

# How do we think about social nets using graphs?

Monday, January 10, 2022 4:26 PM

Overarching question: What kinds of "social networks" phenomena can we describe using graph theory?

Deep challenge: social networks are

- Huge
- Complex
- Messy
- Hard to measure

So in using graph theory (which is regimented, precise, abstract) to study social nets, need to be careful to ask the right questions.

Major concept to study in this course:

- Local properties of nodes/links  $\leftrightarrow$  global properties of graphs/networks
- However, what local properties matter?

Motivating question:

- 1960s, Mark Granovetter found empirically that people tended to hear about new jobs from distant acquaintances, not close friends. Can we develop graph theory to understand why?

## Triadic Closure

Tuesday, January 11, 2022 2:26 PM

Q: what kinds of structure might social networks have?

Already conjectured: global structure

↳ giant component

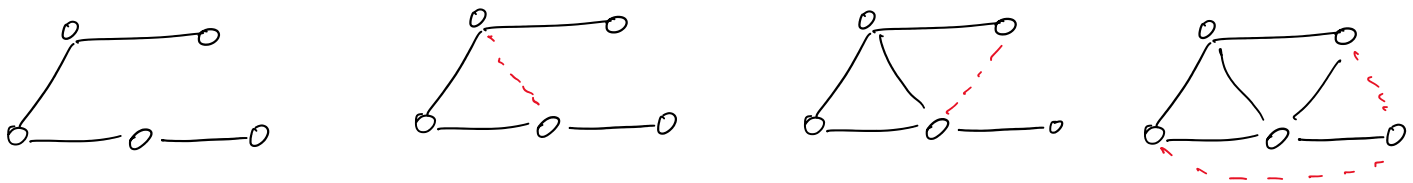
↳ small diameter

what about local structure?

Idea: If I'm close friends w/ Alice and Bob, then

1. eventually they'll run into each other
2. they're more likely to trust each other
3. I may have an incentive to introduce them.

Thus, over time, we may see networks evolve like this:



Concept: "Triadic Closure."

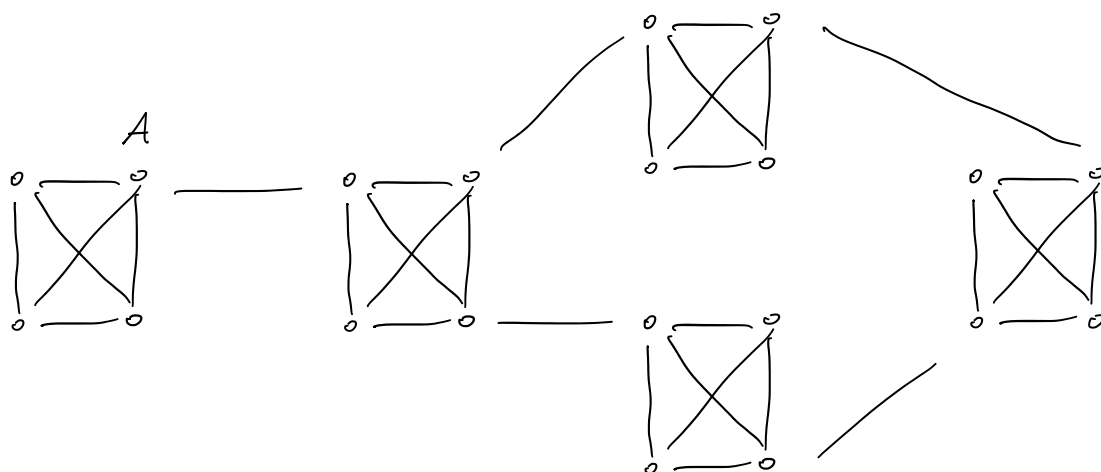
Now: what kind of statement is this?

- qualitative
- intuitive
- Cartoony
- can prompt further investigation

## What about hearing about jobs?

Tuesday, January 11, 2022 2:37 PM

Triadic closure: clustering leads to clustering,  
so we might expect nets like this:



many clusters that are tightly connected,  
joined by fewer links.

Q: if new info comes to someone, is that  
person likely to be in my group?

only a  $\frac{1}{5}$  chance!

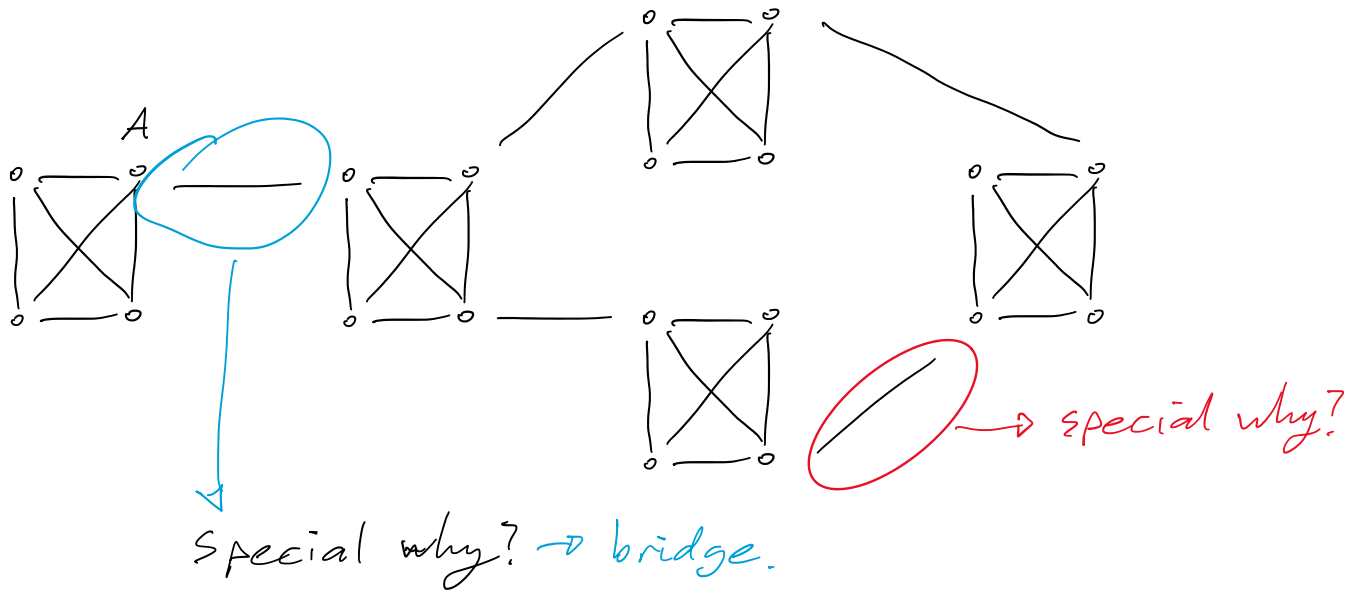
↳ if info spreads along network links, this  
means info is likely to come to me  
via one of those weird connector  
links.

Q: what makes them special?

# Bridges

Tuesday, January 11, 2022

2:51 PM



Def: if deleting a link makes a graph disconnected, the link is called a bridge.

Q: how many bridges above? 1.

Def: if deleting a link makes its endpoints farther apart, the link is called a local bridge.

Alternate:  $(A, B)$  is a local bridge if  $A$  and  $B$  have no common friends.

From before: local bridges are crucial for spreading new information!

Intuition: everyone in my local group already knows what I know!

## But what about job info?

Wednesday, January 12, 2022 10:02 AM

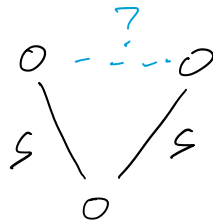
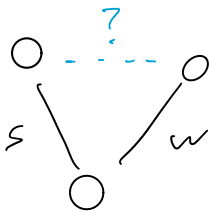
Need to distinguish betw. strong/weak friends.  
Label each edge! Note: have not done this yet!

↳ but will be very normal  
someday.

s = "strong", close friend

w = "weak", acquaintance

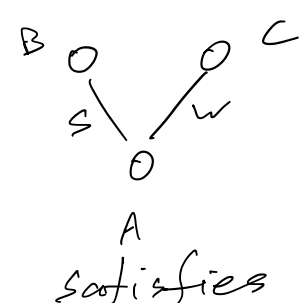
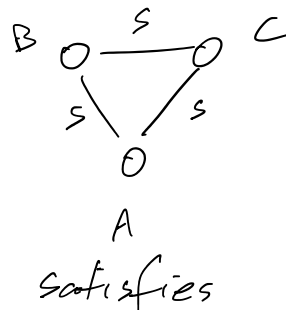
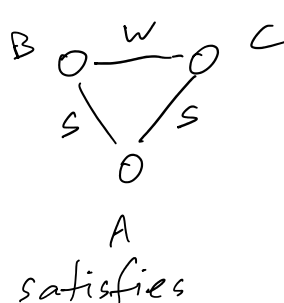
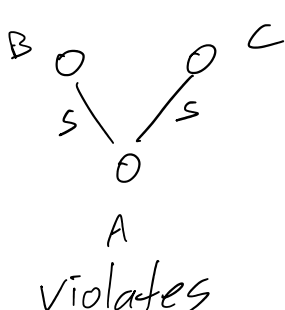
Q: when does triadic closure make the most sense?



↑  
this triad would be the most  
likely to close.

Approach: formalize this, chase the consequences!

Def: Node A violates the strong Triadic Closure  
Property (STC) if has strong links to both B  
and C, but B and C have no link between  
them.



# Does STC make sense?

Wednesday, January 12, 2022 10:02 AM

Perspective: STC is deliberately extreme.

"Caricature" of a real phenomenon we expect to see in real networks.

Note: not nearly as extreme as it could be!  
could say "all triads are closed."

instead, STC says "some triads are closed."

General approach in this class: think abt  
overarching problem, then formalize a "cartoon"  
model of it!

Q: How do you know which cartoon model is good?

↳ very hard to say ahead of time!

↳ learn the "art" over time

↳ judge assumption's value somewhat by consequences

Q: What make consequences interesting?

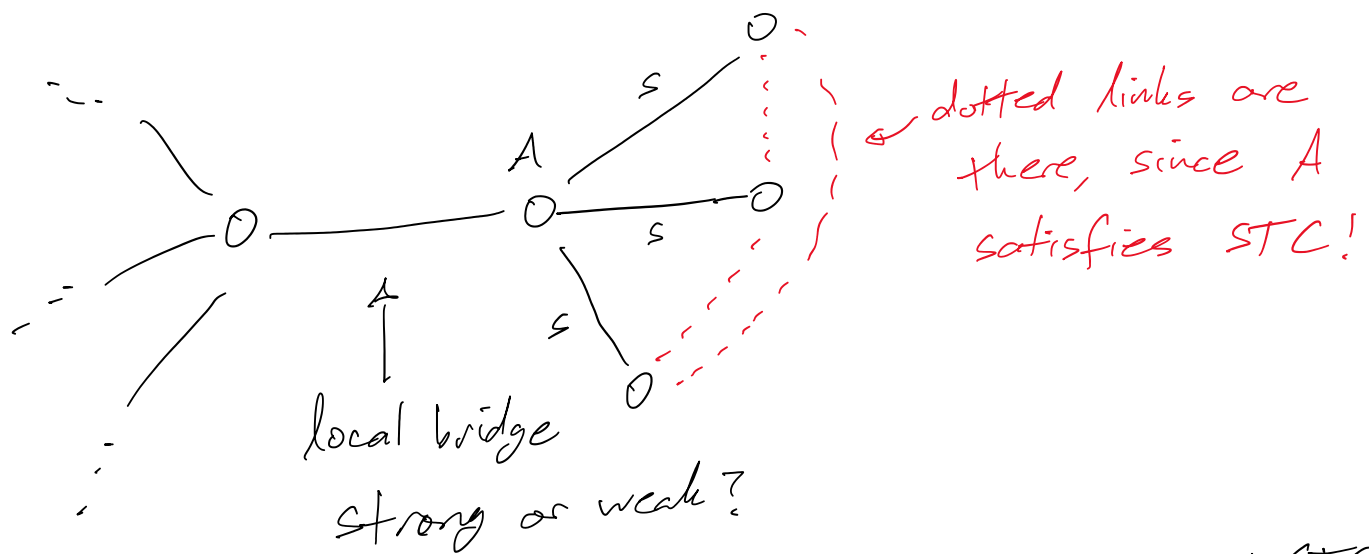
- not already obvious
- counterintuitive
- "robust"
- explain real observed phenomenon

# Consequences of STC

Wednesday, January 12, 2022 10:02 AM

Fact: if node A has  $\geq 2$  strong ties & satisfies STC, then any local bridge it is connected to is a weak tie.

Hypothesis I: node A has  $\geq 2$  strong ties



MUST be weak, since strong + STC  $\Rightarrow$  not a local bridge.

So: — local bridges are likely sources of new info

— local bridges are likely to be weak ties

— thus, a disproportionate amt of new info comes via weak ties.

# Think about the theorem more

Wednesday, January 12, 2022 10:02 AM

Fact: if node  $A$  has  $\geq 2$  strong ties + satisfies STC,  
 $P$   $Q$

then any local bridge it is connected to  
is a weak tie.

$R$

$$P \wedge Q \Rightarrow R.$$

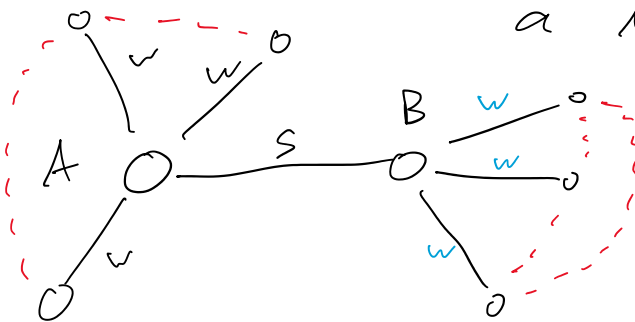
Approach: when you see new theorem, pick apart  
its assumptions so you understand it.

Q1: what if  $P$  fails? (i.e.,  $P = \text{False}$ ).

then  $P \wedge Q = \text{False}$ , so  $R$  could be false.

$P = \text{False}$ :  $A$  has  $\leq 1$  strong tie.

Can we make that strong tie  
a local bridge?



Sure!

But what does that  
say abt  $B$ ?

only 1 strong tie  
also!



## Think about the theorem more pt 2

Wednesday, January 12, 2022 10:02 AM

Fact: if node A has  $\geq 2$  strong ties + satisfies STC,  
 $P$   $Q$

then any local bridge it is connected to  
is a weak tie.

$R$

$$P \wedge Q \Rightarrow R.$$

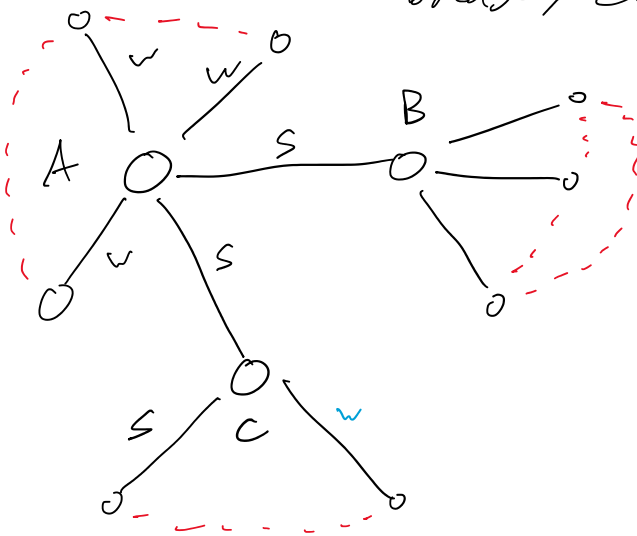
Approach: when you see new theorem, pick apart  
its assumptions so you understand it.

Q1: what if Q fails? (i.e.,  $Q = \text{False}$ ).

then  $P \wedge Q = \text{False}$ , so  $R$  could be false.

$Q = \text{False}$ : A violates STC.

Now can we make a strong local  
bridge, even if A has  $\geq 2$  strong?



Sure!

But what does that  
say abt B + C?  
both also must violate  
either P or Q  
(or both)!

## Last note on this

Wednesday, January 12, 2022 10:02 AM

Read section 3.3 in the EK book: has a lot of info on empirical validation of this idea!

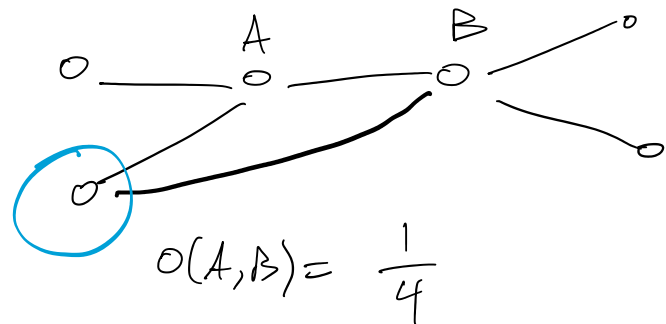
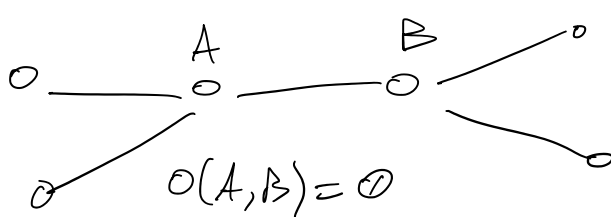
Empirical challenge is interesting; in real life

- Ties are not just strong/weak; continuum
- Edges could be "soft" local bridges; important but not sharply so

Cell phone call data:

- Proxy for strength: total time spent on calls
- Proxy for bridge-ness: "neighborhood overlap" of edge:

$$O(A,B) = \frac{\# \text{ nodes connected to both } A, B}{\# \text{ nodes connected to at least one of } A, B}$$



low overlap  $\leftrightarrow$  "local bridge-ish"

- Interesting fact: tie strength is positively correlated to neighborhood overlap! (see Fig 3.7 in EK book)

