CS 4720/5720 Design and Analysis of Algorithms

Homework #2

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Answers to homework problems:

1. Consider the following graphs in Figure 1.1

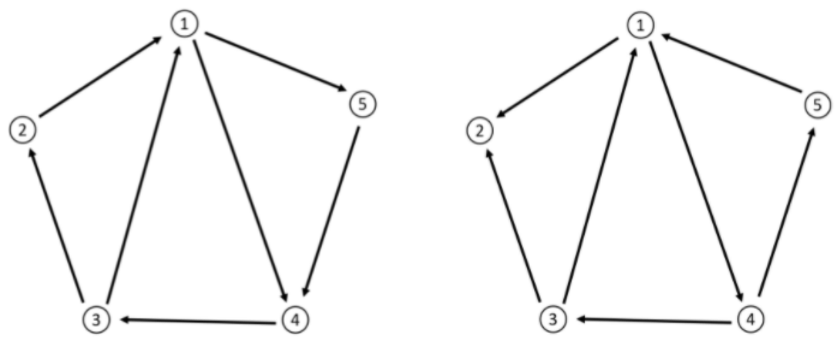

Figure 1.1

   (a) Are these graphs strongly connected? Explain why or why not.

   (b) Are these graphs aperiodic? If a graph is periodic, compute its period.

Answers:

1. Consider the following graphs in Figure 1.1

   a In Figure 1.1 we see two different directed graphs. To check if a graph is strongly connected or not a *depth first search* can be done on the graph starting from some node, x_i, in set, X, of nodes that exists in the graph G. The depth search is a pretty common algorithm the steps through the graph trying to visiting each pair of nodes through the edges that exist. If there exists a path in between any pair of nodes we call the graph strongly connected. As we can see looking at the graph on the left that there exists a path in-between all the nodes that traverses along the outside of the graph. So we can say that this graph is strongly connected. As for the graph on the left we see that node 2 is only visited and has no edges leaving it so their fore any pair of nodes that is looking for a path from node 2 to other node does not exsist and we can call this graph weakly connected.

   b As for checking if the two graphs in Figure 1.1 are aperiodic or not, in mathematical areas of graph theory, directed graphs are said to be aperiodic if there is no integer, k, that is grater than 1 that divides the length of every cycle of the graph. This is to say that for the matrix made up for the graph on the left,

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix},$$

we can call it periodic

$$\begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

2. Consider the following weighted adjacency matrix in Figure 1.2:

$A =$

0.5	0	0.5	0
0.1	0.8	0.1	0
0	0.05	0.85	0.1
0.23	0.07	0	0.7

Figure 1.2

(a) Is this matrix row-stochastic?
(b) Draw the graph corresponding to this matrix.
(c) Is the graph strongly connected?
(d) Is the graph aperiodic?

Answers:

2. Consider the following weighted adjacency matrix in Figure 1.2:

a Is this matrix row-stochastic?
b Draw the graph corresponding to this matrix.
c Is the graph strongly connected?
d Is the graph aperiodic?

3. Consider the following graph in Figure 1.3

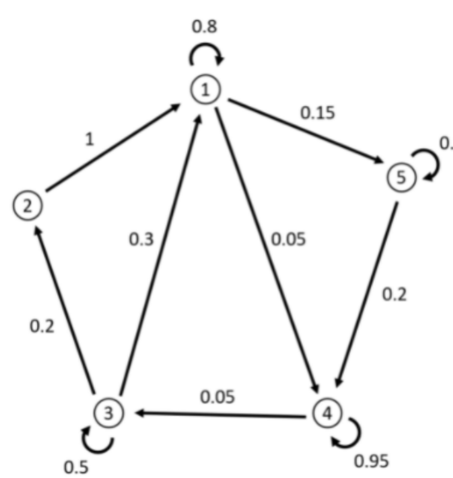

Figure 1.3

(a) Write the weighted adjacency matrix corresponding to this graph. Verify that the graph you wrote is row-stochastic.

(b) By hand, using matrix multiplication, perform 1 step of the DeGroot opinion dynamic model on this graph with initial opinions of (1, 1, 0.5, 0, 0).

(c) In the DeGroot opinion dynamics model, which node's initial opinion gets the most weight in society's final opinion? Which node gets the least weight? (it is highly recommended that you use computer code to answer this question).

Answers:

3. Consider the following graph in Figure 1.3

   (a) Write the weighted adjacency matrix corresponding to this graph. Verify that the graph you wrote is row-stochastic.

   (b) By hand, using matrix multiplication, perform 1 step of the DeGroot opinion dynamic model on this graph with initial opinions of (1, 1, 0.5, 0, 0).

   (c) In the DeGroot opinion dynamics model, which node's initial opinion gets the most weight in society's final opinion? Which node gets the least weight? (it is highly recommended that you use computer code to answer this question).

1. Consider again the graph from problem 2 (Figure 1.2).

   (a) By hand, using matrix multiplication, perform 1 step of the Friedkin-Johnsen opnion dynamic model on this graph with initial opinions of (1, 1, 0.5, 0, 0) and lambda values of (0.9, 0.1, 0.8, 1, 0.5).

   (b) As we discussed in class, in the Friedkin-Johnsen model, nodes usually have differing opinions even after a long time. Keeping the lambda values from part A, experiment with this graph and try to find an assignment of initial opinions that results in the most different limiting opinions. That is, what equilibrium on this graph can have the most disagreement out of all equilibria? You will likely want to use compute code to solve this problem. Full credit is possible if you show understanding and effort; I will award 1 bonus point if you can show that your answer is the most disagreement possible.

Answers:

4. Consider again the graph from problem 2 (Figure 1.2).

   (a) By hand, using matrix multiplication, perform 1 step of the Friedkin-Johnsen opnion dynamic model on this graph with initial opinions of (1, 1, 0.5, 0, 0) and lambda values of (0.9, 0.1, 0.8, 1, 0.5).

   (b) As we discussed in class, in the Friedkin-Johnsen model, nodes usually have differing opinions even after a long time. Keeping the lambda values from part A, experiment with this graph and try to find an assignment of initial opinions that results in the most different limiting opinions. That is, what equilibrium on this graph can have the most disagreement out of all equilibria? You will likely want to use compute code to solve this problem. Full credit is possible if you show understanding and effort; I will award 1 bonus point if you can show that your answer is the most disagreement possible.