Homework I

Deadline: 2019-3-15

Reminder. Homework must be done using MATLAB publish for coding problems and using MATLAB Publish/LATEX for calculation and analysis problems.

1. (10 pts) In a reference book, there is an example about Newton's method for root finding, showing in Figure 1. The author claims "starting from $x_0 = 0.25$ gives a divergent set of iterates" based on the second table. Whether the claim is right? Verify your answer numerically.

Example 8.4 (Newton's method failure). Suppose $f(x) = x^5 - 3x^4 + 25$. Newton's method applied to this function gives the iteration

$$x_{k+1} = x_k - \frac{x_k^5 - 3x_k^4 + 25}{5x_k^4 - 12x^3}.$$

These iterations converge when x_0 is sufficiently close to the root $x^* \approx -1.5325$. For instance, the iterates starting from $x_0 = -2$ are shown below:

k	0	1	2	3	4
x_k	-2	-1.687500	-1.555013	-1.533047	-1.532501

Farther away from this root, however, Newton's method can fail. For instance, starting from $x_0 = 0.25$ gives a divergent set of iterates:

k	0	1	2	3	4
x_k	0.25	149.023256	119.340569	95.594918	76.599025

Figure 1: Example from reference book.

2. Let $f(x) = e^x - x - 2$ and consider the problem of finding a root of $f(x)$ by fixed point iteration.

   - (5 pts) Show that there exists $x_* \in (1, 2)$ such that $f(x_*) = 0$.

   Consider the following two different formulations of $g(x)$:

   $$g(x) = e^x - 2 \quad \text{or} \quad g(x) = \ln(x + 2).$$

   - (5 pts) Show that for each $g(x)$ above, $x = g(x)$ implies $f(x) = 0$.
   - (20 pts) Discuss the convergence of fixed point iteration applied to $x = g(x)$ with $x_0 > x_*$ for the two different $g(x)$ given above.

3. (15 pts) Given a number $a \neq 0$, write out the Newton iteration for computing $1/a$ *without calculating division* in each iteration. Will the Newton's iteration converge starting from any initial value, and why?

4. (15 pts) Reproduce the table in the slide “Secant Method” of “02rootfinding.pdf”. Note that you do not need to draw exactly the same table as in the slide, but only need to present your numerical results in an elegant way.