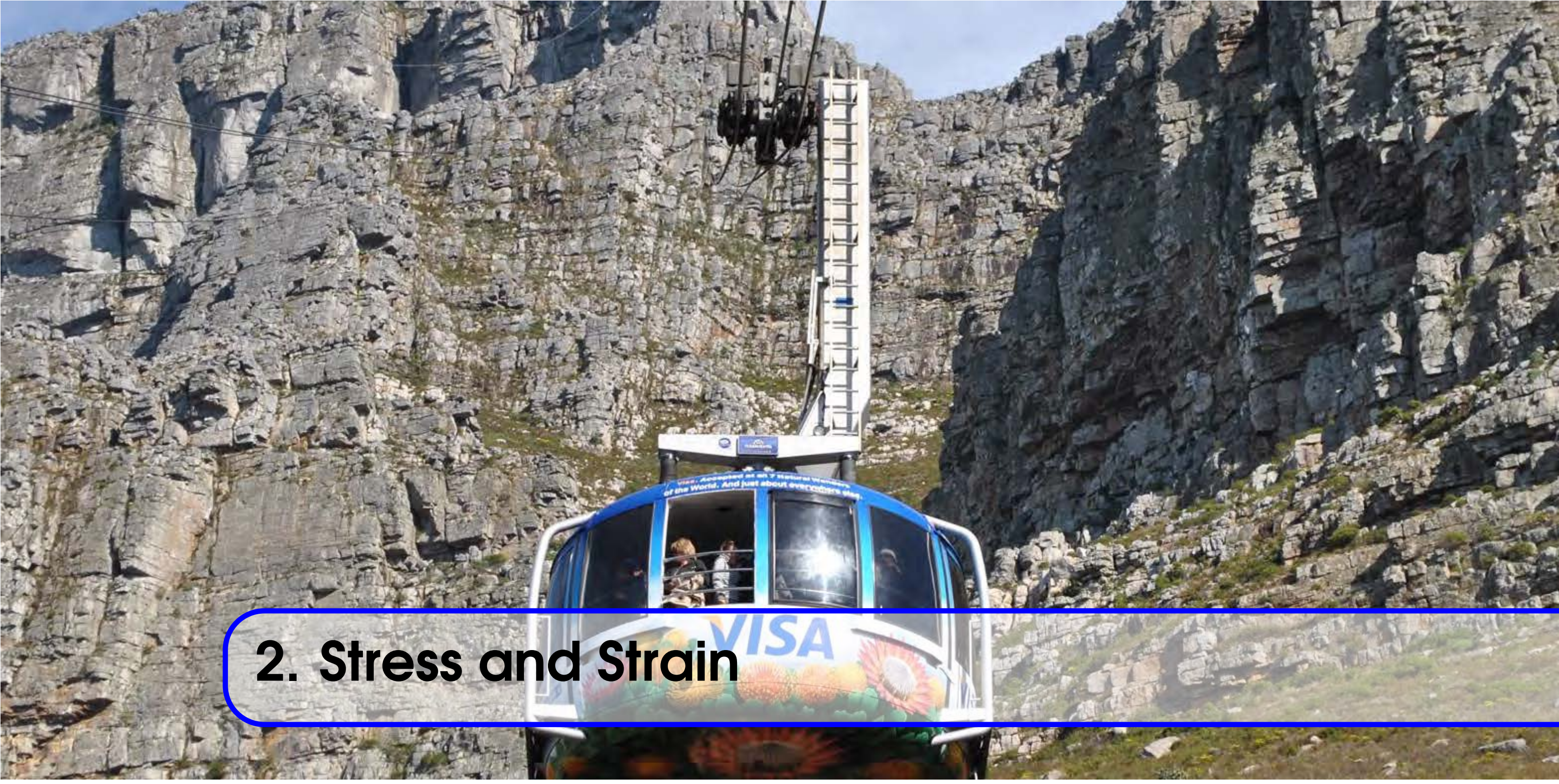

2. Stress and Strain

View of the cable car from lower cable station on Table Mountain. An understanding of stresses and strains in the design of the cables and other structural members is essential in their design.

2.1 Introduction

In this course we make certain assumptions about the materials that we analyse, they are **isotropic** and **homogeneous**. Isotropic materials have the same properties in all directions. These properties can be mechanical like strength or physical like thermal conductivity. Therefore if a mechanical load is applied in any direction the response of an isotropic material to that load will be the same. A material that is not isotropic is anisotropic like a fiber reinforced composite. This composite is stronger parallel to the fibres than perpendicular to the fibres. Homogeneous materials have a uniform composition with no second phase[1]. A material that is not homogeneous is heterogeneous like concrete which is mixture of sand, stone, gravel with a binder cement. A well mixed concrete is also isotropic. An example of a homogeneous material which is anisotropic is rolled steel which is stronger in the direction of rolling.

2.2 Normal Stress

One of the most important concepts in mechanics is that of stress, it is represented by the Greek letter σ (sigma). It is the force per unit area or the intensity of the internal force at a point.

[1] A phase should not be confused with a state of matter which can be solid, liquid or gas. The same state can have different phases such as oil mixed with water which is in the liquid state but has two phases, the oil and the water. A single phase however can only be a single state.

Consider a rectangular bar with uniform cross section in figure 2.1(a) which is subject to an axial force F. This is a **tension force** which elongates the bar. The **normal stress** is the stress experienced by the cross section A and is defined as follows:

$$\textbf{Normal Stress} = \frac{\text{Force normal to an area}}{\text{Area over which the force acts}}$$

Let's consider a small area ΔA and a very small force ΔF which is the resultant internal force acting normal to this area in figure 2.1(b). As ΔA tends to zero this corresponding force ΔF also approaches zero. The stress on this cross section is defined as:

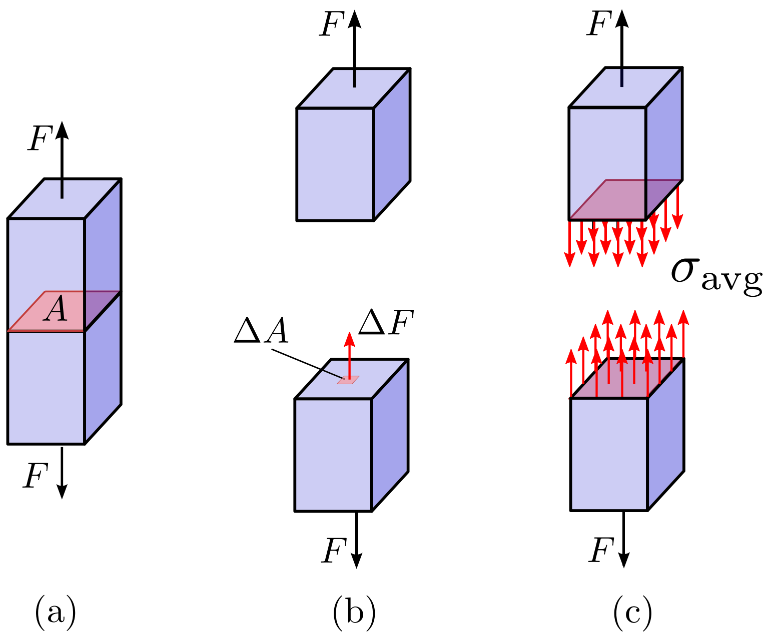

Figure 2.1: Illustration of axial forces and stresses

$$\boxed{\sigma = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A}} \tag{2.1}$$

The average force intensity over the whole cross section as shown in figure 2.1(c) is:

$$\boxed{\sigma_{avg} = \frac{F}{A}} \tag{2.2}$$

Here F is the load in Newtons (N) and A is the area in square meters (m^2). The unit of stress is the Pascal (Pa). The Pascal is a very small quantity. Usually stress in engineering applications is expressed in megapascals (MPa) where 1 MPa = 1 000 000 Pa. A useful method of calculating stress is to express force in Newtons and area in square millimetres to get:

$$\boxed{1\,\text{MPa} = \frac{1\,\text{N}}{1\,\text{mm}^2}} \tag{2.3}$$

The following sign convention is followed when a tensile force acts on a body as seen in figure 1.1 the resulting **tensile normal stress** is positive and when a compressive force acts on a body **compressive normal stress** is negative.

■ **Example 2.1** Determine the average normal stress in each of the 20 mm diameter bars of the truss shown in figure 2.2(a).

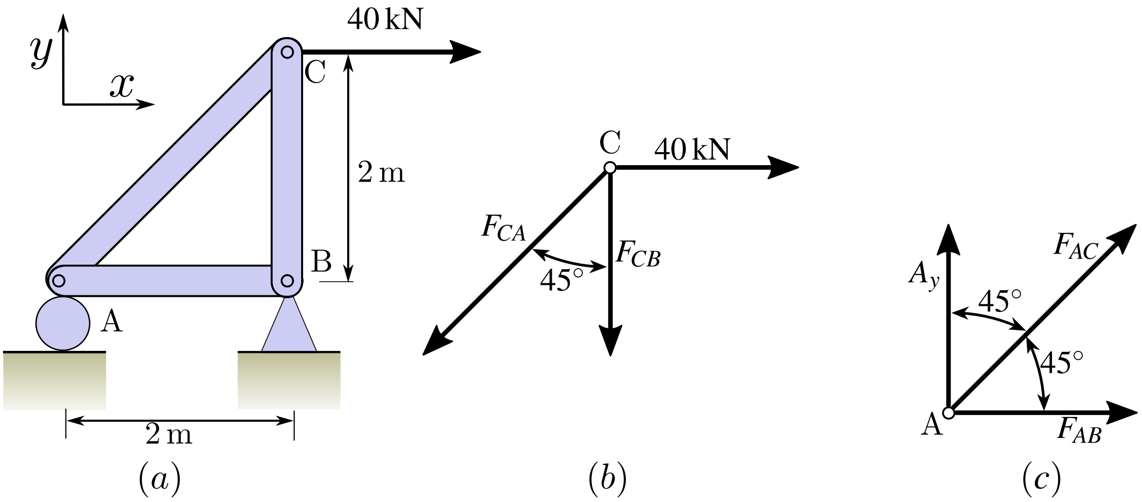

Figure 2.2: Truss with (b) joint C and (c) joint A isolated

Solution
This example is calculated using in units of N, mm and MPa.
In a similar way to example 1.1 the angles BAC = 45° and ACB = 45° because the length of AB = BC.
To start the angle BAC is calculated from the beam dimensions, knowing that BC is perpendicular to AB and the lengths of AB and BC are equal. It follows BAC = 45°. Similarly angle ACB = 45°.

Joint C is the only joint where there is a known force so we begin there using the method of joints.
Joint C
$\Sigma F_x = 0 \Rightarrow 40\,000\,\text{N} - F_{CA}\sin 45 = 0 \Rightarrow F_{CA} = 56\,568\,\text{N}$
$\Sigma F_y = 0 \Rightarrow -F_{CB} - F_{CA}\cos 45 = 0 \Rightarrow F_{CB} = -40\,000\,\text{N}$ compression

Joint A
$\Sigma F_x = 0 \Rightarrow F_{AB} + F_{AC}\cos 45 = 0 \Rightarrow F_{AB} = -40\,000\,\text{N}$ compression

The area of each cross section A = $\pi\left(\frac{20}{2}\right)^2 = 314.16\,\text{mm}^2$
$\sigma_{AC} = \dfrac{F_{AC}}{A} = \dfrac{56568}{314.16} = 180.1\,\text{MPa}$
$\sigma_{CB} = \sigma_{AB} = \dfrac{F_{CB}}{A} = \dfrac{F_{CB}}{A} = \dfrac{-40000}{314.16} = -127.3\,\text{MPa}$ compression

■

2.3 Normal Strain

When a solid body is subjected to some loading (mechanical or thermal), it undergoes **deformation**. This means it changes size and/or shape which means the lengths and/or the angles in the body can change.

Strain, represented by the Greek letter ε (epsilon), is the intensity of deformation or deformation per unit length just as stress is the force intensity.

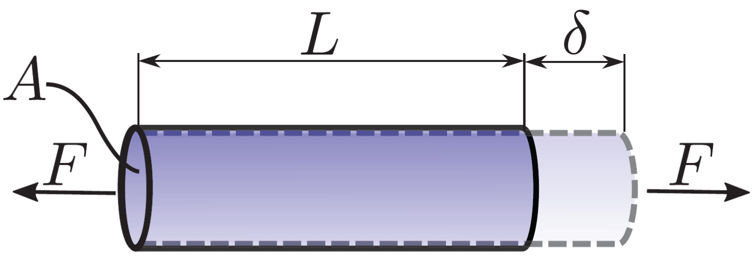

Figure 2.3: Normal strain in a cylinder

The average normal strain is

$$\boxed{\varepsilon_{avg} = \frac{\delta}{L}} \tag{2.4}$$

where δ (delta) is the Greek letter representing the change in length in meters and L is the original length also in meters. A positive value of ε indicates extension of the body and a negative value indicates contraction.

Strain is a dimensionless quantity but is often expressed in terms of ratios of lengths for example m/m. In most engineering application it is convenient to use units of microstrain $\mu\varepsilon$ where 1 $\mu\varepsilon$ = 1×10^{-6} m/m. It is also useful to represent strains in terms of percent. For most metal objects the strain will seldom be bigger than 0.2% which is 0.002 m/m.

2.4 Stress-Strain Diagrams

To design or analyse structures an understanding of the properties of the material being used is needed. The most effective way of determining these properties is in a **tensile test**. We have either a round or flat sample (see insets in figure 2.4) pulled with a controlled displacement which is related to the strain. The force is measured with a load cell and the strain calculated. The plot of the stress against strain output is also shown in the figure 2.4.

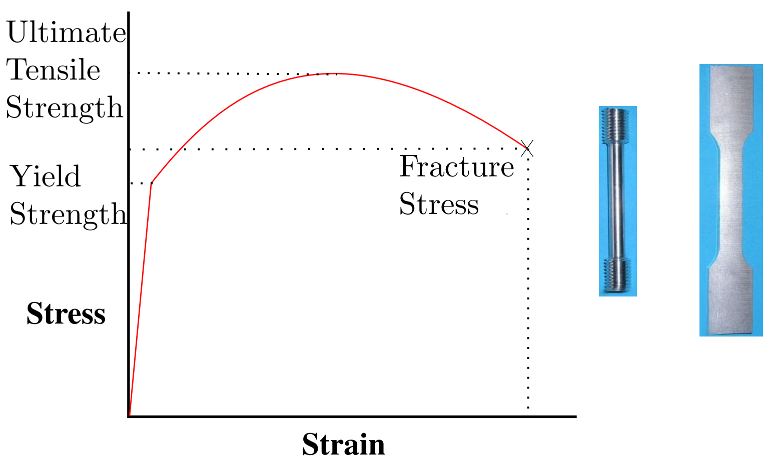

Figure 2.4: Stress versus strain for a typical metal

The **yield strength** can be considered to be the elastic limit in many cases. Any stress applied above the yield strength will permanently deform the material. This is called **plastic deformation**. The

ultimate tensile strength represents the maximum stress the material can tolerate. The fracture stress represents the stress when the material will break completely[2].

In design we would use the yield strength as our maximum stress, however because there are many uncertainties in our loads, our material properties and even our calculations, a factor of safety is used:

$$\boxed{\text{Factor of Safety} = \frac{\text{Material Strength}}{\text{Required Strength}}} \tag{2.5}$$

The material strength is typically the yield strength and the required strength is usually calculated from some mechanics equation based on the required loading.

Factors of safety depend on the application and can range from just over 1 all the way to more than 10. Issues such as the weight, how critical part the part is and uncertainty in the calculations all influence the safety factor used.

2.5 Hooke's Law

Below the yield strength, a material typically exhibits linear elastic behaviour. The relationship between the stress and strain is linear. Elastic behaviour means it will return to the same state after a load is removed. The strain measured is proportional to the applied stress and described by the equation:

$$\boxed{\sigma = \varepsilon E} \tag{2.6}$$

where E is called the **Young's Modulus** or the **Modulus of Elasticity**. The unit for E is Pa which is the same as for stress. For typical engineering materials a more convenient unit is 1 GPa = 1×10^9 Pa. Steel, which is one of the most widely used engineering materials, has a modulus of 200 GPa.

2.6 Poisson's Ratio

When a *tensile force is applied to a material in the longitudinal direction* then there will typically be a contraction of the material in the lateral direction [3]. The ratio of the strain in the lateral direction to strain in the longitudinal direction is called Poisson's Ratio. It is represented by the Greek symbol ν (pronounced nu) and is defined as follows:

$$\boxed{\nu = -\frac{\varepsilon_{lateral}}{\varepsilon_{longitudinal}}} \tag{2.7}$$

Take note of the negative sign. It shows that for positive Poisson's Ratios if there is an extension in the longitudinal direction, there will be a contraction in the lateral direction. Alternatively if there is a contraction in the longitudinal direction there will be extension in the lateral direction.

Most engineering materials have Poisson's Ratio between 0.3 and 0.4. The maximum Poisson's Ratio is 0.5 and indicates the volume will stay constant.

[2]See **MecMovies M3.1** at http://web.mst.edu/ mecmovie/ for a complete explanation of the tensile test

[3]There are some exceptions to this and materials that expand in the lateral direction when pulled in the longitudinal direction are called Auxetic Materials

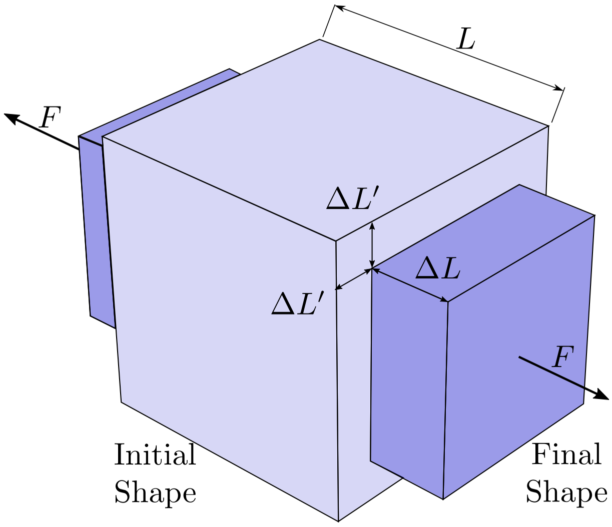

Figure 2.5: Cube is pulled with tensile force F in longitudinal direction

2.7 Shear Stress

Stress discussed in section 2.2 has the load perpendicular to area A over which it acts. If the load acts tangential to the area then this is called a **shear force** V and the stress is called the **shear stress** [4] . The shear stress is denoted by the Greek letter τ (tau) and defined as follows similar to normal stress:

$$\boxed{\tau = \lim_{\Delta A \to 0} \frac{\Delta V}{\Delta A}} \tag{2.8}$$

The average shear stress τ_{avg} is then:

$$\boxed{\tau_{avg} = \frac{V}{A}} \tag{2.9}$$

The stress that is experienced directly below or above the offset block in Figure 2.6 is the **bearing stress** or **compressive stress**.

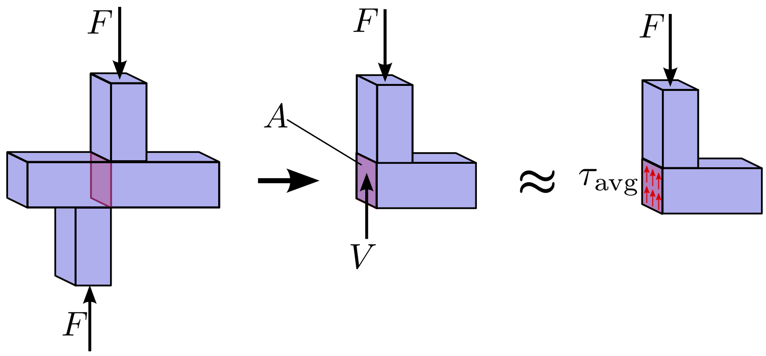

Figure 2.6: Horizontal bar loaded with offset blocks showing the shear stress

Shear stresses are often found in bolts, rivets and pins used to connect various objects.

[4]See **MecMovies M1.3** at http://web.mst.edu/∼mecmovie/

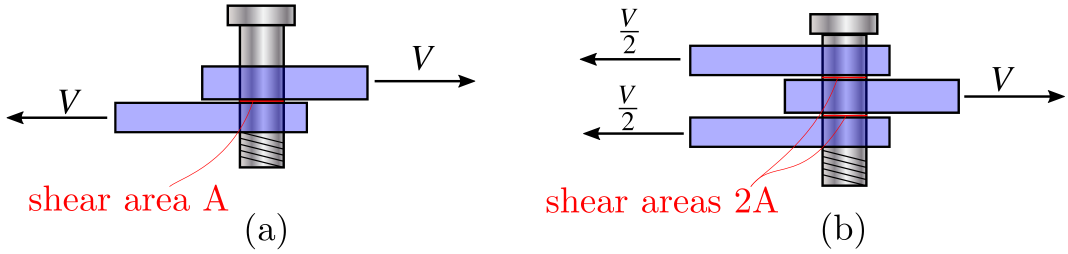

Figure 2.7: Bolts shown in (a) single shear and (b) double shear.

For the **double shear** shown in Figure 2.7 the area is double that for **single shear**. The shear stress in double shear is:

$$\boxed{\tau_{avg} = \frac{V}{2A}} \tag{2.10}$$

■ **Example 2.2** Determine the average shear stress developed in pin B of the truss of figure 2.2. Each pin has a diameter of 25 mm and is in double shear.

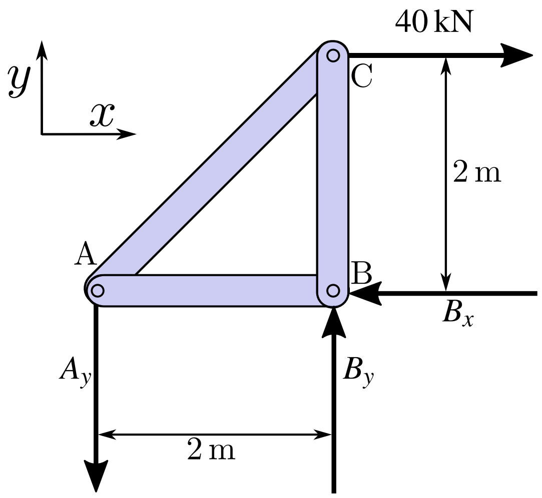

Figure 2.8: Free body diagram for figure 2.2

The shear area $A = \pi\left(\frac{25}{2}\right)^2 = 490.87\,\text{mm}^2$

Next the total force on the joint is calculated.

$\Sigma M_A = 0 \Rightarrow B_y \times 2 - 40000 \times 2 = 0 \Rightarrow B_y = 40\,000\,\text{N}$

$\Sigma F_x = 0 \Rightarrow 40000 - B_x = 0 \Rightarrow B_x = 40\,000\,\text{N}$

Total force on joint is $\sqrt{40000^2 + 40000^2} = 56\,568\,\text{N}$

The average shear stress $\tau_{avg} = \dfrac{V}{2A} = \dfrac{56568}{2 \times 490.87} = 57.62\,\text{MPa}$

■

2.8 Shear Strain

Deformation involving a change of shape is called **shear strain**. This is defined in the figure 2.9 with the Greek letter γ (gamma) and $\tan\gamma = \frac{\delta_s}{L}$. Since the shear strain angle is small, $\tan\gamma = \gamma$. Remember that the angle measurement is in radians. Strain can then be defined this as:

$$\boxed{\gamma = \frac{\delta_s}{L}} \tag{2.11}$$

It is the change in angle between two faces of an element of material.

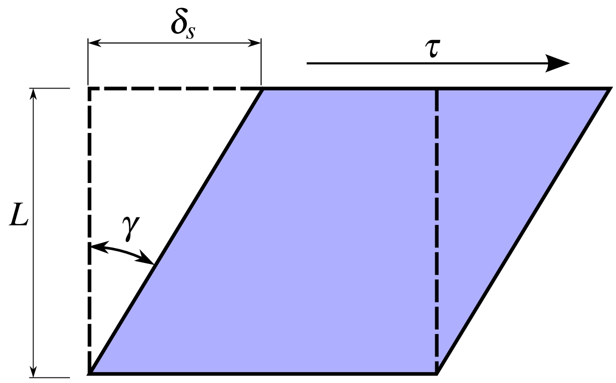

Figure 2.9: Shear strain shown for a block of height L under a shear stress of τ.

The shear strain is related to the shear stress by the equation:

$$\boxed{\tau = \gamma G} \tag{2.12}$$

where G is the **Shear Modulus** or **Modulus of Rigidity**.

For isotropic homogeneous materials we can relate the Young's modulus E, Poisson's ratio ν and Modulus of Rigidity G with:

$$\boxed{G = \frac{E}{2(1+\nu)}} \tag{2.13}$$