Intro to Computer Science Assignment 2

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Problem 2.1
Proof by its cotrapositive:
Lemma: If n is not divisible by 3, then n^2 is not divisible by 3.
Proof :
3 is a prime number, n is not divisible by 3 $\implies$ 3 is not a factor of n $\implies$ 3 is not a factor of $n \cdot n = n^2$ $\implies$ n is not divisible by 3.

Problem 2.2 Lemma: $0^3 + 1^3 + 2^3 + ... + n^3 = \sum_{i=0}^{n} i^3 = [\frac{n(n+1)}{2}]^2$
Proof by induction:
Basis:

$$n = 0 \tag{1}$$

$$\implies \sum_{i=0}^{n} i^3 = 0^3 = 3 \tag{2}$$

$$[\frac{n(n+1)}{2}]^2 = [\frac{0(0+1)}{2}]^2 = 0 \tag{3}$$

$$\implies \sum_{i=0}^{n} i^3 = [\frac{n(n+1)}{2}]^2, n = 0 \tag{4}$$

Inductive Proof: Assume lemma holds for $n = k \implies$

$$\sum_{i=0}^{k} i^3 = [\frac{k(k+1)}{2}]^2 \implies \sum_{i=0}^{k+1} i^3 = \sum_{i=0}^{k} i^3 + (k+1)^3 = [\frac{k(k+1)}{2}]^2 + (k+1)^3 \tag{5}$$

$$= \frac{k^2(k+1)^2}{4} + (k+1)^3 \tag{6}$$

$$= \frac{k^2(k+1)^2}{4} + \frac{(4k+4)(k+1)^2}{4} \tag{7}$$

$$= \frac{(k^2+4k+4)(k+1)^2}{4} \tag{8}$$

$$= \frac{(k+2)^2(k+1)^2}{4} \tag{9}$$

$$= [\frac{(k+1)(k+2)}{2}]^2 \tag{10}$$