Regression Analysis for Airline Stock Price Changes

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1 Introduction

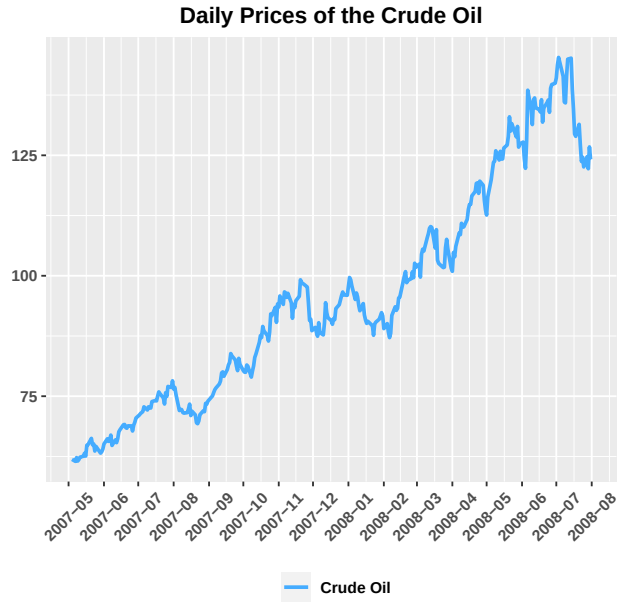
Generally, the price changes of crude oil often have influence on the industries since the industries usually have enormous demands on crude oil. For airline companies, the expenditure on fuel accounts for a considerable proportion of their gross costs. Hence, stock prices of airline companies are usually sensitive to the price changes of crude oil.

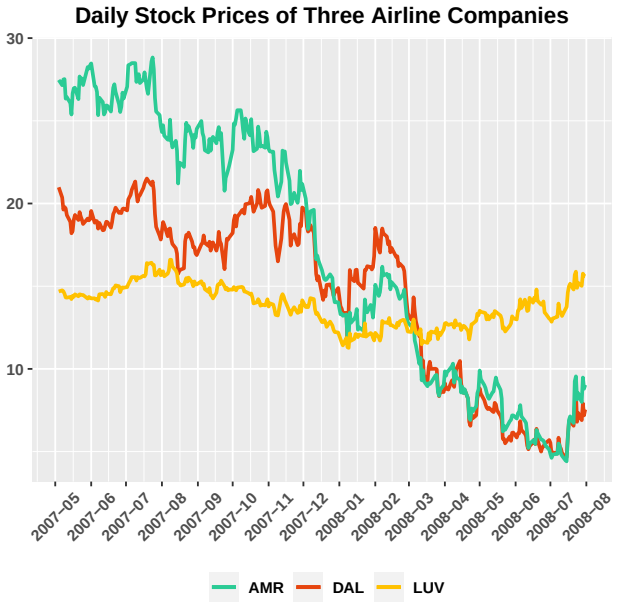
This paper aims to analysis the linear relationship of price changes between crude oil and three airline stocks. Price data of crude oil and three airline stocks from May 2007 to July 2018, which includes stock prices of Delta Airline (DAL), American Airline (AMR) and Southwest Airline (LUV), are used to analyze in this paper. Before we analyze the price data, environment should be set up in R.

2 Data preparation

Data preparation is the process of cleaning and transforming raw data prior to processing, analysis and modeling. In this project, price of crude oil and three airline stocks are key variables to concern about. Load in the dataset from *airline.csv* respectively.

Visualize the price trends of crude oil and three airline stocks. We can find that the price of crude oil was approximately increasing from May 2007 to July 2008, while the stock prices of Delta Airline and American Airline were decreasing sharply in 2008, and the stock prices of Southwest Airline was fluctuating around $14.





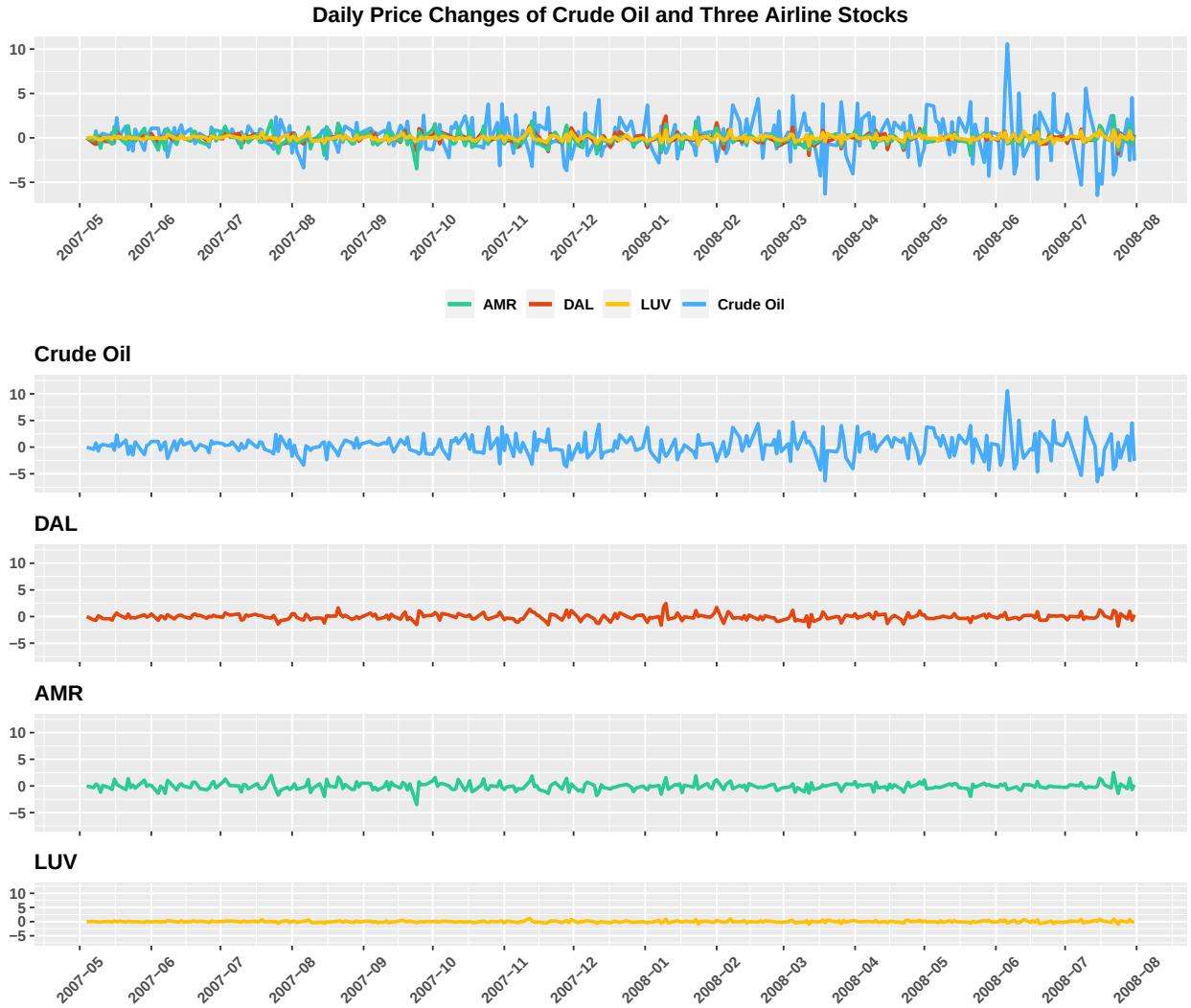
Following are summaries of price trends data.

Table 1: Summary of Price Trends

Prices	Crude.Oil	DAL	AMR	LUV
Min.	61.48	4.64	4.41	11.27
1st Qu.	75.74	8.76	9.19	12.77
Median	92.22	16.98	16.36	13.89
Mean	95.49	14.52	17.19	13.82
3rd Qu.	110.19	18.86	24.39	14.81
Max.	145.31	21.51	28.83	16.60

3 Descriptive Data Analysis

In this project, we are interested in the relationship, particularly linear relationship, between daily price changes of crude oil X_t and daily stock price changes of each airline Y_t. Hence, we should calculate and visualize the price changes from the original dataset.



The price changes of crude oil presents an abnormal trend in 2008. Especially, the price changes began to flutuate violently from March 2008. In 2008, there was a huge growth of the price of crude oil, which was possibly influenced by the financial crisis of 2007 to 2008.

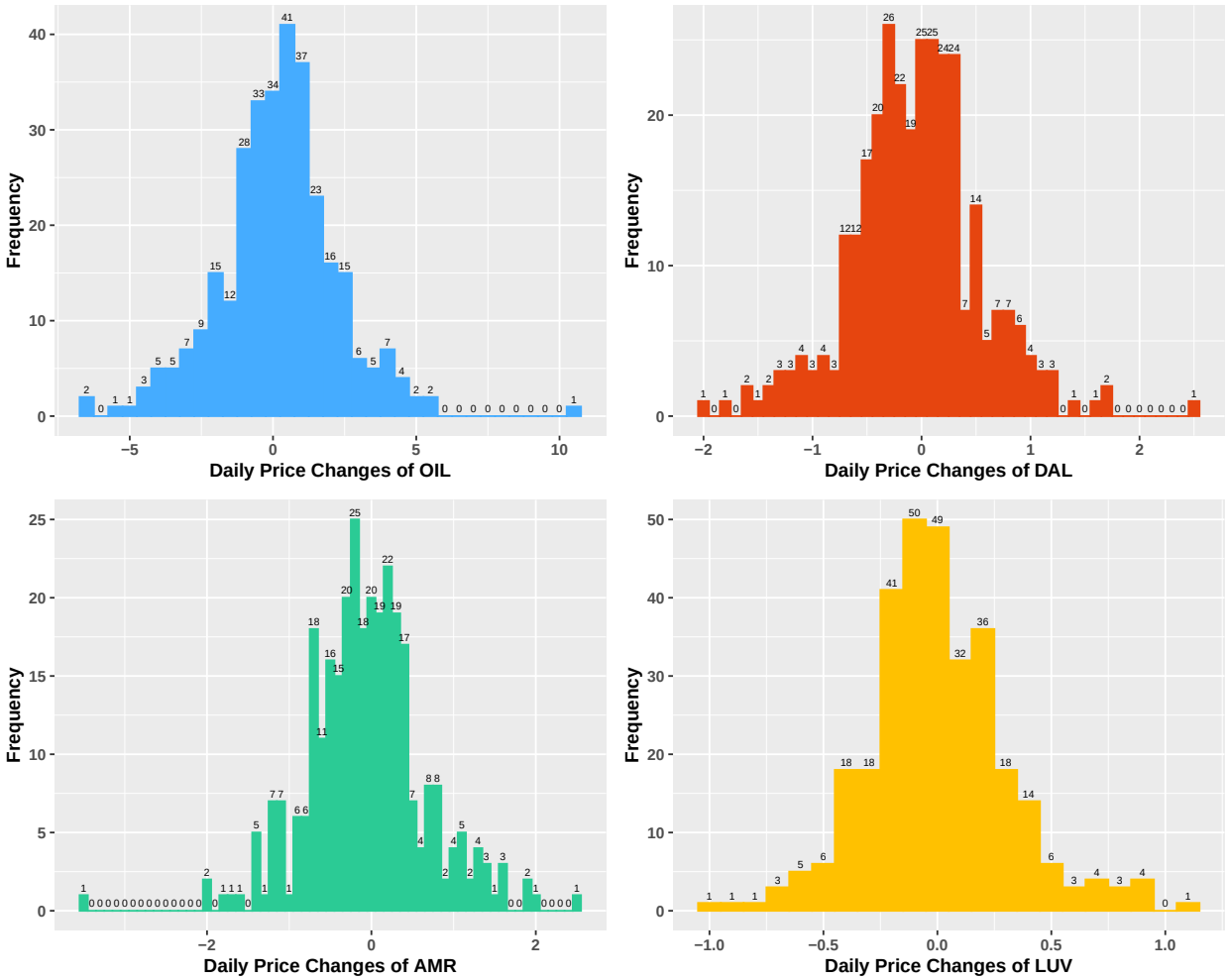
Following are summaries of price changes data.

Table 2: Summary of Price Changes

Price.Changes	Crude.Oil	DAL	AMR	LUV
Min.	-6.48	-1.98	-3.49	-0.98
1st Qu.	-0.87	-0.38	-0.49	-0.18
Median	0.28	-0.02	-0.08	-0.02
Mean	0.20	-0.04	-0.06	0.00
3rd Qu.	1.32	0.27	0.31	0.18
Max.	10.58	2.46	2.52	1.13

In order to fit the the price changes between crude oil and airline stocks, we can plot scatters to predict the general correlation. From the figures below, we find that a great number of points gather around the origin and there are probably negative correlations between crude oil and airline stocks.

Normal distribution is often used for demostration in mathematical modeling. Hence, we assume that the price changes of crude oil and airline stock data satisfy normal distribution. We should use histograms and other normality tests to verify our assumptions.



Excluding the outliers, we find that the distribution of crude oil price changes has a heavier tail than others. The distributions of airline stock prices are closer to normal distributions.

Also, we can compute the estimation of skewness and kurtosis of the given data to verify the assumptions. From the results, the distributions of crude oil, DAl and LUV show positive skewness, while the distribution of AMR shows negative skewness but it is much closer to the normal distribution. Consider the kurtosis, we find that the distributions of crude oil and AMR show heavy tails. In general, all distributions we concerned about are similar to the normal distribution. We should use normality tests to check the normality.

Table 3: Estimation for Skewness and Kurtosis of Price Changes

Type	Skewness	Kurtosis
Crude Oil	0.1652397	2.158807
DAL	0.1371564	1.357304
AMR	-0.0407580	1.939930
LUV	0.3515515	1.150949

Firstly, we use Kolmogorov-Smirnov Test, Cramer-von Mises Test and Jarque-Bera Test to test normality.

Table 4: Statistics in Three Tests for Price Changes

Statistic	Crude Oil	DAL	AMR	LUV
Kolmogorov-Smirnov Test	0.0671522	0.0634506	0.0664633	0.0704655
Cramer-von Mises Test	0.3087982	0.2187327	0.2591243	0.3405950
Jarque-Bera Test	64.2961545	26.0976278	50.9425669	24.6705837

Table 5: P-value in Three Tests for Price Changes

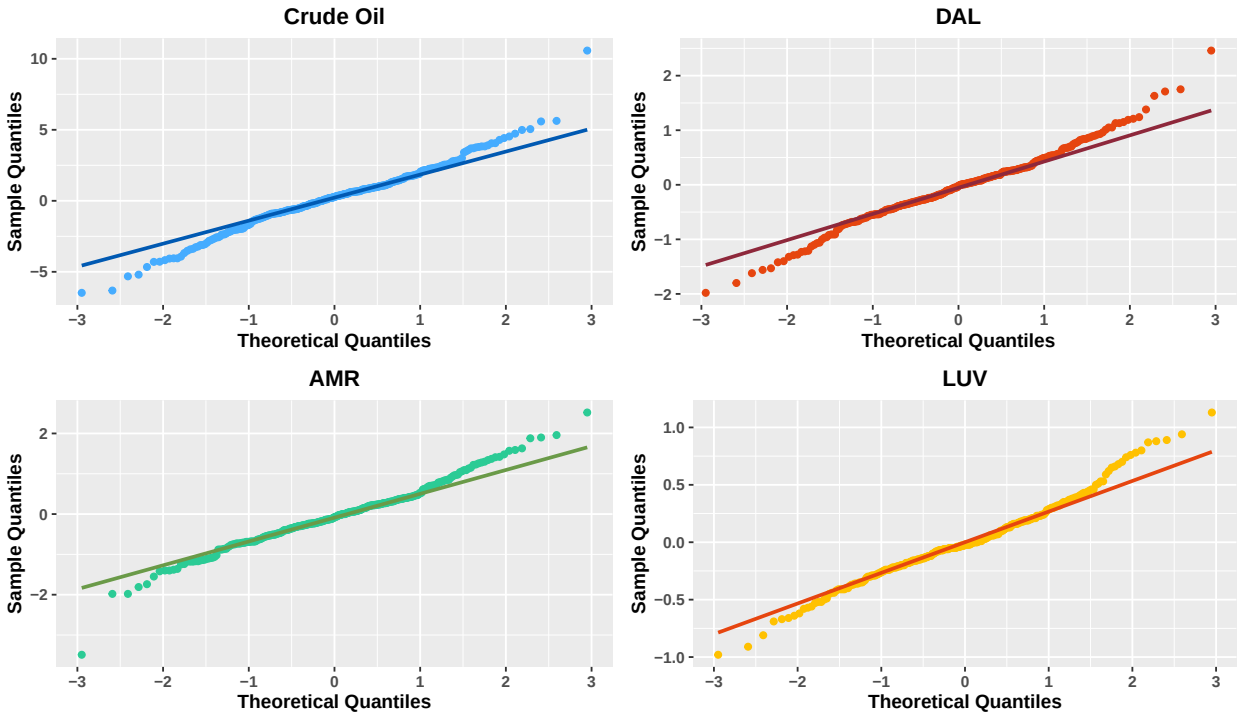
P-value	Crude Oil	DAL	AMR	LUV
Kolmogorov-Smirnov Test	1.18e-01	1.6e-01	1.25e-01	8.85e-02
Cramer-von Mises Test	1.28e-01	2.34e-01	1.77e-01	1.04e-01
Jarque-Bera Test	1.09e-14	2.15e-06	8.67e-12	4.39e-06

In all hypothesis tests for normality, if P-value is smaller than the significance level, then we reject the null hypothesis and consider that the samples are not sampling from normal distribution.

In Kolmogorov-Smirnov Test, only LUV satisfy normal distribution over 0.01 level of significance since others have a greater P-value. But we cannot give a conclusion that these distributions do not satisfy the normal distributions sufficiently because all P-values are not very small.

In Cramer-von Mises Test, we reject all hypotheses over 0.01 level of significance, which means we don't think that these distributions are normal distrbutions.

In Jarque-Bera Test, we have sufficient evidences to conclude that all price changes do not satisfy normal distribution over 0.01 level of significance.



QQ-plot can be used to check normality with the quantiles of the data. The points should locate on the diagonal if the empirical distribution is consistent with the theoretical distribution. From the figure below, we find that all QQ-plots are not of the shape of the straight lines. The upper ends and the lower ends deviated from the diagonal, which show the S-shape curves, indicate the heavy tails.

In total, we conclude that the price changes of crude oil and three airline stocks do not satify normal distribution.

4 Regression Modeling

We are going to explore more precise linear relationship between price changes of crude oil and three airline stocks using linear regression. Based on the properties of time sequences, we should consider the possible relationship between Y_t and X_t, X_{t-1}, $\cdots$, X_1 Y_{t-1}, $\cdots$, Y_1. For simplicity, we only consider relationship between Y_t and X_t, X_{t-1}, Y_{t-1} in this project.

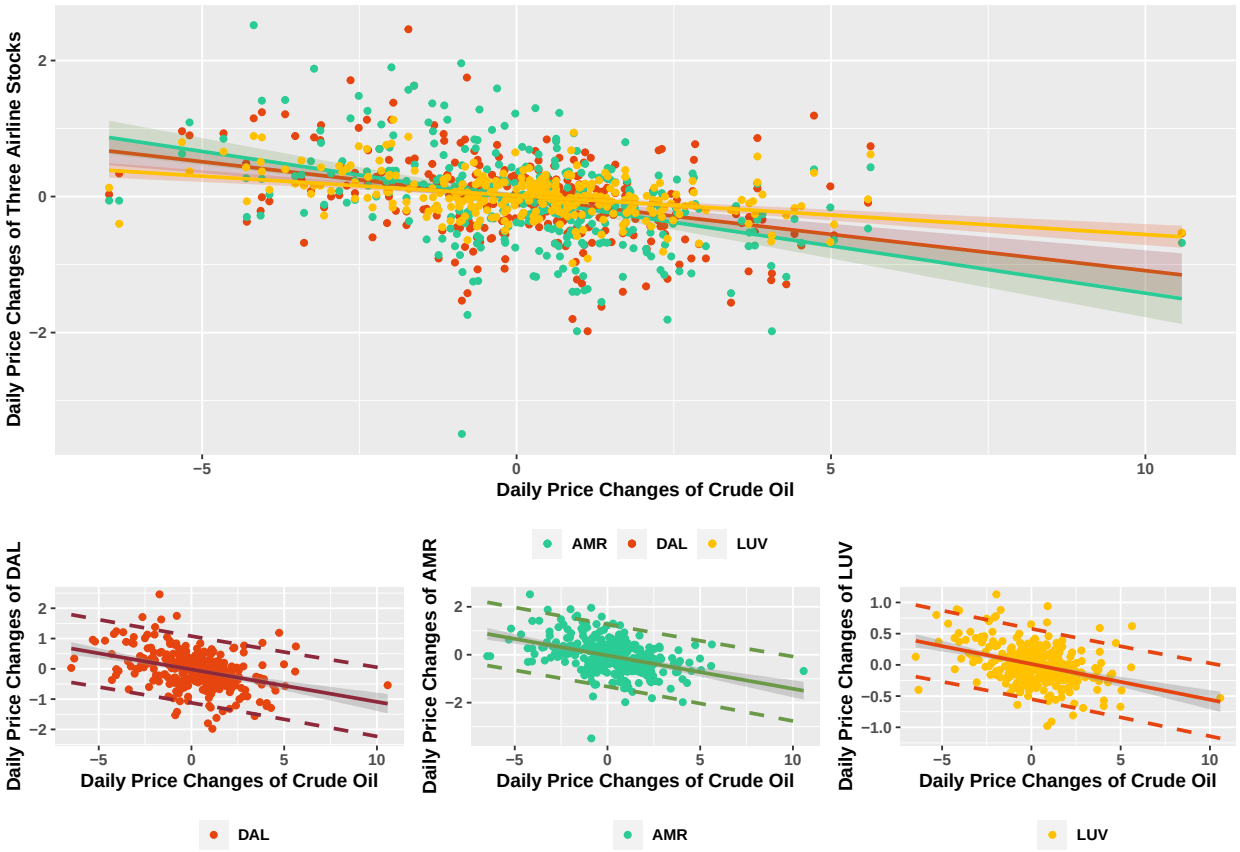
4.1 Simple Linear Regression

4.1.1 Modeling

Simple linear regression only contains the Y_t and X_t terms.

$$Y_t = \beta_0 + \beta_1 X_t + \epsilon_t$$

Build simple linear regression model between the price changes of crude oil and airline stocks, respectively. Visualize the results as below.



Then we summarize the regression results in the table below.

```
## [1] ""
```

Table 6: Statistics in Simple Linear Regression

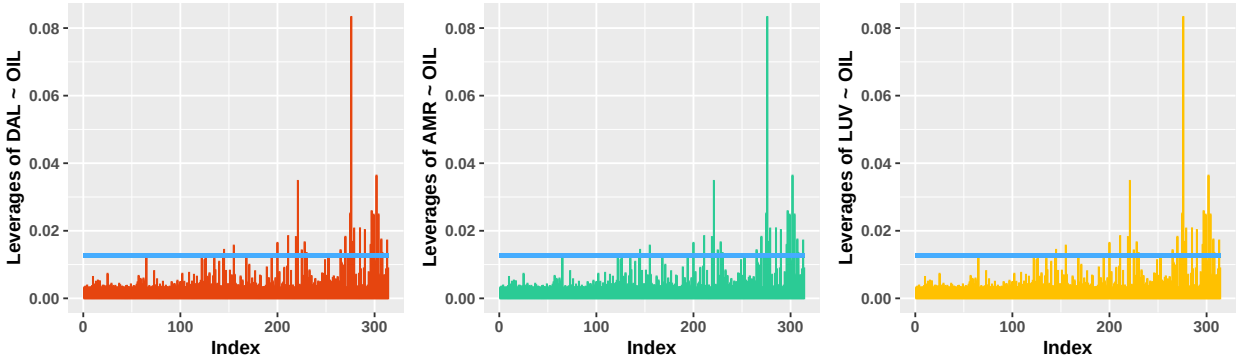
Statistic	DAL	AMR	LUV
Intercept	-0.022	-0.031	0.014
Slope	-0.107	-0.139	-0.057
P-value of Slope	1.48e-11	1.43e-13	1.98e-12
Adjusted R-squared	0.133	0.158	0.144
P-value of F-statistic	1.82e-11	1.43e-13	1.98e-12

For three cases, we find that the price changes of crude oil X_t is a significant variable, whose coefficients are -0.107, -0.139 and -0.057, respectively. If daily price change of crude oil increase \$1, the daily stock price change of DAL, AMR and LUV drop \$0.107, \$0.139 and \$0.057, respectively.

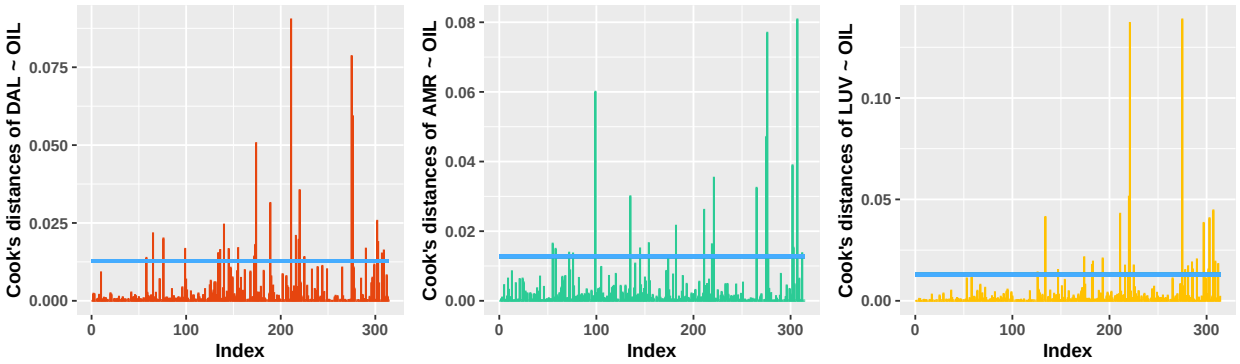
4.1.2 Diagnosis

We use several measures to diagnose models above, for example, leverages, residuals and Cook's distance.

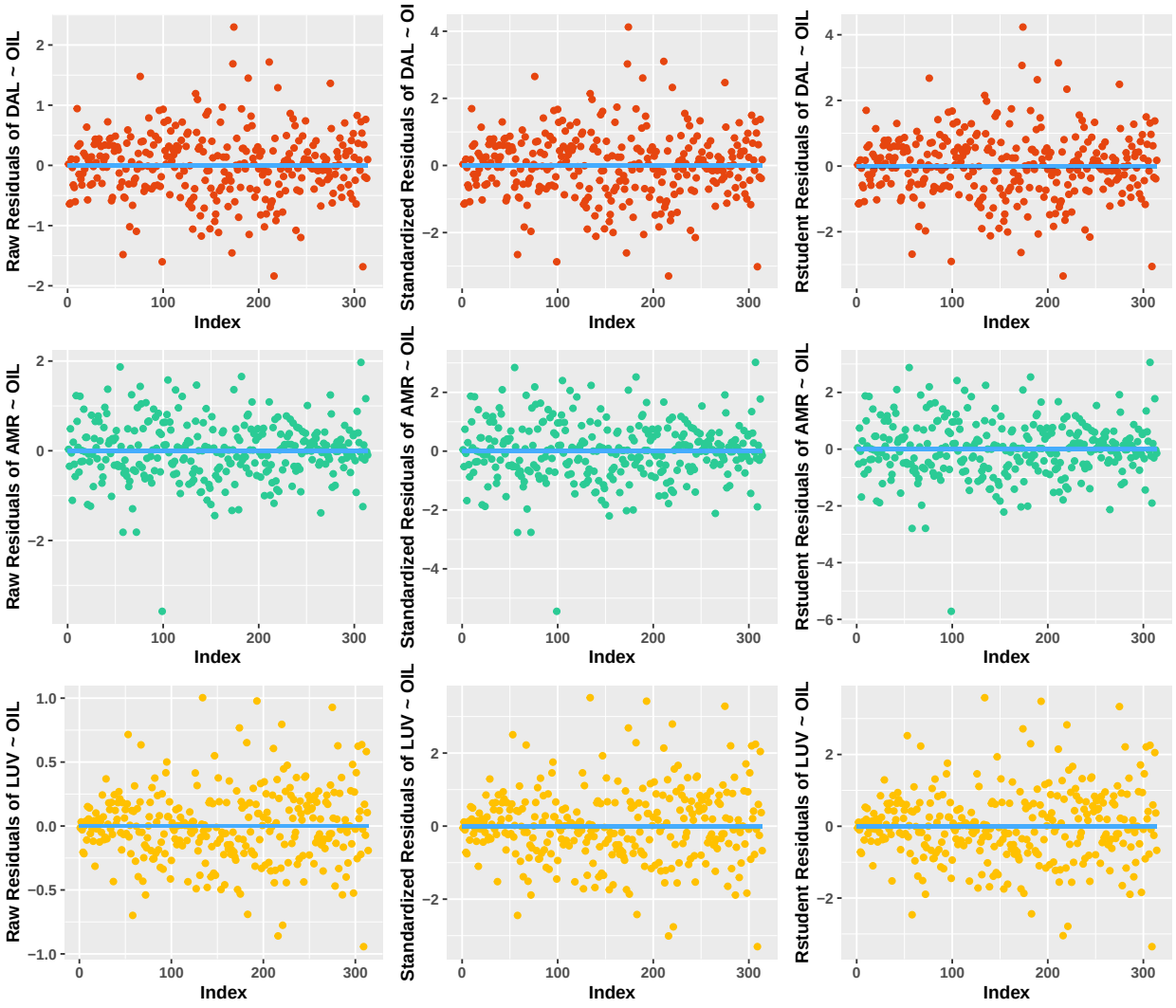
The leverage H_{ii} of the i-th observation measures how much influence Y_i has on its own fitted value $\hat{Y}_i$. If $H_{ii} > 2(p+1)/n$, then it is generally considered to be too large. From the results, we find that the tail data have larger leverage, which means abnormal values.

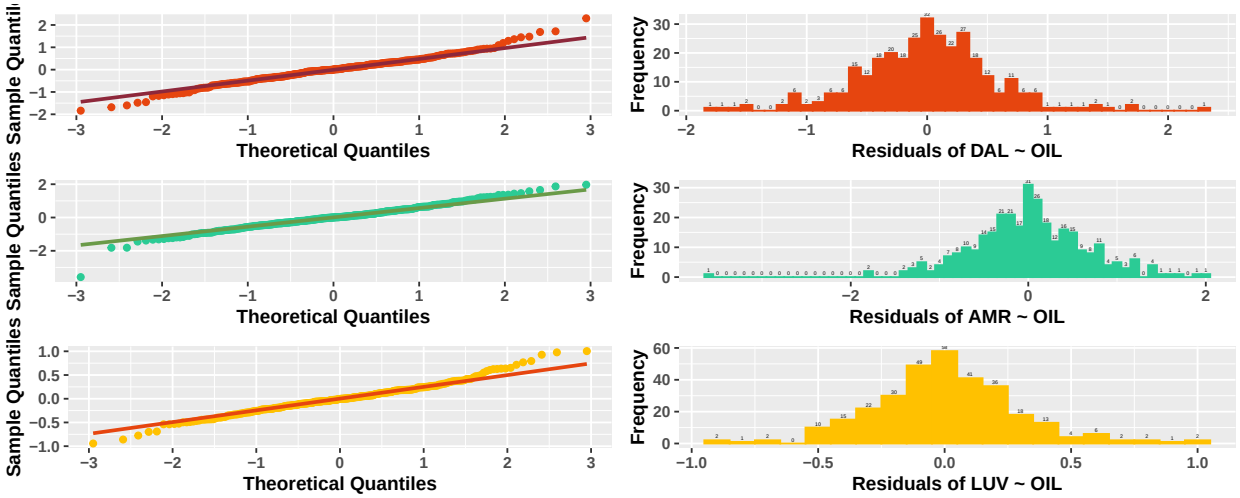


Cook's distance measures how much the fitted values change if the i-th observation is deleted. Large value means high influence of the observation. If $D > 4/(n-p-1)$, then it is considered to be of great influence. From the results, we also find that the tail data have larger Cook's distance, which means abnormal values.



The residuals contain the information of fitting situation. From QQ-plots and histograms, the central data shows a strong normality of the residuals.





4.2 Delayed Linear Regression

4.2.1 Modeling

Linear regression with delayed variable contains the Y_t, X_t and Y_{t-1} terms.

$$Y_t = \beta_0 + \beta_1 X_t + \beta_2 Y_{t-1} + \epsilon_t$$

Build Delayed Linear Regression model between the price changes of crude oil and airline stocks, respectively. Print the results as below.

Table 7: Statistics in Delayed Linear Regression

Statistic	DAL	AMR	LUV
Intercept	-0.017	-0.032	0.015
Slope of X	-0.109	-0.138	-0.057
P-value of Slope of X	6.76e-12	2.32e-13	1.15e-12
Slope of Ym1	0.088	-0.015	-0.189
P-value of Slope of Ym1	0.098	0.769	2.63e-04
Adjusted R-squared	0.138	0.156	0.178
P-value of F-statistic	3.35e-11	1.35e-12	2.36e-14

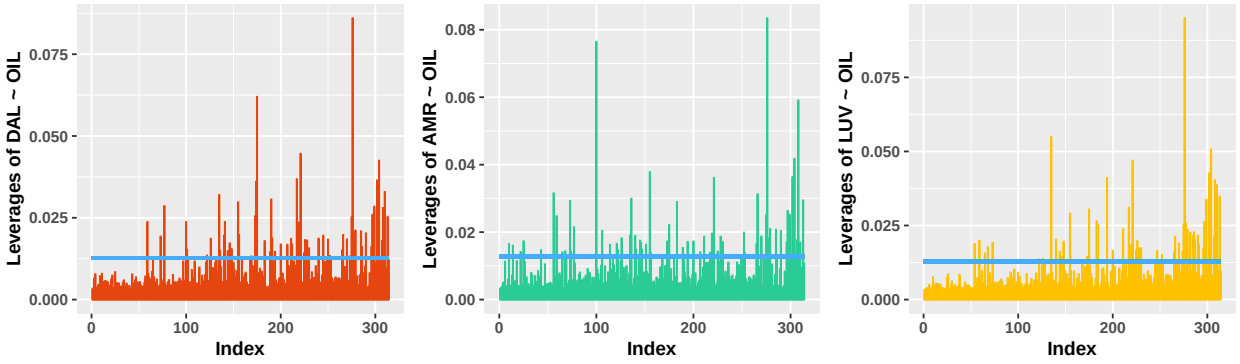
For three cases, we find that the price changes of crude oil X_t is a significant variable, whose coefficients are -0.109, -0.138 and -0.057, respectively. If daily price change of crude oil increase \$1, the daily stock price change of DAL, AMR and LUV drop \$0.109, \$0.138 and \$0.057, respectively.

For DAL and AMR, variables Y_{t-1} are not significant. Only for LUV, the price changes of LUV one day ago Y_{t-1} is a significant variable.

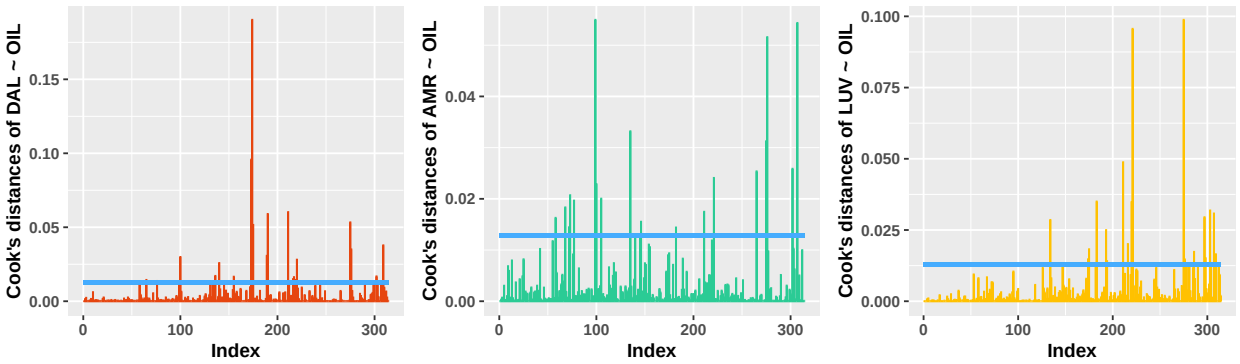
4.2.2 Diagnosis

We also use several measures to diagnose models above, for example, leverages, residuals and Cook's distance.

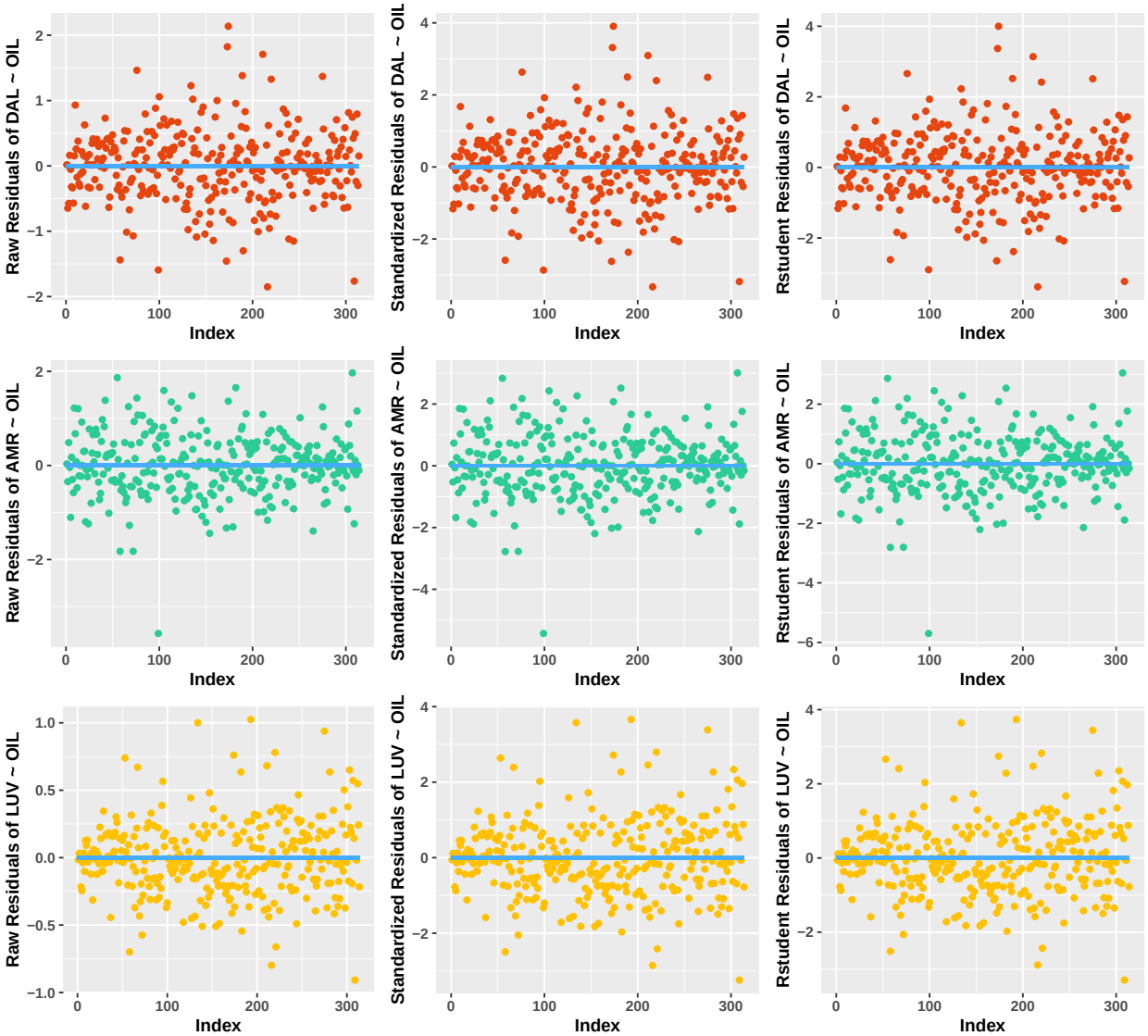
The leverage H_{ii} of the i-th observation measures how much influence Y_i has on its own fitted value $\hat{Y}_i$. If $H_{ii} > 2(p+1)/n$, then it is generally considered to be too large. From the results, we find that the tail data have larger leverage, which means abnormal values.

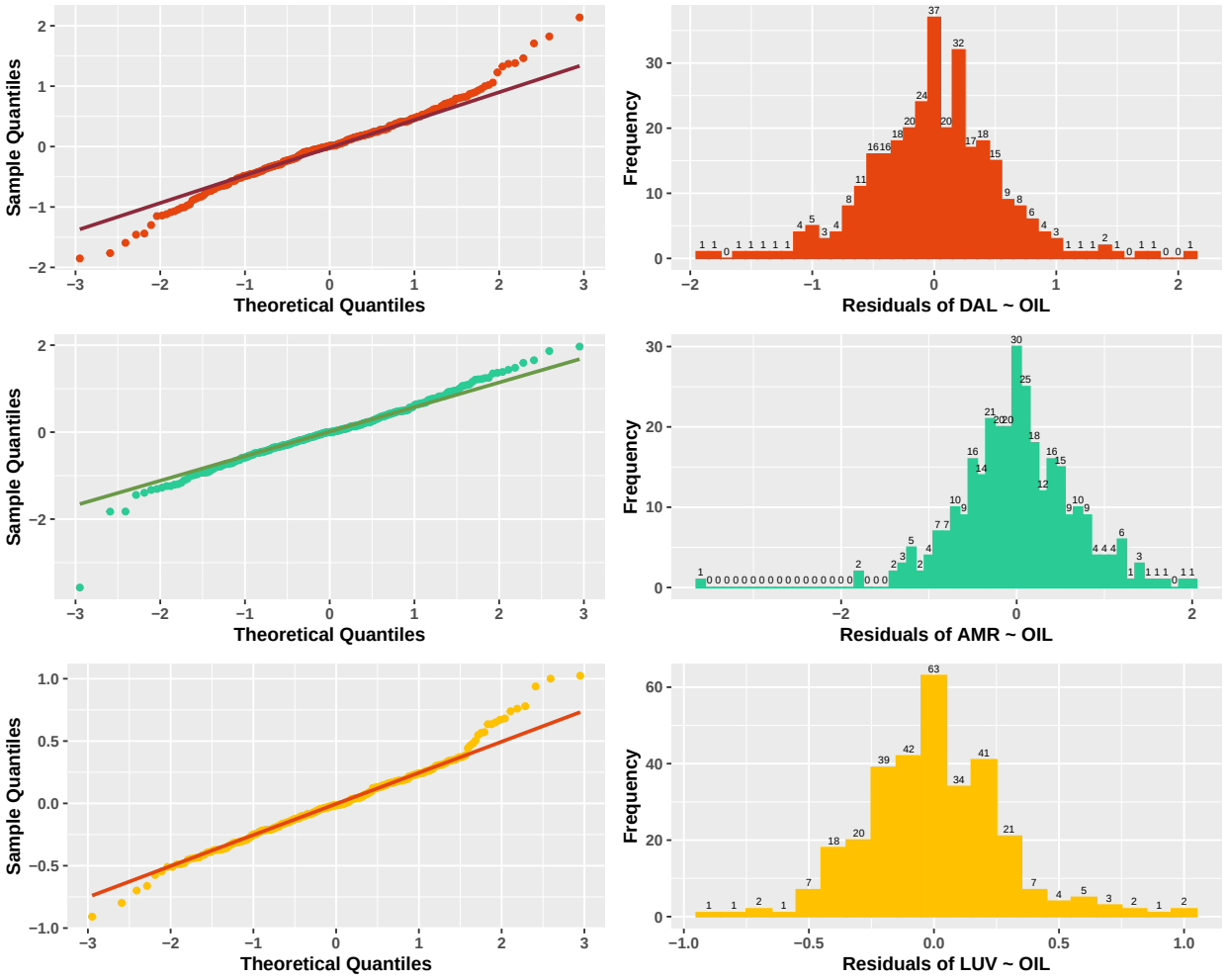


Cook's distance measures how much the fitted values change if the i-th observation is deleted. Large value means high influence of the observation. If $D > 4/(n-p-1)$, then it is considered to be of great influence. From the results, we also find that the tail data have larger Cook's distance, which means abnormal values.



The residuals contain the information of fitting situation. From QQ-plots and histograms, the central data shows a strong normality of the residuals.





4.3 Model Selection

We use AIC to select the model. Summarize the results as below.

Table 8: AIC of Models

Model	AIC
DAL.SLR	529.62500
DAL.DLR	528.84770
AMR.SLR	632.81220
AMR.DLR	634.72500
LUV.SLR	109.75549
LUV.DLR	98.28676

Models had smaller AIC means better. Hence, we should choose *DAL.DLR*, *AMR.SLR* and *LUV.DLR*.

5 Conclusion

In this project, we build linear models for the relationship between daily price changes of crude oil and daily stock price changes of three airlines.

For Delta Airline, the best linear regression model is

$$\hat{Y}_t = -0.017 - 0.109X_t + 0.088Y_{t-1}.$$

For American Airline, the best linear regression model is

$$\hat{Y}_t = -0.031 - 0.139X_t.$$

For Southwest Airline, the best linear regression model is

$$\hat{Y}_t = 0.015 - 0.057X_t - 0.189Y_{t-1}.$$

6 Appendix

The appendix shows the entire codes for this project.

6.1 Introduction

```
library(DistributionUtils)
library(ggplot2)
library(goftest)
library(gridExtra)
library(tseries)
```

6.2 Data Preparation

6.2.1 Data Loading and Processing

```
prices <- read.csv("airline.csv")
prices$Date <- as.Date(as.character(prices$Date), format="%Y%m%d")
changes <- cbind(Date=prices$Date,
                 rbind(c(0, 0, 0, 0), prices[-1, -1] - prices[-dim(prices)[1], -1]))
```

6.2.2 Price Trends

```
OIL.trend <- ggplot(data=prices) +
    geom_line(aes(x=Date, y=OIL, colour="OIL"), size=1, group=0) +
    labs(title="Daily Prices of the Crude Oil", x=NULL, y=NULL) +
    theme(plot.title=element_text(hjust=0.5, face="bold"),
          axis.text=element_text(face="bold"),
          axis.text.x=element_text(angle=45, vjust=0.5),
          legend.position="bottom", legend.title=element_blank(),
          legend.text=element_text(face="bold")) +
    scale_colour_manual(values=c("OIL"="#45ACFF"), labels=c("Crude Oil")) +
    scale_x_date(date_labels="%Y-%m", date_breaks="1 month")

price.trend <- ggplot(data=prices) +
    geom_line(aes(x=Date, y=DAL, colour="DAL"), size=1, group=0) +
    geom_line(aes(x=Date, y=AMR, colour="AMR"), size=1, group=0) +
    geom_line(aes(x=Date, y=LUV, colour="LUV"), size=1, group=0) +
    labs(title="Daily Stock Prices of Three Airline Companies",
         x=NULL, y=NULL) +
    theme(plot.title=element_text(hjust=0.5, face="bold"),
```

```
          axis.text=element_text(face="bold"),
          axis.text.x=element_text(angle=45, vjust=0.5),
          legend.position="bottom", legend.title=element_blank(),
          legend.text=element_text(face="bold")) +
    scale_colour_manual(values=c("DAL"="#E6450F", "AMR"="#2BCB95",
                                 "LUV"="#FFC100"),
                        labels=c("DAL"="DAL", "AMR"="AMR",
                                 "LUV"="LUV")) +
    scale_x_date(date_labels="%Y-%m", date_breaks="1 month")

grid.arrange(OIL.trend, price.trend, ncol=2)

summary(prices)[, -1]
```

6.3 Descriptive Data Analysis

6.3.1 Price Changes

```
price.change <- ggplot(data=changes) +
    geom_line(aes(x=Date, y=OIL, colour="OIL"), size=1, group=0) +
    geom_line(aes(x=Date, y=DAL, colour="DAL"), size=1, group=0) +
    geom_line(aes(x=Date, y=AMR, colour="AMR"), size=1, group=0) +
    geom_line(aes(x=Date, y=LUV, colour="LUV"), size=1, group=0) +
    labs(title="Daily Price Changes of Crude Oil and Three Airline Stocks",
         x=NULL, y=NULL) +
    theme(plot.title=element_text(hjust=0.5, face="bold"),
          axis.text=element_text(face="bold"),
          axis.text.x=element_text(angle=45, vjust=0.5),
          legend.position="bottom", legend.title=element_blank(),
          legend.text=element_text(face="bold")) +
    scale_colour_manual(values=c("OIL"="#45ACFF", "DAL"="#E6450F",
                                 "AMR"="#2BCB95", "LUV"="#FFC100"),
                        labels=c("OIL"="Crude Oil", "DAL"="DAL",
                                 "AMR"="AMR", "LUV"="LUV")) +
    scale_x_date(date_labels="%Y-%m", date_breaks="1 month")

OIL.change <- ggplot(data=changes) +
    geom_line(aes(x=Date, y=OIL, colour="OIL"), size=1, group=0) +
    labs(title="Crude Oil", x=NULL, y=NULL) +
    ylim(-7.5, 12.5) +
    theme(plot.title=element_text(face="bold"),
          axis.text=element_text(face="bold"),
          axis.text.x=element_blank(),
          legend.position="none") +
    scale_colour_manual(values=c("OIL"="#45ACFF"), labels=c("Crude Oil")) +
    scale_x_date(date_labels="%Y-%m", date_breaks="1 month")

DAL.change <- ggplot(data=changes) +
    geom_line(aes(x=Date, y=DAL, colour="DAL"), size=1, group=0) +
    labs(title="DAL", x=NULL, y=NULL) +
    ylim(-7.5, 12.5) +
    theme(plot.title=element_text(face="bold"),
          axis.text=element_text(face="bold"),
```

```
          axis.text.x=element_blank(),
          legend.position="none") +
    scale_colour_manual(values=c("DAL"="#E6450F"), labels=c("DAL")) +
    scale_x_date(date_labels="%Y-%m", date_breaks="1 month")

AMR.change <- ggplot(data=changes) +
    geom_line(aes(x=Date, y=AMR, colour="AMR"), size=1, group=0) +
    labs(title="AMR", x=NULL, y=NULL) +
    ylim(-7.5, 12.5) +
    theme(plot.title=element_text(face="bold"),
          axis.text=element_text(face="bold"),
          axis.text.x=element_blank(),
          legend.position="none") +
    scale_colour_manual(values=c("AMR"="#2BCB95"), labels=c("AMR")) +
    scale_x_date(date_labels="%Y-%m", date_breaks="1 month")

LUV.change <- ggplot(data=changes) +
    geom_line(aes(x=Date, y=LUV, colour="LUV"), size=1, group=0) +
    labs(title="LUV", x=NULL, y=NULL) +
    ylim(-7.5, 12.5) +
    theme(plot.title=element_text(face="bold"),
          axis.text=element_text(face="bold"),
          axis.text.x=element_text(angle=45, vjust=0.5),
          legend.position="none") +
    scale_colour_manual(values=c("LUV"="#FFC100"), labels=c("LUV")) +
    scale_x_date(date_labels="%Y-%m", date_breaks="1 month")

grid.arrange(price.change, OIL.change, DAL.change, AMR.change, LUV.change,
             nrow=6, layout_matrix=rbind(1, 1, 2, 3, 4, 5))

summary(changes)[, -1]
```

6.3.2 Scatter

```
change.scatter <- ggplot(data=changes) +
    geom_point(aes(x=OIL, y=DAL, colour="DAL"), size=1.5) +
    geom_point(aes(x=OIL, y=AMR, colour="AMR"), size=1.5) +
    geom_point(aes(x=OIL, y=LUV, colour="LUV"), size=1.5) +
    labs(title=NULL, x="Daily Price Changes of Crude Oil",
         y="Daily Price Changes of Three Airline Stocks") +
    theme(plot.title=element_blank(),
          axis.title=element_text(face="bold"),
          axis.text=element_text(face="bold"),
          legend.position="bottom", legend.title=element_blank(),
          legend.text=element_text(face="bold")) +
    scale_colour_manual(values=c("DAL"="#E6450F", "AMR"="#2BCB95",
                                 "LUV"="#FFC100"),
                        labels=c("DAL"="DAL", "AMR"="AMR",
                                 "LUV"="LUV"))

DAL.scatter <- ggplot(data=changes) +
    geom_point(aes(x=OIL, y=DAL, colour="DAL"), size=1.5) +
    labs(title=NULL, x=NULL, y=NULL) +
```

```
    theme(plot.title=element_blank(),
          axis.text=element_text(face="bold"),
          legend.position="bottom", legend.title=element_blank(),
          legend.text=element_text(face="bold")) +
    scale_colour_manual(values=c("DAL"="#E6450F"), labels=c("DAL"="DAL"))

AMR.scatter <- ggplot(data=changes) +
    geom_point(aes(x=OIL, y=AMR, colour="AMR"), size=1.5) +
    labs(title=NULL, x=NULL, y=NULL) +
    theme(plot.title=element_blank(),
          axis.text=element_text(face="bold"),
          legend.position="bottom", legend.title=element_blank(),
          legend.text=element_text(face="bold")) +
    scale_colour_manual(values=c("AMR"="#2BCB95"), labels=c("AMR"="AMR"))

LUV.scatter <- ggplot(data=changes) +
    geom_point(aes(x=OIL, y=LUV, colour="LUV"), size=1.5) +
    labs(title=NULL, x=NULL, y=NULL) +
    theme(plot.title=element_blank(),
          axis.text=element_text(face="bold"),
          legend.position="bottom", legend.title=element_blank(),
          legend.text=element_text(face="bold")) +
    scale_colour_manual(values=c("LUV"="#FFC100"), labels=c("LUV"="LUV"))

grid.arrange(change.scatter, DAL.scatter, AMR.scatter, LUV.scatter,
             nrow=3, ncol=3, layout_matrix=rbind(c(1, 1, 1), c(1, 1, 1), c(2, 3, 4)))
```

6.3.3 Histogram

```
OIL.hist <- ggplot(data=changes, aes(x=OIL)) +
    geom_histogram(binwidth=0.5, fill="#45ACFF", color="#45ACFF") +
    geom_text(aes(label=as.character(..count..)),
              stat="bin", binwidth=0.5, vjust=-0.5, size=2) +
    labs(title=NULL, x="Daily Price Changes of OIL", y="Frequency") +
    theme(plot.title=element_blank(),
          axis.title=element_text(face="bold"),
          axis.text=element_text(face="bold"))

DAL.hist <- ggplot(data=changes, aes(x=DAL)) +
    geom_histogram(binwidth=0.1, fill="#E6450F", color="#E6450F") +
    geom_text(aes(label=as.character(..count..)),
              stat="bin", binwidth=0.1, vjust=-0.5, size=2) +
    labs(title=NULL, x="Daily Price Changes of DAL", y="Frequency") +
    theme(plot.title=element_blank(),
          axis.title=element_text(face="bold"),
          axis.text=element_text(face="bold"))

AMR.hist <- ggplot(data=changes, aes(x=AMR)) +
    geom_histogram(binwidth=0.1, fill="#2BCB95", color="#2BCB95") +
    geom_text(aes(label=as.character(..count..)),
              stat="bin", binwidth=0.1, vjust=-0.5, size=2) +
    labs(title=NULL, x="Daily Price Changes of AMR", y="Frequency") +
    theme(plot.title=element_blank(),
```

```
          axis.title=element_text(face="bold"),
          axis.text=element_text(face="bold"))

LUV.hist <- ggplot(data=changes, aes(x=LUV)) +
    geom_histogram(binwidth=0.1, fill="#FFC100", color="#FFC100") +
    geom_text(aes(label=as.character(..count..)),
              stat="bin", binwidth=0.1, vjust=-0.5, size=2) +
    labs(title=NULL, x="Daily Price Changes of LUV", y="Frequency") +
    theme(plot.title=element_blank(),
          axis.title=element_text(face="bold"),
          axis.text=element_text(face="bold"))

grid.arrange(OIL.hist, DAL.hist, AMR.hist, LUV.hist, nrow=2, ncol=2)
```

6.3.4 Skewness and Kurtosis

```
OIL.skew <- skewness(changes$OIL)
OIL.kurt <- kurtosis(changes$OIL)

DAL.skew <- skewness(changes$DAL)
DAL.kurt <- kurtosis(changes$DAL)

AMR.skew <- skewness(changes$AMR)
AMR.kurt <- kurtosis(changes$AMR)

LUV.skew <- skewness(changes$LUV)
LUV.kurt <- kurtosis(changes$LUV)
```

6.3.5 Normality Test

```
OIL.ks <- ks.test(changes$OIL, "pnorm", mean=mean(changes$OIL), sd=sd(changes$OIL))
OIL.cvm <- cvm.test(changes$OIL, "pnorm", mean=mean(changes$OIL), sd=sd(changes$OIL))
OIL.jb <- jarque.bera.test(changes$OIL)

DAL.ks <- ks.test(changes$DAL, "pnorm", mean=mean(changes$DAL), sd=sd(changes$DAL))
DAL.cvm <- cvm.test(changes$DAL, "pnorm", mean=mean(changes$DAL), sd=sd(changes$DAL))
DAL.jb <- jarque.bera.test(changes$DAL)

AMR.ks <- ks.test(changes$AMR, "pnorm", mean=mean(changes$AMR), sd=sd(changes$AMR))
AMR.cvm <- cvm.test(changes$AMR, "pnorm", mean=mean(changes$AMR), sd=sd(changes$AMR))
AMR.jb <- jarque.bera.test(changes$AMR)

LUV.ks <- ks.test(changes$LUV, "pnorm", mean=mean(changes$LUV), sd=sd(changes$LUV))
LUV.cvm <- cvm.test(changes$LUV, "pnorm", mean=mean(changes$LUV), sd=sd(changes$LUV))
LUV.jb <- jarque.bera.test(changes$LUV)
```

6.3.6 Heavy Tail Test

```
OIL.qqplot <- ggplot(data=changes, aes(sample=OIL)) +
    geom_qq(color="#45ACFF") +
    geom_qq_line(color="#0059B2", size=1) +
    labs(title="Crude Oil", x="Theoretical Quantiles", y="Sample Quantiles") +
```

```
    theme(plot.title=element_text(hjust=0.5, face="bold"),
          axis.title=element_text(face="bold"),
          axis.text=element_text(face="bold"))

DAL.qqplot <- ggplot(data=changes, aes(sample=DAL)) +
    geom_qq(color="#E6450F") +
    geom_qq_line(color="#8E283B", size=1) +
    labs(title="DAL", x="Theoretical Quantiles", y="Sample Quantiles") +
    theme(plot.title=element_text(hjust=0.5, face="bold"),
          axis.title=element_text(face="bold"),
          axis.text=element_text(face="bold"))

AMR.qqplot <- ggplot(data=changes, aes(sample=AMR)) +
    geom_qq(color="#2BCB95") +
    geom_qq_line(color="#6A9A48", size=1) +
    labs(title="AMR", x="Theoretical Quantiles", y="Sample Quantiles") +
    theme(plot.title=element_text(hjust=0.5, face="bold"),
          axis.title=element_text(face="bold"),
          axis.text=element_text(face="bold"))

LUV.qqplot <- ggplot(data=changes, aes(sample=LUV)) +
    geom_qq(color="#FFC100") +
    geom_qq_line(color="#E6450F", size=1) +
    labs(title="LUV", x="Theoretical Quantiles", y="Sample Quantiles") +
    theme(plot.title=element_text(hjust=0.5, face="bold"),
          axis.title=element_text(face="bold"),
          axis.text=element_text(face="bold"))

grid.arrange(OIL.qqplot, DAL.qqplot, AMR.qqplot, LUV.qqplot, nrow=2, ncol=2)
```

6.4 Simple Linear Regression

6.4.1 Modeling

```
DAL.SLR <- lm(DAL ~ OIL, data=changes)
DAL.changes <- as.data.frame(cbind("OIL"=changes$OIL,
                                   "DAL"=changes$DAL,
                                   predict(DAL.SLR, interval="prediction")))
DAL.lm <- ggplot(data=DAL.changes) +
    geom_point(aes(x=OIL, y=DAL, color="DAL"), size=1.5) +
    stat_smooth(aes(x=OIL, y=DAL), color="#8E283B", method=lm) +
    geom_line(aes(x=OIL, y=lwr), color="#8E283B", size=1, linetype="dashed") +
    geom_line(aes(x=OIL, y=upr), color="#8E283B", size=1, linetype="dashed") +
    labs(title=NULL, x="Daily Price Changes of Crude Oil",
         y="Daily Price Changes of DAL") +
    theme(plot.title=element_blank(),
          axis.title=element_text(face="bold"),
          axis.text=element_text(face="bold"),
          legend.position="bottom", legend.title=element_blank(),
          legend.text=element_text(face="bold")) +
    scale_colour_manual(values=c("DAL"="#E6450F", "AMR"="#2BCB95",
                                 "LUV"="#FFC100"),
                        labels=c("DAL"="DAL", "AMR"="AMR",
```

```
                                    "LUV"="LUV"))
summary(DAL.SLR)

AMR.SLR <- lm(AMR ~ OIL, data=changes)
AMR.changes <- as.data.frame(cbind("OIL"=changes$OIL,
                                   "AMR"=changes$AMR,
                                   predict(AMR.SLR, interval="prediction")))
AMR.lm <- ggplot(data=AMR.changes) +
    geom_point(aes(x=OIL, y=AMR, color="AMR"), size=1.5) +
    stat_smooth(aes(x=OIL, y=AMR), color="#6A9A48", method=lm) +
    geom_line(aes(x=OIL, y=lwr), color="#6A9A48", size=1, linetype="dashed") +
    geom_line(aes(x=OIL, y=upr), color="#6A9A48", size=1, linetype="dashed") +
    labs(title=NULL, x="Daily Price Changes of Crude Oil",
         y="Daily Price Changes of AMR") +
    theme(plot.title=element_blank(),
          axis.title=element_text(face="bold"),
          axis.text=element_text(face="bold"),
          legend.position="bottom", legend.title=element_blank(),
          legend.text=element_text(face="bold")) +
    scale_colour_manual(values=c("DAL"="#E6450F", "AMR"="#2BCB95",
                                 "LUV"="#FFC100"),
                        labels=c("DAL"="DAL", "AMR"="AMR",
                                 "LUV"="LUV"))
summary(AMR.SLR)

LUV.SLR <- lm(LUV ~ OIL, data=changes)
LUV.changes <- as.data.frame(cbind("OIL"=changes$OIL,
                                   "LUV"=changes$LUV,
                                   predict(LUV.SLR, interval="prediction")))
LUV.lm <- ggplot(data=LUV.changes) +
    geom_point(aes(x=OIL, y=LUV, color="LUV"), size=1.5) +
    stat_smooth(aes(x=OIL, y=LUV), color="#E6450F", method=lm) +
    geom_line(aes(x=OIL, y=lwr), color="#E6450F", size=1, linetype="dashed") +
    geom_line(aes(x=OIL, y=upr), color="#E6450F", size=1, linetype="dashed") +
    labs(title=NULL, x="Daily Price Changes of Crude Oil",
         y="Daily Price Changes of LUV") +
    theme(plot.title=element_blank(),
          axis.title=element_text(face="bold"),
          axis.text=element_text(face="bold"),
          legend.position="bottom", legend.title=element_blank(),
          legend.text=element_text(face="bold")) +
    scale_colour_manual(values=c("DAL"="#E6450F", "AMR"="#2BCB95",
                                 "LUV"="#FFC100"),
                        labels=c("DAL"="DAL", "AMR"="AMR",
                                 "LUV"="LUV"))
summary(LUV.SLR)

change.lm <- ggplot(data=changes) +
    geom_point(aes(x=OIL, y=DAL, colour="DAL"), size=1.5) +
    stat_smooth(aes(x=OIL, y=DAL), colour="#E6450F", fill="#8E283B",
                alpha=0.2, method=lm) +
    geom_point(aes(x=OIL, y=AMR, colour="AMR"), size=1.5) +
    stat_smooth(aes(x=OIL, y=AMR), colour="#2BCB95", fill="#6A9A48",
```

```
                alpha=0.2, method=lm) +
    geom_point(aes(x=OIL, y=LUV, colour="LUV"), size=1.5) +
    stat_smooth(aes(x=OIL, y=LUV), colour="#FFC100", fill="#E6450F",
                alpha=0.2, method=lm) +
    labs(title=NULL, x="Daily Price Changes of Crude Oil",
         y="Daily Price Changes of Three Airline Stocks") +
    theme(plot.title=element_blank(),
          axis.title=element_text(face="bold"),
          axis.text=element_text(face="bold"),
          legend.position="bottom", legend.title=element_blank(),
          legend.text=element_text(face="bold")) +
    scale_colour_manual(values=c("DAL"="#E6450F", "AMR"="#2BCB95",
                                 "LUV"="#FFC100"),
                        labels=c("DAL"="DAL", "AMR"="AMR",
                                 "LUV"="LUV"))

grid.arrange(change.lm, DAL.lm, AMR.lm, LUV.lm, nrow=3, ncol=3,
             layout_matrix=rbind(c(1, 1, 1), c(1, 1, 1), c(2, 3, 4)))
```

6.4.2 Diagnosis

Leverage

```
DAL.lev <- as.data.frame(cbind("Index"=1:dim(changes)[1],
                               "Leverage"=influence(DAL.SLR)$hat))
DAL.lev.plot <- ggplot(data=DAL.lev) +
    geom_bar(aes(x=Index, y=Leverage, color="DAL"),
             stat="identity", width=0.2) +
    geom_segment(aes(x=0, y=2 * 2 / dim(changes)[1],
                     xend=dim(changes)[1], yend=2 * 2 / dim(changes)[1]),
                 size=1, color="#45ACFF") +
    labs(title=NULL, x="Index", y="Leverages of DAL ~ OIL") +
    theme(plot.title=element_blank(),
          axis.title=element_text(face="bold"),
          axis.text=element_text(face="bold"),
          legend.position="none") +
    scale_colour_manual(values=c("DAL"="#E6450F"))

AMR.lev <- as.data.frame(cbind("Index"=1:dim(changes)[1],
                               "Leverage"=influence(AMR.SLR)$hat))
AMR.lev.plot <- ggplot(data=AMR.lev) +
    geom_bar(aes(x=Index, y=Leverage, color="AMR"),
             stat="identity", width=0.2) +
    geom_segment(aes(x=0, y=2 * 2 / dim(changes)[1],
                     xend=dim(changes)[1], yend=2 * 2 / dim(changes)[1]),
                  size=1, color="#45ACFF") +
    labs(title=NULL, x="Index", y="Leverages of AMR ~ OIL") +
    theme(plot.title=element_blank(),
          axis.title=element_text(face="bold"),
          axis.text=element_text(face="bold"),
          legend.position="none") +
    scale_colour_manual(values=c("AMR"="#2BCB95"))

LUV.lev <- as.data.frame(cbind("Index"=1:dim(changes)[1],
```

```
                                "Leverage"=influence(LUV.SLR)$hat))
LUV.lev.plot <- ggplot(data=LUV.lev) +
    geom_bar(aes(x=Index, y=Leverage, color="LUV"),
             stat="identity", width=0.2) +
    geom_segment(aes(x=0, y=2 * 2 / dim(changes)[1],
                     xend=dim(changes)[1], yend=2 * 2 / dim(changes)[1]),
                 size=1, color="#45ACFF") +
    labs(title=NULL, x="Index", y="Leverages of LUV ~ OIL") +
    theme(plot.title=element_blank(),
          axis.title=element_text(face="bold"),
          axis.text=element_text(face="bold"),
          legend.position="none") +
    scale_colour_manual(values=c("LUV"="#FFC100"))

grid.arrange(DAL.lev.plot, AMR.lev.plot, LUV.lev.plot, ncol=3)
```

Cook's Distance

```
DAL.cook <- as.data.frame(cbind("Index"=1:dim(changes)[1],
                                "Cook"=cooks.distance(DAL.SLR)))
DAL.cook.plot <- ggplot(data=DAL.cook) +
    geom_bar(aes(x=Index, y=Cook, color="DAL"),
             stat="identity", width=0.2) +
    geom_segment(aes(x=0, y=4 / (dim(changes)[1] - 2),
                     xend=dim(changes)[1], yend=4 / (dim(changes)[1] - 2)),
                 size=1, color="#45ACFF") +
    labs(title=NULL, x="Index", y="Cook's distances of DAL ~ OIL") +
    theme(plot.title=element_blank(),
          axis.title=element_text(face="bold"),
          axis.text=element_text(face="bold"),
          legend.position="none") +
    scale_colour_manual(values=c("DAL"="#E6450F"))

AMR.cook <- as.data.frame(cbind("Index"=1:dim(changes)[1],
                                "Cook"=cooks.distance(AMR.SLR)))
AMR.cook.plot <- ggplot(data=AMR.cook) +
    geom_bar(aes(x=Index, y=Cook, color="AMR"),
             stat="identity", width=0.2) +
    geom_segment(aes(x=0, y=4 / (dim(changes)[1] - 2),
                     xend=dim(changes)[1], yend=4 / (dim(changes)[1] - 2)),
                 size=1, color="#45ACFF") +
    labs(title=NULL, x="Index", y="Cook's distances of AMR ~ OIL") +
    theme(plot.title=element_blank(),
          axis.title=element_text(face="bold"),
          axis.text=element_text(face="bold"),
          legend.position="none") +
    scale_colour_manual(values=c("AMR"="#2BCB95"))

LUV.cook <- as.data.frame(cbind("Index"=1:dim(changes)[1],
                                "Cook"=cooks.distance(LUV.SLR)))
LUV.cook.plot <- ggplot(data=LUV.cook) +
    geom_bar(aes(x=Index, y=Cook, color="LUV"),
             stat="identity", width=0.2) +
    geom_segment(aes(x=0, y=4 / (dim(changes)[1] - 2),
```

```
                    xend=dim(changes)[1], yend=4 / (dim(changes)[1] - 2)),
                size=1, color="#45ACFF") +
    labs(title=NULL, x="Index", y="Cook's distances of LUV ~ OIL") +
    theme(plot.title=element_blank(),
          axis.title=element_text(face="bold"),
          axis.text=element_text(face="bold"),
          legend.position="none") +
    scale_colour_manual(values=c("LUV"="#FFC100"))

grid.arrange(DAL.cook.plot, AMR.cook.plot, LUV.cook.plot, ncol=3)
```

Residual

```
DAL.res <- as.data.frame(cbind("Index"=1:dim(changes)[1],
                               "Residual"=residuals(DAL.SLR)))
DAL.rsd <- as.data.frame(cbind("Index"=1:dim(changes)[1],
                               "Rstandard"=rstandard(DAL.SLR)))
DAL.rst <- as.data.frame(cbind("Index"=1:dim(changes)[1],
                               "Rstudent"=rstudent(DAL.SLR)))
DAL.res.plot <- ggplot(data=DAL.res) +
    geom_point(aes(x=Index, y=Residual, color="DAL"), size=1.5) +
    geom_segment(aes(x=0, y=0,
                     xend=dim(changes)[1], yend=0),
                 size=1, color="#45ACFF") +
    labs(title=NULL, x="Index", y="Raw Residuals of DAL ~ OIL") +
    theme(plot.title=element_blank(),
          axis.title=element_text(face="bold"),
          axis.text=element_text(face="bold"),
          legend.position="none") +
    scale_colour_manual(values=c("DAL"="#E6450F"))
DAL.rsd.plot <- ggplot(data=DAL.rsd) +
    geom_point(aes(x=Index, y=Rstandard, color="DAL"), size=1.5) +
    geom_segment(aes(x=0, y=0,
                     xend=dim(changes)[1], yend=0),
                 size=1, color="#45ACFF") +
    labs(title=NULL, x="Index", y="Standardized Residuals of DAL ~ OIL") +
    theme(plot.title=element_blank(),
          axis.title=element_text(face="bold"),
          axis.text=element_text(face="bold"),
          legend.position="none") +
    scale_colour_manual(values=c("DAL"="#E6450F"))
DAL.rst.plot <- ggplot(data=DAL.rst) +
    geom_point(aes(x=Index, y=Rstudent, color="DAL"), size=1.5) +
    geom_segment(aes(x=0, y=0,
                     xend=dim(changes)[1], yend=0),
                 size=1, color="#45ACFF") +
    labs(title=NULL, x="Index", y="Rstudent Residuals of DAL ~ OIL") +
    theme(plot.title=element_blank(),
          axis.title=element_text(face="bold"),
          axis.text=element_text(face="bold"),
          legend.position="none") +
    scale_colour_manual(values=c("DAL"="#E6450F"))

AMR.res <- as.data.frame(cbind("Index"=1:dim(changes)[1],
```

```
                                "Residual"=residuals(AMR.SLR)))
AMR.rsd <- as.data.frame(cbind("Index"=1:dim(changes)[1],
                                "Rstandard"=rstandard(AMR.SLR)))
AMR.rst <- as.data.frame(cbind("Index"=1:dim(changes)[1],
                                "Rstudent"=rstudent(AMR.SLR)))
AMR.res.plot <- ggplot(data=AMR.res) +
    geom_point(aes(x=Index, y=Residual, color="AMR"), size=1.5) +
    geom_segment(aes(x=0, y=0,
                     xend=dim(changes)[1], yend=0),
                 size=1, color="#45ACFF") +
    labs(title=NULL, x="Index", y="Raw Residuals of AMR ~ OIL") +
    theme(plot.title=element_blank(),
          axis.title=element_text(face="bold"),
          axis.text=element_text(face="bold"),
          legend.position="none") +
    scale_colour_manual(values=c("AMR"="#2BCB95"))
AMR.rsd.plot <- ggplot(data=AMR.rsd) +
    geom_point(aes(x=Index, y=Rstandard, color="AMR"), size=1.5) +
    geom_segment(aes(x=0, y=0,
                     xend=dim(changes)[1], yend=0),
                 size=1, color="#45ACFF") +
    labs(title=NULL, x="Index", y="Standardized Residuals of AMR ~ OIL") +
    theme(plot.title=element_blank(),
          axis.title=element_text(face="bold"),
          axis.text=element_text(face="bold"),
          legend.position="none") +
    scale_colour_manual(values=c("AMR"="#2BCB95"))
AMR.rst.plot <- ggplot(data=AMR.rst) +
    geom_point(aes(x=Index, y=Rstudent, color="AMR"), size=1.5) +
    geom_segment(aes(x=0, y=0,
                     xend=dim(changes)[1], yend=0),
                 size=1, color="#45ACFF") +
    labs(title=NULL, x="Index", y="Rstudent Residuals of AMR ~ OIL") +
    theme(plot.title=element_blank(),
          axis.title=element_text(face="bold"),
          axis.text=element_text(face="bold"),
          legend.position="none") +
    scale_colour_manual(values=c("AMR"="#2BCB95"))

LUV.res <- as.data.frame(cbind("Index"=1:dim(changes)[1],
                                "Residual"=residuals(LUV.SLR)))
LUV.rsd <- as.data.frame(cbind("Index"=1:dim(changes)[1],
                                "Rstandard"=rstandard(LUV.SLR)))
LUV.rst <- as.data.frame(cbind("Index"=1:dim(changes)[1],
                                "Rstudent"=rstudent(LUV.SLR)))
LUV.res.plot <- ggplot(data=LUV.res) +
    geom_point(aes(x=Index, y=Residual, color="LUV"), size=1.5) +
    geom_segment(aes(x=0, y=0,
                     xend=dim(changes)[1], yend=0),
                 size=1, color="#45ACFF") +
    labs(title=NULL, x="Index", y="Raw Residuals of LUV ~ OIL") +
    theme(plot.title=element_blank(),
          axis.title=element_text(face="bold"),
```

```
          axis.text=element_text(face="bold"),
          legend.position="none") +
    scale_colour_manual(values=c("LUV"="#FFC100"))
LUV.rsd.plot <- ggplot(data=LUV.rsd) +
    geom_point(aes(x=Index, y=Rstandard, color="LUV"), size=1.5) +
    geom_segment(aes(x=0, y=0,
                     xend=dim(changes)[1], yend=0),
                 size=1, color="#45ACFF") +
    labs(title=NULL, x="Index", y="Standardized Residuals of LUV ~ OIL") +
    theme(plot.title=element_blank(),
          axis.title=element_text(face="bold"),
          axis.text=element_text(face="bold"),
          legend.position="none") +
    scale_colour_manual(values=c("LUV"="#FFC100"))
LUV.rst.plot <- ggplot(data=LUV.rst) +
    geom_point(aes(x=Index, y=Rstudent, color="LUV"), size=1.5) +
    geom_segment(aes(x=0, y=0,
                     xend=dim(changes)[1], yend=0),
                 size=1, color="#45ACFF") +
    labs(title=NULL, x="Index", y="Rstudent Residuals of LUV ~ OIL") +
    theme(plot.title=element_blank(),
          axis.title=element_text(face="bold"),
          axis.text=element_text(face="bold"),
          legend.position="none") +
    scale_colour_manual(values=c("LUV"="#FFC100"))

grid.arrange(DAL.res.plot, DAL.rsd.plot, DAL.rst.plot,
             AMR.res.plot, AMR.rsd.plot, AMR.rst.plot,
             LUV.res.plot, LUV.rsd.plot, LUV.rst.plot,
             nrow=3, ncol=3)
```

QQ-plot and Histogram

```
DAL.res.qq <- ggplot(data=DAL.res, aes(sample=Residual)) +
    geom_qq(color="#E6450F") +
    geom_qq_line(color="#8E283B", size=1) +
    labs(title=NULL, x="Theoretical Quantiles", y="Sample Quantiles") +
    theme(plot.title=element_blank(),
          axis.title=element_text(face="bold"),
          axis.text=element_text(face="bold"))
DAL.res.hist <- ggplot(data=DAL.res, aes(x=Residual)) +
    geom_histogram(binwidth=0.1, fill="#E6450F", color="#E6450F") +
    geom_text(aes(label=as.character(..count..)),
              stat="bin", binwidth=0.1, vjust=-0.5, size=2) +
    labs(title=NULL, x="Residuals of DAL ~ OIL", y="Frequency") +
    theme(plot.title=element_blank(),
          axis.title=element_text(face="bold"),
          axis.text=element_text(face="bold"))

AMR.res.qq <- ggplot(data=AMR.res, aes(sample=Residual)) +
    geom_qq(color="#2BCB95") +
    geom_qq_line(color="#6A9A48", size=1) +
    labs(title=NULL, x="Theoretical Quantiles", y="Sample Quantiles") +
    theme(plot.title=element_blank(),
```

```
          axis.title=element_text(face="bold"),
          axis.text=element_text(face="bold"))
AMR.res.hist <- ggplot(data=AMR.res, aes(x=Residual)) +
    geom_histogram(binwidth=0.1, fill="#2BCB95", color="#2BCB95") +
    geom_text(aes(label=as.character(..count..)),
              stat="bin", binwidth=0.1, vjust=-0.5, size=2) +
    labs(title=NULL, x="Residuals of AMR ~ OIL", y="Frequency") +
    theme(plot.title=element_blank(),
          axis.title=element_text(face="bold"),
          axis.text=element_text(face="bold"))

LUV.res.qq <- ggplot(data=LUV.res, aes(sample=Residual)) +
    geom_qq(color="#FFC100") +
    geom_qq_line(color="#E6450F", size=1) +
    labs(title=NULL, x="Theoretical Quantiles", y="Sample Quantiles") +
    theme(plot.title=element_blank(),
          axis.title=element_text(face="bold"),
          axis.text=element_text(face="bold"))
LUV.res.hist <- ggplot(data=LUV.res, aes(x=Residual)) +
    geom_histogram(binwidth=0.1, fill="#FFC100", color="#FFC100") +
    geom_text(aes(label=as.character(..count..)),
              stat="bin", binwidth=0.1, vjust=-0.5, size=2) +
    labs(title=NULL, x="Residuals of LUV ~ OIL", y="Frequency") +
    theme(plot.title=element_blank(),
          axis.title=element_text(face="bold"),
          axis.text=element_text(face="bold"))

grid.arrange(DAL.res.qq, DAL.res.hist,
             AMR.res.qq, AMR.res.hist,
             LUV.res.qq, LUV.res.hist,
             nrow=3, ncol=2)
```

6.5 Delayed Linear Regression

6.5.1 Modeling

```
delay.changes <- cbind(changes,
                       "OILm1"=c(0, changes$OIL[-dim(changes)[1]]),
                       "DALm1"=c(0, changes$DAL[-dim(changes)[1]]),
                       "AMRm1"=c(0, changes$AMR[-dim(changes)[1]]),
                       "LUVm1"=c(0, changes$LUV[-dim(changes)[1]]))

DAL.DLR <- lm(DAL ~ OIL + DALm1, data=delay.changes)
AMR.DLR <- lm(AMR ~ OIL + AMRm1, data=delay.changes)
LUV.DLR <- lm(LUV ~ OIL + LUVm1, data=delay.changes)

knitr::kable(
    x=data.frame(
        "Statistic"=c("Intercept", "Slope of X", "P-value of Slope of X",
                      "Slope of Ym1", "P-value of Slope of Ym1",
                      "Adjusted R-squared", "P-value of F-statistic"),
        "DAL"=c(-0.017, -0.109,
                format(6.76e-12, scientific=T, digits=3, nsmall=3), 0.088,
```

```
                0.098, 0.138,
                format(3.35e-11, scientific=T, digits=3, nsmall=3)),
        "AMR"=c(-0.032, -0.138,
                format(2.32e-13, scientific=T, digits=3, nsmall=3), -0.015,
                0.769, 0.156,
                format(1.35e-12, scientific=T, digits=3, nsmall=3)),
        "LUV"=c(0.015, -0.057,
                format(1.15e-12, scientific=T, digits=3, nsmall=3), -0.189,
                format(2.63e-4, scientific=T, digits=3, nsmall=3), 0.178,
                format(2.36e-14, scientific=T, digits=3, nsmall=3))),
    caption="Statistics in Delayed Linear Regression",
    col.names=c("Statistic", "DAL", "AMR", "LUV"),
    align=c("c", "c", "c", "c"))
```

6.5.2 Diagnosis

Leverage

```
DAL.DLR.lev <- as.data.frame(cbind("Index"=1:dim(delay.changes)[1],
                                   "Leverage"=influence(DAL.DLR)$hat))
DAL.DLR.lev.plot <- ggplot(data=DAL.DLR.lev) +
    geom_bar(aes(x=Index, y=Leverage, color="DAL"),
             stat="identity", width=0.2) +
    geom_segment(aes(x=0, y=2 * 2 / dim(changes)[1],
                     xend=dim(changes)[1], yend=2 * 2 / dim(changes)[1]),
                 size=1, color="#45ACFF") +
    labs(title=NULL, x="Index", y="Leverages of DAL ~ OIL") +
    theme(plot.title=element_blank(),
          axis.title=element_text(face="bold"),
          axis.text=element_text(face="bold"),
          legend.position="none") +
    scale_colour_manual(values=c("DAL"="#E6450F"))

AMR.DLR.lev <- as.data.frame(cbind("Index"=1:dim(delay.changes)[1],
                                   "Leverage"=influence(AMR.DLR)$hat))
AMR.DLR.lev.plot <- ggplot(data=AMR.DLR.lev) +
    geom_bar(aes(x=Index, y=Leverage, color="AMR"),
             stat="identity", width=0.2) +
    geom_segment(aes(x=0, y=2 * 2 / dim(changes)[1],
                     xend=dim(changes)[1], yend=2 * 2 / dim(changes)[1]),
                 size=1, color="#45ACFF") +
    labs(title=NULL, x="Index", y="Leverages of AMR ~ OIL") +
    theme(plot.title=element_blank(),
          axis.title=element_text(face="bold"),
          axis.text=element_text(face="bold"),
          legend.position="none") +
    scale_colour_manual(values=c("AMR"="#2BCB95"))

LUV.DLR.lev <- as.data.frame(cbind("Index"=1:dim(delay.changes)[1],
                                   "Leverage"=influence(LUV.DLR)$hat))
LUV.DLR.lev.plot <- ggplot(data=LUV.DLR.lev) +
    geom_bar(aes(x=Index, y=Leverage, color="LUV"),
             stat="identity", width=0.2) +
    geom_segment(aes(x=0, y=2 * 2 / dim(changes)[1],
```

```
                    xend=dim(changes)[1], yend=2 * 2 / dim(changes)[1]),
                size=1, color="#45ACFF") +
    labs(title=NULL, x="Index", y="Leverages of LUV ~ OIL") +
    theme(plot.title=element_blank(),
          axis.title=element_text(face="bold"),
          axis.text=element_text(face="bold"),
          legend.position="none") +
    scale_colour_manual(values=c("LUV"="#FFC100"))

grid.arrange(DAL.DLR.lev.plot, AMR.DLR.lev.plot, LUV.DLR.lev.plot, ncol=3)
```

Cook's Distance

```
DAL.DLR.cook <- as.data.frame(cbind("Index"=1:dim(delay.changes)[1],
                                    "Cook"=cooks.distance(DAL.DLR)))
DAL.DLR.cook.plot <- ggplot(data=DAL.DLR.cook) +
    geom_bar(aes(x=Index, y=Cook, color="DAL"),
             stat="identity", width=0.2) +
    geom_segment(aes(x=0, y=4 / (dim(changes)[1] - 2),
                     xend=dim(changes)[1], yend=4 / (dim(changes)[1] - 2)),
                 size=1, color="#45ACFF") +
    labs(title=NULL, x="Index", y="Cook's distances of DAL ~ OIL") +
    theme(plot.title=element_blank(),
          axis.title=element_text(face="bold"),
          axis.text=element_text(face="bold"),
          legend.position="none") +
    scale_colour_manual(values=c("DAL"="#E6450F"))

AMR.DLR.cook <- as.data.frame(cbind("Index"=1:dim(delay.changes)[1],
                                    "Cook"=cooks.distance(AMR.DLR)))
AMR.DLR.cook.plot <- ggplot(data=AMR.DLR.cook) +
    geom_bar(aes(x=Index, y=Cook, color="AMR"),
             stat="identity", width=0.2) +
    geom_segment(aes(x=0, y=4 / (dim(changes)[1] - 2),
                     xend=dim(changes)[1], yend=4 / (dim(changes)[1] - 2)),
                 size=1, color="#45ACFF") +
    labs(title=NULL, x="Index", y="Cook's distances of AMR ~ OIL") +
    theme(plot.title=element_blank(),
          axis.title=element_text(face="bold"),
          axis.text=element_text(face="bold"),
          legend.position="none") +
    scale_colour_manual(values=c("AMR"="#2BCB95"))

LUV.DLR.cook <- as.data.frame(cbind("Index"=1:dim(delay.changes)[1],
                                    "Cook"=cooks.distance(LUV.DLR)))
LUV.DLR.cook.plot <- ggplot(data=LUV.DLR.cook) +
    geom_bar(aes(x=Index, y=Cook, color="LUV"),
             stat="identity", width=0.2) +
    geom_segment(aes(x=0, y=4 / (dim(changes)[1] - 2),
                     xend=dim(changes)[1], yend=4 / (dim(changes)[1] - 2)),
                 size=1, color="#45ACFF") +
    labs(title=NULL, x="Index", y="Cook's distances of LUV ~ OIL") +
    theme(plot.title=element_blank(),
          axis.title=element_text(face="bold"),
```

```
          axis.text=element_text(face="bold"),
          legend.position="none") +
    scale_colour_manual(values=c("LUV"="#FFC100"))

grid.arrange(DAL.DLR.cook.plot, AMR.DLR.cook.plot, LUV.DLR.cook.plot, ncol=3)
```

Residual

```
DAL.DLR.res <- as.data.frame(cbind("Index"=1:dim(delay.changes)[1],
                                   "Residual"=residuals(DAL.DLR)))
DAL.DLR.rsd <- as.data.frame(cbind("Index"=1:dim(delay.changes)[1],
                                   "Rstandard"=rstandard(DAL.DLR)))
DAL.DLR.rst <- as.data.frame(cbind("Index"=1:dim(delay.changes)[1],
                                   "Rstudent"=rstudent(DAL.DLR)))
DAL.DLR.res.plot <- ggplot(data=DAL.DLR.res) +
    geom_point(aes(x=Index, y=Residual, color="DAL"), size=1.5) +
    geom_segment(aes(x=0, y=0,
                     xend=dim(changes)[1], yend=0),
                 size=1, color="#45ACFF") +
    labs(title=NULL, x="Index", y="Raw Residuals of DAL ~ OIL") +
    theme(plot.title=element_blank(),
          axis.title=element_text(face="bold"),
          axis.text=element_text(face="bold"),
          legend.position="none") +
    scale_colour_manual(values=c("DAL"="#E6450F"))
DAL.DLR.rsd.plot <- ggplot(data=DAL.DLR.rsd) +
    geom_point(aes(x=Index, y=Rstandard, color="DAL"), size=1.5) +
    geom_segment(aes(x=0, y=0,
                     xend=dim(changes)[1], yend=0),
                 size=1, color="#45ACFF") +
    labs(title=NULL, x="Index", y="Standardized Residuals of DAL ~ OIL") +
    theme(plot.title=element_blank(),
          axis.title=element_text(face="bold"),
          axis.text=element_text(face="bold"),
          legend.position="none") +
    scale_colour_manual(values=c("DAL"="#E6450F"))
DAL.DLR.rst.plot <- ggplot(data=DAL.DLR.rst) +
    geom_point(aes(x=Index, y=Rstudent, color="DAL"), size=1.5) +
    geom_segment(aes(x=0, y=0,
                     xend=dim(changes)[1], yend=0),
                 size=1, color="#45ACFF") +
    labs(title=NULL, x="Index", y="Rstudent Residuals of DAL ~ OIL") +
    theme(plot.title=element_blank(),
          axis.title=element_text(face="bold"),
          axis.text=element_text(face="bold"),
          legend.position="none") +
    scale_colour_manual(values=c("DAL"="#E6450F"))

AMR.DLR.res <- as.data.frame(cbind("Index"=1:dim(delay.changes)[1],
                                   "Residual"=residuals(AMR.DLR)))
AMR.DLR.rsd <- as.data.frame(cbind("Index"=1:dim(delay.changes)[1],
                                   "Rstandard"=rstandard(AMR.DLR)))
AMR.DLR.rst <- as.data.frame(cbind("Index"=1:dim(delay.changes)[1],
                                   "Rstudent"=rstudent(AMR.DLR)))
```

```
AMR.DLR.res.plot <- ggplot(data=AMR.DLR.res) +
    geom_point(aes(x=Index, y=Residual, color="AMR"), size=1.5) +
    geom_segment(aes(x=0, y=0,
                     xend=dim(changes)[1], yend=0),
                 size=1, color="#45ACFF") +
    labs(title=NULL, x="Index", y="Raw Residuals of AMR ~ OIL") +
    theme(plot.title=element_blank(),
          axis.title=element_text(face="bold"),
          axis.text=element_text(face="bold"),
          legend.position="none") +
    scale_colour_manual(values=c("AMR"="#2BCB95"))
AMR.DLR.rsd.plot <- ggplot(data=AMR.DLR.rsd) +
    geom_point(aes(x=Index, y=Rstandard, color="AMR"), size=1.5) +
    geom_segment(aes(x=0, y=0,
                     xend=dim(changes)[1], yend=0),
                 size=1, color="#45ACFF") +
    labs(title=NULL, x="Index", y="Standardized Residuals of AMR ~ OIL") +
    theme(plot.title=element_blank(),
          axis.title=element_text(face="bold"),
          axis.text=element_text(face="bold"),
          legend.position="none") +
    scale_colour_manual(values=c("AMR"="#2BCB95"))
AMR.DLR.rst.plot <- ggplot(data=AMR.DLR.rst) +
    geom_point(aes(x=Index, y=Rstudent, color="AMR"), size=1.5) +
    geom_segment(aes(x=0, y=0,
                     xend=dim(changes)[1], yend=0),
                 size=1, color="#45ACFF") +
    labs(title=NULL, x="Index", y="Rstudent Residuals of AMR ~ OIL") +
    theme(plot.title=element_blank(),
          axis.title=element_text(face="bold"),
          axis.text=element_text(face="bold"),
          legend.position="none") +
    scale_colour_manual(values=c("AMR"="#2BCB95"))

LUV.DLR.res <- as.data.frame(cbind("Index"=1:dim(delay.changes)[1],
                                   "Residual"=residuals(LUV.DLR)))
LUV.DLR.rsd <- as.data.frame(cbind("Index"=1:dim(delay.changes)[1],
                                   "Rstandard"=rstandard(LUV.DLR)))
LUV.DLR.rst <- as.data.frame(cbind("Index"=1:dim(delay.changes)[1],
                                   "Rstudent"=rstudent(LUV.DLR)))
LUV.DLR.res.plot <- ggplot(data=LUV.DLR.res) +
    geom_point(aes(x=Index, y=Residual, color="LUV"), size=1.5) +
    geom_segment(aes(x=0, y=0,
                     xend=dim(changes)[1], yend=0),
                 size=1, color="#45ACFF") +
    labs(title=NULL, x="Index", y="Raw Residuals of LUV ~ OIL") +
    theme(plot.title=element_blank(),
          axis.title=element_text(face="bold"),
          axis.text=element_text(face="bold"),
          legend.position="none") +
    scale_colour_manual(values=c("LUV"="#FFC100"))
LUV.DLR.rsd.plot <- ggplot(data=LUV.DLR.rsd) +
    geom_point(aes(x=Index, y=Rstandard, color="LUV"), size=1.5) +
```

```
    geom_segment(aes(x=0, y=0,
                     xend=dim(changes)[1], yend=0),
                 size=1, color="#45ACFF") +
    labs(title=NULL, x="Index", y="Standardized Residuals of LUV ~ OIL") +
    theme(plot.title=element_blank(),
          axis.title=element_text(face="bold"),
          axis.text=element_text(face="bold"),
          legend.position="none") +
    scale_colour_manual(values=c("LUV"="#FFC100"))
LUV.DLR.rst.plot <- ggplot(data=LUV.DLR.rst) +
    geom_point(aes(x=Index, y=Rstudent, color="LUV"), size=1.5) +
    geom_segment(aes(x=0, y=0,
                     xend=dim(changes)[1], yend=0),
                 size=1, color="#45ACFF") +
    labs(title=NULL, x="Index", y="Rstudent Residuals of LUV ~ OIL") +
    theme(plot.title=element_blank(),
          axis.title=element_text(face="bold"),
          axis.text=element_text(face="bold"),
          legend.position="none") +
    scale_colour_manual(values=c("LUV"="#FFC100"))

grid.arrange(DAL.DLR.res.plot, DAL.DLR.rsd.plot, DAL.DLR.rst.plot,
             AMR.DLR.res.plot, AMR.DLR.rsd.plot, AMR.DLR.rst.plot,
             LUV.DLR.res.plot, LUV.DLR.rsd.plot, LUV.DLR.rst.plot,
             nrow=3, ncol=3)
```

QQ-plot and Histogram

```
DAL.DLR.res.qq <- ggplot(data=DAL.DLR.res, aes(sample=Residual)) +
    geom_qq(color="#E6450F") +
    geom_qq_line(color="#8E283B", size=1) +
    labs(title=NULL, x="Theoretical Quantiles", y="Sample Quantiles") +
    theme(plot.title=element_blank(),
          axis.title=element_text(face="bold"),
          axis.text=element_text(face="bold"))
DAL.DLR.res.hist <- ggplot(data=DAL.DLR.res, aes(x=Residual)) +
    geom_histogram(binwidth=0.1, fill="#E6450F", color="#E6450F") +
    geom_text(aes(label=as.character(..count..)),
              stat="bin", binwidth=0.1, vjust=-0.5, size=2) +
    labs(title=NULL, x="Residuals of DAL ~ OIL", y="Frequency") +
    theme(plot.title=element_blank(),
          axis.title=element_text(face="bold"),
          axis.text=element_text(face="bold"))

AMR.DLR.res.qq <- ggplot(data=AMR.DLR.res, aes(sample=Residual)) +
    geom_qq(color="#2BCB95") +
    geom_qq_line(color="#6A9A48", size=1) +
    labs(title=NULL, x="Theoretical Quantiles", y="Sample Quantiles") +
    theme(plot.title=element_blank(),
          axis.title=element_text(face="bold"),
          axis.text=element_text(face="bold"))
AMR.DLR.res.hist <- ggplot(data=AMR.DLR.res, aes(x=Residual)) +
    geom_histogram(binwidth=0.1, fill="#2BCB95", color="#2BCB95") +
    geom_text(aes(label=as.character(..count..)),
```

```
              stat="bin", binwidth=0.1, vjust=-0.5, size=2) +
    labs(title=NULL, x="Residuals of AMR ~ OIL", y="Frequency") +
    theme(plot.title=element_blank(),
          axis.title=element_text(face="bold"),
          axis.text=element_text(face="bold"))

LUV.DLR.res.qq <- ggplot(data=LUV.DLR.res, aes(sample=Residual)) +
    geom_qq(color="#FFC100") +
    geom_qq_line(color="#E6450F", size=1) +
    labs(title=NULL, x="Theoretical Quantiles", y="Sample Quantiles") +
    theme(plot.title=element_blank(),
          axis.title=element_text(face="bold"),
          axis.text=element_text(face="bold"))
LUV.DLR.res.hist <- ggplot(data=LUV.DLR.res, aes(x=Residual)) +
    geom_histogram(binwidth=0.1, fill="#FFC100", color="#FFC100") +
    geom_text(aes(label=as.character(..count..)),
              stat="bin", binwidth=0.1, vjust=-0.5, size=2) +
    labs(title=NULL, x="Residuals of LUV ~ OIL", y="Frequency") +
    theme(plot.title=element_blank(),
          axis.title=element_text(face="bold"),
          axis.text=element_text(face="bold"))

grid.arrange(DAL.DLR.res.qq, DAL.DLR.res.hist,
             AMR.DLR.res.qq, AMR.DLR.res.hist,
             LUV.DLR.res.qq, LUV.DLR.res.hist,
             nrow=3, ncol=2)
```

6.6 Model Selection

```
AIC(DAL.SLR, DAL.DLR)
AIC(AMR.SLR, AMR.DLR)
AIC(LUV.SLR, LUV.DLR)
```