Intro to Computer Science Assignment 7

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a)

$$\varphi(W, X, Y, Z) = (\neg W \wedge X \wedge Y) \vee (X \wedge Y \wedge Z) \vee (W \wedge \neg X) \vee (W \wedge Y)$$

The truth table:

minterm	W	X	Y	Z	$\varphi(W, X, Y, Z)$
-	0	0	0	0	0
-	0	0	0	1	0
-	0	0	1	0	0
-	0	0	1	1	0
-	0	1	0	0	0
m_5	0	1	0	1	1
-	0	1	1	0	0
m_7	0	1	1	1	1
m_8	1	0	0	0	1
m_9	1	0	0	1	1
m_{10}	1	0	1	0	1
m_{11}	1	0	1	1	1
-	1	1	0	0	0
-	1	1	0	1	0
m_{14}	1	1	1	0	1
m_{15}	1	1	1	1	1

$$\begin{aligned}\varphi(W, X, Y, Z) = {} & (\neg W \wedge X \wedge \neg Y \wedge Z) \vee \\ & (\neg W \wedge X \wedge Y \wedge Z) \vee \\ & (W \wedge \neg X \wedge \neg Y \wedge \neg Z) \vee \\ & (W \wedge \neg X \wedge \neg Y \wedge Z) \vee \\ & (W \wedge \neg X \wedge Y \wedge \neg Z) \vee \\ & (W \wedge \neg X \wedge Y \wedge Z) \vee \\ & (W \wedge X \wedge Y \wedge \neg Z) \vee \\ & (W \wedge X \wedge Y \wedge Z)\end{aligned}$$

b) original table

minterm	W	X	Y	Z	$\varphi(W,X,Y,Z)$
m_5	0	1	0	1	1
m_7	0	1	1	1	1
m_8	1	0	0	0	1
m_9	1	0	0	1	1
m_{10}	1	0	1	0	1
m_{11}	1	0	1	1	1
m_{14}	1	1	1	0	1
m_{15}	1	1	1	1	1

sort this table by the number of true variables

minterm	W	X	Y	Z	$\varphi(W,X,Y,Z)$
m_8	1	0	0	0	1
m_5	0	1	0	1	1
m_9	1	0	0	1	1
m_{10}	1	0	1	0	1
m_7	0	1	1	1	1
m_{11}	1	0	1	1	1
m_{14}	1	1	1	0	1
m_{15}	1	1	1	1	1

combine

minterm	W	X	Y	Z	$\varphi(W,X,Y,Z)$
$m_{8,9}$	1	0	0	-	1
$m_{8,10}$	1	0	-	0	1
$m_{5,7}$	0	1	-	1	1
$m_{7,15}$	-	1	1	1	1
$m_{9,11}$	1	0	-	1	1
$m_{10,11}$	1	0	1	-	1
$m_{11,15}$	1	-	1	1	1
$m_{14,15}$	1	1	1	-	1

even further

minterm	W	X	Y	Z	$\varphi(W,X,Y,Z)$
$m_{7,15}$	-	1	1	1	1
$m_{5,7}$	0	1	-	1	1
$m_{8,9,10,11}$	1	0	-	-	1
$m_{10,11,14,15}$	1	-	1	-	1

c) chart

prime	m_8	m_5	m_9	m_{10}	m_7	m_{11}	m_{14}	m_{15}
$m_{7,15}$	-	-	-	-	*	-	-	*
$m_{5,7}$	-	*	-	-	*	-	-	-
$m_{8,9,10,11}$	*	-	*	*	-	*	-	-
$m_{10,11,14,15}$	-	-	-	*	-	*	*	*

clearly, only $m_{5,7}$, $m_{8,9,10,11}$, $m_{10,11,14,15}$ is essential.
we write down $(\neg W \wedge X \wedge Z) \vee (W \wedge \neg X) \vee (W \wedge Y)$