[HW1]

1. Naive Definition of Probability.

1.

(a). " > "

Fix the 1^{st} dice result, we know that for "21", it shall be greater or equal to 3 $(3+18=21)$; for "22", it shall be greater or equal to 4 $(4+18=22)$.

$\Rightarrow$ When we tune "3" to "4", we can have more possibilities for the remaining three dices in "21" case.

(b) " = "

Pr. that a random 2 letters word is a palindrom (A):

$$P(A) = (26 \times 1) / 26^2 = \frac{1}{26}$$

Pr. ... 3 letters ... (B):

$$P(B) = (26 + 26 \times 25 \times 1) / 26^3$$

$$= 1/26$$

2.

(a). $Pr(\text{flush}) = \dfrac{\binom{4}{1}\left[\binom{13}{5}-1\right]}{\binom{52}{5}}$

(b). $Pr(\text{Two pair}) = \dfrac{\binom{13}{2}\binom{4}{2}^2\binom{11}{1}\binom{4}{1}}{\binom{52}{5}}$

3.

(a). # move up = 111

\# move right = 110.

$$\Rightarrow \binom{111+110}{111} = \binom{221}{111}$$ ✓

(b). From (0, 0) to (110, 111):

\# move up = 111

\# move right = 110.

From (110, 111) to (210, 211):

\# move up = 211 − 111 = 100

\# move right = 210 − 110 = 100

$$\Rightarrow \binom{111+110}{111}\binom{100+100}{100} = \binom{221}{111}\binom{200}{100}$$ ✓

4.

The probability is that

$$\frac{26!}{26 + 26\times 25 + \cdots + 26\times\cdots\times 1}$$

$$= \frac{1}{1+1+\frac{1}{2!}+\frac{1}{3!}+\cdots+\frac{1}{25!}}$$

By Taylor expansion on e at 0,

$$e = 1+1+\frac{1}{2!}+\frac{1}{3!}+\cdots$$ ✓

Therefore, the Pr. is approximatly 1/e.

II. Story Proofs

5. $\sum_{k=0}^{n} \binom{n}{k} = 2^n$.

I can't image a good story. However,

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}.$$

Let $a=b=1$. Then,

$$2^n = \sum_{k=0}^{n} \binom{n}{k}. \quad \text{Q.E.D.}$$

6. $$\frac{(2n)!}{2^n \cdot n!} = (2n-1)(2n-3)\cdots 3 \cdot 1.$$

It is a partnership story. Check the book.

7. $$\binom{n}{k} + \binom{n}{k-1} = \binom{n+1}{k},$$

for all positive integers n, k, with $n \geq k$.

(a).

$$\binom{n}{k} + \binom{n}{k-1} = \frac{n!}{k!(n-k)!} + \frac{n!}{(k-1)!(n-k+1)!}$$

$$= \frac{(n-k+1)n! + kn!}{k!(n-k+1)!}$$

$$= \frac{(n+1)!}{k!(n+1-k)!}$$

$$= \binom{n+1}{k}$$

(b). There are $n+1$ people, one of them is pre-designated as "president", others are citizens.

To count # outcomes such that we select k people from these n+1 people,

- Right side: {straightforward}.
- Left side: we divide it into 2 situations:
  1. one is any k are from n citizens;
  2. another is k-1 are from n citizens and plus the only choice of "president".