} = \frac{\mathbf{V}_s}{R + \dfrac{1}{j\omega C}}$$

Step 2
Transfer to Phasor Domain

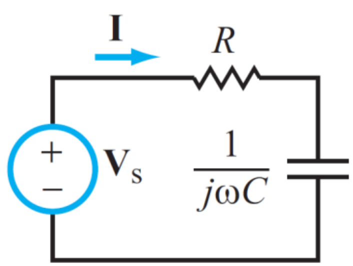
$i \rightarrow \mathbf{I}$

$v \rightarrow \mathbf{V}$

$R \rightarrow \mathbf{Z}_R = R$

$L \rightarrow \mathbf{Z}_L = j\omega L$

$C \rightarrow \mathbf{Z}_C = 1/j\omega C$



$\mathbf{V}_s = 12 e^{-j135^\circ}$ (V)

Step 5
Transform Solution
Back to Time Domain

$$i(t) = \mathfrak{Re}[\mathbf{I} e^{j\omega t}]$$
$$= 6 \cos(\omega t - 105^\circ)$$
(mA)

Example: RL Circuit

The voltage source of the circuit shown in Fig. 7-8(a) is given by

$$v_s(t) = 15\sin(4\times 10^4 t - 30^\circ)\ \text{V}.$$

Also, $R = 3\ \Omega$ and $L = 0.1$ mH. Obtain an expression for the voltage across the inductor.

Solution:

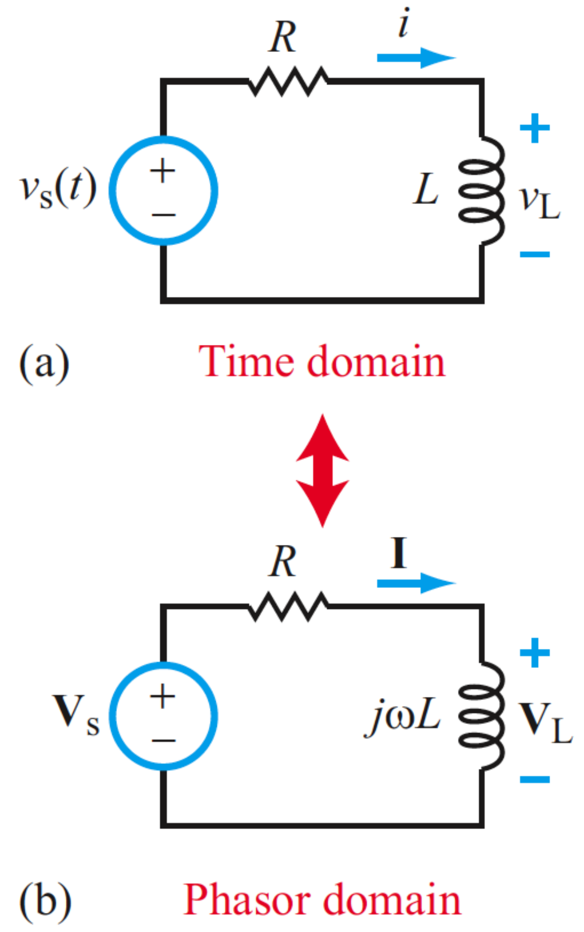
Step 1: Convert $\boldsymbol{v_s(t)}$ to the cosine reference

$$\begin{aligned} v_s(t) &= 15\sin(4\times 10^4 t - 30^\circ) \\ &= 15\cos(4\times 10^4 t - 30^\circ - 90^\circ) \\ &= 15\cos(4\times 10^4 t - 120^\circ)\ \text{V}, \end{aligned}$$

and its corresponding phasor $\mathbf{V}_s$ is given by

$$\mathbf{V}_s = 15e^{-j120^\circ}\ \text{V}.$$

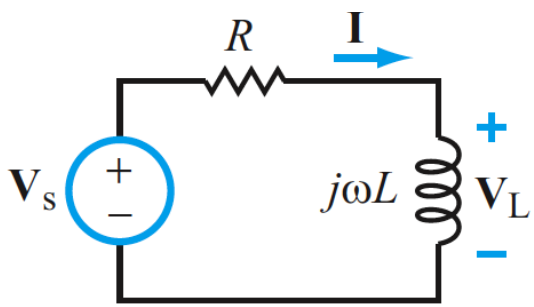
Step 2: Transform circuit to the phasor domain



Example: RL Circuit cont.

Step 3: Cast KVL in phasor domain

$$R\mathbf{I} + j\omega L\mathbf{I} = \mathbf{V}_s.$$



(b) Phasor domain

Step 4: Solve for unknown variable

$$\mathbf{I} = \frac{\mathbf{V}_s}{R + j\omega L} = \frac{15e^{-j120^\circ}}{3 + j4 \times 10^4 \times 10^{-4}}$$

$$= \frac{15e^{-j120^\circ}}{3 + j4} = \frac{15e^{-j120^\circ}}{5e^{j53.1^\circ}} = 3e^{-j173.1^\circ} \text{ A.}$$

The phasor voltage across the inductor is related to $\mathbf{I}$ by

$$\mathbf{V}_L = j\omega L\mathbf{I}$$

$$= j4 \times 10^4 \times 10^{-4} \times 3e^{-j173.1^\circ}$$

$$= j12e^{-j173.1^\circ}$$

$$= 12e^{-j173.1^\circ} \cdot e^{j90^\circ} = 12e^{-j83.1^\circ} \text{ V,}$$

where we replaced j with e^{j90°.

Example: RL Circuit cont.

Step 5: Transform solution to the time domain

The corresponding time-domain voltage is

$$
\begin{aligned}
v_{\mathrm{L}}(t) &= \mathfrak{Re}[\mathbf{V}_{\mathrm{L}} e^{j\omega t}] \\
&= \mathfrak{Re}[12 e^{-j83.1^\circ} e^{j4\times 10^4 t}] \\
&= 12 \cos(4 \times 10^4 t - 83.1^\circ)\ \mathrm{V}.
\end{aligned}
$$

Impedance and Admittance

Impedance is voltage/current

$$\mathbf{Z} = R + jX$$

R = resistance = Re(Z)

X = reactance = Im(Z)

Admittance is current/voltage

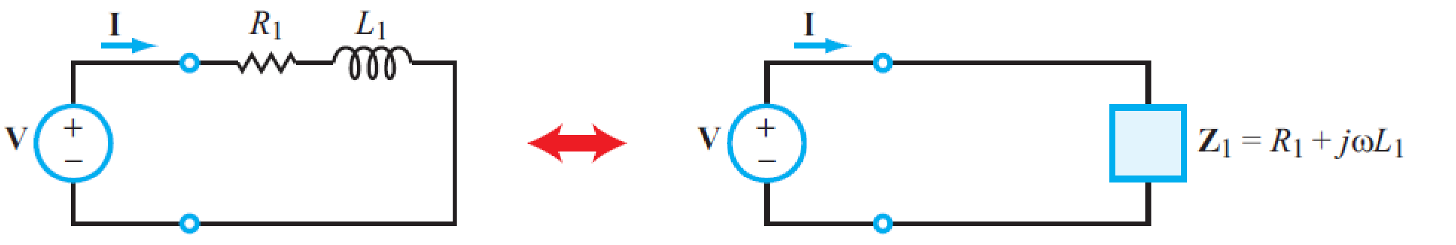
$$\mathbf{Y} = \frac{1}{\mathbf{Z}} = G + jB$$

G = conductance = Re(Y)

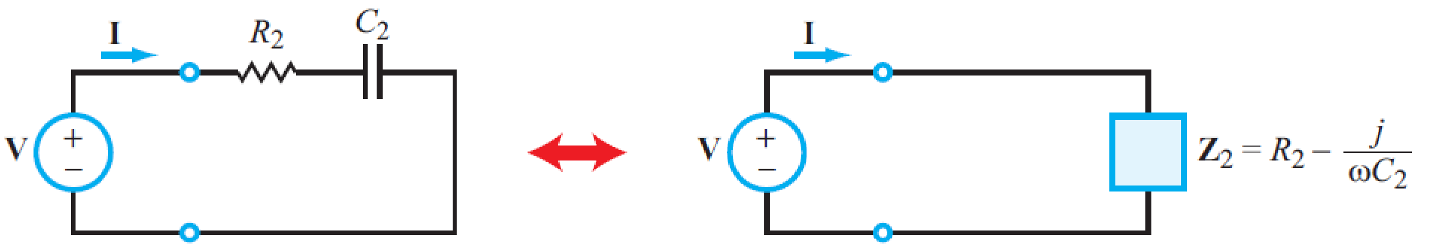
B = susceptance = Im(Y)

Resistor	$\mathbf{Z} = R$	$\mathbf{Y} = 1/R$
Inductor	$\mathbf{Z} = j\omega L$	$\mathbf{Y} = 1/j\omega L$
Capacitor	$\mathbf{Z} = 1/j\omega C$	$\mathbf{Y} = j\omega C$

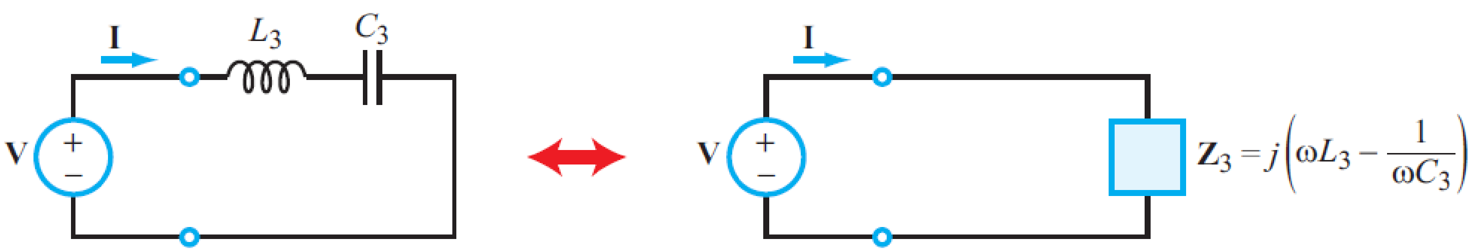
Impedance Transformation



(a) RL



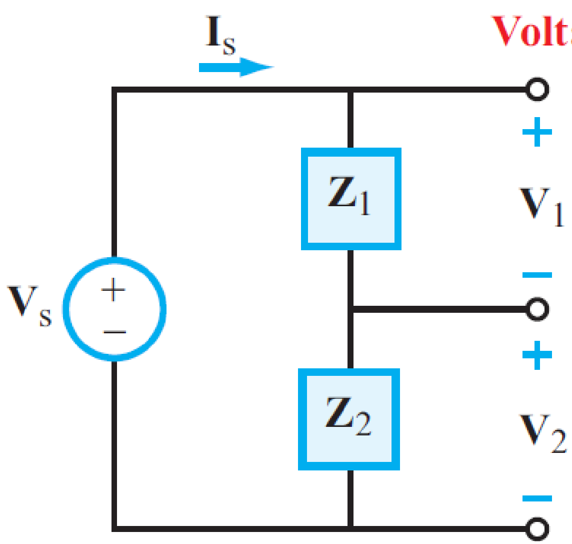
(b) RC



(c) LC

Voltage & Current Division

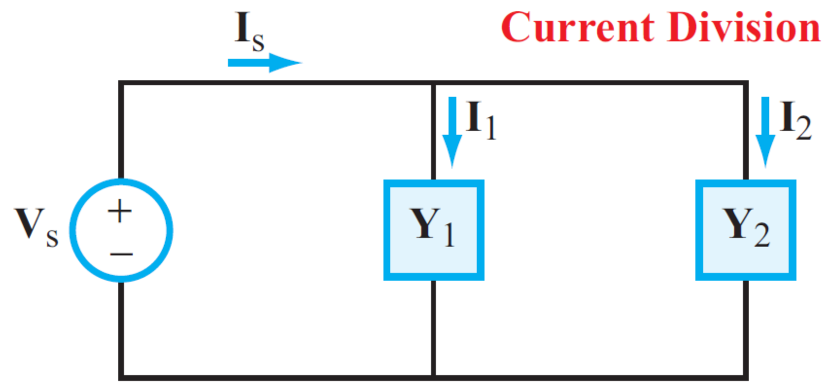
Voltage Division



$$\mathbf{V}_1 = \left(\frac{\mathbf{Z}_1}{\mathbf{Z}_1 + \mathbf{Z}_2}\right)\mathbf{V}_s$$

$$\mathbf{V}_2 = \left(\frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}\right)\mathbf{V}_s$$

Current Division

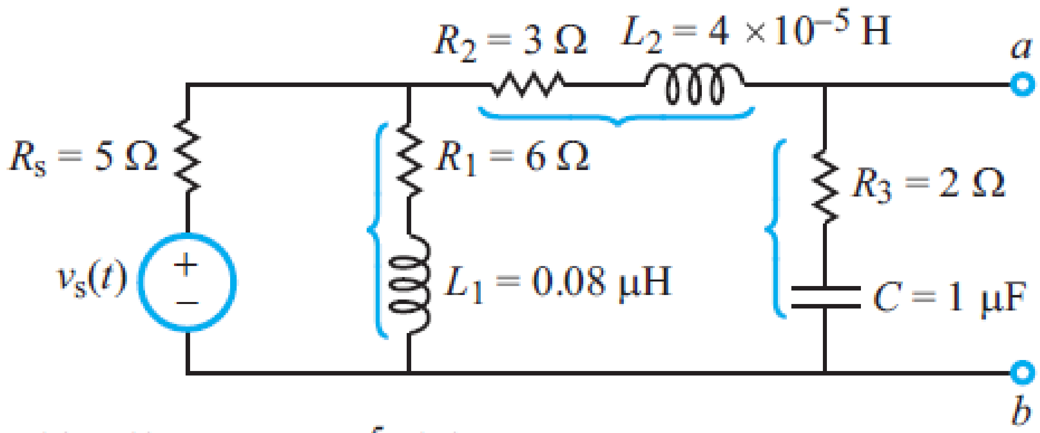


$$\mathbf{I}_1 = \left(\frac{\mathbf{Y}_1}{\mathbf{Y}_1 + \mathbf{Y}_2}\right)\mathbf{I}_s \qquad \mathbf{I}_2 = \left(\frac{\mathbf{Y}_2}{\mathbf{Y}_1 + \mathbf{Y}_2}\right)\mathbf{I}_s$$

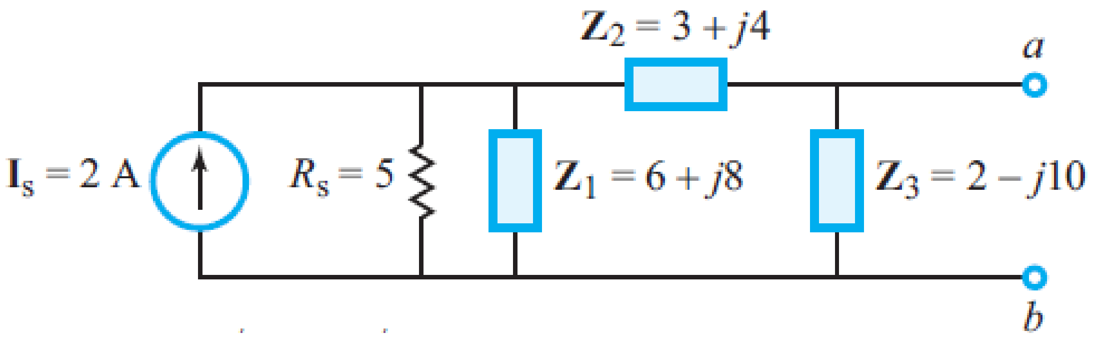
Linear circuit techniques

- We can now apply all the techniques we learned before (for dc circuits in the time domain) to ac circuits in the phase domain:
  - Superposition
  - Thevenin / Norton Equivalents

Example: Thévenin Circuit

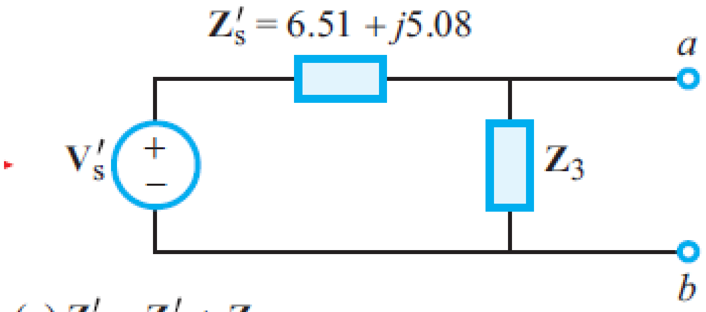


(a) $v_s(t) = 10 \cos 10^5 t$ (V)

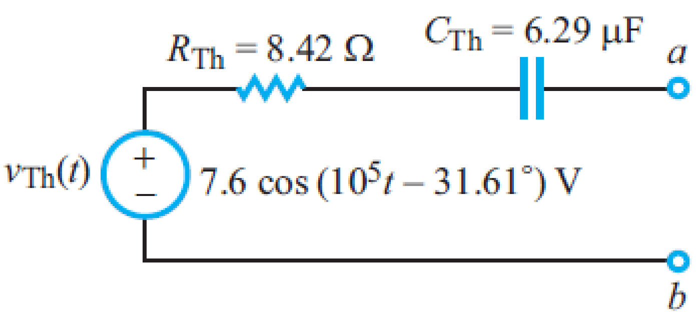


Slide 5-17

Example: Thévenin Circuit (cont'd)



(e) $\mathbf{Z}'_s = \mathbf{Z}'_1 + \mathbf{Z}_2$



(f) Thévenin equivalent

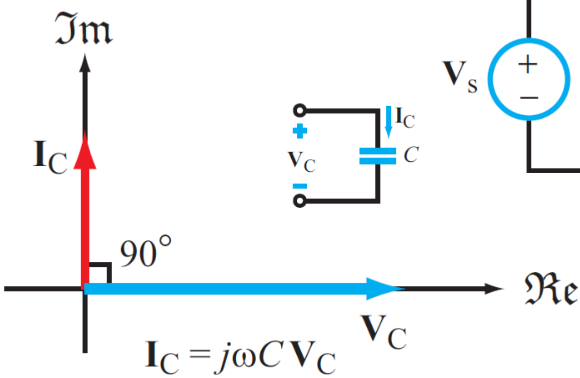
$$\mathbf{Z}_{\mathrm{Th}} = \mathbf{Z}'_s \parallel \mathbf{Z}_3$$

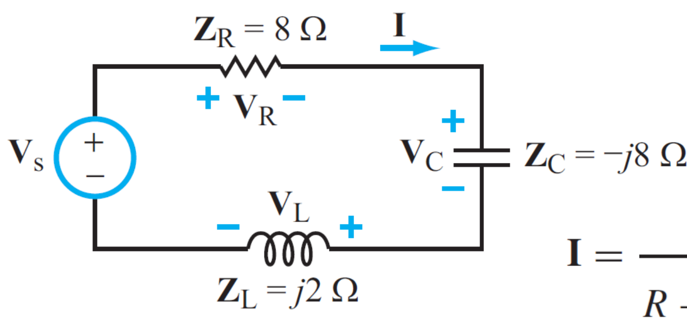
$$= \frac{(6.51 + j5.08)(2 - j10)}{(6.51 + j5.08) + (2 - j10)} = (8.42 - j1.59)\ \Omega$$

$$\boxed{R_{\mathrm{Th}} = 8.42\ \Omega, \qquad C_{\mathrm{Th}} = \frac{1}{1.59\omega} = 6.29\ \mu\mathrm{F}}$$

Phasor Diagrams

Capacitor

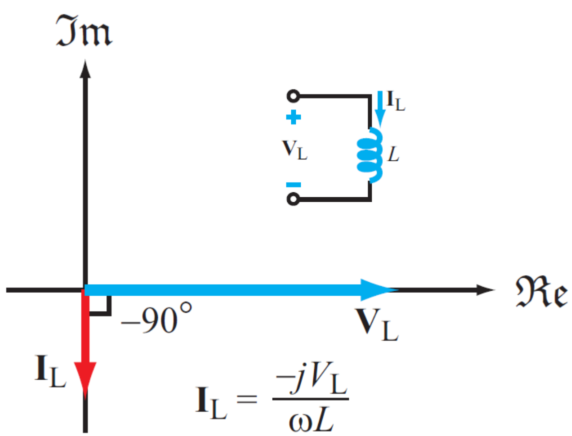


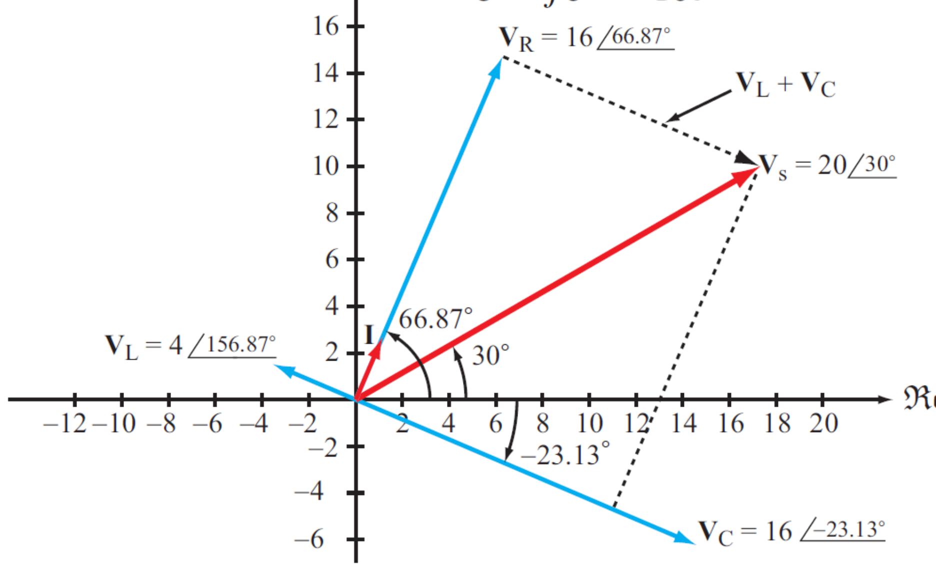


$$\mathbf{I} = \frac{\mathbf{V}_s}{R + j\omega L - \dfrac{j}{\omega C}} = \frac{20e^{j30^\circ}}{8 + j2 - j8}$$

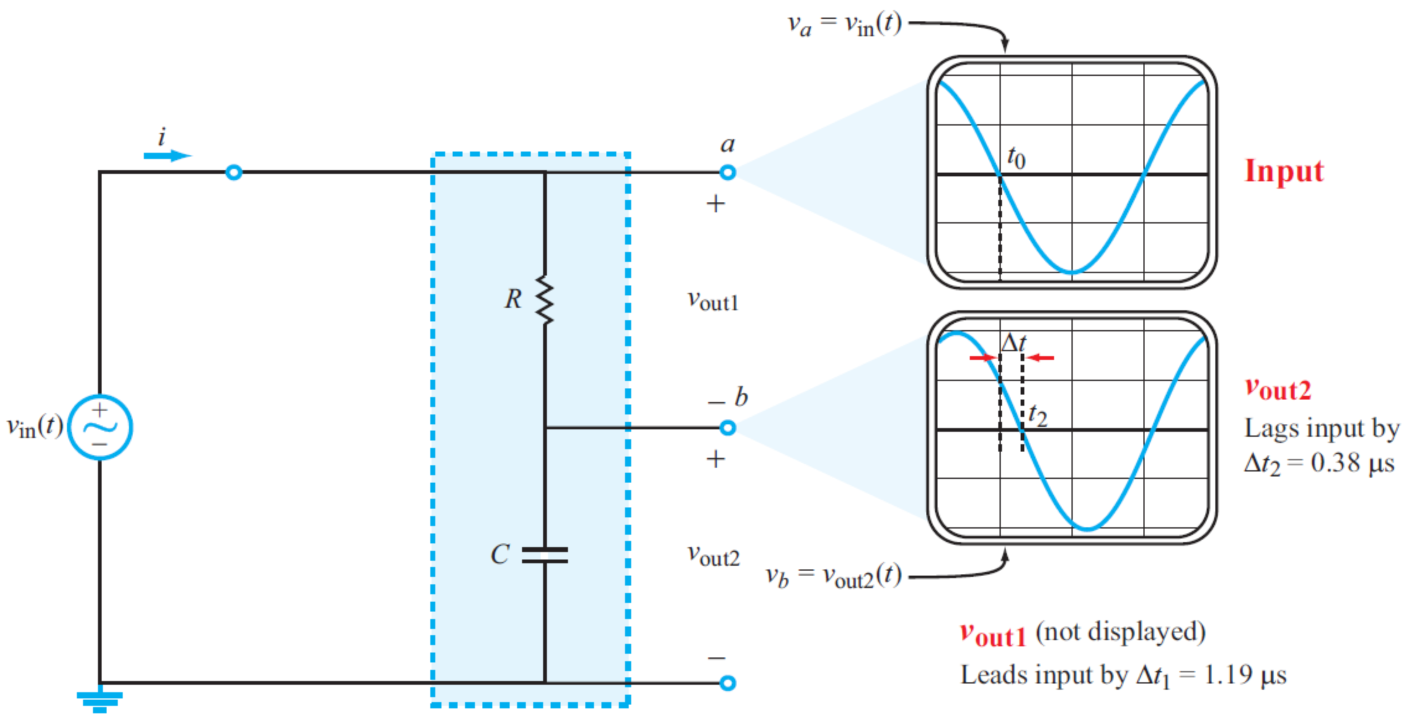
$$= \frac{20e^{j30^\circ}}{8 - j6} = \frac{20e^{j30^\circ}}{10e^{-j36.87^\circ}} = 2e^{j66.87^\circ}\ \text{A}$$

Inductor

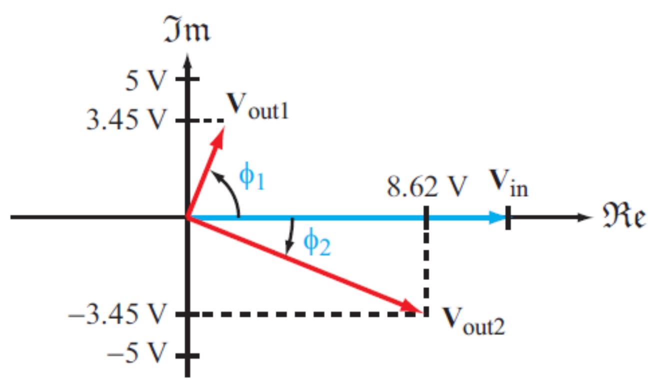




Phasor Shift Circuits



(a) Time-domain waveforms



(c) Phasors $\mathbf{V}_{in}$, $\mathbf{V}_{out1}$, and $\mathbf{V}_{out2}$ in the complex plane

Power Supply Circuit

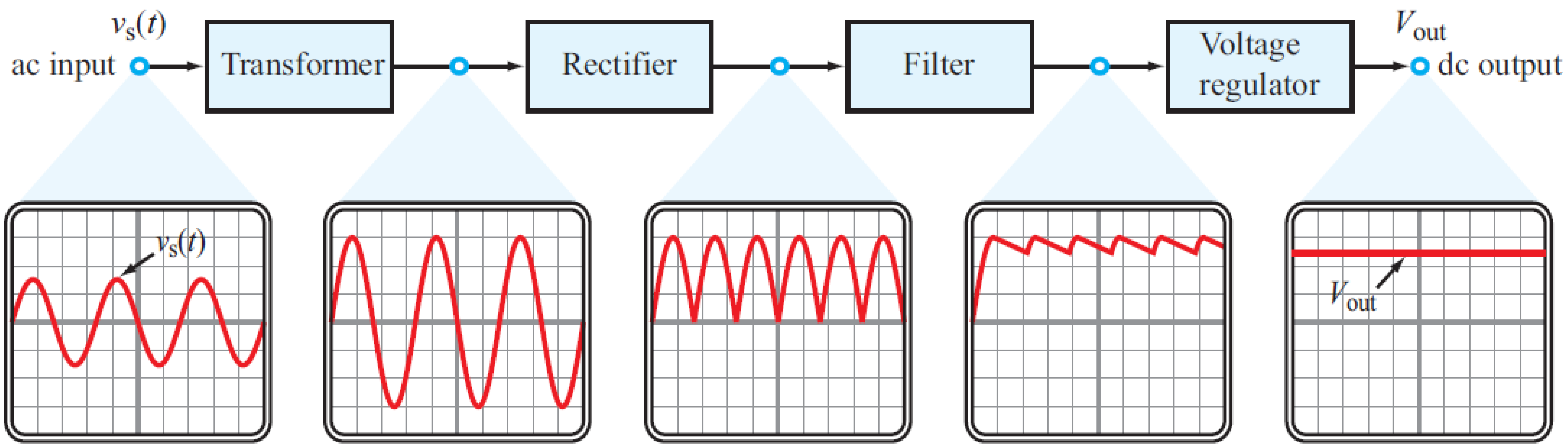


Figure 7-31: Block diagram of a basic dc power supply.

Ideal Transformer

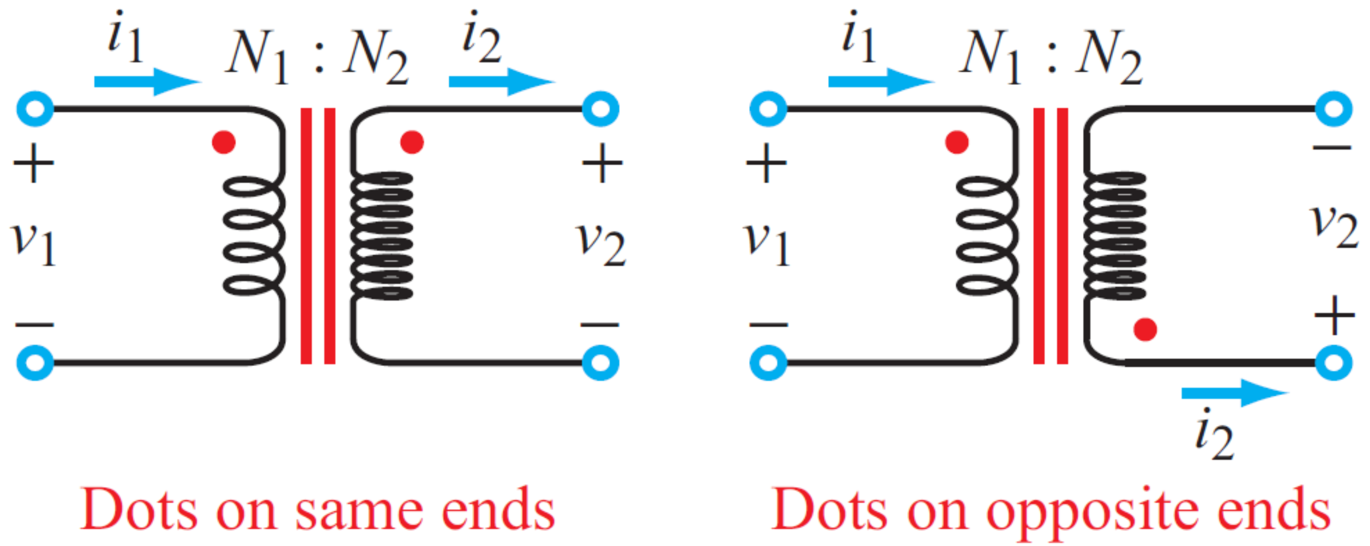


Figure 7-32: Schematic symbol for an ideal transformer. Note the reversal of the voltage polarity and current direction when the dot location at the secondary was moved from the top end of the coil to the bottom end. For both configurations:

$$\frac{v_2}{v_1} = \frac{N_2}{N_1} \qquad \frac{i_2}{i_1} = \frac{N_1}{N_2}$$

Half-Wave Rectifier

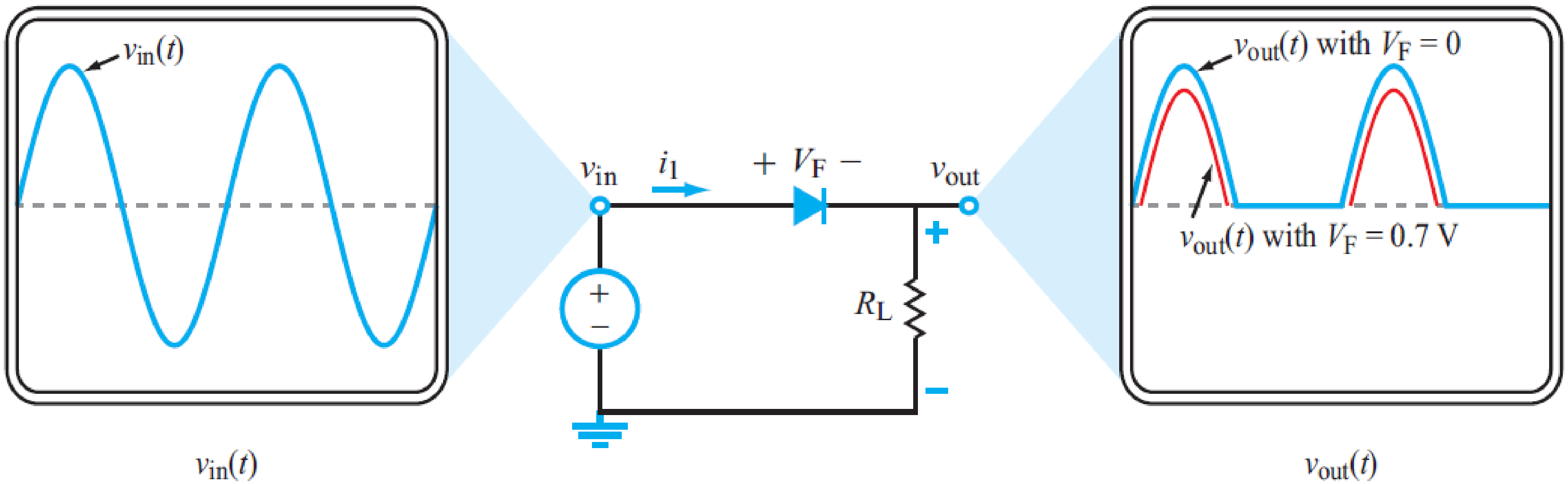
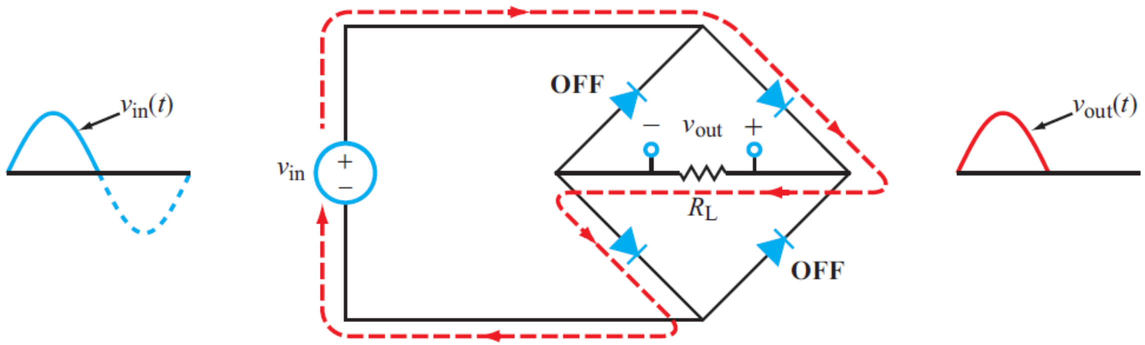


Figure 7-33: Half-wave rectifier circuit.

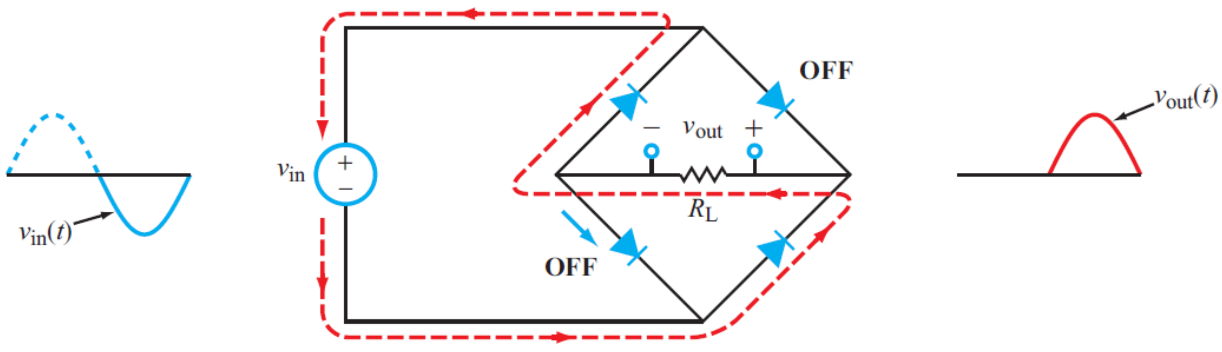
Full-Wave Rectifier

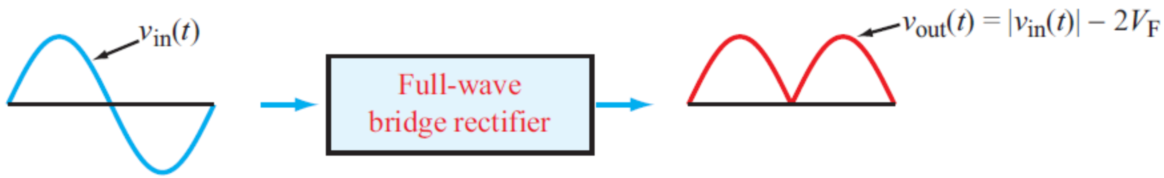
Current flow during first half of cycle



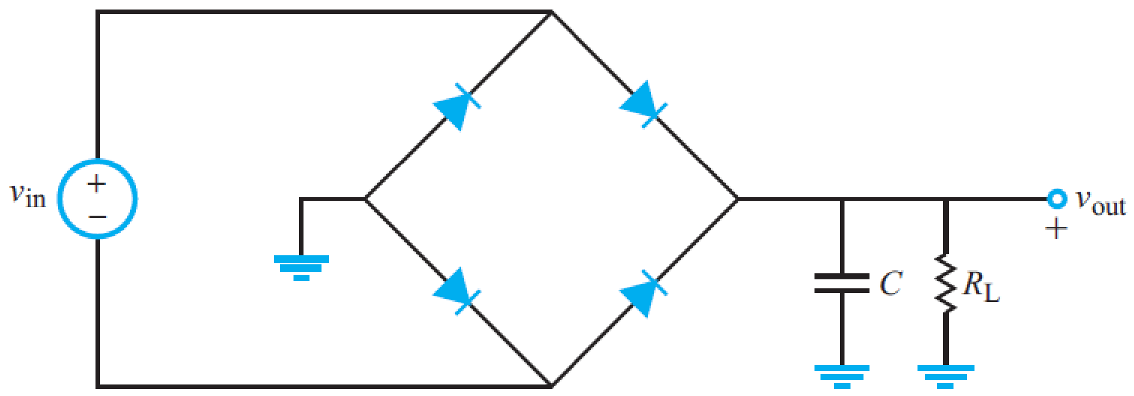
(a) Positive half cycle

Current flow during second half of cycle

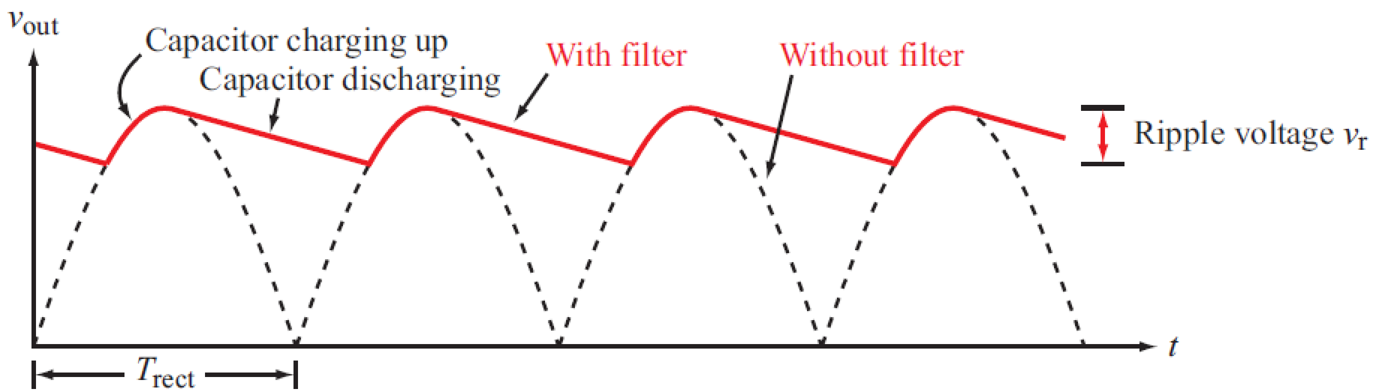




Smoothing RC Filter



(a) Bridge rectifier with filter



(b) Filtered output

Complete Power Supply

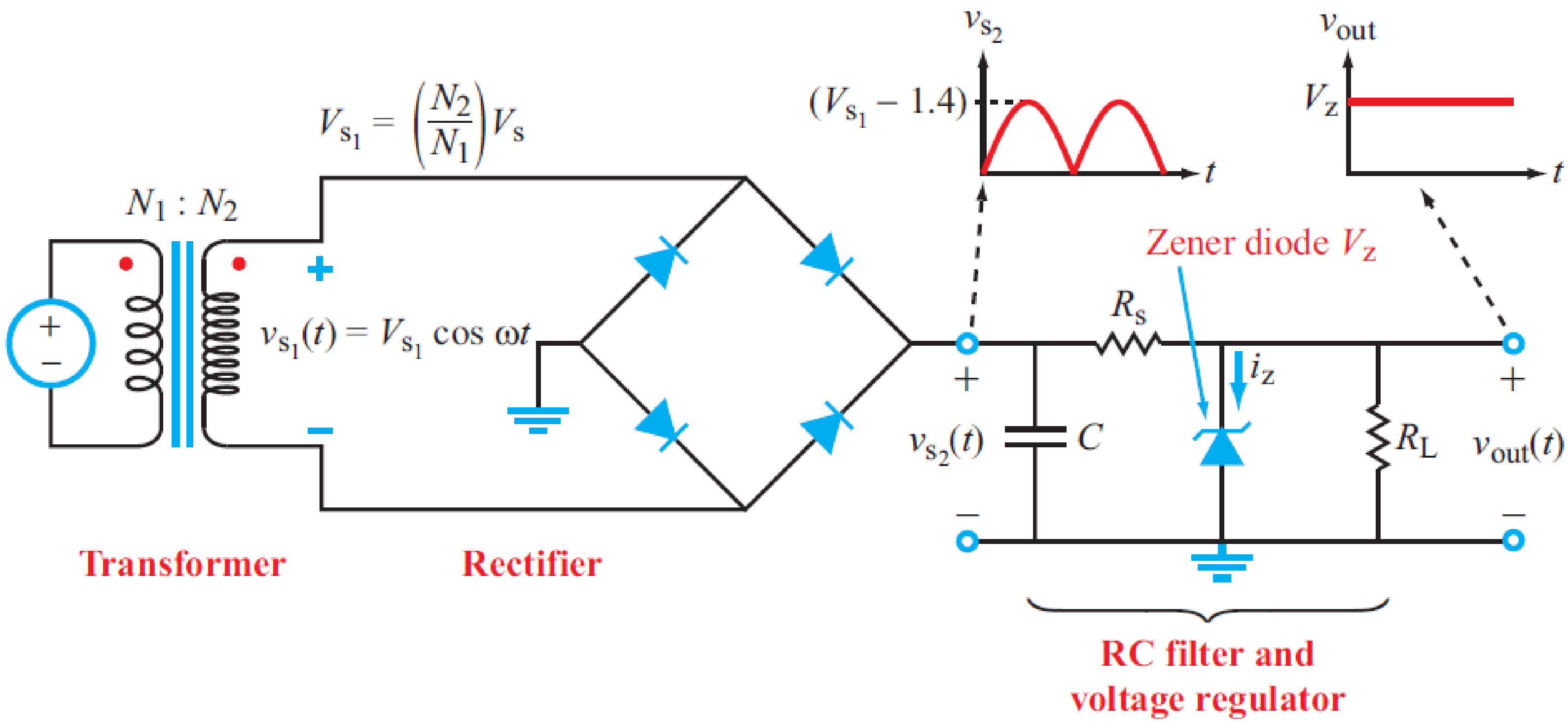


Figure 7-36: Complete power-supply circuit.