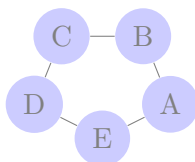


CS 4720/5720 Design and Analysis of Algorithms Homework #1 Student: (Robert Denim Horton)

Answers to homework problems:

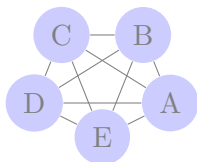
1.

- (a) Given the formal definition of a pivotal node, a node can be defined as such when the node of interest exists along every shortest possible path in a given pair of nodes. For example, given a graph with the set of nodes S_0 with defined nodes $\{A, B, C, D, E\}$. The graph could be represented as;



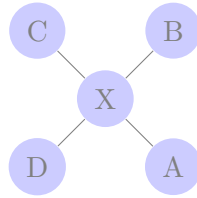
As we can see in this diagram all the nodes are pivotal nodes. Starting with node pivotal node A , its only pivotal node pair is E and B , for node B its only pivotal node pair is C and A , for node C its only pivotal node pair is B and D , its only pivotal node pair is C and E , and lastly for node E its only pivotal node pair is D and A . Here we can see that every node that exists has at least one pair of nodes that also exists in the graph.

- (b) For a pivotal node to have two different pairs of pivotal nodes, we can again define a node with two different pairs of pivotal nodes as a node that is along the fastest path for two nodes but with the added implication that this node is along one other pair of pivotal nodes. With the same set, S_0 , from part a we can construct a graph to be represented as,



We can see that for every node in the graph, it has at least two different pairs of pivotal nodes. Node A has its first pivotal nodes as E and B and second pair of pivotal nodes C and D , node B has its first set of pivotal nodes C and A and second pair of pivotal nodes E and D , node C has its first set of pivotal nodes E and A and its second set of pivotal nodes B and D , node D has its first set of pivotal nodes B and A and second pair of pivotal nodes C and E , node E has its first pair of pivotal nodes D and A and its second set of pivotal nodes C and B and its second set of pivotal nodes A and D .

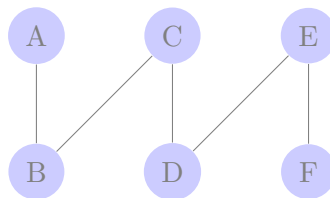
- (c) For a graph comprised of atleast 4 nodes where a single node is pivotla for any pair of nodes that comprise a pivotal pair. So with a different set, S_1 , of nodes $\{A, B, C, D, X\}$ we can use a graph to represnt a graph where node X is the node of intrest,



Here we that node X has a pivotla pair with every node that is not X in the graph. So node X has pivotal pairs A and C , pivotal pair B and D , pivotal pair A and B , pivotal pair C and D , pivotal pair D and A , and pivotal pair C and B . So six possible pairs in total.

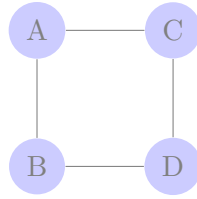
2.

- (a) With the formal definetion of a special type of node called a *gatekeeper node*, we can identify gatekeeper node(s) as a node that must be connected by two other nodes that have no other possbile paths or connections between the two connected nodes and are limited to traversing through the gatkeeper node to get to one another Given a set of nodes, S_2 , comprised of nodes $\{A, B, C, D, E, F\}$, we can build a graph with a subset of gatekeeper nodes that comprises more than half the set of nodes in S_1 .



As we can see, the subset of nodes $\{B, C, D, E\}$ are all gatekeeper nodes tha must be traversed in order for specific pair of nodes to travel to one another via edges. Further explaining we see that for the of nodes A and C they must travers through node B , qualifying node B as a gatekeeper node. The same can be said for nodes C , D , and E each be a gatekeeper for their own pair of nodes that they are connected to. C is a gatekeeper for nodes B and D , D is a gatekeeper for nodes C and E , and E is a gatekeeper for nodes D and F .

- (b) Extendeding the deffinetion of a gatekeeper there are also special kinds of gatekeeper nodes called an *local gatekeeper node*. These types typed of nodes are also connected by a pair of nodes but does not necessarily be traversed to get from one of node pairs to the other. Given the set, S_3 , of nodes $\{A, B, C, D\}$ we can build a graph that consists of **only local gatekeeper nodes**. The graph would look like;

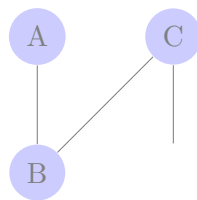


As can be seen from this graph, we notice that every pair of nodes that aren't directly connected have multiple paths or traversal to get to one another. For example, nodes A and D have two paths of traversal to get to one another. They can traverse through the first local gatekeeper, B , or they can traverse to each other through the second local gatekeeper C . The logic can be tested on any pairs of nodes in this graph, that are not directly connected, and the conclusion will be that for every one of these pairs of nodes, there are two local gatekeeper nodes. After stepping through the logic for every pair of nodes we will find that every node in this graph is a local gatekeeper node.

3.

- (a) To first construct such a graph let's first formally define *diameter* and *average distance* of a graph. The **diameter** of a graph can be found by simply finding the longest shortest path that connects to nodes in the graph. To find the **average distance** for a smaller and simpler looking graph, we can look through each pair of nodes that has a connection (or series of connections) and then add up all the edges that connect each combination of possible pairs of nodes. We then divide this sum by the number of nodes that exist on the graph. With the provided knowledge of how to find the *diameter* and *average distance* we can then start to build a graph where the diameter is at least three times larger than the average distance.

Given a set, S_4 , of nodes $\{A, B, C, D\}$ we construct the first part of the graph where the nodes A , B , and C are closely connected by a single node in between consecutive pairs. as provided below,



- (b) Describe how you could extend your construction to produce graphs in which the diameter exceeds the average distance by as large a factor as you'd like. (That is, for every number c , can you produce a graph in which the diameter is more than c times as large as the average distance?)