Homework II

Deadline: 2019-3-31

Reminder. Homework must be done using MATLAB publish for coding problems and using MATLAB Publish/LATEX for calculation and analysis problems.

1. (a) (15 pts) Reproduce plot as the one in the slide titled "Divergence Case: Runge Phenomenon" of "03interpolation.pdf" for $f(x) = \frac{1}{1+25x^2}$ over $[-1, 1]$. That is, do **Lagrange interpolation** for $f(x)$ with $n = 10$ using equally spaced points, and then plot $f(x)$ and the interpolation polynomials in the same figure.

   (b) (5 pts) Let $T_n(x) = \cos(n \arccos(x))$ be the Chebshev polynomial of degree n over $[-1, 1]$. Compute the roots of $T_n(x)$. **Hint:** $T_n(x)$ has n distinct roots.

   (c) (15 pts) Do the Lagrange interpolation of $f(x) = \frac{1}{1+25x^2}$ over $[-1, 1]$ with $n = 10$, but this time use the roots of the Chebshev polynomial in (b) as the interpolation points rather than the equally spaces points in (a). In order to interpolate a polynomial of degree n, we need to use the roots of $T_{n+1}(x)$. Plot $f(x)$ and the interpolation polynomials in the same figure. What is your observation compared with the figure from (a)?

2. (10 pts) Show that $H_k(x)$ and $K_k(x)$ in the Hermite interpolation satisfy

$$H_k(x_i) = \begin{cases} 1 & i = k \\ 0 & i \neq k \end{cases}, \quad H'_k(x_i) = 0,$$

$$K_k(x_i) = 0, \quad K'_k(x_i) = \begin{cases} 1 & i = k \\ 0 & i \neq k. \end{cases}$$

3. (15 pts) Starting from $\phi_0 = 1$, construct a system of orthogonal polynomials $\{\phi_0, \phi_1, \phi_2, \phi_3\}$ on the interval $(-1, 1)$ with respect to the weight function $w(x) = 1$. Plots the polynomials and verify that the roots of each polynomial are distinct and lie in the interval $(-1, 1)$.

4. (15 pts) Do natural cubic spline interpolation through points

$$(1, 16), \ (2, 18), \ (3, 21), \ (4, 17), \ (5, 15), \ (6, 12).$$

   That is, reproduce the plot in "Cubic Spline Interpolation" of "03interpolation.pdf".