Homework 8

DATA130021 Financial Econometrics

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Problem 1

Suppose there are two risky assets, C and D. The tangency portfolio is 65% C and 35% D, and the expected return and standard deviation of the return on the tangency portfolio are 5% and 7%, respectively. Suppose also that the risk-free rate of return is 1.5%. If you want the standard deviation of your return to be 5%, what proportions of your capital should be in the risk-free asset, asset C and asset D?

Solution:

From the setting, we have the following notations:

$$\mu_T = 0.05,\ \sigma_T = 0.07,\ \mu_f = 0.015,\ \omega_1 = 0.65,\ \omega_2 = 0.35,\ \sigma_R = 0.05.$$

Then the ω of the investment to the tangency portfolio with $\sigma_R = 0.05$ is

$$\omega = \frac{\sigma_R}{\sigma_T} = \frac{0.05}{0.07} = \frac{5}{7},$$

and the proportions of the capital in the risk-free asset, asset C and asset D should be:

$$\omega_f = 1 - \omega = \frac{2}{7} \approx 0.286,$$

$$\omega_C = \omega\omega_1 = \frac{5}{7} \times 0.65 \approx 0.464,$$

$$\omega_D = \omega\omega_2 = \frac{5}{7} \times 0.35 = 0.25.$$

Problem 2

What is the beta of a portfolio if $\mathbb{E}(R_P) = 16$, $\mu_f = 5.5\%$ and $\mathbb{E}(R_M) = 11\%$?

Solution:

From the setting, we have the following notations:

$$\mu_R = 0.16,\ \mu_M = 0.11,\ \mu_f = 0.055.$$

Then the beta of this portfolio is

$$\beta_R = \frac{\sigma_R}{\sigma_M} = \frac{\mu_R - \mu_f}{\mu_M - \mu_f} = \frac{0.16 - 0.055}{0.11 - 0.055} = \frac{21}{11} \approx 1.909.$$

Problem 3

Suppose that the riskless rate of return is 4% and the expected market return is 12%. The standard deviation of the market return is 11%. Suppose as well that the covariance of the return on Stock A with the market return is $165\%^2$.

(a) What is the beta of Stock A?

(b) What is the expected return on Stock A?

(c) If the variance of the return on Stock A is $\mathbf{220}\%^2$, what percentage of this variance is due to market risk?

Solution:

From the setting, we have the following notations:

$$\mu_f = 0.04,\ \mu_M = 0.12,\ \sigma_M = 0.11,\ \sigma_{AM} = 0.0165.$$

(a) The beta of Stock A is

$$\beta_A = \frac{\sigma_{AM}}{\sigma_M^2} = \frac{0.0165}{0.11^2} = \frac{15}{11} \approx 1.364.$$

(b) The expected return on Stock A is

$$\mathbb{E}(R_A) = \mu_A = \mu_f + \beta_A(\mu_M - \mu_f) = 0.04 + \frac{15}{11} \times (0.12 - 0.04) \approx 0.149.$$

(c) From the setting, we know that $\sigma_A^2 = \mathbf{0.022}$ and $\sigma_A^2 = \beta_A^2 \sigma_M^2 + \sigma_{\epsilon,A}^2$. Then the percentage of this variance due to market risk is

$$\text{ratio} = \frac{\beta_A^2 \sigma_M^2}{\sigma_A^2} = \frac{1.364^2 \times 0.11^2}{\mathbf{0.022}} = \frac{0.0225}{\mathbf{0.022}} \approx 102.27\%.$$

(There is something wrong with the assumption, which is marked in bold.)

Problem 4

Use the stock return data of Apple, Amazon and Google from 2015/01/01 to 2016/12/31 to find the tangency portfolio weight and the minimum variance. Plot the efficient frontier.

Solution:

Codes and the efficient frontier are shown as follow. The tangency portfolio weights of Apple, Amazon and Google are -0.117, 0.928, 0.189, respectively. The minimum variance is 0.0132.

```
library(quadprog)

AAPL <- read.csv("./AAPL.csv")
AAPL <- diff(AAPL$Close) / AAPL$Close[-dim(AAPL)[1]]
AMZN <- read.csv("./AMZN.csv")
AMZN <- diff(AMZN$Close) / AMZN$Close[-dim(AMZN)[1]]
GOOG <- read.csv("./GOOG.csv")
GOOG <- diff(GOOG$Close) / GOOG$Close[-dim(GOOG)[1]]
returns <- data.frame("AAPL"=AAPL, "AMZN"=AMZN, "GOOG"=GOOG)
N <- dim(returns)[2]

mu <- apply(returns, MARGIN=2, FUN=mean)
Sigma <- cov(returns)
sigma <- sqrt(diag(Sigma))

N.point <- 3000
mu.min <- -0.001
mu.max <- max(mu)

A <- cbind(rep(1, N), mu)
mu.p <- seq(mu.min, mu.max, length=N.point)
sd.p <- mu.p
weights <- matrix(0, nrow=N.point, ncol=N)

for(i in 1:length(mu.p)){
    b <- c(1, mu.p[i])
    result <- solve.QP(Dmat=2 * Sigma, dvec=rep(0, N), Amat=A, bvec=b, meq=2)
    sd.p[i] <- sqrt(result$value)
    weights[i, ] <- result$solution
}

mu.f <- 0
sharpe <- (mu.p - mu.f) / sd.p
ind.max.sharpe <- (sharpe == max(sharpe))
ind.min.var <- (sd.p == min(sd.p))
ind.eff.front <- (mu.p > mu.p[ind.min.var])

weights[ind.max.sharpe, ]
```

```
## [1] -0.1169175  0.9281079  0.1888095
```

```
sd.p[ind.min.var]
```

```
## [1] 0.01319717
```

```r
plot(0, 0, xlim=c(0, 0.025), ylim=c(-0.0015, 0.0025), pch=8, cex=1.5, lwd=2,
     main="Efficient Frontier", xlab="Risk", ylab="Expected Returns")
lines(sd.p[ind.eff.front], mu.p[ind.eff.front], type="l", lty=1, lwd=2)
lines(sd.p[!ind.eff.front], mu.p[!ind.eff.front], type="l", lty=2, lwd=2)
abline(0, max(sharpe), lwd=2, col="blue")
points(sigma, mu, pch=19, cex=1.5, lwd=2, col="red")
points(sd.p[ind.max.sharpe], mu.p[ind.max.sharpe], pch=8, cex=1.5, lwd=2)
points(sd.p[ind.min.var], mu.p[ind.min.var], pch=3, cex=1.5, lwd=2)
```