# Intro to Computer Science Assignment 3

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**Problem 3.1** Lemma: The number of elements in the power set $P(s)$ of a finite set S with n elements is $2^n$
Proof:

**Basis:** Given a empty set $A = \emptyset$, its power set $P(A) = \{\emptyset\}$

**Inductive proof:** Assume Lemma holds for set $A$ with k element $\implies$ the size of its power set $P(A)$ is $2^k \implies$ for set with a extra element $m$, $B = \{m\} \cup A$ has $k+1$ elments $\implies$ in its power set $P(B)$, it includes all the element in $P(A)$ and with a a set with its element set unioned with element $m$ $\implies P(B) = P(A) \cup \{C \cup \{m\} \mid C \in P(A)\} \implies$ the size of $P(B)$ is twice the size of $P(A) \implies$ the size of $P(B)$ is equal to $2^{k+1}$

**Problem 3.2**

**a)**

- It is not reflexive: $a = b \implies \neg a \neq b$
- It is symmetric: $a = b \implies b = a$
- It is transitive: $a = b \wedge b = c \implies a = c$

**b)**

- It is reflexive: $\mid a - a \mid = 0 \leq 3 \implies \forall a \in A.(a, a) \in R$
- It is symmetric: $\mid a - b \mid \leq 3 \iff \mid b - a \mid \leq 3$
- It is not transitive: $\mid a - b \mid \leq 3 \wedge \mid b - c \mid \leq 3 \not\Longrightarrow \mid a - c \mid \leq 3$ . A example: $a = 3, b = 6, c = 9 \implies \mid a - b \mid \leq 3, \mid b - c \mid \leq 3, \mid a - c \mid \geq 3$

**c)**

- It is reflexive: $\forall a \in \mathbb{Z}.(a\, mod\, 10) = (b\, mod\, 10)$
- It is symmetric: $\forall a, b \in \mathbb{Z}.(a\, mod\, 10) = (b\, mod\, 10)$
  $\implies (b\, mod\, 10) = (a\, mod\, 10) \implies \forall (a, b) \in R, (b, a) \in R$

**Problem 3.3** in problem3.hs file

**Problem 3.4** in problem4.hs file (in the comment)