24352: Homework #4

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August 30, 2021

Problem 1

State space representation from differential equations.

a)

$2\ddot{x} + 4\dot{x} + 4x = 3u$, where the state vector is $\begin{bmatrix} x \\ \dot{x} \end{bmatrix}$. The out put is x.

Sol.

$\ddot{x} + 2\dot{x} + 2x = 1.5u \Rightarrow \begin{cases} \dot{x} = 0x + \dot{x} \\ \ddot{x} = -2x - 2\dot{x} + 1.5u \end{cases}$, thus

$$\dot{\mathbf{x}} = \begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -2 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ 1.5 \end{bmatrix} u \tag{1}$$

$$y = \begin{bmatrix} x \end{bmatrix} = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} u \tag{2}$$

We get:

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -2 & -2 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0 \\ 1.5 \end{bmatrix}, \mathbf{C} = \begin{bmatrix} 1 & 0 \end{bmatrix}, \mathbf{D} = \begin{bmatrix} 0 \end{bmatrix} \tag{3}$$

b)

$2\ddot{x} + 4\dot{x} + 4x = 3u$, where the state vector is $\begin{bmatrix} x + \dot{x} \\ \dot{x} \end{bmatrix}$. The out put is x.

Sol.

$\begin{cases} \dot{x} + \ddot{x} = \dot{x} + (-2x - 2\dot{x} + 1.5u) = -2(x + \dot{x}) + \dot{x} + 1.5u \\ \ddot{x} = -2x - 2\dot{x} + 1.5u = -2(x + \dot{x}) + 0\dot{x} + 1.5u \\ x = (x + \dot{x}) + (-1)\dot{x} \end{cases}$, thus

$$\dot{\mathbf{x}} = \begin{bmatrix} \dot{x} + \ddot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} x + \dot{x} \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 1.5 \\ 1.5 \end{bmatrix} u \tag{4}$$

$$y = \begin{bmatrix} x \end{bmatrix} = \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} x + \dot{x} \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} u \tag{5}$$

We get:

$$\mathbf{A} = \begin{bmatrix} -2 & 1 \\ -2 & 0 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 1.5 \\ 1.5 \end{bmatrix}, \mathbf{C} = \begin{bmatrix} 1 & -1 \end{bmatrix}, \mathbf{D} = \begin{bmatrix} 0 \end{bmatrix} \tag{6}$$

c)

Using the A,B,C,D matrices found in parts (a) and (b), use Matlab's step command to simulate the step responses to both systems. How do these step responses compare and does that comparison make sense?

Sol.

Matlab code:

```
t = 0:0.01:10;

A = [0, 1; -2 -2];
B = [0; 1.5];
C = [1 0];
D = [0];
sys1 = ss(A,B,C,D);

A = [-2, 1; -2 0];
B = [1.5; 1.5];
C = [1 -1];
D = [0];
sys2 = ss(A,B,C,D);

[y1,t] = step(sys1,t);
[y2,t] = step(sys2,t);
plot(t,y1,'r',t,y2,'b--','linewidth',2)
grid minor
title('Response to step')
xlabel('Time (s)')
ylabel('Amplitude')
legend('y1', 'y2')
```

Result:
We can tell from Fig.1 that the step responses are exactly the same. This make sense because we are representing the same system, and the output is the same for the two representations.

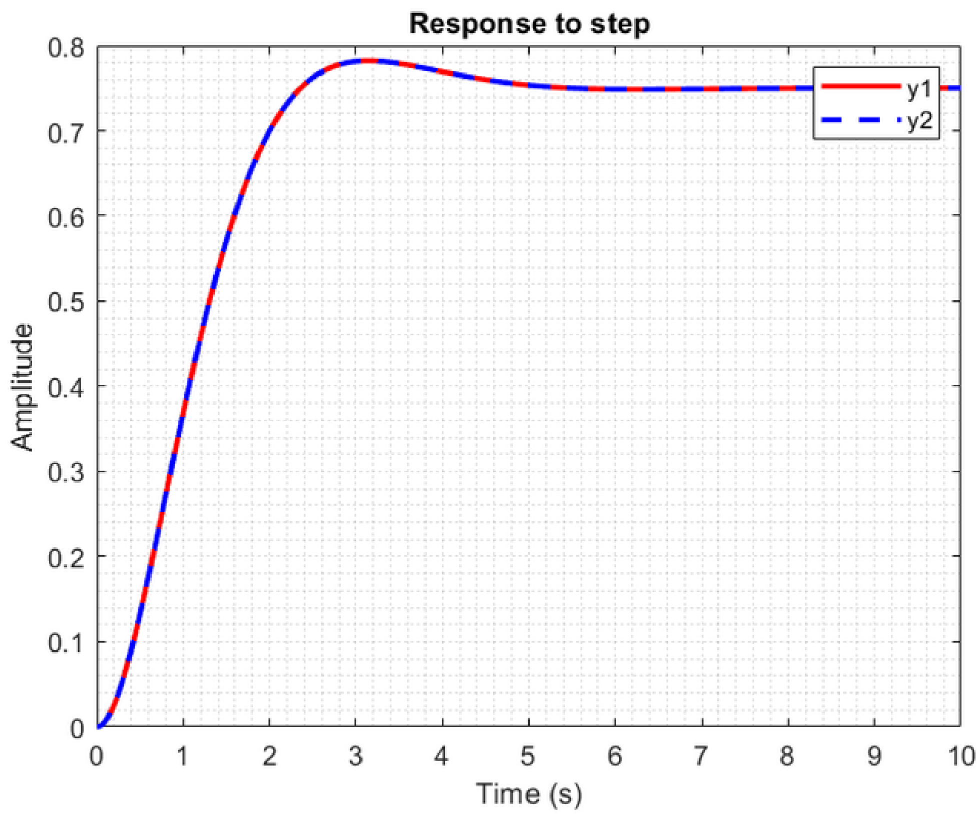
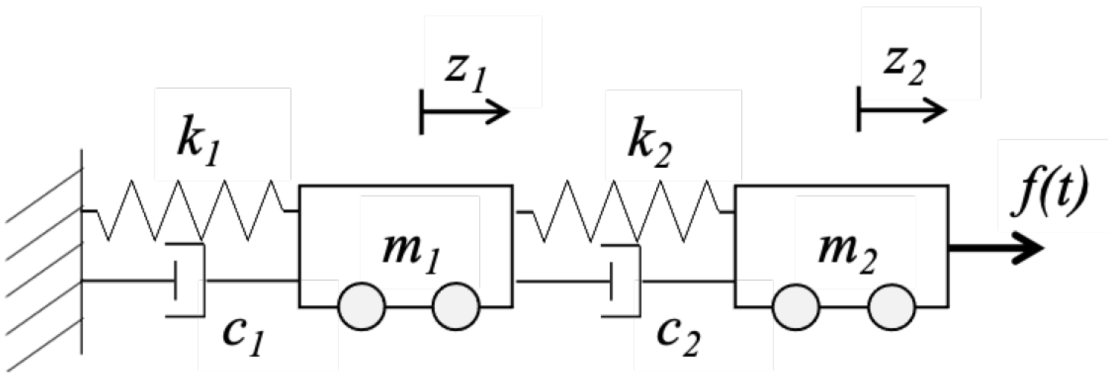


Figure 1: Step Response

Problem 2

State space representation of a 2-mass system.
For the following mechanical system, there is one input, f, and two outputs: the position of the first mass, z_1, and the distance between the masses, $z_1 - z_2$.



a)

Write the state equation, $\dot{x} = Ax + Bu$, where x is a state vector that you choose, A is the system matrix, B is the input matrix, and u is the input.

Sol.

First we identify all energy storage elements:

Energy Storage Elements	State variable
m_1 (Kinetic)	$\dot{z_1}$
m_2 (Kinetic)	$\dot{z_2}$
k_1 (Potential)	z_1
k_2 (Potential)	$z_2 - z_1$

From all energy storage elements we define the state vector: $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} z_1 \\ z_2 - z_1 \\ \dot{z_1} \\ \dot{z_2} \end{bmatrix}$, and input $u = f$

Draw FBD to analyze system dynamics:

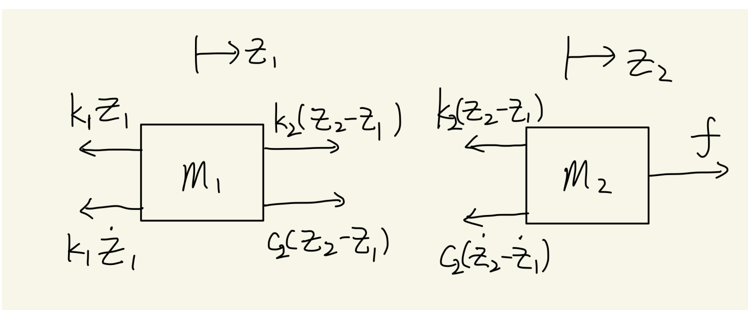


Figure 2: Free body diagram

We get the following dynamic equations:

$$\begin{cases} m_1\ddot{z_1} = k_2(z_2 - z_1) + c_2(\dot{z_2} - \dot{z_1}) - k_1 z_1 - c_1\dot{z_1} \\ m_2\ddot{z_2} = f - k_2(z_2 - z_1) - c_2(\dot{z_2} - \dot{z_1}) \end{cases}$$

$$\Rightarrow \begin{cases} \dot{x_1} = \dot{z_1} \\ \dot{x_2} = \dot{z_2} - \dot{z_1} \\ \dot{x_3} = \ddot{z_1} = \frac{k_2}{m_1}(z_2 - z_1) + \frac{c_2}{m_1}(\dot{z_2} - \dot{z_1}) - \frac{k_1}{m_1}z_1 - \frac{c_1}{m_1}\dot{z_1} \\ \dot{x_4} = \ddot{z_2} = \frac{f}{m_2} - \frac{k_2}{m_2}(z_2 - z_1) - \frac{c_2}{m_2}(\dot{z_2} - \dot{z_1}) \end{cases}$$

$$\Rightarrow \begin{cases} \dot{x_1} = 0x_1 + 0x_2 + x_3 + 0x_4 + 0u \\ \dot{x_2} = 0x_1 + 0x_2 + (-1)x_3 + x_4 + 0u \\ \dot{x_3} = -\frac{k_1}{m_1}x_1 + \frac{k_2}{m_1}x_2 - \frac{c_1+c_2}{m_1}x_3 + \frac{c_2}{m_1}x_4 + 0u \\ \dot{x_4} = 0x_1 - \frac{k_2}{m_2}x_2 + \frac{c_2}{m_2}x_3 - \frac{c_2}{m_2}x_4 + \frac{1}{m_2}u \end{cases}$$

We can write the state space equation as:

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ -\frac{k_1}{m_1} & \frac{k_2}{m_1} & -\frac{c_1+c_2}{m_1} & \frac{c_2}{m_1} \\ 0 & -\frac{k_2}{m_2} & \frac{c_2}{m_2} & -\frac{c_2}{m_2} \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{m_2} \end{bmatrix} u \tag{7}$$

b)

Write the output equation, $y = Cx + Du$.

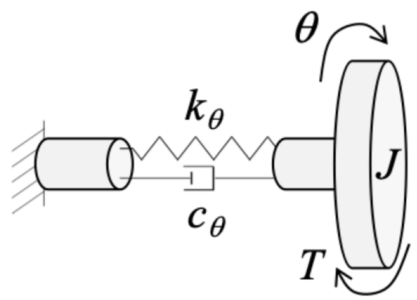
Sol.

The output vector: $\mathbf{y} = \begin{bmatrix} z_1 \\ z_1 - z_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ -x_2 \end{bmatrix}$, thus:

$$\mathbf{y} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u \tag{8}$$

Problem 3

Changing the system matrix with state feedback.
The rotary mass-spring-damper system shown below has an input torque $u = T$ and output position θ with a rotational spring k_θ and rotational damper c_θ



a)

Find the state space representation of this system.

Sol.

Define the state vector: $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix}$, input $u = T$, and output $y = \theta$.
We can write the equation of motion as: $J\ddot{\theta} = -k_\theta\theta - c_\theta\dot{\theta} + T \Rightarrow \ddot{\theta} = -\frac{k_\theta}{J}\theta - \frac{c_\theta}{J}\dot{\theta} + \frac{1}{J}T$

$$\begin{aligned} \dot{\mathbf{x}} &= \begin{bmatrix} 0 & 1 \\ -\frac{k_\theta}{J} & -\frac{c_\theta}{J} \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ \frac{1}{J} \end{bmatrix} u \\ y &= \begin{bmatrix} 1 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \end{bmatrix} u \end{aligned} \tag{9}$$

b)

Now assume that your input torque T is a function of the system state so that $T = R - \mathbf{kx}$ where $\mathbf{k}$ is a vector $\begin{bmatrix} k_1 & k_2 \end{bmatrix}$ and $\mathbf{x}$ is the state vector. Find the new state equation assuming a new input, $u = R$.

Sol.

Substitute u in Eq.9 with $\left(R - \begin{bmatrix} k_1 & k_2 \end{bmatrix} \mathbf{x}\right)$ we get:

$$
\begin{aligned}
\dot{\mathbf{x}} &= \begin{bmatrix} 0 & 1 \\ -\frac{k_\theta}{J} & -\frac{c_\theta}{J} \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ \frac{1}{J} \end{bmatrix} \left(R - \begin{bmatrix} k_1 & k_2 \end{bmatrix} \mathbf{x}\right) \\
&= \left(\begin{bmatrix} 0 & 1 \\ -\frac{k_\theta}{J} & -\frac{c_\theta}{J} \end{bmatrix} - \begin{bmatrix} 0 \\ \frac{1}{J} \end{bmatrix} \begin{bmatrix} k_1 & k_2 \end{bmatrix} \right) \mathbf{x} + \begin{bmatrix} 0 \\ \frac{1}{J} \end{bmatrix} R \\
&= \begin{bmatrix} 0 & 1 \\ -\frac{k_\theta + k_1}{J} & -\frac{c_\theta + k_2}{J} \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ \frac{1}{J} \end{bmatrix} R \\
&= \begin{bmatrix} 0 & 1 \\ -\frac{k_\theta + k_1}{J} & -\frac{c_\theta + k_2}{J} \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ \frac{1}{J} \end{bmatrix} u \\
y &= \begin{bmatrix} 1 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \end{bmatrix} \left(R - \begin{bmatrix} k_1 & k_2 \end{bmatrix} \mathbf{x}\right) \\
&= \begin{bmatrix} 1 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \end{bmatrix} u
\end{aligned} \tag{10}
$$

c)

Assuming $J = 1\ kg \cdot m^2$, $c_\theta = 2\ N \cdot m \cdot s/rad$, $k_\theta = 5\ N \cdot m/rad$, simulate the response to a step input of $0.1\ N \cdot m$ for the original system and a step of $0.64\ N \cdot m$ for the new system with $k = \begin{bmatrix} 27 & 6 \end{bmatrix}$.

Sol.

Matlab code:

```matlab
% original system
t = 0:0.01:10;
J = 1;
c = 2;
k_theta = 5;
K = 0.1; % step input amplitude

A = [0, 1; -(k_theta/J) -(c/J)];
B = [0; 1/J];
C = [1 0];
D = [0];
sys1 = ss(A,B,C,D) * K;

% new system
k_1 = 27;
k_2 = 6;
K = 0.64;
A = [0, 1; -(k_theta+k_1)/J -(c+k_2)/J];
B = [0; 1/J];
C = [1 0];
D = [0];
sys2 = ss(A,B,C,D) * K;

% plot step responce
[y1,t] = step(sys1,t);
[y2,t] = step(sys2,t);
plot(t,y1,'r',t,y2,'b--','linewidth',2)
```

```
grid minor
title('Response to step')
xlabel('Time (s)')
ylabel('Amplitude')
legend('y1', 'y2')
```

Result:

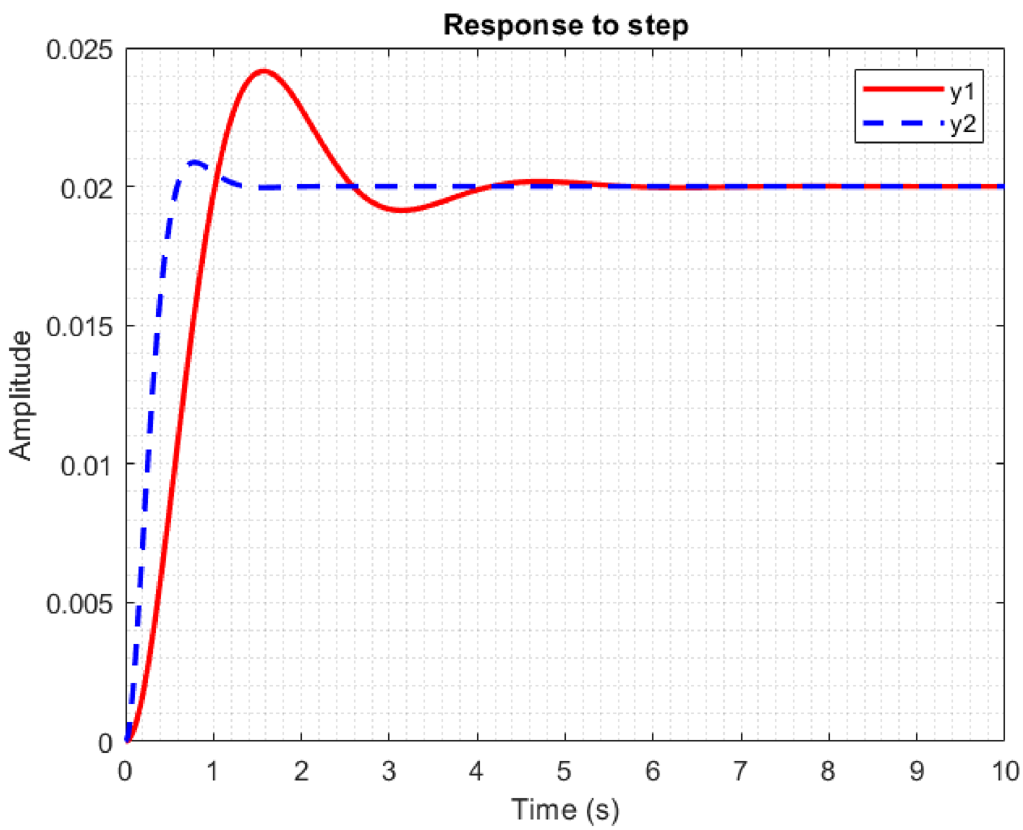
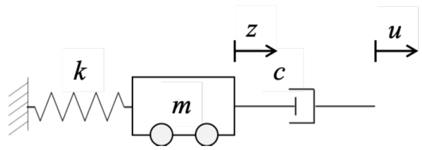


Figure 3: Step Response

Problem 4

State space representation with input derivatives.
For the following mechanical system, input is the displacement u and output is the displacement z.



a)

Find the state space representation of this system.

Sol.

Draw FBD to analyze system dynamics:

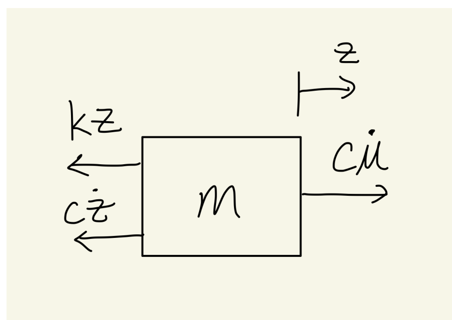


Figure 4: Free body diagram

We get the following dynamic equation: $m\ddot{z} = -kz - c\dot{z} + c\dot{u} \Rightarrow \ddot{z} = -\frac{k}{m}z - \frac{c}{m}\dot{z} + \frac{c}{m}\dot{u}$
Because we have derivative of input, we can define the state vector as: $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} z \\ \dot{z} - \frac{c}{m}u \end{bmatrix}$, and output $y = z$.
From $\ddot{z} = -\frac{k}{m}z - \frac{c}{m}\dot{z} + \frac{c}{m}\dot{u}$

$$\Rightarrow \begin{cases} \dot{x_1} = \dot{z} = \left(\dot{z} - \frac{c}{m}u\right) + \frac{c}{m}u = x_2 + \frac{c}{m}u \\ \dot{x_2} = \ddot{z} - \frac{c}{m}\dot{u} = -\frac{k}{m}z - \frac{c}{m}\dot{z} + \frac{c}{m}\dot{u} - \frac{c}{m}\dot{u} = -\frac{k}{m}z - \frac{c}{m}(x_2 + \frac{c}{m}u) = -\frac{k}{m}x_1 - \frac{c}{m}x_2 - \left(\frac{c}{m}\right)^2 u \end{cases}$$

We get:

$$\begin{aligned} \dot{\mathbf{x}} &= \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{c}{m} \end{bmatrix} \mathbf{x} + \begin{bmatrix} \frac{c}{m} \\ -\left(\frac{c}{m}\right)^2 \end{bmatrix} u \\ y &= \begin{bmatrix} 1 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \end{bmatrix} u \end{aligned} \tag{11}$$

b)

Assume that $m = 10kg$, $c = 20N - s/m$, $k = 40N/m$, and the input displacement is a step displacement of $0.2m$. Use Matlab to plot the response $z(t)$. Include your code and plot. Is there anything unusual about this step response? Why is this the case for this system?
Matlab code:

```
t = 0:0.01:10;
m = 10;
c = 20;
k = 40;
K = 0.2; % step input amplitude

A = [0, 1; -(k/m) -(c/m)];
B = [c/m; -(c/m)^2];
C = [1 0];
D = [0];
sys = ss(A,B,C,D) * K;
step(sys,t)
```

Result:

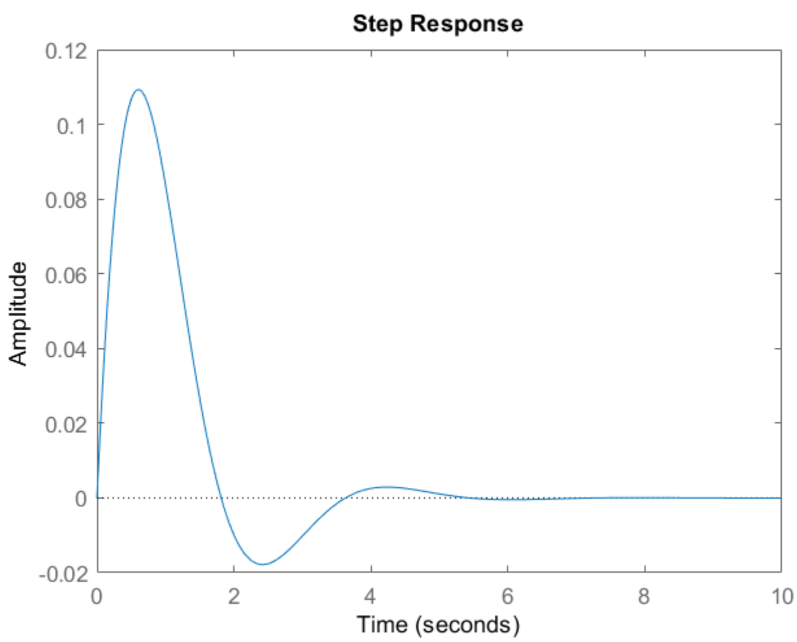


Figure 5: Step Response

The step response is as expected. The final value goes to zero, because the first derivative of the input u only changes at time 0_+, thus energy is added to the system only at time 0_+. And because this is a damped system, total energy in the system will eventually dissipate to zero.

Problem 5

Transfer function to state space.
For each of the transfer functions shown, write the corresponding differential equation and provide a state space representation.

a)

$$\frac{\Theta_1(s)}{T_1(s)} = \frac{100}{s^4 + 20s^3 + 10s^2 + 7s + 100} \tag{12}$$

Sol.

Convert the given transfer function to standard format: $\frac{\Theta_1(s)}{T_1(s)} = \frac{b_1 s^3 + b_2 s^2 + b_3 s + b_4}{s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4}$,
where $b_1 = b_2 = b_3 = 0$, $b_4 = 100$, $a_1 = 20$, $a_2 = 10$, $a_3 = 7$, $a_4 = 100$.
We have:

$$\begin{aligned}
\dot{\mathbf{x}} &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -a_4 & -a_3 & -a_2 & -a_1 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u \\
&= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -100 & -7 & -10 & -20 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u \\
y &= \begin{bmatrix} b_4 & b_3 & b_2 & b_1 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \end{bmatrix} u \\
&= \begin{bmatrix} 100 & 0 & 0 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \end{bmatrix} u
\end{aligned} \tag{13}$$

Matlab code:

```matlab
clc,clear
t = 0:0.01:10;
%% a
NUM = 100;
DEN = [1 20 10 7 100];
sys_a_tf = tf(NUM, DEN);
[A_, B_, C_, D_] = tf2ss(NUM, DEN);
A = [0 1 0 0;
     0 0 1 0;
     0 0 0 1;
     -100 -7 -10 -20];
B = [0;0;0;1];
C = [100 0 0 0];
D = 0;
sys_a_ss = ss(A,B,C,D);
sys_a_tf2ss = ss(A_,B_,C_,D_);

[y1,t] = step(sys_a_tf,t);
[y2,t] = step(sys_a_ss,t);
[y3,t] = step(sys_a_tf2ss,t);
plot(t,y1,'r',t,y2,'b:',t,y3,'g--','linewidth',2)
grid minor
```

```
title('Response to step')
xlabel('Time (s)')
ylabel('Amplitude')
legend('transfer function', 'state space','tf2ss')
```

Result:

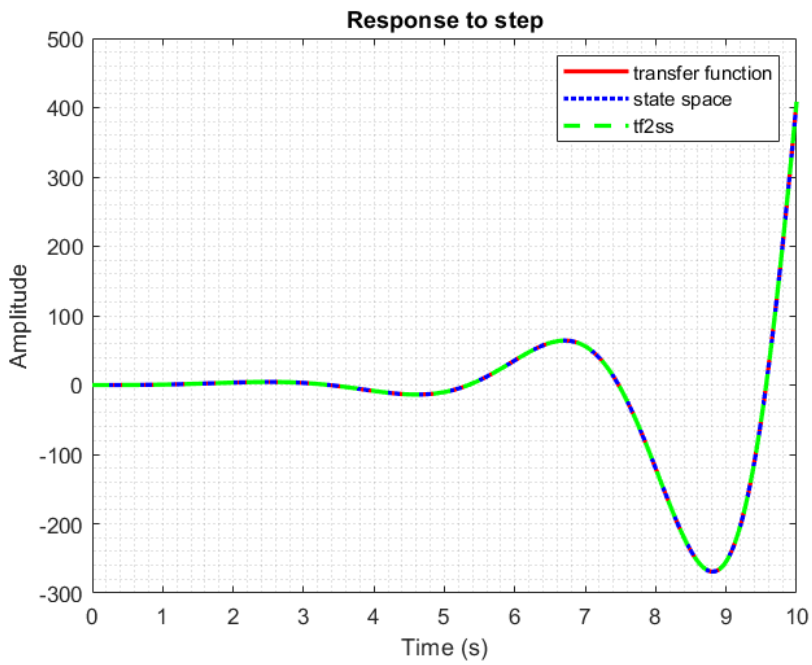


Figure 6: Step Response

b)

$$\frac{Y_1(s)}{F_1(s)} = \frac{8s+10}{s^3+s^2+5s+13} \tag{14}$$

Sol.

Convert the given transfer function to standard format: $\frac{Y_1(s)}{F_1(s)} = \frac{b_1 s^2 + b_2 s + b_3}{s^3 + a_1 s^2 + a_2 s + a_3}$, where $b_1 = 0$, $b_2 = 8$, $b_3 = 10$, $a_1 = 1$, $a_2 = 5$, $a_3 = 13$.
We have:

$$\begin{aligned}
\dot{\mathbf{x}} &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_3 & -a_2 & -a_1 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \\
&= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -13 & -5 & -1 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \\
y &= \begin{bmatrix} b_3 & b_2 & b_1 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \end{bmatrix} u \\
&= \begin{bmatrix} 10 & 8 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \end{bmatrix} u
\end{aligned} \tag{15}$$

Matlab code:

```
clc,clear
t = 0:0.01:10;
%% b
NUM = [8 10];
DEN = [1 1 5 13];
sys_b_tf = tf(NUM, DEN);
[A_,B_, C_, D_] = tf2ss(NUM, DEN);
A = [0 1 0;
     0 0 1;
    -13 -5 -1];
B = [0;0;1];
C = [10 8 0];
D = 0;
sys_b_ss = ss(A,B,C,D);
sys_b_tf2ss = ss(A_,B_,C_,D_);

[y1,t] = step(sys_b_tf,t);
[y2,t] = step(sys_b_ss,t);
[y3,t] = step(sys_b_tf2ss,t);
plot(t,y1,'r',t,y2,'b:',t,y3,'g--','linewidth',2)
grid minor
title('Response to step')
xlabel('Time (s)')
ylabel('Amplitude')
legend('transfer function', 'state space','tf2ss','location','best')
```

Result:

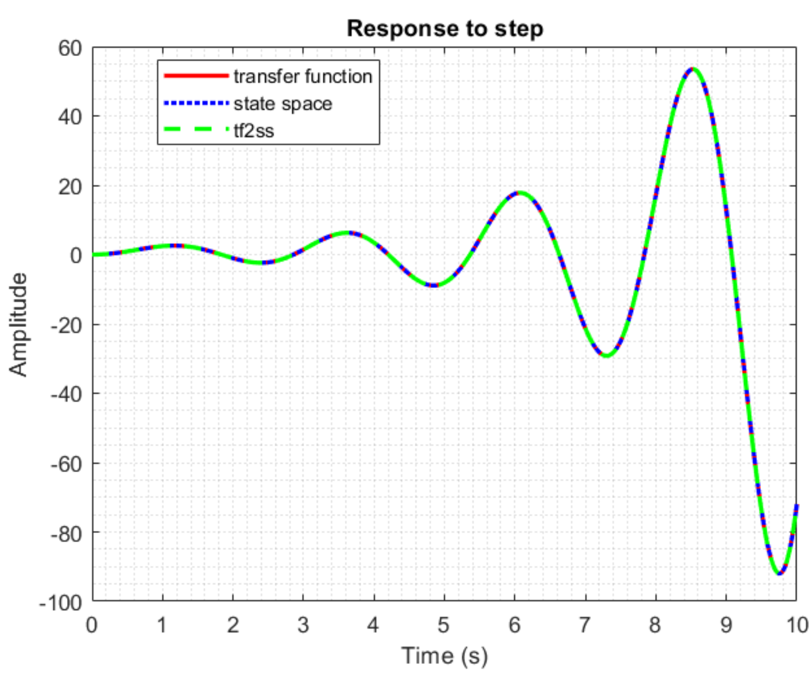


Figure 7: Step Response

Problem 6

State space to transfer function.
In general, state space representations are not unique as discussed in lecture. You are given the following systems.

$$\begin{bmatrix}\dot{x}\\ \ddot{x}\end{bmatrix} = \begin{bmatrix}-5 & 0\\ 0 & -1\end{bmatrix}\begin{bmatrix}x\\ \dot{x}\end{bmatrix} + \begin{bmatrix}3\\ 1\end{bmatrix}u \tag{16}$$

$$y = \begin{bmatrix}7 & 0\end{bmatrix}\begin{bmatrix}x\\ \dot{x}\end{bmatrix} \tag{17}$$

$$\begin{bmatrix}\dot{x}\\ \ddot{x}\end{bmatrix} = \begin{bmatrix}-5 & 0\\ 0 & -1\end{bmatrix}\begin{bmatrix}x\\ \dot{x}\end{bmatrix} + \begin{bmatrix}3\\ 0\end{bmatrix}u \tag{18}$$

$$y = \begin{bmatrix}7 & 3\end{bmatrix}\begin{bmatrix}x\\ \dot{x}\end{bmatrix} \tag{19}$$

a)

Analytically (by hand!) show that these systems will result in the same transfer function $\frac{Y(s)}{U(s)}$.

Sol.

Start by doing Laplace transform on state equation in Eq.16 and Eq.17:

$$\mathcal{L}\{\dot{\mathbf{x}}\} = sX(s) = AX(s) + BU(s) \tag{20}$$

$$\mathcal{L}\{y\} = Y(s) = CX(s) + DU(s) \tag{21}$$

From Eq.20 we get:

$$X(s) = (sI - A)^{-1}BU(s) \tag{22}$$

Replace $X(s)$ in Eq.21 with Eq.22

$$Y(s) = (C(sI - A)^{-1}B + D)U(s) \tag{23}$$

So we have:

$$\begin{aligned}\frac{Y(s)}{U(s)} &= C(sI - A)^{-1}B + D\\ &= \begin{bmatrix}7 & 0\end{bmatrix}(\begin{bmatrix}s & 0\\ 0 & s\end{bmatrix} - \begin{bmatrix}-5 & 0\\ 0 & -1\end{bmatrix})^{-1}\begin{bmatrix}3\\ 1\end{bmatrix} + 0\\ &= \begin{bmatrix}7 & 0\end{bmatrix}\left(\frac{1}{(s+1)(s+5)}\begin{bmatrix}s+1 & 0\\ 0 & s+5\end{bmatrix}\right)\begin{bmatrix}3\\ 1\end{bmatrix}\\ &= \begin{bmatrix}7 & 0\end{bmatrix}\begin{bmatrix}\frac{3}{(s+5)}\\ \frac{1}{(s+1)}\end{bmatrix}\\ &= \frac{21}{s+5}\end{aligned} \tag{24}$$

Now we do the same to Eq.18 and Eq.19:

$$\begin{aligned}\frac{Y(s)}{U(s)} &= C(sI-A)^{-1}B+D \\ &= \begin{bmatrix}7 & 3\end{bmatrix}(\begin{bmatrix}s & 0\\0 & s\end{bmatrix}-\begin{bmatrix}-5 & 0\\0 & -1\end{bmatrix})^{-1}\begin{bmatrix}3\\0\end{bmatrix}+0 \\ &= \begin{bmatrix}7 & 3\end{bmatrix}\left(\frac{1}{(s+1)(s+5)}\begin{bmatrix}s+1 & 0\\0 & s+5\end{bmatrix}\right)\begin{bmatrix}3\\0\end{bmatrix} \\ &= \begin{bmatrix}7 & 3\end{bmatrix}\begin{bmatrix}\frac{3}{(s+5)}\\0\end{bmatrix} \\ &= \frac{21}{s+5}\end{aligned} \tag{25}$$

We get the same transfer function $\frac{Y(s)}{U(s)} = \frac{21}{s+5}$.

b)

Matlab code:

```
clc,clear
t = 0:0.01:10;
% system 1
A = [-5 0;
     0 -1];
B = [3;1];
C = [7 0];
D = 0;
sys_ss_1 = ss(A,B,C,D);
[NUM, DEN] = ss2tf(A,B,C,D);
sys_ss2tf_1 = tf(NUM, DEN);

% system 2
A = [-5 0;
     0 -1];
B = [3;0];
C = [7 3];
D = 0;
sys_ss_2 = ss(A,B,C,D);
[NUM, DEN] = ss2tf(A,B,C,D);
sys_ss2tf_2 = tf(NUM, DEN);

% hand tf
NUM = 21;
DEN = [1 5];
sys_tf = tf(NUM, DEN);

[y1,t] = step(sys_ss_1,t);
[y2,t] = step(sys_ss_2,t);
[y3,t] = step(sys_ss2tf_1,t);
```

```
[y4,t] = step(sys_ss2tf_2,t);
[y5,t] = step(sys_tf,t);
plot(t,y1,'r',t,y2,'r:',t,y3,'g',t,y4,'g:',t,y5,'b','linewidth',2)
grid minor
title('Response to step')
xlabel('Time (s)')
ylabel('Amplitude')
legend('ss(1)', 'ss(2)','ss2tf(1)','ss2tf(2)','hand tf','location','best')
```

Result:

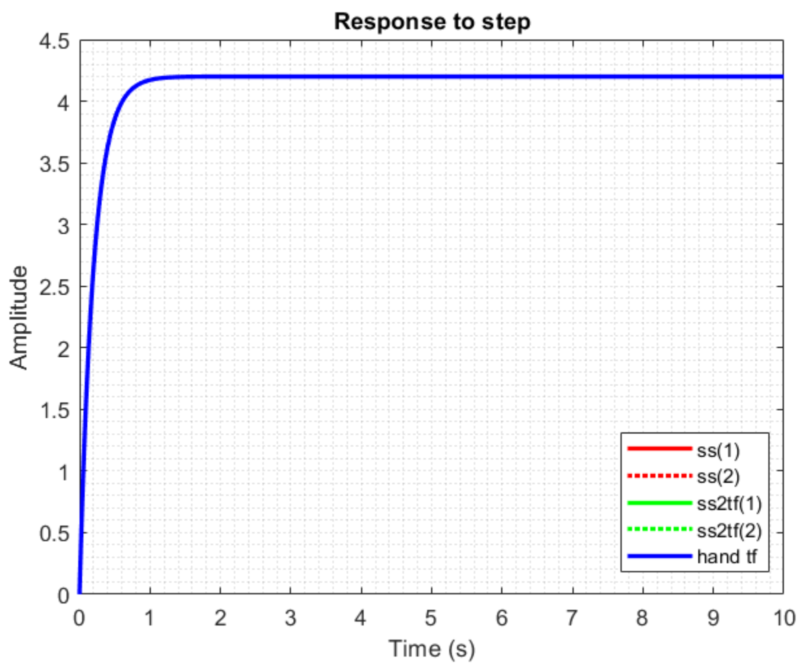


Figure 8: Step Response