Selection vs. Social Influence

Monday, January 10, 2022 4:26 PM

Overarching theme: real networks exhibit homophily; individuals "look like" their friends. Why?

One possibility is "Selection": factors conspire to put similar people together

- Schelling segregation is our key example of this
- Also more pedestrian things like club membership

Another possibility is "social influence." For example:

- I look like my friends because I adopt their ideas over time, and they adopt mine.
- I look like my friends because as a group, we differentiate ourselves from "other" groups.

In this section of the class,

- we'll take an overview of how these processes could be modeled
- Discover some power tools from linear algebra that make this easy to think about
- We'll end with epidemiological models.

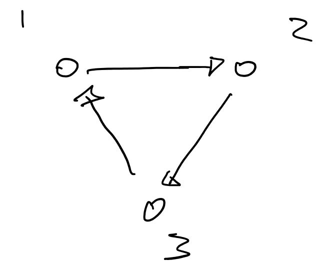
Common feature of all these models: they are all "spreading" processes on networks.

- Spreading opinions/ideas
- Spreading diseases
- Spreading customs/culture

A simple starter model

Wednesday, February 23, 2022 10:03 AM

Let's look at directed graphs.



Graph is $G = (V, E)$

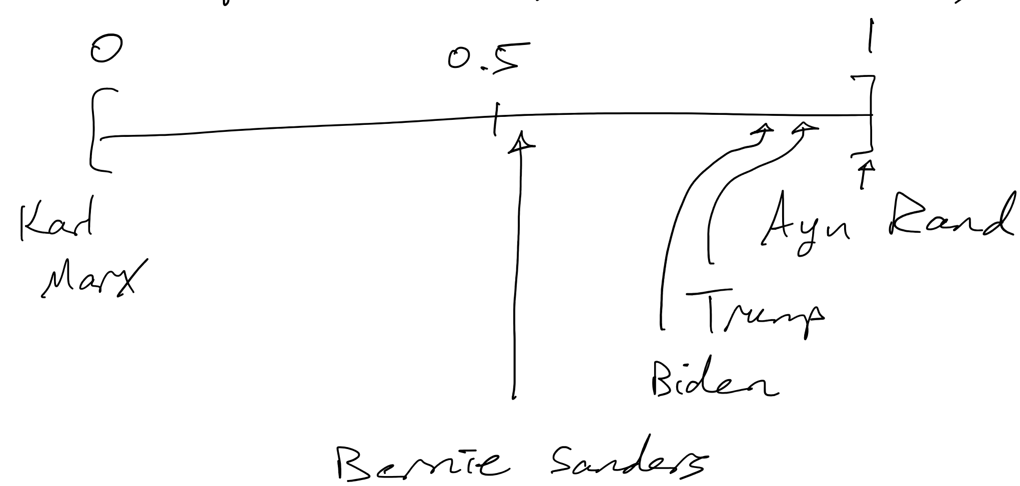
- Nodes are individuals/agents
- An edge (i, j) means that

  node i <u>listens to</u> node j.

- Each node $i \in V$ has an <u>opinion</u> $x_i \in [0, 1]$.

  note: "opinion" is on a continuum.

  so if topic is "capitalism is good,"



Q: how do opinions spread?

Along listening links!

Assumption: society is civil.

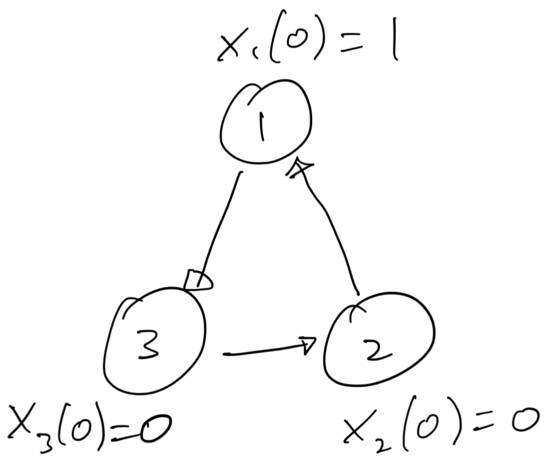
Networks where opinions don't converge

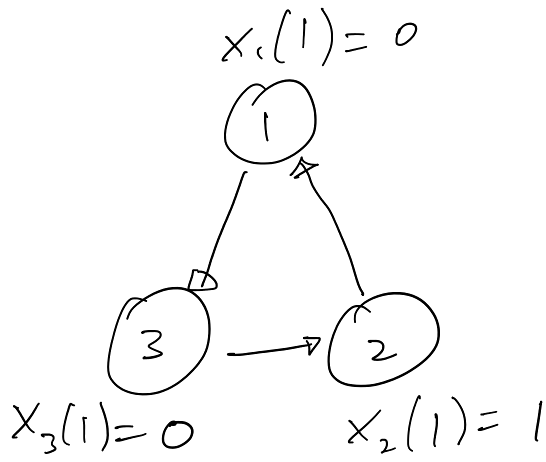
Wednesday, February 23, 2022 10:03 AM

t = time step. $t \in \{0, 1, \dots\}$

$X_i(t)$ = node i's opinion @ time t.

Idea: if I'm listening to a node, I adopt its opinion in the next time step.





t	0	1	2	3
$X_1(t)$	1	0	0	1
$X_2(t)$	0	1	0	0
$X_3(t)$	0	0	1	0

- - - - - - - . .

Keeps on cycling forever.

Q: can this model ever settle down?

Concept: <u>Convergence</u>.

Informal: do opinions eventually settle down?

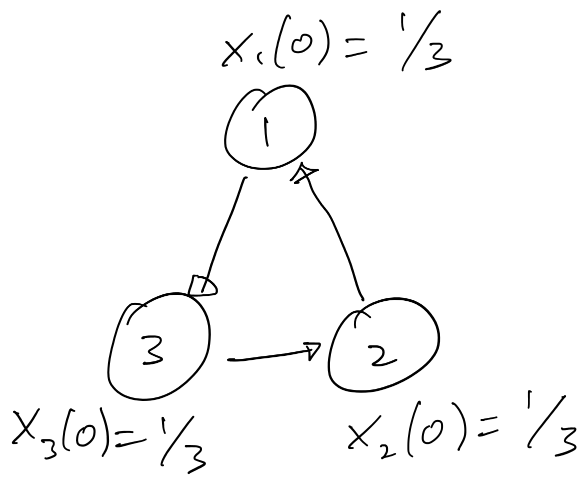
Not yet! something abt model or network or initial condition caused nonconvergence.

Convergence due to initial cond. and graph structure

Wednesday, February 23, 2022 10:03 AM

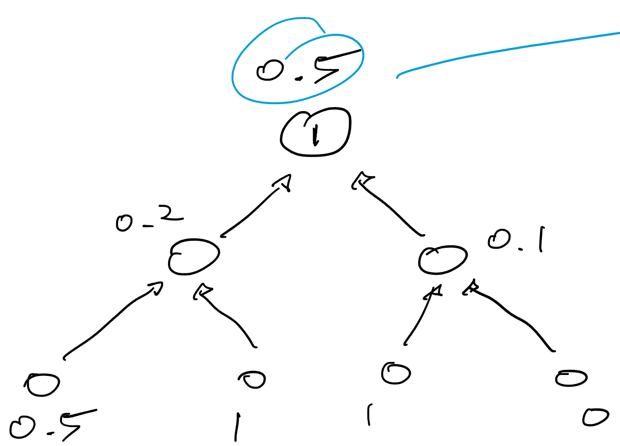
Q: what could yield convergence?

- Initial condition?



If $x_i(0) = x_j(0)$ for all i, j, then convergence is trivial.

- Graph structure?

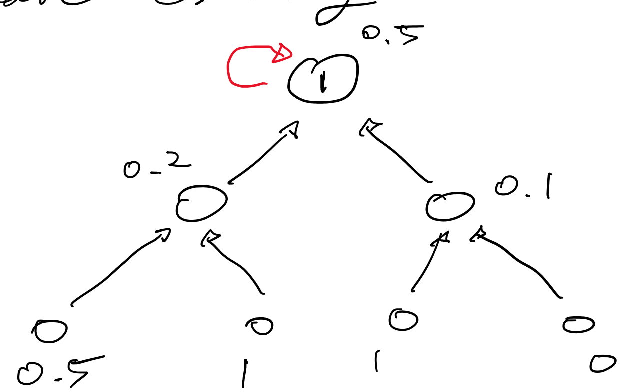


Idea: The 0.5 will eventually spread to all other nodes.

But what is $X_1(t=1)$?

Our model implicitly requires each node to have exactly one outgoing edge!

FIX: self-loop!



Interpret: node 1 simply retains its opinion each time step.

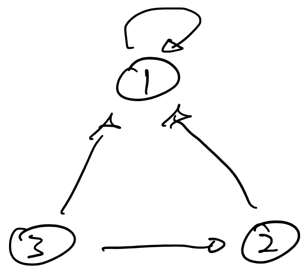
What if I listen to 2 people?

Wednesday, February 23, 2022 10:03 AM

Model issue: each node can only listen to 1 other.

makes no sense!

What should happen in a graph like:



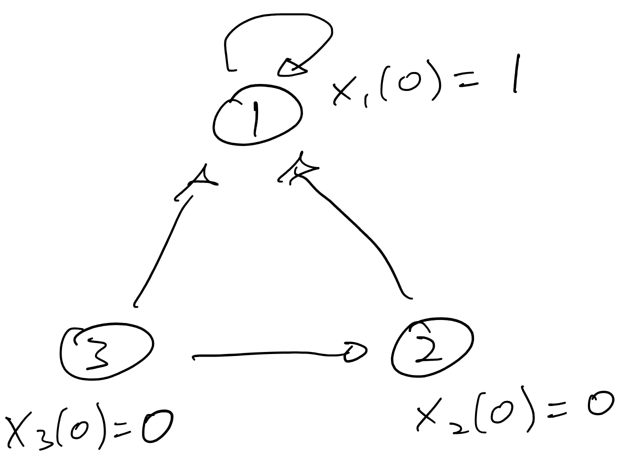
1 ignores everyone

2 listens to 1

3 listens to <u>both</u> 1 and 2.

Simple idea: if I have multiple outgoing edges, I <u>average</u> those nodes' opinions to create mine.

So: $X_3(t+1) = \dfrac{X_2(t) + X_1(t)}{2}$.



t	0	1	2	3
$x_1(t)$	1	1	1	1
$X_2(t)$	0	1	1	1
$X_3(t)$	0	1/2	1	1

<u>Converged.</u> to what? opinion of node 1.

opinions of nodes 2, 3 were eventually erased!

Q: why? no paths leaving node 1.

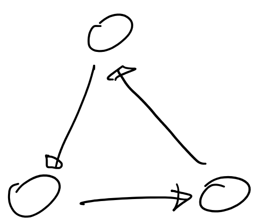
Strong Connectivity

Wednesday, February 23, 2022 10:03 AM

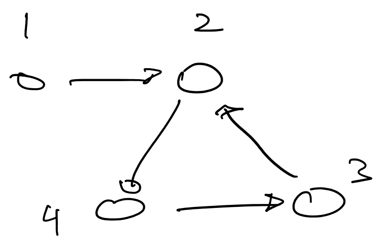
Q: When can nodes' opinions be erased?

A: when some node(s) have no paths leaving.

<u>Def</u>: A directed graph is called <u>strongly-connected</u> if there is a directed path from every node to every other node.

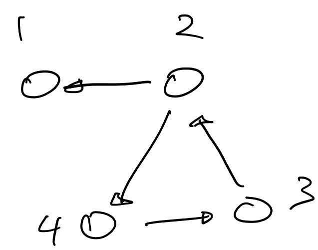


Strongly Conn.



<u>not</u> strongly conn.

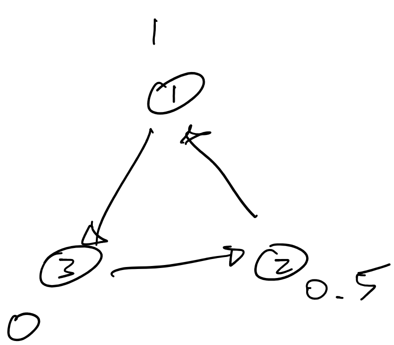
(no path from $3 \to 1$)



<u>not</u> strongly conn.

(no path from $1 \to 3$)

Fact: if a graph is strongly-connected, then no node's opinion is erased by these averaging opinion dynamics.



t	0	1	2	3	
x_1	1	0	1/2	1	
x_2	1/2	1	0	1/2	- - - -
x_3	0	1/2	1	0	

Periodicity

Wednesday, February 23, 2022 10:03 AM

But ~~strong~~ connectivity clearly not enough for convergence.

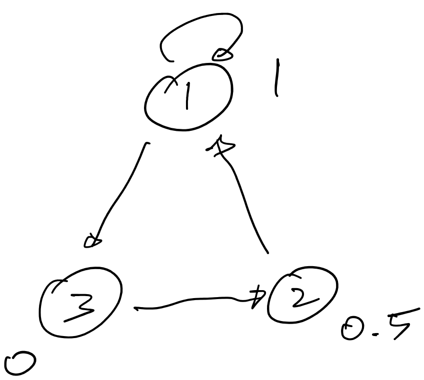
Why no convergence?

Idea: the previous graph is <u>periodic</u>; all cycles have length 3.

(that's why the opinions repeat every 3 time steps).

How could we break this periodicity?

$$x_1(t+1) = \frac{x_1(t) + x_3(t)}{2}$$



note: ∃ 2 cycles:

length - 1

length - 3

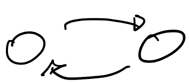
t	0	1	2	3					
x_1	1	1/2	1/2	3/4	5/8	9/16	21/32		5/8 ...
x_2	1/2	1	1/2	1/2	3/4	5/8	9/16	....	5/8 ...
x_3	0	1/2	1	1/2	1/2	3/4	5/8		5/8 ...

Convergence! Eventually, all nodes converge to same opinion.

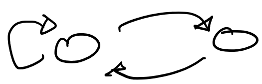
Aperiodicity

Wednesday, February 23, 2022 10:03 AM

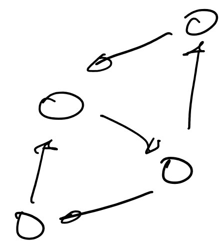
Def: A graph is <u>periodic</u> if there is an integer $k > 1$ that divides the length of every cycle. If a graph is not periodic, it is called <u>aperiodic</u>.



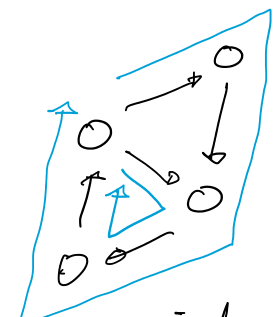
periodic
$k = 2$



aperiodic
cycle lengths
1, 2



periodic
$k = 3$



aperiodic
cycle len:
3, 4.

Fact: If a graph is <u>strongly connected</u> and <u>aperiodic</u>, then the averaging opinion dynamics <u>converge</u> to a common opinion that is a function of every node's initial opinion.