1. Introduction

The construction of Green Point Stadium, would be impossible without a knowledge of Mechanics.

1.1 Mechanics of Solids

Mechanics of Solids or Mechanics of Materials extends the ideas presented in Statics. The main assumptions of force and moment equilibrium in Statics still apply in Mechanics of Solids. The main difference being that in a Statics course it is assumed that bodies are rigid and cannot change size or shape. In a Mechanics course bodies can deform. In the real world bodies are deformable, and a study of mechanics allows calculation of these deformations which also allows predictions to be made about when things will break.

1.2 Presenting Quantities

There are two issues which will be addressed when representing any engineering quantity: with what unit will it be presented and how many digits or significant figures will be used.

1.2.1 Units

In science and math courses SI[1] derived units (Pa, N,..) and base units (K, m, s...) were the recommended way of representing units. Then any extreme number was represented using exponentiation for example 3.456×10^{-7} or 1.234×10^{5}. In science courses you can deal with objects on the scale of the universe to smaller than atoms, so avoiding prefixes helps to prevent errors in conversion.

In engineering we seldom go to these scales and it is most useful to work with units and prefixes (MPa, mm^2, kN) that are firstly more convenient to individual problems and secondly guide the engineer's intuition to whether a solution calculated is valid. An important restriction is that any unit and prefix should be consistently used from beginning to end in a problem. Changing units can invariably lead to errors.

1.2.2 Significant Figures

In this course it is recommended that all final numerical answers will be presented to four significant figures. In problems with many calculation steps, answers to intermediate steps can be presented with five significant figures to allow for checking and avoiding errors with rounding. For example 1234.5, 1234.0 and 0.012345 are all five significant figures.

[1] from the French Système international d'unités, the modern form of the metric system.

1.3 Review of Statics

To develop any ideas in Mechanics an understanding of Statics is required. The essential ideas of statics are summarised here. An in depth understanding is assumed and good sources of further information are by Beer[1] and Hellaby[2].

1.3.1 Equations of Equilibrium

All bodies in this course are assumed to be in static equilibrium. This means that the sum of the moments and the sum of the forces are equal to zero. This is described in vector form by:

$$\boxed{\begin{aligned} \sum \mathbf{F} &= 0 \\ \sum \mathbf{M} &= 0. \end{aligned}} \tag{1.1}$$

It is often more useful to represent these equations in component form in a cartesian coordinate system to help sum these moments and forces and determine any unknown quantities:

$$\boxed{\begin{array}{ccc} \sum F_x = 0, & \sum F_y = 0, & \sum F_z = 0, \\ \sum M_x = 0, & \sum M_y = 0, & \sum M_z = 0. \end{array}} \tag{1.2}$$

1.3.2 Freebody Diagrams

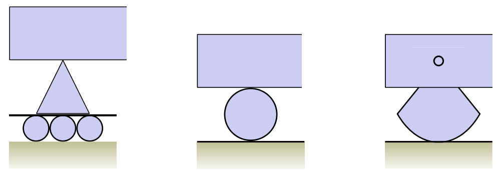
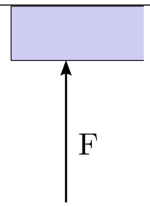
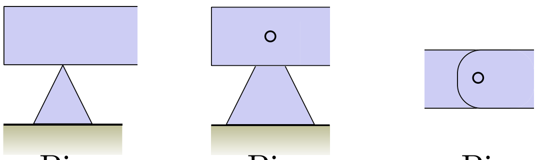
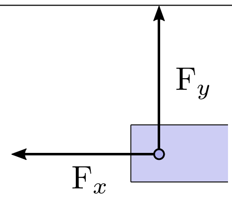
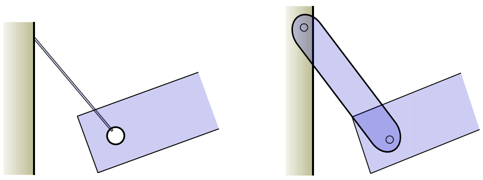
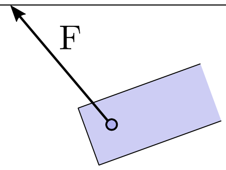
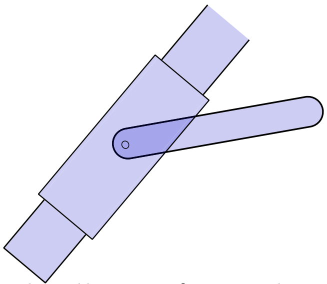
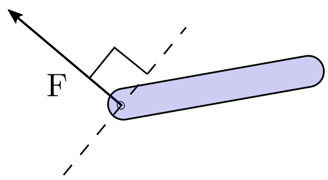
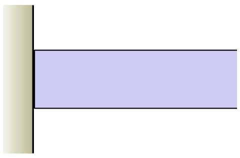
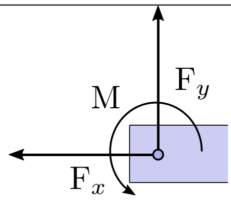
It would be impossible to calculate all unknown quantities from static equilibrium without the ability to draw **free-body diagrams**. Below are a summary of the steps to be followed when constructing such a diagram.

1. Determine the body which is important in analysis and separate it from any bodies attached to it.
2. Indicate all external applied forces and moments at their exact locations on the separated body. **Do not include internal forces.**
3. Show the forces and moments which oppose the displacement and rotation of the separated body. Surface forces that occur between two bodies in contact are called **reactions**. These reactions are shown in Table 1.1. From Newton's third law the force exterted by one body on a second body will be equal and opposite to the force exerted by the second body on the first body. The load may be applied anywhere on the bodies to cause these reactions.
4. Use the **static sign convention** in which forces have the same sign as the coordinate directions in which they are defined. Moment vectors are considered positive if they act in an anticlockwise direction.
5. Include dimensions on figure which will help in determining moments.

1.4 Analysis of Trusses

Trusses are an important class of engineering structure. The truss members are connected by pin jointed connections. Pin joints can resist or transfer forces but not moments between members. This means that the sum of forces at a pin is not zero but the sum of moments about a pin is zero. These members are considered to be slender. That means their length is much greater than their width or breadth.

Table 1.1: Support Reactions for Two Dimensional Bodies

Support or connection	Reaction
 Roller Roller Rocker	 One unknown force F
 Pin Pin Pin	 Two unknown forces F_x and F_y
 Short cable Short link or rod	 One unknown force F
 Pinned collar on frictionless rod	 One unknown force F
 Clamped, fixed or rigid	 One unknown moment M and two unknown forces F_x and F_y

When analysing truss members it is assumed that they can only resist a load along their length known as an **axial load**. They are two-force members. The forces that act on these members are equal, opposite and collinear. They can either be in tension and acted upon by a tensile force as in Figure 1.1(a) or in compression and acted on by a compressive force as in Figure 1.1(b).

Another assumption made about trusses is that the weights of the members are considered negligible i.e. not considered in calculations. This is usually a reasonable assumption since the weight of truss

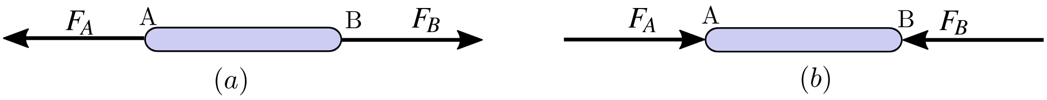


Figure 1.1: Two force members in tension(a) or compression(b)

member is small in comparison to the other loads it should carry.

Trusses where all the forces and members are in the same plane are called **planar trusses** whereas trusses which fill out three dimensions are called **space trusses**.

The basic building block of a truss is a triangle which is a **rigid** structure. It will not collapse under loading and the deformation due to internal strains are considered negligible. It is also called internally stable as seen in Figure 1.2(a). In contrast Figure 1.2(b) is unstable and will collapse under the loading shown.

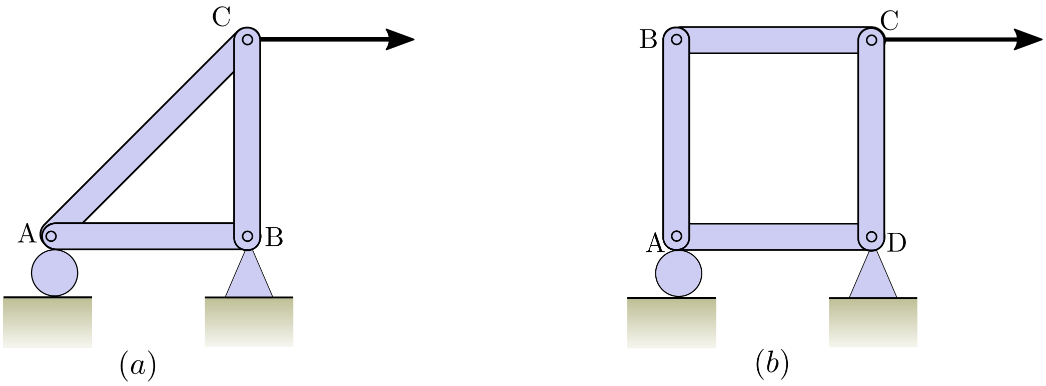


Figure 1.2: Internally stable or rigid structure(a) and unstable structure(b) are shown.

Trusses cannot resist a load perpendicular to their length otherwise known as a **lateral load**. In Figure 1.3 if ACD was a continuous member then the structure would no longer be a truss but a **frame**.

To solve for various forces in trusses there are two methods for calculating the internal forces: Method of Joints and Method of Sections. It should be noted using these methods do not include calculating the external forces which may be required.

1.4.1 Method of Joints

If an entire truss is in equilibrium then every pin in that truss must be in equilibrium. This can be represented by drawing a free body diagram of that pin and writing the equilibrium equations for each pin. Working in 2-dimensions, we have two equilibrium equations to satisfy $\sum F_x = 0$ and $\sum F_y = 0$. This idea is illustrated in the following example.

Example 1.1 Calculate the forces in all the joints in Figure 1.3.

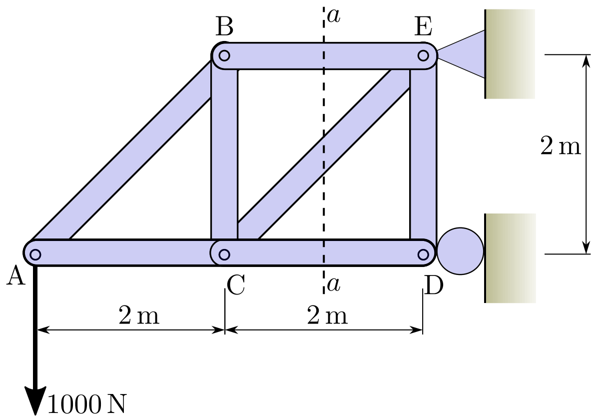


Figure 1.3: Truss structure with a concentrated load at A and simply supported at D and E. Members AC and CD are separate members but collinear. BC and ED are perpendicular to ACD.

Free body diagrams of the pins in Figure 1.3 are shown in Figure 1.4. To start the angle BAC is

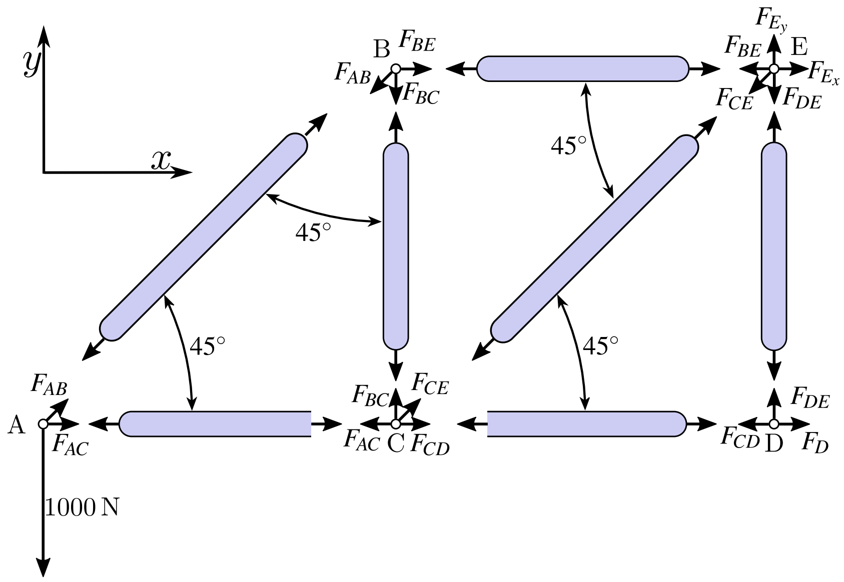


Figure 1.4: Method of joints illustrated for Figure 1.3 with all members assumed to be in tension.

calculated from the beam dimensions, knowing that BC is perpendicular to AC and the lengths of AC and BC are equal. It follows angle BAC = 45°. Similarly angle ECD = 45°.

Joint A is the only joint where there is a known force so we begin there. We start by assuming up and to the right is positive as indicated by the coordinate axes. Let's also assume all the joints are in tension. If any force turns out to be negative then the joint is in compression.

Joint A

$\Sigma F_y = 0 \Rightarrow F_{AB}\sin 45 - 1000 = 0 \Rightarrow F_{AB} = 1414.2\,\mathrm{N}$

$\Sigma F_x = 0 \Rightarrow F_{AB}\cos 45 + F_{AC} = 0 \Rightarrow F_{AC} = -1000.0\,\mathrm{N}$

Negative sign indicates F_{AC} is an opposite direction to that shown in Figure 1.4 so F_{AC} is in compression. We move onto Joint B since we now know a force F_{AB}. We know F_{AC} as well however there are more unknowns at Joint C so we do it later.

Joint B
$\Sigma F_x = 0 \Rightarrow -F_{AB}\sin 45 + F_{BE} = 0 \Rightarrow F_{BE} = 1000.0\,\mathrm{N}$
$\Sigma F_y = 0 \Rightarrow -F_{AB}\cos 45 - F_{BC} = 0 \Rightarrow F_{BC} = -1000.0\,\mathrm{N}$
F_{BC} is negative so member in compression and direction drawn is wrong.
Joint C
$\Sigma F_y = 0 \Rightarrow F_{BC} + F_{CE}\cos 45 = 0 \Rightarrow F_{CE} = 1414.2\,\mathrm{N}$
$\Sigma F_x = 0 \Rightarrow -F_{AC} + F_{CE}\sin 45 + F_{CD} = 0 \Rightarrow F_{CD} = -2000.0\,\mathrm{N}$
Joint D
$\Sigma F_x = 0 \Rightarrow -F_{CD} + F_D = 0 \Rightarrow F_D = -2000.0\,\mathrm{N}$
There is *no force in the vertical direction since Joint D is a roller* so F_{DE} is zero and there is no axial force in member DE. Member DE is therefore called a **zero-force** member.
Joint E
$\Sigma F_y = 0 \Rightarrow F_{E_y} - F_{CE}\cos 45 = 0 \Rightarrow F_{E_y} = 1000.0\,\mathrm{N}$
$\Sigma F_x = 0 \Rightarrow -F_{BE} - F_{CE}\cos 45 + F_{E_x} = 0 \Rightarrow F_{E_x} = -2000.0\,\mathrm{N}$

■

1.4.2 Method of Sections

The method of joints is useful when all the forces in a truss structure have to be determined. If only a few forces have to be determined, the method of sections may be more efficient. A section of the truss is isolated and a free body diagram is drawn for that section. The following example illustrates this idea. The isolated section will be in moment equilibrium.

■ **Example 1.2** Calculate the force in members CD and CE in Figure 1.3.

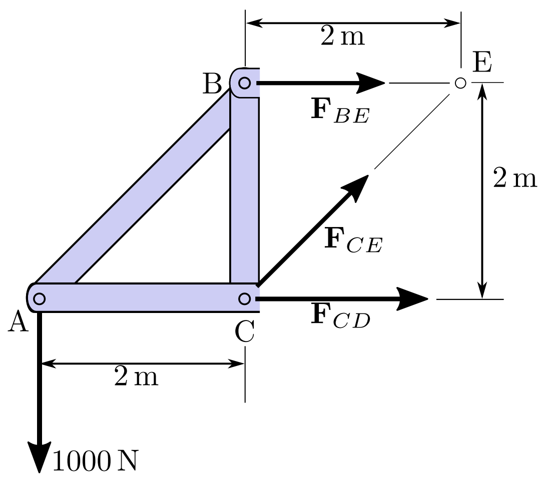


Figure 1.5: Method of sections through a-a in Figure 1.3

The whole section should be in moment equilibrium. Assume counterclockwise moments are positive and F_{CD} is positive as indicated in Figure 1.5.
$\Sigma M_E = 0 \Rightarrow 2 \times F_{CD} + (2+2) \times 1000 = 0 \Rightarrow F_{CD} = -2000.0\,\mathrm{N}$
This indicates F_{CD} is negative and direction is opposite to what is shown in Figure 1.5.
$\Sigma F_y = 0 \Rightarrow -1000 + F_{CE}\cos 45 = 0 \Rightarrow F_{CE} = 1414.2\,\mathrm{N}$ ■

1.5 Distributed Loading

Loads applied to structures are not applied to points but distributed over lines, areas and even volumes. In many instances it is convenient to replace the distributed load with a single resultant force which is **statically equivalent**. By statically equivalent we mean that the sum of forces and sum of moments for the two load cases are the same. The reactions at the same support is the same for statically equivalent systems.

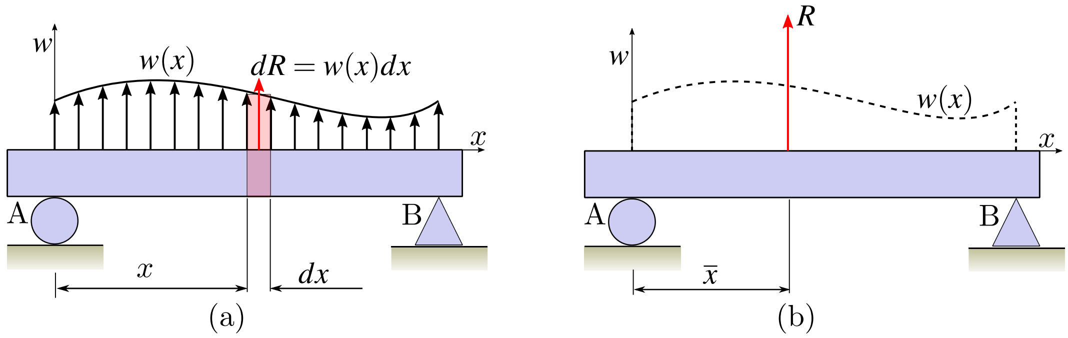


Figure 1.6: Distributed load (a) and statically equivalent force with location (b) shown.

The magnitude of the resultant force, R, is the sum of the differential forces $dR = w(x)dx$:

$$R = \int w(x)dx \tag{1.3}$$

The moment of the forces about a normal axis must be the same for the two cases.

$$R\bar{x} = \int xw(x)dx$$

$$\Rightarrow \bar{x} = \frac{\int xw(x)dx}{\int w(x)dx} \tag{1.4}$$

The resultant is located at the **centroid** of the area under the distributed load. These ideas are illustrated with the following example.

■ **Example 1.3** Determine the magnitude and location of the equivalent resultant force acting on the shaft in Figure 1.7.

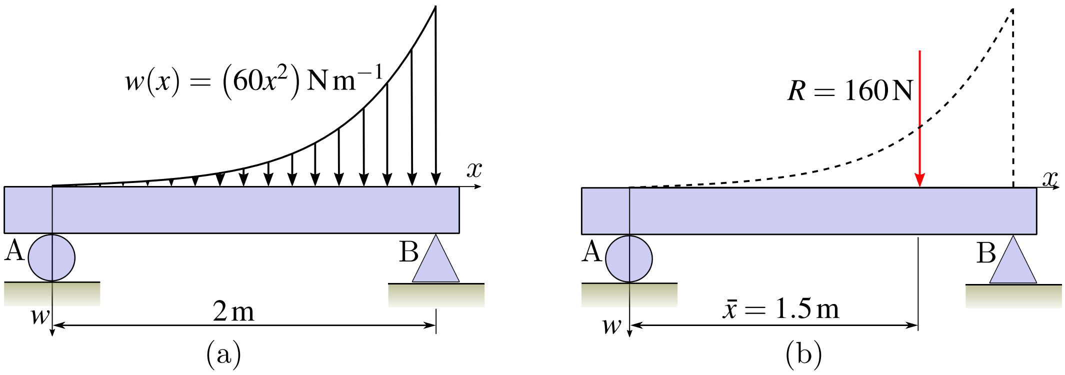


Figure 1.7: Distributed load(a) and statically equivalent force with location(b) shown.

The problem can be solved by integration since $w(x)$ is given.

$$R = \int w(x)dx = \int_0^2 60x^2 dx = 160.0\,\mathrm{N}$$

The location of $\bar{x}$ is measured from A

$$\bar{x} = \frac{\int x w(x) dx}{\int w(x) dx} = \frac{\int_0^2 x 60x^2 dx}{\int_0^2 60x^2 dx} = 1.500\,\mathrm{m}$$

■