

CS 4720/5720 Design and Analysis of Algorithms

Homework #2

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Answers to homework problems:

1. Consider the following graphs in Figure 1.1

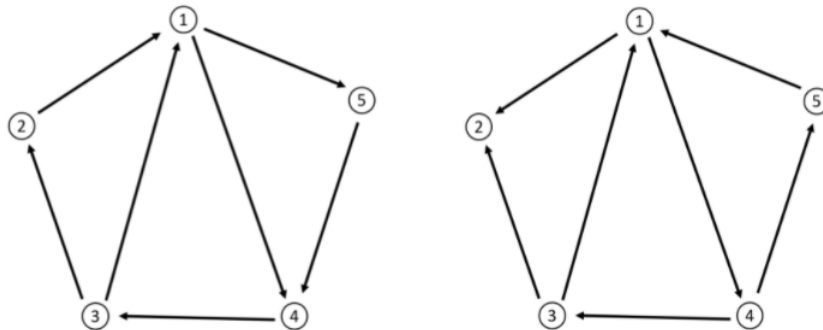


Figure 1.1

- (a) Are these graphs strongly connected? Explain why or why not.
 (b) Are these graphs aperiodic? If a graph is periodic, compute its period.
2. Consider the following weighted adjacency matrix in Figure 1.2:

$$A = \begin{array}{|c|c|c|c|} \hline 0.5 & 0 & 0.5 & 0 \\ \hline 0.1 & 0.8 & 0.1 & 0 \\ \hline 0 & 0.05 & 0.85 & 0.1 \\ \hline 0.23 & 0.07 & 0 & 0.7 \\ \hline \end{array}$$

Figure 1.2

- (a) Is this matrix row-stochastic?
 (b) Draw the graph corresponding to this matrix.
 (c) Is the graph strongly connected?
 (d) Is the graph aperiodic?
3. Consider the following graph in Figure 1.3

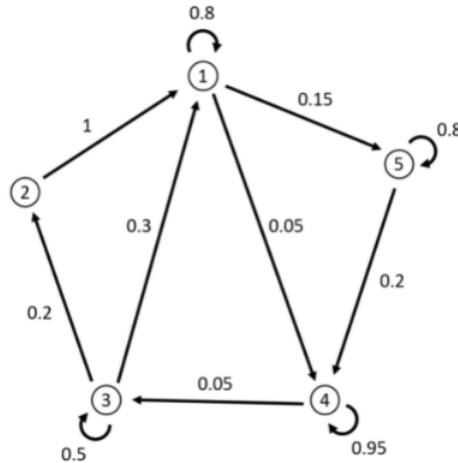


Figure 1.3

- (a) Write the weighted adjacency matrix corresponding to this graph. Verify that the graph you wrote is row-stochastic.
 - (b) By hand, using matrix multiplication, perform 1 step of the DeGroot opinion dynamic model on this graph with initial opinions of $(1, 1, 0.5, 0, 0)$.
 - (c) In the DeGroot opinion dynamics model, which node's initial opinion gets the most weight in society's final opinion? Which node gets the least weight? (it is highly recommended that you use computer code to answer this question).
4. Consider again the graph from problem 2 (Figure 1.2).
- (a) By hand, using matrix multiplication, perform 1 step of the Friedkin-Johnsen opinion dynamic model on this graph with initial opinions of $(1, 1, 0.5, 0, 0)$ and lambda values of $(0.9, 0.1, 0.8, 1, 0.5)$.
 - (b) As we discussed in class, in the Friedkin-Johnsen model, nodes usually have differing opinions even after a long time. Keeping the lambda values from part A, experiment with this graph and try to find an assignment of initial opinions that results in the most different limiting opinions. That is, what equilibrium on this graph can have the most disagreement out of all equilibria? You will likely want to use compute code to solve this problem. Full credit is possible if you show understanding and effort; I will award 1 bonus point if you can show that your answer is the most disagreement possible.