Assignment-1

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Q1

A "computational" KR model can represent knowledge and is "computer processable". For ontology, as Staab and Studer said, "An ontology is a formal, explicit specification of a shared conceptualization". For ontology, we fix vocabulary relevant to domain and fix meaning of terms in vocabulary so that we can use these vocabulary to stand for another domain (aka. knowledge or real world), this is why ontology can represent knowledge. Ontology is computational because it uses syntax to represent knowledge and syntax is "computer processable" and can be reasoned by computer.

Q2

"He is running fast.", this sentence can not be modeled in the formal languages. I think a logic-based KR language should not be designed as expressive as possible. For expressiveness, more expressive power comes with computational cost. For example, first-order logic is more expressive than zero-order logic, but first-order logic is undecidable. Again for instance, higher-order logic's model-theoretic properties are less well-behaved than those of first-order logic. Therefore, a KR language should be expressive enough to modeling something in certain circumstance but not as expressive as possible in any circumstances.

Q3

(1)

Timo is a cow.

Tom is "LivestockOwner", and "LivestockOwner" should hasLivestock cow or sheep. Tom hasLivestock Timo that is not sheep. We can infer Timo is a cow.

(2)

No, Fido is a sheep.

Tom hasLivestock Timo and Fido. LivestockOwner must have cow and sheep.   Timo is cow, so Fido is sheep.

(3)

No.

We can know that LivestockOwner should hasLivestock some cow, but not if a person has some cow, he is a LivestockOwner.

(4)

Zero.

LivestockOwner only has one or more cats.

Q4

(1)

1

concept names: Couple , Chinese
role names: hasChild

$$Chinese \sqcap Couple \sqsubseteq \leq 3 hasChild.\top$$

2

concept names: Course, Professor
role names: taught_by, working_at
nominals: ML, SFM, NJU

$$\{ML\} \sqsubseteq Course \sqcap \exists taught\_by.\{SFM\}$$
$$\{SFM\} \sqsubseteq Professor \sqcap \exists working\_at.\{NJU\}$$

3

concept names: university, school, department

role names: as_member_of

nominals: NJU

$$\{NJU\} \sqsubseteq university$$
$$\{NJU\} \sqsubseteq \exists has\_member.school \sqcup department$$

4

concept names: student

role names: has_student

nominals:NJU

$$\{NJU\} \sqsubseteq \geq 30000 has\_student.student$$

5

concept names: undergraduates, graduates, teachers

role names: has_member

nominals: AI School

$$\exists member\_of.\{AISchool\} \sqsubseteq undergraduates \sqcup graduates \sqcup teachers$$

6

concept names:countries

role names:citizenOf, consists of

$$\exists citizenOf.\top \sqsubseteq countries$$

(2)

5

$$\forall x(memberof(x, AISchool) \to (undergraduates(x) \vee graduates(x) \vee teachers(x)))$$

6

$$\forall x(\exists y.citizenOf(x, y) \to countries(x))$$

Q5

1

Prove:

We construct an ontology: $A \sqsubseteq \neg A$, assume there is a model $\mathcal{I}$, then $A^{\mathcal{I}} \subseteq (\neg A)^{\mathcal{I}} = \Delta^{\mathcal{I}} \setminus A^{\mathcal{I}}$, contradiction occur, so this ontology has no model.

2

Disprove:

Assume there is an ontology that has only finite models and one of the models is $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$. Then if $x \in \Delta^{\mathcal{I}}$, $\Delta'^{\mathcal{I}} = \Delta^{\mathcal{I}} \setminus \{x\} \cup \{x'\}$, x' is fresh, and $\mathcal{I}' = (\Delta'^{\mathcal{I}}, \cdot^{\mathcal{I}})$ is also a model for this ontology. We can fine infinite fresh elements, so there are infinite models.

3

Prove:

From **1** and **2**, we can infer **3** is true.

4

Prove:

A satisfiable class **C** w.r.t. $\mathcal{T}$ exists a model $\mathcal{I}$ of $\mathcal{T}$ and some $d \in \Delta^{\mathcal{I}}, d \in C^{\mathcal{I}}$, so $\mathcal{I}$ is not empty.

5

Disprove:

Assume an unsatisfiable class **C** have a non-empty interpretation $\mathcal{I}$ in some model, but at this case by definition , **C** is satisfiable, contradiction occur.

6

Prove:

From **5**, we know an unsatisfiable class **C** has only empty interpretation, and empty set is subclass of any other class.

Q6

$(\neg A)^{\mathcal{I}} = \{f, h, i\}$

$(\exists r.(A \sqcup B))^{\mathcal{I}} = \{d, e, f\}$

$(\exists s.\exists s.\neg A)^{\mathcal{I}} = \{d, e\}$

$(\neg A \sqcap \neg B)^{\mathcal{I}} = \{h, i\}$

$(\forall r.(A \sqcup B))^{\mathcal{I}} = \{d, e, f, g, h, i\}$

$(\leq 1 s.\top)^{\mathcal{I}} = \{e, f, g, h, i\}$

Q7

(1)

$(Q \sqcap \geq 2 r.P)^{\mathcal{I}} = \emptyset$

$(\forall r.Q)^{\mathcal{I}} = \{b, c, d, e\}$

$(\neg \exists r.Q)^{\mathcal{I}} = \{b, c, e\}$

$(\forall r.\top \sqcap \exists r^{-}.P)^{\mathcal{I}} = \{b, d, e\}$

$(\exists r^{-}.\bot)^{\mathcal{I}} = \emptyset$

(2)

$(A \sqcap B)^{\mathcal{I}} = \emptyset$

$(\exists r.B)^{\mathcal{I}} = \{1, 2\}$

$(\exists r.(A \sqcap B))^{\mathcal{I}} = \emptyset$

$(\top)^{\mathcal{I}} = \{1, 2, 3, 4, 5, 6\}$

$(A \sqcap \exists r.B)^{\mathcal{I}} = \{1, 2\}$

These are true:

- $\mathcal{I} \models A \equiv \exists r.B$
- $\mathcal{I} \models A \sqcap B \sqsubseteq \top$
- $\mathcal{I} \models \exists r.A \sqsubseteq A \sqcap B$

Q8

1

Hold.
for any $\mathcal{I}$, if $(C \sqsubseteq D)^{\mathcal{I}}$, then $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$. As $(\exists r.C)^{\mathcal{I}} = \{x \in \Delta^{\mathcal{I}} | \exists y \in \Delta^{\mathcal{I}}, y \in C^{\mathcal{I}}, (x,y) \in r^{\mathcal{I}}\} \subseteq \{x \in \Delta^{\mathcal{I}} | \exists y \in \Delta^{\mathcal{I}}, y \in D^{\mathcal{I}}, (x,y) \in r^{\mathcal{I}}\}$, so $\exists r.C \sqsubseteq \exists r.D$ holds.

2

Do not hold.
We construct a $\mathcal{I}$. $\Delta^{\mathcal{I}} = \{a\}, C^{\mathcal{I}} = \emptyset, r^{\mathcal{I}} = \{(a,a)\}$, then $(\exists r.C)^{\mathcal{I}} = \emptyset \nsupseteq \{a\} = (\leq 1r.\top)^{\mathcal{I}}$.

3

Hold.
for any I, $(\leq 0r.\top)^{\mathcal{I}} = \{x \in \Delta^{\mathcal{I}} || y \in \Delta^{\mathcal{I}} | (x,y) \in r^{\mathcal{I}}| \leq 0\} = \{x \in \Delta^{\mathcal{I}} | \text{no y such that } (x,y) \in r^{\mathcal{I}}\}$. $(\forall r.\bot)^{\mathcal{I}} = \{x \in \Delta^{\mathcal{I}} | \text{no y such that } (x,y) \in r^{\mathcal{I}}\}$, so $(\leq 0r.\top)^{\mathcal{I}} = (\forall r.\bot)^{\mathcal{I}}$, so $\leq 0r.\top$ is equivalent to $\forall r.\bot$.

4

Do not hold.
We construct a $\mathcal{I}$. $\Delta^{\mathcal{I}} = \{a,b\}, A^{\mathcal{I}} = \{a\}, B^{\mathcal{I}} = \{b\}, r^{\mathcal{I}} = \{(a,a),(a,b)\}$. Then $(\forall r.(A \sqcup B))^{\mathcal{I}} = \{a,b\} \nsubseteq \{b\} = ((\forall r.A) \sqcup (\forall r.B))^{\mathcal{I}}$.

5

Hold.
for any interpretation $\mathcal{I}$:
$x \in (\exists r.(A \sqcup B))^{\mathcal{I}}$ if and only if $\exists y \in (A \sqcup B)^{\mathcal{I}}$ such that $(x,y) \in r^{\mathcal{I}}$($y \in (A \sqcup B)^{\mathcal{I}}$ means $y \in A^{\mathcal{I}} \text{or } y \in B^{\mathcal{I}}$), if and only if $\exists y \in A^{\mathcal{I}}$ such that $(x,y) \in r^{\mathcal{I}}$ or $\exists y \in B^{\mathcal{I}}$ such that $(x,y) \in r^{\mathcal{I}}$, if and only if $x \in ((\exists r.A) \sqcup (\exists r.B))^{\mathcal{I}}$.

Q9

Construct an interpretation $\mathcal{I}$,Let $\mathcal{I}$ be defined by:

- $\Delta^{\mathcal{I}} = \{a,b,c,d,e,f\}$
- $Person^{\mathcal{I}} = \{a,b,c,d,e,f\}$
- $Parent^{\mathcal{I}} = \{a,c\}$

- $Mother^{\mathcal{I}} = \{a\}$
- $hasChild^{\mathcal{I}} = \{(a, b), (c, d), (e, f)\}$

Then:
$Parent^{\mathcal{I}} = \{a, c\} \subseteq \{a, c, e\} = (\exists hasChild.Person)^{\mathcal{I}}$
and:
$Mother^{\mathcal{I}} = \{a\} \subseteq \{a, c\} = Parent^{\mathcal{I}}$
and so $\mathcal{I} \models \mathcal{T}$.
but:
$Parent^{\mathcal{I}} = \{a, c\} \nsubseteq \{a\} = Mother^{\mathcal{I}}$
so $\mathcal{I} \not\models Parent \sqsubseteq Mother$

Q10

(1)

- if $X \sqsubseteq_{\mathcal{T}} Y$, then for every model $\mathcal{I}$ of $\mathcal{T}$, we have $X^{\mathcal{I}} \subseteq Y^{\mathcal{I}}$. Assume there is a model $\mathcal{I}$ and some $d \in \Delta^{\mathcal{I}}$, $d \in (X \sqcap \neg Y)^{\mathcal{I}} = X^{\mathcal{I}} \cap (\Delta^{\mathcal{I}} \setminus Y^{\mathcal{I}})$, that is said $d \in X^{\mathcal{I}}$ but $d \notin Y^{\mathcal{I}}$, contradiction occur.
- if $X \sqcap \neg Y$ is not satisfiable w.r.t. $\mathcal{T}$, for every model $\mathcal{I}$ of $\mathcal{T}$, some $d \in \Delta^{\mathcal{I}}$, $d \notin (X \sqcap \neg Y)^{\mathcal{I}}$, so if $d \in X^{\mathcal{I}}$, then $d \notin (\neg Y)^{\mathcal{I}}, d \in Y^{\mathcal{I}}$. That is said $X^{\mathcal{I}} \subseteq Y^{\mathcal{I}}, X \sqsubseteq_{\mathcal{T}} Y$.

(2)

- if X is satisfiable w.r.t. $\mathcal{T}$, there exists a model $\mathcal{I}$ of $\mathcal{T}$ and some $d \in \Delta^{\mathcal{I}}$ with $d \in X^{\mathcal{I}}$. So $X^{\mathcal{I}} \nsubseteq \emptyset = (\bot)^{\mathcal{I}}$, also $X \not\sqsubseteq \bot$.
- if $X \not\sqsubseteq \bot$, there exists a model $\mathcal{I}$ of $\mathcal{T}$, and $X^{\mathcal{I}} \nsubseteq \emptyset = (\bot)^{\mathcal{I}}$, so $X^{\mathcal{I}}$ is not empty, for some $d \in \Delta^{\mathcal{I}}$, we have $d \in X^{\mathcal{I}}$, this means X is satisfiable w.r.t. $\mathcal{T}$.

Q11

(1)

Proof:
Since $(\mathcal{I}_1, d_1) \sim (\mathcal{I}_2, d_2)$, there is a bisimulation $\bigotimes$ between $\mathcal{I}_1$ and $\mathcal{I}_2$ such that $d_1 \sim d_2$. We prove the theorem by induction on the structure of C. Since, up to equivalence, any $\mathcal{ALC}$ concept can be constructed using only the constructors conjunction, negation, and existential quantification, , we

consider only these constructors in the induction step. The base case is the one where C is a concept name.

- Assume that $C = A$. Then $d_1 \in A^{\mathcal{I}_1}$, if and only if $d_2 \in A^{\mathcal{I}_2}$, is an immediate consequence of $d_1 \bigotimes d_2$.
- Assume that $C = D \sqcap E$. Then $d_1 \in (D \sqcap E)^{\mathcal{I}_1}$ if and only if $d_1 \in D^{\mathcal{I}_1}$ and $d_1 \in E^{\mathcal{I}_1}$ (due to semantics), if and only if $d_2 \in D^{\mathcal{I}_2}$ and $d_2 \in E^{\mathcal{I}_2}$ (due to induction hypothesis), if and only if $d_2 \in (D \sqcap E)^{\mathcal{I}_2}$ (due to semantics).
- Assume that $C = \neg F$. Then $d_1 \in (\neg F)^{\mathcal{I}_1}$ if and only if $d_1 \notin F^{\mathcal{I}_1}$ (due to semantics), if and only if $d_2 \notin F^{\mathcal{I}_2}$ (due to induction hypothesis), if and only if $d_2 \in (\neg F)^{\mathcal{I}_2}$ (due to semantics).
- Assume that $C = \exists r.G$. Then $d_1 \in (\exists r.G)^{\mathcal{I}_1}$ if and only if there exist $d_1' \in G^{\mathcal{I}_1} and (d_1, d_1') \in r^{\mathcal{I}_1}$ (due to semantics), if and only if there exist $d_2' \in \Delta^{\mathcal{I}_2}$ such that $d_1' \bigotimes d_2'$ (due to definition1(i), we have $d_2' \in G^{\mathcal{I}_2}$) and $(d_2, d_2') \in r^{\mathcal{I}_2}$(due to definition1(ii)), if and only if $d_2 \in (\exists r.G)^{\mathcal{I}_2}$.

End.

(2)

Construct $\mathcal{ALCQ}$ concept $A = \geq 2r.\top$ and $\Delta^{\mathcal{I}_1} = \{a, b, c\}$, $r^{\mathcal{I}_1} = \{(a, b), (a, c)\}$, $\Delta^{\mathcal{I}_2} = \{d, e\}$, $r^{\mathcal{I}_2} = \{(d, e)\}$. $(\mathcal{I}_1, a) \sim (\mathcal{I}_2, d)$. We have $a \in A^{\mathcal{I}_1} \text{but } d \notin A^{\mathcal{I}_2}$, from **(1)**, we know $\mathcal{ALC}$ do not support this.

(3)

if $(\mathcal{I}_1, d_1) \sim (\mathcal{I}_2, d_2)$, for a $\mathcal{ALC}$-TBox $\mathcal{T}$, we try to **show:** $\mathcal{I}_1 \models \mathcal{T}$ if and only if $\mathcal{I}_2 \models \mathcal{T}$. For $C \sqsubseteq D$ in $\mathcal{T}$, if $\mathcal{I}_1 \models C \sqsubseteq D$, let $d_1 \in C^{\mathcal{I}_1}$, we have $d_2 \in C^{\mathcal{I}_2}$, so $\mathcal{I}_2 \models C \sqsubseteq D$, vice versa. Construct $\Delta^{\mathcal{I}_1} = \{a, b, c\}$, $r^{\mathcal{I}_1} = \{(a, b), (b, c), (a, c)\}$, $\Delta^{\mathcal{I}_2} = \{d, e, f\}$, $r^{\mathcal{I}_2} = \{(d, e), (e, f)\}$. For $\mathbf{transitive(r)}$ which is in Tbox, we have $\mathcal{I}_1 \models transitive(r)$ but $\mathcal{I}_2 \not\models transitive(r)$, from our proof above we know this do not happen in $\mathcal{ALC}$-Tbox.

Q12

(1)

The axiom count is 801, the logical axiom count is 322. The axiom consist of logical axiom and non-logical axiom, such as comment. The logical axiom is the axiom that can be used in reasoning.

(2)

use nominals:

- Country EquivalentTo DomainThing and ({ America, England, France, Germany, Italy })

use negations:

- VegetarianPizza EquivalentTo Pizza and (not (hasTopping some SeafoodTopping)) and (not (hasTopping some MeatTopping))
- NonVegetarianPizza EquivalentTo Pizza and (not VegetarianPizza)

declare a sub-property of an object property:

- hasBase SubPropertyOf hasIngredient
- hasTopping SubPropertyOf hasIngredient
- isBaseOf SubPropertyOf isIngredientOf
- isToppingOf SubPropertyOf isIngredientOf

declare an inverse property:

- hasBase InverseOf isBaseOf
- hasTopping InverseOf isToppingOf
- hasIngredient InverseOf isIngredientOf

(3)

Because IceCream EquivalentTo owl:Nothing. "IceCream SubClassOf hasTopping some FruitTopping" and "hasTopping Domain Pizza" infer IceCream and Pizza in the same class, but in fact they are in DisjointClasses. Contradiction occur.

(4)

- The inferred superclass for class "CajunSpiceTopping" is "SpicyTopping".
- The inferred superclass for class "SloppyGiuseppe" are "CheesyPizza", "InterestingPizza", "MeatyPizza", "SpicyPizza" and "SpicyPizzaEquivalent".

(5)

An error occur “Internal reasoner error (See the logs for more info).” The object property "hasIngredient" is not functional.

Q13

see 201300069 邓嘉宏.owl