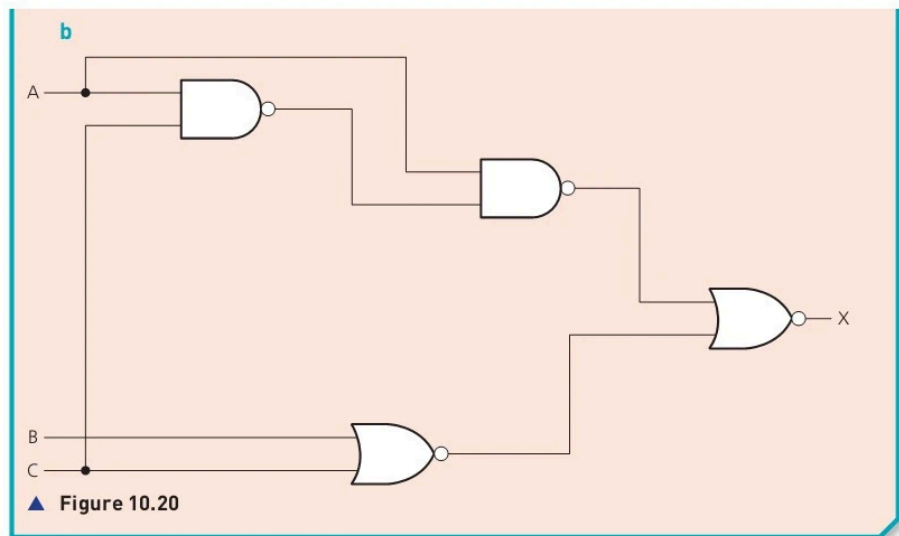


## 10 BOOLEAN LOGIC



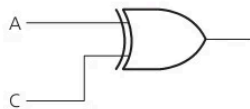
### ? Example 3

A logic circuit can be represented by the following logic expression:  $(A \text{ XOR } C) \text{ OR } (\text{NOT } C \text{ NAND } B)$

Produce a logic circuit and a truth table from the above statement.

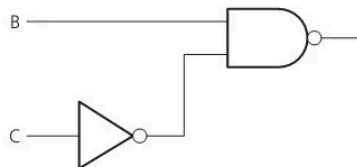
In this example we have a connecting logic gate which is **OR**.

So, if we produce one half of the circuit from  $(A \text{ XOR } C)$  we get:



▲ Figure 10.21

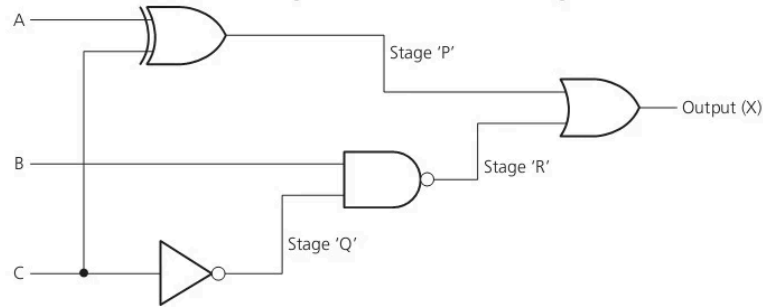
The other half of the circuit is found from  $(\text{NOT } C \text{ NAND } B)$ :



▲ Figure 10.22

## 10.3 Logic circuits, logic expressions, truth tables and problem statements

If we now combine these together to form the final logic circuit:



▲ Figure 10.23

The truth table is shown:

▼ Table 10.13

Input values			Values at stages:			Output
A	B	C	'P'	'Q'	'R'	X
0	0	0	0	1	1	1
0	0	1	1	0	1	1
0	1	0	0	1	0	0
0	1	1	1	0	1	1
1	0	0	1	1	1	1
1	0	1	0	0	1	1
1	1	0	1	1	0	1
1	1	1	0	0	1	1

### ? Example 4

Look at the two truth tables below; in each case produce a logic expression and the corresponding logic circuit:

a

▼ Table 10.14

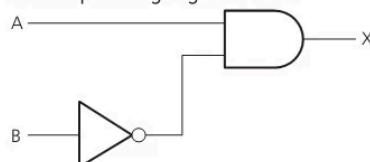
Inputs		Output
A	B	X
0	0	0
0	1	0
1	0	1
1	1	0

To produce the logic statement, we only concern ourselves with the truth table row where the output value is 1. In this case,  $A = 1$  and  $B = 0$  which gives the logical expression:

**A AND NOT B**

So we have the logic expression: **A AND NOT B**

(Note that this could be written as  $A \cdot \bar{B}$  in Boolean.) It is now possible to draw the corresponding logic circuit:



▲ Figure 10.24

## 10 BOOLEAN LOGIC

b

▼ Table 10.15

Inputs		Output
A	B	X
0	0	0
0	1	1
1	0	0
1	1	1

This time we have **two** rows where the output is 1; this gives the following logical expressions:

(NOT A AND B)

(A AND B)

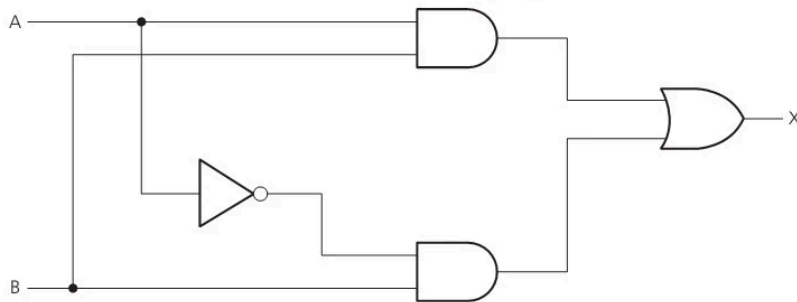
We now join these together using an OR gate to give:

**(NOT A AND B) OR (A AND B)**

So we have the logic expression: (NOT A AND B) OR (A AND B)

(This could be written as:  $\bar{A} \cdot B + A \cdot B$ )

It is now possible to draw the corresponding logic circuit:



▲ Figure 10.25



## Example 5

a Which Boolean expression is represented by the following truth table?

▼ Table 10.16

Input values			Output
A	B	C	X
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

We only need to consider those rows where the output is a 1. This gives us the following three logic expressions:

(NOT A AND NOT B AND NOT C)  
 (A AND NOT B AND NOT C)  
 (A AND B AND NOT C)

If we now join the three expressions with an OR gate, we end up with the final logic expression:

**(NOT A AND NOT B AND NOT C) OR (A AND NOT B AND NOT C) OR (A AND B AND NOT C)**

### 10.3 Logic circuits, logic expressions, truth tables and problem statements

- b i** Which logic expression is represented by the following truth table?  
**ii** Show that your logic expression in part **i** is the same as:  $(B \text{ AND } C) \text{ OR } (A \text{ AND } C) \text{ OR } (A \text{ AND } B)$

▼ **Table 10.17**

A	B	C	X
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

- i** We only need to consider those rows where the output is a 1. This gives us the following four Boolean expressions:

(NOT A AND B AND C)  
 (A AND NOT B AND C)  
 (A AND B AND NOT C)  
 (A AND B AND C)

If we now join the four expressions with an OR gate we end up with the following logic expression:

$(\text{NOT A AND B AND C}) \text{ OR } (A \text{ AND NOT B AND C}) \text{ OR } (A \text{ AND B AND NOT C}) \text{ OR } (A \text{ AND B AND C})$

- ii** To show that  $(B \text{ AND } C) \text{ OR } (A \text{ AND } C) \text{ OR } (A \text{ AND } B)$  produces the same output as that shown in part **i** we need to produce a new truth table and show that the output is the same as the one in the given truth table:

▼ **Table 10.18**

A	B	C	B AND C	A AND C	A AND B	$(B \text{ AND } C) \text{ OR } (A \text{ AND } C) \text{ OR } (A \text{ AND } B)$	X
0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0
0	1	0	0	0	0	0	0
0	1	1	1	0	0	1	1
1	0	0	0	0	0	0	0
1	0	1	0	1	0	1	1
1	1	0	0	0	1	1	1
1	1	1	1	1	1	1	1

As the second truth table shows, the outputs from logic expressions are both the same; thus the logic expression  $(B \text{ AND } C) \text{ OR } (A \text{ AND } C) \text{ OR } (A \text{ AND } B)$  gives the same output as the logic expression in part **i**.