THE PRIVATE PROVISION OF PUBLIC GOODS UNDER UNCERTAINTY: A SYMMETRIC-EQUILIBRIUM APPROACH

DONALD C. KEENAN
University of Georgia

ILTAE KIM
Chonnam National University

RONALD S. WARREN, JR.
University of Georgia

Abstract

Various studies have examined whether increased uncertainty about the non-Nash response of others to an individual's voluntary contribution to a public good affects that individual's contribution so as to mitigate the free-rider problem. We extend this single-agent approach to the analysis of a symmetric equilibrium. We provide conditions on group size and endogenous relative risk aversion that imply increased equilibrium contributions in response to greater uncertainty about the productivity of each individual's contribution to the actual level of the public good. These results enable us to broaden the circumstances in which the theory predicts that increased uncertainty reduces free riding.

1. Introduction

An extensive theoretical literature, summarized and assessed in Cornes and Sandler (1996), examines a single individual's voluntary contribution to the

Donald C. Keenan, Department of Economics, Terry College of Business, University of Georgia, Athens, Georgia 30602, USA (keenan@terry.uga.edu). Iltae Kim, Department of Economics, Chonnam National University, Gwangju 500-757, Republic of Korea (kit2603@chonnam.ac.kr). Ronald S. Warren, Jr., Department of Economics, Terry College of Business, University of Georgia, Athens, Georgia 30602, USA (warren@terry.uga.edu).

We thank John Conley, Todd Sandler, and an anonymous referee for helpful comments.
Received April 19, 2004; Accepted April 21, 2006.

Journal of Public Economic Theory, 8 (5), 2006, pp. 863–873.

provision of a public good in a setting of uncertainty. A particular interest of this body of work is whether uncertainty increases the level of contributions compared to that present under certainty, thereby reducing the degree of free riding. A traditional distinction is drawn in this literature between uncertainty about the contributions of others to the provision of the public good and uncertainty about the response of others to an individual's own public-good contribution. The first type of uncertainty embodies the concept of Nash conjectures, whereas the second type involves non-Nash conjectures. Because of the distinct way the uncertainty enters in each of the two cases, an analysis assuming Nash conjectures amounts to studying the effects of additive risk, while the use of non-Nash conjectures involves examining the consequences of multiplicative risk.

Even in a representative-agent setting, however, an analysis along these lines is clearly incomplete, since one wants to describe the interaction of such agents with others so that the choices of all agents then become endogenous. This requires investigating a symmetric equilibrium within an explicitly game-theoretic framework. Little work has been done in such an overtly equilibrium setup; the analyses of Gradstein, Nitzan, and Slutsky (1992, 1993) are noteworthy exceptions.[1] One interpretation of the earlier research from this point of view is that it provides a preliminary step toward a full equilibrium analysis, and that the results of analyzing a single individual should precede those examining individuals' interactions.

This paper advances the equilibrium analysis of the private provision of public goods under uncertainty by showing that the approach taken by the earlier literature proves fruitful. Novel results obtained from analyzing the purely individual problem are easily translated into results for the symmetric equilibrium of a representative-agent, game-theoretic model in which the technology of providing the public good is uncertain. Section 2 presents a model of individual contributions to a public good when the response of others to an increase in such contributions is uncertain. We provide conditions on risk preferences that determine the compensated and uncompensated effects on an individual's contribution of an increase in uncertainty about others' contributions. In Section 3, we assume that the source of this uncertainty about the contributions of others is the unknown productivity of each person's contribution in determining the actual level of the public good. We then state assumptions about group size and risk preferences that imply, in a symmetric equilibrium of identical individuals, increased contributions as

[1]Eichberger and Kelsey (2002) and Bailey, Eichberger, and Kelsey (2005) study an equilibrium model of the private provision of public goods under uncertainty, but make the distinction between risk and ambiguity, where the latter assumes that the probabilities are unknown. Bailey, Eichberger, and Kelsey cite experimental work surveyed in Andreoni and Croson (in press) in support of the idea that the alleviation of free riding is due more to uncertainty than to altruism, the other explanation commonly offered.

a result of greater uncertainty about this productivity. Section 4 summarizes our findings and provides concluding remarks.

2. Individual Contributions with Non-Nash Conjectures

An individual from a group of size n maximizes the value of a von Neumann–Morgenstern utility function $U_i(c_i, Q)$, where c_i is the level of consumption of a private numeraire good, and Q is the total quantity of a pure public good, amounting to the sum of the ith individual's contribution g_i and the contributions $G_i = \sum_{j \neq i}^{n} g_j$ of the $n - 1$ other individuals. Hence, $Q = \sum_{i=1}^{n} g_i$, and so $Q = G_i + g_i$. The ith individual's budget constraint requires that expenditures on the private good plus contributions to the public good equal the individual's total income, so that

$$c_i + p g_i = m_i, \tag{1}$$

where m_i is the ith individual's income, and p is the per unit price of the public good. It is assumed that U is increasing in its arguments, continuous, strictly concave (implying risk aversion), and thrice-continuously differentiable. For notational simplicity, we henceforth suppress the individual's index i when examining a single individual.

We begin our analysis by assuming non-Nash behavior (Sandler, Sterbenz, and Posnett 1987, Shogren 1990), so that the individual is concerned with how variations in his own contribution to the public good affect the public-good contributions of others. Thus, the conjectural variation $\Phi = dG/dg \neq 0$, indicating an individual's conjecture concerning the response of others' contributions to a change in his own level of public-good provision, is treated as a random variable. The total quantity Q of the public good then consists of the initial, certain provision $\bar{G}$ of the public good by others, the individual's own contribution g, and the change Φg in others' contributions caused by his own provision of the public good, so that $Q = \bar{G} + g(1 + \Phi)$.

The individual's problem is to choose g to maximize

$$E[V(g, \Phi)] = \int U[m - pg, \bar{G} + g(1 + \Phi)]\, dF(\Phi), \tag{2}$$

where Φ is a random variable defined on the interval $[-1, \infty]$, with cumulative distribution function $F(\Phi)$, and V is defined implicitly in Equation (2). The first-order and second-order conditions for a maximum are, respectively,

$$E[V_g] = E[-pU_c + (1 + \Phi) U_Q] = 0, \tag{3}$$

and

$$E[V_{gg}] = E\left[p^2 U_{cc} - 2p(1 + \Phi) U_{cQ} + (1 + \Phi)^2 U_{QQ}\right] < 0. \tag{4}$$

To arrive at the comparative-statics results, it is convenient to work with the random variable $Z = (1 + \Phi) g$, consisting of the individual's contribution g to the public good plus the uncertain change in others' contributions caused by the individual's own provision. Since the purpose of Z is to serve as a proxy for Φ, we hold g constant when analyzing the direct effects of changes in Z.

Since uncertainty affects expected utility multiplicatively, it is useful to employ the Arrow–Pratt index of relative risk aversion to measure risk preferences. In the case of a single random choice variable Q, this index is

$$\begin{aligned} R(Q) &= -QU_{QQ}(Q)/U_Q(Q) \\ &= -(Q/\mathrm{MU}_Q)[\partial(\mathrm{MU}_Q)/\partial Q], \end{aligned} \tag{5}$$

where, of necessity, the marginal value of the random variable Q is measured cardinally in terms of the chosen utility index. In the current two-good problem, however, it is possible to measure instead the marginal value of the random total quantity of the public good Q ordinally in terms of the nonrandom private consumption good c, that is, in terms of the marginal rate of substitution $\mathrm{MRS}_{c,Q}$ of Q for c rather than in terms of the cardinal marginal utility MU_Q of Q. This observation leads naturally to the multivariate, ordinal index of relative risk aversion

$$R^*(c, \bar{G}+Z) = -(Z/\mathrm{MRS}_{c,Z})[\partial(\mathrm{MRS}_{c,Z})/\partial Z], \tag{6}$$

which, when expressed in terms of the derivatives of the utility function, takes the form

$$\begin{aligned} R^*(c, \bar{G}+Z) = &-ZU_{QQ}(c, \bar{G}+Z)/U_Q(c, \bar{G}+Z) \\ &+ ZU_{Qc}(c, \bar{G}+Z)/U_c(c, \bar{G}+Z). \end{aligned} \tag{7}$$

When Equation (7) is evaluated at a deterministic optimum, where $p = U_g/U_c = U_Z(1+\Phi)/U_c$, we arrive at the index of endogenous relative risk aversion

$$\begin{aligned} R^*(Z) = &-ZU_{QQ}(c, \bar{G}+Z)/U_Q(c, \bar{G}+Z) \\ &+ pgU_{cQ}(c, \bar{G}+Z)/U_Q(c, \bar{G}+Z). \end{aligned} \tag{8}$$

This measure of risk preferences will prove valuable in the subsequent analysis in which Z is taken to be the random variable.[2] Because c and g are endogenously determined in the individual's choice problem, they are notationally suppressed. Similarly, the exogenous constant $\bar{G}$ is suppressed as well.

Performing a Slutsky-type decomposition for the certainty case, it is easily shown that an individual's contribution g to the public good increases with a decrease in Φ when $R^* > 1$. That is,

[2]By invoking the corresponding endogenous measure of absolute risk aversion, rather than the relative measure, the same procedure can be used to generate results concerning an individual's response to uncertainty in the case of Nash conjectures. Most results in the literature, as well as some novel conditions, can be easily generated in this manner.

$$\begin{aligned}\partial g/\partial\Phi &= -[(ZU_{ZZ}) - (ZU_{Zc})(U_Z/U_c) + U_Z]/V_{gg} \\ &= \left[(U_Z/V_{gg})(R^* - 1)\right] \\ &= [g(\partial g/\partial G)] - (U_Z/V_{gg}),\end{aligned} \tag{9}$$

where $V_{gg} < 0$ by the second-order condition for an optimum when there is no uncertainty.[3] Equation (9) is, of course, a Slutsky-type decomposition, the first term being the income effect. Note that the increase in G induced by the increase in Φ is equivalent to an increase in income from the standpoint of choosing consumption c. Thus, c increases exactly when it is normal, and so it follows from the budget equation and the fact that income and prices have not actually changed that then and only then must g fall. Since the well-known condition $\partial \mathrm{MRS}_{c,Q}/\partial Q < (>) 0$ characterizes c as normal (inferior), the definition in Equation (6) shows that the sign of the income effect depends entirely on the sign of R^*.

The intuition for the role of R^* in determining the total effect of an increase in Φ is similarly straightforward. Just as the *sign* of the index of endogenous relative risk aversion R^* completely determines the sign of the income effect, the *size* of R^* determines the strength of the income effect. On the other hand, R^* has no effect on either the sign or the strength of the opposing substitution effect. It follows that, for a sufficiently high value of R^*, the magnitude of the income effect will exceed that of the substitution effect.

We now analyze the compensated effect on an individual's public-good contribution of an increase in uncertainty about others' responses to the individual's own level of public-good provision. Diamond and Stiglitz (1974) define a mean-utility-preserving spread of the distribution for G such that the resulting distribution of utility is stochastically dominated in the second degree by the original distribution, but where the mean of the utility distribution remains unchanged. Using their procedure, we obtain the following result:

PROPOSITION 1: *An individual's contribution g to the public good increases (decreases) with a mean-utility-preserving spread of the distribution for the conjectural variation Φ if and only if $R^*(Z)$ is a decreasing (increasing) function of Z.*

Proof: Using the general comparative-statics result from Diamond and Stiglitz (1974), we find that the effect of increased uncertainty on the individual's provision of the public good is positive (negative) if

[3] This follows from $\partial g/\partial G = -\,(U_{QQ} - pU_{cQ})/V_{gg} < 0$ in the case of no uncertainty.

$$\partial(gU_{QQ}/U_Q)/\partial g > (<)0.$$

Differentiation of this expression yields

$$\begin{aligned}
&\partial(gU_{QQ}/U_Q)/\partial g \\
&\quad = g(1+\Phi)\big\{U_{QQQ}/U_Q - [p/(1+\Phi)](U_{QQc}/U_Q) \\
&\qquad\qquad - (U_{QQ}/U_Q)^2 + [(p/(1+\Phi)]\big(U_{QQ}U_{Qc}/U_Q^2\big)\big\} + U_{QQ}/U_Q \\
&\quad = Z\big[U_{QQQ}/U_Q - (U_{QQ}/U_Q)^2\big] - pg\big(U_{QQc}/U_Q - U_{QQ}U_{Qc}/U_Q^2\big) + U_{QQ}/U_Q \\
&\quad = -R_Z^*(Z). \quad \blacksquare
\end{aligned}$$

This is a complete (necessary and sufficient) characterization of the response to a compensated increase in multiplicative risk in terms of the endogenous index of relative risk aversion. This finding is to be contrasted to results provided below concerning the uncompensated effect, which typically provide only sufficient conditions.

Of course, the sign of $R_z^*(Z)$ may not be known *a priori*, so it is useful to present stronger conditions that are sufficient to sign $R_z^*(Z)$ and which are expressed in terms of more familiar measures of risk preferences. Consider the Arrow–Pratt index of relative risk aversion which, in the present context, is

$$\begin{aligned}
R(c, \bar{G}+Z) &= -ZU_{QQ}(c, \bar{G}+Z)/U_Q(c, \bar{G}+Z) \\
&= ZA(c, \bar{G}+Z) \\
&= -ZU_{ZZ}/U_Z, \qquad (10)
\end{aligned}$$

where $A(c, \bar{G}+Z)$ is, of course, the corresponding Arrow–Pratt index of absolute risk aversion.

COROLLARY 1: *If the Arrow–Pratt index of relative risk aversion $R(c, \bar{G}+Z)$ is decreasing in Z and increasing in c, then a mean-utility-preserving spread of the distribution for the conjectural variation Φ increases the individual's contribution g to the public good.*

It follows, given additive separability, that if the Arrow–Pratt index of relative risk aversion is decreasing in Z [i.e., $R_Z(\bar{G}+Z) < 0$], then an individual's contribution g to the public good increases in response to a mean-utility-preserving spread of the distribution for the conjectural variation Φ.

We now analyze the uncompensated effects of increased uncertainty in the conjectural variation Φ. The appropriate concept of increased uncertainty for such an analysis is described by a Rothschild and Stiglitz (1970) spread in the distribution for G, where the mean of this distribution is held constant but the probability weight in the tails of the distribution increases.

PROPOSITION 2: *An individual's public-good contribution g increases (decreases) with a mean-preserving spread in the distribution for the conjectural variation* Φ *if the index of endogenous relative risk aversion* $R^*(Z)$ *is decreasing (increasing) in* Z [*i.e.,* $R^*_Z < (>) 0$] *and if that index exceeds (is less than) unity* [*i.e.,* $R^*(Z) > (<) 1$].

Proof: Using the comparative-statics result in Rothschild and Stiglitz (1971), we find that an individual's public-good contribution g increases (decreases) if

$$\partial\left(g^2 U_{QQ}\right)/\partial g > (<) 0.$$

Differentiation of this expression shows that

$$\begin{aligned}\partial\left(g^2 U_{QQ}\right)/\partial g &= -g[pgU_{cQQ} - g(1+\Phi)U_{QQQ} - 2U_{QQ}] \\ &= -gU_Q\left\{R^*_Z - [R^*(Z) - 1]A(c, \bar{G} + Z)\right\}. \quad \blacksquare \qquad (11)\end{aligned}$$

The intuition for this result is straightforward. In switching from an analysis of Diamond–Stiglitz compensated effects of increased risk to a consideration of Rothschild–Stiglitz uncompensated effects, the mean of the conjectural variation Φ is reduced to its original level. Thus, in addition to the compensated effect that is determined by R^*_Z, there is a relative-price effect induced by the restoration of a lower mean for Φ. As our earlier observations in the certainty case revealed, if and only if $R^* > 1$ will this relative-price effect lead to an increase in the individual's contribution g to the public good.[4]

Once again, we are able to use the basic result to state more familiar conditions on risk preferences that are sufficient to ensure that increased uncertainty about the conjectural variation Φ increases an individual's public-good contribution. As Sandler, Sterbenz, and Posnett (1987) observe, risk aversion ($U_{QQ} < 0$) and prudence ($U_{QQQ} > 0$, as defined by Kimball 1990) are not sufficient to determine the effect on an individual's public-good contribution of

[4]It is instructive to compare the foregoing analysis with that of Dardanoni (1988), who provides a similar treatment of both multiplicative (non-Nash) and additive (Nash) uncertainty. There are two noteworthy contrasts between our results and his. First, Dardanoni (1988) does not explicitly invoke a measure of endogenous risk aversion, and therefore is unable to associate his two central assumptions on preferences with the magnitude and monotonicity, respectively, of such a measure. As a consequence, Dardanoni does not clearly relate his two assumptions to one another. Second, while Dardanoni presents a Slutsky-type decomposition of the effect of an increase in multiplicative risk, he is unable to determine whether the substitution effect or the income effect dominates. However, this determination becomes feasible once it is recognized that the mean impact of the two separate effects can be determined together by a single assumption regarding the magnitude of R^*.

increased risk about Φ when there are non-Nash conjectures, even assuming additively separable utility. We are, however, able to state the following result using slightly stronger assumptions about risk preferences:

COROLLARY 2: *In the case of additively separable utility, if the Arrow–Pratt index of relative risk aversion is decreasing in* $Z[i.e., R_Z(\bar{G}+Z)<0]$ *and exceeds unity* $[R(\bar{G}+Z)>1]$*, then a mean-preserving spread in the distribution for the conjectural variation* Φ *increases the individual's contribution* g *to the public good.*

Thus, we are able to delineate conditions on risk preferences that are sufficient to reduce free-riding behavior in the presence of uncertainty about the contributions of others. These findings contrast sharply with the largely ambiguous results reported in Sandler, Sterbenz, and Posnett (1987).

3. Individual Contributions in a Symmetric Equilibrium

One shortcoming of the approach taken above is that the contributions of others to the public good are treated as an exogenous random variable even though these contributions are, in a true Nash equilibrium, the endogenous choices of others. Indeed, in a representative-agent setting, the contributions of others should be exactly the same as those of the individual under investigation. It turns out, however, that the single-agent approach taken in Section 2 can be extended with a minimum of complications to a symmetric-equilibrium analysis by appropriately specifying an underlying exogenous source of individuals' uncertainty about others' contributions to the public good.

Consider, for example, the model analyzed by Gradstein, Nitzan, and Slutsky (1992, 1993) and discussed in Cornes and Sandler (1996, pp. 182–184). In that model, a distinction is drawn between an individual's contribution (or "input") g_i to the public good Q and the resulting effective "output" q_i of Q. The "technology" used in generating the public good is given by $q_i=\alpha g_i$, where α is an uncertain productivity parameter. As before, each individual's utility is given by $U_i(c_i, Q)$ but now $Q=Q_j+q_i$, where $Q_j=\sum_{j\neq i} q_j$.

In a symmetric equilibrium, $E_{g_i}[U(g_i^*, \sum_{j\neq i} g_j^*)]=0$ with $g_i^*=g_j^*=g^*$. Let the probability distribution for α be completely parameterized by r, so that a shift in the distribution for α is denoted dr. The effect of a shift in the distribution for α on the equilibrium individual contribution g^* to the public good is

$$dg^*/dr = dg_i^*/dr = -E_r\big[U_{g_i}^*\big]\big/\big\{E\big[U_{g_ig_i}^*\big]+(n-1)E\big[U_{g_ig_j}^*\big]\big\}. \qquad (12)$$

The first term in the denominator of Equation (12) is $(\alpha^2 EU_{QQ} - \alpha pEU_{cQ}) + (p^2\, EU_{cc} - \alpha pEU_{cQ})$, which is negative by the second-order condition for an expected-utility maximum. If one makes the plausible assumption that the similar second term in the denominator, $(n-1)(\alpha^2\, EU_{QQ} - \alpha pEU_{cQ})$, is also negative, then the sign of dg^*/dr is determined by the sign of the numerator in Equation (12).

In the present setting, productivity risk enters multiplicatively, as in the single-agent model with non-Nash conjectures. Thus, we are able to proceed with the comparative-statics analysis of a compensated increase in uncertainty in a symmetric equilibrium exactly in the manner of Proposition 1, above.

PROPOSITION 3: *Every individual's contribution g^* to the public good increases (decreases) with a mean-utility-preserving spread of the distribution for individual productivity in producing the public good if $R^*(Z)$ is a decreasing (increasing) function of Z, given the assumption $\alpha EU_{QQ} - pEU_{cQ} < 0$.*

Proof:

$$
\begin{aligned}
\partial(U_{\alpha\alpha}/U_\alpha)/\partial g_i &= \partial(ngU_{QQ}/U_Q)/\partial g_i \\
&= n\alpha g\big[(U_{QQQ}/U_Q) - pU_{QQc}/\alpha U_Q - (U_{QQ}/U_Q)^2 \\
&\qquad + pU_{QQ}U_{Qc}/\alpha U_Q^2\big] + U_{QQ}/U_Q \\
&= nZ\big[(U_{QQQ}/U_Q) - (U_{QQ}/U_Q)^2\big] \\
&\quad - png\big[(U_{QQc}/U_Q) - \big(U_{QQ}U_{Qc}/U_Q^2\big)\big] + U_{QQ}/U_Q \\
&= -nR_Z^*(Z). \quad \blacksquare
\end{aligned}
$$

Corollary 1 can also be applied, in a straightforward way, to the present case of a symmetric equilibrium.

We now state the effect on the equilibrium contribution g^* of an uncompensated increase in uncertainty about individual productivity, in the manner prescribed by Proposition 2.

PROPOSITION 4: *Every individual's contribution g^* to the public good increases (decreases) with a mean-preserving spread in the distribution for individual productivity in producing the public good if $R^*(Z)$ is decreasing (increasing) in Z and if $nR^*(Z) > (<)1$, given the assumption $\alpha EU_{QQ} - pEU_{cQ} < 0$.*

Proof:

$$
\begin{aligned}
\partial(U_{\alpha\alpha})/\partial g_i &= \partial\big[(ng)^2U_{QQ}\big]/\partial g_i \\
&= -(ng)^2(pU_{QQc} - \alpha U_{QQQ}) + 2ngU_{QQ} \\
&= -ngU_Q\big\{nR_Z^* - [nR^*(Z) - 1]A(c, Q_i + Z)\big\}. \quad \blacksquare
\end{aligned}
$$

With a sufficiently large number of individuals, $nR^*(Z)$ necessarily exceeds 1, given $R^*(Z)$. Therefore, large numbers ameliorate the free-riding problem in the face of an uncompensated increase in productivity risk. Because the sign of the switch from compensated to uncompensated effects is assuredly positive for a sufficiently large group, an increase in each individual's public-good contributions in response to an uncompensated increase in uncertainty about productivity in producing the public good is assured by appropriately signing the compensated term, which occurs exactly when $R_z^* < 0$. Easily interpreted conditions for this to hold were noted above, following Proposition 1.

4. Concluding Remarks

Assuming non-Nash behavior, the response of others to an individual's own public-good contribution is uncertain, so the conjectural variation term is effectively a multiplicative random variable. Our analysis of this model reveals that the compensated effect of an increase in the uncertainty about others' responses to an individual's own contribution increases (decreases) that contribution if the individual's attitude toward risk is characterized by decreasing (increasing) endogenous relative risk aversion. We are also able to provide conditions on preferences, expressed in terms of both the monotonicity and magnitude of the index of endogenous relative risk aversion, which determine the sign of the uncompensated effect of an increase in the uncertainty about others' contributions to the public good. These results, in turn, yield conditions, expressed in terms of more traditional measures of risk preferences, which are sufficient to imply alleviation of the free-rider problem in response to the increased uncertainty.

Since the public-good contributions of others are endogenous in a full Nash equilibrium, we then extend the representative-agent approach to a symmetric-equilibrium analysis. Specifically, we introduce multiplicative uncertainty about the productivity of individuals' contributions in producing the actual level of the public good as the underlying, exogenous source of an individual's uncertainty about others' contributions. Using the results derived for the non-Nash case, we establish conditions on group size and the index of endogenous relative risk aversion that are necessary and sufficient (sufficient) for a compensated (uncompensated) increase in this type of productivity risk to increase equilibrium individual contributions to the public good. Our analysis thus establishes a set of plausible circumstances in which a more uncertain environment unambiguously mitigates the free-riding problem in the theory of public goods.

References

ANDREONI, J., and R. CROSON (2008) Partners versus strangers: Random rematching in public goods experiments, in *Handbook of Experimental Economics Results*, Vol. 1, C. R. Plott and V. L. Smith, eds. Amsterdam: North-Holland.

BAILEY, R. W., J. EICHBERGER, and D. KELSEY (2005) Ambiguity and public good provision in large societies, *Journal of Public Economic Theory* **7**, 741–759.

CORNES, R., and T. SANDLER (1996) *The Theory of Externalities, Public Goods and Club Goods*, 2nd ed., Cambridge: Cambridge University Press.

DARDANONI, V. (1988) Optimal choices under uncertainty: The case of two-argument utility functions, *Economic Journal* **98**, 429–450.

DIAMOND, P. A., and J. E. STIGLITZ (1974) Increases in risk and in risk aversion, *Journal of Economic Theory* **8**, 337–360.

EICHBERGER, J., and D. KELSEY (2002) Strategic complements, substitutes, and ambiguity: The implications for public goods, *Journal of Economic Theory* **106**, 436–466.

GRADSTEIN, M., S. NITZAN, and S. SLUTSKY (1992) The effect of uncertainty on interactive behavior, *Economic Journal* **102**, 554–561.

GRADSTEIN, M., S. NITZAN, and S. SLUTSKY (1993) Private provision of public goods under price uncertainty, *Social Choice and Welfare* **10**, 371–382.

KIMBALL, M. (1990) Precautionary savings in the small and in the large, *Econometrica* **58**, 53–73.

ROTHSCHILD, M., and J. E. STIGLITZ (1970) Increasing risk: I. A definition, *Journal of Economic Theory* **2**, 225–243.

ROTHSCHILD, M., and J. E. STIGLITZ (1971) Increasing risk II: Its economic consequences, *Journal of Economic Theory* **3**, 66–84.

SANDLER, T., F. P. STERBENZ, and J. POSNETT (1987) Free riding and uncertainty, *European Economic Review* **31**, 1605–1617.

SHOGREN, J. F. (1990) On increased risk and the voluntary provision of public goods, *Social Choice and Welfare* **7**, 221–229.