Course Name: Homework #1

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Problem 1

Give an appropriate positive constant c such that $f(n) \leq c \cdot g(n)$ for all $n > 1$.[1]

1. $f(n) = n^2 + n + 1$, $g(n) = 2n^3$
2. $f(n) = n\sqrt{n} + n^2$, $g(n) = n^2$
3. $f(n) = n^2 - n + 1$, $g(n) = n^2/2$

Solution

We solve each solution algebraically to determine a possible constant c .

Part One

$$\begin{aligned} n^2 + n + 1 = \\ &\leq n^2 + n^2 + n^2 \\ &= 3n^2 \\ &\leq c \cdot 2n^3 \end{aligned}$$

Thus a valid c could be when $c = 2$.

Part Two

$$\begin{aligned} n^2 + n\sqrt{n} = \\ &= n^2 + n^{3/2} \\ &\leq n^2 + n^{4/2} \\ &= n^2 + n^2 \\ &= 2n^2 \\ &\leq c \cdot n^2 \end{aligned}$$

Thus a valid c is $c = 2$.

Part Three

$$\begin{aligned} n^2 - n + 1 = \\ &\leq n^2 \\ &\leq c \cdot n^2/2 \end{aligned}$$

Thus a valid c is $c = 2$.

Problem 2

Let $\Sigma = \{0, 1\}$. Construct a DFA A that recognizes the language that consists of all binary numbers that can be divided by 5.

Let the state q_k indicate the remainder of k divided by 5. For example, the remainder of 2 would correlate to state q_2 because $7 \mod 5 = 2$.

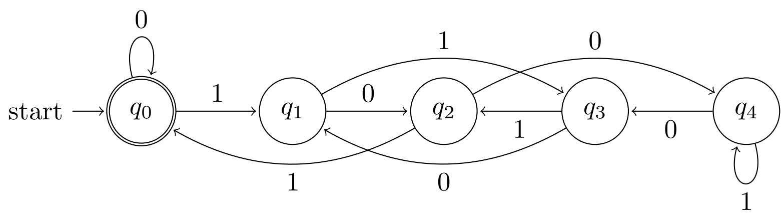


Figure 1: DFA, A, this is really beautiful, ya know?

Justification

Take a given binary number, x. Since there are only two inputs to our state machine, x can either become $x0$ or $x1$. When a 0 comes into the state machine, it is the same as taking the binary number and multiplying it by two. When a 1 comes into the machine, it is the same as multiplying by two and adding one.
Using this knowledge, we can construct a transition table that tell us where to go:

	$x \mod 5 = 0$	$x \mod 5 = 1$	$x \mod 5 = 2$	$x \mod 5 = 3$	$x \mod 5 = 4$
$x0$	0	2	4	1	3
$x1$	1	3	0	2	4

Therefore on state q_0 or ($x \mod 5 = 0$), a transition line should go to state q_0 for the input 0 and a line should go to state q_1 for input 1. Continuing this gives us the Figure 1.

Problem 3

Write part of **Quick-Sort**($list, start, end$)

```
function Quick-Sort(list, start, end)
    if start ≥ end then
        return
    end if
    mid ← Partition(list, start, end)
    Quick-Sort(list, start, mid − 1)
    Quick-Sort(list, mid + 1, end)
end function
```

Algorithm 1: Start of QuickSort

Problem 4

question

solution

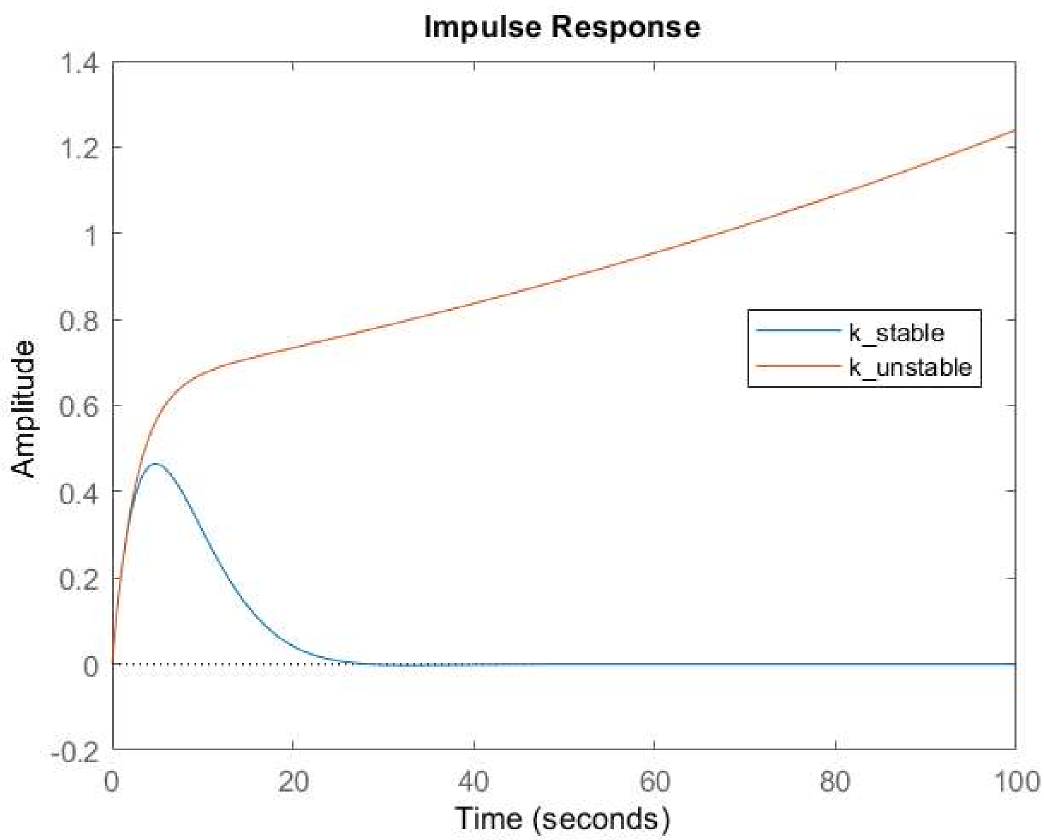
partA

Homework 3

Question 1

```
clc,clear;

k_stable = 10; % N * m^-1
k_unstable = 9.8;
g = 9.81;
m = 1;    % kg
c = 1.5; % N * s * m^-1
l = 2; % m
l1 = 1; % m
t = 0:0.1:100; % 10 seconds of time
figure;
hold on
for k = [k_stable k_unstable]
    G = tf(1, [m * l^2 c * l1^2 (k*l1^2 - m*g)]);
    impulse(G, t)
end
legend("k_{stable}", "k_{unstable}", 'Location', "best")
```



Question 2

References

[1] Van Brummelen J, O'Brien M, Gruyer D, et al. Autonomous vehicle perception: The technology of today and tomorrow[J]. Transportation research part C: emerging technologies, 2018, 89: 384-406.