Differential Equations Assignment Report

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Variant: №10

O.D.E. and initial conditions:
$$\begin{cases} y' = \sin(2x)/2 - y*\cos(x) \\ y(0) = 1 \\ x \in [0, 5.2] \end{cases}$$

GitHub: https://github.com/DenisLevkovets/DE

Solution:

$y' = \sin(2x)/2 - y*\cos(x)$

$y'+y*\cos(x) = \sin(2x)/2$

Solve complementary eq.

$y'+y*\cos(x) = 0$

$y'/y = -\cos(x)$

$dy/y = -\cos(x)dx$

$\ln|y| = -\sin(x) + C$

$y = e^{-\sin(x)} * C1 \to e^{-\sin(x)} * C1(x)$

Find the parameter function:

$y' = -\cos(x) * e^{-\sin(x)} * C1(x) + e^{-\sin(x)} * C1'(x)$

$-\cos(x) * e^{-\sin(x)} * C1(x) + e^{-\sin(x)} * C1'(x)+ e^{-\sin(x)} * C1(x) *\cos(x) = \sin(2x)/2$

$e^{-\sin(x)} * C1'(x)= \sin(2x)/2$

$C1'(x) = \sin(2x)/(2*e^{-\sin(x)})$

$C1(x) = \sin(2x)/(2*e^{-\sin(x)})dx$

$C1(x) = e^{\sin(x)} *(\sin(x) -1) + C2$

$y = e^{-\sin(x)} * (e^{\sin(x)} *(\sin(x) -1)+ C2)= C2 * e^{-\sin(x)} + \sin(x) -1$

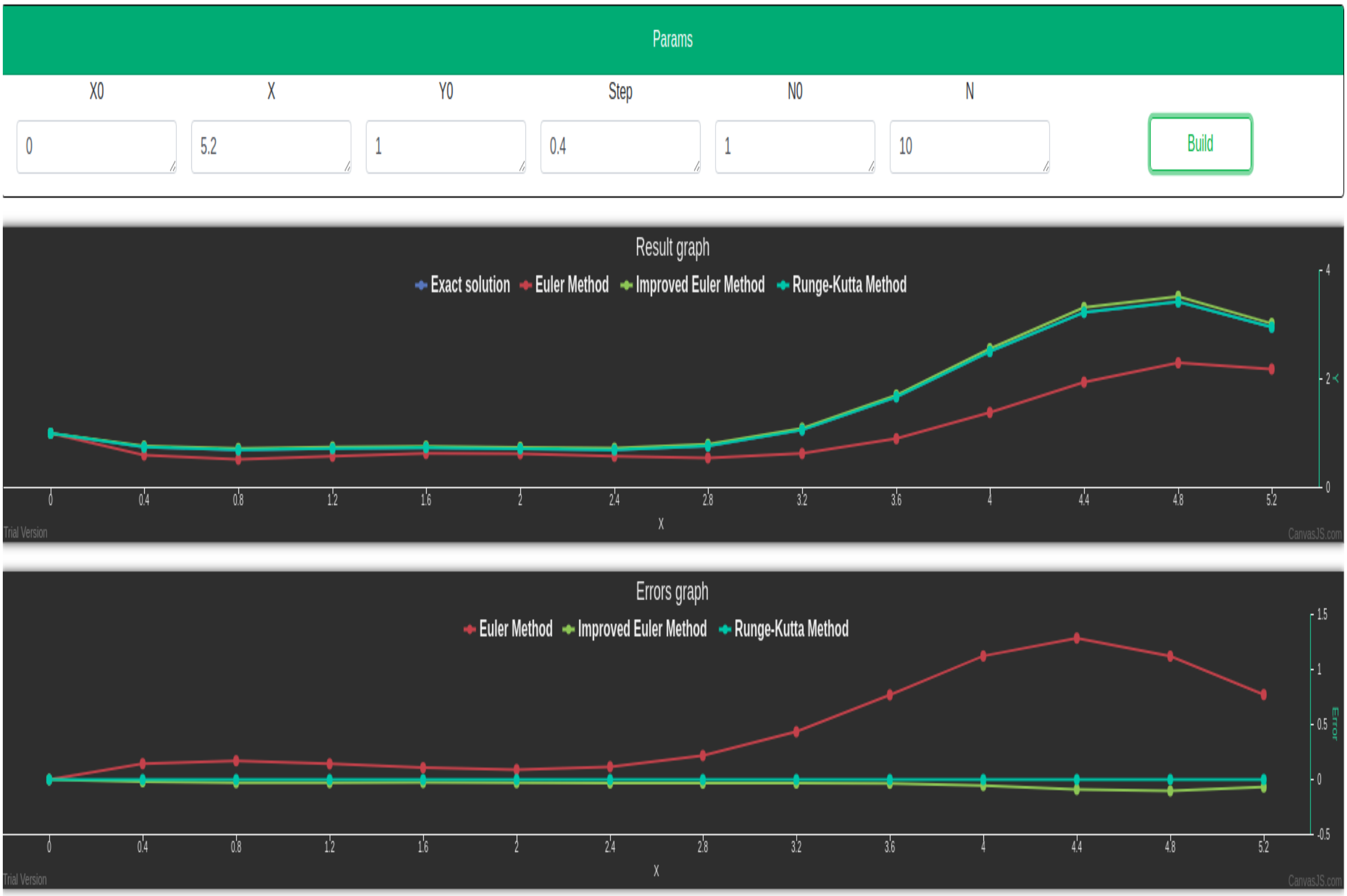
Find Constant

$y(0)=1$

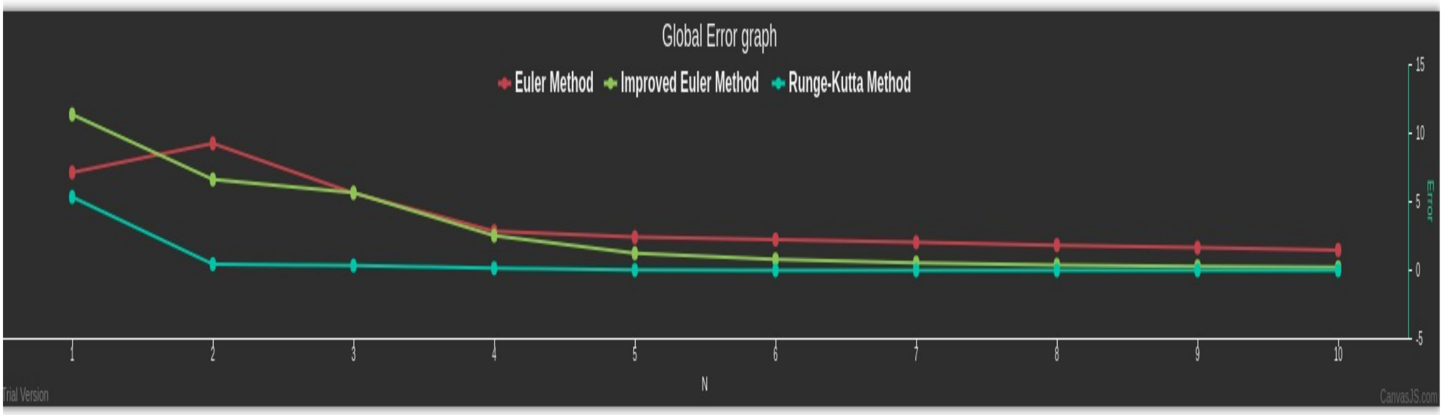
$1 = C2 * e^{-\sin(0)} + \sin(0) - 1 = C2 * e^{0} + 0 - 1 \Rightarrow C2 = 2$

$y = 2 * e^{-\sin(x)} + \sin(x) - 1$

Graphs:

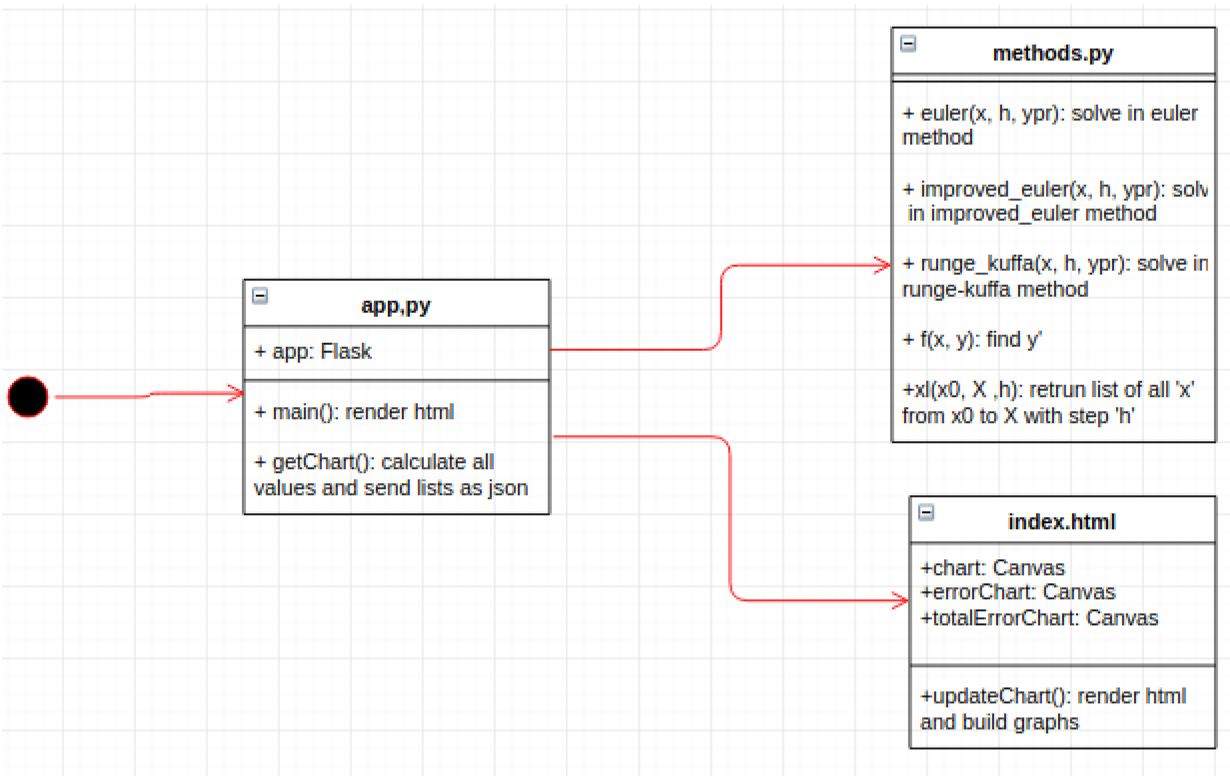


We can notice that Improved Euler and Runge-Kuffa Methods have very low error with exact solution.



It is global error for N=[1..10]. We can notice that when we take small N, than our step is very big and there is big error with exact solution. That's why to have low error we should calculate with small step.

UML



Requirements:

$ pip install flask