Математическая и прикладная статистика

Лабораторная работа

Белоброцкий Денис Витальевич, Магистратура, 1 курс

Вариант

Количество гласных букв = 10

Количество согласных букв = 16

Вариант = 06

Исходный код

https://github.com/DenisBelobrotski/AppliedStatisticsLab

Задание 1

Модули

```
import math
import numpy as np
from scipy import stats
from scipy import special
import matplotlib.pyplot as plt
# %matplotlib inline
```

Чтение данных (data_1_var_06.txt)

```
data_file_name = "data_1_var_06.txt"

def parse_data(file_name):
    result_data = []
    result_data_len = 0
    all_data_parsed = False

    with open(file_name, "r") as file:
        numbers = file.read().split(",")
        numbers_len = len(numbers)
        for current in numbers:
            result_data.append(float(current.strip()))
        result_data_len = len(result_data)
        all_data_parsed = (numbers_len == result_data_len)

    return result_data, result_data_len, all_data_parsed
```

```
data, dataLen, allDataParsed = parse_data(data_file_name)

print("Parsed numbers count = " + str(dataLen))
print("Is all data parsed = " + str(allDataParsed))
sortedData = sorted(data)
```

```
Parsed numbers count = 345
Is all data parsed = True
```

а) Выборочное среднее

```
def calc_sample_mean(in_data):
    in_data_len = len(in_data)
    mean_sum = 0.0
    for number in in_data:
        mean_sum += number
    return mean_sum / in_data_len

sampleMean = calc_sample_mean(data)
print("Sample mean = " + str(sampleMean))
```

```
Sample mean = 3.5859199999999984
```

б) Выборочная дисперсия

```
def calc_variance_sum(in_data, mean):
    variance_sum = 0.0
    for number in in_data:
        variance_sum += (number - mean)**2
    return variance_sum

def calc_sample_variance(in_data, mean):
    in_data_len = len(in_data)
    return calc_variance_sum(in_data, mean) / in_data_len

sampleVariance = calc_sample_variance(data, sampleMean)
print("Sample variance = " + str(sampleVariance))
```

```
Sample variance = 1.2073530875130443
```

в) Исправленная дисперсия

```
def calc_unbiased_sample_variance(in_data, mean):
    in_data_len = len(in_data)
    return calc_variance_sum(in_data, mean) / (in_data_len - 1)

unbiasedSampleVariance = calc_unbiased_sample_variance(data, sampleMean)
print("Unbiased sample variance = " + str(unbiasedSampleVariance))
```

```
Unbiased sample variance = 1.210862834860466
```

г) Размах выборки

```
dataMin = sortedData[0]
dataMax = sortedData[-1]
dataRange = dataMax - dataMin
print("Min = " + str(dataMin))
print("Max = " + str(dataMax))
print("Range = " + str(dataRange))
```

```
Min = 0.6819
Max = 6.5951
Range = 5.913200000000001
```

д) Медиана

```
median = sortedData[dataLen // 2]
if dataLen % 2 == 0:
    median += sortedData[dataLen // 2 - 1]
    median /= 2

print("Median = " + str(median))
```

```
Median = 3.5788
```

е) Квартили

```
lowerQuartile = sortedData[dataLen // 4]
upperQuartile = sortedData[3 * dataLen // 4]

print("Q1 = " + str(lowerQuartile))
print("Q2 = " + str(median))
print("Q3 = " + str(upperQuartile))
```

```
Q1 = 2.8036
Q2 = 3.5788
Q3 = 4.3214
```

ж) Выборочная квантиль уровня 1/3

```
quantile_1_3 = sortedData[dataLen // 3]

print("Q = " + str(quantile_1_3))
```

```
Q = 3.0461
```

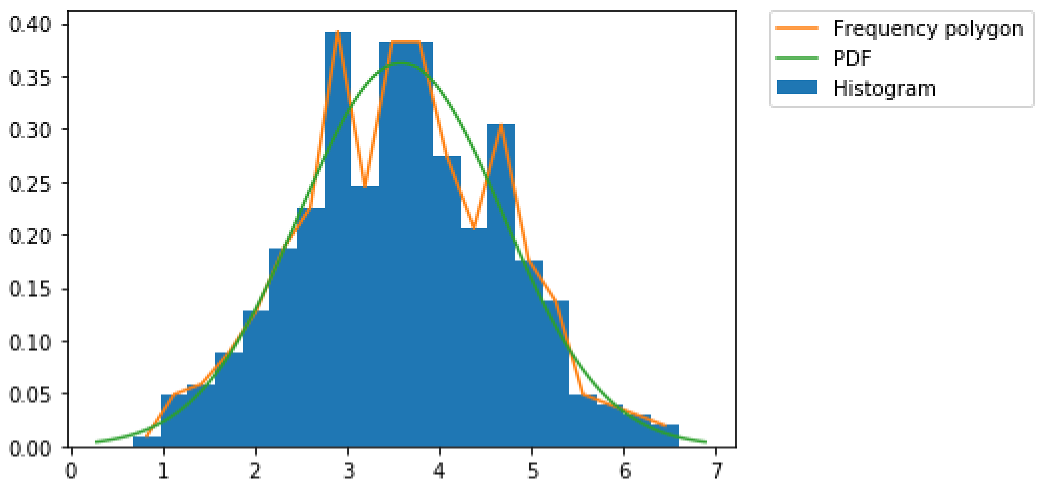
з) Гистограмма, полигон частот, плотность нормального распределения.

PDF - Probability Density Function (Функция плотности распределения)

```
def norm_pdf(x, mu, sigma):
    return (1.0 / (sigma * math.sqrt(2 * math.pi))) * math.exp(-0.5 * ((x - mu) /
sigma)**2)

def norm_pdf_list(x, mu, sigma):
    result = x.copy()
    for i in range(len(x)):
        result[i] = norm_pdf(result[i], mu, sigma)

    return result

def draw_pdf(hist_bins_count, mu, sigma):
    n, bins, patches = plt.hist(data, hist_bins_count, density=True,
label="Histogram")
    bins = np.delete(bins, -1)
```

```
    for i in range(hist_bins_count):
        bins[i] += patches[i].get_width() / 2

    plt.plot(bins, n, label="Frequency polygon")

    nodes_count = 100
    nodes = np.linspace(mu - 3 * sigma, mu + 3 * sigma, nodes_count)
    plt.plot(nodes, norm_pdf_list(nodes, mu, sigma), label="PDF")

    plt.legend(bbox_to_anchor=(1.05, 1), loc='upper left', borderaxespad=0.0)

    print("mu = ", str(mu))
    print("sigma = ", str(sigma))
    print("hist bins count = ", str(hist_bins_count))

draw_pdf(20, sampleMean, math.sqrt(unbiasedSampleVariance))
```

```
mu =  3.5859199999999984
sigma =  1.1003921277710351
hist bins count =  20
```

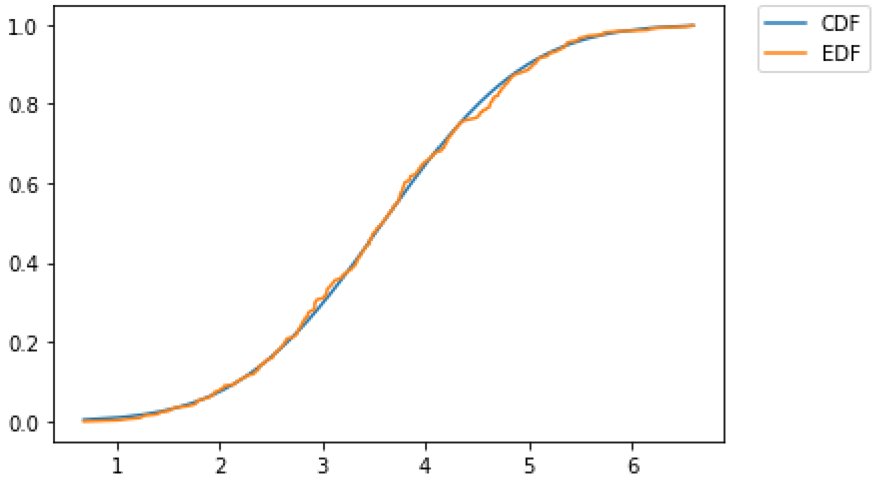


и) Эмпирическая функция распределения и функция распределения нормального закона

CDF - Cumulative distribution function (Теоритическая функция распределения)
EDF - Empirical distribution function (Эмпирическая функция распределения)

```
def norm_cdf(x, mu, sigma):
    return 0.5 * (1 + math.erf((x - mu) / (sigma * 1.4142)))

def norm_cdf_list(in_data, mu, sigma):
    out_data = in_data.copy()
    for i in range(len(out_data)):
        out_data[i] = norm_cdf(out_data[i], mu, sigma)
    return out_data
```

```
def edf(in_data):
    out_data = in_data.copy()
    out_data_len = len(out_data)
    for i in range(out_data_len):
        out_data[i] = i / out_data_len
    return out_data

def draw_cdf(mu, sigma):
    plt.plot(sortedData, norm_cdf_list(sortedData, mu, sigma), label="CDF")
    plt.plot(sortedData, edf(sortedData), label="EDF")
    plt.legend(bbox_to_anchor=(1.05, 1), loc='upper left', borderaxespad=0.0)

draw_cdf(sampleMean, math.sqrt(unbiasedSampleVariance))
```



к) Доверительные интервалы для среднего и дисперсии с вероятностью 0.99

```
def confidence_interval_mean(mean, variance, data_len, probability):
    bias = math.sqrt(variance / (data_len - 1)) * stats.t.ppf(1 - probability / 2,
data_len - 1)
    return mean - bias, mean + bias

def confidence_interval_variance(variance, data_len, probability):
    lower_bound = (data_len - 1) * variance / stats.chi2.ppf((1 + probability) /
2, data_len - 1)
    upper_bound = (data_len - 1) * variance / stats.chi2.ppf((1 - probability) /
2, data_len - 1)
    return lower_bound, upper_bound

print("Mean confidence interval = " + str(confidence_interval_mean(sampleMean,
unbiasedSampleVariance, dataLen, 0.99)))
print("Sample mean = " + str(sampleMean))
print()
print("Variance confidence interval = " +
str(confidence_interval_variance(unbiasedSampleVariance, dataLen, 0.99)))
```

```
print("Unbiased sample variance = " + str(unbiasedSampleVariance))
```

```
Mean confidence interval = (3.5851758590018594, 3.5866641409981375)
Sample mean = 3.5859199999999984

Variance confidence interval = (1.0029449329842604, 1.4865973537588315)
Unbiased sample variance = 1.210862834860466
```

Задание 2

```
significance = 0.05
test_hists_bins_count = 20
kolmogorov_distribution_quantile = 1.36
a1_distribution_quantile = 0.46
a2_distribution_quantile = 2.49
chi2_distribution_quantile = stats.chi2.ppf(1 - significance,
test_hists_bins_count - 1)
norm_distribution_quantile = stats.norm.ppf(1 - significance)
```

a) При помощи критерия Колмогорова проверить гипотезу о том, что данные имеют нормальный закон распределения со средним 10 и дисперсией 5.5. Уровень занчимости 0.05.

```
def kolmogorov_test_normal(sorted_data, mean, variance, quantile):
    data_len = len(sorted_data)
    cdf_result = norm_cdf_list(sortedData, mean, math.sqrt(variance))
    edf_result = edf(sortedData)

    max_distance = 0
    for i in range(data_len):
        # max_distance = max(max_distance, math.fabs(cdf_result[i] -
edf_result[i]))
        max_distance = max(max_distance, math.fabs(cdf_result[i] - (i + 1) /
data_len))

    result_statistic = math.sqrt(data_len) * max_distance
    return result_statistic, (result_statistic < quantile)

kolmogorov_test_normal_statistic, kolmogorov_test_normal_passed = \
    kolmogorov_test_normal(sortedData, 10.0, 5.5,
kolmogorov_distribution_quantile)
print("Kolmogorov test:")
print("statistic = ", str(kolmogorov_test_normal_statistic))
print("quantile = ", str(kolmogorov_distribution_quantile))
```

```
print("passed = ", str(kolmogorov_test_normal_passed))
```

```
Kolmogorov test:
statistic =  17.604856029131053
quantile =  1.36
passed =  False
```

б) При помощи критерия Крамера-Мизеса проверить гипотезу о том, что данные имеют нормальный закон распределения со средним 10 и дисперсией 5.5. Уровень занчимости 0.05.

```
def cramer_mises_test_normal(sorted_data, mean, variance, quntile):
    data_len = len(sorted_data)
    cdf_result = norm_cdf_list(sortedData, mean, math.sqrt(variance))

    result_statistic = 1 / (12 * data_len)
    for i in range(data_len):
        result_statistic += (cdf_result[i] - (2 * i + 1)/(2 * data_len))**2

    return result_statistic, (result_statistic < quntile)

cramer_mises_test_normal_statistic, cramer_mises_test_normal_passed = \
    cramer_mises_test_normal(sortedData, 10.0, 5.5, a1_distribution_quantile)
print("Cramer-Mises test:")
print("statistic = ", str(cramer_mises_test_normal_statistic))
print("quantile = ", str(a1_distribution_quantile))
print("passed = ", str(cramer_mises_test_normal_passed))
```

```
Cramer-Mises test:
statistic =  111.35222279248605
quantile =  0.46
passed =  False
```

в) При помощи критерия Андерсона-Дарлинга проверить гипотезу о том, что данные имеют нормальный закон распределения со средним 10 и дисперсией 5.5. Уровень занчимости 0.05.

```
def anderson_darling_test_normal(sorted_data, mean, variance, quntile):
    data_len = len(sorted_data)
    cdf_result = norm_cdf_list(sortedData, mean, math.sqrt(variance))
```

```
    result_statistic = -data_len
    for i in range(data_len):
        coef = (2 * i + 1)/(2 * data_len)
        result_statistic += -2 * (coef * math.log(cdf_result[i]) + (1 - coef) *
math.log(1 - cdf_result[i]))

    return result_statistic, (result_statistic < quntile)

anderson_darling_test_normal_statistic, anderson_darling_test_normal_passed = \
    anderson_darling_test_normal(sortedData, 10.0, 5.5, a2_distribution_quantile)
print("Anderson-Darling test:")
print("statistic = ", str(anderson_darling_test_normal_statistic))
print("quantile = ", str(a2_distribution_quantile))
print("passed = ", str(anderson_darling_test_normal_passed))
```

```
Anderson-Darling test:
statistic =  1402.6543578852445
quantile =  2.49
passed =  False
```

Чтение данных второй выборки (data_1_var_07.txt)

```
data2_file_name = "data_1_var_07.txt"

data2, data2Len, allData2Parsed = parse_data(data2_file_name)

print("Parsed numbers count = " + str(data2Len))
print("Is all data parsed = " + str(allData2Parsed))
sortedData2 = sorted(data2)
```

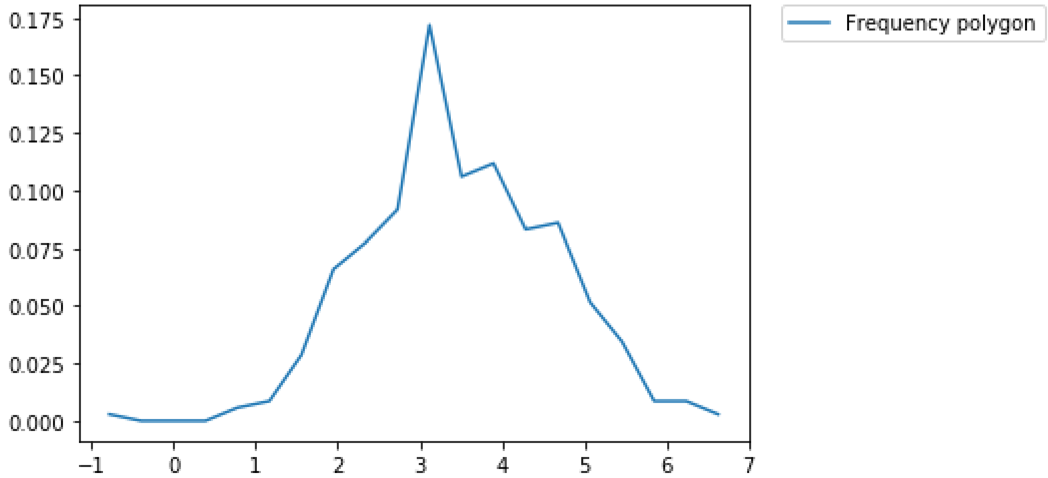
```
Parsed numbers count = 349
Is all data parsed = True
```

г) При помощи критерия хи-квадрат проверить гипотезу, что вторая выборка имеет нормальное распределение. Уровень значимости 0.05.

```
def create_hist(in_data, intervals_count, lower_bound, upper_bound, normalized =
False):
    out_hist = [0] * intervals_count
    intervals_positions = [0] * intervals_count
    hist_step = (upper_bound - lower_bound) / intervals_count
    cur_border = lower_bound + hist_step
```

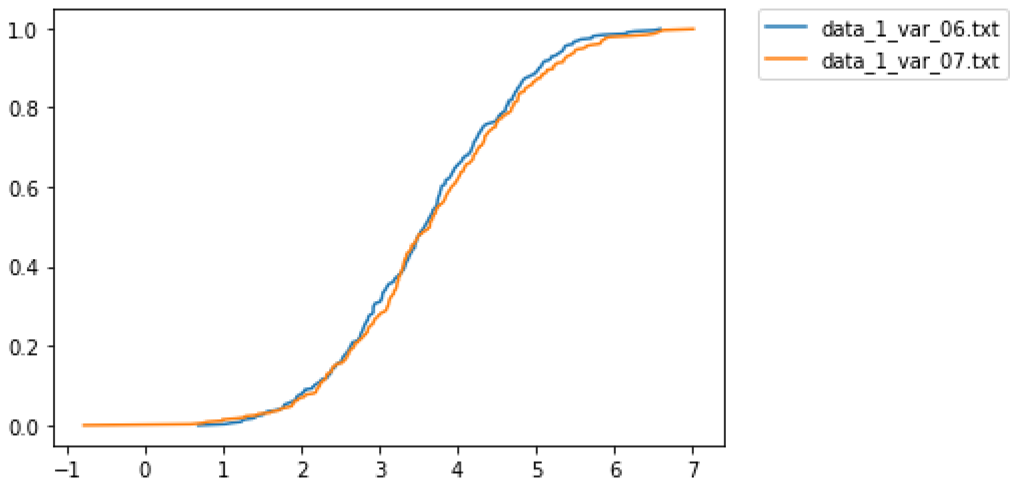
```
    cur_interval_index = 0
    in_data_len = len(in_data)

    for i in range(intervals_count):
        intervals_positions[i] = lower_bound + hist_step * i

    for current_number in in_data:
        if current_number < cur_border:
            out_hist[cur_interval_index] += 1
        elif cur_interval_index < intervals_count - 1:
            cur_interval_index += 1
            cur_border += hist_step

    if normalized:
        for i in range(intervals_count):
            out_hist[i] /= in_data_len

    return out_hist, intervals_positions

def pearson_test_normal(in_data, bins_count, quantile):
    mean = calc_sample_mean(in_data)
    sqrd_variance = math.sqrt(calc_unbiased_sample_variance(in_data, mean))
    in_data_len = len(in_data)
    in_data_hist, _ = create_hist(in_data, bins_count, in_data[0], in_data[-1],
False)
    step = (in_data[-1] - in_data[0]) / bins_count
    cur_border = in_data[0]
    chi_sum = 0

    for i in range(bins_count):
        prev_border = cur_border
        cur_border = in_data[0] + (i + 1) * step
        expected_count = in_data_len * (norm_cdf(cur_border, mean, sqrd_variance)
- norm_cdf(prev_border, mean, sqrd_variance))
        chi_sum += (in_data_hist[i] - expected_count)**2 / expected_count

    return chi_sum, (chi_sum < quantile)

hist, interval_positions = create_hist(sortedData2, test_hists_bins_count,
sortedData2[0], sortedData2[-1], True)
plt.plot(interval_positions, hist, label="Frequency polygon")
_ = plt.legend(bbox_to_anchor=(1.05, 1), loc='upper left', borderaxespad=0.0)

pearson_test_normal_statistic, pearson_test_normal_passed = \
    pearson_test_normal(sortedData2, test_hists_bins_count,
chi2_distribution_quantile)
print("Pearson test:")
print("hist intervals = ", str(test_hists_bins_count))
print("statistic = ", str(pearson_test_normal_statistic))
print("quantile = ", str(chi2_distribution_quantile))
print("passed = ", str(pearson_test_normal_passed))
```

```
Pearson test:
hist intervals =  20
statistic =  30.06662489471957
quantile =  30.14352720564616
passed =  True
```



Сравнение эмпирических функций распределения двух выборок

```
edf_1 = edf(sortedData)
edf_2 = edf(sortedData2)

plt.plot(sortedData, edf_1, label="data_1_var_06.txt")
plt.plot(sortedData2, edf_2, label="data_1_var_07.txt")
_ = plt.legend(bbox_to_anchor=(1.05, 1), loc='upper left', borderaxespad=0.0)
```



д) Проверить гипотезу однородности выборок при помощи критерия Колмогорова-Смирнова. Уровень значимости 0.05.

```
def apply_edf(in_data, in_data_edf, argument):
    in_data_len = len(in_data)
    result = 0

    if argument > in_data[-1]:
        result = 1
    elif argument > in_data[0]:
        for i in range(in_data_len):
            if argument <= in_data[i]:
                percentage = (argument - in_data[i - 1]) / (in_data[i] - in_data[i
- 1])
                result = in_data_edf[i - 1] + percentage * (in_data_edf[i] -
in_data_edf[i - 1])
                break
    else:
        result = 0

    return result

def calc_smirnov_test_distance(in_data_1, in_data_2, in_data_2_edf):
    in_data_1_len = len(in_data_1)
    distance = 0

    for i in range(in_data_1_len):
        distance = max(distance, math.fabs(
            (i + 1) / in_data_1_len -
            apply_edf(in_data_2, in_data_2_edf, in_data_1[i])))

    return distance

def smirnov_test(in_data_1, in_data_2, quantile):
    in_data_1_len = len(in_data_1)
    in_data_2_len = len(in_data_2)
    in_edf_1 = edf(in_data_1)
    in_edf_2 = edf(in_data_2)

    distance_positive = calc_smirnov_test_distance(in_data_1, in_data_2, in_edf_2)
    distance_negative = calc_smirnov_test_distance(in_data_2, in_data_1, in_edf_1)
    distance = max(distance_positive, distance_negative)
    statistic_coef = math.sqrt(in_data_1_len * in_data_2_len / (in_data_1_len +
in_data_2_len))
    statistic = statistic_coef * distance

    return statistic, (statistic < quantile)

smirnov_test_statistic, smirnov_test_passed = \
    smirnov_test(sortedData, sortedData2, kolmogorov_distribution_quantile)
print("Smirnov test:")
print("statistic = ", str(smirnov_test_statistic))
print("quantile = ", str(kolmogorov_distribution_quantile))
print("passed = ", str(smirnov_test_passed))
```

```
Smirnov test:
statistic =  0.7671199254742549
quantile =  1.36
passed =  True
```

e) Проверить гипотезу однородности выборок при помощи критерия Розенблатта. Уровень значимости 0.05.

```
def calc_ranks(in_data):
    out_data = in_data.copy()
    out_data_len = len(out_data)
    for i in range(out_data_len):
        out_data[i] = i
    return out_data

def rosenblatt_test(in_data_1, in_data_2, quantile):
    in_data_1_len = len(in_data_1)
    in_data_2_len = len(in_data_2)
    ranks_1 = calc_ranks(in_data_1)
    ranks_2 = calc_ranks(in_data_2)

    sum_1 = 0
    for i in range(in_data_1_len):
        sum_1 += (ranks_1[i] - (i + 1))**2
    sum_1 /= in_data_2_len

    sum_2 = 0
    for j in range(in_data_2_len):
        sum_2 += (ranks_2[j] - (j + 1))**2
    sum_2 /= in_data_1_len

    squared_omega = 1. / (in_data_1_len * in_data_2_len) * ((1. / 6) + sum_1 +
sum_2) - (2. / 3)
    statistic = (in_data_1_len * in_data_2_len) / (in_data_1_len + in_data_2_len)
* squared_omega

    mean = 1. / 6 * (1 + 1 / (in_data_1_len + in_data_2_len))
    variance = 1. / 45 * (1 + 1 / (in_data_1_len + in_data_2_len)) * (1 + 1 /
(in_data_1_len + in_data_2_len) - 3. / 4 * (1 / in_data_1_len + 1 /
in_data_2_len))

    corrected_statistic = (statistic - mean) / math.sqrt(45. * variance) + 1. / 6

    return corrected_statistic, (corrected_statistic < quantile)

# https://en.wikipedia.org/wiki/Cram%C3%A9r%E2%80%93von_Mises_criterion
def rosenblatt_test_2(in_data_1, in_data_2, quantile):
    in_data_1_len = len(in_data_1)
    in_data_2_len = len(in_data_2)
    ranks_1 = calc_ranks(in_data_1)
```

```
    ranks_2 = calc_ranks(in_data_2)

    sum_1 = 0
    for i in range(in_data_1_len):
        sum_1 += (ranks_1[i] - (i + 1))**2

    sum_2 = 0
    for j in range(in_data_2_len):
        sum_2 += (ranks_2[j] - (j + 1))**2

    n = in_data_1_len
    m = in_data_2_len

    u = n * sum_1 + m * sum_2

    t = u / (n * m * (n + m)) - (4 * m * n - 1) / (6 * (m + n))

    return t, (t < quantile)

rosenblatt_test_statistic, rosenblatt_test_passed = \
    rosenblatt_test(sortedData, sortedData2, a2_distribution_quantile)
print("Rosenblatt test:")
print("statistic = ", str(rosenblatt_test_statistic))
print("quantile = ", str(a2_distribution_quantile))
print("passed = ", str(rosenblatt_test_passed))
```

```
Rosenblatt test:
statistic =  -115.74373038178214
quantile =  2.49
passed =  True
```

ж) Проверить гипотезу однородности выборок при помощи критерия Уилкоксона (Манна-Уитни). Уровень значимости 0.05.

```
def wilcoxon_mann_whitney_test(in_data_1, in_data_2, quantile):
    n = len(in_data_1)
    m = len(in_data_2)

    result_sum = 0
    for i in range(n):
        for j in range(m):
            if in_data_1[i] < in_data_2[j]:
                result_sum += 1

    mean = (n * m) / 2
    variance = n * m * (n + m + 1) / 12

    statistic = (result_sum - mean) / math.sqrt(variance)
```

```
    return statistic, (statistic < quantile)

wilcoxon_mann_whitney_test_statistic, wilcoxon_mann_whitney_test_passed = \
    wilcoxon_mann_whitney_test(sortedData, sortedData2,
norm_distribution_quantile)
print("Wilcoxon-Mann-Whitney test:")
print("statistic = ", str(wilcoxon_mann_whitney_test_statistic))
print("quantile = ", str(norm_distribution_quantile))
print("passed = ", str(wilcoxon_mann_whitney_test_passed))
```

```
Wilcoxon-Mann-Whitney test:
statistic =  0.8026188969873727
quantile =  1.6448536269514722
passed =  True
```

```
def pearson_test_2_experiments(in_data_1, in_data_2, bins_count, quantile):
    in_data_1_len = len(in_data_1)
    in_data_2_len = len(in_data_2)
    in_data_len = in_data_1_len + in_data_2_len

    lower_bound = min(in_data_1[0], in_data_2[0])
    upper_bound = max(in_data_1[-1], in_data_2[-1])

    hist_1, _ = create_hist(in_data_1, bins_count, lower_bound, upper_bound,
False)
    hist_2, _ = create_hist(in_data_2, bins_count, lower_bound, upper_bound,
False)

    sum_1 = 0
    for j in range(bins_count):
        frequency = (hist_1[j] + hist_2[j]) / in_data_len
        if frequency < 1.0e-05:
            continue
        sum_1 += (hist_1[j] - in_data_1_len * frequency)**2 / (in_data_1_len *
frequency)

    sum_2 = 0
    for j in range(bins_count):
        frequency = (hist_1[j] + hist_2[j]) / in_data_len
        if frequency < 1.0e-05:
            continue
        sum_2 += (hist_2[j] - in_data_2_len * frequency)**2 / (in_data_2_len *
frequency)

    statistic = sum_1 + sum_2

    return statistic, (statistic < quantile)
```

```
pearson_test_2_experiments_statistic, pearson_test_2_experiments_passed = \
    pearson_test_2_experiments(sortedData, sortedData2, test_hists_bins_count,
chi2_distribution_quantile)
print("Pearson test:")
print("statistic = ", str(pearson_test_2_experiments_statistic))
print("quantile = ", str(chi2_distribution_quantile))
print("passed = ", str(pearson_test_2_experiments_passed))
```

```
Pearson test:
statistic =  18.04004731707197
quantile =  30.14352720564616
passed =  True
```