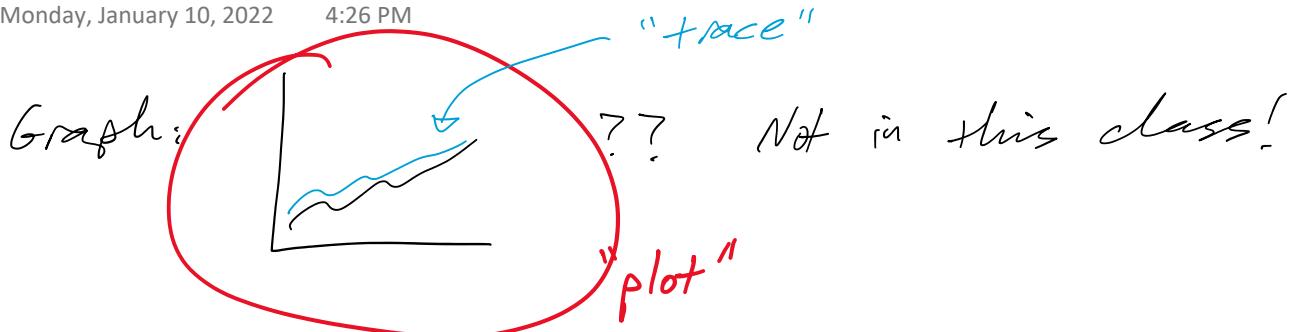


What is a graph?

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4:26 PM



Graph: data structure for expressing "pair" relationships.

To watch how I do this in Algorithms on YouTube: [CS 5720 Lecture 12: Graph Refresher](#)

What is a pair relationship?

- Friends
- Enemies
- start/end of a road
- a course and its prerequisite

:

stuff that we can draw a network for!

Graph, formally:

a tuple (V, E)

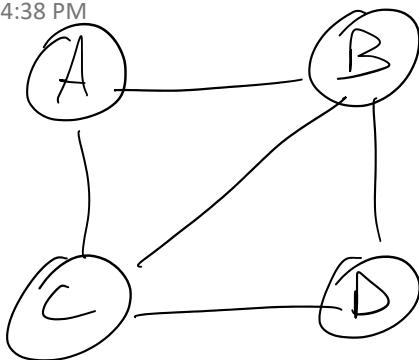
\hookrightarrow set of "edges" (links)
 \hookrightarrow set of "vertices" "connections"
(nodes, agents)
"stuff to connect"

Graph Basics 1

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Example:



$$V = \{A, B, C, D\}$$

no rules here!

- people
- access pts
- road intersections
- DB entries

$$E = \{(A, B), (A, C), (B, C), (B, D), (C, D)\}$$

rule: every element of E is a pair of elements of V .

$$\text{Formally: } E \subseteq V \times V$$

Aside: set theory

given sets A, B :

make new sets:

$A \cap B$ - intersection

$A \cup B$ - union

$A \setminus B$ - set minus

$A \times B$ - Cartesian product
(set of all ordered pairs (x, y) where $x \in A, y \in B$.)

state facts abt sets:

$x \in A$

membership

$A \subseteq B$

subset

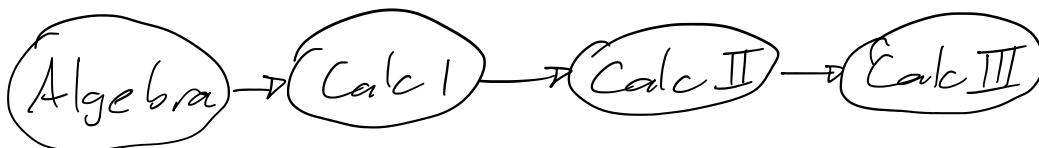
Graph Basics 2

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Q: Does the order of the pair matter in edge?
No, unless it does.

Definition: A Directed graph is a graph where edge direction matters. Formally: if $G = (V, E)$ directed, then $(x, y) \in E \not\Rightarrow (y, x) \in E$.

Informal: draw arrows on edges!



$(Alg, Calc) \in E$, but $(Calc, Alg) \notin E$

Examples? • Followers on social media

- Road networks
- Org charts

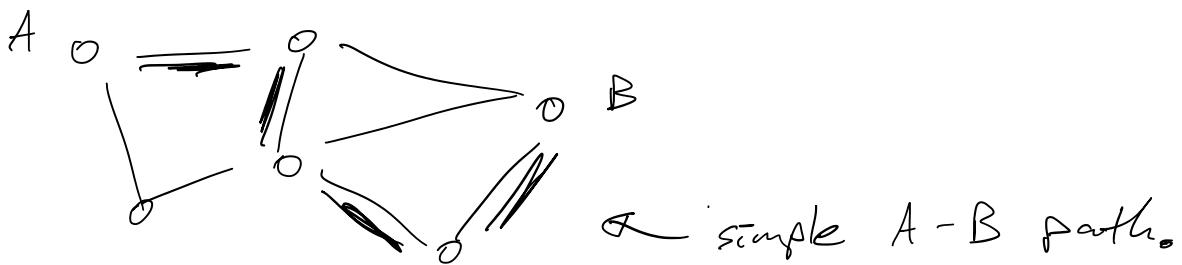
Definition: A graph is Undirected if edge direction doesn't matter: $(x, y) \in E \Leftrightarrow (y, x) \in E$.

- Facebook friends
- Neighbors in a neighborhood
- only say "a relationship exists," not much more.
- Vastly easier to analyze than directed.

Paths and cycles

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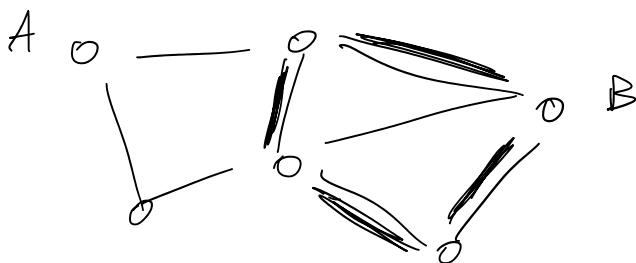
Def: a path in a graph is a sequence of vertices connected end-to-end by edges.



why? "simple" path means no repeated vertices
• transportation: this is part of Google maps!

• social networks: ideas spread along paths!

Def: a cycle is a path whose only repeated vertices are the start/end.



Def: a graph is acyclic if it contains no cycles.

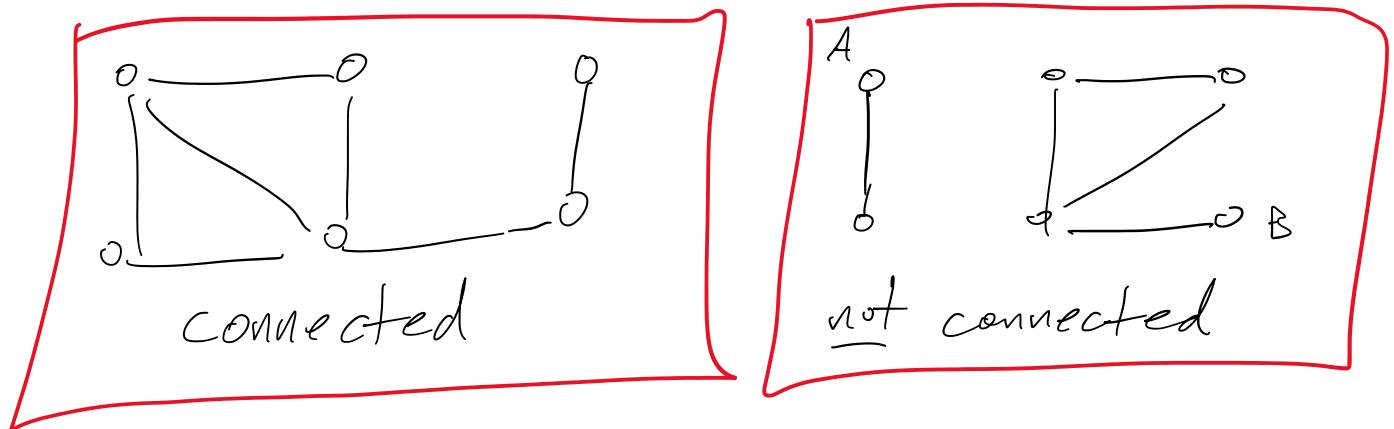
why: A graph w/ cycles has redundant paths!

Connectivity

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Now: high level structure!

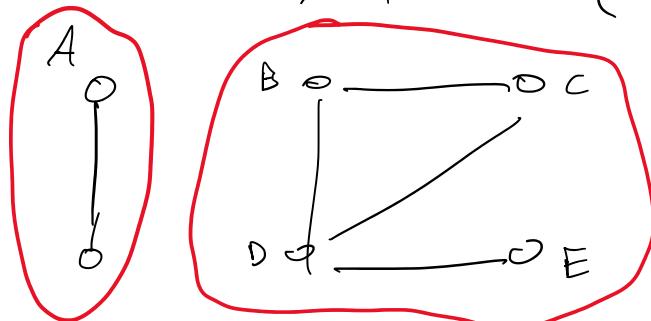
Def: a graph is connected if there is a path connecting every pair of nodes.



Def: Given $G = (V, E)$, a subset $V' \subseteq V$ is a component if both of these are true:

1) there is a path between every pair of nodes in V' , and

2) adding any other node to V' would make 1) false. (V' is "maximal"!)



2 components

(why is BCD not a component?)

"Giant component": a single component which contains most of the nodes in the graph.

Distance

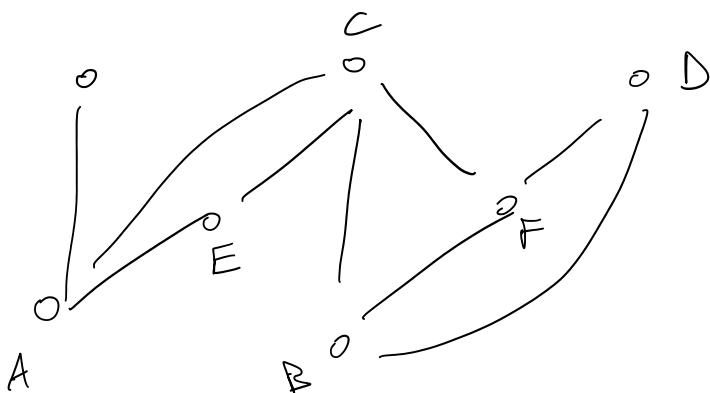
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High level: how many "hops" to go from A to B?

Def: Length of a path is # of edges in the path.

Def: Distance from A \rightarrow B is length of shortest A-B path.

write $D(A, B)$ to mean distance from A \rightarrow B.



Q: what is $D(A, B)$ in this graph?

By def, length of shortest path. what are the paths?

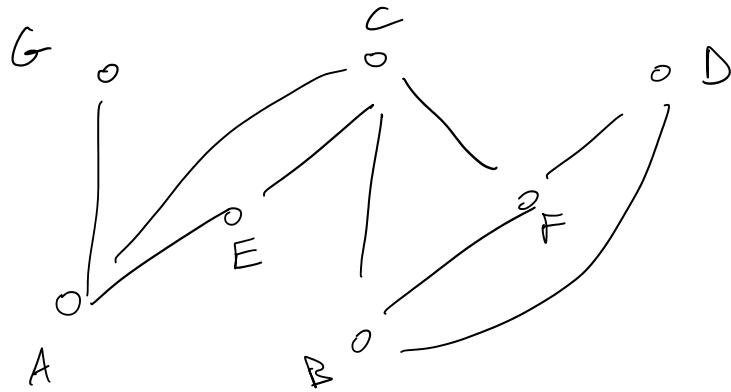
- 5 A E C F D B
- 4 A E C F B
- 3 A E C B
- 4 A C F D B
- 3 A C F B
- 2 A C B

the algo suggested by
the definition is
"exhaustive search"
1. enumerate all answers
2. return the best

Q: is there a better way?

Breadth-first search (BFS)

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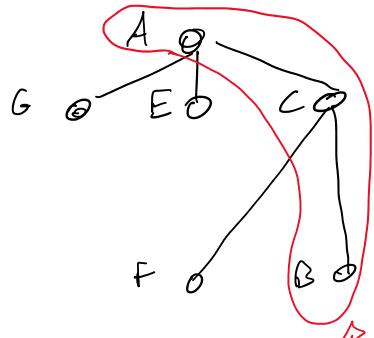


BFS idea:

0. start at A.
1. check all nodes 1 hop from A (A's neighbors)
2. " " " 2 hops from A
(neighbors of nodes from the nodes from prev. step)
3. check all nodes 3 hops from A
(neighbors of nodes from the nodes from prev. step)
⋮
when you find B (target node), return
step # you ended on.

Q: how do you keep track? FIFO/Queue!

Track Queue and search tree:

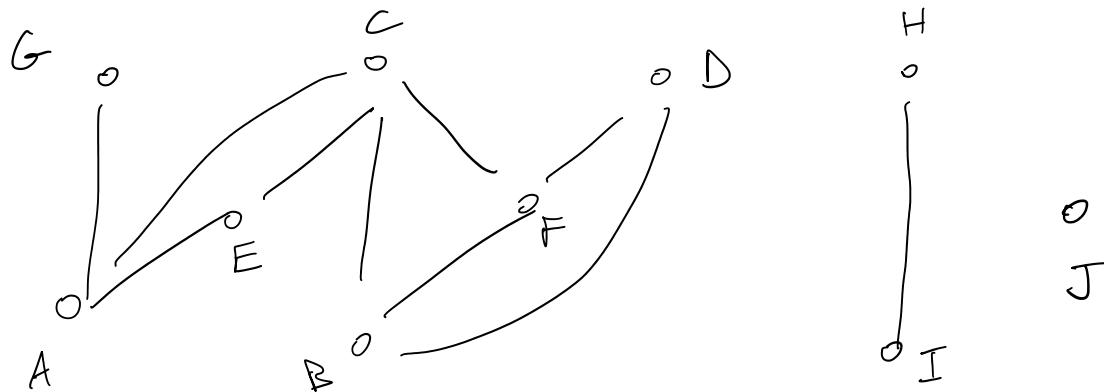


A
A G E C ← enqueue A's nbrs
G E C ← G has no nbrs
E C ← E's nbrs already checked
C F B ← B found, terminate
ans. = length of this path.

Power of BFS: connectivity, component count, acyclicity

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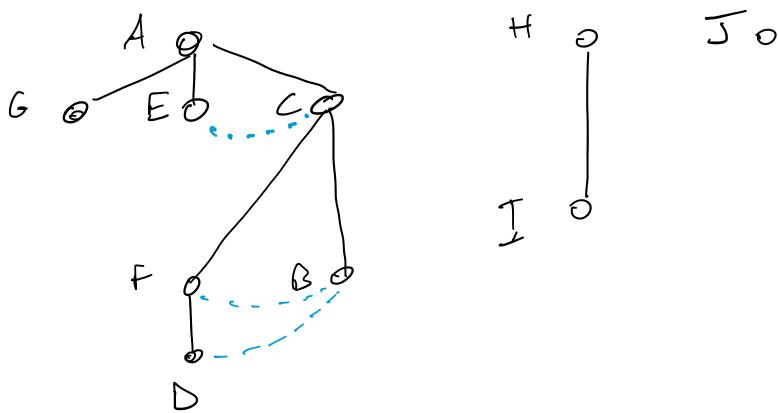
Fact: BFS more general than just finding dist.



Q: is this graph connected?

BFS can tell you!

modification: don't stop when you hit B; continue until no more edges, then restart.



- Graph is connected if you never need to restart!
- # components = # restarts + 1 !

Finally, BFS can tell us if the graph is acyclic.

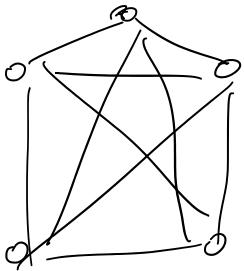
what are dotted edges in search tree?

→ edges that BFS skipped.

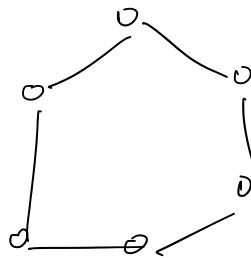
If search tree has no cross-edges, graph is acyclic.

Small-world phenomena

Tuesday, January 11, 2022 11:15 AM
Def: the diameter of a network is the length of the longest distance between any 2 nodes.



diameter: 1



diameter: 3

Q: what is the diameter of the global social network?

- intuitively, maybe quite short!

Countries of origin of people I know on a first-name basis:

- Nigeria
- Colombia
- Honduras
- India
- Pakistan
- Australia
- UK
- South Africa
- Togo
- Italy
- Iran
- China

- Empirically, short!

Milgram, 1960's: median path was 6

- Collaboration dist: Erdős # me and

Kalita: 10

Atyabi: 9

Boulle: 7

Zhou: 6

ECE { Plett: 6
Trimboli: 5