3. Stress Transformations

Colour variations seen in plastic cutlery is due to the photoelastic effect. The manufacturing process causes stresses in the cutlery. Polarised light interacts with the stresses and causes the different colours.

3.1 Introduction

In previous sections normal and shear stresses were calculated in various situations. These stresses interact with each other in a complex fashion. Occasionally stresses need to be calculated on planes at some angle to the applied forces. This may be the case when you need to calculate the stress on a crack orientated at an angle to the applied load. You may need to know the stress normal to the crack to know whether the crack will open or close.

If you know how to calculate the stress in any arbitrary orientation you may need to know the orientation and magnitude of the maximum stress. This will help in knowing when a structure will fail. Calculation of these maximum stresses and their directions is unlike calculating the forces and their directions. Forces are vectors and stresses are higher rank **tensors**[1].
In this section a more general definition of normal stress is required which is related to a coordinate axis:

$$\boxed{\sigma_x = \sigma_{xx} = \lim_{\Delta A \to 0} \frac{\Delta F_x}{\Delta A_x}} \tag{3.1}$$

In σ_{xx}, the first x subscript indicates the normal of the area over which force acts and the second x subscript indicates the force direction. The repeated subscript, in this case x, is often dropped for convenience.

For normal stresses the area normal and the force direction are parallel. The sign convention for normal stress was established previously with tensile stresses being positive and compressive stresses being negative.

[1] A description of tensors is beyond the scope of this course. They are geometric objects which are a generalisation of scalars and vectors which are tensors of rank 0 and 1, respectively. Stress is a tensor of rank 2.

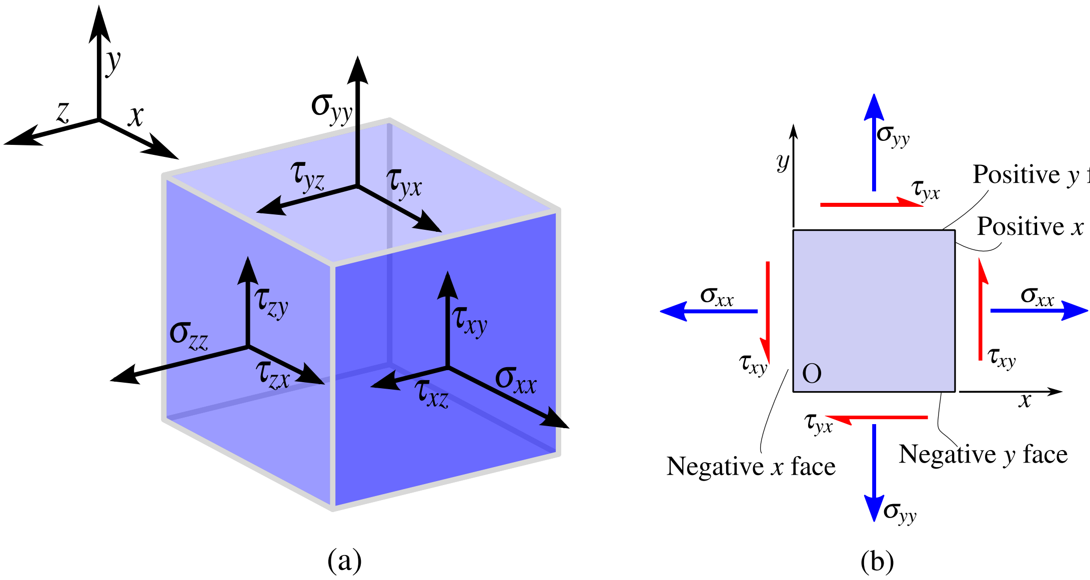

Figure 3.1: Stress depicted in (a) three dimensions and on (b) two dimensional stress element.

In a similar fashion to normal stress, shear stress can be represented as follows:

$$\boxed{\tau_{xy} = \sigma_{xy} = \lim_{\Delta A \to 0} \frac{\Delta V_y}{\Delta A_x}} \tag{3.2}$$

Here the shear force ΔV_y and the area normal are perpendicular to each other. This is unlike normal stress where these vector quantities are parallel to each other. The sign conventions for the shear stress is as follows:

Shear Stress is Positive

- acts in the positive coordinate direction on a positive face of the stress element, or
- acts in the negative coordinate direction on a negative face of the stress element.

Shear Stress is Negative

- acts in the positive coordinate direction on a negative face of the stress element, or
- acts in the negative coordinate direction on a positive face of the stress element.

For example all shear stresses are all positive in figure 3.1(b). The **stress element** in figure 3.1(b) is a two-dimensional graphical representation of a three-dimensional object.

3.1.1 Shear Stress Equilibrium

If an object is in static equilibrium then any part isolated from it must be under static equilibrium no matter the size or shape. Let's consider a small volume element depicted in figure 3.2 with a shear stress τ_{xy} that is known. The positive and negative z-normal faces are stress free. The shear stresses τ_{yx}, τ'_{xy} and τ'_{yx} are unknown.

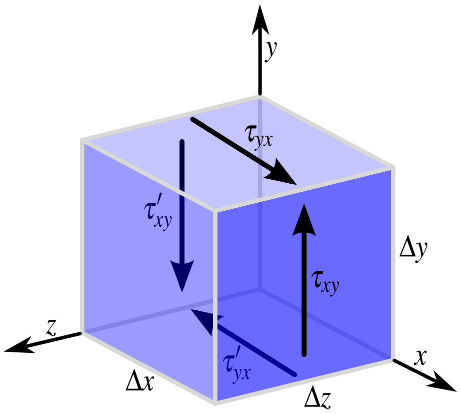

Figure 3.2: Shear stress acting on a small volume element.

Since the volume element is in equilibrium the sum of forces and moments about any point is zero. Summing forces in the x direction:

$$\begin{aligned} \Sigma F_x &= \tau_{yx}\Delta x\Delta z - \tau'_{yx}\Delta x\Delta z = 0 \\ &\Rightarrow \tau_{yx} = \tau'_{yx}. \end{aligned} \tag{3.3}$$

Summing forces in the y direction:

$$\begin{aligned} \Sigma F_y &= \tau_{xy}\Delta y\Delta z - \tau'_{xy}\Delta y\Delta z = 0 \\ &\Rightarrow \tau_{xy} = \tau'_{xy}. \end{aligned} \tag{3.4}$$

Taking moments about the z-axis:

$$\begin{aligned} \Sigma M_z &= (\tau_{xy}\Delta y\Delta z)\,\Delta x - (\tau_{yx}\Delta x\Delta z)\,\Delta y = 0 \\ &\Rightarrow \tau_{xy} = \tau_{yx}. \end{aligned} \tag{3.5}$$

Combining equations (3.3), (3.4) and (3.5) gives:

$$\tau_{xy} = \tau_{yx} = \tau'_{xy} = \tau'_{yx}. \tag{3.6}$$

So the shear stress on any of the four faces would be the same. This property is called the *complementary* property of shear.

Stresses occur in three dimensions, represented with three axes, x, y and z. For simplicity out of plane stresses are assumed to be zero in a condition called **plane stress**:

$$\boxed{\sigma_{zz} = \tau_{zx} = \tau_{zy} = 0} \tag{3.7}$$

From equation (3.6) we see then that $\tau_{xz} = \tau_{yz} = 0$ for plane stress.

3.2 Plane Stress Transformations using Equilibrium Equations

A state of stress can be completely described by three components in two-dimensions , σ_{xx}, σ_{yy} and τ_{xy} acting on a plane with x and y coordinate axes. Often we want to represent it on a different plane represented by normal n and tangential t coordinate axes giving stress components σ_n, σ_t and τ_{nt}.[2] Changing from one set of axes to another is termed a **stress transformation**.

■ **Example 3.1** Consider an element with a shear stress of 110 MPa and a tensile stress of 185 MPa in the horizontal plane. Determine the normal and shear stresses at a slope 60° to the horizontal.

Solution
Sketch a free-body diagram of a wedge shaped portion of the stress element, shown in figure 3.3(a). The wedge shape is formed with a plane cutting the element at the 30° to the vertical. Let the area of the inclined surface be dA. Remember we are looking at a two dimensional representation of what is a three-dimensional stress state. The angle 30° the new plane makes to the vertical is also the angle the normal rotates from x to n. The area of the vertical surface will be $dA\cos 30$ and the area of the horizontal surface will be $dA\sin 30$.

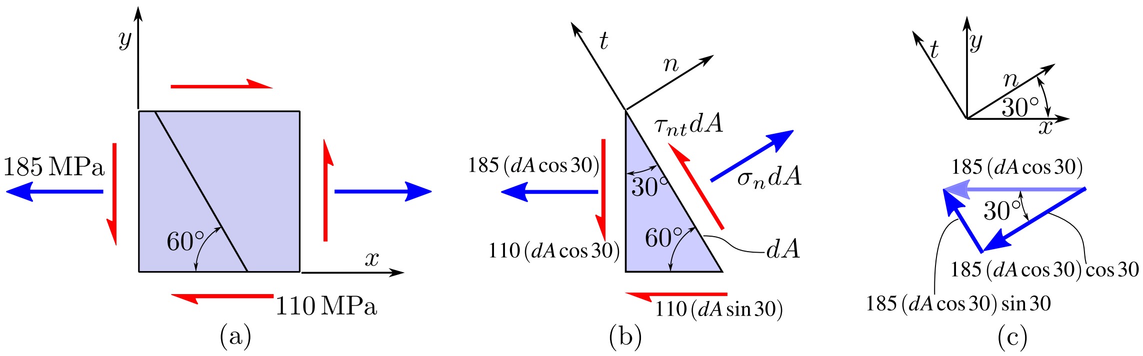

Figure 3.3: Illustration of (a) plane stress element and (b) determination of stress on an arbitrarily chosen plane at 60° clockwise to the horizontal. The resolution of the x direction force is shown in (c).

The forces on these areas are the product of the areas and the stresses. For example the force in the x direction is $185(dA\cos 30)$. The horizontal force is resolved into components in the n and t axes, see figure 3.3(c). The shear force is also resolved in a similar way.

Summing forces parallel to the inclined plane in the t-direction.

$$\begin{aligned}\Sigma F_t =& \tau_{nt}\mathrm{d}A - 110\,(\mathrm{d}A\cos 30)\cos 30 + 110\,(\mathrm{d}A\sin 30)\sin 30\\ &+ 185\,(\mathrm{d}A\cos 30)\sin 30 = 0\end{aligned}$$

All the dA factors disappear which gives: $\tau_{nt} = -25.1$ MPa. The shear is negative so it is in a opposite direction to what is shown in figure 3.3(b).

Summing forces in the normal to the inclined surface.

$$\begin{aligned}\Sigma F_n =& \sigma_n\mathrm{d}A - 110\,(\mathrm{d}A\cos 30)\sin 30 - 110\,(\mathrm{d}A\sin 30)\cos 30\\ &- 185\,(\mathrm{d}A\cos 30)\cos 30 = 0\end{aligned}$$

The normal stress $\sigma_n = 234$ MPa. This is positive and therefore in tension. It is indicated in the correct direction in figure 3.3(b). ■

[2] Many authors use the notation $\sigma_{x'}$, $\sigma_{y'}$ and $\tau_{x'y'}$ to represent σ_n, σ_t and τ_{nt}, respectively.

3.3 General Equations of Plane Stress Transformation

The method of transforming the normal and shear stresses from *x*-*y* coordinates to *n*-*t* coordinates can be developed in a general fashion and expressed as a set of stress transformation equations. [3]

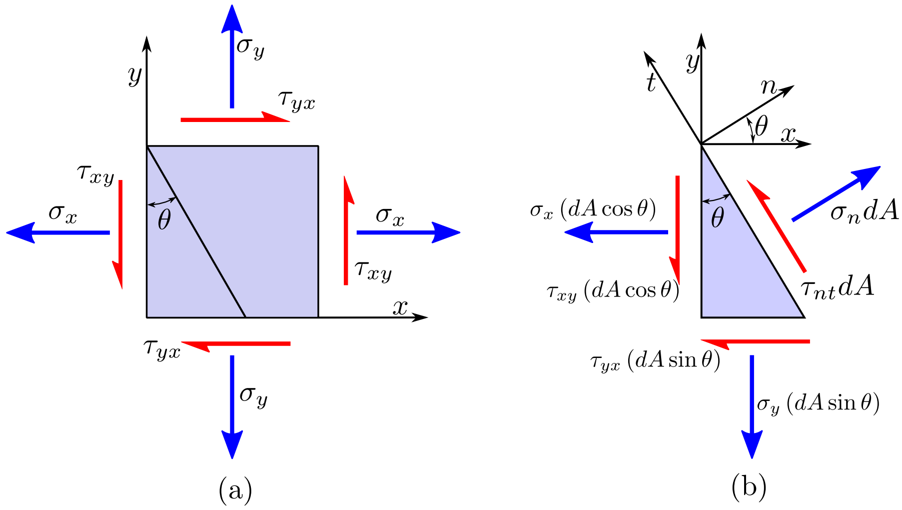

Figure 3.4: Drawing of a (a) Stress element with an arbitrary plane at an angle θ to vertical and (b) free-body diagram of wedge shaped element.

Consider the free body diagram of figure 3.4(b). Using the equations of equilibrium by summing forces in the n and t axes directions. Note we drop the repeated subscript in σ_{xx} and make it σ_x

$$\begin{aligned}\Sigma F_n =& \sigma_\mathrm{n} dA - \tau_{xy}\,(dA\cos\theta)\sin\theta - \tau_{yx}\,(dA\sin\theta)\cos\theta\\ &- \sigma_x\,(dA\cos\theta)\cos\theta - \sigma_y\,(dA\sin\theta)\sin\theta = 0\\ \Rightarrow& \sigma_\mathrm{n} = \sigma_x\cos^2\theta + \sigma_y\sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta\end{aligned}$$

and

$$\begin{aligned}\Sigma F_t =& \tau_{nt} dA - \tau_{xy}\,(dA\cos\theta)\cos\theta + \tau_{yx}\,(dA\sin\theta)\sin\theta\\ &+ \sigma_x\,(dA\cos\theta)\sin\theta - \sigma_y\,(dA\sin\theta)\cos\theta = 0\\ \Rightarrow& \tau_\mathrm{nt} = -(\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}\left(\cos^2\theta - \sin^2\theta\right)\end{aligned}$$

Using the following trigonometric identities:

$$\sin(2\theta) = 2\sin\theta\cos\theta; \qquad \sin^2\theta = \frac{1-\cos(2\theta)}{2}; \qquad \cos^2\theta = \frac{1+\cos(2\theta)}{2}$$

We obtain the following formulae called the **plane stress transformation equations**:

$$\boxed{\sigma_\mathrm{n} = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta} \tag{3.8}$$

and

$$\boxed{\tau_\mathrm{nt} = -\left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta} \tag{3.9}$$

[3] see UCT MecMovies 3.8 for an animated derivation of the plane stress transformations

■ **Example 3.2** Consider an element on the surface of an aircraft. What would the stress be on an element inclined at 60° clockwise to the vertical?

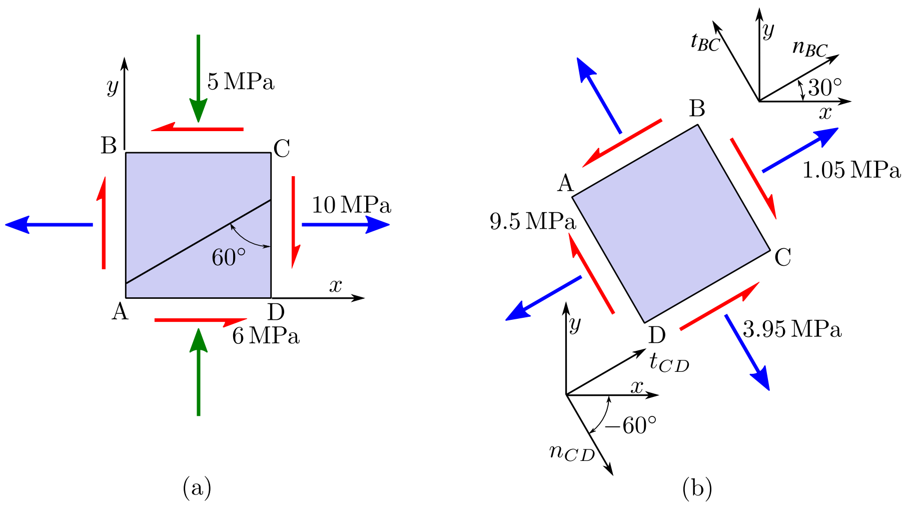

Figure 3.5: (a) Element on the surface of an aircraft and (b) the element transformed .

Solution
First consider the stresses according to the established sign conventions.

$$\sigma_x = 10\,\text{MPa}; \qquad \sigma_y = -5\,\text{MPa}; \qquad \tau_{xy} = -6\,\text{MPa}$$

The transformed element has two perpendicular faces. Consider the transformation to each of these faces in planes CD and BC separately shown in figure 3.5(b).

Plane BC
Using equation (3.8) to calculate the normal stress and use the angle that plane BC is transformed in figure 3.5(b)

$$\sigma_{\text{n}} = \frac{10-5}{2} + \frac{10+5}{2}\cos\left(2(30)\right) - 6\sin\left(2(30)\right) = 1.05\,\text{MPa}$$

Using equation (3.9) to calculate the shear stress

$$\tau_{\text{nt}} = -\frac{10+5}{2}\sin\left(2(30)\right) - 6\cos\left(2(30)\right) = -9.5\,\text{MPa}$$

Considering the shear sign convention the shear on face BC is pointing in the negative t_{BC} direction on the positive n_{BC} face.

Plane CD
Again using equation (3.8) to calculate the normal stress with the angle −60° instead of 30°.

$$\sigma_{\text{n}} = \frac{10-5}{2} + \frac{10+5}{2}\cos\left(2(-60)\right) - 6\sin\left(2(-60)\right) = 3.95\,\text{MPa}$$

Check the previous shear stress with equation (3.9).

$$\tau_{\text{nt}} = -\frac{10+5}{2}\sin\left(2(-60)\right) - 6\cos\left(2(-60)\right) = 9.5\,\text{MPa}$$

Consider our shear sign convention again, the shear on face AD is pointing in the positive t_{CD} direction on the positive n_{CD} face. Notice the shear stress changed signs according to the axes which were considered. ■

3.4 Principal Stresses

As seen in the previous section stresses at a point vary according to the plane direction in which we consider the stresses. In engineering it is very important to determine the largest normal and shear stresses in a component to determine when it will fail. In this section we will determine the magnitude of these stresses and the direction in which they will act. To get the maximum, the normal stress equation (3.8) is differentiated with respect to the angle θ and see where it is zero. This gives:

$$\frac{d\sigma_{\mathrm{n}}}{d\theta} = -\frac{\sigma_x - \sigma_y}{2} 2\sin 2\theta + \tau_{xy} 2\cos 2\theta = 0 \tag{3.10}$$

Solving this equation and with the solution $\theta = \theta_p$ the following is obtained:

$$\boxed{\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2}} \tag{3.11}$$

The tan function recurs every 180° so $2\theta_{p1}$ and $2\theta_{p2}$ are 180° apart and therefore the two roots θ_{p1} and θ_{p2} will be 90° apart, i.e. $\theta_{p1} = \theta_{p2} \pm 90°$. The two roots are called the **principal angles**. Equation (3.11) is represented geometrically in figure 3.6.

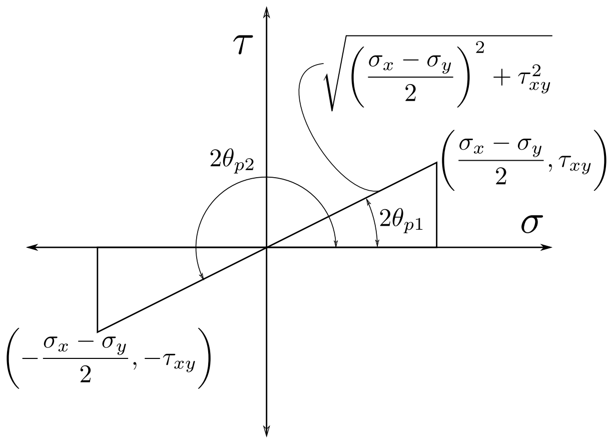

Figure 3.6: Illustration of principal angles in equation (3.11)

The values of θ_{p1} and θ_{p2} must be substituted into equation (3.8) to give the maximum and minimum normal stresses called the **principal stresses**, σ_{p1} and σ_{p2}.

$$\boxed{\sigma_{p1,p2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}} \tag{3.12}$$

Here σ_{p1} is the major principal stress which is greater than minor principal stress σ_{p2}. The plane on which these stresses act are the **principal stress planes**. Looking at the right hand side of equation (3.9) for shear stress ($-\frac{\sigma_x-\sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$), it is exactly half the right hand side of equation (3.10) for the normal stress derivative. Then the normal stress derivative and the shear stress will be zero at the same time therefore **the shear stress will be zero on the principal stress plane**.

3.5 Maximum In-Plane Shear Stress

As with obtaining the principal stress we differentiate the appropriate stress equation and set to zero to get the maximum in-plane shear stress τ_{max}. Differentiate equation (3.9) to give

$$\frac{d\tau_{\mathrm{nt}}}{d\theta} = -(\sigma_x - \sigma_y)\cos 2\theta - 2\tau_{xy}\sin 2\theta = 0 \tag{3.13}$$

The solution gives the orientation $\theta = \theta_s$ of the plane where the shear is either maximum or minimum.

$$\boxed{\tan 2\theta_s = -\frac{(\sigma_x - \sigma_y)/2}{\tau_{xy}}} \tag{3.14}$$

As with the principal angles the θ_s angles are 90° apart. Notice that equation (3.11) and equation (3.14) are negative reciprocals of each other therefore the angles $2\theta_s$ and $2\theta_p$ are 90° apart from each other. The angles θ_s and θ_p are 45° apart, i.e. $\theta_s = \theta_p \pm 45°$. The maximum shear plane is therefore 45° to the principal plane.

Substitute θ_s into equation (3.9) gives the magnitude of the in-plane shear stress τ_{max}.

$$\boxed{\tau_{\max} = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}} \tag{3.15}$$

The maximum shear stress is ambiguous in sign. Unlike normal stress where tension and compression has different effects on material behaviour, a different sign in shear stress has no difference in fact the same shear stress will have a different sign but same magnitude if taken on an orthogonal plane.

Unlike the principal plane where no shear stress acts normal stress acts on the plane of maximum shear stress and equals:

$$\boxed{\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}} \tag{3.16}$$

The σ_{avg} will be the same on both orthogonal $\tau_{\max}$ planes.

Another relation which relates principal and maximum shear stress is given from equation (3.12) and equation (3.15):

$$\boxed{\tau_{\max} = \frac{\sigma_{p1} - \sigma_{p2}}{2}} \tag{3.17}$$

■ **Example 3.3** A stress element has a tensile stress of 50 MPa, a compressive stress of 10 MPa and a shear stress of 40 MPa as shown in figure 3.7.

Determine the principal stresses and at which angles they occur, the maximum shear stress and the angle at which it occurs. Also determine the normal stress on the plane of maximum shear stress.

Solution
The stresses are according to the sign conventions: $\sigma_x = 50\,\text{MPa}$; $\sigma_y = -10\,\text{MPa}$; $\tau_{xy} = 40\,\text{MPa}$

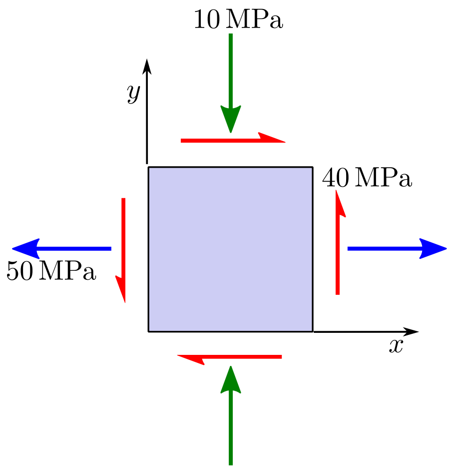

Figure 3.7: Element and stresses shown

Principal Angles
Substitute stress values into equation (3.11) to give:

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{4}{3}$$

$2\theta_p = 53.1^\circ$ and $180^\circ + 53.1^\circ = 233.1^\circ$
$\theta_p = 26.6^\circ$ and 116.6° which gives the two principal angles.

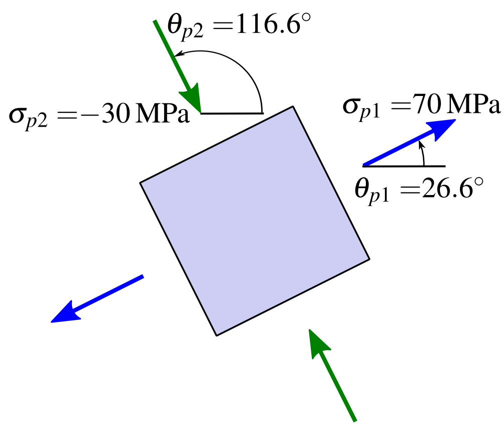

Figure 3.8: Principal stress shown for figure 3.7.

Principal Stresses
Substitute stress values and principal angles into equation (3.8) to give:

$$\sigma_\text{p} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta_p + \tau_{xy}\sin 2\theta_p$$

$\sigma_\text{p1} = 70\,\text{MPa}$ corresponds to $\theta_{p1} = 26.6^\circ$ and $\sigma_\text{p2} = -30\,\text{MPa}$ corresponds to $\theta_{p1} = 116.6^\circ$. We know that $\sigma_\text{p1} = 70\,\text{MPa}$ corresponds to the major principal stress since it is larger than all the normal stresses. Similarly $\sigma_\text{p2} = -30\,\text{MPa}$ since it is smaller than all the normal stresses.

The principal stresses can be checked using equation (3.12). If equation (3.12) was used directly the principal angle corresponding to each principal stress would not be known.

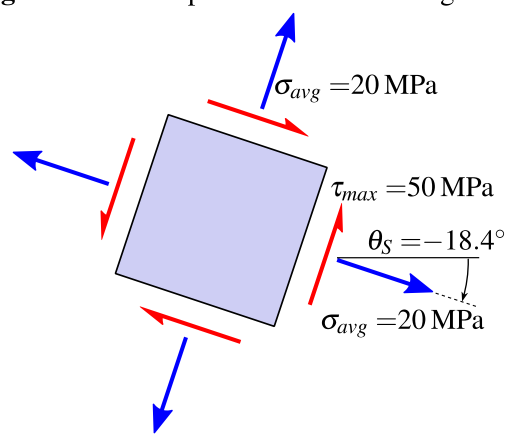

Figure 3.9: Maximum shear stress shown

Maximum Shear Stress
This is calculated from equation (3.15)

$$\tau_\text{max} = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \pm 50\,\text{MPa}.$$

The maximum shear angle can be obtained knowing that it is 45° from the principal angle so $\theta_S = \theta_\text{p} - 45^\circ = -18.4^\circ$.

The shear sign can be checked by substituting -18.4° into equation (3.9) to see its positive. The normal stress is obtained from equation (3.16) to give 20 MPa. ■

3.6 Mohr's Circle for Plane Stress

The basic equations of stress transformation derived previously can be interpreted graphically in a way that is easy to remember and convenient to use. It also aids in rapid understanding of the physical significance of the transformation equations.

To derive the circle equation write equation (3.8) and equation (3.9) in the form with the trigonometric functions on one side [4]:

$$\sigma_{\mathrm{n}} - \frac{\sigma_x + \sigma_y}{2} = \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

and

$$\tau_{\mathrm{nt}} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

Both equations above are squared, added and simplified to give an equation with no θ:

$$\boxed{\left(\sigma_{\mathrm{n}} - \frac{\sigma_x + \sigma_y}{2}\right)^2 + \tau_{\mathrm{nt}}^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \tag{3.18}$$

This is the equation for a circle in terms of variables σ_{n} and τ_{nt}. If we let:

$$\boxed{C = \frac{\sigma_x + \sigma_y}{2}} \tag{3.19}$$

then C is centre of the circle located on the horizontal σ axis. Also let

$$\boxed{R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}} \tag{3.20}$$

which is the square root of the right hand side of equation (3.18). This is the radius of the circle. Equation (3.18) is then rewritten as

$$\boxed{(\sigma_{\mathrm{n}} - C)^2 + \tau_{\mathrm{nt}}^2 = R^2} \tag{3.21}$$

This equation represents a circle with radius R and center C. This is called **Mohr's Circle** after Otto Mohr[5].

[4] See MecMovies 12.15 for a step by step guide of the derivation of Mohr's circle stress transformation equations

[5] Otto Christian Mohr was a German structural engineer who developed this circle approach in 1882

3.6.1 Construction of Mohr's Circle

The following steps can be used to plot Mohr's Circle. We assume that we have been given σ_x, σ_y and τ_{xy} of an element in plane stress.
The sign convention for plotting stresses are as follows:

1. Normal stresses are positive in tension and negative in compression as in previous sections
2. Shear stress that turn the element face clockwise are plotted above the σ-axis and shear stresses that turn the face anti-clockwise are plotted below the σ-axis, see figure 3.10.

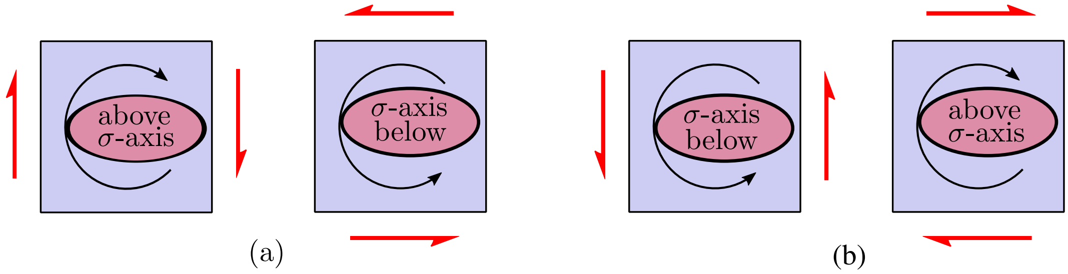

Figure 3.10: Sign convention for shear stresses used in constructing Mohr's Circle: (a) is negative shear and (b) is positive shear.

1. Draw the coordinate axes with σ as the horizontal axis or abscissa and with τ as the vertical axis or ordinate [6].
2. Locate the centre of the circle C with coordinate $\left(\sigma_{\text{avg}}, 0\right)$ where $\sigma_{\text{avg}} = \frac{\sigma_x+\sigma_y}{2}$.
3. Plot a point A, (σ_x, τ_{xy}) which may relate to two opposite faces on the stress element; however it is easier to specify the stresses on the positive x face. This represents $\theta = 0°$. Importantly **σ is positive to the right and τ which turns a face anti-clockwise is below the σ-axis**.
4. Plot a point B, (σ_y, τ_{yx}) which would be on the opposite side of the σ-axis to τ_{xy}. The plane going through this point represents $\theta = 90°$.
5. Draw line AB through C which represents the diameter of the circle
6. Draw Mohr's circle through points A and B with centre C. The circle has radius R.

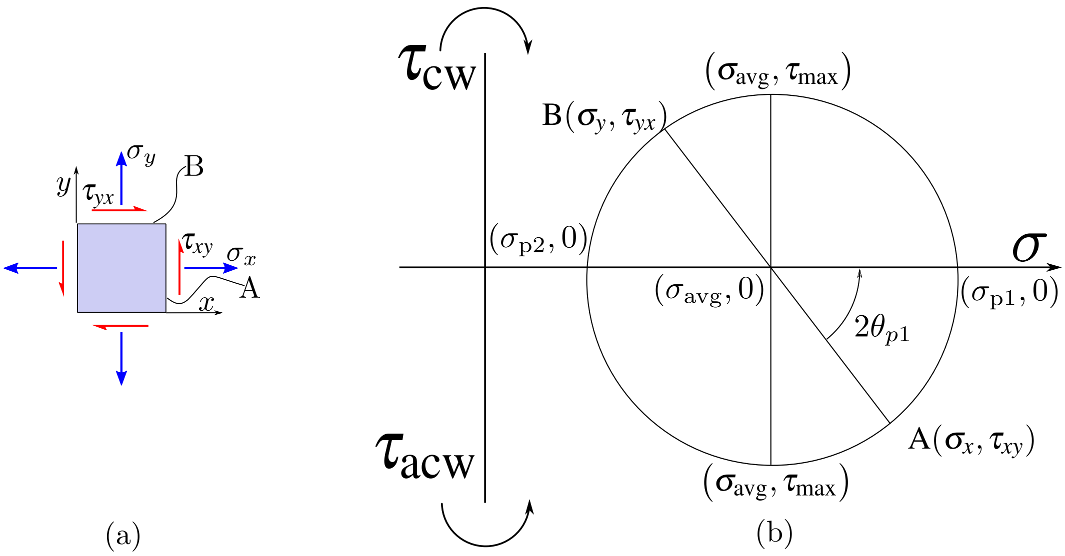

Figure 3.11: Stress element (a) and Mohr's circle of stress (b) shown.

The angle $2\theta_p$ in figure 3.11 is twice the principal angle. Any angle in Mohr's Circle space represents twice that angle in real space.

[6] MecMovies 12.16 presents a step by step guide for plotting Mohr's Circle.

■ **Example 3.4** An element in plane stress is subjected to stresses $\sigma_x = 100\,\text{MPa}$, $\sigma_y = 34\,\text{MPa}$ and $\tau_{xy} = 28\,\text{MPa}$, as shown in figure 3.12. Using Mohr's circle, determine the following quantities: (a) the principal stresses and angles, (b) the maximum shear stress and angle, and (c) the stresses acting on an element inclined at an angle $\theta = 40°$ anticlockwise. (Consider only the in-plane stresses, and show (c) on a sketch of a properly oriented element.)

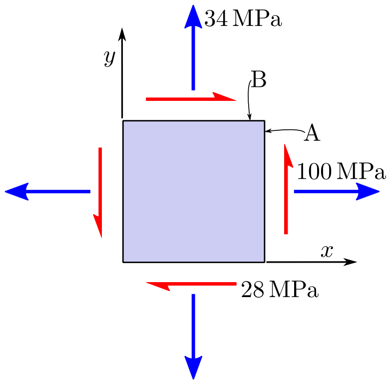

Figure 3.12: Plane stress element with various loadings.

Solution

1. The normal stress and shear axes are seen in figure 3.13
2. Centre of Mohr's circle from equation (3.19): $C = \dfrac{100+34}{2} = 67\,\text{MPa}$
3. Point A is at $(100, 28)$ which is below the σ axis since the shear on the x-face turns the element anti-clockwise
   Point B is at $(34, 28)$ which is above the is above the σ axis since the shear on the x-face turns the element anti-clockwise
4. Line AB is drawn which goes through point C in figure 3.13
5. The radius of the circle is determined using equation (3.20):

$$R = \sqrt{\left(\frac{100-34}{2}\right)^2 + 28^2} = 43.2\,\text{MPa}$$

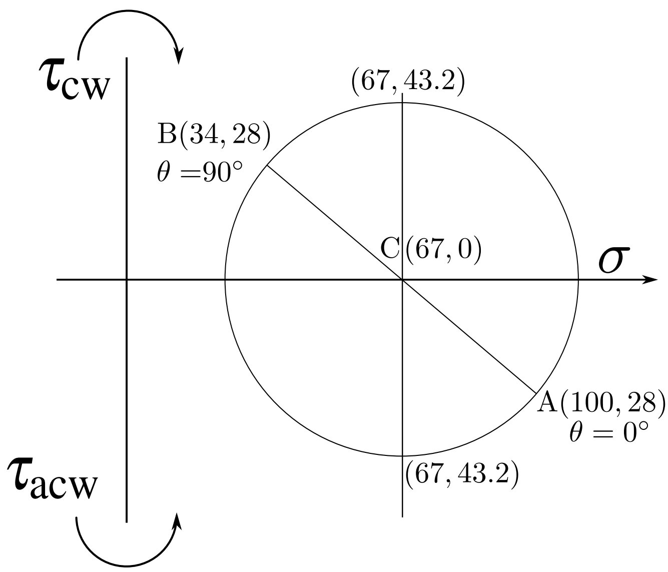

Figure 3.13: Mohr's Circle for stress element shown in figure 3.12

(a) The principal stresses are:

$$\sigma_{p1} = 67 + 43.2 = 110.2\,\text{MPa}$$

$$\sigma_{p2} = 67 - 43.2 = 23.7\,\text{MPa}$$

Principal angle (maximum) from circle geometry

$$\theta_{p1} = \frac{1}{2}\tan^{-1}\frac{100-67}{28} = 20.15^\circ$$

Principal angle (minimum)

$$\theta_{p2} = \theta_{p1} + 90 = 110.2^\circ$$

(b) Maximum shear stress from the ordinate of circle below centre:

$$\tau_{max} = 43.2\,\text{MPa}$$

Shear angle of maximum stress is $20.15^\circ - 45^\circ = -24.85^\circ$

(c) If an element is rotated 40° then we rotate 80° on Mohr's Circle from point A to D.
The angle ACP1 is $\tan^{-1}\frac{100-67}{28} = 40.3^\circ$
so angle DCP1$= 80^\circ - 40.3^\circ = 39.7^\circ$

(Point D)

$$\sigma_n = 67 + (43)(\cos 39.7^\circ) = 100\,\text{MPa}$$

$$\tau_{nt} = (43)(\sin 39.7^\circ) = 27.5\,\text{MPa}$$

Stresses represented by point D' correspond to an angle $\theta = 130^\circ$ (or $2\theta = 260^\circ$):

(Point D')

$$\sigma_t = 67 + (43)(\cos 130^\circ) = 33.9\,\text{MPa}$$

$$\tau_{tn} = (43)(\sin 130^\circ) = 27.5\,\text{MPa}$$

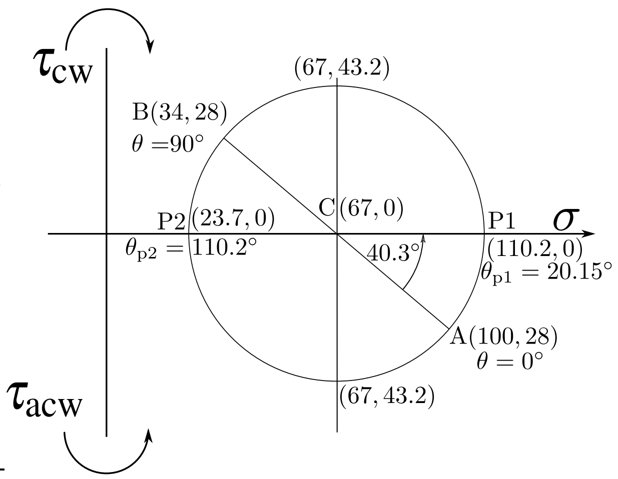

Figure 3.14: Principal stresses shown

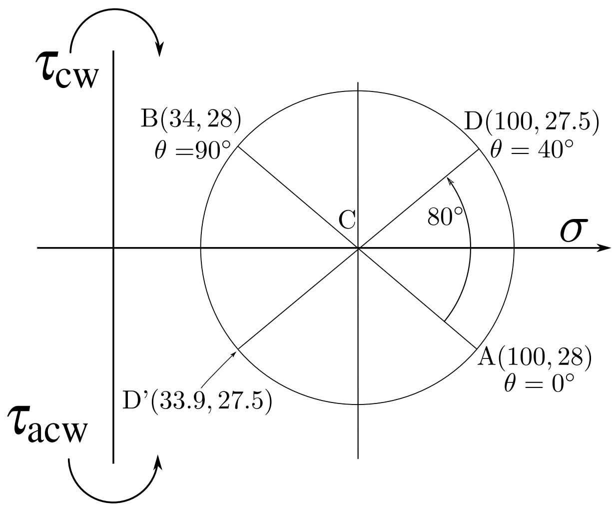

Figure 3.15: Transformation of stresses on Mohr's Circle

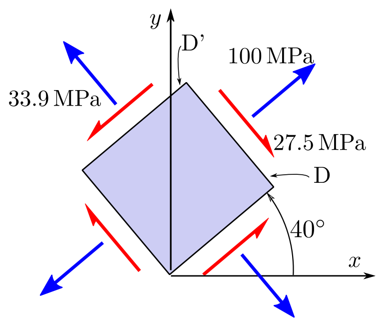

Figure 3.16: Transformed element

■