Intro to Computer Science Assignment 4

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Problem 4.1

a) $a, b \in \sum^*$, and R is the relations. Clearly, $(a, a) \in R$ since a text is a prefix of its self. $(a, b) \in R.(b, a) \in R \iff a = b$ Since if a is a prefix of b, b is the prefix of a, then they are the same(self-contained). $(a, b) \in R.(b, c) \in R \implies (a, c) \in R$ Since a text's prefix's prefix must be the prefix of the text.

b) $a, b \in \sum^*$, and R is the relations. Clearly, $(a, a) \notin R$ since, by definiton, a proper prefix cannot equal to the string. $(a, b) \in R \implies b = aq, q \neq \emptyset \implies aq = b \implies a \neq bq \implies (b, a) \notin R$

c) Yes. all are comparable.

Problem 4.2

a) $f : A \mapsto B, g : B \mapsto C, g \circ f$ is bijective. Suppose $x_1, x_2 \in A$ such that $f(x_1) = f(x_2) \implies g(f(x_1)) = g(f(x_2))$
Since $g \circ f$ is bijective. $\implies x_1 = x_2 \implies$ f is injective.
Given $y \in C \implies \exists x \in A$ such that $g(f(x)) = y$
Let $z = f(x) \implies \forall y \in C \exists z \in B$ such that $g(z) = y$

b) Given $f : \{1\} \mapsto \{1, 2, 3\}, x \mapsto x, g : \{1, 2, 3\} \mapsto \{4, 5, 6\}, x \mapsto 2x$
Clearly, f is injective and g is bijective, but $g \circ f$ is not surjective

c) Given $f : \{1\} \mapsto \{1, 2, 3\}, x \mapsto x, g : \{1, 2, 3\} \mapsto \{4\}, x \mapsto 4$ Clearly f is not surjective and g is not injective, but $g \circ f : \{1\} \mapsto \{4\}$ is bijective.