Week5

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$$F(X)=\begin{cases}0, x\le 0\\ \dfrac{x}{a}, 0<x<a\\ 1, x\ge a\end{cases}$$

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(1)

$P(X\le 3)=F(3)=1-e^{-1.2}$

(2)

$P(X>4)=1-F(4)=e^{-1.6}$

(3)

$P(3\le X\le 4)=F(4)-F(3)=e^{-1.2}-e^{-1.6}$

(4)

$P(X\le 3||X\ge 4)=P(X\le 3)+P(X>4)=1-e^{-1.2}+e^{-1.6}$

(5)

$P(X=2.5)=0$

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(1)

$P(X<2)=F(2)=\ln 2$

$P(0<X\le 3)=F(3)-F(0)=1$

$P(2 < X < 5/2) = F(5/2) - F(2) = \ln\frac{5}{4}$

(2)

$F'(X) = f(x)$

$$\therefore f(x) = \begin{cases} 0, x < 1 \\ \frac{1}{x}, 1 \le x < e \\ 0, x \ge e \end{cases}$$

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(1)

当$x < 1$时$, F(x) = 0$

当$1 \le x \le 2$时$, F(x) = \int_1^x (2 - \frac{2}{t^2})dt = 2x + \frac{2}{x} - 4$

当$x > 2$时$, F(x) = 1$

$$\therefore F(x) = \begin{cases} 0, x < 1 \\ 2x + \frac{2}{x} - 4, 1 \le x \le 2 \\ 0, x > 2 \end{cases}$$

(2)

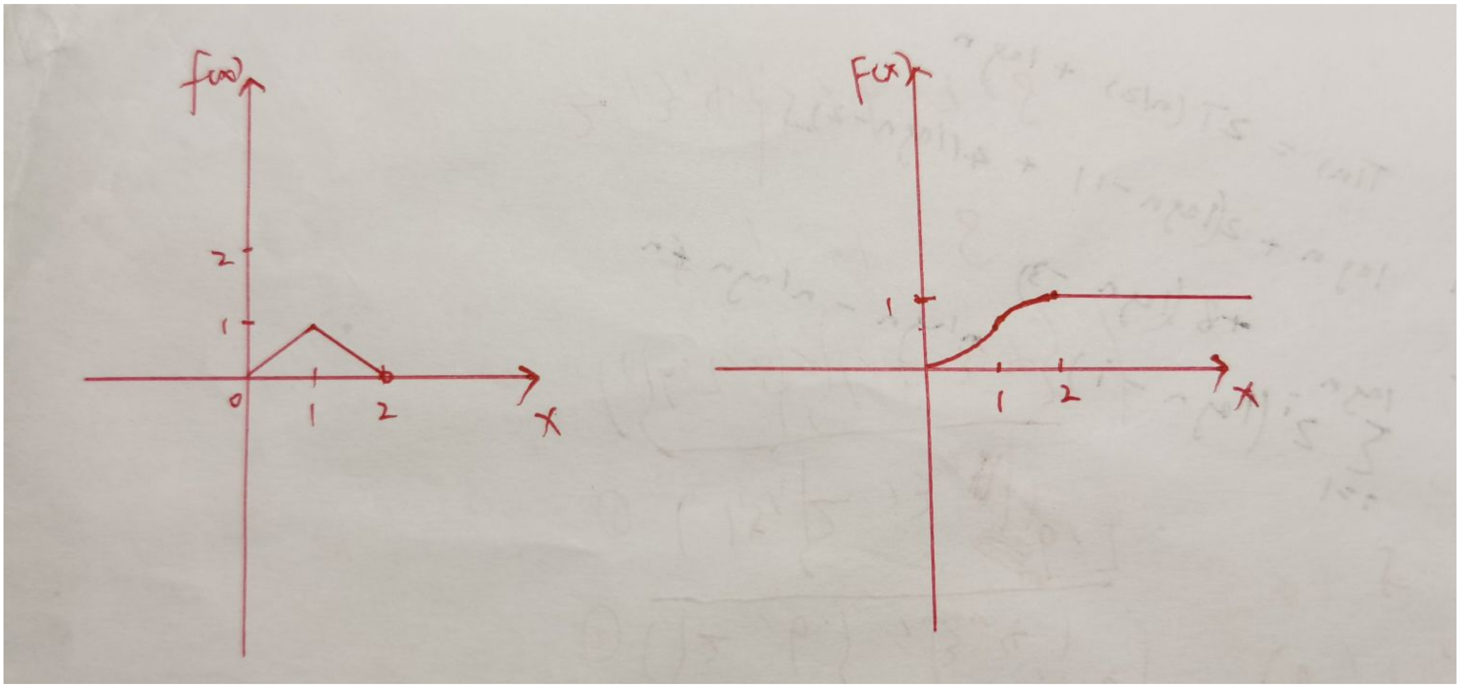
当$x < 0, F(x) = 0$

当$0 \le x < 1, F(x) = \int_0^x tdt = \frac{1}{2}x^2$

当$1 \le x < 2, F(x) = \frac{1}{2} + \int_1^x (2-t)dt = 2x - \frac{1}{2}x^2 - 1$

当$x \ge 2, F(X) = 1$

$$\therefore F(x)=\begin{cases}0, x<0\\ \frac{1}{2}x^2, 0\le x<1\\ 2x-\frac{1}{2}x^2-1, 1\le x<2\\ 1, x\ge 2\end{cases}$$



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对单个器件, $P(X>1500)=\int_{1500}^{+\infty}\frac{1000}{x^2}=\frac{2}{3}$

令事件A表示5只器件中至少有2只寿命大于1500小时

$P(A)=1-(\frac{1}{3})^5-5\times\frac{2}{3}\times(\frac{1}{3})^4=\frac{232}{243}$

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一次中,不离开窗口的概率 : $F(10)=1-e^{-2}$

分布律 : $P(Y=k)=\binom{5}{k}e^{-2k}(1-e^{-2})^{5-k}$

$P(Y\ge 1)=1-(1-e^{-2})^5$

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该方程有实根要求：$K \leq -1$或$K \geq 2$

P(有实根) $= F(5) - F(2) = \frac{3}{5}$

P115 18

$$E(X) = \int_0^{+\infty} \frac{x^2}{\sigma^2} e^{\frac{-x^2}{2\sigma^2}} dx = \int_0^{+\infty} e^{\frac{-x^2}{2\sigma^2}} dx = \pi$$

$$\because E(X^2) = \int_0^{+\infty} \frac{x^3}{\sigma^2} e^{\frac{-x^2}{2\sigma^2}} dx = \int_0^{+\infty} e^{\frac{-x^2}{2\sigma^2}} dx^2 = 2\sigma^2$$

$$\therefore D(X) = E(X^2) - (E(X))^2 = 2\sigma^2 - \pi^2$$

4.2

设X表示长方形的宽

$$E(X) = \int_0^2 \frac{1}{2} x dx = 1$$

周长的期望：$E(20/X + 2X) = \int_0^2 x + \frac{10}{x} dx$,不存在.

所以, 期望和方差不存在.

4.3

$$\because \int_0^{\infty} Ae^{-x} dx = A = 1$$

$$\therefore A = 1$$

$$E(Y) = \int_0^{+\infty} e^{-3x} dx = \frac{1}{3}$$

4.4

$$\because \int_{-\infty}^{+\infty} e^{\frac{-t^2}{2}} dt = \sqrt{2\pi}$$

令$t = \frac{t-u}{\sigma}$

则：$\int_{-\infty}^{+\infty} e^{\frac{-(t-u)^2}{2\sigma^2}} d\frac{t-u}{\sigma} = \sqrt{2\pi} = \frac{1}{\sigma}\int_{-\infty}^{+\infty} e^{\frac{-(t-u)^2}{2\sigma^2}} dt$

$\therefore \int_{-\infty}^{+\infty} e^{\frac{-(t-u)^2}{2\sigma^2}} dt = \sqrt{2\pi}\sigma$