Gen IMS II Homework 1

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Task 1

a)

Starting from the basic formula, we have:

$$Q = CV_C \tag{1}$$
$$\frac{dQ}{dt} = i \tag{2}$$
$$V_S = V_C + V_R \tag{3}$$
$$V_R = iR \tag{4}$$

Using the formula above, we can differentiate (1) with respect to t

$$\frac{dQ}{dt} = C\frac{dV_C}{dt} = i \tag{5}$$
$$\tag{6}$$

Thus, equation (4) can be substituted using (5)

$$V_R = RC\frac{dV_C}{dt} \tag{7}$$

Plug into (3), we have first order seperable ODE

$$RC\frac{dV_C}{dt} + V_C = V_S \tag{8}$$

b)

We can solve (9) by seperation of variable:

$$RC\frac{dV_C}{V_C - V_S} = -dt \tag{9}$$

Integrate both side, we have

$$ln(V_C + V_S) = -\frac{t}{RC} + constant \tag{10}$$

Take the exponential function on both side

$$V_C - V_S = Be^{-\frac{t}{RC}} \tag{11}$$

Also, plug-in the initial condition, we have:

$$0 - V_S = B \tag{12}$$
$$B = -V_S \tag{13}$$

The solution is:

$$V_C(t) = V_S - V_S e^{-\frac{t}{RC}} \tag{14}$$
$$A = V_S \tag{15}$$
$$B = -V_S \tag{16}$$
$$\lambda = -\frac{1}{RC} \tag{17}$$
$$\tag{18}$$

c)

Using the following python code, we can generate a graph

```
import numpy as np
import matplotlib.pyplot as plt
import math

def graph(formula, points):
    x = points
    y = formula(x)
    plt.plot(x, y)
    plt.show()

graph(lambda t: 10 - 10 * np.exp(-t/(1000 * 100* 10**-6)), np.linspace(0, 0.7, 200))
```