Machine Learning 4

Yiping Deng, Shalom-David Anifowoshe

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1 K-means algorithm

K-means is a iterative algorithm that basically does the following:

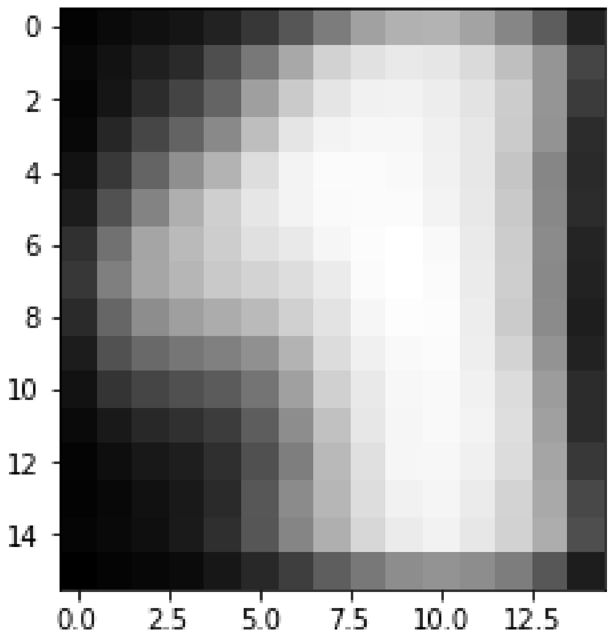
- picks k centers(usually done by picking first k data).
- groups the dataset by centers
- recomputes the centers(taking the mean value)
- repeats...

Mathematically speaking, for a given sets of clusters, we have $S = \{S_1, S_2, ...S_k\}$ with its corresponding mean value $\bar{X} = \{\bar{x_1}, \bar{x_2}, ...\bar{x_k}\}$. k-means algorithms aims to

$$\arg\min_S \sum_{i=1}^{k} \sum_{v \in S_i} norm(x, \bar{x_i})$$

1.1 $k = 1$

For $k = 1$, the algorithm calculates the mean value of the vectors. Here the number of iterations were insignificant because it was only making one cluster. The image below shows the $k = 1$ case:



With $k = 1$, we can prove that the only center will just be the mean value of the set.

Proof.

$$\arg\min_S \sum_{i=1}^{k} \sum_{v \in S_i} norm(x, \bar{x_i}) = \{\{x_1, x_2, ...x_k\}\}$$

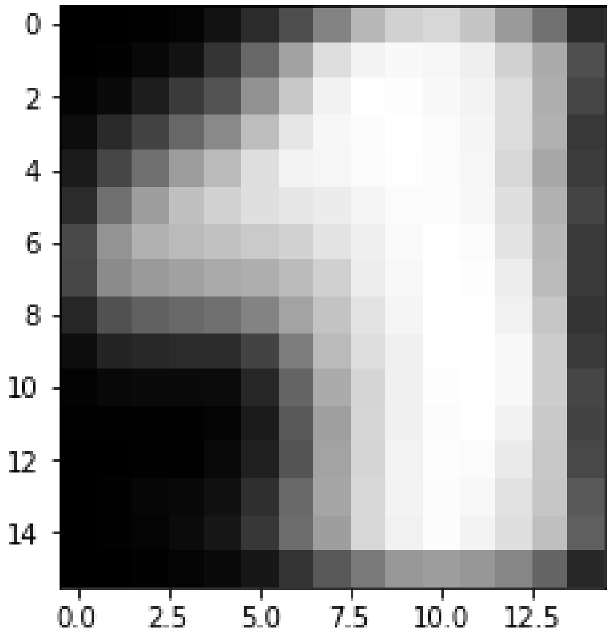
Thus, we can conclude the center:

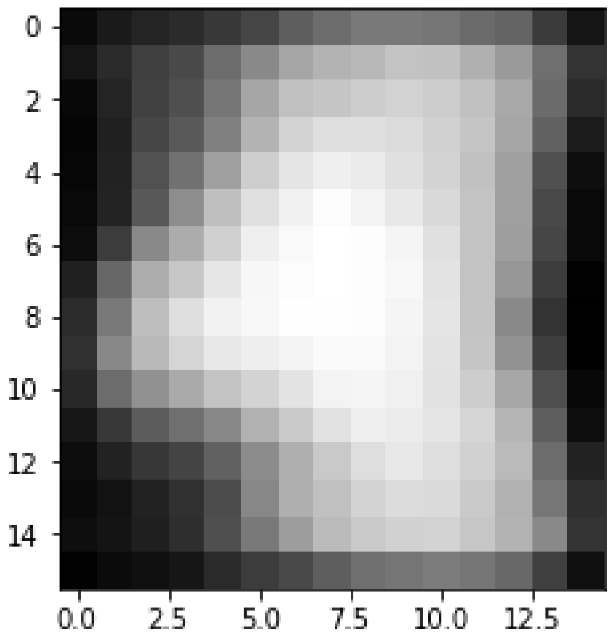
$$\bar{x_1} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

Which is exactly the mean value. □

1.2 $k = 2$

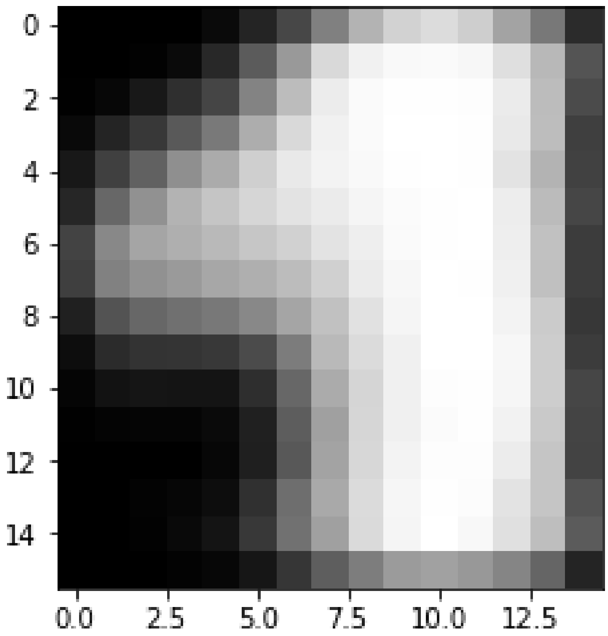
With this case, the algorithm splits the dataset into two clusters and iteratively computes there centers, here the number of iterations makes a difference because after each iteration the algorithm approaches convergence.

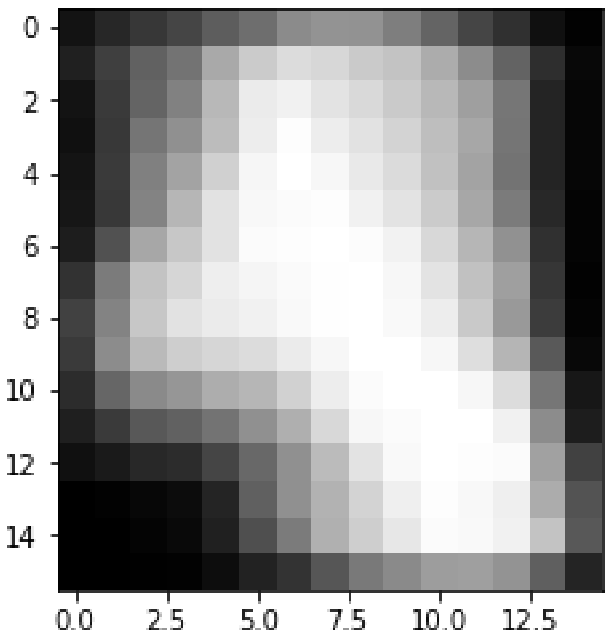


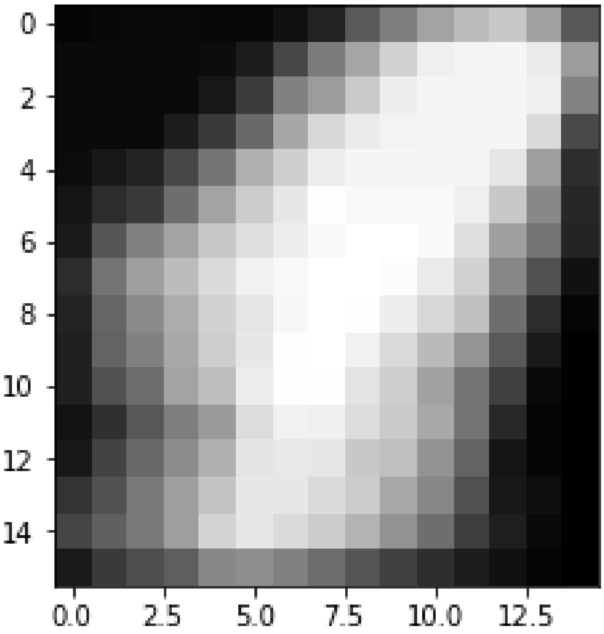


1.3 $k = 3$

Very similar logic from the $k = 2$ case applies here for the K = 3 case, but however 3 clusters are now being used. The number of iterations is positivly correlated with the likely hood of convergence.

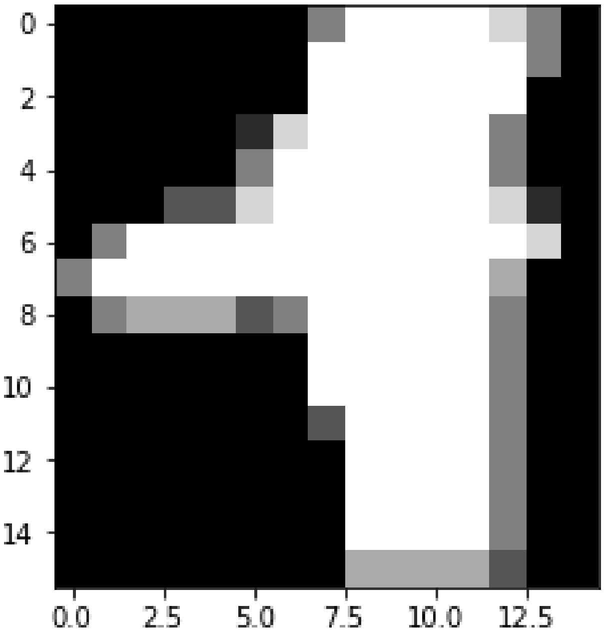


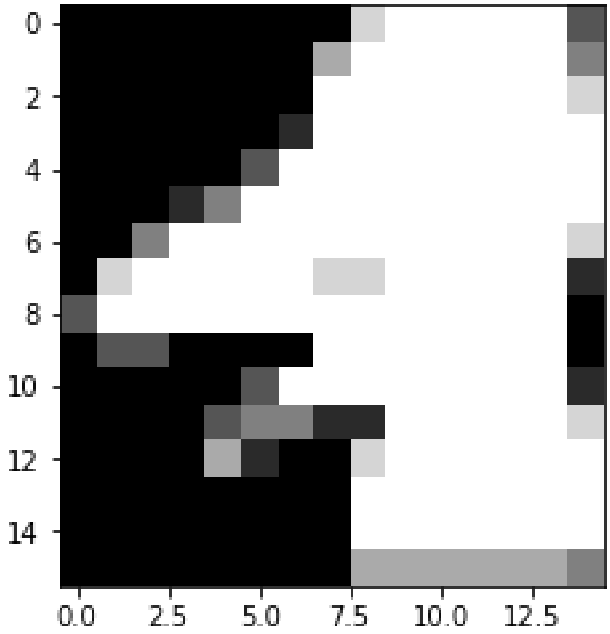


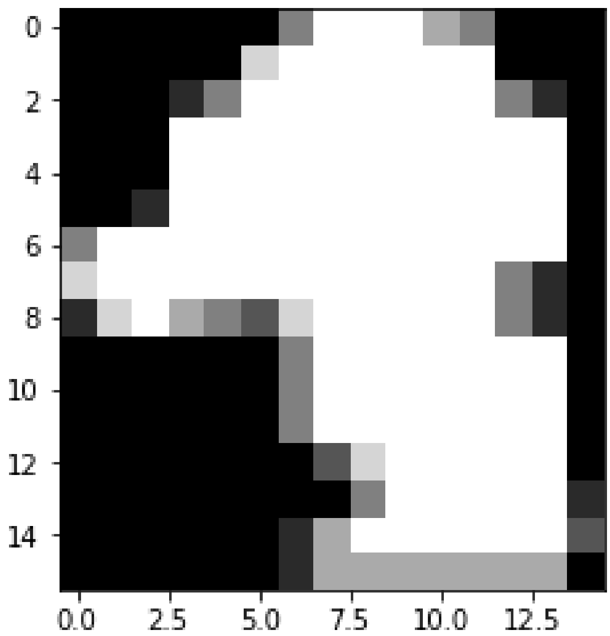


1.4 $k = 200$

For the K = 200 case, all 200 points from the data set form their own cluster. Here the distance of each point to it's center is 0 as each cluster is unique to each point. Additionally, here the number of iterations do not change the output because the algorithm converges after the first iteration.
We will simply include 3 graphs here for presentation.







for $k = 200$, clearly $\arg\min$ obtains its minimum 0 at its first iteration.

Proof.

$$\arg\min_S \sum_{i=1}^{k} \sum_{v \in S_i} norm(x, \bar{x_i}) = \{\{x_1\}, \{x_2\}, ...\{x_n\}\}$$

Thus,

$$\bar{x_i} = \frac{1}{1} \sum_{x \in S_i} x = x_i$$

Which is picked at its first place. □