Written Part of Project 4: Car-Tracking

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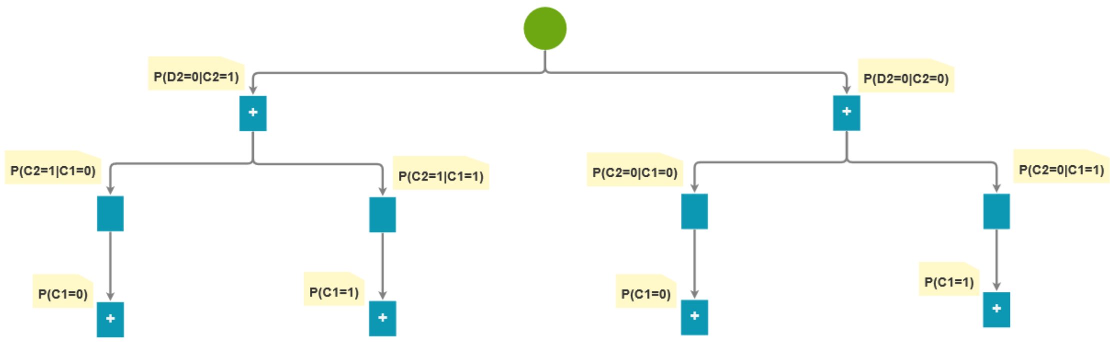
PROBLEM 1

a.

$$P(C_2|D_2=0) \propto P(C_2, D_2=0)$$
$$\propto \underbrace{P(D_2=0|C_2)}_{f_1(C_2)} \sum_{C_1} \underbrace{P(C_2|C_1)}_{f_2(C_1,C_2)} \underbrace{P(C_1)}_{f_3(C_1)}$$

Factor graph:

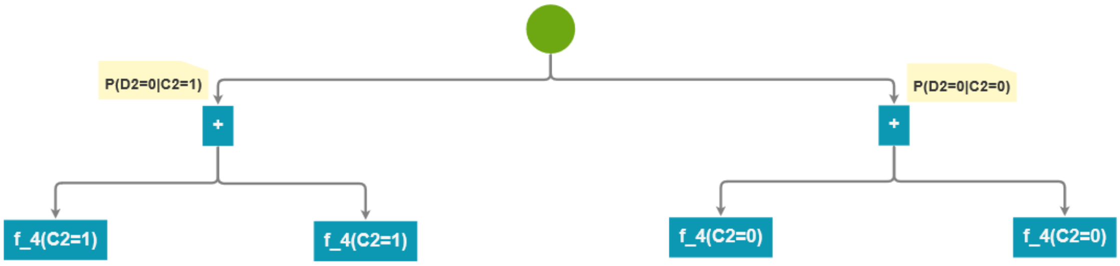


Frist eliminate C_1:

$$f_4(C_2=1) = \sum_{C_1} P(C_2=1|C_1) * P(C_1) = 0.5(\epsilon) + 0.5(1-\epsilon)$$

$$f_4(C_2=0) = \sum_{C_1} P(C_2=0|C_1) * P(C_1) = 0.5(\epsilon) + 0.5(1-\epsilon)$$

For the above two equal to each other, thus we have $P(C_2|D_2=0) \propto f_1(C_2)$.
Factor graph:



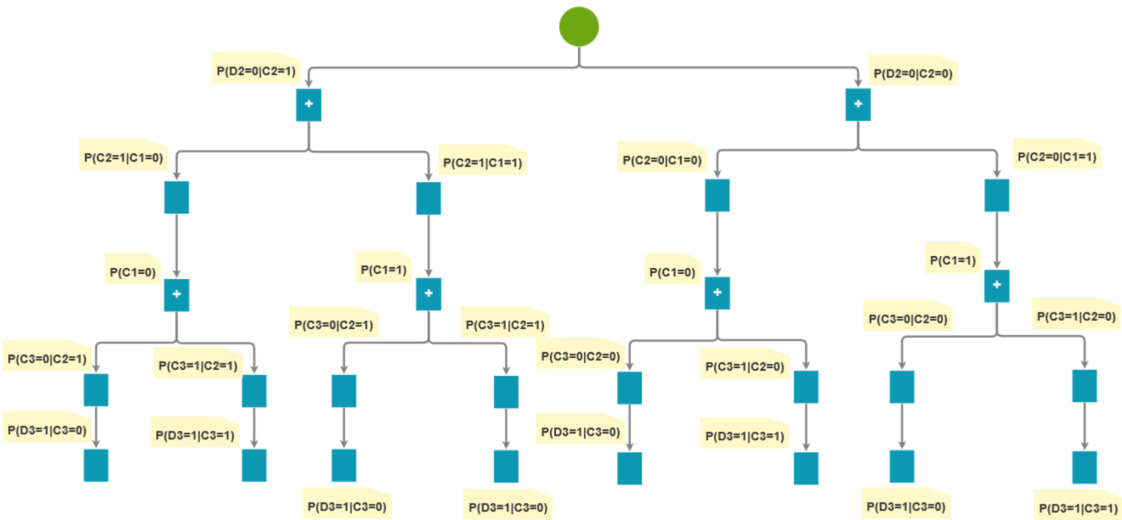
Then we do a normalization:

$$
\begin{aligned}
P(C_2=1|D_2=0) &= \frac{P(D_2=0|C_2=1)}{P(D_2=0|C_2=1)+P(D_2=0|C_2=0)} \\
&= \frac{\eta}{\eta+1-\eta} \\
&= \eta
\end{aligned}
$$

b.

$$
\begin{aligned}
P(C_2|D_2=0,D_3=1) &\propto P(C_2,D_2=0,D_3=1) \\
&\propto \underbrace{P(D_2=0|C_2)}_{f_1(C_2)}\sum_{C_1}\underbrace{P(C_2|C_1)}_{f_2(C_1,C_2)}\underbrace{P(C_1)}_{f_3(C_1)}\sum_{C_3}\underbrace{P(C_3|C_2)}_{f_4(C_2,C_3)}\underbrace{P(D_3=1|C_3)}_{f_5(C_3)}
\end{aligned}
$$

Factor graph:

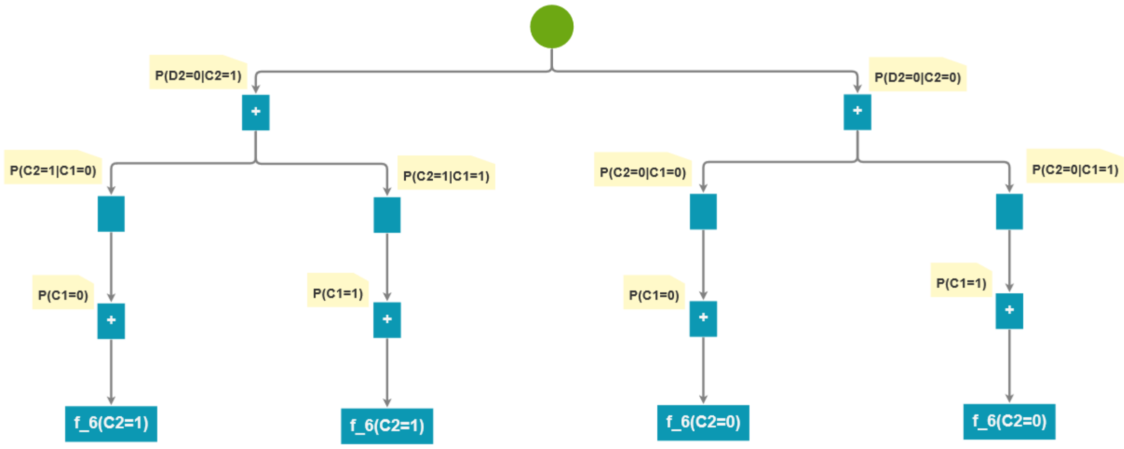


Frist eliminate C_3:

$$f_6(C_2=1)=\sum_{C_3}P(C_3|C_2=1)*P(D_3=1|C_3)=\eta\epsilon+(1-\eta)(1-\epsilon)$$

$$f_6(C_2=0)=\sum_{C_3}P(C_3|C_2=0)*P(D_3=1|C_3)=(1-\eta)\epsilon+\eta(1-\epsilon)$$

Factor graph:
For $f_6(C_2)$ dosen't involve C_1, so we let $f_6(C_2)$ stay out of the sum operation.



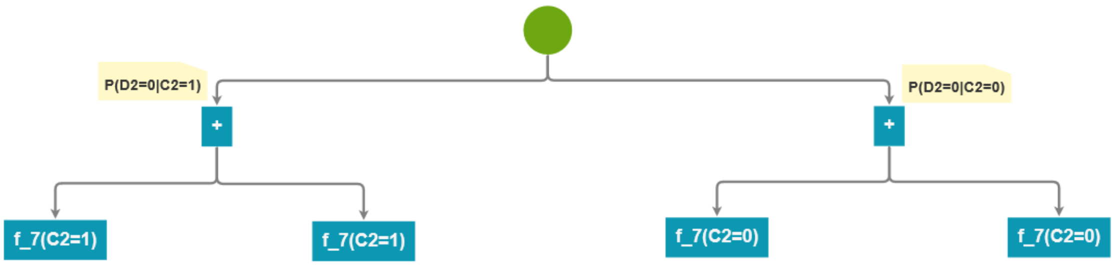
$$P(C_2|D_2=0,D_3=1) \propto f_1(C_2) f_6(C_2) \sum_{C_1} f_2(C_1, C2) f_3(C_1)$$

Then eliminate C_1:

$$f_7(C_2=1) = \sum_{C_1} P(C_2=1|C_1) * P(C_1) = 0.5(\epsilon) + 0.5(1-\epsilon)$$

$$f_7(C_2=0) = \sum_{C_1} P(C_2=0|C_1) * P(C_1) = 0.5(\epsilon) + 0.5(1-\epsilon)$$

Factor graph:



Again $f_7(C_2)$ has two elements which are equal, so we can have:

$$P(C_2|D_2=0,D_3=1) \propto f_1(C_2) f_6(C_2)$$

Then we do a normalization:

$$\begin{aligned} P(C_2=1|D_2=0,D_3=1) &= \frac{P(D_2=0|C_2=1) f_6(C_2=1)}{P(D_2=0|C_2=1) f_6(C_2=1) + P(D_2=0|C_2=0) f_6(C_2=0)} \\ &= \frac{\epsilon\eta^2 + \eta(1-\eta)(1-\epsilon)}{\epsilon\eta^2 + 2\eta(1-\eta)(1-\epsilon) + \epsilon(1-\eta)^2} \end{aligned}$$

c.

Suppose $\epsilon = 0.1$ and $\eta = 0.2$

i.

By the answers of previous questions, we can get the outcomes:

$$P(C_2 = 1|D_2 = 0) = 0.2 \quad \text{and} \quad P(C_2 = 1|D_2 = 0, D_3 = 1) \approx 0.4157$$

ii.

We know after giving the information of sensor at 3_{rd} time step $D_3 = 1$ the probability of the car being at position 1 increases. The main reason is that η is small from which we can know the car is at position 1 with relatively high probability at 3_{rd} time step if we know $D_3 = 1$. And again ϵ is also small from which we know the car is less likely to change from 0 to 1 from 2_{nd} time step to 3_{rd} time step. Thus the sensor $D_3 = 1$ increases the probability of the car being at position 1 at 2_{nd} time step.

iii.

By let

$$\frac{\epsilon\eta^2 + \eta(1-\eta)(1-\epsilon)}{\epsilon\eta^2 + 2\eta(1-\eta)(1-\epsilon) + \epsilon(1-\eta)^2} = \eta$$

we have $\epsilon = 0.5$

Explain: Once we set $\epsilon = 0.5$, it means no matter what position the car is at 2_{nd} time step, the position at 3_{rd} time step will always be 1 or 0 with same probability which is 0.5. This tells us whether $D_3 = 1$ or not has no effect on the position of the car at 2_{nd} time step although it still affects the position at 3_{rd} time step.

PROBLEM 2, 3 AND 4

The rest of part are all implemented by code.
Please check my code for the details.