

CS 4740 Networks, Crowds, and Markets

Homework # 6

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Answers to homework problems:

1. Say whether the following claim is true or false, and provide a brief (1-3 sentence) explanation for your answer.

Claim: If player A in a two-person game has a dominant strategy s_A , then there is a pure strategy Nash equilibrium in which player A plays s_A and player B plays a best response to s_A .

Answers:

1. Well, it is known that since player A has a dominant strategy the player A's strategy will always have a higher value than player B regardless of what ever decision player B makes. We also know that there to be Nash equilibrium then there exists a best outcome that either player can experience dependent of what ever decision the other player, B, makes. However, player A has a dominant strategy and means that both players are not guaranteed to make decision resulting in Nash equilibrium.
3. Find all pure strategy Nash equilibria in the game below. In the payoff matrix below the rows correspond to player A's strategies and the columns correspond to player B's strategies. The first entry in each box is player A's payoff and the second entry is player B's payoff.

		Player B	
		L	R
Player A	U	1, 2	3, 2
	D	2, 4	0, 2

Answers:

3. Formally defining Nash equilibrium, the book says, "... suppose that Player 1 chooses a strategy S and Player 2 chooses a strategy T. We say that this pair of strategies, (S, T), is a Nash equilibrium if S is a best response to T, and T is a best response to S.
4. Below is a table made to show best results for row and column choices from the payoff matrix.

		Player B	
		<i>L</i>	<i>R</i>
Player A	<i>U</i>	1, 2	3, 2
	<i>D</i>	2, 4	0, 2

To begin finding all the Nash equilibria we will simply evaluate the best strategy for each player depending on which action the other player performs.

So first evaluating player A's best strategy we know that if player B chooses column L then player A must choose row D because 2 is better than 1. If player B chooses column R then player A's best choice would be row U since 3 is greater than 0.

Now evaluating Player B's best strategy, we know that if player A chooses Row D then player B's best choice is to pick column L because 4 is better than 2. When Player A chooses row U, then player B can choose either choice and get the same reward of 2.

So, we can classify the best possible outcomes for both players as;

- Player A chooses U and player B chooses R
- Player A chooses D and player B chooses L

These pairs of combined choices, (U, R) and (D, L) can be considered as Nash equilibrium since either one of them would be a best response to whatever choice the other player might make.

4. Consider the two-player game with players, strategies and payoffs described in the following game matrix.

		Player B		
		<i>L</i>	<i>M</i>	<i>R</i>
Player A	<i>t</i>	0, 3	6, 2	1, 1
	<i>m</i>	2, 3	0, 1	7, 0
	<i>b</i>	5, 3	4, 2	3, 1

- Does either player have a dominant strategy? Explain briefly (1-3 sentences).
- Find all pure strategy Nash equilibria for this game.

Answers:

4. Below is a table made to show best results for row and column choices from the payoff matrix.

		Player B		
		<i>L</i>	<i>M</i>	<i>R</i>
Player A	<i>t</i>	0, 3	6, 2	1, 1
	<i>m</i>	2, 3	0, 1	7, 0
	<i>b</i>	5, 3	4, 2	3, 1

- (a) Considering each player's best strategy we can evaluate each scenario of choices made by both players.

When considering player A's best strategy we look at all the moves player B can choose and then pick player A's best choice accordingly. If player B chooses *L* then player A's best choice will be any of the choices *b* because 5 is greater than 2 and 0. If player B chooses *M* then player A's best choice will be *t* since 6 is the greatest reward. If player B chooses *R* then player A's best choice will be *m* since 7 is greater than 3 and 1.

When considering player B's best strategy we look at all the moves player A can choose and pick player B's best choice accordingly. If player A chooses *t* then player B's best choice will be *L* since 3 is greater than 2 or 1. If player A chooses *m* then player B's best choice will once again be *L* since 3 is greater than 1 and 0. If player A chooses *b* then player B's best choice will be *L* again because 3 is greater than 2 and 1.

As we can see regardless of what action player A makes player B will always choose *L*. From this we can say that player B has a dominant strategy, *L*.

- (b) Continuing off the information that was gathered in part (a), we found that a Nash equilibrium, (B, L) , occurs when player A chooses *b* and player B chooses *L* since both values return values from these choices sum to 8 being the highest total return amongst players.

5. Consider the following two-player game in which each player has three strategies.

		Player B		
		<i>L</i>	<i>M</i>	<i>R</i>
Player A	<i>U</i>	1, 1	2, 3	1, 6
	<i>M</i>	3, 4	5, 5	2, 2
	<i>D</i>	1, 10	4, 7	0, 4

Find all the (pure strategy) Nash equilibria for this game.

Answers:

5. Below is a table made to show best results for row and column choices from the payoff matrix.

		Player B		
		<i>L</i>	<i>M</i>	<i>R</i>
Player A	<i>U</i>	1, 1	2, 3	1, 6
	<i>M</i>	3, 4	5, 5	2, 2
	<i>D</i>	1, 10	4, 7	0, 4

For Player A's best strategy, if player B chooses *L* then player A must choose *M* since 3 is greater than 1. If player B chooses *M* then player A must choose *M* since 5 is greater than 4 and 2. If player B chooses *R* then player A must choose *M* since 2 is greater than 0 and 1.

For Player B's best strategy, if player A chooses *U* then player B must choose *R* since 6 is greater than 3 and 1. If player A chooses *M* then player B must choose *M* since 5 is greater than 4 and 2. If player A chooses *D* then player B must choose *L* since 10 is greater than 4 and 1.

Hence, we can see that move *M* for both players result in a Nash equilibrium since this would be the best response to either one of the players moves.