Homework II

Deadline: 2019-4-30

Reminder. Homework must be done using MATLAB publish for coding problems and using MATLAB Publish/LATEX for calculation and analysis problems.

1. (10 pts) Reproduce the figure in the slide titled "Forward Euler: Example" of "07ODE.pdf".
2. (10 pts) Reproduce the figure in the slide titled "Trapezoidal Method: Example" of "07ODE.pdf".
3. (10 pts) Show that the local truncation error of the Trapezoidal method for the ODE initial value problem is $O(h^3)$.
4. (10 pts) Derive the Adams-Moulton method for $k = 3$, i.e. the AM formula in the slide titled "The Admas Family" of "07ODE.pdf".
5. (10 pts) Prove the following result: The Fourier Transform of $g(t) = h(t)f(t)$ is given by
$$\hat{g}(\omega) = \frac{1}{2\pi}(\hat{h} \star \hat{f})(\omega).$$
6. (10 pts) Let $f = [f_0, \cdots, f_{n-1}] \in \mathcal{C}^n$ and let $\hat{f} = [\hat{f}_0, \cdots, \hat{f}_{n-1}]$ be the discrete Fourier transform of f. Show that
$$\|f\|_2^2 = \|\hat{f}\|_2^2/n.$$
7. (10 pts) Prove the following equation:
$$\hat{f}_{2k+1} = \sum_{j=0}^{n/2-1} e^{-i2\pi j/n}\left(f_j - f_{j+n/2}\right)e^{-i2\pi kj/(n/2)},$$
which appears in the derivation of FFT.