Gen IMS II Homework 6

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Task 1

First, the transfer function

$$G(s) = \frac{R}{\frac{1}{Cs} + R} = \frac{RCs}{1 + RCs}$$

Plug-in the wave input jw, we have:

$$G(j\omega) = \frac{RC\omega j}{1 + RC\omega j}$$

Then we calculate the norm to find out the change in magnitude:

$$\begin{aligned} \|G(j\omega)\|^2 &= \|\frac{RC\omega j(1 - RC\omega j)}{1 + R^2C^2\omega^2}\|^2 \\ &= \frac{R^2C^2\omega^2}{1 + R^2C^2\omega^2} \\ \|G(j\omega)\| &= \frac{RC\omega}{\sqrt{1 + R^2C^2\omega^2}} \end{aligned}$$

This implies that

$$\lim_{\omega \to \infty} \|G(j\omega)\| = 1$$

$$\lim_{\omega \to 0} \|G(j\omega)\| = 0$$

We can now use Matlab to plot this function:

```
%% initial condition
R = 1e3;
C = 1e-6;
syms omega;
%% normal plot
fplot(log10(omega), 20 * log10(sqrt((R^2 * C^2 * omega^2)/(1 + R^2 * C^2 * omega^2))), [0.001, 10000
hold on
fplot(20 * log10(1))
fplot(20 * log10(0.0000001))
hold off
%% end of file
```

Figure 1: Code for ploting Task 1

```
%% initial condition
R = 1e3;
C = 1e-6;
syms omega;
%% normal plot
fplot(log10(omega), 20 * log10(1 / sqrt(1 + R^2 * C^2 * omega^2)), [0.001, 100000])
hold on
fplot(20 * log10(1))
fplot(20 * log10(0.00001))
hold off
%% end of file
```

Figure 3: Code for ploting Task 2

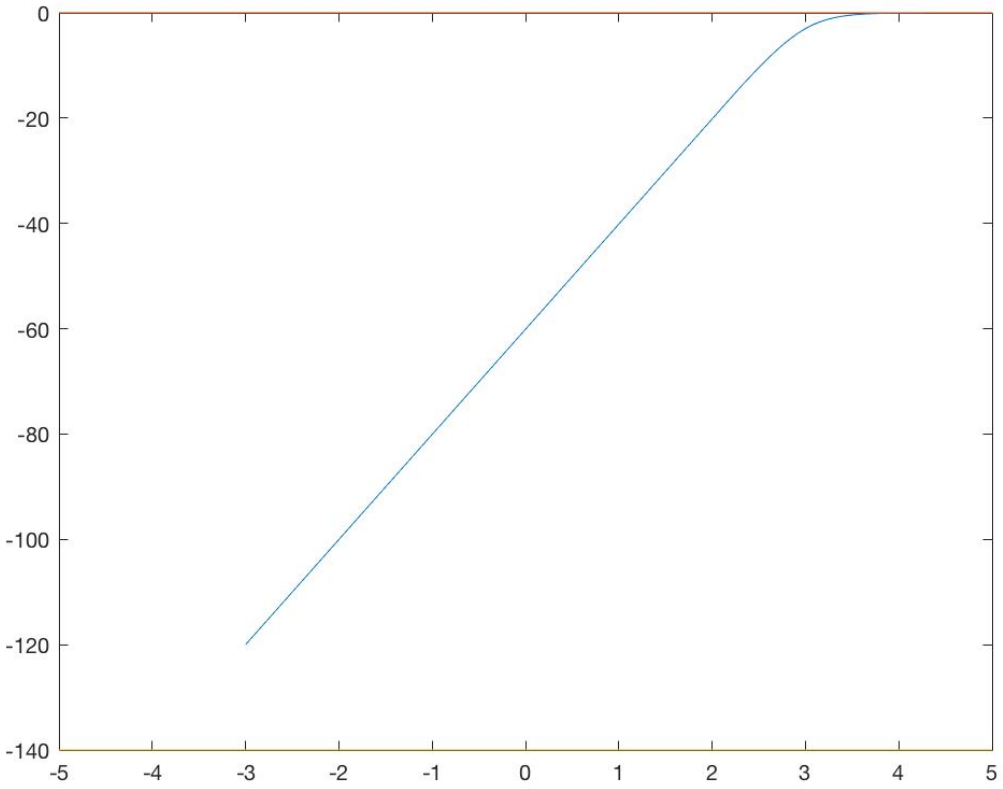

Figure 2: Plot of $\|G(jw)\|$

Task 2

First, the transfer function

$$G(s) = \frac{\frac{1}{Cs}}{R + \frac{1}{Cs}} = \frac{1}{1 + RCs}$$

Then, you have

$$\|G(j\omega)\| = \|\frac{1}{1 + RC\omega j}\| = \frac{1}{\sqrt{1 + R^2C^2w^2}}$$

Using the following Matlab code, have

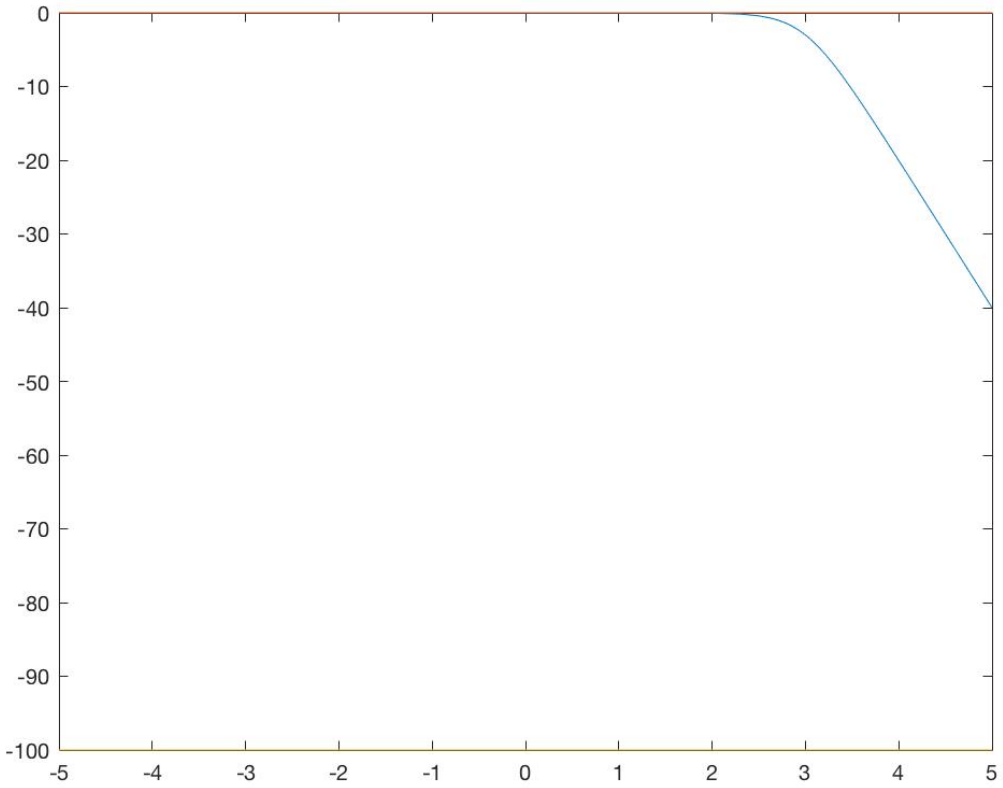

Figure 4: Plot of $\|G(jw)\|$

Task 3

From the above solution, we have

$$\|G_{band}(j\omega)\| = \|G_{high}(j\omega)\| \cdot \|G_{low}j\omega\|$$

Using the Matlab code, we have

```
%% initial condition
R = 1e3;
C = 1e-6;
syms omega;
%% normal plot
fplot(log10(omega), 20 * log10(1 / sqrt(1 + R^2 * C^2 * omega^2) * sqrt((R^2 * C^2 * omega^2
hold on
fplot(20 * log10(1))
fplot(20 * log10(0.00001))
hold off
%% end of file
```

Figure 5: Code for Task3

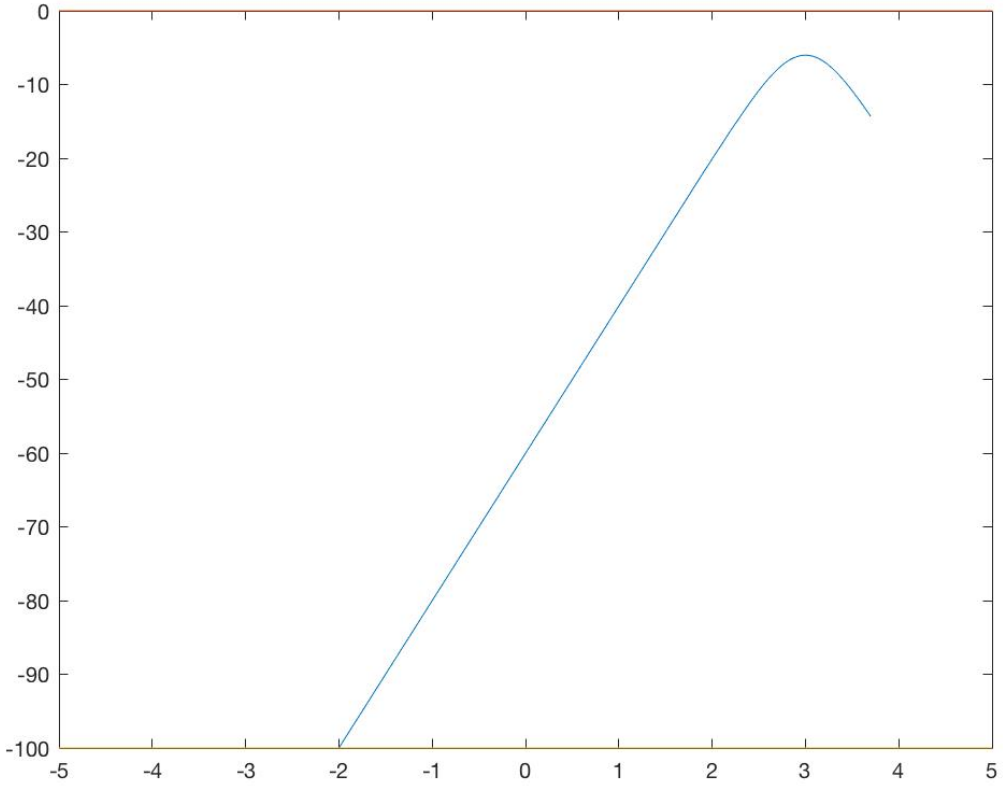

Figure 6: Plot of $\|G(jw)\|$