Measurement Protocol for Spiral Air-Core Electromagnets

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1 Preamble

Firstly, I need to make one thing clear. The described procedure is not an ideal way to measure inductance, but it is one of the two that I can think of, having current equipment. The most optimal way to measure the inductance is to use a *measuring device made for this exact purpose*, that is being an RLC meter. Chinese market offers multitude of solutions starting from several hundred euros. It's a worthy investment if STELAR wishes to standartize the measurements and create a more straightforward workflow and certification processes.

2 Prerequisites

In order to follow this guide, one must know how to use a VNA: calibration, measurements, data analysis. One would also benefit from understanding parallel RLC frequency response (Z_{11} parameters, phase behaviour near resonance)

3 Introduction

This document describes the procedure for measuring and extracting inductance values from spiral air-core multi-layer electromagnets using Vector Network Analyzer (VNA) impedance measurements. The method employs a two-step curve fitting approach that accounts for frequency-dependent skin effect in the resistance and extracts the inductance from the inductive reactance in the sub-resonant frequency regime.

3.1 Physical Model

The Device Under Test (DUT, air core single/multi-layer electromagnet) is modeled as a parallel RLC circuit[1] with the following characteristics:

- **Resistance (frequency-dependent):** The AC resistance increases with frequency due to skin effect:

$$R_{\text{ac}}(\omega) = \alpha \cdot \exp\left(\beta\sqrt{\omega}\right) \tag{1}$$

  where $\alpha \approx R_{\text{dc}}$ and $\beta = \frac{(1+j)d}{\sqrt{2}\rho}$ - skin effect parameter (**not skin depth** δ).

- **Inductance:** The inductive reactance is linear with angular frequency:

$$X_L(\omega) = \omega L \tag{2}$$

[1]It must be noted that the parallel RLC model works well *only* within restrains specified below. Readers are incentivized to come up with their own models if parallel RLC yields inadequate results.

- **Capacitance:** Parasitic inter-turn and layer-to-layer capacitance causes self-resonance at frequency f_{SRF}:

$$f_{\mathrm{SRF}} = \frac{1}{2\pi\sqrt{LC}} \tag{3}$$

The complex impedance is:

$$Z(\omega) = R_{\mathrm{ac}}(\omega) + j\omega L \tag{4}$$

valid for $f \ll f_{\mathrm{SRF}}$ (typically $f < f_{\mathrm{SRF}}/12$ to $f_{\mathrm{SRF}}/15$).

4 Equipment Requirements

- Vector Network Analyzer (VNA) with impedance measurement capability (e.g., S-parameter conversion to Z-parameters)
- Calibration kit appropriate for the DUT impedance range
- Measurement leads/cables with known characteristics
- Device Under Test (DUT): spiral air-core multi-layer electromagnet, or other RLC-model friendly device
- Connection fixtures for DUT mounting (crocodile)

5 Measurement Procedure

5.1 Step 1: Determine Self-Resonant Frequency (SRF)

The self-resonant frequency is the critical parameter that defines the valid measurement range. At f_{SRF}, the DUT transitions from inductive to capacitive behavior.

Connect the DUT to the VNA (initial calibration at VNA ports is sufficient for this step[2]).

Configure the VNA for impedance magnitude and phase measurements (or S11 with Smith chart display).

Set a wide frequency sweep covering the expected SRF range. For typical spiral air-core electromagnets, start with 100 kHz to 10 MHz.

Observe the impedance behavior:

- **Below SRF:** Impedance magnitude $|Z|$ increases with frequency (inductive region)
- **At SRF:** Maximum impedance is reached
- **Above SRF:** Impedance magnitude decreases with frequency (capacitive region)

Identify the SRF from the VNA display:

- **Impedance magnitude:** Peak in $|Z(f)| \rightarrow\ k\Omega, M\Omega, etc.$
- **Phase:** *Transition through* $0°$ *(or* $180°$ *phase flip*[3])

[2] that being said, one must be mindful of measurement range, i.e. the calibration must be satisfactory for selected range

[3] this is the most straightforward way to find SRF of parallel RLC element, prefer if possible

- **Smith chart:** Crossing from inductive (upper) to capacitive (lower) half-plane through the right-most (open circuit) point ($|Z| \to \infty$)

Record f_{SRF} precisely. Refine the frequency span around the resonance if needed for better resolution.

Important: The SRF value must be manually entered into the Python script in the variable `f_res`.

5.2 Step 2: VNA Calibration at DUT Plane

Proper calibration[4] is **critical** for accurate inductance extraction. The calibration must be performed at the DUT connection plane[5], not at the VNA input ports.

5.2.1 Why Calibration at DUT Plane?

Measurement leads introduce:

- Series inductance (L_{leads})
- Series resistance (R_{leads})
- Parasitic capacitance

These parasitic elements will be included in the measured impedance if calibration is only performed at the VNA ports, resulting in incorrect inductance values.

5.2.2 Calibration Procedure

2.1 Attach the measurement leads/cables to the VNA.

2.2 Perform a full 1-port calibration (OPEN, SHORT[6], LOAD) at the **DUT connection plane** (i.e., at the end of the measurement leads where the DUT will connect).

2.3 Use calibration standards appropriate for the expected impedance range:

  - **OPEN:** Disconnect leads (open circuit)
  - **SHORT:** Short-circuit at DUT plane
  - **LOAD:** $50\,\Omega$ termination (or appropriate reference impedance)

2.4 Set the frequency range for calibration to cover the measurement frequencies (determined in Step 3).

2.5 Verify calibration quality by checking residual errors (some VNAs provide calibration verification metrics).

5.3 Step 3: Select Measurement Frequencies

The valid measurement range must satisfy $f \ll f_{\text{SRF}}$ to ensure the parallel RLC model reduces to series RL behavior.

[4] not to be confused with calibration from p.5.1; this calibration is actually critical and may lead to overestimation of inductance.

[5] what is DUT plane? it is the input point of the device under test. So the calibration **must account** for the inductance and resistance of the leads

[6] Good luck finding one of those, by the way. My golden coated calibration kit which came with VNA was taken for parts, I guess, so create your own shorts

5.3.1 Frequency Selection Criteria

$$f_{\max} < \frac{f_{\mathrm{SRF}}}{12} \quad \text{to} \quad \frac{f_{\mathrm{SRF}}}{15} \tag{5}$$

Example: If $f_{\mathrm{SRF}} = 4.89\,\mathrm{MHz}$:

- Maximum measurement frequency: $f_{\max} \approx 408\,\mathrm{kHz}$ (using $f_{\mathrm{SRF}}/12$)
- Recommended range: $50\,\mathrm{kHz}$[7] to $400\,\mathrm{kHz}$

5.3.2 Number and Distribution of Points

- Use at least 7–10 frequency points for reliable curve fitting
- Distribute points logarithmically or linearly across the frequency range
- Include lower frequencies ($\sim 50\,\mathrm{kHz}$) to capture near-DC resistance
- Ensure the highest frequency is still well below $f_{\mathrm{SRF}}/12$

5.4 Step 4: Impedance Measurements

4.1 With the VNA calibrated at the DUT plane, connect the DUT.

4.2 Configure the VNA to measure and display:

  - Real part of impedance: $\mathrm{Re}(Z_{11})$
  - Imaginary part of impedance: $\mathrm{Im}(Z_{11})$

4.3 Set the frequency points as determined in Step 3.

4.4 For each frequency point, record:

  - Frequency: f (Hz)
  - Real impedance: $\mathrm{Re}(Z_{11})$ (Ω)
  - Imaginary impedance: $\mathrm{Im}(Z_{11})$ (Ω)

4.5 Verify measurement stability:

  - Check that $\mathrm{Im}(Z_{11}) > 0$ (inductive behavior)
  - Verify that $\mathrm{Im}(Z_{11})$ increases approximately linearly with frequency
  - Ensure $\mathrm{Re}(Z_{11})$ increases with frequency (skin effect)

4.6 Export or manually record the data.

Caution: If measurements show $\mathrm{Im}(Z_{11}) < 0$, you are likely above above the SRF (overshoot). Reduce the maximum measurement frequency.

[7] lowest available frequency available with current VNA model SV4401A

5.5 Step 5: Data Entry into Python Script

Edit the Python script to include your measurement data:

```
# Set the SRF (from Step 1)
f_res = 4.89e6  # Hz (example: 4.89 MHz)

# Enter measurement data (from Step 4)
frequencies = np.array([477e3, 400e3, 352e3, 300e3,
                        250e3, 200e3, 150e3, 103e3,
                        50e3])  # Hz

Z_real = np.array([2.09, 1.84, 1.7, 1.52, 1.31,
                   1.15, 0.95, 0.7, 0.428])  # Ohms

Z_imag = np.array([50.2, 42.5, 37.2, 32, 26.6,
                   21.5, 16.2, 11.2, 5.33])  # Ohms
```

Notes:

- Frequencies must be in Hz (use scientific notation, e.g., `400e3` for 400 kHz)
- Arrays must have the same length
- Order doesn't matter (script handles unsorted data)

5.6 Step 6: Execute the Script

6.1 Save the modified Python script.

6.2 Run the script:

```
python rlc_fit.py
```

6.3 The script will output fitting results to the console:

- Skin effect parameters: α (approximate DC resistance) and β
- Inductance: L with uncertainty
- Derived capacitance: C (from f_{SRF} and L)

6.4 Examine the generated plots (4 subplots):

- **Top-left:** Real part fit (resistance with skin effect)
- **Top-right:** Imaginary part fit (inductive reactance)
- **Bottom-left:** Linearity check ($\mathrm{Im}(Z)$ vs. ω)
- **Bottom-right:** Fit residuals[8]

[8] the inductance fit still shows some systematic trends, the origins of which remain a mystery to the author. The magnitude of them, however, is so small, that it makes no difference **at the moment.** If you find the error in the model, feel free to e-mail me at denis.burov@proton.me

6 Results Interpretation and Validation

6.1 Expected Output

The console output provides:

```
TWO-STEP FIT: RESISTANCE + INDUCTANCE
======================================================================
Input:
  f_resonance (known) = 4890.0 kHz
  Measurement range: 0.05 - 0.48 MHz

Step 1 - Skin effect (Re(Z) only):
  alpha = 0.XXX +/- 0.XXX Ohm
  beta = X.XXX +/- X.XXX x10^-3
  R_dc = X.XX Ohm

Step 2 - Inductance (Im(Z) only):
  L = XX.XX +/- X.XX uH

Derived:
  C = XX.XX +/- X.XX nF (from f_res)
  f_res (from L,C) = XXXX.X kHz (check)
  R_ac(2 MHz) = XX.XX Ohm
======================================================================
```

6.2 Quality Checks

1. **Recalculated SRF:** The f_{res} calculated from fitted L and derived C should match the input f_{SRF} within a few percent. Large discrepancies indicate:

   - Measurement frequencies too close to SRF
   - Poor calibration
   - Inappropriate model (e.g., significant core losses, close to SRF)

2. **Residuals plot:** Residuals should be randomly distributed around zero[9] with magnitude much smaller than the measured values. Systematic trends indicate model inadequacy.

3. **Linearity check:** The $\text{Im}(Z)$ vs. ω plot should show excellent linearity. Deviations suggest proximity to resonance or measurement errors.

4. **Skin effect:** The real part should show exponential growth: $e^{\sqrt{\omega}}$. At low frequencies, $R_{\text{ac}} \approx R_{\text{dc}} = \alpha$.

5. **Uncertainty:** Check that parameter uncertainties are small relative to fitted values (typically $< 5\%$ for good measurements).

[9]although they aren't quite random at the moment, see other footnote

6.3 Common Issues and Troubleshooting

Symptom	Likely Cause & Solution
Poor fit to Im(Z) (non-linear)	Measurement frequencies too close to SRF. Reduce $f_{\max}$ to $< f_{\mathrm{SRF}}/15$.
Large residuals in Re(Z)	Skin effect model may be inadequate. Check conductor geometry and consider proximity effects.
Negative Im(Z) values	Above SRF or calibration error. Re-check SRF and calibration.
Recalculated f_{SRF} doesn't match	Model breakdown: either too close to resonance, or DUT has distributed parameter effects not captured by lumped RLC model.
Large uncertainty in L	Insufficient number of measurement points or limited frequency range. Increase number of points and expand frequency span (within valid range).

7 Summary of Best Practices

- Always identify f_{SRF} first with a wide frequency sweep
- Calibrate at the DUT plane, not at VNA ports[10]
- Keep measurement frequencies below $f_{\mathrm{SRF}}/12$ (preferably $f_{\mathrm{SRF}}/15$)
- Use at least 7–10 frequency points distributed across the valid range
- Verify inductive behavior: Im(Z) > 0 and ***increasing*** with frequency
- Check fit quality through residuals and linearity plots
- Validate results by comparing recalculated f_{SRF} with measured value

8 References

- Skin effect in conductors: Terman, F.E., *Radio Engineers' Handbook*, McGraw-Hill (1943)
- VNA calibration techniques: Agilent Application Note 1287-3, *Exploring the Architectures of Network Analyzers*
- Self-resonance in inductors: Grandi et al., *Stray Capacitances of Single-Layer Solenoid Air-Core Inductors*, IEEE Trans. IM (1999)

[10] what does "at the DUT plane" mean? Read present manual again