Book of abstracts of the

8^{th} International Algebraic Conference in Ukraine

Dedicated to the memory of
Professor Vitaliy Mikhaylovich Usenko

Editors

Yu. A. Drozd, V. V. Kirichenko, B. V. Novikov

July 5 – 12, 2011
Lugansk Taras Shevchenko
National University
Ukraine

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С23

8-ма Міжнародна алгебраїчна конференція в Україні: збірник тез (англійською мовою) — Луганськ: Видавництво Луганського національного університету імені Тараса Шевченка, 2011. — 320 с.

С23 **8-ма Міжнародна алгебраїчна конференція в Україні:** збірник тез (англійською мовою) — Луганськ: Видавництво Луганського національного університету імені Тараса Шевченка, 2011. — 320 с.

У збірнику містяться матеріали 8-ої Міжнародної алгебраїчної конференції в Україні, присвяченої 60-річчю від дня народження професора Віталія Михайловича Усенка.

Тези поділені на наступні тематичні розділи: алгебраїчні аспекти теорії диференціальних рівнянь; алгебраїчна геометрія та топологія; аналітична та алгебраїчна теорія чисел; комп'ютерна алгебра та дискретна математика; групи та алгебраїчна динаміка; зображення та лінійна алгебра; кільця та модулі; напівгрупи та алгебраїчні системи.

Book of abstracts of the 8th International Algebraic Conference in Ukraine dedicated to the 60th anniversary of Professor Vitaliy Mikhaylovich Usenko.

Abstracts in the book are divided to the following topical sections: algebraic aspects of the theory of differential equations; algebraic geometry and topology; analytic and algebraic theory of numbers; computer algebra and discrete mathematics; groups and algebraic dynamics; representations and linear algebra; rings and modules; semigroups and algebraic systems.

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Підписано до друку рішенням Вченої ради
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In Memory of
Vitaliy Mikhaylovich Usenko
(1951 — 2006)

Contents

Topical section III

31

Analytic and Algebraic Theory of Numbers

Topical section IV

Computer Algebra and Discrete Mathematics 51

TOPICAL SECTION VII

RINGS AND MODULES

183

General information

The 8th International Algebraic Conference in Ukraine dedicated to the 60th anniversary of Professor Vitaliy Mikhaylovich Usenko will take place on July 5—12, 2011 in Lugansk, Ukraine.

The conference will include the following topical sections:

1. Algebraic Aspects of the Theory of Differential Equations;
2. Algebraic Geometry and Topology;
3. Analytic and Algebraic Theory of Numbers;
4. Computer Algebra and Discrete Mathematics;
5. Groups and Algebraic Dynamics;
6. Representations and Linear Algebra;
7. Rings and Modules;
8. Semigroups and Algebraic Systems.

We plan plenary talks (45 min), section talks (25 min) and short communications (15 min). The official languages of the conference are Ukrainian, Russian and English.

Organizers

The Conference is organized jointly by:

Institute of Mathematics
of National Academy of Sciences of Ukraine,

Kyiv Taras Shevchenko National University
(Ukraine),

Institute of Applied Mathematics and Mechanics
of National Academy of Sciences of Ukraine,

Francisk Skorina Gomel State University (Belarus),

Slavyansk State Pedagogical University (Ukraine),

Lugansk Taras Shevchenko National University
(Ukraine).

1. Local organizing committee

Chairman: S. V. Savchenko.

Vice-chairman: I. A. Mikhaylova.

Members:

Yu. M. Arlinskiĭ, S. M. Chuyko, Ya. M. Dymarskii, G. A. Mogylniy, O. V. Revenko, V. V. Shvyrov, A. V. Zhuchok, Yu. V. Zhuchok.

2. Program committee

Co-chairmans: Yu. A. Drozd (Ukraine), V. V. Kirichenko (Ukraine).

Vice-chairman: B. V. Novikov (Ukraine).

Members:

V. I. Andriychuk (Ukraine)	V. A. Artamonov (Russia)	V. V. Bavula (United Kingdom)
Yu. V. Bodnarchuk (Ukraine)	V. Dlab (Canada)	M. Dokuchaev (Brazil)
W. Dudek (Poland)	V. Futornyj (Brazil)	R. I. Grigorchuk (USA)
I. S. Grunskii (Ukraine)	P. M. Gudivok (Ukraine)	A. I. Kashu (Moldova)
M. Ya. Komarnytskyj (Ukraine)	V. A. Kozlovskii (Ukraine)	L. A. Kurdachenko (Ukraine)
F. M. Lyman (Ukraine)	F. Marko (USA)	V. S. Monakhov (Belarus)
M. S. Nikitchenko (Ukraine)	A. Yu. Ol'shanskii (USA)	S. A. Ovsienko (Ukraine)
M. O. Perestyuk (Ukraine)	A. P. Petravchuk (Ukraine)	I. V. Protasov (Ukraine)
Yu. M. Ryabukhin (Moldova)	A. M. Samoilenko (Ukraine)	M. M. Semko (Ukraine)
V. V. Sharko (Ukraine)	L. A. Shemetkov (Belarus)	I. P. Shestakov (Brazil)
A. N. Skiba (Belarus)	O. M. Stanzhitsky (Ukraine)	A. Stolin (Sweden)
V. I. Sushchansky (Poland)	P. D. Varbanets (Ukraine)	M. V. Zaicev (Russia)
M. M. Zarichnyi (Ukraine)	E. I. Zelmanov (USA)	A. N. Zubkov (Russia)

Conference schedule

1. Program overview

MONDAY, JULY 4	
09:00—17:00	Arrival day, registration of participants
TUESDAY, JULY 5	
08:00—12:00	Arrival day, registration of participants
12:00—14:10	Opening ceremony, plenary talks
13:15—13:25	☕☕☕ Coffee break ☕☕☕
14:10—15:40	Excursion at Lugansk Taras Shevchenko National University
18:00—20:00	Welcome Party
WEDNESDAY, JULY 6	
09:00—13:45	Plenary talks
11:35—12:05	☕☕☕ Coffee break ☕☕☕
13:45—15:30	Dinner
15:30—18:05	Section talks
16:50—17:05	☕☕☕ Coffee break ☕☕☕
THURSDAY, JULY 7	
09:00—13:45	Plenary talks
11:35—12:05	☕☕☕ Coffee break ☕☕☕
13:45—15:30	Dinner
15:30—18:05	Section talks
16:50—17:05	☕☕☕ Coffee break ☕☕☕
departure at 7:00	Excursion to Artyomovsk Winery (biggest Eastern European enterprise producing sparkling wine)

FRIDAY, JULY 8

09:00–13:45	Plenary talks
11:35–12:05	☕☕☕ Coffee break ☕☕☕
13:45–15:30	Dinner
15:30–17:00	Section talks
16:30–16:40	☕☕☕ Coffee break ☕☕☕
18:00–22:00	Conference dinner

SATURDAY, JULY 9

09:00–13:45	Plenary talks
11:35–12:05	☕☕☕ Coffee break ☕☕☕
13:45–15:30	Dinner
15:30–18:05	Section talks
16:50–17:05	☕☕☕ Coffee break ☕☕☕

SUNDAY, JULY 10

09:00–13:45	Plenary talks
11:35–12:05	☕☕☕ Coffee break ☕☕☕
13:45–15:30	Dinner
15:30–17:25	Section talks
16:30–16:45	☕☕☕ Coffee break ☕☕☕
17:25–19:00	Lugansk city sightseeing tour

MONDAY, JULY 11

09:00–13:45	Plenary talks
11:35–12:05	☕☕☕ Coffee break ☕☕☕
13:45–15:30	Dinner
15:30–18:05	Section talks
16:50–17:05	☕☕☕ Coffee break ☕☕☕

TUESDAY, JULY 12

10:00	Conference closing

2. Program of plenary talks

TUESDAY, JULY 5

12:00—12:30	Anatolii Zhuchok	Opening ceremony. «About Professor Vitaliy Mikhaylovich Usenko»
12:30—13:15	Alexander Olshanskii	Dehn and space functions of groups
13:25—14:10	Alexei Kashu	On characteristic submodules and operations over them

WEDNESDAY, JULY 6

09:00—09:45	Leonid Kurdachenko	On the transitivity of normality and related topics
09:55—10:40	Yuriy Bodnarchuk	Simple modules with a torsion over the Lie algebra of unitriangular polynomial derivations in two variables
10:50—11:35	Yury Arlinskii	Operator-norm approximation of holomorphic one-parameter semigroups of contractions in Hilbert spaces
12:05—12:50	Mykola Komarnytskyj	On ultraproducts of the Noetherian V-monoids with zero
13:00—13:45	Vasyl Andriychuk	On the Tate-Poitou exact sequence for finite modules over pseudoglobal fields

THURSDAY, JULY 7

09:00—09:45	Yuriy Drozd	Tilting for non-commutative curves
09:55—10:40	Frantisek Marko	Bideterminants for Schur superalgebras
10:50—11:35	Mykola Pratsiovytyi	Alternating Luroth series representations for real numbers and their application
12:05—12:50	Andriy Oliynyk	Groups defined by automata over rings
13:00—13:45	Andrii Gatalevych, Volodymyr Shchedryk, Bogdan Zabavsky	Bezout rings and rings of elementary divisors

FRIDAY, JULY 8

09:00—09:45	Alex Martsinkovsky	Approximation, duality, and stability
09:55—10:40	Anatoliy Petravchuk	Polynomial Lie algebras of rank one
10:50—11:35	Vladimir Bavula	An analogue of the Conjecture of Dixmier is true for the algebra of polynomial integro-differential operators
12:05—12:50	Anatoliy Prykarpatsky	On the algebraic structure of dynamical systems on discrete manifolds
13:00—13:45	Yuriy Leonov	Regular wreath products of finite p-groups and Kaloujnine groups

SATURDAY, JULY 9

09:00—09:45	Vyacheslav Futorny	Representations of Lie algebra of vector fields on a torus
09:55—10:40	Vladimir Sergeichuk	Deformations of forms
10:50—11:35	Anatolii Zhuchok	Structural properties of dimonoids
12:05—12:50	Vasyl Petrychkovych	Factorizations in rings of block triangular matrices
13:00—13:45	Oleg Gutik	On semigroups of (almost) monotone injective partial selfmaps of integers with cofinite domains and images

SUNDAY, JULY 10

09:00—09:45	Vladimir Sharko	Crossed modules and its applications
09:55—10:40	Nikolay Vorobyov	On the problems of the structure of Fitting classes
10:50—11:35	Alexander Zubkov	Standard homological properties for supergroups
12:05—12:50	Yakov Dymarskii	Topological problems in the theory of eigenfunctions for nonlinear boundary value problems
13:00—13:45	Hossein Hedayati	Congruences on Γ-semigroups

MONDAY, JULY 11

09:00—09:45	Vitalij Bondarenko	Critical posets and posets with nonnegative Tits form
09:55—10:40	Olga Dashkova	On modules over group rings of soluble groups with the condition $max - nnd$
10:50—11:35	Leonid Bedratyuk	Castelnuovo-Mumford regularity of algebras of SL_2-invariants
12:05—12:50	Volodymyr Lyubashenko	A-infinity-morphisms with several entries
13:00—13:45	Antanas Laurinčikas	Universality of Hurwitz zeta-functions

In Memory of Vitaliy Mikhaylovich Usenko

(01.04.1951 – 06.03.2006)

Vitaliy Mykhailovych Usenko was born on April 1st 1951 in Blagodatnoye village of Volnovakhslii district in Donetsk oblast. In 1973 he graduated from the Department of Mechanics and Mathematics of Kyiv Taras Shevchenko State University. Then he worked as an engineer at the research sector of Donetsk Polytechnic Institute and as an assistant of the Department of Applied Mathematics.

In 1979 V. M. Usenko started postgraduate course at Kharkiv Institute of Radio-electronics. An outstanding algebraist, Professor Lazar Matveiievich Gluskin was his research supervisor. After completing postgraduate course in 1982 he came back to Donetsk Polytechnic Institute to the position of assistant of the Department of Higher Mathematics.

In 1984 Vitaliy M. Usenko defended candidate of sciences dissertation on the topic: «Subdirect Products of Monoids». In 1985 he moved together with his family to Slavyansk town of Donetsk Region where he took the position of head of the Department of Algebra and Geometry of Slavyansk State Pedagogical Institute. 1994–1996 he was the dean of Physics and Mathematics Faculty of this institute. After having defended dissertation on the topic «Semigroups and Near-Rings of Transformations» (research supervisor – Prof. V. V. Kirichenko) in 2000, he gained his Doctor Degree, and in a year he was awarded the rank of a professor.

Since September 2001 Professor Usenko had been working at Luhansk Taras Shevchenko State Pedagogical University. Here he at first was in charge of the Department of Algebra and Mathematical Analysis, since 2003 he was the head of the Department of Algebra and Discrete Mathematics whose establishment was initiated by Vitaliy Mykhailovych.

In 2005 Professor Usenko was invited to read lectures in the University of Antioquia (Columbia). The year business trip was not completed, on March 6th 2006 he passed away due to cardiac failure. Prof. Usenko died when his talent was in its prime having left many interesting plans, projects and incomplete works. His early death is a big loss for Ukrainian mathematics.

Research activity of Vitaliy Usenko was very fruitful. Its main fields were the semi-group theory and the near-ring theory. V. M. Usenko described congruences of monoid of semilinear transformations (generalization of the famous Maltsev's theorem about congruences of matrix semigroups), introduced and studies the construction of semidirect product of groups with united subgroup. He constructed the category of semigroup pairs and a category of convolutions of a semigroup pair; he described free and universal objects of these categories. The category of semigroup pair is the extension the concept of category of group pairs by B. I. Plotkin, while description of free objects of this category is a generalized answer to the question posed by A. H. Kurosh concerning the structure of free objects of the group pair category. The theory of general products of Popp was extended to semigroups as well as the results of Redei and Kon concerning the structure of general product of two cyclic groups. Apart from these results Prof. Usenko described structural properties of endomorphism semigroup of a free semigroup which define it up to isomorphism. This description was obtained in terms of the theory of ideal extensions and is a generalization of the non-commutative case of L. M. Gluskin's results concerning endomorphism semigroups of linear spaces and modules.

He also made a significant contribution in solution of the problem of near-ring classification. He constructed the category of NR-conjugations of group and near-ring, he described universal objects of this category. In terms of the NR-product construction the structure of symmetric near-ring on a free group was described which adds and reinforces the famous results about structure of symmetric near-rings, obtained in works of Berman and Silverman, Meldrum and others, the results of Zeamer about transformation near-rings of free groups.

Vitaliy Mykhailovych described the structure of endomorphism semigroups of completely 0-simple semigroup with the help of wreath product. This completes the results of L. M. Gluskin, Preston, Mann and Tamura about homomorphisms and congruences of completely 0-simple semigroups, and generalizes the Rees' theorem about the automorphism group of completely 0-simple semigroup.

These and other results of Professor Usenko are acknowledged in the mathematical world. Vitaliy Mykhailovych is a kind of person to be called «man of science». His life journey is the confirmation of it. Having headed the Department of Algebra and Geometry in Slavyansk, he created his talented team that later in professional spheres was named «Algebraic school of Usenko».

V. M. Usenko's scientifically-organizing activity was rather fruitful. He was the initiator and the main organization man of the First International Algebraic Conference in Ukraine (Slavyansk, 1998) which was dedicated to the memory of his supervisor and tutor L. M. Gluskin. Very few people knew that he met the considerable part of expenses for the organization of this conference. And it has always been the case. To invest money in the science and not to cash on it was typical for Vitaliy Mykhailovych.

The most outstanding and fruitful life period of the scientist was in Luhansk. In Luhansk National Pedagogical University he grounded and headed the laboratory of theoretical and applied problems of mathematics, for which work the outstanding mathematicians of Ukraine were involved. At the initiative of Professor Usenko, with the help of the laboratory staff and with the material support of the university administration were organized a journal edition of the international level «Algebra and Discrete Mathematics», and also the «Ukrainian Mathematical Bulletin». Thanks to Vitaliy Mykhailovych in Luhansk National Pedagogical University was based the postgraduate study with the specialization in «Algebra and Number Theory» and were opened the scientific and metrological seminars for postgraduate students. Later (in 2004) on the basis of laboratory of theoretical and applied problems of mathematics was grounded the Branch of Institute of Applied Mathematics and Mechanics of National Academy of Sciences of Ukraine. In 2002 on the basis of Luhansk National Pedagogical University Professor Usenko organized an international conference on application of algebraic methods in the discrete mathematics. The conference had a high scientific level: there were many interesting reports and Vitaliy

Mykhailovych planned the further organization of such conferences, but to put it into practice was not possible.

Vitaliy Usenko is the author of about hundred scientific works. Under his supervision there were prepared and defended eight candidate's dissertations, but the number of the mathematicians who consider him to be their teacher is rather larger. His followers work in various high educational establishments of Ukraine. In Luhansk National Pedagogical University in a short time professor Usenko succeeded to base an algebraic team. These docents and assistants consider him to be their teacher: I. A. Mikhaylova, A. V. Zhuchok and Yu. V. Zhuchok, V. V. Shvyrov, A. V. Radchuk.

Vitaliy Mykhailovych Usenko was a talented mathematician and a teacher, an outstanding organization man, a wise head, a reliable companion, a kind, cheerful and merry person. The best memory of him is the living out of his ideas and the realization of his uncompleted plans.

Yu. A. Drozd, A. I. Kashu, V. V. Kirichenko,
M. Ya. Komarnytskyj, L. A. Kurdachenko, I. A. Mikhaylova,
B. V. Novikov, A. P. Petravchuk, A. B. Popov,
M. V. Pratsiovytyi, I. V. Protasov, M. M. Semko,
L. A. Shemetkov, V. I. Sushchansky, P. D. Varbanets,
A. V. Zhuchok, Yu. V. Zhuchok

List of the basic publications of V. M. Usenko

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Topical section I

Algebraic Aspects of the Theory of Differential Equations

8th International Algebraic Conference
July 5—12 **(2011)**, Lugansk, Ukraine

PLENARY TALK

Operator-norm approximation of holomorphic one-parameter semigroups of contractions in Hilbert spaces

Yury Arlinskiĭ

Let $\{T(t)\}_{t\geq 0}$ be a one-parameter C_0-semigroup of contractions in a separable Hilbert space. There are several approximations of $T(t)$ in strong operator topology [1], which justify the representation $T(t) = \exp(-tA)$, $t \geq 0$, when A is the unbounded generator of $T(t)$. If the semigroup admits holomorphic contractive continuation into the sector $\mathcal{S}(\varphi) = \{z \in \mathbb{C} : |\arg z| \leq \varphi\}$ $(\varphi \in \pi/2)$, the Euler approximation

$$T(t) = \lim_{n\to\infty} (I + tA/n)^{-n}$$

is valid in the operator-norm topology (see [2] and references therein). In the talk I shall present results concerning the operator-norm approximations and corresponding estimates in the Iosida and Dunford-Segal formulas.

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8th International Algebraic Conference
July 5—12 **(2011)**, Lugansk, Ukraine

CONTRIBUTED ABSTRACT

Многообразия потенциалов семейства периодических спектральных задач

Ю. Евтушенко

Мы рассматриваем семейство симметрических периодических краевых задач второго порядка на отрезке. В качестве функционального параметра семейства выступает вещественный потенциал. Хорошо известно, что спектр такой краевой задачи вещественен и состоит только из изолированных собственных значений, которые не более, чем двукратны. Нас интересуют те потенциалы, которым отвечают именно двукратные собственные значения фиксированного номера. Будет описана параметризация указанного подсемейства, восходящая к результатам Н.Е. Жуковского [1]. Эта параметризация позволяет снабдить подсемейство структурой гладкого подмногообразия коразмерности два. Попутно будут получены новые классы интегрируемых линейных дифференциальных уравнений с периодическими потенциалами.

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SECTION TALK

Testing problems for the asymptotically critical exponential autoregression processes

O. N. Ie

Let $\xi^n = (\xi_1, \xi_2, \ldots, \xi_n)$, $n \geq 2$ be an observation of the exponential autoregression process $\xi_i = \theta\xi_{\iota-1} + \varepsilon_i$, $i = 1, 2, ...,$ where $\xi_0 = 0$, $\theta \in (0, \infty)$ is an unknown parameter and $\varepsilon_1, \varepsilon_2, \ldots,$ are i.i.d. variables which have exponential distribution with density $p(x) = e^{-x}$ when $x \geq 0$ and $p(x) = 0$ when $x < 0$. Denote by $\mathbf{P}_\theta^n$ a measure, which gives a distribution of the observation ξ^n. We consider the problem of testing of simple hypotheses H^n and $\tilde{H}^n$ under the observation ξ^n where H^n and $\tilde{H}^n$ mean that a distribution of the observation ξ^n defines by the measures $\mathbf{P}_\theta^n$ and $\mathbf{P}_{\tilde{\theta}}^n$ respectively, as $\theta \neq \tilde{\theta}$.

Let $p_\theta(x_1, ..., x_n)$ be a density of the measure $\mathbf{P}_\theta^n$ with respect to the Lebeg's measure. Introduce the Hellinger integral $H_n(\varepsilon)$ of order $\varepsilon \in (-\infty, \infty)$ for the measures $\mathbf{P}_{\tilde{\theta}}^n$ and $\mathbf{P}_\theta^n$ setting [1]

$$H_n(\varepsilon) = H\left(\varepsilon; \mathbf{P}_{\tilde{\theta}}^n, \mathbf{P}_\theta^n\right) = \int_0^\infty \ldots \int_0^\infty p_{\tilde{\theta}}^\varepsilon(x_1, ..., x_n)\; p_\theta^{1-\varepsilon}(x_1, ..., x_n)\; dx_1 ... dx_n.$$

The following theorem about an asymptotical behaviour of the Hellinger integral $H_n(\varepsilon)$ as $n \to \infty$ is valid.

Theorem 1. *Let* $\theta_n = 1 - \Delta_n$, $\Delta_n > 0, \tilde{\theta}_n = 1 - \tilde{\Delta}_n$, $\tilde{\Delta}_n > 0$ *and* $\tilde{\Delta}_n = c\Delta_n, 0 < c < 1$. *If* θ_n *and* $\tilde{\theta}_n$ *depend on* n *such that* $\Delta_n \to 0$ *and* $n\Delta_n \to \infty$ *as* $n \to \infty$. *Then for all* $\varepsilon \in (-\infty, +\infty)$ *there exists limit*

$$\lim_{n\to\infty} n^{-1} \ln H_n(\varepsilon) = \kappa(\varepsilon),$$

where $\kappa(\varepsilon) = -\ln\left(1 + (1-\varepsilon)\frac{(1-c)}{c}\right)$ *for all* $\varepsilon \in \left[0, \frac{1}{1-c}\right)$ *and* $\kappa(\varepsilon) = \infty$ *for all* $\varepsilon \notin \left[0, \frac{1}{1-c}\right)$.

Introduce the likelihood ratio

$$z_n(x_1, ..., x_n) = \frac{p_{\tilde{\theta}}(x_1, ..., x_n)}{p_\theta(x_1, ..., x_n)}, x_i \in (-\infty, \infty) \quad for \quad all \;\; i = 1, 2, ..., n.$$

Let δ_n be a Neyman-Pearson test of level $\alpha_n \in (0, 1)$ for the testing of H^n and $\tilde{H}^n$ under the observation ξ^n. Then (see [1])

$$\delta_n = \mathrm{I}(\Lambda_n > d_n) + q_n \mathrm{I}(\Lambda_n = d_n),$$

where $\mathrm{I}(A)$ is an indicator of the set A, $\Lambda_n = \ln z_n(\xi_1, ..., \xi_n)$, $d_n \in (-\infty, +\infty)$ and $q_n \in [0, 1]$ are the parameters of the test δ_n, defined by the condition $\mathbf{E}_\theta^n\, \delta_n = \alpha_n$. Here $\mathbf{E}_\theta^n$ means an expectation with respect to the measure $\mathbf{P}_\theta^n$. By β_n we denote 2nd type error probability for the test δ_n.

Then the large deviation theorems are prove for Λ_n as $n \to \infty$ both under the hypothesis H^n and under the hypothesis $\tilde{H}^n$. On the basis of large deviation theorems we establish the relation between exponents of the rates of decrease for the error probabilities α_n and β_n of Neyman-Pearson test δ_n as $n \to \infty$. In this case we use general methods of solution of this problem developed in the papers [2-5] for general binary statistical experiments.

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SHORT COMMUNICATION

Operators in divergence form and their extremal extensions

Yury Arlinskiĭ and Yury Kovalev

Let H be a separable Hilbert space. We considered the problem of characterizations of the Friedrichs and Kreĭn nonnegative selfadjoint extensions for densely defined operators of the form $\mathcal{A} = L_2^* L_1$, where L_1 and L_2 are densely defined and closed operators in H taking values in a Hilbert space $\mathfrak{H}$ and possessing the property $L_1 \subset L_2$. Such kind of operators $\mathcal{A}$ we call *operators in divergence form*. Some conditions for the equality $(L_2^* L_1)^* = L_1^* L_2$ are obtained. Our main results are applied to the following differential operators in the Hilbert space $L_2(\mathbb{R})$:

$$\operatorname{dom}(A_0) = \left\{ f \in W_2^2(\mathbb{R}) : f(y) = 0,\ y \in Y \right\},\ A_0 := -\frac{d^2}{dx^2},$$
$$\operatorname{dom}(A') = \left\{ g \in W_2^2(\mathbb{R}) : g'(y) = 0,\ y \in Y \right\},\ A' := -\frac{d^2}{dx^2},$$
$$\operatorname{dom}(H_0) = \left\{ f \in W_2^2(\mathbb{R}) : f(y) = 0,\ f'(y) = 0,\ y \in Y \right\},\ H_0 := -\frac{d^2}{dx^2}.$$

Here $W_2^1(\mathbb{R})$ and $W_2^2(\mathbb{R})$ are Sobolev spaces, Y is finite or infinite monotonic sequence of points in $\mathbb{R}$, satisfying the condition $\inf\{|y'-y''|,\ y', y'' \in Y,\ y' \neq y''\} > 0$. The operators A_0, A', and H_0 are densely defined and nonnegative with finite (the set Y is finite) or infinite defect indices (the set Y is infinite) and are basic for investigations of Hamiltonians on the real line corresponding to the δ, δ' and $\delta - \delta'$ interactions, respectively, [1].

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SHORT COMMUNICATION

On estimates for a minimal differential operator being the tensor product of two others

D. Limanskii

Suppose that Ω is a domain in $\mathbb{R}^n$ and $p \in [1, \infty]$. For a given differential operator $P(D)$, $D := (D_1, \dots, D_n)$, $D_k := -i\partial/\partial x_k$, we denote by $\mathscr{L}^0_{p,\Omega}(P)$ the linear space of minimal differential polynomials $Q(D)$ satisfying the estimate

$$\|Q(D)f\|_{L^p(\Omega)} \leq C_1 \|P(D)f\|_{L^p(\Omega)} + C_2 \|f\|_{L^p(\Omega)}, \qquad f \in C_0^\infty(\Omega), \tag{1}$$

with some constants $C_1, C_2 > 0$ independent of f.

If $p = 2$ and Ω is bounded, Hörmander [1] showed that for the tensor product

$$P(D) = P_1(D) \otimes P_2(D) = P_1(D_1, \dots, D_{p_1}, 0, \dots, 0)\, P_2(0, \dots, 0, D_{p_1+1}, \dots, D_n) \tag{2}$$

of two differential operators P_1 and P_2 acting on different variables the space $\mathscr{L}^0_{2,\Omega}(P)$ is equal to the tensor product of the spaces $\mathscr{L}^0_{2,\Omega}(P_1)$ and $\mathscr{L}^0_{2,\Omega}(P_2)$.

Here we consider the case of $p = \infty$, $\Omega = \mathbb{R}^n$, and an operator $P(D)$ of the form (2). The statement closed to Hörmander's has been obtained in [2] for elliptic operators P_1 and P_2 which full symbols are nondegenerated (i. e., they have no real zeros). The latter condition is essential: in the case of the product $P_1 \otimes P_2$ of two homogeneous elliptic operators the space $\mathscr{L}^0_{\infty,\mathbb{R}^n}(P)$ contains no nontrivial differential monomials though each of $\mathscr{L}^0_{\infty,\mathbb{R}^n}(P_1)$ and $\mathscr{L}^0_{\infty,\mathbb{R}^n}(P_2)$ is maximal possible [2] (see also [4]).

The next result presents a complete description of $\mathscr{L}^0_{\infty,\mathbb{R}^n}(P_1 \otimes P_2)$ in this case.

Theorem. *[3] Let $P(D)$ be a differential operator of the form (2), and let $P_1(D)$ and $P_2(D)$ be homogeneous elliptic operators of orders l and m respectively. Then the space $\mathscr{L}^0_{\infty,\mathbb{R}^n}(P_1 \otimes P_2)$ is minimal possible, i. e., the inclusion $Q \in \mathscr{L}^0_{\infty,\mathbb{R}^n}(P)$ is equivalent to the equality $Q(D) = c_1 P(D) + c_2,\ c_1, c_2 \in \mathbb{C}$.*

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Contributed abstract

Оценка решения вблизи времени обострения для квазилинейного параболического уравнения с источником и неоднородной плотностью

А. В. Мартыненко, В. Н. Шраменко

Рассматривается квазилинейное параболическое уравнение с источником и неоднородной плотностью следующего вида

$$\rho(x)\frac{\partial u}{\partial t} = div(u^{m-1}|Du|^{\lambda-1}Du) + \rho(x)u^p.$$

При условии, что $\lambda > 0$, $m+\lambda-2 > 0$, $p > m+\lambda-1$, $\rho(x) = |x|^{-l}$, $0 \leq l < \lambda+1 < N$, $p < p^*(l) = m+\lambda-1+(\lambda+1-l)/(N-l)$ получена точная универсальная (т.е. не зависящая от начальной функции) оценка решения вблизи времени обострения:

$$u(x,t) \leq \gamma(T-t)^{-\frac{1}{p-1}}$$

при $t \in (\frac{T}{2}, T)$ и $|x| \leq \frac{1}{2}(T-t)^{\frac{1}{H}}$, где γ- постоянная, которая зависит только от параметров задачи l, m, λ, p, N и

$$H = \frac{(p-1)(\lambda+1-l)}{p-m-\lambda+1}.$$

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SECTION TALK

Applications of generating functions and Hilbert series to the center-focus problem

M. N. Popa, V. V. Pricop

We examine the center-focus problem [1] for the differential system

$$\dot{x} = cx + dy + \sum_{j+l=m>1} a_{jl}x^j y^l, \ \dot{y} = ex + fy + \sum_{j+l=m>1} b_{jl}x^j y^l \tag{1}$$

with Lyapunov's function [1]

$$U(x,y) = K_2 + \sum_{k=3}^{\infty} f_k(x,y),$$

where $K_2 = (cx + dy)y - (ex + fy)x$ is the center-affine comitant [2] of the system (1), and $f_k(x,y)$ are homogeneous polynomials of degree k in relation to x and y.

Suppose that under the system (1) exist so constants $G_1, G_2, G_3, ..$ that the identity take place

$$\frac{dU}{dt} = \sum_{k=2}^{\infty} G_{k-1}K_2^k.$$

Are shown that the constants $G_1, G_2, G_3, ..$ generates some isobar polynomials [3] of coefficients of a system (1), that are the coefficients of center-affine (unimodular) comitants [1], [4] of given type [4], belonging to finite dimensional linear spaces of comitants of the same type. With generalized Hilbert series [4] of graded algebra of unimodular comitants of a system (1) is determinate the form of generalized and common generating function of linear space, with the gradual decomposition after above linear spaces. We bring some estimations of Krull's dimension for the graded algebra built with these spaces for the system (1) for $m = 2$. Are shown that this dimension is comparable with a finite number of focal sizes [5], that differ the center of the focus.

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SECTION TALK

Locally nilpotent derivations of polynomial algebra in three variables which are sums of four $\mathbb{Z}^3$-homogeneous

Pavlo Prokofiev

Locally nilpotent derivations of polynomial algebra are a source of numerous exotic examples of automorphisms of affine space. We can mention Nagata's and Anick's automorphisms among most important.

Taking into consideration natural $\mathbb{Z}^3$-grading on the algebra of polynomial differential operators we can put such question: what kind of homogeneous summands should we take in order to get locally nilpotent derivation as their sum. This task includes problem of characterization of Newton polygons of locally nilpotent derivations.

The following theorem describes all locally nilpotent derivations of polynomial algebra in three variables which are sums of four $\mathbb{Z}^3$-homogeneous.

Theorem 1. *Let D be locally nilpotent derivations of polynomial algebra in three variables which is the sum of four $\mathbb{Z}^3$-homogeneous, and at least one of its summands is not elementary. Than D has one of the following forms after corresponding renaming of variables up to a constant factor :*

$$\left(x_1x_3^{m_3}+\alpha x_2^{k_2+1}\right)^2 x_3^{n_3}\left((k_2+1)x_2^{k_2}\frac{\partial}{\partial x_1}-\frac{x_3^{m_3}}{\alpha}\frac{\partial}{\partial x_2}\right), \tag{1}$$

$$\left(x_1+\alpha x_2^{k_2+1}x_3^{m_3}\right)^2 x_3^{n_3}\left((k_2+1)x_2^{k_2}x_3^{m_3}\frac{\partial}{\partial x_1}-\frac{1}{\alpha}\frac{\partial}{\partial x_2}\right), \tag{2}$$

$$\left((k_2+1)x_1+\beta x_2^{k_2+1}\right)x_3^{l}\left(x_2^{k_2}\frac{\partial}{\partial x_1}-\frac{1}{\beta}\frac{\partial}{\partial x_2}\right)+\alpha\frac{\partial}{\partial x_3}, \tag{3}$$

$$\left((k_2+1)x_1+\beta x_2^{k_2+1}\right)\left(x_2^{k_2}\frac{\partial}{\partial x_1}-\frac{1}{\beta}\frac{\partial}{\partial x_2}\right)+\alpha x_1^{v_1}x_2^{v_2}\frac{\partial}{\partial x_3}, \tag{4}$$

where $m_3, n_3, l, k_2, v_1, v_2 = 0, 1, 2, \ldots, \alpha, \beta \in \mathbb{F}^*$

The form of derivations which are sums of four elementary summands is also described.

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Topical section II

Algebraic Geometry and Topology

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SHORT COMMUNICATION

Submanifold of compact operators' with fixed Jordan bloks

A. Bondar

The manifold of symmetric real matrices with fixed multiplicities of eigenvalues for the first time was considered by V.I. Arnold in the work [1]. The results of Arnold were generalized for the case of compact real self-adjoint operators by the group of the Japanese mathematicians in the article [2]. They introduced a special local diffeomorphism that maps Arnold's submanifold to a flat subspace. Ya. Dymarskii developed the aforementioned works into a full theory [3]. In report we will describe the smooth structure for compact operators' submanifold such that eigenvalues of the fixed multiplicity conforms several Jordan cages. The work is based on the results obtained V.I. Arnold in the article [4] and a local diffeomorphism type [2].

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CONTRIBUTED ABSTRACT

On birational composition of quadratic forms over finite field

A. Bondarenko

Let $f(X)$ and $g(Y)$ be nonsingular quadratic forms having dimension m and n respectively over field K, $\operatorname{char} K \neq 2$. If the product $f(X)g(Y)$ is birationaly equivalent over field K to the quadratic form $h(Z)$ over field K with dimension $m+n$, then we will say that quadratic forms $f(X)$ and $g(Y)$ form a birational composition $h(Z)$ over field K. The first results on the problem of birational composition trace back to Hurwitz, who studied the problem of "sum of squres". The classic results of Hurwitz and Radon on this problem are well known (cf. [1, 2]). The review [3] is dedicated to the results and methods in studying such birational compositions. Pfister [4] considered such identities as well, but he supposed that Φ_i is a rational function of $x_1, \ldots, x_m, y_1, \ldots, y_n$. First general theorems on birational compositions of quadratic forms were obtained in [5]. The complete solution of the problem of birational composition over local fields was obtained in [6]. The main aim of the given report is the solution of the problem of birational composition over finite fields. If $f(X)$ or $g(Y)$ is isotropic over $\mathbb{F}_q$, then we obtain the answer from Theorem 1 from [5]: the product $f(X)g(Y)$ is birationally equivalent to $h(Z)$ over $\mathbb{F}_q$ if and only if $h(Z)$ is an arbitrary non-zero quadratic form of dimension $m+n$ with isotropic over $\mathbb{F}_q$ nonsingular part. In the following theorem we consider anisotropic case.

Theorem 1. *Let $f(X)$ and $g(Y)$ be anisotropic quadratic forms of dimension m and n over finite field $\mathbb{F}_q$, where $\operatorname{char} \mathbb{F}_q \neq 2$, $m \leq n$. Then $n \leq 2$, birational composition $h(Z)$ of $f(X)$ and $g(Y)$ always exists and uniquely defined up to equivalence over $\mathbb{F}_q$: a) if $m = n = 1$, then $h(z_1, z_2) = c_1 c_2 z_1^2$, where $c_1 \in D_{\mathbb{F}_q}(f)$, $c_2 \in D_{\mathbb{F}_q}(g)$; b) if $1 \leq m \leq n \leq 2$, then $h(z_1, \ldots, z_{m+n}) = z_1^2 - \alpha z_2^2$, where $\alpha \in \mathbb{F}_q^* \setminus \mathbb{F}_q^{*2}$.*

Naturally, the following question appears: for which fields K there exists birational composition of quadratic forms over K?

The following necessary condition for existence of birational composition of arbitrary quadratic forms $f(X)$ and $g(Y)$ over K is proved: the index $[K^* : K^{*2}] \leq 2$. We have reasons to suppose that this condition is also sufficient.

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Section talk

Construction of lattices over singularities with large conductor

Olena Drozd-Koroleva

The main goal of this work is to generalize thre results of Dieterich [1] that describe lattices (Cohen-Macauley modules) over singularities of algebraic curves with large conductor.

Definition (Singularity with large conductor)**.** *Let* $\mathbf{A}$ *be a complete local commutative noetherian ring of Krull dimension* 1 *without nilpotent elements,* $\mathfrak{m}$ *be its maximal ideal and* $\mathbb{K} = \mathbf{A}/\mathfrak{m}$ *be its residue field. Denote by* $\mathbf{Q}$ *the complete quotient ring of* $\mathbf{A}$ *and by* $\tilde{\mathbf{A}}$ *the integral closure of* $\mathbf{A}$ *in* $\mathbf{Q}$*. Denote by* $\mathbf{J} = \operatorname{rad} \tilde{\mathbf{A}}$ *the Jacobson radical of the ring* $\tilde{\mathbf{A}}$ *and by* $\mathbb{C} = \operatorname{ann}_{\mathbf{A}} \tilde{\mathbf{A}}/\mathbf{A}$ *the* conductor *of* $\tilde{\mathbf{A}}$ *in* $\mathbf{A}$*, that is the largest ideal of the ring* $\tilde{\mathbf{A}}$ *contained in* $\mathbf{A}$*.*

1. *We call* $\mathbf{A}$ *a* singularity with large conductor *(*SLC*) if* $\mathbf{J} pe \mathfrak{m} \operatorname{spe} \mathbf{J}^2$.
2. *We call a finitely generated* $\mathbf{A}$*-module* M *an* $\mathbf{A}$-lattice *if it is a* torsion free module*, that is the natural homomorphism* $M \to \mathbf{Q}\mathbf{A}M$ *is a monomorphism. It is equivalent to* M *being an* $\mathbf{A}$-Cohen-Macauley module*. The category of all* $mathbf{A}$*-lattices will be denoted by* $\operatorname{Lat} \mathbf{A}$.

Dieterich [1] discovered conditions of representation finiteness and tameness of the category $\operatorname{Lat} \mathbf{A}$ for SLC $\mathbf{A}$ in case when $\mathbf{A}$ has «*geometrical nature*», namely is an algebra over an algebraically closed field that coincides with the residue field $\mathbb{K}$.

Dieterich results were somewhat limited by the condition of field $\mathbb{K}$ being algebraically closed as well as $\mathbf{A}$ being a $\mathbb{K}$-algebra. My goal is to formulate a generalization of the theorem without these restrictions. It is especially useful for the theory of integral representations where the characteristic of the field $\mathbf{A}$ is zero and the characteristic of the residue field is positive. Unfortunately it is impossible to transfer Dietrich's proof directly. But some steps have been made to reach the goal:

1. The *sandwich-category* $\mathcal{T}$ has been identified with the bimodule category $\operatorname{El}(\mathcal{B})$, where $\mathcal{B} = \Lambda$ considered as bimodule over the $\mathbb{K}$-algebra $\mathbb{K} \times \Lambda$, where Λ acts from the left and $\mathbb{K}$ from the right side. This category is abelian, $\operatorname{Ext}^2_{\mathcal{T}} = 0$ and it ntains sufficient number of projective objects.
2. We considere the bimodule $\mathcal{E}$ over the category $\mathcal{T}$ definied by the rule $\mathcal{E}(M, N) = \operatorname{Ext}^1_{\mathcal{T}}(M, XN)$.
3. We check for a number of cases that the bimodule category $\operatorname{El}(\mathcal{E})$ is representation finite if $\nu(\mathbf{A}) < 4$. We prove that it is wild when $\nu(\mathbf{A}) > 4$.

In my talk I will also describe the further steps that are to be done on the way to prove the generalized classification theorem.

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CONTRIBUTED ABSTRACT

Kahn correspondence for Cohen—Macaulay modules and vector bundles and application to curve singularities

Volodymyr Gavran

Let X be a normal complete surface singularity with the singular point x. From [3] it is known that X has a resolution, i.e. there exists a birational projective morphism $\pi : \widetilde{X} \to X$ such that $\widetilde{X}$ is smooth. For an arbitrary cycle C on $\widetilde{X}$ we define the functor $R_C : \mathrm{MCM}(X) \to \mathrm{VB}(C)$ from the category of Cohen—Macaulay modules over X to the category of vector bundles over C as $R_C(M) = (\pi^* M)^{\vee\vee} \otimes \mathcal{O}_C$ for any $M \in \mathrm{MCM}(X)$, where $\mathcal{F}^\vee = \mathcal{H}om_{\mathcal{O}_{\widetilde{X}}}(\mathcal{F}, \mathcal{O}_{\widetilde{X}})$ is the dual sheaf to $\mathcal{F} \in \widetilde{X}$.

For the case when X is a scheme over an algebraically closed field it is proved in [2] that for some special cycle Z, which is called a reduction cycle, the following theorem holds.

Theorem 1. [2, Theorem 1.4] *Let $\pi : (\widetilde{X}, E) \to (X, x)$ be a resolution of a normal surface singularity, and let$Z$ be a reduction cycle on $\widetilde{X}$. Then the functor R_Z maps non-isomorphic objects from $\mathrm{MCM}(X)$ to non-isomorphic ones from $\mathrm{VB}(Z)$ and a vector bundle $F \in \mathrm{VB}(Z)$ is isomorphic to $R_Z M$ for some M if and only if it is generically generated by global sections and there is an extension of F to a vector bundle F_2 on $2Z$ such that the exact sequence*

$$0 \longrightarrow F(-Z) \longrightarrow F_2 \longrightarrow F \longrightarrow 0$$

induces a monomorphism $\mathrm{H}^0(E, F(Z)) \to \mathrm{H}^1(E, F)$.

I have proved that Theorem 1 holds in the case of "abstract" normal surface singularities, that is spectra of arbitrary complete noetherian local normal rings of Krull dimension 2.

Using this fact I have extended the Drozd—Greuel tameness criterion for curve singularities over an algebraically closed field [1, Theorem 1] to "abstract" curve singularities. Namely, the following theorem holds.

Theorem 2. *Let A be a complete noetherian local reduced ring of Krull dimension 1 such that its resudue field is perfect and of characteristic not equal to 2. Suppose that A is of infinite Cohen-Macaulay type. Then it is of tame type if and only if it dominates one of the singularities T_{pq}.*

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8th International Algebraic Conference
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A-infinity-morphisms with several entries

V. Lyubashenko

It is well-known that operads play a prominent part in the study of A_∞-algebras. In particular, A_∞-algebras in the conventional sense [2] are algebras over the **dg**-operad A_∞, a resolution (a cofibrant replacement) of the **dg**-operad As of associative non-unital **dg**-algebras. What about morphisms $A \to B$ of A_∞-algebras? It is shown in [1] that they are maps over certain bimodule over the **dg**-operad A_∞. This bimodule is a resolution (a cofibrant replacement) of the corresponding As-bimodule.

Here we address morphisms with several arguments $f : A_1, \dots, A_n \to B$ of A_∞-algebras. We explain that they are maps over certain $n \wedge 1$-operad A_∞-module F_n. The latter means an $\mathbb{Z}^n_{\geq 0}$-graded complex with n left and one right pairwise commuting actions of A_∞.

Furthermore, it is a resolution (a cofibrant replacement) of the corresponding notion for associative **dg**-algebras without unit.

The unital case is quite similar to the non-unital one. There is an operad A_∞^{hu} governing homotopy unital A_∞-algebras. Homotopy unital morphisms $A \to B$ are controlled by an operad A_∞^{hu}-bimodule [1].

In the current article we describe the $n \wedge 1$-operad A_∞^{hu}-module F_n^{hu} responsible for homotopy unital A_∞-morphisms $f : A_1, \dots, A_n \to B$. We see that it is a resolution (a cofibrant replacement) of the corresponding $n \wedge 1$-operad module over the operad of associative unital **dg**-algebras.

The **dg**-operad of A_∞-algebras has two useful forms. The first, A_∞, is already presented as a resolution of the operad As. The second is easy to remember, because all generators have the same degree 1 and the expression for the differential contains no oscillating signs. These two are related by an isomorphism of operads that changes the degrees in a prescribed way. Structure equations for this isomorphism use certain signs. These signs reappear in the formula for the differential in the first operad. There are similar duplicates of other operads and modules over them: F_n, A_∞^{hu}, F_n^{hu}, etc.

The definition and main properties of $n \wedge 1$-operad modules pop out in the study of lax Cat-span multicategories — one more direction treated in the article as a category theory base of the whole subject. These lax multicategories generalize strict multicategories associated with the monad of free strict monoidal category. Composition of A_∞- and A_∞^{hu}-morphisms of several arguments is presented as convolution of a certain colax Cat-span multifunctor viewed as a coalgebra and the lax Cat-span multifunctor Hom viewed as an algebra. This gives the multicategory of A_∞- or A_∞^{hu}-algebras.

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SHORT COMMUNICATION

On the Tate pairing associated to an isogeny between abelian varieties over pseudofinite fields

V. Nesteruk

Let $\phi\colon A \to B$ be an isogeny of abelian varieties defined over field k, $\overline{k}$ be an algebraic extension of k, $\hat{\phi}$ be dual isogeny. If k is a finite field, P. Bruin [1] defined the Tate pairing associated to ϕ

$$\ker \hat{\phi}(k) \;\times\; \mathrm{coker}\,(\phi(k)) \longrightarrow k^*$$

as follows. There is a canonical isomorphism ϵ_ϕ from $\ker \hat{\phi}$ to the Cartier dual $(\ker \phi)^\vee$ of $(\ker \phi)$. For $x \in \ker \hat{\phi}(k), y \in \mathrm{coker}\,(\phi(k))$ put $(x, y) \mapsto (\epsilon_\phi x)(\sigma a - a)$, where σ is the generator of absolute Galois group $\mathrm{Gal}\,(\overline{k}/k)$ and $a \in A(\overline{k})$ is any element with $(\phi(a) \bmod \phi(A(k))) = y$.

Let C be an absolutely irreducible projective curve defined over finite field k, J is the Jacobian of curve C over k, n is a positive integer, $(n, \mathrm{char}(k)) = 1$ and $\mu_n(k)$ denotes the group of n-th roots of unity in $\overline{k}^*$. For divisor classes $x \in \mathrm{J}[n](k)$ and $y \in \mathrm{J}(k)/n\mathrm{J}(k)$ there are coprime divisors D and R such that $x = [D]$ and $y = [R] + n\mathrm{J}(k)$, and there exist a function $f \in k(C)$ such that $(f) = nD$. The Tate pairing $t_n(x, y)\colon\ \mathrm{J}[n](k) \times \mathrm{J}(k)/n\mathrm{J}(k) \longrightarrow k^*$ is defined by $t_n(x, y) = f(R)$ [3].

The Frey-Rück pairing [2] $\{.,.\}\colon\ \mathrm{J}[n](k) \times \mathrm{J}(k)/n\mathrm{J}(k) \longrightarrow \mu_n(k)$ is defined by $\{x, y \bmod n\mathrm{J}(k)\}_n = f(E)^{(q-1)/n}$, where $f(E) = \prod_{P \in C(\overline{k})} f(P)^{n_P}$ if $E = \sum_{P \in C(\overline{k})} n_P P$. The Tate and Frey-Rück pairings may be defined over pseudofinite fields as well.

P. Bruin [1] and E. Schaefer [4] shoved that the perfectness of Tate pairing and of the Frey-Rück pairing follow from that of the Tate pairing associated to an isogeny.

We prove the perfectness of the Tate pairing associated to an isogeny over pseudofinite fields. Namely,

Theorem. *Let ϕ be an isogeny between abelian varieties over a pseudofinite field k. Let m be order of* $\ker \phi$. *Suppose that k contains m-th roots of 1. Then the Tate pairing associated to ϕ is perfect.*

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Short communication

On the Brauer group of algebraic function fields of genus zero over pseudoglobal fields

O. Novosad

Let F be a pseudoglobal field with $\mathrm{char}\,(F) \neq 2$ (i.e. an algebraic function field in one variable with a pseudofinite [1] constant field). Let K be an algebraic function field in one variable over a field F. By a place of K/F, we mean a normalized discrete valuation on K which is trivial on $F^* = F \setminus \{0\}$. Define

$$\mathbb{P}(K/F) = \{P \mid P \text{ is a place of } K/F\}.$$

We denote by $\bar{V}_P$ the residue field with respect to $P \in \mathbb{P}(K/F)$. If K has genus 0, it has the form $K = F(x, \sqrt{ax^2+b})$ where $a, b \in F^*$ and x is transcendental over F. Such a K is determined up to isomorphism by the quaternion algebra $Q = (a, b/F)$, we write $F(Q)$ for K.

The following result is the first step in our study of the Brauer group $\mathrm{Br}\,(F)$ of a genus zero extension of a pseudoglobal field. Notice that analogous result was proved by I. Han for global field in [2].

Theorem 1. *Let F be a pseudoglobal field. Suppose that Q is a quaternion division algebra over F. Let $K = F(Q)$. Then*

$$\bigcap_{P\in\mathbb{P}(K/F)} \mathrm{Br}\,(\bar{V}_P/F) = \bigcap_{\deg\,(P)=2} \mathrm{Br}\,(\bar{V}_P/F) =$$

$$= \left\{ [Q'] \;\middle|\; \begin{array}{c} Q' \textit{ is a quaternion algebra over } F \\ \textit{with } \mathrm{supp}\,(Q') \subseteq \mathrm{supp}\,(Q) \end{array} \right\} \subseteq \mathrm{Br}\,(F).$$

The cardinality of this set is 2^{n-1} where $n = |\mathrm{supp}\,(Q)|$.

The proof is based on the argument used in [1] in the case of global field K and on the Hasse principle for the Brauer group of a pseudoglobal field [3].

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CONTRIBUTED ABSTRACT

Subgroups of the free products of paratopological groups

N. M. Pyrch

It was proved in [1] that for any family $\{G_i : i \in I\}$ of paratopological groups there exists the free product $\prod\limits_{i\in I}^{*} G_i$ of this family.

Subgroup H of paratopological group G is called homomorphic retract of G if there exists a continuous homomorhism $r\colon G \to H$ such that $r(h) = h$ for all $h \in H$.

Proposition 1. *Let $\{G_i : i \in I\}$ be a family of paratopological groups and let H_i be a homomporphic retract of G_i for each $i \in I$. Then the subgroup H of $\prod\limits_{i\in I}^{*} G_i$ generated by the set $\bigcup\limits_{i\in I} H_i$ is topologically isomorphic to the free topological product $\prod\limits_{i\in I}^{*} H_i$.*

Subset A of topological space X is called sequentially dense in X if for any point $x \in X$ there exists a sequence $\{a_n\}_{n=1}^{\infty}$ points from A converging to x.

Proposition 2. *Let $\{G_i : i \in I\}$ be a family of paratopological groups and let H_i be a subgroup of G_i for each $i \in I$. Then the subgroup H of $\prod\limits_{i\in I}^{*} G_i$ generated by the set $\bigcup\limits_{i\in I} H_i$ is (sequentially) dense in $\prod\limits_{i\in I}^{*} G_i$ if and only if each H_i is (sequentially) dense in G_i.*

Theorem 1. *Let G_i be a family of T_1- paratopological groups. Then $d(\prod\limits_{i\in I}^{*} G_i) = d(\bigcup\limits_{i\in I} G_i)$ (here $\bigcup\limits_{i\in I} G_i$ is the union of topological spaces G_i with the one common point $e \in G_i$).*

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SECTION TALK

The primitive points on elliptic cones

A. S. Radova

We consider an elliptic cone

$$x^2 + Ay^2 = z^2, \tag{1}$$

where A is a positive squarefree integer.

Let $F_A(N)$ denote the number of primitive points (x, y, z), $0 < z \leq N$ on the cone (1).

Let $\rho_A(N)$ be the number of primitive representations of n as

$$n = x^2 + Ay^2,$$

$(x, y) = 1$.
It is well known that there is a finite set α of values A for which

$$\tfrac{1}{w}\rho_A(n^2)$$

(with some integer w) is a multiplicative function.

Then for every $A \in \alpha$ the asymptotic formula

$$F_A(N)\colon = \sum_{\substack{n\leq N\\ n=x^2+Ay^2\\ \varphi_1<\arg(x+\sqrt{-A}y)\leq\varphi_2}} \rho_A(n^2) = c(A)(\varphi_2-\varphi_1)N + O(N^{\frac{1}{2}}logN) \tag{2}$$

holds.

Assuming the fulfilment of the Riemann conjecture for those A for which $Q(\sqrt{-A})$ is one-class field we obtain the analogue of the asymptotic formula of work [1]

$$G_A(N)\colon = \sum_{\substack{u^2+Av^2=w^2\\ 0<uv\leq\frac{2}{\sqrt{A}}N^2}} 1 = c_0(A)N^2 + c_1N^{\frac{2}{3}} + O(N^{\frac{37}{82}}), \tag{3}$$

where $c_0(A)$, c_1A are computable constants.

Moreover, we investigate in the report the problem of the least value $h = h(N)$, for which the equation

$$x^2 + Ay^2 = z^2$$

has a solution with $(x, y) = 1$, $N - h < z \leq N$.

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SECTION TALK

Finite Hjelmslev planes

Olga Schensnevich

Klingenberg has earried out studies of Desarguesian and Pascalian Hjelmslev planes [2]. Emphasis in placed on finite Hjelmslev - planes.

An H - plane π' is defined as a collection of points and lines together with an incidence relation subject to the following rules:

I. For each pair of distinct points there exists at least one line that passes through them.

II. For each pair of distinct lines there exists at least one point of intersection.

A pair of lines will be said to be neighbor in case they intersect in more than one point, and nonneighbor otherwise. This will be denoted by $l \circ m$ and $l \oslash m$, respectively. A similar definition and notation will be used for points.

III. If k, l, m, are concurrent lines such that $k \circ l$ and $l \oslash m$, then $k \oslash m$.

IV. If $k \circ l$ and $l \oslash m$, then $km \circ lm$, where km denoted an arbitrary point of intersection of the lines k and m.

V. If $P \circ Q$ and $Q \oslash R$, then $PR \circ QR$.

VI. There exist points P_1,P_2,P_3, P_4 such that $P_i \oslash P_j$, and $P_iP_k \oslash P_iP_j$, whenever i,j,k are all different.

Klingenberg has shown that with each π' is associated a projective plane π as follows: The relation of neighbor is first shown to be an equivalence relation. The equivalence classes become the points and lines of π. A class of points $\mathfrak{U}$ is defined as incident on a class of lines $\mathfrak{B}$ if and only if there exists a point P in $\mathfrak{U}$ and a line k in $\mathfrak{B}$ such that P is incident on k.

Definition. *A finite H - plane π' will be called* uniform *in case there exists a line k in π' for which $\lambda(k)$ is an integer and π' is not a projective plane.*

Theorem. *Let π' be a finite, uniform H-plane. Then λ is independent of k. In fact (i) $\lambda = t$, (ii) $s = t^2$, and (iii) the incidence matrix A of π' is a group-divisible, regular design with two associate classes and parameters $v = t^4 + t^3 + t^2$, $n = t^2$, $m = t^2 + t + 1$, $r = t^2 + t$, $k = t^2 + t$, $\lambda_2 = 1$, $\lambda_1 = t^2$.*

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Crossed modules and its applications

V. Sharko

A G-**crossed module** is a triple (C, ∂, G), where C and G are groups, $\partial : C \longrightarrow G$ is a homomorphism, G acts on C on the left and homomorphis ∂ is to satisfy the conditions:

a) $\partial(gc) = g(\partial)g^{-1}$ for all $g \in G, c \in C$,

b) $cdc^{-1} = (\partial) \cdot d$ for all $c, d \in C$.

A **morphism** (α, β) from the crossed module (C, ∂, G) to (C', ∂', G') is a pair of group homomorphisms $\alpha : C \longrightarrow C'$ and $\beta : G \longrightarrow G'$ such that $\beta \cdot \partial = \partial' \cdot \alpha$ and $\alpha(g \cdot c) = \beta(g) \cdot \alpha(c)$ $(g \in G, c \in C)$.

Let **CM** denote this category of crossed modules. If $\beta = Id$ on $G = G'$, we say that α is a G-**morphism** and denote this category by $\mathbf{CM}_G$.

Fix a group G, a G-crossed module C is said to be **projective** if it is projective in the category $\mathbf{CM}_G$.

A **projective crossed chain complex** is sequence of groups and homomorohisms

$$e \leftarrow \pi \stackrel{\partial_1}{\leftarrow} G \stackrel{\partial_2}{\leftarrow} C_2 \stackrel{\partial_3}{\leftarrow} C_3 \leftarrow \ldots \stackrel{\partial_n}{\leftarrow} C_n$$

such that:

a) (C_2, ∂_2, G) is projective G-crossed module,

b) for each $i \geq 3$ the module C_i is projective $\mathbb{Z}[\pi]$-module, ∂_i is homomorphism of $\mathbb{Z}[\pi]$-modules, ∂_2 commutes with the action of the group G and $\partial_3(C_3)$ is a $\mathbb{Z}[\pi]$-module,

c) $\partial_i \cdot \partial_{i+1} = 0$.

Let $d(P)$ be a additive function given on a category of finite generated projective modules. Then for pair of the modules $C \supset D$, where C is projective, p-$rank(C, D)$ is define.

Theorem 1. *Let* (C, ∂_i, G): $e \leftarrow \pi \stackrel{\partial_1}{\leftarrow} G \stackrel{\partial_2}{\leftarrow} C_2 \stackrel{\partial_3}{\leftarrow} C_3 \leftarrow \ldots \stackrel{\partial_n}{\leftarrow} C_n$ *be a projective crossed chain complex.* (C, d) *is* p*-minimal if and only if* p-$rank(C_i, \partial_i(C_{i+1})) = 0$ *and additive.*

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SHORT COMMUNICATION

Normal functor of finite degree in the asymptotic category and ultrametric

O. Shukel'

The asymptotic dimension asdim of a metric space was introduced by M. Gromov [1]. Recall that a metric space X has asymptotic dimension $\le n$, $n \in \mathbf{N} \cup \{0\}$ (denoted by $\operatorname{asdim} X \le n$) if for every $D > 0$ there exists a uniformly bounded cover $\mathcal{U}$ of X such that $\mathcal{U} = \mathcal{U}_0 \cup \cdots \cup \mathcal{U}_n$, where all $\mathcal{U}_i$ are D-disjoint.

Asymptotic category is described in [2]. E. Shchepin [3] introduced the notion of normal functor in the category **Comp** of compact Hausdorff spaces and continuous maps. The counterpart of the notion of a normal functor in the asymptotic category is offered in [4]. It is proved that this functor preserves the class of asymptoticaly zero-dimensionaly metric spaces [4] and preserves the class of metric spaces of finite asymptotic dimension [5].

We consider the analogue of the metric construction suggested in [4] for ultrametric spaces in the asymptotic category.

For every ultrametric space (X, d) an ultrametric $\hat{d}$ on the space $F(X)$ is defined as following.

Given $a, b \in F(X)$, we let

$$\begin{aligned}\hat{d}(a,b) = \inf\{\max\{d(f_{2i-1}, f_{2i}) \mid\ & f_{2i-1}, f_{2i} \colon A_i \to X \text{ are such that} \\ & \text{there exist } c_i \in F(A_i),\ \operatorname{supp}(c_i) = A_i,\ i = 1, \dots, m, \text{ with} \\ & a = F(f_1)(c_1),\ F(f_2)(c_1) = F(f_3)(c_2), \\ & \dots, \\ & F(f_{2m-1})(c_m) = F(f_{2m-2})(c_{m-1}),\ F(f_{2m})(c_m) = b\}.\end{aligned}$$

We prove that function $\hat{d}$ is an ultrametric and we may say that the considered functor preserves the class of ultrametric spaces. Note that every ultrametric space is coarsely equivalent to asymptoticaly zero-dimensionaly metric space.

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8th International Algebraic Conference
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Short communication

The torsion of Brauer group over pseudoglobal field

L. Stakhiv

It is now known [1-2] that if K is a global field, then the n-torsion subgroup of its Brauer group $\mathrm{Br}\,(K)$ equals the relative Brauer group $\mathrm{Br}\,(L/F)$ of an abelian extension L/F. C. D. Popescu, J. Sonn, and A. R. Wadsworth conjectured [3] that this property characterizes the global fields within the class of infinite fields which are finitely generated over their prime fields and shoved as a first step towards proving this conjecture that if K is a non-global infinite field, which is finitely generated over its prime field and $\ell \neq \mathrm{char}\,(F)$ is a prime number such that $\mu_{\ell^2} \subseteq F^*$, then there does not exist an abelian extension L/F such that ${}_\ell\mathrm{Br}\,(F) = \mathrm{Br}\,(L/F)$. Nevertheless, for a pseudoglobal field K (i.e. K is an algebraic function field in one variable with pseudofinite [4] constant field) the following results still hold:

Theorem 1. *Let K be a pseudoglobal field of characteristic p, ℓ a prime different from p, r a positive integer. Then there exists an abelian ℓ-extension L/K of exponent ℓ^r such that the local degree $[LK_v : K_v]$ is equal to ℓ^r for every prime v of K.*

Theorem 2. *Given a pseudoglobal field K and a positive integer n, there exists an abelian extension L/K (of exponent n) such that the n-torsion subgroup of the Brauer group of K is equal to the relative Brauer group of L/K.*

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CONTRIBUTED ABSTRACT

Derived categories of coherent sheaves over non-commutative nodal curves

D. E. Voloshyn

We consider certain non-commutative projective curves such that all their singularities are nodal algebras [1, 2]. *Non-commutative nodal curves* are non-commutative analogues of projective configurations considered in [3]. We say that a *vector bundle* over a non-commutative curve is a locally projective coherent sheaf of modules over this curve. There is a conjecture that non-commutative nodal curves are unique in some class of non-commutative curves such that an effective classification for objects of the categories of vector bundles and the derived categories of vector bundles can be obtained. We study the derived categories of vector bundles over these curves (see [4] about the categories of vector bundles). The work uses the technique of "matrix problems" more exactly representations of bunches of semi-chains [5].

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PLENARY TALK

Standard homological properties for supergroups

A. N. Zubkov

Let K be a field of characteristic $p \neq 2$. Those vector spaces (over the filed K) which are graded by $\mathbb{Z}_2 = \{0, 1\}$ form a tensor category, SMod_K, with the canonical symmetry. Objects defined in this symmetric tensor category are called with the adjective 'super' attached. For example, an algebra object in SMod_K is called a *superalgebra*. All superalgebras including Hopf superalgebras are assumed to be supercommutative. A *K-functor* (resp., a *supergroup*) is a set-valued (resp., group-valued) functor defined on the category SAlg_K of superalgebras. A group K-functor G, that is represented by a (Hopf) superalgebra $K[G]$, is called an *affine supergroup*. In other words, G is a group K-functor and an *affine superscheme* simultaneously. It is called an *algebraic supergroup*, provided $K[G]$ is finitely generated. In what follows all supergroups are supposed to be affine.

If H is a closed subsupergroup of G, that is $K[H] = K[G]/I_H$ for a Hopf superideal in $K[G]$, then a *sheafification* of a *naive quotient* $R \to G(R)/H(R), R \in \mathsf{SAlg}_K$, with respect to a *Grothendieck topology of fppf coverings*, is called a *sheaf quotient* of G over H and it is denoted by $G\tilde{/}H$.

Let G_{ev} denote a *largest even* subsupergroup of G, that is $K[G_{ev}] = K[G]/K[G]K[G]_1$. Observe that $K[G_{ev}]$ represents also an affine group $G_{res} = G|_{\mathsf{Alg}_K}$, where Alg_K is a full subcategory of SAlg consisting of all commutative K-algebras.

Remind that a local K-functor X is called a *superscheme* iff X has an *open covering* by affine subsuperschemes $X_i, i \in I$ (cf. [2]). If the index set I is finite and each $K[X_i]$ is a Noetherian superalgebra, then X is called a *Noetherian superscheme*.

Recently, author (in collaboration with A.Masuoka) has proved that for any algebraic supergroup G and its (closed) subsupergroup H, the sheaf quotient $G\tilde{/}H$ satisfies the following properties (cf. [3]):

(Q1) $G\tilde{/}H$ is a Noetherian supersheme.

(Q2) The quotient morphism $\pi : G \to G\tilde{/}H$ is affine and faithfully flat. It infers that for any affine open subsuperscheme $U \subseteq G\tilde{/}H$ its pre-image $V = \pi^{-1}(U)$ is an affine H-invariant subsuperscheme and $K[V]$ is a faithfully flat $K[U]$-module. In particular, $K[U] \simeq K[V]^H$.

(Q3) $G\tilde{/}H$ is affine iff $G_{ev}\tilde{/}H_{ev}$ is affine iff $G_{res}\tilde{/}H_{res}$ is affine.

We are going to discuss how to use the above result to build the standard approach to cohomologies of vector bundles on $G\tilde{/}H$. For example, the following result was formulated in [1] without proof. Set $X = G\tilde{/}H$ and let $\mathcal{O}_X$ is a *structure sheaf* of a superscheme X.

Theorem 1. *A category of quasi-coherent $\mathcal{O}_X G$-supermodules is equivalent to the category of H-supermodules.*

Some other applications will be also discussed.

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Topical section III

Analytic and Algebraic Theory of Numbers

8th International Algebraic Conference
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SHORT COMMUNICATION

On the defect in valuation theory

Kamal Aghigh

Let v be a henselian valuation of any rank of a field K and $\bar{v}$ be the extension of v to a fixed algebraic closure $\overline{K}$ of K. A finite extension (K', v') of (K, v) will be called tame if

1. It is defectless, i.e. $def(k'/K)$ is one, with $def(k'/K)$ given by $[k' : K]/ef$, where e, f are respectively the index of ramification and the residual degree of v'/v.
2. The residue field of v' is a separable extension of the k_v , residue field of v.
3. The ramification index of v'/v is not divisible by the characteristic of k_v.

In 1998 Khanduja gave a characterization of a tame field, i.e., every finite extension of (K, v) is tamely ramified and defectless.In 2010, Kuhlmann describe the role of defect plays in deep open problems and present several examples of algebraic extensions with non trivial defect. In this talk we survey on these items and will give some hint on a open problem.

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CONTRIBUTED ABSTRACT

$g(\alpha)$ and $\widehat{g}(\alpha)$ functions

Z. Dadayan

Indian mathematic S.S. Pillai introduced and studied the arithmetic function over $\mathbb{N}$

$$P(n) = \sum_{k=1}^{n} \gcd(k, n). \tag{1}$$

He proved that $P(n)$ is a multiplicative function. Moreover, Pillai obtained a non-trivial asymptotic formula for the summatory function for $P(n)$.

For the last 10 years O. Bordelles and K.A. Broughan in their papers obtained a new estimates for an error term in asymptotic formula. Also they considered an asymptotic behaviour the sum

$$\sum_{n \leqslant x} \frac{P(n)}{n^a} (x \to \infty),$$

where a is a fix real parameter.

Let $\alpha \in \mathbb{Z}[i]$. We define the following functions

$$g(\alpha) = \sum_{\beta (mod\, \alpha)} N(\gcd(\beta, \alpha)), \quad \widehat{g}(\alpha) = \sum_{\beta (mod\, \alpha)} \frac{1}{N(\gcd(\beta, \alpha))}.$$

We investigate the distribution of values of the functions $g(\alpha)$ and $\widehat{g}(\alpha)$ over the ring of the Gaussian integers $\mathbb{Z}[i]$. We construct asymptotic formulas for the summatory functions

$$\sum_{N(\alpha) \leqslant x} \frac{g(\alpha)}{N^a(\alpha)} \text{ and } \sum_{N(\alpha) \leqslant x} \frac{\widehat{g}(\alpha)}{N^a(\alpha)} \ (a \in \mathbb{R}).$$

Theorem 1. *For $x \to \infty$ we have*

$$\sum_{N(\alpha) \leqslant x} \frac{g(\alpha)}{N^a(\alpha)} = \begin{cases} \frac{\pi^2 x^{2-a} \log x}{(2-a) Z(2)} - \frac{\pi^2 x^{2-a}}{(2-a)^2 Z(2)} + \frac{c x^{2-a}}{(2-a) Z(2)} + \frac{Z^2(a-1)}{Z(a)} + O\left(x^{\frac{3}{2}-a} \log x\right), \ if\, a > \frac{3}{2}, a \neq 2; \\ \frac{\pi^2 x^{2-a} \log x}{(2-a) Z(2)} - \frac{\pi^2 x^{2-a}}{(2-a)^2 Z(2)} + \frac{c x^{2-a}}{(2-a) Z(2)} + O\left(x^{\frac{3}{2}-a} \log x\right), \ if\, a \leqslant \frac{3}{2}. \end{cases}$$

Theorem 2. *For $x \to \infty$ we have*

$$\sum_{N(\alpha) \leqslant x} \frac{\widehat{g}(\alpha)}{N^2(\alpha)} = \frac{\pi Z(3) \log x}{Z(2)} + \frac{C_0 Z(3)}{Z(2)} - \frac{\pi Z'(2) Z(3)}{Z^2(2)} + O\left(x^{-\frac{2}{3}}\right),$$

where $Z(s)$ is the Hecke's zeta-function and c, C_0 are the absolute constants.

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8th International Algebraic Conference
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Plenary talk

Universality of the Hurwitz zeta function

A. Laurinčikas

The Hurwitz zeta-function $\zeta(s,\alpha)$, $s=\sigma+it$, $0<\alpha\leq 1$, is defined, for $\sigma>1$, by

$$\zeta(s,\alpha)=\sum_{m=0}^{\infty}\frac{1}{(m+\alpha)^s},$$

and is analytically continued to the whole complex plane, except for a simple pole at $s=1$ with residue 1. It is known that the function $\zeta(s,\alpha)$ with transcendental and rational $\alpha\neq 1, \frac{1}{2}$ is universal in the sense that its shifts $\zeta(s+i\tau,\alpha)$ uniformly on compact subsets of the strip $D=\{s\in\mathbb{C}:\ \frac{1}{2}<\sigma<1\}$ approximate any analytic function.

In the report, we discuss the universality of composite functions $F(\zeta(s,\alpha))$. Denote by $H(D)$ the space of analytic functions on D equipped with the topology of uniform convergence on compacta. For example we have the following statement [1]. For $a_1,\dots,a_r\in\mathbb{C}$ denote

$$H_{a_1,\dots,a_r}(D)=\left\{g\in H(D):\ (g(s)-a_j)^{-1}\in H(D),\ j=1,\dots,r\right\}.$$

Theorem. *Suppose that the number α is transcendental and the function $F:\ H(D)\to H(D)$ is continuous and $F(H(D))=H_{a_1,\dots,a_r}(D)$. For $r=1$, let $K\subset D$ be a compact subset with connected complement, and let $f(s)$ be a continuous and $\neq a_1$ function on K. For $r\geq 2$, let $K\subset D$ be compact subset, and $f(s)\in H_{a_1,\dots,a_r}(D)$. Then, for every $\varepsilon>0$,*

$$\liminf_{T\to\infty}\frac{1}{T}\mathrm{meas}\left\{\tau\in[0,T]:\ \sup_{s\in K}|F(\zeta(s+i\tau,\alpha))-f(s)|<\varepsilon\right\}>0.$$

Also, the universality of $F(\zeta(s),\zeta(s,\alpha))$, where $\zeta(s)$ is the Riemann zeta function, is considered [2].

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8th International Algebraic Conference
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SHORT COMMUNICATION

Approximation of analytic functions by shifts of zeta-functions

R. Macaitienė

Approximation of analytic functions is old and important problem of the theory of functions. By the Mergelyan theorem, every continuous function on a compact subset $K \subset \mathbb{C}$ which is analytic in the interior of K can be approximated with a given accuracy uniformly on K by polynomials. Moreover, it is known that there exist functions whose shifts approximate on compact subsets of some region any analytic functions. First this was observed by S. M. Voronin who proved the above approximation property for the Riemann zeta-function. Also, other zeta and L-functions have a similar property which is called an universality.

Our talk is devoted to simultaneous approximation of a collection of analytic functions by shifts of zeta-functions having and having no the Euler product over primes. The first type of zeta-functions is represented by zeta-functions $\zeta(s, F)$, $s = \sigma + it$, of new forms defined by

$$\zeta(s,F) = \prod_{p|N}\left(1 - \frac{c(p)}{p^s}\right)^{-1} \prod_{p\nmid N}\left(1 - \frac{c(p)}{p^s} + \frac{1}{p^{2s-1}}\right)^{-1}, \quad \sigma > \frac{\kappa+1}{2},$$

where $c(p)$ are the Fourier coefficients of the form F, and N and κ denote the level and weight of F. Periodic Hurwitz zeta-functions $\zeta(s, \alpha; \mathfrak{a})$ represent zeta-functions without Euler's product. For periodic sequences of complex numbers $\mathfrak{a} = \{a_m : m \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}\}$ and α, $0 < \alpha \le 1$, they are defined by

$$\zeta(s,\alpha;\mathfrak{a}) = \sum_{m=0}^{\infty} \frac{a_m}{(m+\alpha)^s}, \quad \sigma > 1.$$

The functions $\zeta(s, F)$ are entire while the point $s = 1$ is a possible simple pole of the functions $\zeta(s, \alpha; \mathfrak{a})$. The shifts $\zeta(s+i\tau, F)$ and $\zeta(s+i\tau, \alpha; \mathfrak{a})$ approximate analytic functions uniformly on compact subsets of the strip $\{s \in \mathbb{C} : \frac{1}{2} < \sigma < 1\}$.

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8th International Algebraic Conference
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SHORT COMMUNICATION

Incompatible parities in Fermat's equation

Mario De Paz, Enzo Bonacci

Among the unexplored properties of the binomial expansion (ref. [1],[2]) with relevant influences in limiting Fermat triples until an almost impossible condition of existence (ref. [3],[4],[5]), the most promising seemed the criterion of incompatible parities (ref. [6]). Such achievement was found stimulating by a pool of skilled mathematicians, ruled by Professor Umberto Cerruti from the University of Turin, who decided to analyze our mathematical strategies in detail. In their official revision of our work (ref. [7]) they gave us some suggestions in order to improve it and the present talk is the result of that enriching discussion.

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8th International Algebraic Conference
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Взвешенная функция унитарных делителей

П. Попович

Натуральное называется унитарным делителем n, если d/n и $(d, \frac{n}{d}) = 1$.

Обозначим через $\tau_k^*(n)$ количество представителей n в виде произведения k множителей $n = d_1 \cdot ... \cdot d_k$, при чем $(d_i, d_j) = 1$ при $i \neq j; i, j = 1, 2, ..., k$.

Мы изучаем проблему распределения значений функции $\tau_k^*(n)$, взвешенную функцией $\omega(n)$, где $\omega(n)$- число различных простых делителей n.

В [1],[2] была построена асимптотическая формула для сумматорной функции, ассоциированной с функцией $\tau(n)\omega(n)$, где $\tau(n)$- функция числа делителей n. Мы строим аналогичные формулы для $\tau_k^*(n)\omega(n), k = 2, 3, ...$ В частности, доказано утверждение.

Теорема. *При $x \to \infty$справедлива асимптотическая формула*

$$\sum_{n \leqslant x} \tau_2^*(n)\omega(n) = Ax\log(x)\log\log x \sum_{j=0}^{M} \frac{a_j}{(\log\log x)^j} + O(\frac{x\log x}{(\log\log x)^M})$$

с вычислимыми коэффициентами a_j

$$\sum_{n \leqslant x} \tau_k^*(n)\omega(n) = (x\,Q_{k-1}(\log x)\log\log x + x\,R_{k-1}(\log x))(1 + O(x)), k = 3$$

$$\sum_{n \leqslant x} \tau_k^*(n)\omega(n) = (x\,Q_{k-1}(\log x)\log\log x + x\,R_{k-1}(\log x))(1 + O(x\,(\log x)^{\frac{k^2-6k+3}{k-3}})), k \geqslant 4$$

Дальнейшее улучшение остаточного члена связано с улучшением оценки k-ых моментов дзета-функции Римана на половинной прямой.

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СВЕДЕНИЯ ОБ АВТОРАХ

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Section talk

Systems of two-symbol encoding for real numbers and their applications

M. Pratsiovytyi

We consider different systems of expansion (representation) for fractional part of real number using two-symbol alphabet $\{0,1\}$ with zero and non-zero redundancy: 1) classical binary representation; 2) negabinary representation; 3) Q_2-representation; 4) Q_2^*-representation; 5) mediant representation; 6) representation by incomplete sums of convergent series; 7) A_2-continued fractions; 8) cylindrical representation; 9) Markov representation; 10) two-symbol s-adic representation et al. Criterions for rationality and irrationality of a number are given. We study the geometry of such representations (geometric meaning of numerals, properties of cylindrical sets, basic metric relation etc.), formulate basic facts of corresponding metric theories. In particular, we give normal properties of numbers, topological and metric properties of sets of numbers with conditions on symbols (tail sets, sets with deleted combinations of symbols et al.). We compare facts of corresponding metric theories and establish relations with different numeration systems.

We consider applications of the above mentioned representations in probabilistic number theory, fractal geometry, fractal and mathematical analysis, theory of probability distributions. The problem on Lebesgue structure (i.e., content of discrete, singular and absolutely continuous components) of the random variable defined by probability distributions of digits in some representation is solved completely (we consider schemes when digits are independent, when digits form a Markov chain, when infinite sequence of digits form a Markov chain and other digits are independent et al.). Fractal properties of probability distribution functions of the above mentioned random variables are described. In particular, we study if these functions preserve fractal dimension.

We prove the fact that it is possible to give equivalent definition of fractal Hausdorff-Besicovitch dimension using only cylindrical sets corresponded to any of the above mentioned representation. We consider fractal coordinate systems on the unit interval generated by cylindrical sets of corresponded representations as well as fractal transformations. We use these coordinate systems for modeling and studying of functions with complicated local structure and dynamical systems with chaotic trajectories.

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CONTRIBUTED ABSTRACT

On one class of continuous functions with complicated local structure containing singular and non-differentiable functions

M. Pratsiovytyi, A. V. Kalashnikov

Let $1 < s$ be a given positive integer, $[0,1] \ni x = \frac{\alpha_1}{s} + \frac{\alpha_2}{s^2} + \ldots + \frac{\alpha_n}{s^n} + + \ldots \equiv \Delta^s_{\alpha_1\alpha_2\ldots\alpha_n\ldots}$, $A = \{0, 1, \ldots, s-1\}$, and let $p_0, p_1, \ldots, p_{s-1}$ be real numbers such that $p_0 + p_1 + \ldots + p_{s-1} = 1$, $\beta_0 = 0$, $\beta_k = \sum\limits_{i=0}^{k-1} p_i > 0$, $p_* = \max\limits_i |p_i| < 1$, $k = 1, \ldots, s-1$.

Theorem 1. *There exist only one function F such that it satisfies a system of equations*

$$f\left(\Delta^s_{i\alpha_1\alpha_2\ldots\alpha_n\ldots}\right) = \beta_i + p_i f\left(\Delta^s_{\alpha_1\alpha_2\ldots\alpha_n\ldots}\right), i = \overline{0, s-1}, \tag{1}$$

is defined in every point from $[0,1]$ and bounded, moreover it can be represented in the form

$$F(x) = \beta_{\alpha_1} + \sum_{k=2}^{\infty} \left(\beta_{\alpha_k} \prod_{j=1}^{k-1} p_{\alpha_j} \right) \text{ where } x = \Delta^s_{\alpha_1\alpha_2\ldots\alpha_n\ldots}. \tag{2}$$

Function F defined by equality (2) is continuous in all points from $[0,1]$, $F(0) = 0$, $F(1) = 1$, if $p_i = \dfrac{1}{s}$ for all $i \in A$ then $F(x) = x$.

Lemma 1. *If $p_i \geqslant 0$ then function F is a continuous non-decreasing probability distribution function on the interval $[0,1]$.*

Let $\mu_F(\Delta^s_{c_1\ldots c_m}) = f(\Delta^s_{c_1\ldots c_m(s-1)}) - f(\Delta^s_{c_1\ldots c_m(0)})$.

Lemma 2. *If $p_i \geqslant 0$, $i = 0, \ldots, s-1$, then equality $\mu_F(\Delta^s_{c_1c_2\ldots c_m}) = \prod\limits_{i=1}^{m} p_{c_i}$ holds.*

Theorem 2. *If $p_i > 0$ for any $i \in A$ then graph Γ of function $y = F(x)$ is a self-affine set in the space $\mathbb{R}^2$, moreover $\Gamma = f_1(\Gamma) \cup f_2(\Gamma) \cup \ldots \cup f_s(\Gamma)$ where*

$$f_1 : \begin{cases} x' = \frac{1}{s}x, \\ y' = p_0 y, \end{cases} \qquad f_i : \begin{cases} x' = \frac{1}{s}x + \frac{i-1}{s}, \\ y' = \beta_{i-1} + p_{i-1}y. \end{cases}$$

Self-affine dimension of graph is a solution of equation

$$\left(\frac{p_0}{s}\right)^{\frac{x}{2}} + \left(\frac{p_1}{s}\right)^{\frac{x}{2}} + \ldots + \left(\frac{p_{s-1}}{s}\right)^{\frac{x}{2}} = 1.$$

Theorem 3. *For Lebesgue integral, the following equality holds:*

$$\int_0^1 F(x)dx = \frac{1}{s-1}\left(\beta_1 + \beta_2 + \ldots + \beta_{s-1}\right).$$

Theorem 4. *If at least one number $p_i \neq \frac{1}{s}$ exists among numbers $p_0, p_1, ..., p_{s-1}$ then function F does not preserve fractal Hausdorff—Besicovitch dimension.*

Theorem 5. *If there exist p_i and p_k such that $p_i p_k < 0$ then function F does not have derivative in any point from $[0, 1]$.*

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8th International Algebraic Conference
July 5—12 **(2011)**, Lugansk, Ukraine PLENARY TALK

Alternating Luroth series representations for real numbers and their application

M. Pratsiovytyi, Yu. Khvorostina

It is known [1], that any real number $x \in (0,1]$ can be represented in the form of a finite or an infinite alternating Luroth series

$$x = \frac{1}{a_1} + \sum_{n>2} \frac{(-1)^{n-1}}{a_1(a_1+1)\dots a_{n-1}(a_{n-1}+1)a_n}, \; a_n \in N.$$

Each irrational number has a unique infinite and non periodic $\widetilde{L}$-expansion and each rational number has a finite or a periodic $\widetilde{L}$-expansion. If number x can be represented in the form the infinite series, then we will write symbolically $x = \widetilde{L}(a_1, a_2, \dots, a_n, \dots)$. If number x can be represented in the form the finite series, then we will write $x = \widetilde{L}(a_1, a_2, \dots, a_n)$.

Definition. *Let $(c_1, c_2, \dots, c_n)$ is given ordered set of positive integer numbers. The cylinder of n-th rank with the base $c_1, c_2, \dots, c_n$ is called the set*

$$\Delta^{\widetilde{L}}_{c_1c_2\dots c_n} = \{x : x = \widetilde{L}(c_1, c_2, \dots, c_n, a_{n+1}, a_{n+2} \dots), a_{n+i} \in N, \forall i \in N\}.$$

The cylindrical sets are important in the metric theory. We obtain the following properties of the cylindrical sets: 1. $\Delta^{\widetilde{L}}_{c_1\dots c_n} = \bigcup\limits_{i=1}^{\infty} \Delta^{\widetilde{L}}_{c_1\dots c_n i}$.

2. $\sup \Delta^{\widetilde{L}}_{c_1\dots c_{2m-1}i} = \inf \Delta^{\widetilde{L}}_{c_1\dots c_{2m-1}(i+1)} = \widetilde{L}(c_1, \dots, c_{2m-1}, i+1)$;
   $\inf \Delta^{\widetilde{L}}_{c_1\dots c_{2m}i} = \sup \Delta^{\widetilde{L}}_{c_1\dots c_{2m}(i+1)} = \widetilde{L}(c_1, \dots, c_{2m}, i+1)$.
3. $|\Delta^{\widetilde{L}}_{c_1\dots\dots c_n}| \equiv \operatorname{diam}\Delta^{\widetilde{L}}_{c_1\dots\dots c_n} = \dfrac{1}{(c_1+1)c_1\dots(c_n+1)c_n} \le \dfrac{1}{2^n} \to 0 \;\; (n \to \infty)$.
4. $\dfrac{|\Delta^{\widetilde{L}}_{c_1c_2\dots c_n i}|}{|\Delta^{\widetilde{L}}_{c_1c_2\dots c_n}|} = \dfrac{1}{(i+1)i}$.

$\widetilde{L}$-expansion is $\widetilde{Q}_\infty$-expansion (see [2]), if $q_k = \dfrac{1}{k(k+1)}$.

In the report we offer results of studying of metric, topological and fractal properties of sets of real numbers with different restrictions on usage of $\widetilde{L}$-symbols, such that tailing sets, sets with limiting $\widetilde{L}$-symbols, sets $C[\widetilde{L}, \{V_k\}]$, sets which is defined by frequencies of usage of symbols and others. And we consider mathematical objects with a difficult local structure (operators, dynamical systems, singular functions and others). The comparative analysis of the solution of the basic measure problems of the positive and alternating Luroth series representations for real numbers, have been done.

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8th International Algebraic Conference
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Topological, metric, probabilistic and fractal theories of representations for numbers by alternating Lüroth series

M. Pratsiovytyi, Yu. Zhykhareva

Representation of the number $x \in (0,1]$ in the form

$$x = \frac{1}{d_1+1} + \frac{1}{d_1(d_1+1)(d_2+1)} + ... + \frac{1}{d_1(d_1+1)d_2(d_2+1)...d_{n-1}(d_{n-1}+1)(d_n+1)} + ...,$$

where d_n is a fixed infinite set of positive integers, is called the L-expansion of number x or expansion in an alternating Lüroth series. We write it briefly in the form

$$x = \Delta^L_{d_1 \ldots d_m \ldots}$$

and call it by L-representation of the number x. Positive integer d_n is called the nth L-symbol of the number x.

Since any number $x \in (0,1]$ has a unique L-representation, d_n is a function of x that is $d_n = d_n(x)$.

In the talk, we study the Lebesgue structure, topological, metric and fractal properties of the random variable

$$\xi = \Delta^L_{\tau_1 \tau_2 \ldots \tau_k \ldots}$$

defined by probability distributions of random L-symbols τ_k (we consider schemes when a) τ_k are mutually independent, b) they form a Markov chain, c) $\{\tau_{k_m}\}$ form a Markov chain et al.). Properties of spectrum and essential support of density of the random variable ξ are described in detail.

We give necessary and sufficient conditions when the probability distribution function of the random variable ξ preserve the Hausdorff-Besicovitch dimension, has nontrivial density, belongs to certain type of pure singular distribution.

In the talk, we also consider a generalization of the L-representation namely Q^*_∞-representation of numbers $x \in [0,1]$:

$$x = \beta_{\alpha_1 1} + \sum_{k=2}^{\infty} \left[\beta_{\alpha_k k} \prod_{j=1}^{k-1} q_{\alpha_j j} \right] = \Delta^{Q^*_\infty}_{\alpha_1 \alpha_2 \ldots \alpha_k \ldots}$$

where $\alpha_k \in \mathbb{N}$, $\|q_{ik}\|$ is a matrix with the following properties: $q_{ik} > 0$, $q_{0k} + \ldots + q_{ik} + \ldots = 1$ for any $k, i \in \mathbb{N}$, $\beta_{\alpha_k k} = q_{0k} + \ldots + q_{(\alpha_k - 1)k}$.

We apply the Q^*_∞-representation to definition of continuous singular and nondifferentiable functions by systems of functional equations as well as to definition and studying of fractal sets of Besicovitch-Eggleston type and sets of numbers such that they have not a frequency of at least one symbol.

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8th International Algebraic Conference
July 5–12 **(2011)**, Lugansk, Ukraine SHORT COMMUNICATION

Joint universality of some zeta-functions

S. Račkauskienė, D. Šiaučiūnas

Let, as usual, $\zeta(s)$, $s = \sigma + it$, denote the Riemann zeta-function, $\zeta(s, F)$ be the zeta-function attached to a holomorphic normalized Hecke eigen cusp form F, and let $\zeta(s, \alpha; \mathfrak{a})$ be a periodic Hurwitz zeta-function

$$\zeta(s, \alpha; \mathfrak{a}) = \sum_{m=0}^{\infty} \frac{a_m}{(m+\alpha)^s}, \quad \sigma > 1,$$

where α, $0 < \alpha \le 1$, is a fixed parameter and $\mathfrak{a} = \{a_m : m \in \mathbb{N} \cup \{0\}\}$ is a periodic sequence of complex numbers. In the report, we discuss an approximation of a collection of analytic functions by shifts

$$\Big(\zeta(s+i\tau), \zeta(s+i\tau, \alpha_1; \mathfrak{a}_{11}), \dots, \zeta(s+i\tau, \alpha_1; \mathfrak{a}_{1l_1}), \dots, \\ \zeta(s+i\tau, \alpha_r; \mathfrak{a}_{r1}), \dots, \zeta(s+i\tau, \alpha_r; \mathfrak{a}_{rl_r})\Big)$$

as well as by

$$\Big(\zeta(s+i\tau, F), \zeta(s+i\tau, \alpha_1; \mathfrak{a}_{11}), \dots, \zeta(s+i\tau, \alpha_1; \mathfrak{a}_{1l_1}), \dots, \\ \zeta(s+i\tau, \alpha_r; \mathfrak{a}_{r1}), \dots, \zeta(s+i\tau, \alpha_r; \mathfrak{a}_{rl_r})\Big).$$

Here the numbers $\alpha_1, \dots, \alpha_r$ are algebraically independent over the field of rational numbers, and $\mathfrak{a}_{11}, \dots, \mathfrak{a}_{1l_1}, \dots, \mathfrak{a}_{r1}, \dots, \mathfrak{a}_{rl_r}$ are periodic sequences of complex numbers. For example, under some rank hypothesis on the coefficients of the sequences $\mathfrak{a}_{j,l}$, $j = 1, \dots, r$, $l = 1, \dots, l_j$, the following statement is valid [1].

For $j = 1, \dots, r$, $l = 1, \dots, l_j$, let K_{jl} and K be compact subset of the strip $\{s \in \mathbb{C} : \frac{1}{2} < \sigma < 1\}$ with connected complement, the function $f_{jl}(s)$ be continuous on K_{jl} and analytic in the interior of K_{jl}, and let the function $f(s)$ is continuous and non-vanishing on K, and analytic in the interior of K. Then, for every $\varepsilon > 0$,

$$\liminf_{T\to\infty} \frac{1}{T} \mathrm{meas}\Big(\tau \in [0, T] : \sup_{s\in K} |\zeta(s+i\tau) - f(s)| < \varepsilon, \\ \sup_{1\le j\le r} \sup_{1\le l\le l_j} \sup_{s\in K_{jl}} |\zeta(s+i\tau, \alpha_j; \mathfrak{a}_{jl}) - f_{jl}(s)| < \varepsilon\Big) > 0.$$

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8th International Algebraic Conference
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Contributed abstract

Discrete universality of the composite function

J. Rašytė

It is well known that the Riemann zeta-function $\zeta(s)$, $s=\sigma+it$, is universal in the sense that its shifts $\zeta(s+i\tau)$ or $\zeta(s+imh)$, $\tau\in\mathbb{R}$, $h>0$, $m\in\mathbb{N}\cup\{0\}$, approximate any analytic function uniformly on compact subsets of some region. In the case of shifts $\zeta(s+imh)$, the above property is called a discrete universality of $\zeta(s)$.

In the report, we consider the discrete universality of composite functions $F(\zeta(s))$. Let $D=\{s\in\mathbb{C}:\frac{1}{2}<\sigma<1\}$, and $H(D)$ denote the space of analytic functions on D equipped with the topology of uniform convergence on compacta. Among other results, we have the following theorem. For $a_1,\dots,a_r\in\mathbb{C}$ and $C\in\mathbb{C}$, let $H_{C;a_1,\dots,a_r}(D)=\{g\in H(D):(g(s)-a_j)^{-1}\in H(D),\ j=1,\dots,r,\ \text{or } g(s)\equiv C \text{ if } \exists a_j=C\}$.

Theorem. *Suppose that the number* $\exp\left\{\frac{2\pi k}{h}\right\}$ *is irrational for all* $k\in\mathbb{Z}\setminus\{0\}$*, and that* $F:H(D)\to H(D)$ *is a continuous function such that* $F(S)=H_{F(0);a_1,\dots,a_r}(D)$*. For* $r=1$*, let* $K\subset D$ *be a compact subset with connected complement, and* $f(s)$ *be a continuous and* $\neq a_1$ *function on* K*, and analytic in the interior of* K*. For* $r\geq 2$*, let* $K\subset D$ *be a compact subset, and* $f(s)\in H_{F(0);a_1,\dots,a_r}(D)$*. Then, for every* $\varepsilon>0$*,*

$$\liminf_{N\to\infty}\frac{1}{N+1}\sharp\{0\leq m\leq N:\sup_{s\in K}|F(\zeta(s+imh)-f(s))|<\varepsilon\}>0.$$

If $r=1$ and $a_1=0$, then we have the universality of $e^{\zeta(s)}$ and $\zeta^N(s)$, $N\in\mathbb{N}$. If $r=2$ and $a_1=-1$, $a_2=1$, then the universality of $\sin\zeta(s)$ follows.

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8th International Algebraic Conference
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SHORT COMMUNICATION

Multifold Kloosterman sums over $\mathbb{Z}[i]$

O. Savastru, S. Varbanets

We study two types of the multifold Kloosterman sums over the ring $\mathbb{Z}[i]$:

$$K_N(\alpha_0, \alpha_1, \dots, \alpha_{N-1}; \gamma) := \sum_{j=0}^{N-1} \sum_{\substack{x_j \in \mathbb{Z}_\gamma^*[i] \\ x_0 \dots x_N \equiv 1 \,(\mathrm{mod}\ \gamma)}} e^{2\pi i \Re(\frac{1}{\gamma} \sum\limits_{l=0}^{N-1} \alpha_l x_l)}, \tag{1}$$

$$\widetilde{K}_N(\alpha_0, \alpha_1, \dots, \alpha_{N-1}; q, h) := \sum_{j=0}^{N-1} \sum_{\substack{x_j \in \mathbb{Z}_q^*[i] \\ N(x_0 \dots x_N) \equiv h \,(\mathrm{mod}\ q)}} e^{2\pi i \Re(\frac{1}{q} \sum\limits_{l=0}^{N-1} \alpha_l x_l)}, \tag{2}$$

where $\alpha_0, \dots, \alpha_{N-1}, \gamma \in \mathbb{Z}[i]$, $h, q \in \mathbb{N}$.

We obtained non-trivial estimates for $K_N(\alpha_0, \alpha_1, \dots, \alpha_{N-1}; \gamma)$ and $\widetilde{K}_N(\alpha_0, \alpha_1, \dots, \alpha_{N-1}; q, h)$, in particular, for the norm Kloosterman sums $\widetilde{K}_N$ under condition $(\alpha_j, p) = 1$, p is a prime number, the following estimate

$$\widetilde{K}_N(\alpha_0, \alpha_1, \dots, \alpha_{N-1}; p^m, h) \leqslant (8N-2) p^{\frac{2N+1}{2} m} \tag{3}$$

holds.

The estimate (3) is "the rooted estimate" and apparently in general case is the best possible.

These results generalize the results for the norm Kloosterman sums for $N = 2$ from [1].

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8th International Algebraic Conference
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Short communication

Analogue of Vinogradov's theorem over the ring of Gaussian integers

S. Sergeev, S. Kurbat

The following assertion is proved.

Theorem 1. *Let θ be irrational,*

$$\left|\theta - \frac{a}{q}\right| \leq \frac{1}{q^2}, (a,q) = 1, q \leq x\left(\log x\right)^{-10}$$

Then

$$\tilde{\pi}\left(x,\theta\right) << \left(x^{\frac{2}{3}}q^{\frac{1}{3}} + x^{\frac{5}{6}}q^{-\frac{1}{3}} + x^{\frac{11}{12}}q^{-\frac{1}{6}}\right)\left(\log x\right)^2$$

This result generalizes the result of Dupain, Hall and Tenenbaum on an expansion of the Vinogradov's theorem on the trigonometric sum over the prime numbers.

Corollary 1. *Let θ satisfies Theorem 1. Then*

$$\tilde{\pi}\left(x,\theta\right) = o\left(\tilde{\pi}\left(x\right)\right) := \sum_{N(p)\leq x} 1$$

Theorem 2. *Let $f\left(w\right)$ be a multiplicative function over $\mathbb{Z}\left[i\right]$, $\left|f\left(w\right)\right| \leq 1$. We proved, that for almost all irrationals $\theta \in [0;1]$ we have for $\delta < \frac{k}{\log\log x} < 2-\delta,\ 0<\delta<1$, and $x \to \infty$*

$$\sum_{\substack{w \in \mathbb{Z}\left[i\right] \\ \Omega\left(w\right) = k \\ N\left(w\right) \leq x}} f\left(w\right) e^{2\pi i Re\theta w} = o\left(\tilde{\pi}_k\left(x\right)\right)$$

In the set of multiplicative functions over $\mathbb{Z}$ satisfying $\left[f\left(w\right)\right] \leq 1$ such result have been proved by Indlekofer and Katei.

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8th International Algebraic Conference
July 5–12 **(2011)**, Lugansk, Ukraine SECTION TALK

Canonical $\widehat{\varphi}$-representation of numbers and its applications

N. Vasylenko

Let $\mathcal{A} = \{0,1\}$ and $L = \mathcal{A} \times \mathcal{A} \times \ldots \times \mathcal{A} \times \ldots$. Consider the functional correspondence f between the set L and $\mathbb{R}^1$: $L \ni (\varepsilon_n) \xrightarrow{f} x \in \mathbb{R}^1$ given by the equality

$$x = \sum_{n=1}^{\infty} \varepsilon_n \widehat{\varphi}^{n-1} = \varepsilon_1 \widehat{\varphi}^0 + \varepsilon_2 \widehat{\varphi}^1 + \cdots + \varepsilon_n \widehat{\varphi}^{n-1} + \cdots, \ \widehat{\varphi} = \frac{1-\sqrt{5}}{2}. \tag{1}$$

Theorem 1. *The range of function f is the closed interval $[-1, \varphi]$ where $\varphi = -\widehat{\varphi}^{-1}$.*

Definition 1. *Representation of the number x from $[-1, \varphi]$ in the form (1) is called the* $\widehat{\varphi}$-representation ($\widehat{\varphi}$-expansion) *of this number. We denote symbolically this expression by*

$$x = \Delta_{\varepsilon_1 \varepsilon_2 \ldots \varepsilon_n \ldots} \tag{2}$$

and call it by $\widehat{\varphi}$-representation *of this number.*

For the above mentioned representation of numbers we introduce the notion of cylindrical set (cylinder) and study their properties as well as solve some related problems [1]. In particular, the question about the number of different $\widehat{\varphi}$-representations of real number $x \in [-1, \varphi]$ is studied exhaustively.

Theorem 2. *All points from $[-1, \varphi]$, except for countable set of points, have a continuum set of different $\widehat{\varphi}$-representations.*

Theorem 3. *The endpoints of $[-1, \varphi]$ have a unique $\widehat{\varphi}$-representation. The endpoints of cylinder of any rank, except for endpoints of $[-1, \varphi]$, have a countable set of different $\widehat{\varphi}$-representations.*

The fact that (Lebesgue) almost all numbers $x \in [-1, \varphi]$ can be represented in the form of (1) by continuum set of ways have some advantages as well as disadvantages. For example, there are inconveniences for comparison of two numbers. We introduce the "simplest" $\widehat{\varphi}$-representation, the so-called canonical representation.

Definition 2. *$\widehat{\varphi}$-representation of number x from $[-1, \varphi]$ is called the canonical representation if $\varepsilon_k \varepsilon_{k+1} \varepsilon_{k+2} \neq 001$ for any $k \in \mathbb{N}$.*

Theorem 4. *Any real number $x \in [-1, \varphi]$ has a canonical representation moreover it is unique.*

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8th International Algebraic Conference
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Contributed abstract

Second moment of error term in the divisor problem over $M_n(\mathbb{Z})$

I. Velichko

Let $M_k(\mathbb{Z})$ denotes the ring of integer matrices of order k, $GL_k(\mathbb{Z})$ is the unite group of $M_k(\mathbb{Z})$. We denote the number of different (to association) representations of matrix $C \in M_k(\mathbb{Z})$ in the form $C = A_1A_2,\ A_1, A_2 \in M_k(\mathbb{Z})$ and $C = A_1A_2A_3,\ A_1, A_2, A_3 \in M_k(\mathbb{Z})$ as $\tau_k(C)$ and $\tau_k^{111}(C)$ accordingly. It is interesting to investigate asymptotic behavior of the sums

$$T_k^*(x) =: {\sum_{n\le x}}' \ {\sum_{G\in M_k(\mathbb{Z}),\ |det\, G|=n}}'' \tau_k(G), \quad T_k^{111}(x) =: \sum_{n\le x} \ {\sum_{G\in M_k(\mathbb{Z}),\ |det\, G|=n}}'' \tau_k^{111}(G)$$

for different k (sum $\sum'$ is taken throughout all square-free n, sum $\sum''$ is taken throughout all matrices G accurate to integer unimodular factor).

G.Bhowmik and H.Menzer [1], H.-Q.Liu [2], A.Ivič [3], N.Fugelo and I.Velichko [4] studied an asymptotic behavior of the similar sums, but only for two factors.

Using Perron's summation formula, we can obtain the following estimates:

$$T_k^*(x) = xP_2(\log x) + O(x^{1/2}log^5 x),$$
$$T_2^{111}(x) = xP_5(\log x) + O(x^{69/88+\varepsilon}).$$

Our aim is to estimate the second moments of the error terms for the functions $T_k^*(x)$ and $T_2^{111}(x)$.

Theorem 1. *Let $\Delta_1(x) = T_k^*(x) - xP_2(\log x)$, $\Delta_2(x) = T_2^{111}(x) - xP_5(\log x)$, then for $x \to \infty$ the following estimates*

$$\int_1^x \Delta_1^2(x)dx \ll x^{121/70}, \quad \int_1^x \Delta_2^2(x)dx \ll x^{24/10}$$

holds.

Using the last theorem, it can be easily shown that for almost all $x \le x_0$ remainder terms $\Delta_1(x_0) \ll x_0^{51/140}$, $\Delta_2(x_0) \ll x_0^{7/10}$.

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SECTION TALK

Application of triangular matrix calculus in a combinatorial analysis and number theory

R. Zatorsky

Application of parafunctions of triangular matrices [1] to the problem of paths at an inclined diagram [2] and to the Polya problem of reduction of a square matrix permanent to a determinant of a matrix [3] are considered. Using parafunctions of triangular matrices, recurrent fractions, which are natural n-dimensional generalizations of chain fractions [4], algorithms for formal power series [5]-[6], positional notation systems of k-th order, based on k-dimensional vectors, are constructed. It is also shown that our systems of the first order coincide with classical positional notation systems [7].

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Topical section IV

Computer Algebra and Discrete Mathematics

8th International Algebraic Conference
July 5—12 **(2011)**, Lugansk, Ukraine

SECTION TALK

Krein parameters of self-complementary strongly regular graph

A. Rahnamai Barghi

Although the Krein parameters of any association scheme are non-negative real numbers, the dual of association schemes are not association scheme in general. Any symmetric association scheme of class 2 arises naturally from strongly regular graphs. In this talk, we study relationship between intersection numbers of association schemes of class 2 and Krein parameters. We show that Krein parameters of a self-complementary strongly regular graph are non-negative integers.

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Short communication

Матричная однонаправленная функция

А. Я. Белецкий, Р. П. Мегрелишвили

Основная цель работы состоит в построении однонаправленной функции на примитивных двоичных матрицах n-го порядка M и разработке алгоритмов синтеза таких матриц. К примитивным будем относить двоичные матрицы M, последовательность степеней которых в кольце вычетов по $\mod 2$ образует последовательность максимальной длины (m-последовательность). Следовательно, матрицы M могут рассматриваться как образующие мультипликативной группы, порядок L_n которой определяется выражением

$$L_n = 2^n - 1. \tag{1}$$

Идея формирования однонаправленных функций на основе двоичных примитивных матриц M впервые, по-видимому, была изложена в [1]. Суть протокола обмена данными по открытому каналу связи между двумя абонентами компьютерной сети **A** (Алисой) и **B** (Бобом), в ходе которой образуется секретный ключ криптографической защиты информации K, сводится к следующему. Пусть V и M — открытые двоичные вектор-строка и примитивная матрица порядка n.

1. Абонент **A** вырабатывает случайный показатель x, вычисляет вектор $V_a = V \cdot M^x$ и посылает его абоненту **B**.

2. Абонент **B** вырабатывает случайный показательy, вычисляет вектор $V_b = V \cdot M^y$ и посылает его абоненту **A**.

3. Алиса вычисляет ключ $K_a = V_b \cdot M^x$.

4. Боб вычисляет ключ $K_b = V_a \cdot M^y$.

Вполне очевидно, что по завершении протокола оба абонента будут располагать одинаковым секретным ключом K, так как

$$K_a = V \cdot M^y \cdot M^x = K_b = V \cdot M^x \cdot M^y. \tag{2}$$

Из соотношений (1) и (2), во-первых, явствует важность коммутативности множества M при его генерации и, во-вторых, становится очевидным необходимость в разработке разнообразных методов синтеза гарантированно невырожденных примитивных двоичных матриц M высокого порядка.

В работе [1] приведен достаточно простой рекуррентный способом формирования матриц M. Предложенные матрицы хотя и являются примитивными и коммутативными, но их применение в криптографии затруднено в силу строгой нечетности порядков этих матриц.

В данном докладе рассмотрен прямой (исключающий рекурсию) способ формирования гарантированно невырожденных примитивных и коммутативных матриц, основанный на применении обобщенных преобразований (кодов) Грея [2,3]. В табл. 1 приведена группа простых операторов Грея (g).

Обозначение оператора g	Выполняемая операция
e (или 0)	Сохранение исходной комбинации
1	Инверсная перестановка
2	Прямое кодирование по Грею левостороннее
3	Обратное кодирование по Грею левостороннее
4	Прямое кодирование по Грею правостороннее
5	Обратное кодирование по Грею правостороннее

Таблица 1. Полная группа простых операторов Грея

Из элементов полной группы простых операторов Грея, представленных в табл. 1, можно сформировать так называемые составные коды Грея (G), образуемые произведением простых (элементарных) кодов Грея.

$$G = \prod_{j=1}^{k} g_j$$

где g_j – простой КГ, выбираемый из полной группы $\{\overline{g_0,\ g_5}\}$, а k– порядок СКГ.

Как показали результаты компьютерных расчетов, интересными свойствами обладают матрицы, отвечающие отдельным СКГ. Замечательная особенность таких матриц состоит в том, что порядок L_n циклических групп, порождаемых операторами G, определяется соотношением: $L_n = 2^m - 1$, $m \leq n$, где n — порядок матрицы.

Более того, существуют такие значения порядка n и такие СКГ G, для которых степени соответствующих им матриц M составляют последовательность максимальной длины. В табл. 2 приведены примеры таких СКГ.

Порядок матрицы (n)			
32	64	128	256
2244424	22533435	2425535	22533435
2442224	22534335	2433534	22534335
12242253	24334225	2435334	24334225
12242443	25224334	22524224	25224334
12252242	222524424	22533334	2222535224

Таблица 2. Составные коды Грея, формирующие примитивные матрицы

Целесообразность применения в криптографии и в других приложениях матриц, отвечающих составным кодам Грея, объясняется рядом замечательных свойств, которыми они обладают. Во-первых, матрицы, порождаемые СКГ любого порядка, чрезвычайно просто генерировать. Во-вторых, такие матрицы являются гарантированно невырожденными. В-третьих, для них легко вычисляются обратные матрицы. Данное свойство особенно важно для построения матричных алгоритмов шифрования [4]. В-четвертых, как установлено на основании компьютерного моделирования, для произвольных порядков n матриц существуют такие СКГ, которые доставляют соответствующим матрицам свойство примитивности. Это свойство проявляется в том, что порядок циклических групп, формируемых этими матрицами, достигает максимального значения, равного $2^n - 1$. И, наконец, в-пятых, если некоторая матрица M является примитивной, то это свойство сохраняется инвариантным к группам линейных Q — преобразований над строками и столбцами матриц M и Ω — преобразований над СКГ G. В состав Ω-группы входят такие операторы линейных преобразований над G: циклического сдвига (S), обращения (I), инверсии (R) и сопряжения (C), а также произвольные комбинации этих операторов. Q-группу линейных преобразований над матрицами M составляют операторы «дружной перестановки» строк и столбцов матрицы, частным случаем которых являются операторы «дружного циклического сдвига» строк и столбцов матрицы M. Такие преобразования порождают так называемые подобные матрицы.

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Short communication

Синтез псевдослучайных последовательностей максимального периода

А. Я. Белецкий, Е. А. Белецкий

Генераторы псевдослучайных последовательностей (ПСП) строятся, как правило, на основе регистров сдвига с линейными обратными связями (РСЛОС). При этом длина периода ПСП будем максимальной, если обратные связи в РСЛОС создаются примитивными неприводимыми полиномами (НП) $n-$й степени. НП φ_n является примитивным (ПНП), если степени образующего элемента $\omega = 10$ (являющегося полиномом первой степени)ω^kв кольце вычетов по модулю φ_n формируют последовательность длины $L_n = 2^n - 1$.

Известны два типа РСЛОС-генераторов ПСП над ПНП: генераторы по схемам Галуа и Фибоначчи [1]. Обратные связи в регистрах этих генераторов задаются матрицами преобразований, которые обозначим G и F соответственно. Матрицы G и F связаны оператором правостороннего транспонирования (транспонирования относительно вспомогательной диагонали матрицы), который обозначим $\perp$. Следовательно, $G \xleftrightarrow{\perp} F$, т.е. $F = G^\perp$ и $G = F^\perp$.

Пусть $\varphi_n = 0_n 0_{n-1} 0_{n-2} \cdots 0_1 0_0$ есть ПНП $n-$й степени, в котором $0_i \in \{0, 1\}$, $i = \overline{1,\ n-1}$. Поскольку в двоичном НП $\varphi_n = \varphi_0 = 1$, то φ_n= $10_{n-1}0_{n-2}\cdots 0_1 1$.
Матрицы $G^{(10)}_{\varphi_n}$ и $F^{(10)}_{\varphi_n}$ составляются по схемам, представленными соотношениями (1).

$$G^{(10)}_{\varphi_n} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ \dots \\ n-1 \\ n \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & \dots & n-1 & n \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & \dots & 0 & 1 \\ 1 & 0 & 0 & \dots & 0 & 0_1 \\ 0 & 1 & 0 & \dots & 0 & 0_2 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 0 & 0_{n-2} \\ 0 & 0 & 0 & \dots & 1 & 0_{n-1} \end{pmatrix} \end{array}, \quad F^{(10)}_{\varphi_n} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ \dots \\ n-1 \\ n \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & \dots & n-1 & n \end{array} \\ \begin{pmatrix} 0_{n-1} & 0_{n-2} & 0_{n-3} & \dots & 0_1 & 1 \\ 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & 0 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & \dots & 1 & 0 \end{pmatrix} \end{array} \quad (1)$$

Генераторы ПСП можно составить также на основании k-х степеней матриц (1), вычисляемых в кольце вычетов по mod 2, причем показатель k должен быть взаимно прост с длиной периода L_n. Кроме того, множество матриц (генераторов) формирования ПСП может быть расширено за счет введения подобных матриц. Матрица *, подобная матрице , образуется в результате преобразования $M^* = P \cdot M \cdot P^{-1}$, где $P-$произвольная матрица перестановки.

Матрицы (1) являются не единственным способом задания функций возбуждения триггеров РСЛОС-генераторов ПСП. Альтернативными вариантами (1) можно рассматривать матрицы G и F, в которых образующий полином (ОП) $\omega = 10$ замещается полиномом первой степени $\omega = 11$ [2]. Следствие предлагаемой замены ОП проявляется в том, что выход каждого разряда РСЛОС становится замкнутым на его вход. Для иллюстрации отмеченной особенности на рис. 1 показана структурная схема генератора Галуа, составленная для ПНП φ_8= 100101011 и ОП $\omega = 11$.

Переход от $G^{(10)}_{\varphi_n}$ к $G^{(11)}_{\varphi_n}$ реализуется элементарной операцией

$$G^{(11)}_{\varphi_n} = G^{(10)}_{\varphi_n} \oplus E. \quad (2)$$

На основании соотношений (1) и (2) приходим к следующим значениям функций возбуждения $k-$го разряда ($D-$триггера) РСЛОС над ПНП φ_n и ОП $\omega = 11$.

$$D_k = k \oplus (k-1) \oplus n \cdot 0_{k-1}, k = \overline{1,\ n}. \quad (3)$$

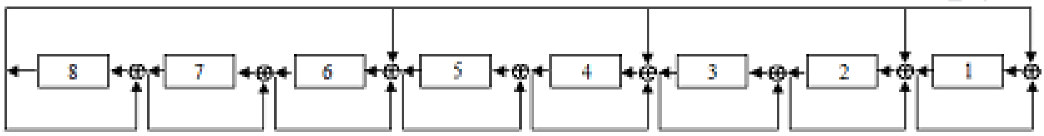

Рис. 1. Генератор Галуа (φ_8= 100101011 и ОП $\omega = 11$)

В правой части выражения (3) первые две компоненты отвечают откликам соответствующих разрядов регистра.

Переходим к рассмотрению алгоритма формирования функций возбуждения РСЛОС-генераторов ПСП над ПНП φ_n и ОП второй степени $\omega = 1\omega_1\omega_0$, $\omega_i \in \{1,\ 0\}$, $i = 0,\ 1$. Матрица Галуа составляется за два этапа. На первом этапе матрица $G_{\varphi_n}^{(1\omega_1\omega_0)}$ предварительно записывается в виде:

$$G_{\varphi_n}^{(1\omega_1\omega_0)} = \begin{array}{c} \\ 1\\2\\3\\4\\5\\\dots\\n-1\\n \end{array} \begin{array}{c} \begin{array}{ccccccc} 1 & 2 & 3 & \dots & n-2 & n-1 & n \end{array} \\ \begin{pmatrix} \omega_0 & 0 & 0 & \dots & 0 & 1 & \omega_1 \oplus 0_{n-1} \\ \omega_1 & \omega_0 & 0 & \dots & 0 & 0_1 & 1 \\ 1 & \omega_1 & \omega_0 & \dots & 0 & 0_2 & 0_1 \\ 0 & 1 & \omega_1 & \dots & 0 & 0_3 & 0_2 \\ 0 & 0 & 1 & \dots & 0 & 0_4 & 0_3 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & \omega_1 & \omega_0 \oplus 0_{n-2} & 0_{n-3} \\ 0 & 0 & 0 & \dots & 1 & \omega_1 \oplus 0_{n-1} & \omega_0 \oplus 0_{n-2} \end{pmatrix} \end{array}. \qquad (4)$$

Если $\omega_1 \oplus u_{n-1}$= 0, то (4) является конечной матрицей и второй этап не выполняется, а функции возбуждения триггеров регистра определяются соотношениями:

$$D_k = (k-2) \oplus (k-1) \cdot \omega_1 \oplus k \cdot \omega_0 \oplus (n-1) \cdot a_{k-1} \oplus n \cdot a_{k-2}, k = \overline{1,\ n}, \qquad (5)$$

причем

$$a_{k-2} = \begin{cases} \omega_1 \oplus a_{n-1}, & \text{если} \ \ k = 1; \\ \omega_0 \oplus a_{n-2}, & \text{если} \ \ k = n. \end{cases}$$

В том случае, когда $\omega_1 \oplus 0_{n-1} = 1$, функции D_k также определяются по формуле (5), но элементы $n-$го столбца матрицы (4) оказываются зависимыми от переменных ПНП $a_{n-1},\ a_{n-2}$ и образующего полинома ω_0, ω_1.

В докладе обсуждается технология синтеза генераторов ПСП над примитивными неприводимыми полиномами φ_n относительно образующих полиномов ω степени, превышающей 2. Приводятся примеры структурных схем генераторов Галуа и Фибоначчи, а также подобных им генераторов, и дается сравнительный анализ статистических свойств ПСП, формируемых такими генераторами.

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SECTION TALK

Implementing categories with generic programming techniques

A. Chentsov

The importance of category theory for computing continues to grow. Recently categories have been proposed as foundations for ontologies [1]. In order to progress on this path category theory need adequate and effective formalization. Such formalization has been a matter of research since late 1970s. In general one can distinguish following universal approaches to categories formalization[2]:

- ontological;
- logico-algebraic;
- computational.

First is a basis for formalization of other systems, second is developed to assist in proving categorical facts, the last one is most appropriate for effective application of categorical machinery to specific problems.

If we consider implementation of categories as a computer algebra system it has specific characteristics. Firstly due to very high degree of abstraction it is inherently polymorphic [3]. Secondly categorical concepts are introduced by gradual refinement and they form taxonomies. Finally the theorems and proofs in the theory has computational nature which can be expressed as (generic) algorithms. All of these features are cornerstones of generic programming.

This work considers computational approach to category theory formalization by means of generic programming techniques. The main goal is to develop efficient library implementing categories in programming language with generics, e.g. C++. We discuss pros and contras of such implementation and compare it to other approaches.

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SHORT COMMUNICATION

Minimizing the graphs models presented strongly connected components

V. Chepurko

Introduction. There are various types of discrete functional models such as automata, graphs with marked vertices, etc. Graphs with distinguished vertices are one of the basic models when considering these two directions. First — this is the problem associated with the analysis of the operating environment by wandering on them agents (mobile robots, machines, search engines, etc.) Second — it's a flowchart programs. The problem of equivalence of programs is one of the most important tasks of programming theory[1].

$G = (V, E, M, \mu)$ — deterministic oriented graph G[2]. V — set of vertices, E — set of arcs $E \subseteq V \times V$, M - set of labels, $\mu : S \to M$ — marking function vertices. $p = g_1, \dots, g_k$ — path in the graph G. $\mu(p) = \mu(g_1), \dots, \mu(g_k)$ — mark the path. Language of vertex g is L_g — a set of marks of all the paths from the vertices of g. Vertices with the same languages are equivalent.

We consider the problem of converting a deterministic directed graph G with marked vertices in an equivalent graph G' with the minimum number of vertices. In this case, the languages of all the vertices of G must be equal languages of vertices of G', $\{L_(g)\}_(g) \in G = \{L_(g')\}_(g') \in G'$. The solution of the minimization problem reduces to finding a partition vertices of the graph into classes of equivalent vertices, i.e., vertices with the same L_g. Partition is gross, i.e. each partition class can not be broken down into subclasses.

The idea:

- Choose a vertex is the end of one of the lenses, consider it a zero level of the graph. From the selected vertex minimization algorithm perform acyclic graph[3] until it reaches the initial. Second pass. Looking for the period among the nodes and elements. If the period is found uniting the corresponding vertices of the graph and the lens.

Conclusion. The new algorithm with $O(e)$ time complexity for minimizing graphs with marked vertices for Strongly connected component is proposed, e-number of arcs.

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8th International Algebraic Conference
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SHORT COMMUNICATION

On a subgraph induced at a labeling of a graph by the subset of vertices with an interval spectrum

N. N. Davtyan, A. M. Khachatryan, R. R. Kamalian

We consider nonoriented connected finite graphs without loops and multiple edges. $V(G)$, $E(G)$ and $V'(G)$ denote the sets of vertices, edges and pendent vertices of a graph G, respectively. The diameter of a graph G is denoted by $d(G)$. A vertex $x \in V(G)$ is called a peripheral vertex of a graph G, if there exists a vertex $y \in V(G)$ on the distance $d(G)$ from x. A nonempty finite subset of consecutive positive integers is called an interval. A function $\varphi : E(G) \to \{1, 2, ..., |E(G)|\}$ is called a labeling of a graph G, if for arbitrary different edges $e' \in E(G)$ and $e'' \in E(G)$ $\varphi(e') \neq \varphi(e'')$. For a graph G, and for any $V_0 \subseteq V(G)$, we denote by $< V_0 >_G$ the subgraph of a graph G induced by the subset V_0 of its vertices. If G is a graph, x is any vertex of G, φ is an arbitrary labeling of G, then the set $S_G(x, \varphi) \equiv \{\varphi(e)/e \in E(G), e \text{ is incident with } x\}$ is called a spectrum of a vertex x of a graph G at a labeling φ, and let us set $V_{int}(G, \varphi) \equiv \{x \in V(G)/\ S_G(x, \varphi) \text{ is an interval}\}$. $\lambda(G)$ denotes the set of all such labelings φ of a graph G, for which $V_{int}(G, \varphi) \neq \varnothing$.

For arbitrary integers n and i, satisfying the inequalities $n \geq 3, 2 \leq i \leq n-1$, and for any sequence $A_{n-2} \equiv (a_1, a_2, ..., a_{n-2})$ of nonnegative integers, we define the sets $V_n[i, A_{n-2}]$ and $E_n[i, A_{n-2}]$ as follows:

$$V_n[i, A_{n-2}] \equiv \begin{cases} \{y_{i,1}, ..., y_{i,a_{i-1}}\}, & \text{if } a_{i-1} > 0 \\ \varnothing, & \text{if } a_{i-1} = 0, \end{cases}$$

$$E_n[i, A_{n-2}] \equiv \begin{cases} \{(x_i, y_{i,j})/1 \leq j \leq a_{i-1}\}, & \text{if } a_{i-1} > 0 \\ \varnothing, & \text{if } a_{i-1} = 0. \end{cases}$$

For $\forall n \in N, n \geq 3$, and for any sequence $A_{n-2} \equiv (a_1, a_2, ..., a_{n-2})$ of nonnegative integers, we define a graph $P_n[A_{n-2}]$ as follows:

$$V(P_n[A_{n-2}]) \equiv \{x_1, ..., x_n\} \cup \left(\bigcup_{i=2}^{n-1} V_n[i, A_{n-2}] \right),$$

$$E(P_n[A_{n-2}]) \equiv \{(x_i, x_{i+1})/1 \leq i \leq n-1\} \cup \left(\bigcup_{i=2}^{n-1} E_n[i, A_{n-2}] \right).$$

A graph G is called a galaxy, if either $G \cong K_2$, or there exist $n \in N, n \geq 3$ and a sequence A_{n-2} of nonnegative integers $a_1, a_2, ..., a_{n-2}$, for which $G \cong P_n[A_{n-2}]$.

Theorem 1. *For any graph G and for $\forall \varphi \in \lambda(G)$ $< V_{int}(G, \varphi) >_G$ is a forest, each connected component H of which satisfies one of the following two conditions: 1) $H \cong K_1$, and the only vertex of the graph H may or may not belong to the set $V'(G)$, 2) H is a galaxy, satisfying one of the following three conditions: a) $V'(H) \subseteq V'(G)$, b) exactly one vertex of the set $V'(H)$, which is a peripheral vertex of H, doesn't belong to the set $V'(G)$, c) exactly two vertices of the set $V'(H)$, with $d(H)$ as the distance between them, don't belong to the set $V'(G)$.*

Corollary 1. *For any graph G with $V'(G) = \varnothing$, and for $\forall\varphi \in \lambda(G)$, an arbitrary connected component of the forest $< V_{int}(G,\varphi) >_G$ is a simple chain.*

Corollary 2. *A labeling, which provides every vertex of a graph G with an interval spectrum, exists iff G is a galaxy.*

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Section talk

Конфигурации сопряженных подстановок

И. И. Дериенко

На множестве $Q = \{1, 2, 3, \ldots, n\}$ будем рассматривать симметрическую группу подстановок S_n.

Определение 1. *Цикловым типом подстановки* ϕ *будем называть следующий набор чисел:* $C(\phi) = \{l_1, l_2, \ldots, l_n\}$, *где* l_i *есть количество циклов длины* i.

Каждый набор $\{l_1, l_2, \ldots, l_n\}$ будем обозначать Z_i.

Определение 2. *Две подстановки* $\phi, \psi \in S_n$ *называются сопряженными, если существует подстановка* $\rho \in S_n : \rho\phi\rho^{-1} = \psi$.

Имеет место следующая теорема [1].

Теорема 1. *Подстановки* ϕ, ψ *тогда и только тогда сопряжены, когда*

$$C(\phi) = C(\psi)$$

Определение 3. *Флоком* F_i *будем называть подмножество симметрической группы* S_n, *все элементы которого имеют один и тот же цикловой тип* Z_i. *Так если* $\psi, \psi \in F_i$, *тогда* $C(\phi) = C(\psi) = Z_i$.

На каждом флоке F_i выделим произвольную подстановку σ, которую будем называть стволовой подстановкой. Основным понятием этой работы будет понятие конфигурации. Приведем описание этого объекта. Для данного n зафиксируем некоторый цикловой тип Z_i. Рассмотрим соответствующий ему флок F_i. Выделим на F_i стволовую подстановку σ и некоторую подстановку $\hat{\phi}_0$. Далее с помощью σ определим следующее рекуррентное соотношение: $\phi_k\sigma\phi_k^{-1} = \phi_{k+1}$, $k = 0, 1, 2, \ldots$, где $\hat{\phi}_0$ есть некоторая подходящая подстановка такая, что $Z\left(\hat{\phi}_0\right) = Z(\sigma)$, т.е. все элементы этой последовательности будут принадлежать F_i. На рисунке 1 приведен пример конфигурации.

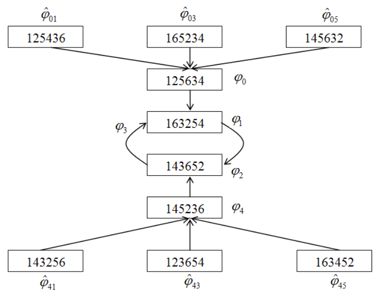

Рис. 1. *Пример конфигурации*

Автор считает, что предложенная алгебраическая структура «конфигурация» является достаточно богатой и может быть моделью физических, химических и биологических процессов. Так для $n = 6$ флоков будет 11, а количество конфигураций пока неизвестно. Эти объекты могут быть математической моделью образования кристаллов, а также образования органов живого организма.

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8th International Algebraic Conference
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О конечных частично упорядоченных множествах

Т. И. Дорошенко

Диаграммой конечного частично упорядоченного (ч.у.) множества $P = \{p_1, \ldots, p_n\}$ называется колчан $Q(P)$ с множеством вершин $VQ(P) = \{1, \ldots, n\}$ и множеством стрелок $AQ(P)$, заданных по правилу: существует одна стрелка из вершины i в вершину j тогда и только тогда, когда $p_i < p_j$ и из соотношения $p_i \leq p_k \leq p_j$ следует, что или $k = i$ или $k = j$. Мы используем терминологию и обозначения [1].

Рассмотрим квадратную матрицу $U_n = \begin{pmatrix} 1 & \ldots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \ldots & 1 \end{pmatrix}$, все элементы которой единицы. Очевидно, что матрица $\mathcal{E}_p = [Q(P)] = \begin{pmatrix} 0 & U_n & 0 & \ldots & \ldots & 0 \\ 0 & 0 & U_n & 0 & \ldots & 0 \\ \vdots & & \ddots & \ddots & \ddots & \vdots \\ \vdots & & & \ddots & \ddots & 0 \\ \vdots & & & & 0 & U_n \\ 0 & \ldots & \ldots & \ldots & \ldots & 0 \end{pmatrix}$ является матрицей смежности диаграммы ч.у. множества ширины n. Напомним, что черепичный порядок $\Lambda = \{\mathcal{O}, \mathcal{E}(\Lambda)\}$ называется (0,1)-порядком, если $\mathcal{E}(\Lambda)$ является (0,1)-матрицей. Матрица смежности колчана черепичного порядка, построенного по матрице $\mathcal{E}_p$ имеет вид:

$$\begin{vmatrix} 0 & U_n & 0 & \ldots & \ldots & 0 \\ 0 & 0 & U_n & 0 & \ldots & 0 \\ \vdots & & \ddots & \ddots & \ddots & \vdots \\ \vdots & & & \ddots & \ddots & 0 \\ 0 & \ldots & \ldots & \ldots & 0 & U_n \\ U_n & 0 & \ldots & \ldots & \ldots & 0 \end{vmatrix}$$

и $w(P) = inx\, P = n$, где $w(P)$ — ширина ч.у. множества P.

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СВЕДЕНИЯ ОБ АВТОРАХ

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8th International Algebraic Conference
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New computational algorithms for some non-rigid group

Naser Ghafooriadl, Ali Moghani

To enumerate isomers of the fluxional molecules, a useful theorem for maturity and the integer-valued characters of finite groups was introduced by the second author. If G_i and G_j be any subgroups of an arbitrary finite group G, a subduced representation denoted $G(/G_i) \downarrow G_j$ is known as a subgroup of the coset representation $G(/G_i)$ that contains only the elements associated with the elements in G_j. A unit subduced cycle index (USCI) is delineated as $Z(G(/G_i) \downarrow G_j, S_d) = \prod_{g\in\Omega} s_{d_g}^{(ij)}$ where Ω is a transversal set for the double coset decompositions concerning G_i and G_j for $i, j = 1, 2, 3, ..., s$ and $s_{d_g}^{ij} = |G_i|/|g^{-1}G_i g \cap G_j|$.

In this paper we introduce new algorithms for computing all the USCIs for some non-rigid groups.

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8th International Algebraic Conference
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SHORT COMMUNICATION

Finite representations of algebraic systems

I. S. Grunsky, I. I. Maksimenko

Earlier [1,2] authors have studied representations of unstructured objects, which are defined by sets of descriptors. In this article the algebraic system of objects $(\mathbf{A}, \leq, \vee, \wedge, \mathrm{n})$, is considered, where $\leq$ is a preorder, $\vee, \wedge$ - idempotent, commutative and associative binary operations, $\mathrm{n} : \mathbf{A} \to \mathbf{N}^{+} \cup \{\infty\}$ - nondecreasing function of complexity, and valid are axioms:

1. for any objects $A, B \in \mathbf{A}$, correlations $A \leq A \vee B$, $A \wedge B \leq A$ are fulfilled;
2. for any objects $A_1, A_2, B \in \mathbf{A}$ if $A_1 \leq B$, $A_2 \leq B$, follows that $A_1 \vee A_2 \leq B$;
3. for any objects $A_1, A_2, B \in \mathbf{A}$ if $A_1 \not\leq B$, $A_2 \not\leq B$, follows that $A_1 \wedge A_2 \not\leq B$;
4. for any objects $A, B \in \mathbf{A}$, assume that $n(A \vee B) = max(n(A), n(B))$ and $n(A \wedge B) = min(n(A), n(B))$.

Each object is uniquely defined by sets of fragments $Fr(A) = \{B \in \mathbf{A} | \mathrm{B} \leq \mathrm{A}\}$ and cofragments $CoFr(A) = \{B \in \mathbf{A} | \mathrm{B} \not\leq \mathrm{A}\}$. We assume that objects A and B are equivalent $(A \cong B)$, if $Fr(A) = Fr(B)$.

`Proposition 1` The system $(\mathbf{A}, \leq, \vee, \wedge, \mathrm{n})$ is upper semilattice.

An object A is finite, if $n(A)$ is finite, and infinite in contrary case. Object C separates non-equivalent objects A and B ($C \in S(A, B)$) if ($C \leq A$ and $C \not\leq B$) or ($C \not\leq A$ and $C \leq B$).

The system $(\mathbf{A}, \leq, \vee, \wedge, \mathrm{n})$ is finitely separable if the finite separating object exists for any two non-equivalent objects.

By analogy with article [1] let us introduce the distance $\beta(A, B) = 0$ if $A \cong B$ and $\beta(A, B) = 1/k$, $k = inf\{n(C) | C \in S(A, B)\}$ in contrary case. Let us describe the limit set of $F \subseteq \mathbf{A}$ as $LimF = \{B \in \mathbf{A} | \mathrm{inf_C}\{\beta(\mathrm{B}, \mathrm{C}) | \mathrm{C} \in \mathrm{F}, \mathrm{C} \ncong \mathrm{B}\} = 0\}$.

Let $A_0 \in \mathbf{A}$ and $F \subseteq \mathbf{A}$. A pair of objects (A, B) let us term a representation for A_0 and F, if $(A, B) \in Fr(A_0) \times CoFr(A_0)$, and if for any $A' \in F$ from the conditions $(A, B) \in Fr(A') \times CoFr(A')$ follows that $A_0 \cong A'$.

Let us call the system $(\mathbf{A}, \leq, \vee, \wedge, \mathrm{n})$ locally closed, if any sets $Fr(A)$ and $CoFr(A)$ are closed as regards to denumerable number of operations $\vee$ and $\wedge$, respectively.

`Theorem 2` Locally closed and finitelly separable system $(\mathbf{A}, \leq, \vee, \wedge, \mathrm{n})$, $A_0 \in \mathbf{A}$ and $F \subseteq \mathbf{A}$ is given. A finite representation for A_0 and F exists if and only if $A_0 \notin LimF$.

We introduce the operation * as follows $(A_1, B_1) * (A_2, B_2) = (A_1 \vee A_2, B_1 \wedge B_2)$ for any two representations $(A_1, B_1), (A_2, B_2)$ for A_0 and F.

`Proposition 3` The set of representations for A_0 and F is commutative idempotent semigroup regarding to operation *.

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8th International Algebraic Conference
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SECTION TALK

On a second-order recurrence formula

E. Haji-Esmaili, S. H. Ghaderi

This paper introduces a formula for direct calculation the term a_n of the second-order recurrence $\{a_n\}$ defined by $a_n = Xa_{n-m} + Ya_{n-2m}$, in which X, Y and m are fixed constants and the value of two arbitrary terms like a_{n-km} and a_{n-lm} are known where k and l are positive integers and $k > l$. This formula is

$$a_n = \frac{H(k-1)}{H(k-l-1)}a_{n-lm} + (H(k) - \frac{H(k-1)H(k-l)}{H(k-l-1)})a_{n-km} \tag{1}$$

where

$$H(k) = X^k + \sum_{i=1}^{[\frac{k}{2}]}\sum_{j=i}^{k-i}\binom{j-1}{i-1}X^{k-2i}Y^i \tag{2}$$

We will discuss about some applications of this formula.

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8th International Algebraic Conference
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On some generalization of Pólya's enumeration theorem

Yu. Ishchuk

In 1937 there appeared a paper [1] that was to have a profound influence on the future of combinatorial analysis. It was one of George Pólya's most famous papers, and in it Pólya presented to the combinatorial world a powerful theorem which reduced to a matter of routine the solution of a wide range of problems. Moreover, this theorem contained within it the potential for growth and generalization in many directions.

Pólya's Theorem relates to the enumeration of mathematical objects called "patterns", which can be defined as equivalence classes of mappings from a set $D = \{d_1, d_2, \ldots, d_m\}$ (the set of "figures") to the set $R = \{r_1, r_2, \ldots, r_n\}$ (the set of "colours"). The maps $f, g \in R^D$ are called equivalent if there is a $\sigma \in G$ such that $f\sigma = g$, where G is a subgroup of S_m.

The first of several extensions and generalizations of Pólya Theorem was that of deBruijn [2]. His generalization consisted in the introduction of another group H of S_n, which permutes the colours, in addition to the group G of permutations of the figures. Two patterns were then regarded as equivalent if one could be obtained from the other by permuting colours and figures by appropriate permutations.

The brief account of Pólya's Theorem will be given and the solutions of some enumeration problems using GAP [5] will be proposed.

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Short communication

Enumeration of 2-color N-diagrams with one black cycle

A. A. Kadubovskiy

We determine the number of nonequivalent 2-color chord diagrams of order n with one black cycle under the action of two groups: C_n — a cyclic group of order n, and D_{2n} — a dihedral group of order $2n$.

Definition 1. *A configuration (graph) on a plane that consists of a circle and n chords that connects $2n$ different points on it is called a chord diagram of order n, or, briefly, an n-diagram (see, e.g., [1]).*
An n-diagram whose arcs of the circle are colored in two colors so that any two neighboring arcs have different colors is called a 2-color chord diagram.

All 2-color diagrams are constructed on a unit circle (in R^2) with fixed clockwise enumeration of $2n$ points on it, which are the vertices of a regular $2n$-polygon.

Definition 2. *A 2-color diagram that does not contain (contain) chords that connect points with numbers of the same parity is called an O-diagram (N-diagram).*
A sequence of chords and black (white) arcs that form a homeomorphic image of the oriented circle is called a b-cycle (w-cycle) of a 2-color diagram with given direction on the circle (see, e.g., [2]).
Denote the set of all N-diagram with n chords that have 1 black cycles by $\Im^N_{1,n}$.

Theorem 1. *The number d^*_n of nonequivalent diagrams from the class $\Im^N_{1,n}$ under the action of a cyclic group C_n can be calculated by the relation*

$$n \cdot d^*_n - d_n - \sum_{i|n,\ i\neq n} \phi^2\left(\frac{n}{i}\right)(i-1)!\left(2^{i-1}-1\right)\left(\frac{n}{i}\right)^{i-1} = \begin{cases} 0, & n \text{ odd}, \\ \left(\frac{n}{2}\right)! 2^{n-2}, & n \text{ even}, \end{cases} \tag{1}$$

where $d_n = \left|\Im^N_{1,n}\right| = (n-1)!\left(2^{n-1}-1\right)$, $\phi(q)$ is the Euler totient function (the number of natural numbers smaller than q and coprime with it).

Theorem 2. *The number d^{**}_n of nonequivalent diagrams from the class $\Im^N_{1,n}$ under the action of a dihedral group D_{2n} can be calculated by the relation*

$$2d^{**}_n - d^*_n = \begin{cases} 2^{\frac{n-1}{2}}\left(\frac{n-1}{2}\right)!\left(2^{\frac{n-1}{2}}-1\right), & n \text{ odd}, \\ 2^{\frac{n}{2}-2}\left(\frac{n}{2}-1\right)!\left(2^{\frac{n}{2}}+\left(2^{\frac{n}{2}-1}-1\right)\left(\frac{n}{2}+1\right)\right), & n \text{ even}. \end{cases} \tag{2}$$

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SHORT COMMUNICATION

On classification of (3,3,2)-type functional equations

Halyna V. Kraynichuk

Every quasigroup (i.e. invertible) function is a composition of binary invertible functions defined on the same set[1]. The problem is: *describe all such decompositions of the same invertible function*. The problem occurs in various branches of mathematics. For example, if a groupoid satisfies an identity $\omega = \upsilon$, then ω and υ are decompositions of a function.

If we consider repetition-free composition only, then the problem has been solved by F.M. Sokhatsky [2], who has proved that every two full decompositions of a finite-valued strongly dependent function are almost the same (every invertible function is strongly dependent). There is a number of articles concerning the study of the repetition-free case of the problem, but the author has not come across any article devoted to the repetition case of the problem. We consider two of the possible decompositions of a ternary function: repetition-free decompositions (for example, $g_1(x, g_2(y, z))$) and decompositions having two appearances of a subject variable (for example, $g_3(g_4(x, y), g_5(x, z))$). Equating these decompositions, we obtain a solution of one of the following type of functional equations (f.equ.): 1) *balanced*, i.e. (2;2;2)-type f.equ. (every subject variable has two appearances); 2) *distributive-like*, i.e. (3;2;2)-type f.equ.; 3) *Bol-Moufang type*, i.e. (4;2;2)-type f.equ.; 4) (3;3;2)-type f.equ. In the example given above the quintuple $(g_1, g_2, g_3, g_4, g_5)$ is a solution of the f.equ. of generalized left distributivity

$$F_1(x, F_2(y, z)) = F_3(F_4(x, y), F_5(x, z)).$$

To find its all solutions over quasigroups of a set is a well known open problem.

Full classification of balanced f.equ. has two classes; that of distributive-like f.equ. has five classes (Sokhatsky, here); that of Bol-Moufang type f.equ. has eight classes.

`Theorem 1.` *Every functional equation of the type* $(3, 3, 2)$ *is parastrophically equivalent to at least one of 25 finding functional equations.*

The following functional equation is taken from the list of these 25 f.equ.:

$$F_1(x; F_2(x; F_3(y; z))) = F_4(y; F_5(y; F_6(x; z))). \qquad (1)$$

`Theorem 2.` *A sequence* $(f_1, \ldots, f_6)$ *of quasigroup operations, defined on a set* Q *is a solution of* (1) *iff there exist substitutions* $\alpha, \beta, \nu, \theta, \pi$ *of* Q *and an abelian group* $(Q; +)$ *such that* $f_2(x; y) = f_1^r(x; \alpha x + \pi y)$, $f_3(y; z) = \pi^{-1}(\beta z + \alpha y)$, $f_5(y; x) = f_4^r(y; \alpha y + \theta x)$, $f_6(x; z) = \theta^{-1}(\alpha x + \beta z)$.

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Contributed abstract

Algebras with infinitary partial quasiary operations and their application in logic

Mykola (Nikolaj) Nikitchenko

Let V be a set, possibly infinite, of names (variables), A be an abstract set. By ${}^V A$ we denote the set of partial functions from V to A. Elements of this set are called partial assignments (nominats in terms of [1]). A partial function $f :{}^V A \to A$ is called a partial V-quasiary operation on A. This operation is infinitary if V is infinite. Let S be a set, possibly infinite, of operation symbols; $\mu : S \to ({}^V A \to A)$ be a total meaning (interpretation) mapping. A pair (A, μ) is called (V, S)-algebra of partial quasiary operations. Such algebras appear in logic when infinitary partial assignments of individual variables are considered. In this case a term of the first-order language semantically represents partial quasiary operation and a formula represents partial quasiary predicate. Thus, properties of algebras with such operations determine the properties of corresponding logics.

The aim of the talk is to present the main definitions and properties of algebras with partial quasiary operations; to construct and investigate equational calculus for such algebras; to apply obtained results to the problems of compositional completeness of infinitary logics.

For algebras in hand we define the following notions: subalgebra, congruence relation, quotient algebra, various kinds of homomorphisms . Then we define the class of terms with finite depth and describe a special algebra of terms. Contrary to algebras with finite-ary operations, in the infinitary case we should consider a term algebra as partial. Equational theory is defined by a set of equalities. A special infinitary equational calculus is constructed; its soundness and completeness are proved.

The constructed infinitary equational logic is used for describing complete classes of compositions considered as quasiary operations on the set of predicates. A simpler proof is obtained for the infinitary propositional logic of partial predicates [2], new results are proved for the infinitary logics of equitone predicates and their variants. Taking into consideration the fact that program semantics can be specified in algebras of quasiary functions, we can also use obtained results for investigation of program properties.

In the whole, algebras with infinitary partial quasiary operations can be useful in different kinds of logics, and, in particular, in program specification logics.

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CONTRIBUTED ABSTRACT

Experimental algorithm for the maximum independent set problem

A. D. Plotnikov

We develop an experimental algorithm for the exact solving of the maximum independent set problem. The algorithm consecutively finds the maximal independent sets of vertices in an arbitrary undirected graph such that the next such set contains more elements than the preceding one. For this purpose, we use a technique, developed by Ford and Fulkerson for the finite partially ordered sets, in particular, their method for partition of a poset into the minimum number of chains with finding the maximum antichain. In the process of solving, a special digraph is constructed, and a conjecture is formulated concerning properties of such digraph. This allows to offer of the solution algorithm. Its theoretical estimation of running time equals to is $O(n^8)$, where n is the number of graph vertices. The offered algorithm was tested by a program on random graphs. The testing the confirms correctness of the algorithm.

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Short communication

Elements of high order in finite fields

R. Popovych

A problem of fast construction of a primitive element for the given finite field is notoriously difficult in computational number theory. That is why one considers less restrictive question: to find an element with provable high order. It is sufficient in this case to obtain a lower bound of the order [1,3,4].

We denote F_q finite field with q elements where q is a power of prime number p. Let $r > 2$ be a prime number coprime with q and a multiplicative order of q modulo r equals to $r-1$. Set $F_q(\theta) = F_q[x]/\Phi_r(x)$ where $\Phi_r(x)$ is r-th cyclotomic polynomial and $\theta = x(mod\Phi_r(x))$. The element $\theta + \theta^{-1}$ is called a Gauss period of type $((r-1)/2, 2)$. It allows to obtain normal base [1]. $U(C, d)$ denotes a number of such integer partitions $u_1, ..., u_C$ of an integer C for which $u_1, ..., u_C \leq d$.

We prove that for any integer e, any integer f coprime with r, any non-zero element a in the field F_q the multiplicative order of the element $\theta^e(\theta^f + a)$ is at least $U(r-2, p-1)$. In particular, a multiplicative order of the element $\theta + \theta^{-1} = \theta^{-1}(\theta^2 + 1)$ is at least $U(r-2, p-1)$. This bound improves the previous bound of O.Ahmadi, I.E.Shparlinski and J.F.Voloch given in [1]. We show that if $a^2 \neq 1, -1$ then a multiplicative order of element $\theta^e(\theta^f + a)$ is least $[U((r-3)/2, p-1)]^2/2$. We also prove that if $a^2 \neq 1$ then an order of group $< \theta + \theta^{-1}, (a\theta + 1)(\theta + a)^{-1} >$ is at least $[U(r-2, p-1) \cdot U((r-3)/2, p-1)]/2$ and construct a generator of this cyclic group.

Using results from [2,5] we give explicit lower bounds for a multiplicative order of the elements and some numerical examples of the bounds.

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CONTRIBUTED ABSTRACT

Algebraic characterization of kaleidoscopical graphs

K. D. Protasova, T. M. Provotar

Let $\Gamma(V, E)$ be a connected graph with the set of vertices V and the set of edges E, d be the path metric on V, $B(v, r) = \{u \in V : d(v, u) \leq r\}, v \in V, r \in \omega = \{0, 1, ...\}$.

A graph $\Gamma(V, E)$ is called *kaleidoscopical* [4] if there exists a coloring (a surjective mapping) $\chi : V \longrightarrow \kappa$, κ is a cardinal, such that the restriction $\chi|B(v, 1) : B(v, 1) \longrightarrow \kappa$ is a bijection on each unit ball $B(v, 1), v \in V$. For kaleidoscopical graphs see also [2, Chapter 6] and [3].

Let G) be a group, X be a transitive G-space with the action $G \times X \longrightarrow X, (g, x) \longmapsto gx$. A subset A of X, $|A| = \kappa$ is said to be a *kaleidoscopical configuration* [1] if there exists a coloring $\chi : X \longrightarrow \kappa$ such that, for each $g \in G$, the restriction $\chi|gA : gA \longrightarrow \kappa$ is a bijection.

We note that kaleidoscopical graphs and kaleidoscopical configurations can be considered as partial cases of kaleidoscopical hypergraphs defined in [2, p.5]. Recall that a *hypergraph* is a pair $(X, \mathfrak{F})$ where X is a set, $\mathfrak{F}$ is a family of subsets of X.

A hypergraph $(X, \mathfrak{F})$ is said to be *kaleidoscopical* if there exists a coloring $\chi : X \longrightarrow \kappa$ such that, for each $F \in \mathfrak{F}$, the restriction $\chi \mid F : F \longrightarrow \kappa$ is a bijection.

Clearly, a graph $\Gamma(V, E)$ is kaleidoscopical if and only if the hypergraph $(V, \{B(v, 1) : v \in V\})$ is kaleidoscopical. A subset A of a G-space X is kaleidoscopical if and only if the hypergraph $(X, \{g(A) : g \in G\})$ is kaleidoscopical.

We say that two hypergraphs $(X_1, \mathfrak{F}_1)$, $(X_2, \mathfrak{F}_2)$ with kaleidoscopical colorings $\chi_1 : X_1 \longrightarrow \kappa$, $\chi_2 : X_2 \longrightarrow \kappa$ are *kaleidoscopically isomorphic* if there is a bijection $f : X_1 \longrightarrow X_2$ such that

- $\forall A \subseteq X_1 : A \in \mathfrak{F}_1 \Longleftrightarrow f(A) \in \mathfrak{F}_2$;
- $\forall x \in X_1 : \chi_1(x) = \chi_2(f(x))$.

We describe an algebraic construction which up to isomorphisms gives all kaleidoscopical graphs.

The *kaleidoscopical semigroup* $KS(\kappa)$ is a semigroup in the alphabet κ determined by the relations $xx = x$, $xyx = x$ for all $x, y \in \kappa$. For our purposes, it is convenient to identify $KS(\kappa)$ with the set of all non-empty words in κ with no factors xx, xyx where $x, y \in \kappa$.

For every $x \in \kappa$, the set $KG(\kappa, x)$ of all words from $KS(\kappa)$ with the first and the last letter x is a subgroup (with the identity x) of the semigroup $KS(\kappa)$. To obtain the inverse element to the word $w \in KG(\kappa, x)$ it suffices to write w in the inverse order. The group $KG(\kappa, x)$ is called the *kaleidoscopical group* in the alphabet κ with the identity x.

Theorem 1. *For any cardinal κ, the following statements hold:*

- *The only idempotents of the semigroup $KS(\kappa)$ are the words x, xy where $x, y \in \kappa$, $x \neq y$.*
- *The kaleidoscopical group $KG(k, x)$ is a free group with the set of free generators*

$$\{xyzx : y, z \in \kappa \backslash \{x\}, y \neq z\}.$$

- *The kaleidoscopical semigroup $KS(\kappa)$ is isomorphic to the sandwich product $L(x) \times KG(\kappa, x) \times R(x)$ with the multiplication*

$$(l_1, g_1, r_1)(l_2, g_2, r_2) = (l_1, g_1 r_1 l_2 g_2, r_2),$$

where $L(x) = \{yx : y \in \kappa\}$, $R(x) = \{xy : y \in \kappa\}$.

We fix $x \in \kappa$, denote by æ(w) the first letter of the word $w \in KS(\kappa)$ and say that an equivalence $\sim$ on $KS(\kappa)$ is *kaleidoscopical* if, for all $w, w' \in KS(\kappa)$ and $y \in \kappa$,

$$w \sim w' \Longrightarrow \text{æ}(w) = \text{æ}(w') \wedge yw = yw',$$

$$w \sim w' \Longleftrightarrow wx \sim w'x.$$

Theorem 2. *For every kaleidoscopical graph $\Gamma(V, E)$ with kaleidoscopical coloring $\chi : V \longrightarrow \kappa$, there exists a kaleidoscopical equivalence $\sim$ on the semigroup $KS(\kappa)$ such that $\Gamma(V, E)$ is kaleidoscopically isomorphic to $\Gamma(\kappa, \sim)$. Every kaleidoscopical equivalence $\sim$ on $KS(\kappa)$ is uniquely determined by some subgroup of the group $KG(\kappa, x)$.*

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CONTRIBUTED ABSTRACT

Algebras associated with labelled graphs

E. Pryanichnikova

The fundamental result in the finite automata theory is the Kleene's theorem, which states the equivalences between finite automata and regular languages. In this paper we introduce and study generally labelled graphs, directed graphs in which both vertices and transitions are labelled with elements of two arbitrary sets. As a main result of this paper we prove an analog of Kleene's theorem for these graphs and characterize their behaviour.

Let X and Y be two arbitrary sets. Generally labelled graph is a quintuple $\mathbf{G} = (G, I, F, \alpha, \beta)$ where $G = (V, E, s, t)$ is a multigraph with finite set of vertices V, set of transitions E and two functions $s, t : E \to V$, the source and target functions; I and F are subsets of V, called the set of initial and final states, respectively; $\alpha : V \to X$ is a mapping associating with each vertex its label; $\beta : E \to Y$ is a mapping associating with each transition its label.

To obtain a characterization of behavior of generally labelled graphs we introduce algebra $(Z, +, \cdot, *, o)$ with binary operations $+$ and $\cdot$, unary operation $*$ and constant o satisfying following properties.

1. $(Z, +, o)$ is a commutative idempotent monoid, $(Z, \cdot)$ is a semigroup, $\cdot$ distributes over $+$ on both sides, o is a two-sided annihilator for $\cdot$

2. Each sequence of elements of the set Z has a supremum with respect to the partial order $\leq$. Order $\leq$ refers to the natural order on Z: $a \leq b$ if an only if $a + b = b$, $a, b \in Z$.

3. For each sequence $\{x_n\}_{n \in N}$ of elements of the set Z an infinitary summation operator $\sum x_n$ gives the supremum of this sequence. Operator $\sum$ satisfied infinitary associativity, commutativity, idempotence and distributivity laws.

4. a^* is defined by supremum of the sequence $\{a, a \cdot a, a \cdot a \cdot a, ...\}$: $a^* = \sum_{n \geq 1} a^n$ where $a^1 = a$, $a^{n+1} = a^n \cdot a$, $n = 1, 2, ...$

Special case of an introduced algebra $(Z, +, \cdot, *, o)$ is an idempotent semiring.

An analog of Kleene's theorem for generally labelled graphs and algebra $(Z, +, \cdot, *, o)$ was proved. We define the conditions for the class of all possible behaviour of generally labelled graphs with given parameters to be representable by terms of some algebra. It has been shown that this algebra is not necessary unique.

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Contributed abstract

On the use of environment topology for self-localization of mobile agents

S. V. Sapunov

The problem of self-localization of a mobile agent (MA) in an environment modeled by a graph with labeled vertices is considered. This problem concerns the classical in theoretical cybernetics problematic of interaction of controlling and controlled systems [1, 2]. This problem is actual in connection with problems of navigation of autonomous mobile robots [3].

The finite simple graph $G = (V, E, M, \mu)$ with the finite set of vertices V, finite set of edges E, finite set of labels M and surjective labeling function $\mu : V \to M$ is called the labeled digraph. The sequence of vertices labels $\mu(g_1) \ldots \mu(g_k)$ corresponding to some path $g_1 \ldots g_k$ in graph G is called a word generated by vertex g_1. We will denote by L_g the set of all words generated by vertex $g \in V$. Two vertices $g, h \in V$ is said to be ε-equivalent iff $L_g = L_h$. The finite set of words $W_g \subseteq M^+$ is called linguistic identifier (LI) of vertex $g \in V$ if for any vertex $h \in V$ equality $W_g \cap L_g = W_g \cap L_h$ is fulfilled iff $g = h$. We will denote by S_g the subgraph of graph G generated by all vertices that are accessible from vertex $g \in V$. Two vertices $g, h \in V$ is said to be σ-equivalent iff $S_g \cong S_h$. It is demonstrated that $\sigma \subseteq \varepsilon$ and inverse inclusion is not fulfilled. The labeled graph D_g is called topological identifier (TI) of vertex $g \in V$ if for any vertex $h \in V$ isomorphism $D_g \cap S_g \cong D_g \cap S_h$ is fulfilled iff $g = h$. Polynomial construction methods for LI and TI of vertices are proposed. It is shown that the homomorphic image of the growing labeled tree corresponding to LI of vertex $g \in V$ is TI of this vertex and the converse is generally incorrect.

A concept of experiments with graphs, where a MA checks if given sets of words are a part of graph's language, is introduced. The experiment detecting the vertex where the MA started walking is called distinguishing. Construction and realization methods for distinguishing experiments with deterministic graphs based on checking the σ-equivalence of vertices are proposed. It is shown that time complexity of the proposed algorithms is polynomial from number of vertices of the graph.

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SHORT COMMUNICATION

Restoration of the mosaic structure by two agents

N. K. Shatokhina, P. A. Shatokhin

The problem of discovering the structure of a mosaic-like graph without holes is examined. The graph consists of equilateral triangles and it is explored by two automata (further called as agents). The first agent executes motions on the graph and transmit information to the second agent. The second describes structure of graph according to received information.

Each displacement of the first agent over the edges of the graph is characterized by certain direction of its motion. Designations to these directions are introduced; so any displacement of the agent over the edges generates the strings of the symbols, which correspond to the selected directions. The properties of the system of the displacement of the first agent are described.

The special cases of the mosaics of this form are determined, that are further called as base mosaics. The properties of the base mosaics are examined. Designations of the strings, which correspond to the routes of their clockwise and counterclockwise rounds, are given.

In the algorithm of the work of the first agent it is assumed that the agent randomly can be placed into any node of the graph; therefore algorithm can consist of two stages. In the first stage, if necessary, the agent reaches the boundary of the graph, and in the second stage the agent goes around graph counterclockwise on the boundary edges until it returns into that node, where it has first come out to the boundary.

Algorithm of the second agent analyzes the string of directions, that was generated and transmitted by the first agent. The string is examined from the end. At first is selected the sub-string, which describes a separate strongly-connected component. Then each substring is represented by the combination of the base mosaics designations.

It is shown that the agents will recognize any graph G of the considered type with an accuracy up to isomorphism. Estimation of the time and capacitive complexities of algorithms is also given.

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Section talk

On classification of distributive-like functional equations

Fedir M. Sokhatsky

This is continuation of [1]. A functional equation is said to be *distributive-like*, if it has three subject variables, one of which has three appearances, two others have two appearances each.

Full classification of distributive-like functional equations over quasigroup operations of a set up to parastrophic equivalency is given in the following statements (for the notions and notations see [2]).

Theorem 1. Any generalized distributive-like functional equation without squares is parastrophically equivalent to exactly one of the following five functional equations:

$$\begin{aligned}
F_1(x;F_2(y;z)) &= F_3(F_4(x;y);F_5(x;z));\\
F_1(y;F_2(x;z)) &= F_3(F_4(y;F_5(x;z));x);\\
F_1(F_2(x;y);y) &= F_3(x;F_4(F_5(x;z);z));\\
F_1(F_2(x;y);y) &= F_3(F_4(x;z);F_5(x;z));\\
F_1(y;F_2(x;z)) &= F_3(y;F_4(x;F_5(x;z))).
\end{aligned}$$

The first of the functional equations is called a *functional equation of generalized left distributivity. "To find its all solutions over quasigroup functions of a set"* is a well known open problem. A partial case of this problem has been solved by V.D. Belousov [3]. All other four equations have been solved in [4].

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SECTION TALK

Invertibility criterion for composition of two quasigroup operations

Fedir M. Sokhatsky, Iryna V. Fryz

Repetition-free composition of quasigroups is a quasigroup, but it is not true for repetition composition. Naturally there exists certain interest in finding an invertibility criterion: relationships under which composition of quasigroups is invertible. An invertibility criterion for repetition compositions of the same arity quasigroups follows from the results of V.D.Belousov[1], G.B.Belyavskaya [2], but it is unknown for different arities.

Definition 1. *Let g and h be quasigroup operations of the arities $n+1$ and $k+1$ respectively and υ be an arbitrary monotonically ascendant mapping from $\overline{0,k} := \{0,\dots,k\}$ to $\overline{0,n}$, where $k \leqslant n$. The terms $g(x_0,\dots,x_n)$ and $h(x_{\upsilon 0},\dots,x_{\upsilon k})$ when $x_j = a_j$ for all $j \neq \upsilon m, \upsilon p$, where $p \neq m$, define a pair of binary operations, which will be called υ-*respective $\{m;p\}$-retracts.

Definition 2. *Let g and h be operations of the arity $n+1$ and $k+1$ defined on Q. They are called* orthogonal of the type (m,υ), *if for all $p \in \overline{0,k}\backslash\{m\}$ their arbitrary pair of υ-respective $\{m;p\}$-retracts is orthogonal.*

Theorem 1. *Let υ be an arbitrary monotonically ascendant mapping of the set $\overline{0,k}$ into $\overline{0,n}$, where $k \leqslant n$ and g, h are invertible operations, and let*

$$f(x_0,\dots,x_n) = g(x_0,\dots,x_{\upsilon m-1}, h(x_{\upsilon 0},\dots,x_{\upsilon k}), x_{\upsilon m+1},\dots,x_n) \tag{1}$$

be fulfilled. Then the invertibility of operation f is equivalent to the orthogonality of the type (m,υ) of g and $h^{(m)}$.

Theorem 2. *Let f be defined by (1), and let*

$$g(x_0,\dots,x_n) = \alpha_0 x_0 + \dots + \alpha_n x_n + a, \quad h(x_{\upsilon 0},\dots,x_{\upsilon k}) := \beta_0 x_{\upsilon 0} + \dots + \beta_k x_{\upsilon k} + b,$$

where $(Q;+)$ is a group and $\alpha_0,\dots,\alpha_n,\beta_0,\dots,\beta_k$ are its automorphisms. The operation f is invertible if and only if for every $p \in \overline{0,k}\backslash\{m\}$ there exists $c \in Q$ such that $I_c\alpha_{\upsilon p} + \alpha_{\upsilon m}\beta_p$, when $p < m$, and $\alpha_{\upsilon m}\beta_p + I_c\alpha_{\upsilon p}$, when $p > m$, is a substitution of Q, where $I_c(x) := -c+x+c$.

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Contributed abstract

Algorithm of finite graph exploration by a collective of agents

A. Stepkin

Introduction. The problem of exploration of an environment giving by a finite graph is widely studied in the literature in various contexts [1]. An algorithm of exploration of unknown graph [2] by three agents is proposed. The first two agents-researchers (AR) traverse on unknown connected undirected graph $G = (V, E)$ without loops and multiple edges. They can read and change colors of graph elements and transfer information about their movements and colorings to the third agent-experimenter.

The aim of the paper is to create an algorithm of functioning of these agents that leads to recovering of the graph.

Functions of agents:

1. agent-researcher (agent with limited memory, which moving on graph):
   - perceives marks of all elements in the neighborhood of the node;
   - moves on graph from node v to node u by edge (v, u);
   - can change color of nodes, edges and incidentors;
2. agent-experimenter (stationary agent with unlimited growing internal memory):
   - conveys, receives, identifies messages from ARs;
   - builds a graph representation based on messages from ARs.

Conclusion. The new algorithm with $O(n^2)$ time and $O(n^2)$ space complexity that explores any finite undirected graph with n nodes is proposed. Each agent-researcher uses two different marks (in total three colors). The method is based on depth-first traversal method.

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SHORT COMMUNICATION

On reduction of exponent matrix

S. Tsiupii

A notion of an exponent matrix is arisen from Ring theory. Some rings are given by exponent matrices, for example, tiled orders are. Exponent matrices are applied also in encoding theory, by planning of many factor experiments etc. We give a sufficient condition for an exponent matrix to be reduced.

Let $M_n(Z)$ be the ring of square $n \times n$-matrices over the ring of integers.

Definition 1. *A matrix $\mathcal{E} = (\alpha_{ij})$ from the ring $M_n(Z)$ is called an exponent matrix if the following conditions hold:*

(i) $\alpha_{ii} = 0$ *for all* $i = 1, 2, \dots, n$;

(ii) $\alpha_{ik} + \alpha_{kj} \geq \alpha_{ij}$ *for all* $i, j, k = 1, 2, \dots, n$.

An exponent matrix $\mathcal{E} = (\alpha_{ij})$ is called *reduced* if $\alpha_{ij} + \alpha_{ji} > 0 \quad \forall i \neq j$.

Note that without loss of generality, we can assume that

$$\alpha_{ij} \geq 0 \quad \text{and} \quad \alpha_{1j} = 0 \quad \forall i, j = 1, 2, \dots, n. \tag{1}$$

Theorem 1. *Let an exponent matrix $\mathcal{E}$ satisfies the conditions (1). If elements in first column on $\mathcal{E}$ are different ($\alpha_{i1} \neq \alpha_{j1}$ if $i \neq j$), then $\mathcal{E}$ is reduced.*

Recall that the condition of this theorem is only sufficient. For example, the exponent matrix

$$\mathcal{E} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 2 \\ 3 & 3 & 0 & 2 \\ 3 & 3 & 0 & 0 \end{pmatrix}$$

is reduced but it does not satisfy the theorem condition.

In this paper we used [1].

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SHORT COMMUNICATION

Loops at quivers of exponent matrices

T. Tsiupii

A notion of an exponent matrix is arisen from Ring theory. Exponent matrices are used in applied task, for example, by planning multi factor experiments, by reliability test of different systems et cetera. Each exponent matrix can be associated with some graph called a quiver [1] and we can study such matrices by methods of Graph theory. We give necessary and sufficient conditions of existence a loop at a given vertex of a quiver of exponent $(0,1)$-matrix. We obtain also the number of loops at such quiver.

Let $M_n(Z)$ be the ring of square $n \times n$-matrices over the ring of integers.

Definition 1. *A matrix $\mathcal{E} = (\alpha_{ij})$ from the ring $M_n(Z)$ is called an exponent matrix if the following conditions hold:*

(i) $\alpha_{ii} = 0 \quad$ *for all* $\quad i = 1, 2, \ldots, n;$

(ii) $\alpha_{ik} + \alpha_{kj} \geq \alpha_{ij} \quad$ *for all* $\quad i, j, k = 1, 2, \ldots, n.$

An exponent matrix $\mathcal{E} = (\alpha_{ij})$ is called *reduced* if $\alpha_{ij} + \alpha_{ji} > 0 \quad \forall i \neq j$.

With a reduced exponent $(0,1)$-matrix $\mathcal{E}$ we associate the poset $P_{\mathcal{E}} = \{1, \ldots, n\}$ with the relation $\leq$ defined by the formula: $i \leq j \Leftrightarrow \alpha_{ij} = 0$.

Theorem 1. *Let $\mathcal{E} = (\alpha_{ij})$ be a reduced exponent $(0,1)$-matrix, $Q(\mathcal{E})$ is the quiver of $\mathcal{E}$, $P_{\mathcal{E}} = \{1, \ldots, n\}$ is the poset associated with the matrix $\mathcal{E}$, $Q(P)$ be a diagram of poset P. The quiver $Q(\mathcal{E})$ have a loop at a vertex v iff the corresponding vertex v of the diagram $Q(P)$ is disconnected point.*

The number of loops at the quiver $Q(\mathcal{E})$ is equals to the number of disconnected points of the diagram $Q(P)$.

Theorem 2. *For any integers m and n $(n \geq 2, 0 \leq m \leq n, m \neq n-2)$ there exists a reduced exponent $(0,1)$-matrix with the quiver on n vertices and with m loops.*

A graph on n vertices that has a loop at each vertex except for the one can't be a quiver of a reduced exponent $(0,1)$-matrix.

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CONTRIBUTED ABSTRACT

Linear inversive generator of PRN's

P. Varbanets, S. Varbanets

In the works of Eichenauer, Lehn, Topuzöglu, Niederreiter developed the method of generation of the sequences of pseudo-random numbers(PRN's) from interval $[0, 1)$ based on recursive congruences. We construct similar sequences of PRN's using the following congruential recursion

$$y_{n+1} \equiv ay_n^{-1} + b + cy_n (mod\ p^m), \tag{1}$$

The investigation of properties of the equidistribution and statistical independency of such sequences is carried by virtue of exponential sums on these sequences. In order to obtain the non-trivial estimates of exponential sums we get the next representation of the element of sequence of PRP's in the form of polynomial on initial value y_0 with coefficients depended on number of this element:

Let $\{y_n\}$ is the sequence of PRN's generated by the recursion (1) with conditions $(y_0, p) = (a, p) = 1$, $0 < \nu_p(b) < \nu_p(c)$. There exist the polynomials $F_0(u, v, w)$, $G_0(u, v, w)$ over $\mathbb{Z}$, $F_0(0, v, w) = G_0(0, v, w) = 0$ such that the relations

$$\begin{aligned} y_{2k} = kb + kacy_0^{-1} + (1 - k(k-1)a^{-1}b^2)y_0 + (-ka^{-1}b)y_0^2 + \\ + (-ka^{-1}c + k^2a^{-2}b^2)y_0^3 + p^\alpha F_0(k, y_0, y_0^{-1}), \end{aligned} \tag{2}$$

$$\begin{aligned} y_{2k+1} = (k+1)b + (a - k(k+1)b^2)y_0^{-1} + (-kab)y_0^{-2} + (-ka^2c + k^2ab^2)y_0^{-3} + \\ + (k+1)cy_0 + p^\alpha G_0(k, y_0, y_0^{-1}), \end{aligned} \tag{3}$$

are right for any $k \geq 2m + 1$, where $\alpha := \min(\nu_p(b^3), \nu_p(bc))$;

$F_0(u, v, w)$, $G_0(u, v, w) \in \mathbb{Z}[u, v, w]$, $F_0(0, v, w) = G_0(0, v, w) = 0$, holds.

The relations (2)-(3) allow to proof that the sequence of PRN's generated by (1) passes s-dimensional test on statistical independency for $s = 1, 2, 3, 4$.

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CONTRIBUTED ABSTRACT

Generator of pseudorandom complex numbers in unit circle

P. Varbanets, S. Zadorozhny

Let $0 \le \xi_1, \xi_2 \le 1$, $0 \le \varphi_1, \varphi_2 < 2\pi$. We denote

$$G(\xi, \varphi) := \{z \in \mathbb{C} : \quad \xi_1 < |z|^2 \le \xi_2; \quad \varphi_1 < argz \le \varphi_2\}$$

Let F be an assemble of domains $G(\xi, \varphi)$. For any sequence of complex numbers $\{z_n\}$, $|z_n| \le 1$ we define the discrepancy function $D_N(z_0, z_1, \ldots, z_{N-1}) := D_N$

$$D_N := \sup_{G(\xi,\varphi)\in F} \left| \frac{A_N(G)}{N} - |G| \right|$$

where A_N denotes the cardinality of the set $\{n : 0 \le n \le N-1, z_n \in G(\xi, \varphi)\}$, $|G|$ denotes the volume of G.

We prove the following inequality. Let M be a positive integer, $M > 1$, $y_n \in \mathbb{Z}_M[i]$, $n = 0, 1, 2, \ldots$. Then the discrepancy function $D_N(\frac{y_0}{M}, \frac{y_1}{M}, \ldots, \frac{y_{N-1}}{M})$ satisfies the inequality

$$D_N \le 2\left(1 - \left(1 - \frac{2\pi}{M}\right)^2\right) + \frac{\log^2 M}{M^{3/4}} \sum_{\substack{\alpha\in\mathbb{Z}_M[i] \\ \alpha\neq 0}} \frac{1}{N} \left| \sum_{n=0}^{N-1} \exp\left(2\pi i Re\left(\frac{\alpha y_n}{M}\right)\right) \right|$$

This estimation is applied for the investigation of sequence of pseudorandom numbers generated by the congruential recursive relation

$$y_{n+1} \equiv \alpha y_n^{-1} + \beta \pmod{\mathfrak{p}^m}$$

where $y_n \cdot y_n^{-1} \equiv 1 \pmod{\mathfrak{p}^m}$, $\alpha, \beta \in \mathbb{Z}[i]$, $\mathfrak{p}$ is a Gaussian prime number.

The inversive generator of pseudorandom real numbers has been studied in works of Lehn, Eichenauer, Topozuglo, Neiderriter and others (for example see [1])

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8th International Algebraic Conference
July 5—12 **(2011)**, Lugansk, Ukraine CONTRIBUTED ABSTRACT

Inversive congruential generator over Galois ring

O. Vernygora

P. Sole and D. Zinoviev [1] considered inversive congruential generator over Galois ring of characteristic 2^l. This work generalises inversive congruential generator studied earlier in I. Eichenauer's [2] and H. Niederreiter's [3] works. Interest to inversive generators over Galois ring of characteristic 2^l is called by applications in the coding and cryptography theory.

In this work we consider the inversive congruential generator over Galois ring of characteristic p^l, where p> 2 prime number, and we show, that the corresponding sequence of pseudorandom numbers passes the two-dimensional serial test for statistical independence (unpredictability).

Let g (x) - fundamentally primitive polynomial of degree m from ring $Z_{p^l}[x]$. Through R (l, m) we designate a quotient ring which is Galois ring of characteristic p^l. Let ξ - a primal element in R (l, m), which generates set $\Xi \subset R(l,m), \Xi = \left\{0, 1, \xi, ..., \xi^{p^m-2}\right\}$.

For an element $x \in R(l,m)$ exists two representations:
(1) $x = x^{(0)} + px^{(1)} + p^2x^{(2)} + ... + p^{l-1}x^{(l-1)}$, $x^{(i)} \in \Xi$, i=0,...,l-1
(2) $x = a_0 + a_1\xi + ... + a_{m-1}\xi^{m-1}$, $a_i \in Z_{p^l}$.
Let's define function $\Phi : R(l,m) \to R(l,m)$ so:

$$\Phi(x) = \begin{cases} \varphi(x), & \text{если } x \notin B; \\ \pi(x'), & \text{если } x \in B \text{ и } x \in Orb(x'). \end{cases}$$

Let's consider the sequence of elements defined by a recursion:
(3) $x_{n+1} = \Phi(x_n) = \Phi^{m+1}(x_0), n = 0, 1, ...$
The period of this sequence is equal p^{ml}.
Using representation x_n in the form (2), we will define normalised map $R(l,m) \to [0,1)$:
$R \ni y = a_0 + a_1\xi + ... + a_{m-1}\xi^{m-1} \to \eta(y) = \frac{a_0+a_1p+...+a_{m-1}p^{m-1}}{p^{ml}} \in [0,1)$.

Sequence x_n of period p^{ml} generates sequence $\eta(x_n) \in [0,1)$, which we name inversive congruential sequence of pseudorandom numbers. It is in regular intervals distributed on $[0,1)$ and unpredictable.

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TOPICAL SECTION V

GROUPS AND ALGEBRAIC DYNAMICS

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CONTRIBUTED ABSTRACT

On maximal chains of length 3 in finite groups

D. Andreeva, A. Skiba

All groups are finite. A chain $E_n < E_{n-1} < \ldots < E_1 < E_0 = G$, where E_i is maximal in E_{i-1}, $i = 1, 2, \ldots, n$ is called a maximal chain of G of lenght n. New interesting results about second and third maximal subgroups were obtained last years. In recent work [1] Li Shirong gave a classification of non-nilpotent groups all of whose second maximal subgroups are TI-subgroups. Also in the work [2] the description of non-nilpotent groups in which every 2-maximal subgroup permutes with all 3-maximal subgroups is obtained. Bearing in mind the above results L.A. Shemetkov posed the following question: what can we say about the structure of a finite group G in which every maximal chain of length 3 contains a proper subnormal in G subgroup? The following theorem gives the answer for this question.

Theorem 1. *Let G be a group. Every maximal chain of length 3 of G contains a proper subnormal in G subgroup if and only if G is nilpotent, $|G| = p^\alpha q^\beta r^\gamma$, where $\alpha + \beta + \gamma \leq 3$, or G is a group one of the following types:*

I. $G = P \rtimes Q$, where $P = G^{\mathfrak{N}}$, and either G is a Schmidt group, $|\Phi(P)| \leq p$ and $|Q : Q_G| = q$ or P is a minimal normal subgroup of G and one of the statements hold:

(1) $|Q : Q_G| = q^2$.

(2) $|Q : Q_G| = q$ and any maximal subgroup of Q different from Q_G is cyclic.

II. $G = P \rtimes (QR)$, where $P = G^{\mathfrak{N}}$ is a minimal normal subgroup of G, $|R| = r$, Q is cyclic, $|Q : Q_G| = q$ and either R is normal in G or $|Q| = q$.

III. $G = P \rtimes Q = PA$, $|\Phi(P)| = p$, $A = \Phi(P) \rtimes Q$ is a representative of a unique class of non-normal maximal subgroups of G, A is a Schmidt group and $|Q : Q_G| = q$.

IV. $\Phi(P) = 1$, G is the subdirect product of the non-normal maximal subgroups A and B, where $A = A_p \rtimes Q$ is a Schmidt group with abelian Sylow subgroups and A_p is a minimal normal subgroup of G; $B = B_p \rtimes Q$ and either B is nilpotent, $|B_p| = p$ or $B \simeq A$.

V. $G = P \rtimes (QR)$ and G has only three classes of maximal subgroups representatives of which are a non-normal Hall r'-subgroup A, a non-normal Hall p'-subgroup L and a normal in G subgroup M such that $|G : M| = q$. Moreover, the following hold:

(1) A is a Schmidt group with abelian Sylow subgroups and $|Q : Q_G| = q$.

(2) L is either a Schmidt group with abelian Sylow subgroups or a nilpotent group.

(3) P is a minimal normal subgroup of G, $|R| = r$ and either R is normal in G or *$|Q| = q$.*

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8th International Algebraic Conference
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SHORT COMMUNICATION

Properties of some selfsimilar groups generated by slowmoving automata transformations

A. Antonenko, E. Berkovich

Consider groups $GSlC_2(k) = \langle \alpha_0, \alpha_1, \alpha_2, \ldots, \alpha_{k-1} \rangle$, where $\alpha_0 = (\alpha_0, \alpha_0)\,\sigma$, $\alpha_i = (\alpha_{i-1}, \alpha_i)$, $i = \overline{1, k-1}$ are transformations of 2-symbol alphabet $X = \{0, 1\}$. This group appeared during the studying of slowmoving automata transformations [1],[2]. It is easy to see that the order of all the generators α_i is equal to 2 and $GSlC_2(2) = \langle \alpha_0, \alpha_1 | \alpha_0^2 = \alpha_1^2 = 1 \rangle$. $GSlC_2(k)$ for $k > 2$ has a more complex structure (there are other relations except $\alpha_i^2 = 1$). We prove that $\{\alpha_0, \alpha_1, \alpha_2, \ldots, \alpha_{k-1}\}$ is a minimal generating set and $GSlC_2(k)$ are fractal for any $k \geq 2$. If a word over $\{\alpha_i | i = \overline{0, k-1}\}$ is a relator of $GSlC_2(k)$ than the number of α_i in it for any given i is even. The set of all the words in which the number of α_i for any given i is even is the commutant of $GSlC_2(k)$.

We have found the algorithm of constructing an infinite sequence of relators of $GSlC_2(3)$, which have different levels. This allows us to prove that $GSlC_2(3)$ has no finite presentation.

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CONTRIBUTED ABSTRACT

On free subgroups in some generalized tetrahedron groups

V. V. Beniash-Kryvets, Y. A. Zhukovets

A generalized tetrahedron group is a group with the following presentation:

$$\Gamma(l,m,n,p,q,r) = \langle a,b,c \mid a^l = b^m = c^n = R_1(a,b)^p = R_2(a,c)^q = R_3(b,c)^r \rangle,$$

where each $R_i(x,y)$ is a cyclically reduced word involving both x and y and all powers are integers greater than 1. These groups appear in many algebraic and geometric questions, for example, as subgroups of generalized triangle groups and as fundamental groups of certain orbifolds. A group G is said to satisfy the Tits alternative if G either contains a non-abelian free subgroup of rank 2 or is virtually soluble. This property is named after J. Tits, who established [1] that it is satisfied by the class of linear groups. In particular, every ordinary tetrahedron or triangle group is linear, and so satisfies the Tits alternative. There is a conjecture that the class of generalized tetrahedron groups satisfies the Tits alternative [2, 3]. There are some sufficient conditions for generalized tetrahedron groups to contain a free subgroup (see [2, 4]). In [5] this conjecture has been proved for the class of generalized tetrahedron groups realized by non-spherical triangle of groups. But in following cases the problem is open: 1) $G_1 = \langle a,b,c \mid a^l = b^m = c^n = (a^\alpha b^\beta)^2 = (b^\gamma c^\delta)^2 = R(a,c)^r\rangle$, where $\frac{1}{l}+\frac{1}{m}+\frac{1}{n}+\frac{1}{r} \ge 1$; 2) $G_2 = \langle a,b,c \mid a^l = b^m = c^n = (a^\alpha b^\beta)^2 = (b^\gamma c^\delta)^3 = (a^\eta c^\varphi)^r\rangle$, where $r = 3,4,5$ and $\frac{1}{l}+\frac{1}{m}+\frac{1}{n}+\frac{1}{r} \ge \frac{7}{6}$; 3) $G_3 = \langle a,b,c \mid a^l = b^m = c^n = (a^\alpha b^\beta)^2(b^\gamma c^\delta)^3 = (a^\eta c^\varphi a^{\eta_1} c^{\varphi_1})^2\rangle$, where $\frac{1}{l}+\frac{1}{m}+\frac{1}{n} \ge \frac{2}{3}$. We prove the following theorem.

Theorem 1. *1. Let* $\Gamma_1 = \langle a,b,c \mid a^3 = b^8 = c^2 = R(a,b)^2 = (ac)^2 = (bc)^2 = 1\rangle$, *where* $R(a,b) = a^{u_1}b^{v_1}\dots a^{u_s}b^{v_s}$, $1 \le u_i \le 2$, $1 \le v_i \le 7$, *and either s is odd or $s \le 6$. Then Γ_1 contains a non-abelian free subgroup.*

2. Let $\Gamma_2 = \langle a,b,c \mid a^4 = b^8 = c^2 = R(a,b)^2 = (ac)^2 = (bc)^2 = 1\rangle$, *where* $R(a,b) = a^{u_1}b^{v_1}\dots a^{u_s}b^{v_s}$, $1 \le u_i \le 3$, $1 \le v_i \le 7$, $U = \sum_{i=1}^s u_i$, $V = \sum_{i=1}^s v_i$. *If either U is odd or* $V \not\equiv 4 (\bmod 8)$, *then Γ_2 contains a non-abelian free subgroup.*

In both cases Γ_1 and Γ_2 satisfy the Tits alternative.

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CONTRIBUTED ABSTRACT

Growth of Schreier graphs of automaton groups

I. V. Bondarenko

Let G be a group with a generating set S and acting on a set X. The (simplicial) Schreier graph $\Gamma(G, S, X)$ of the action (G, X) is the graph with the set of vertices X, where two vertices x and y are adjacent if and only if there exists $s \in S \cup S^{-1}$ such that $s(x) = y$. Every automaton group G generated by an automaton A over some alphabet X naturally acts on the space X^ω of right-infinite sequences over X. We get the uncountable family of orbital Schreier graphs $\Gamma_w(G, \mathsf{A})$ for $w \in X^\omega$ of the action of G on the orbit of w. The Schreier graphs Γ_w provide a large source of self-similar graphs and were studied in relation to such topics as spectrum, growth, amenability, topology of Julia sets, etc.

An important class of automaton groups are generated by polynomial automata. These automata were introduced by S. Sidki in [2], who tried to classify automaton groups by the cyclic structure of the generating automaton. A finite invertible automaton is called polynomial if the simple directed cycles away from the trivial state are disjoint. The term "polynomial" comes from the equivalent definition, where a finite automaton is polynomial if the number of paths of length n avoiding the trivial state in the automaton grows polynomially in n. In [1] V. Nekrashevych proved that the orbital Schreier graphs Γ_w of groups generated by polynomial automata are amenable and conjectured that these graphs have subexponential growth. We prove this conjecture.

Theorem 1. *Let G be a group generated by a polynomial automaton of degree m. There exists a constant A such that all orbital Schreier graphs $\Gamma_w(G)$ for $w \in X^\omega$ have subexponential growth not greater than $A^{(\log n)^{m+1}}$.*

The upper bound in the theorem is asymptotically optimal; for every $m \in \mathbb{N}$ we construct a polynomial automaton of degree m, whose all orbital Schreier graphs Γ_w for $w \in X^\omega$ have growth not less than $B^{(\log n)^{m+1}}$ for some constant $B > 1$.

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CONTRIBUTED ABSTRACT

2-groups with three involutions and locally cyclic commutant

I. Chernenko

The groups with three involutions are investigating for many years.

Problem of description metacyclic 2-groups with three involution solved in [1].

In [2], [3] there are complete description the structure of finite 2-groups with three involutions and central commutant.

In our report the theorem 1 is devoted to description of the non-metacyclic 2-groups with three involutions and locally cyclic commutant.

Theorem 1. *Non-metacyclic 2-group with three involutions and locally cyclic commutant are group of one of types:*

1) $G = B \leftthreetimes < x >, \gamma > 1, x^{-1}ax = a^{1+f\cdot 2^k}, w(< x >) \subset Z(G), |[b, x]| \le 2^{min(\alpha,\gamma-1)}$;
2) $G = B \leftthreetimes < x >, \gamma > 1, x^{-1}ax = a^{-(1+f\cdot 2^k)}, \gamma \ge \alpha - k, B \bigcap < x >= w(< a >) = w(< x >)$;
3) $G = B\cdot < x >, a \ge \gamma > 2, x^{-1}ax = a^{-(1+f\cdot 2^k)}, \gamma > \alpha - k, B \bigcap < x >= w(< a >) = w(< x >)$;
4) $G = ((C \times A)\cdot < b >)\cdot < x >, C$ - *quasicyclic 2-group,* A - *non-single locally cyclic 2-group,* $\beta = \gamma = 0$;
5) $G = B \times X, C = X =< x >$ - *non-single locally cyclic 2-group;*
6) $G = B \times X, B \cong Q_\infty, X$ - *non-single locally cyclic 2-group;*
7) $G = B = A\cdot < b >, A$ - *quasicyclic 2-group,* $\beta > 1, [G : A] > 2, \gamma = 0$;
8) $G = (< x > \times A)\cdot < b >, A$ - *quasicyclic 2-group,* $\gamma > 1, w(< x >) \subset Z(G), [b, x] = c \in A, |c| = 2^\sigma, 0 < \sigma < \gamma$.

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CONTRIBUTED ABSTRACT

On some numerical characteristics of permutability subgroups of finite groups

V. A. Chupordya

Last years the interest to researches in the theory of the finite groups connected with finding the probability of this or that fact is increased. The finding such probability was one of the most prominent aspects which have been studied, that two elements of finite group are permutable in given group G. This size is designated as $\mathbf{cp}(G)$ and for it the following statement is known: If G is a nonabelian finite group with $\mathbf{k}(G)$ conjugacy classes, then $\mathbf{cp}(G) = \mathbf{k}(G)/|G| \leq 5/8$. Elementary extensions of this result can be found in [2], [3] and [4]. In [1] it was shown that $\mathbf{cp}(G) \to 0$ as either index or the derived length of the fitting subgroup of G tends to infinity. For some classes of finite groups explicit formulas for a finding $\mathbf{cp}(G)$ have been found.

In [5] have been studied the probability that two subgroups of a finite group G commute. Let G be a finite group and by $\mathfrak{L}(G)$ we denote the subgroup lattice of G, If H and K are the subgroup of G then equality $HK = KH$ is equivalent to $HK \in \mathfrak{L}(G)$. In [5] it was considered the quantity $\mathbf{sd}(G) = \frac{1}{|\mathfrak{L}(G)|^2}\{(H,K) \in \mathfrak{L}(G)^2 | HK = KH\}$, which was called as subgroup commutativity degree of G. It is natural to consider the probability that subgroup of a finite group G is permutable. Recall that subgroup H of G is permutable, if it is permute with every subgroup of G. Let G be a finite group then consider the quantity $\mathbf{sp}(G) = \frac{1}{|\mathfrak{L}(G)|}\{H \in \mathfrak{L}(G) | HK = KH \text{ for all } K \in \mathfrak{L}(G)\}$ which will be called as subgroup permutability degree of G.

It was proved that $\mathbf{sp}(G) \leq \mathbf{sd}(G)$ for any finite group G. By means of computer algebra system GAP the values of $\mathbf{sp}(G)$ and $\mathbf{sd}(G)$ for some symmetric and alternating groups have been received as well as for dihedral groups and several matrix groups. For finite group G with upper central series $< 1 >= G_0 \leq G_1 \leq ... \leq G_k = G$, where $G_{i+1}/G_i \leq Z(G/G_i)$ it was obtained the inequality such as $\mathbf{sp}(G) \geq \frac{1}{|\mathfrak{L}(G)|}(|\mathfrak{L}(G_1)| + \sum_{i=1}^{k-1} |\mathfrak{L}(G_{i+1}/G_i)| - k + 1)$. It was obtained also the explicit formulas to find the values $\mathbf{sp}(D_{2^m})$ and $\mathbf{sp}(Q_{2^m})$, where D_{2^m} - dihedral group, Q_{2^m} - generalized quaternion group.

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Plenary talk

On modules over group rings of soluble groups with the condition $max - nnd$

O. Yu. Dashkova

Let A be a vector space over a field F. The subgroups of the group $GL(F, A)$ of all automorphisms of A are called linear groups. If A has a finite dimension over F then $GL(F, A)$ can be identified with the group of non-singular $n \times n$-matrices, where $n = dim_F A$. Finite dimensional linear groups have played an important role in mathematics and have been well-studied. When A is infinite dimensional over F, the situation is totally different. The study of infinite dimensional linear groups requires some additional restrictions. In [1] it was introduced the definition of a central dimension of an infinite dimensional linear group. Let H be a subgroup of $GL(F, A)$. H acts on the quotient space $A/C_A(H)$ in a natural way. The authors define $dim_F H$ to be $dim_F(A/C_A(H))$. The subgroup H is said to have a finite central dimension if $dim_F H$ is finite and H has an infinite central dimension otherwise.

If $G \leq GL(F, A)$ then A can be considered as a FG-module. The natural generalization of this case is a consideration of the case when A is an $\mathbf{R}G$-module, $\mathbf{R}$ is a ring, the structure of which is similar to a structure of a field. The generalization of the notion of a central dimension of a linear group is the notion of a cocentralizer of a subgroup. This notion was introduced in [2]. Let A be an $\mathbf{R}G$-module where $\mathbf{R}$ is a ring, G is a group. If $H \leq G$ then the quotient module $A/C_A(H)$ considered as an $\mathbf{R}$-module is called the cocentralizer of H in the module A.

The investigation of algebraic systems satisfying the maximal condition still remains very actual. The example of such system is the class of Noetherian modules. Remind that a module is called a Noetherian module if an ordered set of all submodules of this module satisfies the maximal condition. It should be noted that many problems of Algebra require the investigation of some specific Noetherian modules over group rings as well modules over group rings which are not Noetherian but which are similar to Noetherian modules in some sense.

It is considered an $\mathbf{R}G$-module A such that the cocentralizer of a group G in the module A is not a Noetherian $\mathbf{R}$-module. Let $L_{nnd}(G)$ be a system of all subgroups of a group G for which the cocentralizers in the module A are not Noetherian $\mathbf{R}$-modules. Introduce on $L_{nnd}(G)$ the order with respect to the usual inclusion of subgroups. G is said to satisfy the condition $max - nnd$ if $L_{nnd}(G)$ satisfies the maximal condition as an ordered set. The author studies soluble groups with the condition $max - nnd$ and generalize some results on soluble infinite dimensional linear groups [3]. It is considered the case where $\mathbf{R}$ is an integral domain. The analogous problem for $\mathbf{R}G$-module A where $\mathbf{R}$ is a ring of integers it was investigated in [4].

Later on it is considered an $\mathbf{R}G$-module A such that $C_G(A) = 1$, $\mathbf{R}$ is an integral domain.

The main results of this work are the theorems.

Theorem 1. *Let A be an $\mathbf{R}G$-module and suppose that G is a soluble group satisfying the condition $max - nnd$. If the quotient group $G/[G, G]$ is not finitely generated then G satisfies the following conditions:*

(1) *A has the finite series of $\mathbf{R}G$-submodules $< 0 >= C_0 \le C_1 \le C_2 = A$, such that C_2/C_1 is a finitely generated $\mathbf{R}$-module and the quotient group $Q = G/C_G(C_1)$ is a Prüfer q-group for some prime q;*

(2) *$H = C_G(C_1) \cap C_G(C_2/C_1)$ is an abelian normal subgroup of G;*

(3) *a group G has the series of normal subgroups $H \le L \le N \le M \le G$ such that the quotient group G/M is finite, the quotient group M/N is a Prüfer q-group for some prime q, the quotient group N/L is finitely generated, the quotient group L/H is nilpotent, H is an abelian subgroup, the quotient group M/L is abelian and the cocentralizer of the subgroup N in the module A is a Noetherian $\mathbf{R}$-module.*

Let $ND(G)$ be a set of all elements $x \in G$, such that the cocentralizer of a group $< x >$ in the module A is a Noetherian $\mathbf{R}$-module. Then $ND(G)$ is a normal subgroup of G.

Theorem 2. *Let A be an $\mathbf{R}G$-module and suppose that G is a finitely generated soluble group satisfying the condition $max - nnd$. If the cocentralizer of the subgroup $ND(G)$ in the module A is a Noetherian $\mathbf{R}$-module then G has the series of normal subgroups $H \le L \le G$ such that the quotient group G/L is polycyclic, the quotient group L/H is nilpotent and H is an abelian subgroup.*

Theorem 3. *Let A be an $\mathbf{R}G$-module and suppose that G is a finitely generated soluble group satisfying the condition $max - nnd$. If the cocentralizer of the subgroup $ND(G)$ in the module A is not a Noetherian $\mathbf{R}$-module then G contains the normal subgroup L satisfying the following conditions:*

(1) *The quotient group G/L is polycyclic.*

(2) *$L \le ND(G)$ and the cocentralizer of the subgroup L in the module A is not a Noetherian $\mathbf{R}$-module.*

(3) *The quotient group $L/[L, L]$ is not finitely generated.*

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CONTRIBUTED ABSTRACT

Ω_1-foliated τ-closed formations of T-groups

E. N. Demina

An additive group G with zero 0 is called a multioperator T-group with the system of multioperators T (or a T-group for short) whenever we are also given some system T of n-ary algebraic operations on G for some $n > 0$, while $t(0, \dots, 0) = 0$ for all $t \in T$, where 0 appears on the left n times if t is an n-ary operation (see [1]; [2, ch. III]; [3, ch. VI, p. 356]). Suppose $\mathfrak{M}$ is a class of all T-groups satisfying minimality and maximality conditions for T-subgroups, $\mathfrak{I}_1$ is a class of all simple $\mathfrak{M}$-groups, Ω_1 is a nonempty subclass of $\mathfrak{I}_1$, $\Omega_1' = \mathfrak{I}_1 \backslash \Omega_1$, and $\mathfrak{K}(G)$ is the class of all simple $\mathfrak{M}$-groups isomorphic to the compositions factors of a T-group G. If $\mathfrak{K}(G) \subseteq \Omega_1$ then G is called an Ω_1-group. Let $\mathfrak{M}_{\Omega_1}$ stand for the class of all Ω_1-groups belonging to $\mathfrak{M}$, and $O_{\Omega_1}(G) = G_{\mathfrak{M}_{\Omega_1}}$. Assume henceforth that all T-groups under consideration belong to $\mathfrak{M}$. A function $f \colon \Omega_1 \cup \{\Omega_1'\} \to \{$formations of T-groups$\}$ is called an $\Omega_1 F$-function; a function $\varphi \colon \mathfrak{I}_1 \to \{$nonempty Fitting formations of T-groups$\}$ is called an FR-function. A formation $\Omega_1 F(f, \varphi) = (G \in \mathfrak{M} \colon G/O_{\Omega_1}(G) \in f(\Omega_1')$ and $G/G_{\varphi(A)} \in f(A)$ for all $A \in \Omega_1 \cap \mathfrak{K}(G))$ is called an Ω_1-foliated formation of T-groups with Ω_1-satellite f and direction φ. By φ_0 denote a direction φ such that $\varphi(A) = \mathfrak{M}_{A'}$ for every $A \in \mathfrak{I}_1$.

Definition. *Let $\mathfrak{X}$ be a nonempty class of T-groups and for every T-group $G \in \mathfrak{X}$ some system of its T-subgroups $\tau(G)$ is compared. Following [4] we say that τ is a subgroup $\mathfrak{X}$-functor if the following conditions hold: 1) $G \in \tau(G)$ for any T-group $G \in \mathfrak{X}$; 2) for any epimorphism $\psi : A \to B$, where $A, B \in \mathfrak{X}$, and for any T-groups $H \in \tau(A)$ and $K \in \tau(B)$ the inclusions $H^{\psi} \in \tau(B)$ and $K^{\psi^{-1}} \in \tau(A)$ are executed.*

A formation of T-groups $\mathfrak{F}$ is said to be τ-closed if $\tau(G) \subseteq \mathfrak{F}$ for any T-group $G \in \mathfrak{F}$. Let $\tau\Omega_1 F_n^{\varphi}$ be a set of all n-fold Ω_1-foliated τ-closed $\mathfrak{M}$-formations. By $\tau\Omega_1 F_n(\mathfrak{X}, \varphi)$ denote an $\tau\Omega_1 F_n^{\varphi}$-formation generated by a nonempty set of T-groups $\mathfrak{X}$.

Theorem. *Let $\mathfrak{X}$ be a nonempty $\mathfrak{M}$-class. Then n-fold Ω_1-foliated τ-closed $\mathfrak{M}$-formation $\mathfrak{F} = \tau\Omega_1 F_n(\mathfrak{X}, \varphi)$, where $n \in \mathbf{N}$ and $\varphi_0 \leq \varphi$, has the unique minimal $\tau\Omega_1 F_{n-1}^{\varphi}$-satellite f such that $f(\Omega_1') = \tau\Omega_1 F_{n-1}((G/O_{\Omega_1}(G) : G \in \mathfrak{X}), \varphi)$, $f(A) = \tau\Omega_1 F_{n-1}((G/G_{\varphi(A)} : G \in \mathfrak{X}), \varphi)$ for all $A \in \Omega_1 \cap \mathfrak{K}(\mathfrak{X})$, and $f(A) = \varnothing$ if $A \in \Omega_1 \backslash \mathfrak{K}(\mathfrak{X})$.*

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CONTRIBUTED ABSTRACT

Unitriangle automorphism of the ring of polynomials of two variables above the field of characteristic p

Zh. Dovghei

Let K is fixed field, $AutK[x, y]$ is group of automorphisms of the ring of polynomials $K[x, y]$ of variables x, y above the field K. Arbitrary automorphism is simply determined by images of elements $x, y \in K[x, y]$, that by the pair of polynomials $u = \langle a(x, y), b(x, y)\rangle$, which must be such, that a reflection $x \mapsto a(x, y), y \mapsto b(x, y)$ proceeded to bijection $K[x, y]$ for itself, thus a reverse reflection also must be set the same way. In a group of the automorphisms of ring $K[x, y]$ of polynomials are selected two standard sub-groups: 1) subgroup $Aff_2(K)$ of affine automorphisms - transformations of kind $x \mapsto ax+by+\alpha, y \mapsto cx + dy + \beta$. Her $\begin{vmatrix} a & b \\ c & d \end{vmatrix} \neq 0$; 2) subgroup $J_2(K)$ of triangle automorphisms (which is yet named an affine Jonkier group or sub-group of elementary automorphisms $K[x, y]$)— transformations of kind $x \mapsto ax+b, y \mapsto cy+d(x)$. Here $a, b, c \in K, a \neq 0, c \neq 0, d(x) \in K[x]$.

Crossing of subgroups $GL_2(K)$ i $K[x, y]$ consists of linear triangle automorphisms $x \mapsto ax + b, y \mapsto cx + dy$. The group of such automorphisms is standard denoted by the symbol $T_2(K)$. Jonkiera group $J_2(K)$ contains the subgroup of triangle automorphisms $UJ_2(K)$, that to the automorphisms kind (4), for which $a = 1, c = 1$.

Group $UJ_2(K)$ is semidirect product of subgroup of transformations of kind $[a, 0], a \in K$ and normal divisor $[0, a(x)], a(x) \in K(x)$. Each of these sub-groups in the case of the field of characteristic of p is an elementary abelian p—group. If field is finite, the first from them is finite, and second - always infinite.

Theorem 1. *Group $AutK[x, y]$ laid out in free product of its sub-groups $Aff_2(K)$ i $J_2(K)$ above the united subgroup of $T_2(K)$.*

Lemma 1. *A group $UJ_2(K)$ contains exponent p^2.*

Theorem 2. *The center of group $UJ_2(K)$ consists of transformations of kind $\omega = [0, a], a \in K$. The sub-group of p—degrees of group $UJ_2(K)$ coincides with the center of this group.*

Theorem 3. *A group $UJ_2(K)$ Engel group of lengths $p + 1$, that equality $\varepsilon(UJ_2(K)) = p + 1$ takes place.*

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CONTRIBUTED ABSTRACT

Non-periodic groups without free Abelian subgroup of rank 2 with non-Dedekind norm of Abelian non-cyclic subgroups

M. G. Drushlyak, F. M. Lyman

The notion of Σ-norm extends the variety of characteristic subgroups, which are studied in a group. Σ-norm of a group is the intersection of normalizers of all subgroups of system Σ, which have some theoretical-group properties. The study of Σ-norm is interesting from different points of view, for example, the structure of Σ-norm, the nature of its including in a group and the influence of its structure on whole group.

When a system of Abelian non-cyclic subgroups is taking as a system Σ, we get a norm N_G^A of Abelian non-cyclic subgroups. In this article authors continue studing of non-periodic groups with non-Dedekind norm N_G^A of Abelian non-cyclic subgroups.

If $G = N_G^A$, then all Abelian non-cyclic subgroups are normal in the group G. Non-periodic groups of such type were studied in [1].

Theorem 1. *Let G be non-periodic locally soluble group and its norm N_G^A of Abelian non-cyclic subgroups is non-Dedekind mixed, is finite extension of normal Abelian torsion-free subgroup of rank 1 and isn‘t Chernikov IH-group [2]. Then G is finite extension of normal Abelian torsion-free subgroup of rank 1, which centralizer contains all elements of infinite order of the group G and is the product of cyclic p-group or quaternion group of order 8 and Abelian torsion-free subgroup of rank 1. More over, if the norm N_G^A doesn't have Abelian non-cyclic torsion-free subgroups of rank 1, then the group G doesn't have such subgroups in this case.*

Theorem 2. *Let G be non-periodic locally soluble group and its norm N_G^A of Abelian non-cyclic subgroups is non-Abelian torsion-free group. Then $G = N_G^A$.*

Collorary 3. *If G is non-periodic locally soluble group and its norm N_G^A of Abelian non-cyclic subgroups is non-Dedekind group and isn‘t Chernikov IH-group, then $1 \leq [G : N_G^A] < \infty$.*

Next examples show that the condition of non-Dedekindness of the norm N_G^A is important in collorary 3.

Example 1. $G = (\langle a \rangle \leftthreetimes \langle b \rangle) \times C$, where $|a| = \infty$, $|b| = 2$, $b^{-1}ab = a^{-1}$, C is a direct product of infinitely many isomorphic cyclic groups of order 2. In this group $N_G^A = C$ and $[G : N_G^A] = \infty$.

Example 2. $G = (\langle a \rangle \leftthreetimes \langle b \rangle) \times H$, where $|a| = \infty$, $|b| = 2$, $b^{-1}ab = a^{-1}$, H is Hamiltonian group. In this group $N_G^A = H$ and $[G : N_G^A] = \infty$.

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CONTRIBUTED ABSTRACT

Dualities and equivalences in the abelian group theory

Alexander A. Fomin

Dualities and equivalences of categories are important for the abelian group theory. A. G. Kurosh described p-primitive groups by p-adic matrices in 1937. E. Lee Lady generalized his theorem on torsion-free finite-rank modules over discrete valuation rings. Moreover, he noted that the Kurosh's result is an equivalence of two categories.

In 1938 A. I. Maltsev and D. Derry obtained simultaneously and independently two different descriptions of torsion-free finite-rank abelian groups using sequences of p-adic matrices over all prime numbers p.

It is shown in [1,2] that the description by Maltsev is a duality of two categories. Objects of the first category are torsion-free finite-rank groups. Objects of the second category are matrices of special form introduced by Maltsev (perfect matrices in his terminology).

The p-primitive groups by Kurosh are also in particular quotient divisible. It is shown in [2] that the Kurosh theorem admits a generalization on the class of all mixed quotient divisible groups. Moreover, it is an equivalence of two categories. Objects of one of them are mixed quotient divisible groups. Objects of another one are exactly the Maltsev matrices mentioned above.

The composition of the duality by Maltsev and the generalized equivalence by Kurosh coincides remarkably with the duality introduced in [3].

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SHORT COMMUNICATION

On the symmetry groups of plane quasilattices

O. I. Gerasimova, A. A. Dishlis, M. V. Tsybanov

Let Z be an direct sum of m infinite subgroups of additive group of vectors of n-dimentional Euclidean space E_n. When $m = n$ this subgroup is called a lattice of space E_n. Lattices of space E_n are classified by means of their symmetry groups (when $n = 2$ they are called Brave groups). Root lattice (quasilattice) is called lattice (quasilattice) with the basic composed from the simple root [2] of a root system (generalized system of roots [3]). Arbitrary plane lattice (i.e. lattice in E2) with its Brave group differing away from 1-dimensional lattice, is root lattice. There are natural questions of finding both the symmetry groups of all quasilattices and conditions, when this quasilattice is a root one. On this way following result is obtained.

Theorem 1. *Finite symmetry groups of quadratic root lattices become exhausted with dihedral groups $D5$, $D8$, $D12$ and their subgroups.*

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8th International Algebraic Conference
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On inclusion of finite group in solubly saturated formation

S. F. Kamornikov, L. P. Avdashkova

Only finite group are considered. There are used definitions and notations of [1, 2].

There are published some decade papers, in which inclusion of group G in saturated formation $\mathfrak{F}$ is studied on the following scheme:

1) formation $\mathfrak{F}$ contains all supersoluble groups;

2) assume that in G exists a normal subgroup N for which $G/N \in \mathfrak{F}$;

3) a system of subgroups of N has some restriction (normality, generalized normality, cyclicity, permutability, imbeddability, generalized complementability or other).

Analysis of proofs of basic results of such papers is showing, that in most of them saturated formation $\mathfrak{F}$ may be replaced to solubly saturated formation.

Theorem. *Let $\mathfrak{H}$ is solubly saturated subformation of solubly saturated formation $\mathfrak{F}$. Let N is soluble normal subgroup of group G, where $G/N \in \mathfrak{F}$. If any G-chief factor of group $F(G)/\Phi(G)$ is $\mathfrak{H}$-central in G, then $G \in \mathfrak{F}$.*

Corollary 1. *Let $\mathfrak{F}$ is solubly saturated formation, which contains all supersoluble groups. Let N is soluble normal subgroup of group G, where $G/N \in \mathfrak{F}$. If all G-chief factors of group $F(N)/\Phi(G)$ have prime order, then $G \in \mathfrak{F}$.*

Corollary 2. *Let N is soluble normal subgroup of group G, where group G/N is c-supersoluble. If all G-chief factors of group $F(N)/\Phi(G)$ have prime order, then G is c-supersoluble.*

Corollary 3. *Let $\mathfrak{F}$ is solubly saturated formation, which contains all supersoluble groups. Let N is soluble normal subgroup of group G, where $G/N \in \mathfrak{F}$. If all maximal subgroups of Sylow subgroups of $F(N)$ are c-normal of G, then $G \in \mathfrak{F}$.*

Corollary 4. *Let $\mathfrak{F}$ is solubly saturated formation, which contains all supersoluble groups. Let N is soluble normal subgroup of group G, where $G/N \in \mathfrak{F}$. If all Sylow subgroups of $F(N)$ are cyclic, then $G \in \mathfrak{F}$.*

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8th International Algebraic Conference
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Relatively hyperradical and superradical formations of finite groups

Irina Khalimonchik

All the groups considered are finite. In the Kourovka Notebook [1] L.A.Shemetkov formulated the problem of classification of supperradical formations. This problem was studed in papers [2-3]. In [4-5] authors investigated a hyperradical formations.

Let $\mathfrak{F}$ be a non-empty formation of groups. Remind, that a subgroup H of G is called $\mathfrak{F}$-subnormal in G if either $H = G$ or there is a maximal chain $H = H_0 \subset H_1 \subset \cdots \subset H_n = G$ such that $H_i^{\mathfrak{F}} \subseteq H_{i-1}$ for all $i = 1, \cdots, n$.

Definition. Let $\mathfrak{X}$ be a class of groups. Formation $\mathfrak{F}$ of groups is called:

1) superradical in $\mathfrak{X}$ [2], if $\mathfrak{F}$ is S_n-closed formation in $\mathfrak{X}$ and $\mathfrak{F}$ consider every $\mathfrak{X}$-group $G = AB$, where A and B are $\mathfrak{F}$-subnormal $\mathfrak{F}$-subgroups in G.

2) hyperradical in $\mathfrak{X}$ [4], if $\mathfrak{F}$ is S_n-closed formation in $\mathfrak{X}$ and $\mathfrak{F}$ consider every $\mathfrak{X}$-group $G = \langle A, B\rangle$, where A and B are $\mathfrak{F}$-subnormal $\mathfrak{F}$-subgroups in G;

The following theorem determine relation between hyperradical and superradical formations in $\mathfrak{X}$.

Theorem. Let $\mathfrak{X}$ be a saturated S-closed formation. Then the following statements are equivalent:

1) every S-closed saturated subformation from $\mathfrak{X}$ is hyperradical in $\mathfrak{X}$;

2) every S-closed saturated subformation from $\mathfrak{X}$ is superradical in $\mathfrak{X}$;

3) $\mathfrak{X} \subseteq \mathfrak{N}\mathfrak{A}$.

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8th International Algebraic Conference
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Finite groups with nilpotent and Hall subgroups

Kniahina V., Monakhov V.

We consider finite groups only. A Schmidt group is a non-nilpotent group in which every proper subgroup is nilpotent. Review of the results on Schmidt groups and perspectives of its applications in a group theory as of 2001 are provided in paper [1].

If every proper subgroup of a finite group G is Hall, then obviously, $|G|$ is squarefree, i.e. $|G|$ is not divisible by the square of any prime p. In this case, G contains a normal cyclic Hall subgroup H such that G/H is cyclic.

In this note the properties of a finite non-nilpotent group in which every proper subgroup is either Hall or nilpotent are studied. We prove the following theorem.

Theorem. *Let G be a finite non-nilpotent group in which every proper subgroup is either Hall or nilpotent. Then the following statements hold:*

1) If a Sylow p-subgroup P is non-normal in G, then P is cyclic and every its maximal subgroup is contained in the center of G;

2) If a Sylow p-subgroup P is normal in G, then P is either minimal normal in G or non-abelian, $Z(P) = P' = \Phi(P)$ and $P/\Phi(P)$ is a minimal normal subgroup of $G/\Phi(P)$;

3) If P_1 is a normal p-subgroup different from a Sylow p-subgroup of G, then P_1 is contained in the center of G;

4) $G/Z(G)$ contains a normal abelian Hall subgroup $A/Z(G)$ in which every Sylow subgroup is minimal normal in $G/Z(G)$. Moreover, G/A is cyclic and $|G/A|$ is squarefree.

Corollary. *Let G be a finite non-nilpotent group in which every own subgroup is either Hall or nilpotent. Then G contains a nilpotent Hall subgroup H such that G/H is cyclic. In particular, the derived length of G does not exceed 3.*

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Contributed abstract

Conditions of supersolubility of finite groups

V. A. Kovalyova

Throughout this paper, all groups considered are finite.

Let A be a subgroup of a group G, $K \leq H \leq G$. Then we say that A *covers the pair* (K, H) if $AH = AK$; A *avoids* (K, H) if $A \cap H = A \cap K$. A subgroup H of G is said to be quasinormal [1] or permutable [2] in G if $HE = EH$ for all subgroups E of G. The quasinormal subgroups have many interesting properties. In particular, if E is a quasinormal subgroup of G, then for every *maximal* pair of G, that is, a pair (K, H), where K is a maximal subgroup of H, E either covers or avoids (K, H). This observation leads us to the following generalization of the quasinormality.

Definition. Let A be a subgroup of a group G. We say that A *is a generalized quasinormal in* G, if A either covers or avoids every maximal pair (K, H) of G.

The following theorems are proved.

Theorem 1. *A soluble group G is supersoluble if and only if every subnormal subgroup of G is generalized quasinormal in G.*

A subgroup H of a group G is called a primitive [3] or meet-irreducible [4] in G if H cannot be written as a proper intersection of subgroup of G.

Theorem 2. *A group G is supersoluble if every meet-irreducible subgroup of G is generalized quasinormal in G.*

Theorem 3. *A group G is supersoluble if every cyclic subgroup of G with prime order and order 4 is generalized quasinormal in G.*

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CONTRIBUTED ABSTRACT

Self-coincidence and polygons with odd number of points

Yu. I. Kulazhenko

The first applications of the theory of n-ary groups in affine geometry goes back to D. Vakarelov's and S.A. Rusakov's works [1, 2]. Being based on results of the works [2, 3] the author introduced [4] the concept of self-coincidence for elements of n-ary groups and obtained some results related to this topic [4, 5, 6].

In spite of the results in [4, 5, 6], where the self-coincidence of elements of n-ary groups were considered with respect to sequences of points of some special classes of triangles, quadrangle and hexagons, in our talk we consider the self-coincidence of elements with respect to arbitrary polygons with odd number of points.

In what follows, $G =< X, (\ ),^{[-2]} >$ denotes an n-ary group for elements of which we use the term a *point*. For any $a \in X$, the point $(ab^{[-2]} \overset{2n-4}{b} a)$ is called a *point that is symmetric to a point b relatively the point a*. Any sequence which consists of k elements in X is called a k-gon of G.

Let $p \in X$ and $a_1, \ldots, a_k$ be any sequence of points in X. Then we say that the point $S_{a_k}(\ldots(S_{a_2}(S_{a_1}(p)))\ldots)$ is the sequence of symmetries of the point p with respect to this sequence. We say that a point $p \in X$ *self-coincides* if there exists a sequence $a_1, \ldots, a_k$ of other points in X such that $S_{a_k}(\ldots(S_{a_2}(S_{a_1}(p)))\ldots) = p$.

Other notations and definitions may be found in [2, 3].

Definition. *Let $p \in X$ and $a_1, \ldots, a_k$ $(*)$ be any sequence of points in X. Then we say that:*

(I) the point $S_{a_k}(\ldots(S_{a_2}(S_{a_1}(p)))\ldots)$ is a circuit by the point p of the sequence $(*)$.

(II) x_n is the n-th circuit by the point p of the sequence $(*)$ if x_n is defined recursively as follows:

(1) $x_1 = S_{a_k}(\ldots(S_{a_2}(S_{a_1}(p)))\ldots)$, and

(2) $x_i = S_{a_k}(\ldots(S_{a_2}(S_{a_1}(x_{i-1})))\ldots)$ for all $i = 1, 2, \ldots n$

Theorem. *If G is semiabelian and k is odd, then the $2n$-th circuit by the point p of the sequence $(*)$ is a self-coincidence of p.*

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8th International Algebraic Conference
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Groups whose finitely generated subgroups are either permutable or pronormal

L. A. Kurdachenko, I. Ya. Subbotin, T. V. Ermolkevich

A subgroup H of a group G is called pronormal in G if for every element g of G the subgroups H and H^G are conjugate in $\langle H, H^G\rangle$. In the papers [1, 2, 3] the groups, whose subgroups are either subnormal or pronormal, have been considered. A subgroup H of a group G is called permutable if $HK = KH$ for each subgroup K of G. Remark that every permutable subgroup is ascendant [4]. We initiate investigation of groups whose subgroups are either permutable or pronormal. Moreover, we focus on a more general case: we study the groups whose finitely generated subgroups are either permutable or pronormal. As the first step, we describe such locally finite groups. Observe that in the groups whose finitely generated subgroups are permutable, every subgroup is permutable. Some groups, whose finitely generated subgroups are pronormal have been described by I.Ya. Subbotin and N.F. Kuzenny in [5]. Our main result is the following

Theorem. *Let G be a locally finite group, L a locally nilpotent residual of G and D be the locally nilpotent radical of G. If every finitely generated subgroup of G is either pronormal or permutable, then the following hold:*

(i) *$[G, G]$ is abelian;*

(ii) *$D = L \times Z$ where Z is the upper hypercenter of G;*

(iii) *the Sylow p–subgroup of L is the Sylow p–subgroup of G for each $p \in \Pi(L)$;*

(iv) *$[L, G] = L$, $C_L(G) =< 1 >$, and every subgroup of L is G–invariant;*

(v) *G/L is hypercentral and every subgroup of G/L is permutable. In particular, every ascendant subgroup of G is permutable. Furthermore,*

(vi) *if $\Pi(C/D)$ contains two distinct primes, then G/L is a Dedekind group; in this case, every finitely generated subgroup of G is pronormal;*

(vii) *if C/D is an infinite p–group for some prime p, then G/L is a Dedekind group; in this case, every finitely generated subgroup of G is pronormal;*

(viii) *if C/D is a finite p–group for some prime p and G/L is not a Dedekind group, then $G = A \leftthreetimes P$ where P is a Sylow p–subgroup of G, A is a Dedekind group, P is bounded and central-by-finite, and $P/C_P(A)$ is finite.*

Corollary. *Let G be a locally finite group, whose finitely generated subgroups are either pronormal or permutable. Then every finitely generated subgroup of G is pronormal or G is a group of the type (viii) of Theorem.*

Corollary. *Let G be a locally finite group, whose subgroups are either pronormal or permutable. Then every subgroup of G is pronormal or G is a group of type (viii) of Theorem.*

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Contributed abstract

On some generalization of permutable and pronormal subgroups

L. A. Kurdachenko, A. A. Pypka, I. Ya. Subbotin

Let G be a group. A subgroup H of G is called *self-conjugate permutable* subgroup, if H satisfies the following condition: if $HH^g = H^gH$ then $H^g = H$. In a paper [1] T. Foguel has introduced a following generalization of the permutable subgroups. A subgroup H of a group G is called *conjugate permutable* in G if $HH^g = H^gH$ for each element $g \in G$. The concept of self-conjugate permutable subgroup was appear in a paper [2] as dual to concept of conjugate permutable subgroup. In a paper [2] have been obtained some properties of finite groups whose cyclic subgroups are self-conjugate permutable, but not description. In the following theorems we obtain the full description of such groups in some very wide class of groups, which contains all finite groups.

Theorem 1. *Let G be a locally finite group and L be a locally nilpotent residual of G. If every cyclic subgroup of G is self-conjugate permutable, then the following condition holds:*

(i) L is abelian; (ii) $2 \notin \Pi(L)$ and $\Pi(L) \cap \Pi(G/L) = \varnothing$; (iii) G/L is a Dedekind group; (iv) every subgroup of $C_G(L)$ is G-invariant.

Conversely, if a group G satisfies the condition (i)-(iv), then every subgroup of G is self-conjugate permutable.

A group G is said to be *locally graded* if each non-identity finitely generated subgroup of G has a proper subgroup of finite index.

Theorem 2. *Let G be a locally graded group and L be a locally nilpotent residual of G.*

(i) If G is not periodic, then every subgroup of G is self-conjugate permutable if and only if G is abelian.

(ii) If G is periodic, then every subgroup of G is self-conjugate permutable if and only if G satisfies the conditions of Theorem 1.

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8th International Algebraic Conference
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SECTION TALK

Some properties of conjugate-permutable subgroups

L. A. Kurdachenko, N. A. Turbay

A subgroup H of a group G is said to be *permutable in* G, if $HK = KH$ for every subgroup K of G. In a paper [1] T. Foguel has introduced a following generalization of the permutable subgroups. A subgroup H of a group G is called *conjugate − permutable in* G if $HH^g = H^gH$ for each element $g \in G$. In a paper [2] T. Foguel has considered the groups whose cyclic subgroups are conjugate-permutable. Some results concerning the properties of conjugate-permutable subgroups in infinite groups have been obtained in a paper [3]. We obtain the following essential generalization of the results of these papers.

Theorem 1. *Let G be a group and H be a conjugate-permutable subgroup of G. If the family* $\boldsymbol{M} = \{H^{g_1} \dots H^{g_k} \mid g_1, \dots, g_k \in G, k \in \mathbb{N}\}$ *satisfies the maximal condition (as ordered by inclusion set), then H is a descendant subgroup of G.*

Corollary. *Let G be a group and H be a conjugate-permutable subgroup of G. If G satisfies the maximal condition, then H is a descendant subgroup of G.*

If G is a group, then by $\mathbf{Tor}(G)$ we will denote the maximal normal periodic subgroup of G. We recall that if G is a locally nilpotent group, then $\mathbf{Tor}(G)$ is a (characteristic) subgroup of G and $G/\mathbf{Tor}(G)$ is torsion - free.

Theorem 2. *Let G be a group whose cyclic subgroups are conjugate-permutable. Then G is locally nilpotent and hyperabelian. Moreover,* $G/\mathbf{Tor}(G)$ *is nilpotent of class at most 3.*

Theorem 3. *Let G be a Chernikov group and H be a conjugate-permutable subgroup of G. Then H is subnormal in G. Moreover,* $[H, div(G)] =< 1 >$.

Theorem 4. *Let G be a group, H be a subgroup of G and L be a normal subgroup of G such that* $G = LH$. *Suppose that L has an ascending series of H - invariant subgroups whose factors are either finite or abelian minimax group. If H is conjugate-permutable in G, then H is ascendant subgroup of G.*

Corollary. *Let G be a group and H be a conjugate-permutable subgroup of G. Suppose that G has an ascending series of normal subgroups whose factors are either finite or abelian minimax group. Then H is ascendant subgroup of G.*

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8th International Algebraic Conference
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Regular wreath products of finite p-groups and Kaloujnine groups

Yu. Leonov

Regular (standard) wreath product $A \wr B$ of the groups A and B is group of the pairs

$$\{(b, w) \mid b \in B, w \in Fun(B, A)\},$$

where $Fun(B, A)$ — set of all functions from B to A with group operation $(b_1, w_1) \cdot (b_2, w_2) = (b_1 b_2, w_1^{b_2} w_2)$ with $w_1^{b_2}(x) = w_1(b_2 x)$, $x \in B$ (see [1]).

Let $P_{p,n}$ be the Sylow p-subgroup of the symmetric group S_{p^n} which was studied in [2]. We call this group as Kaloujnine group. The set of those groups can be done recursively. Namely,

$$P_{p,0} = \mathbb{Z}_p, \qquad P_{p,n} = P_{p,n-1} \wr \mathbb{Z}_p, \text{ for } n > 0.$$

Wreath product is neither commutative nor associative operation. For this reason and due to important role of wreath products in group theory it is interesting to study group of the form $\mathbb{Z}_p \wr G$, where G — some group.

First step of such investigation is immersing of the group $I_{p,n} = \mathbb{Z}_p \wr P_{p,n}$, $n \geq 0$ to the well known Kaloujnine group $P_{p,m}$ for some natural appropriate m.

Theorem 1. *There exist monomorphism $\Omega_{p,n} : I_{p,n} \to P_{p,m}$, for any $n \geq 0$ and $m = \log_p |P_{p,n}|$, where $||$ — order of p-group. Moreover, m is the least with such property.*

This result for $p = 2$ was presented in [3]. Check that for this theorem we need to use the (faithful) representation of the Kaloujnine group which was done in [4] for $p = 2$ and in [5] for any prime p. Monomorphism $\Omega_{2,n}$ was also studied in the paper [6]. Our considerations allows to regard an immersion of the group $\mathbb{Z}_p \wr G$, $G \leq P_{p,n}$ $(n \geq 0)$ to the group $P_{p,m}$ for appropriate natural m.

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8th International Algebraic Conference
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On the normalizer condition for one infinitely iterated wreath product of cyclic groups

Yu. Leshchenko

A group G is said to satisfy the *normalizer condition* if every proper subgroup $H < G$ is properly contained in its own normalizer, i.e. $H < N_G(H)$ for all $H < G$.

Let $\delta_{ij} = 1$ if $i = j$ and $\delta_{ij} = 0$ if $i \neq j$. For a prime p (assume that $p \neq 2$) we consider the set $FM_\omega(\mathbb{Z}_p)$ of all "almost identity" infinite matrices (a_{ij}), such that $a_{ij} \in \mathbb{Z}_p$; $a_{ij} = \delta_{ij}$ for all but finitely many $(i,j) \in \mathbb{N} \times \mathbb{N}$ (ω is the least infinite ordinal). Matrices from $FM_\omega(\mathbb{Z}_p)$ are called *finitary matrices* over $\mathbb{Z}_p$. Such matrices can be multiplied by the usual rule:

$$(ab)_{ij} = \sum_{k \in \mathbb{N}} a_{ik} b_{kj},$$

since the sum on the right side contains only a finite number of nonzero terms [1]. The set of all invertible finitary matrices with operation of matrix multiplication forms a group, which is called the *finitary linear group* $GL_\omega(\mathbb{Z}_p)$. The *finitary (upper) unitriangular group* is the group $UT_\omega(\mathbb{Z}_p)$ of all finitary matrices over $\mathbb{Z}_p$ such that $a_{ij} = \delta_{ij}$ for all $i \geq j$. Similarly, we can consider the *lower* unitriangular group. Yu. I. Merzlyakov in [1] proved that $UT_\omega(\mathbb{Z}_p)$ does not satisfy the normalizer condition.

We consider the additive group of the field $\mathbb{Z}_p$ as a permutation group, which acts on itself by the right translations, and define $U_\omega(\mathbb{Z}_p)$ as a group of infinite "almost zero" tableaux

$$[a_1(x_2, \ldots, x_k), \ldots, a_n(x_{n+1}, \ldots, x_k), 0, \ldots], \quad k, n \in \mathbb{N}, \quad k > n,$$

where $a_i(x_{i+1}, \ldots, x_k)$ is a polynomial over the field $\mathbb{Z}_p$ reduced modulo the ideal $\langle x_{i+1}^{p-1} - x_{i+1}, x_{i+2}^{p-1} - x_{i+2}, \ldots, x_k^{p-1} - x_k \rangle$. The group $U_\omega(\mathbb{Z}_p)$ acts on the infinite direct product

$$\mathbb{Z}_p^\omega = \{(t_1, \ldots, t_m, 0, \ldots) \mid t_i \in \mathbb{Z}_p, m \in \mathbb{N}\}$$

($\mathbb{Z}_p^\omega$ is the set of all "almost zero" sequences over $\mathbb{Z}_p$). If $u \in U_\omega(\mathbb{Z}_p)$ and $t = (t_i)_{i=1}^\infty \in \mathbb{Z}_p^\omega$ then

$$t^u = (t_1 + a_1(t_2, \ldots, t_k), \ldots, t_n + a_n(t_{n+1}, \ldots, t_k), t_{k+1}, t_{k+2}, \ldots).$$

Theorem. *The group $U_\omega(\mathbb{Z}_p)$ does not satisfy the normalizer condition.*

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CONTRIBUTED ABSTRACT

Finite 2-groups with non-Dedekind norm of Abelian non-cyclic subgroups and non-cyclic center

T. D. Lukashova, M. G. Drushlyak

One of the main themes in the group theory is the study of the influence of systems of subgroups on the structure of a group. Sometimes a group can have many subgroups with some given properties, but the influence of this system of subgroups need not be significant. In other cases, the presence of a single subgroup with given properties can be very influential on the structure of a group. Introduction of the notion of Σ-norm extends enough the variety of such subgroups. Let's remind that Σ-norm is the intersection of normalizers of subgroups of system Σ, which have some theoretical-group properties. Authors continue study properties of the norm N_G^A of Abelian non-cyclic subgroups, when system Σ consists of all Abelian non-cyclic subgroups of a group G.

If a finite 2-group G coinsides with a norm N_G^A, then all Abelian non-cyclic subgroups are normal in a group G. Such groups were studied in [1,2] and were called $\overline{HA}_2$-groups. In this article authors study the case, when $G \neq N_G^A$.

Theorem 1. *Let G be a finite 2-group with non-Dedekind norm N_G^A of Abelian non-cyclic subgroups. If norm N_G^A is non-metacyclic with non-cyclic center, then G has non-cyclic center.*

Theorem 2. *Let G be a finite 2-group with non-Dedekind norm N_G^A of Abelian non-cyclic subgroups and non-cyclic center. Then G is a group of the following types:*

1) G is non-Dedekind $\overline{HA}_2$-group with non-cyclic center, $G = N_G^A$;

2) $G = Q \times H$, where $Q = \langle y, x\rangle$, $|y| = 2^n$, $n \geq 3$, $|x| = 4$, $y^{2^{n-1}} = x^2$, $x^{-1}yx = y^{-1}$, $H = \langle h_1, h_2\rangle$, $|h_1| = |h_2| = 4$, $[h_1, h_2] = h_1^2 = h_2^2$, $N_G^A = \langle y^{2^{n-2}}\rangle \times H$;

3) $G = Q \leftthreetimes H$, where $Q = \langle y, x\rangle$, $|y| = 2^n$, $n \geq 3$, $|x| = 4$, $y^{2^{n-1}} = x^2$, $x^{-1}yx = y^{-1}$, $H = \langle h_1, h_2\rangle$, $|h_1| = |h_2| = 4$, $[h_1, h_2] = h_1^2 = h_2^2$, $[\langle y\rangle, H] = E$, $[H, \langle x\rangle] = \langle x^2\rangle$, $N_G^A = \langle y^{2^{n-2}}\rangle \times H$;

4) $G = H \leftthreetimes Q$, where $Q = \langle y, x\rangle$, $|y| = 2^n$, $n \geq 3$, $|x| = 4$, $y^{2^{n-1}} = x^2$, $x^{-1}yx = y^{-1}$, $H = \langle h_1, h_2\rangle$, $|h_1| = |h_2| = 4$, $[h_1, h_2] = h_1^2 = h_2^2$, $[\langle y\rangle, H] = E$, $[H, \langle x\rangle] = \langle h^2\rangle$, $N_G^A = \langle y^{2^{n-2}}\rangle \times H$;

5) $G = \langle x\rangle\langle b\rangle$, where $|x| = 2^k$, $|b| = 2^m$, $m > 2$, $k \geq m + r$, $1 \leq r < m - 1$, $[x, b] = x^{2^{k-r-1}s} b^{2^{m-1}t}$, $0 < s < 2$, $0 \leq t < 2$, $N_G^A = \langle x^{2^{m-1}}\rangle \leftthreetimes \langle b\rangle$.

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CONTRIBUTED ABSTRACT

On recognizability of groups by the set of element orders

V. D. Mazurov

For a finite group G, denote by $\pi(G)$ the set of prime divisors of $|G|$ and by $\omega(G)$ *the spectrum* of G, i.e. the set of element orders of G. This set is a subset of the set of natural numbers which is closed under divisibility condition and hence is uniquely defined by the set $\mu(G)$ of its maximal under divisibility elements. A group G is said to be *recognizable* by spectrum $\omega(G)$ (shortly, *recognizable*), if every finite group H with $\omega(H) = \omega(G)$ is isomorphic to G. In other words, G is recognizable if $h(G) = 1$ where $h(G)$ is the number of pairwise non-isomorphic groups H which are *isospectral* to G, that is having the same spectrum as G. A group G is said to be *irrecognizable* if $h(G)$ is infinite.

A survey of recent results on recognizability problem of groups by spectrum see in [1].

Theorem 1. *A finite group G is irrecognizable if and only there exists a finite group H isospectral to G with a non-trivial soluble normal subgroup.*

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CONTRIBUTED ABSTRACT

On direct decompositions of partially saturated formations

A. P. Mekhovich, N. N. Vorobyov

All groups considered are finite. All unexplained notations and terminologies are standard (see [1, 2]).

Let $\{\mathfrak{F}_i \mid i \in I\}$ be a set of non-empty subclasses of a class of groups $\mathfrak{F}$ such that $\mathfrak{F}_i \bigcap \mathfrak{F}_j = (1)$ for all distinct $i, j \in I$. We write $\mathfrak{F} = \oplus_{i\in I}\mathfrak{F}_i$ to denote the collection of all groups of the form $A_1 \times \ldots \times A_t$, where $A_1 \in \mathfrak{F}_{i_1}, \ldots, A_t \in \mathfrak{F}_{i_t}$ for some $i_1, \ldots, i_t \in I$. Any representation $\mathfrak{F} = \oplus_{i\in I}\mathfrak{F}_i$ is said to be a *direct decomposition* of $\mathfrak{F}$ (see [1]).

In [3] it is proved that any formation represented in the form of a direct decomposition of some formations is n-multiply saturated if and only if every component of this direct decomposition is n-multiply saturated. However an analogous result is not true for n-multiply composition formations (see [4]). We prove:

Theorem. *Let $\mathfrak{F} = \oplus_{i\in I}\mathfrak{F}_i$, where $\mathfrak{F}_i$ is a formation. Then the formation $\mathfrak{F}$ is n-multiply $(n \geqslant 1)$ ω-saturated if and only if $\mathfrak{F}_i$ is n-multiply ω-saturated for all $i \in I$.*

Corollary 1. [5, Theorem 1] *Let $\mathfrak{F} = \oplus_{i\in I}\mathfrak{F}_i$, where $\mathfrak{F}_i$ is a formation. Then the formation $\mathfrak{F}$ is ω-saturated if and only if $\mathfrak{F}_i$ is ω-saturated for all $i \in I$.*

Corollary 2. [1, Theorem 4.3.8] *Let $\mathfrak{F} = \oplus_{i\in I}\mathfrak{F}_i$, where $\mathfrak{F}_i$ is a formation. Then the formation $\mathfrak{F}$ is n-multiply $(n \geqslant 1)$ saturated if and only if $\mathfrak{F}_i$ is n-multiply saturated for all $i \in I$.*

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SECTION TALK

Differentiable finite-state izometries and izometric polynomials of the ring of integer 2-adic numbers

Denis Morozov

The aim of the report is to construct requirements which describe izometric polynomials of the ring of integer 2-adic numbers.

The results of this report continue investigations of 2-adic group automatous with the 2-adic izometric functions technique. Polynomials build the important class of izometric function that is why we investigate them.

In addition we investigate differentiable finite-state izometries. The class of finite-state izometries is very important class of izometries and is the object of investigation in many scientific researches.

Definition 1. *Define* $S_n(x_1, x_2)$ *as* $S_n(x_1, x_2) = \sum_{k=0}^{n-1} x_1^{n-k-1} \cdot x_2^k$

Example 1. $S_1(x_1, x_2) = 1$, $S_2(x_1, x_2) = x_1 + x_2$, $S_3(x_1, x_2) = x_1^2 + x_1 \cdot x_2 + x_2^2$ *etc.*

Definition 2. *Define function* $\mu(x) = \overline{x}$: $\mu(x) = \begin{cases} 0, x \in 2Z_2 \\ 1, x \in Z_2^* \end{cases}$

Definition 3. $D_f(x_1, x_2) = \frac{f(x_1)-f(x_2)}{x_1-x_2}$.

Lemma 1. *Polynomial* $f(x) \in Z_2[x]$ *is isometry, if and only if* $\forall x_1, x_2 \in Z_2$ $\overline{D_f}(x_1, x_2) = 1$.

Definition 4. *Let's define for the polynomial* $f(x) = a_0 + a_1x + a_2x^2 + ... + a_nx^n$ *values* A_f *and* B_f*:*

$$A_f = \mu\left(\sum_{k=1}^{[\frac{n+1}{2}]} a_{2k}\right), B_f = \mu\left(\sum_{k=2}^{[\frac{n+1}{2}]} a_{2k-1}\right)$$

Theorem 1. $\overline{a}_1 \oplus (A_f \cap (\overline{x}_1 \oplus \overline{x}_2)) \oplus (B_f \cap (\overline{x}_1 \cup \overline{x}_2))$ *is true if and only if* $\overline{a}_1 = 1,\ A_f = 0,\ B_f = 0$

Theorem 2. *According to theorem 1, polynom* $f(x) = a_0 + a_1x + a_2x^2 + ... + a_nx^n$ *is isometry if and only if when* a_1 *is invertible (odd) integer 2-adic number, the sum of coefficients with even numbers greater then 0 is even 2-adic number and the sum of coefficients with odd numbers greater then 1 is even 2-adic number.*

Theorem 3. *Finite-state izometry of the ring* Z_2 *is differentiable if and only if it's parted linear.*

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8th International Algebraic Conference
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Groups defined by automata over rings

Andriy Oliynyk

Let R be a commutative ring with identity. We consider R as an alphabet. An automaton $\mathcal{A} = \langle Q, \varphi, \psi \rangle$ over R consisting of the set Q of inner states, the transition function $\varphi : Q \times R \to Q$ and the output function $\psi : Q \times R \to R$ is called RS-automaton if for each $q \in Q$ there exists an element $r_q \in R$ such that for arbitrary $x \in R$ the equality $\psi(q, x) = x + r_q$ holds. In each state $q \in Q$ the RS-automaton $\mathcal{A}$ defines a permutation on infinite cartesian power R^∞. All such permutations form a group $GA_S(R)$.

Theorem 1. *The group $GA_S(R)$ is isomorphic to the infinitely iterated wreath power of the additive group of R.*

We define some natural subgroups of $GA_S(R)$ and describe their properties.

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CONTRIBUTED ABSTRACT

Non standard metric products and their isometry groups

B. Oliynyk

Let (X_i, d_i), $i = 1, 2$, be metric spaces. Assume that $\Phi : [0, \infty)^2 \to [0, \infty)$ be a function such that the following conditions hold:

(A) $\Phi(p_1, p_2) = 0$ iff $p_1 = p_2 = 0$;

(B) for arbitrary $q_i, r_i, p_i \in [0, \infty)$ such that $q_i \leq r_i + p_i$, $i = 1, 2$ we have the inequality

$$\Phi(q_1, q_2) \leq \Phi(r_1, r_2) + \Phi(p_1, p_2).$$

The function

$$d_\Phi((x_1, x_2), (y_1, y_2)) = \Phi(d_1(x_1, y_1), d_2(x_2, y_2))$$

is a metric on X ([1]).

Definition 1. *The metric space (X, d_Φ) is called a non standard metric product or Φ-product of X_1, X_2.*

Some similar constructions of non standard products of metric spaces were considered in [2], [3].

For each $a_1 \in X_1$, $a_2 \in X_2$ let

$$X^2_{a_1} = \{(a_1, x_2) \mid x_2 \in X_2\}, \quad X^1_{a_2} = \{(x_1, a_2) \mid x_1 \in X_1\}$$

be subspaces of $(X_1 \times X_2, d_\Phi)$. Denote by C_i the set of values of the metric d_i, $i = 1, 2$. Assume that inequalities

$$\inf_{q_1 \in C_1, q_1 \neq 0} \Phi(q_1, 0) > \sup_{q_2 \in C_2} \Phi(0, q_2), \quad \inf_{q_2 \in C_2, q_2 \neq 0} \Phi(0, q_2) > \frac{1}{2} \sup_{q_1 \in C_1} \Phi(q_1, 0). \tag{1}$$

hold.

Theorem 1. *Let $\Phi : [0, \infty)^2 \to [0, \infty)$ be a function such that conditions (A),(B) and inequalities (1) hold. Assume that $\Phi(q_1, q_2) = \Phi(q_1, 0) + \Phi(0, q_2)$. Then*

$$(Isom X, X) \simeq (Isom X^1_{a_2}, X_1) \times (Isom X^2_{a_1}, X_2)$$

for any $(a_1, a_2) \in X_1 \times X_2$.

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Plenary talk

On Dehn and space functions of groups

A. Yu. Olshanskii

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We consider Dehn and space functions $d(n)$ (resp., $s(n)$) of finitely presented groups $G = \langle A \mid R\rangle$. (These functions have a natural geometric analog.) To define these functions we start with a word w over A of length at most n equal to 1 in G and use relations from R for elementary transformations to obtain the empty word; $d(n)$ (resp., $s(n)$) bounds from above the time (resp., tape space or computer memory) one needs to transform any word of length at most n vanishing in G to the empty word.

One of the main results obtained in [2] is the following criterion: A finitely generated group H has decidable word problem of polynomial space complexity if and only if H is a subgroup of a finitely presented group G with a polynomial space function. (Compare with the results on Dehn functions obtained in [1].) Recent results of Dehn functions from [3] will be also formulated in my talk.

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CONTRIBUTED ABSTRACT

Partitions of groups into thin subsets

Igor Protasov

Let G be an infinite group with the identity e, κ be an infinite cardinal $\leqslant |G|$. By $cf|G|$ we denote the cofinality of $|G|$, κ^+ is the cardinal-successor of κ, $[G]^{<\kappa} = \{F \subseteq G : |F| < \kappa\}$.

We say that a subset $A \subseteq G$ is

- κ-*large* if there exists $F \in [G]^{<\kappa}$ such that $G = FA$;
- κ-*small* if $L \setminus A$ is κ-large for every κ-large subset L;
- κ-*thick* if, for every $F \in [G]^{<\kappa}$, there exists $a \in A$ such that $Fa \subseteq A$;
- κ-*thin* if $|gA \cap A| < \kappa$ for every $g \in G \setminus \{e\}$.

By [2, Theorem 4.2], G can be partitioned in κ κ-large subsets. By [2, Theorem 4.1], G can be partitioned in $\aleph_0$ subsets which are κ-small for each cardinal κ such that $\aleph_0 \leqslant \kappa \leqslant cf|G|$. By [2, Theorem 4.3], if either $\kappa < |G|$ or $\kappa = |G|$ and κ is regular then G can be partitioned in $|G|$ κ-thick subsets.

For κ-thin subsets, its modifications, applications and references see [1]. For a subset A of G, we put $cov(A) = \min\{|S| : S \subseteq G, G = SA\}$. By [3, Theorem 5], if A is κ-thin and $\kappa < |G|$ then $cov(A) = |G|$, if $\kappa = |G|$ then $cov(A) \geqslant cfA$. In contrast to κ-thin subsets, every subgroup A of index κ is κ-small and $cov(A) = \kappa$.

Given an infinite group G and infinite cardinal κ, $\kappa \leqslant |G|$, we denote by $\mu(G, \kappa)$ the minimal cardinal μ such that G can be partitioned in μ κ-thin subsets.

In the following theorem we calculate exact values of $\mu(G, \kappa)$ for all G and κ with only one exception: $|G|$ is singular, $\kappa = |G|$ and $cf|G|$ is a non-limit cardinal.

Theorem 1. *For every infinite group G and every infinite cardinal κ, $\kappa \leqslant |G|$, we have*

$$\mu(G,\kappa) = \begin{cases} \gamma & \text{if } |G| \text{ is non-limit cardinal and } |G| = \gamma^+; \\ |G| & \text{if } |G| \text{ is a limit cardinal and either} \\ & \kappa < |G| \text{ or } |G| \text{ is regular;} \\ cf|G| & \text{if } |G| \text{ is singular, } \kappa = |G| \text{ and } cf|G| \text{ is} \\ & \text{a limit cardinal.} \end{cases}$$

If G is regulator, $\kappa = |G|$ and $cf|G|$ is a non-limit cardinal, $|G| = \gamma^+$, then $\mu(G, \kappa) \in \{\gamma, \gamma^+\}$.

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CONTRIBUTED ABSTRACT

Groups having only two types of pronormal subgroups

A. A. Pypka, N. N. Semko (Jr.)

A subgroup H of a group G is called *abnormal* in G, if $g \in < H, H^g >$ for each element $g \in G$. A subgroup H of a group G is called *pronormal* in G, if the subgroups H and H^g are conjugate in $< H, H^g >$. These subgroups have been introduced by P. Hall. Abnormal subgroups are the antipodes for normal subgroups, they are self-formalizing (that is $H = N_G(H)$) and contranormal (that is $H^G = G$). Clearly every abnormal subgroup is pronormal. But every normal subgroups are pronormal. Normal and abnormal subgroups are two opposite poles in a family of all pronormal subgroups. Therefore is it interesting to consider the groups, whose pronormal subgroups either are normal and abnormal.

Theorem 1. *Let G be a periodic group whose pronormal subgroups either are normal or abnormal. If G is almost locally nilpotent, then it is locally nilpotent or satisfies the following conditions:*

(i) $G = Q \leftthreetimes P$, where P is a Sylow p-subgroup of G and Q is a nilpotent Sylow p'-subgroup of G, p is a prime;

(ii) $P = D < x >$ for some element x, a subgroup D is G-invariant and $x^p \in D$ (so that $Q \times D$ is a locally nilpotent radical of G);

(iii) if H is a G-invariant subgroup of Q, then $C_{Q/H}(xH) =< 1 >$;

(iv) $< x >^G = G$.

Theorem 2. *Let G be a group whose pronormal subgroups either are normal or abnormal. If G is hyperfinite, then it is almost locally nilpotent.*

Corollary 1. *Let G be an infinite group whose pronormal subgroups either are normal or abnormal. If G is a periodic FC-group, then it is locally nilpotent.*

Theorem 3. *Let G be a locally finite group whose pronormal subgroups either are normal or abnormal. If G is hypofinite, then it is almost locally nilpotent.*

Corollary 2. *Let G be a locally finite group whose pronormal subgroups either are normal or abnormal. If G is residually finite, then it is almost locally nilpotent.*

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CONTRIBUTED ABSTRACT

On τ-closed n-multiply ω-saturated formations with nilpotent defect 1

A. I. Rjabchenko

All groups are supposed to be finite. Most of the notations are standard and can be found in [1, 2]. Remind some definitions and designations.

Let ω be some nonempty subsets of primes. The map $f\colon \omega\cup\{\omega'\} \to \{\text{groups formations}\}$ is called an ω-local satellite. Suppose $LF_\omega(f) = \{G \mid G/G_{\omega d} \in f(\omega')$ and $G/F_p(G) \in f(p)$ for all $p \in \omega \cap \pi(G)$, where f is an ω-local satellite, $G_{\omega d}$ is a greatest normal subgroup from G with $\pi(H/K) \cap \omega \neq \emptyset$ for all composition factor H/K. If $\mathfrak{F}$ is a formation such that $\mathfrak{F} = LF_\omega(f)$ for some ω-local satellite then $\mathfrak{F}$ is called an ω-saturated formation and f is called an ω-local satellite of formation $\mathfrak{F}$.

ω-Saturated formations, closed under sub-systems, widely known in various applications of the group classes theory. Recall that a Skiba subgroup functor τ [2] associates with every group G a system of its subgroups $\tau(G)$ such that the following conditions hold: 1) $G \in \tau(G)$ for any group G; 2) for any epimorphism $\phi : A \to B$ and any groups $H \in \tau(A)$ and $T \in \tau(B)$ we have $H^\phi \in \tau(B)$ and $T^{-1} \in \tau(A)$. A formation $\mathfrak{F}$ is called τ-closed if $\tau(G) \subseteq \mathfrak{F}$ for any groups $G \in \mathfrak{F}$.

Any formation is called a 0-multiply ω-saturated. For $n \geq 1$ a formation $\mathfrak{F} \neq \emptyset$ is called n-multiply ω-saturated if it has an ω-saturated satellite such that all its non-empty values are $(n-1)$-multiply ω-saturadet formations [1]. In addition if $\mathfrak{F}$ is τ-closed then $\mathfrak{F}$ is called τ-closed n-multiply ω-saturated formation.

τ-Closed n-multiply ω-saturated formation $\mathfrak{F}$ is called a minimal τ-closed n-multiply ω-saturated non nilpotent formation if $\mathfrak{F} \nsubseteq \mathfrak{N}$ but all τ-closed n-multiply ω-saturated subformation from $\mathfrak{F}$ are contained in $\mathfrak{N}$.

The length of lattice τ-closed n-multiply ω-saturated formations from $\mathfrak{F}/^{\omega_n}_{\tau}\mathfrak{F} \cap \mathfrak{N}$ are contained between $\mathfrak{F} \cap \mathfrak{N}$ and $\mathfrak{F}$ is called nilpotent defect (or $\mathfrak{N}^{\omega_n}_{\tau}$-defect) of τ-closed n-multiply ω-saturated formation $\mathfrak{F}$.

We proved the following

Theorem. *Let $\mathfrak{F}$ be a τ-closed n-multiply ω-saturated formation. The nilpotent defect of formation $\mathfrak{F}$ equals 1 if and only if $\mathfrak{F} = \mathfrak{M} \vee^{\omega_n}_{\tau} \mathfrak{H}$, where $\mathfrak{M}$ is nilpotent τ-closed n-multiply ω-saturated formation, $\mathfrak{H}$ is minimal τ-closed n-multiply ω-saturated non nilpotent formation and also:*

1) any nilpotent τ-closed n-multiply ω-saturated subformation from $\mathfrak{F}$ are contained in $\mathfrak{M} \vee^{\omega_n}_{\tau} (\mathfrak{H} \cap \mathfrak{N})$;

2) any non nilpotent τ-closed n-multiply ω-saturated subformation $\mathfrak{F}_1$ from $\mathfrak{F}$ looks like $\mathfrak{H} \vee^{\omega_n}_{\tau} (\mathfrak{F}_1 \cap \mathfrak{N})$.

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CONTRIBUTED ABSTRACT

On products of subnormal and generally subnormal subgroups of finite groups

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All the groups considered in the article are finite.

Let $\mathfrak{F}$ be a non-empty formation. A subgroup H of a group G is said to be:

1) $\mathfrak{F}$-subnormal in G if either $H = G$ or there exists a maximal chain of subgroups $G = H_0 \supset H_1 \supset \ldots \supset H_{n-1} \subseteq H_n = H$ such that $(H_{i-1})^{\mathfrak{F}} \subseteq H_i$ for all $i = 1, 2, \ldots, n$;

2) $\mathfrak{F}$-accessible in G if there a chain of subgroups $G = H_0 \supseteq H_1 \supseteq \ldots \supseteq H_m = H$ such that either H_i is normal in H_{i-1} or $(H_{i-1})^{\mathfrak{F}} \subseteq H_i$ for all $i = 1, 2, \ldots, m$ [1].

By [2] a formation $\mathfrak{F}$ is called a formation with Shemetkov property if every minimal non-$\mathfrak{F}$-group is either a Schmidt group or a group of prime order.

Theorem. *Let $\mathfrak{F}$ be solvable normal hereditary saturated formation. Then the following assertions are equivalent:*

1) $\mathfrak{F}$ contains any solvable group $G = AB$, where $\pi(G) \subseteq \pi(F)$, A is a subnormal $\mathfrak{F}$-subgroup in G and any Sylow subgroup from B is $\mathfrak{F}$-accessible in G;

2) $\mathfrak{F}$ contains any group $G = AB$, where $\pi(G) \subseteq \pi(F)$, A is a subnormal $\mathfrak{F}$-subgroup in G and any Sylow subgroup from B is $\mathfrak{F}$-subnormal in G;

3) $\mathfrak{F}$ contains any solvable group $G = AB$, where A is a subnormal $\mathfrak{F}$-subgroup in G and B is $\mathfrak{F}$-accessible $\mathfrak{F}$-subgroup in G;

4) $\mathfrak{F}$ contains any solvable group $G = AB$, where A is a subnormal $\mathfrak{F}$-subgroup in G and B is $\mathfrak{F}$-subnormal $\mathfrak{F}$-subgroup in G;

5) $\mathfrak{F}$ is a formation with Shemetkov property in a class $\mathfrak{S}$.

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SHORT COMMUNICATION

On n-multiply saturated formations with limited $\mathfrak{H}_n$-defect

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All groups considered are finite. We use terminology and notations from [1, 2].

Every formations is said to be *a 0-multiply saturated*. For $n \geq 1$, the formation $\mathfrak{F}$ is *n-multiply saturated* if it has a local satellite all of whose non-empty values are $(n-1)$-multiply saturated formations.

For any set of groups $\mathfrak{X}$, $l_n\mathrm{form}\mathfrak{X}$ denotes the intersection of all n-multiply saturated formations containing $\mathfrak{X}$. For every n-multiply saturated formations $\mathfrak{M}$ and $\mathfrak{H}$, we set $\mathfrak{M} \vee_n \mathfrak{H} = l_n\mathrm{form}(\mathfrak{M} \cup \mathfrak{H})$. With respect to the operations $\vee_n$ and $\cap$ the set l_n of n-multiply saturated formations forms a modular lattice.

Let $\mathfrak{F}$, $\mathfrak{H}$ be n-multiply saturated formations, $\mathfrak{F} \not\subseteq \mathfrak{H}$. The length of the lattice $\mathfrak{F}/_n\mathfrak{F} \cap \mathfrak{H}$ of n-multiply saturated formations $\mathfrak{X}$ with $\mathfrak{F} \cap \mathfrak{H} \subseteq \mathfrak{X} \subseteq \mathfrak{F}$ is called *a $\mathfrak{H}_n$-defect* of $\mathfrak{F}$. We use $|\mathfrak{F} : \mathfrak{F} \cap \mathfrak{H}|_n$ to denote the $\mathfrak{H}_n$-defect of $\mathfrak{F}$.

A n-multiply saturated formation $\mathfrak{F}$ is called *$\mathfrak{H}_n$-critical* if $\mathfrak{F} \not\subseteq \mathfrak{H}$ but all proper n-multiply saturated subformations of $\mathfrak{F}$ are contained in $\mathfrak{H}$.

Let $\{\mathfrak{F}_i | i \in I\}$ be the set of all proper n-multiply saturated subformations of $\mathfrak{F}$, $\mathfrak{X} = l_n\mathrm{form}(\cup_{i \in I}\mathfrak{F}_i)$. Then $\mathfrak{F}$ is called: 1) *an irreducible* n-multiply saturated formation if $\mathfrak{F} \neq \mathfrak{X}$; 2) *a reducible* n-multiply saturated formation if $\mathfrak{F} = \mathfrak{X}$.

Suppose $\mathfrak{H} = \mathfrak{N}^m$ is the formation of all soluble groups with a nilpotent length $\leq m$.

Theorem 1. *Let $\mathfrak{F}$ be a n-multiply saturated formation, $\mathfrak{F} \not\subseteq \mathfrak{H}$, $n > m$. Then the $\mathfrak{H}_n$-defect of $\mathfrak{F}$ equals 2 if and only if $\mathfrak{F} = \mathfrak{X}_1 \vee_n \mathfrak{X}_2 \vee_n \mathfrak{M}$, where the formation $\mathfrak{X}_i$ is $\mathfrak{H}_n$-critical, $i \in \{1, 2\}$, $\mathfrak{X}_1 \neq \mathfrak{X}_2$, $\mathfrak{M} \subseteq \mathfrak{H}$;*

Theorem 2. *Let $\mathfrak{F}$ be a reducible n-multiply saturated formation, $\mathfrak{F} \not\subseteq \mathfrak{H}$, $n > m$. Then the $\mathfrak{H}_n$-defect of $\mathfrak{F}$ equals 3 if and only if one of the following statements is satisfied: 1) $\mathfrak{F} = \mathfrak{X}_1 \vee_n \mathfrak{X}_2 \vee_n \mathfrak{X}_3 \vee_n \mathfrak{M}$, where the formation $\mathfrak{X}_i$ is $\mathfrak{H}_n$-critical, $\mathfrak{X}_i \neq \mathfrak{X}_j$, $i, j \in \{1, 2, 3\}$, $i \neq j$, $\mathfrak{M} \subseteq \mathfrak{H}$; 2) $\mathfrak{F} = \mathfrak{X} \vee_n \mathfrak{M}$, where $\mathfrak{X}$ is an irreducible n-multiply saturated formation, $|\mathfrak{X} : \mathfrak{X} \cap \mathfrak{H}|_n = 3$, $\mathfrak{M} \subseteq \mathfrak{H}$, and $\mathfrak{M} \not\subseteq \mathfrak{X}$.*

Theorem 3. *Let $\mathfrak{F}$ be an irreducible n-multiply saturated formation, $\mathfrak{F} \not\subseteq \mathfrak{H}$, $n > m$. Then the $\mathfrak{H}_n$-defect of $\mathfrak{F}$ equals 3 if and only if $\mathfrak{F} = l_n\mathrm{form}G$, where $G = [P_1]([P_2] \ldots ([P_{m+1}]N) \ldots)$, $P_1 = C_G(P_1)$, $P_i = C_{H_i}(P_i)$, where $H_i = [P_i]([P_{i+1}] \ldots ([P_{m+1}]N) \ldots)$ for any $i = 2, \ldots, m+1$, $|P_i| = p_i^{a_i}$, $i = 1, \ldots, m+1$, $|N| = q$, $q \neq p_{m+1}$.*

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SHORT COMMUNICATION

On $\mathfrak{G}$-separability of the lattice l_∞^ω of totally ω-saturated formations

V. G. Safonov, V. V. Shcherbina

All groups considered are finite. We use terminology and notations from [1, 2].

Let ω be a non-empty set of primes. A formation $\mathfrak{F}$ is called ω-saturated if $\mathfrak{F}$ contains each group G with $G/O_\omega(G) \cap \Phi(G) \in \mathfrak{F}$. A function of the form $f : \omega \cup \{\omega'\} \to \{group\ formations\}$ is called an ω-local satellite. For an arbitrary satellite f the symbol $LF_\omega(f)$ denotes the class $(G \mid G/G_{\omega d} \in f(\omega')$ and $G/F_p(G) \in f(p)$ for all $p \in \omega \cap \pi(G))$. If a formation $\mathfrak{F}$ is sach that $\mathfrak{F} = LF_\omega(f)$ then $\mathfrak{F}$ is called ω-local formation and f is an ω-local sattelite of $\mathfrak{F}$. By Theorem 1 [3] a formation $\mathfrak{F}$ is ω-local if and only if it is ω-saturated.

Every formations is said to be 0-multiply ω-saturated. For $n \geq 1$, a formation $\mathfrak{F}$ is called n-multiply ω-saturated, if it has a ω-local satellite f such that avery value of f is a $(n-1)$-multiply ω-saturated formation. A formation that is n-multiply ω-saturated for any non-negative integer n is said to be totally ω-saturated.

For any set of groups $\mathfrak{X}$, $l_\infty^\omega \mathrm{form}\mathfrak{X}$ denotes a totally ω-saturated formation generated by a class of groups $\mathfrak{X}$, i.e. $l_\infty^\omega \mathrm{form}\mathfrak{X}$ is the intersection of all totally ω-saturated formations containing $\mathfrak{X}$. For every totally ω-saturated formations $\mathfrak{M}$ and $\mathfrak{H}$, we suppose $\mathfrak{M} \vee_\infty^\omega \mathfrak{H} = l_\infty^\omega \mathrm{form}(\mathfrak{M} \cup \mathfrak{H})$. With respect to the operations $\vee_\infty^\omega$ and $\cap$ the set l_∞^ω of totally ω-saturated formations forms a complete modular lattice.

Let $\mathfrak{X}$ be a non-empty class of groups. A complete lattice θ of formations is said $\mathfrak{X}$-separable if for every term $\nu(x_1, \ldots, x_m)$ of signature $\{\cap, \vee_\theta\}$, θ-formations $\mathfrak{F}_1, \ldots, \mathfrak{F}_m$, and every group $G \in \nu(\mathfrak{F}_1, \ldots, \mathfrak{F}_m)$ are exists $\mathfrak{X}$-groups $A_1 \in \mathfrak{F}_1, \ldots, A_m \in \mathfrak{F}_m$ such that $G \in \nu(\theta \mathrm{form} A_1, \ldots, \theta \mathrm{form} A_m)$.

Skiba [2] proved that the lattice l_n of n-multiply saturated formations is $\mathfrak{G}$-separable for any non-negative integer n and the lattice of soluble totally saturated formations is $\mathfrak{S}$-separable. Further, Safonov [4] established the $\mathfrak{G}$-separability of the lattice l_∞ of totally saturated formations.

Theorem 1. *The lattice l_∞^ω of totally ω-saturated formations is $\mathfrak{G}$-separable.*

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CONTRIBUTED ABSTRACT

About finite DM-groups of the exponent 4

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Let G — finite 2-group and $S(G)$-system its maximum cyclic subgroups. If subgroup $A \in S(G)$ has in group G additions, than G is identified DM-group. The important results on description of the finite DM-groups are received in [1-3]. But opened remained question about construction of the DM-groups of exponent 4.

Let L_4 — a class all finite DM-groups of exponent 4. As is well known [1], finite 2-group $G \in L_4$ if and only if , when are executed conditions:
1) any cyclic subgroup $< g >$ order of 4 have an addition in group G;
2) any involution $f \in \Phi(G)$ is square of certain element $z \in G$.

Obviously class of the groups L_4 is closed comparatively homomorphisms and direct products of the final number of the groups from L_4. If in group $G \in L_4$ exists the cyclic subgroup $T =< t >$ an order 4, which complies with its centralizer, than G is a dihedral group an order 8. Possible show that any noncommutative group $G \in L_4$ generates variety of the groups γ, which is defined by system identities $\sum = \{x^4 = 1; [x, y, z] = 1; [x^2, y] = 1\}$.

Theorem 1. *Let finite group $G \in \gamma$ and $G = \langle g \rangle \lambda D$. Group G is DM-group if and only if, when are executed conditions: 1) D is DM-group; 2) for any element $d \in D$ exists $d_1 \in D$, that is $d^2 = d_1^2$ and $[g, d_1] = 1$.*

Definition. *Finite group G is identified F-group if are executed conditions: 1) $G \in L_4$; 2) G does not decomposed in direct product of their own subgroups; 3) in the group G exists cyclic subgroup $\langle g \rangle$ order 4, for which exists in the G elementary abelian addition.*

We shall consider group $R = \langle g_1, ..., g_k, v | \mid g_i \mid = 4, \mid v \mid = 2, [g_i, g_j] = g_i^2 g_j^2, [g_i, v] = g_i^2, \mid R \mid = 2^{2k+1}, R \in \gamma \rangle$.

Theorem 2. *Any F-group G isomorphic or dihedral group an order 8 or group R.*

We found necessary and sufficient conditions for a finite nonabelian group $G \in L_4$.

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Contributed abstract

Конечные группы, факторизуемые обобщенно субнормальными подгруппами взаимно простых индексов

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Построеная известным немецким математиком Виландом теория субнормальных подгрупп активно используется многими математиками при изучении строения не простых конечных групп.

В теории классов конечных групп естественным обобщением понятия субнормальности является понятие обобщенной субнормальности.

В 1978 г. Кегель[1] и Л.А. Шеметков[2] поставили задачу о построении теории обобщенно субнормальных подгрупп аналогичной теории субнормальных подгрупп. Данная задача сразу привлекла пристальное внимание специалистов по теории конечных групп. Благодоря работам многих известных математиков сформировался круг проблем, связанных с данной задачей.

Одной из первых проблем, вызвавшей бурное развитие данного направления, была проблема Шеметкова о классификации насыщенных сверхрадикальных формаций, т.е. таких формаций $\mathfrak{F}$, которые замкнуты относительно относительно произведения обобщенно субнормальных $\mathfrak{F}$-подгрупп. Полное решение данной проблемы в классе конечных разрешимых групп было получено в работе [3].

В настоящей работе получено описание непустых наследственных формаций $\mathfrak{F}$, замкнутых относительно произведения обобщенно субнормальных $\mathfrak{F}$-подгрупп взаимно простых индексов. Все рассматриваемые группы конечны и разрешимы.

Теорема. *Пусть $\mathfrak{F}$ — непустая наследственная формация, тогда следующие утверждения эквивалентны:*

1) *формация $\mathfrak{F}$ содержит любую группу $G = AB$, где A и B — $\mathfrak{F}$-субнормальные $\mathfrak{F}$-подгруппы и индексы $\mid G : A \mid$, $\mid G : B \mid$ взаимно просты;*

2) *любая минимальная не $\mathfrak{F}$-группа G одного из следующих типов:*

   а) *G — группа простого порядка q, где $q \notin \pi(\mathfrak{F})$;*

   б) *G — бипримарная p-замкнутая группа($p \in \pi(G)$), $G_p = G^{\mathfrak{F}}$ и $\pi(G) \subseteq \pi(\mathfrak{F})$;*

   в) *G — p-группа, где $p \in \pi(\mathfrak{F})$.*

Следствие. *Бипримарная группа G сверхразрешима тогда и только тогда, когда любая её силовская подгруппа H обладает максимальной цепью $H = H_0 \subseteq H_1 \subseteq \ldots \subseteq H_{n-1} \subseteq H_n = G$ такой, что $\mid H_i : H_{i-1} \mid$ — простые числа для любого $i = 1, 2, ..., n$.*

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Short communication

On a theorem of Doerk and Hawkes

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A group class is called a formation if it is closed under forming quotients and finite subdirect products. If $\mathfrak{F}$ is a non-empty formation, each finite group G has a smallest normal subgroup whose quotient belongs to $\mathfrak{F}$; this is called the $\mathfrak{F}$-residual of G and is denoted by $G^{\mathfrak{F}}$. K. Doerk and T. Hawkes proved in [1] that if A and B are finite groups and $G = A \times B$ is their direct product, then $G^{\mathfrak{F}} = A^{\mathfrak{F}} \times B^{\mathfrak{F}}$, provided that $\mathfrak{F}$ consists of soluble groups. In [1] it was constructed an example which shows that the equality $G^{\mathfrak{F}} = A^{\mathfrak{F}} \times B^{\mathfrak{F}}$ need no longer hold when $\mathfrak{F}$ contains insoluble groups. In this article we show that there are cases when the mentioned equality holds for some formations containing insoluble groups.

All groups considered are finite. If $\mathfrak{F}$ is a formation, then $E_\Phi\mathfrak{F}$ is the class of all groups G such that $G/\Phi(G)$ belongs to $\mathfrak{F}$. If ω is a set of primes, then ω' is the set of primes not in ω. If $\mathfrak{F}$ is a formation and ω is a set of primes, then we define $E_{\Phi_{\omega'}}\mathfrak{F}$ as follows: $G \in E_{\Phi_{\omega'}}\mathfrak{F}$ if and only if there exists a normal ω'-subgroup $K \trianglelefteq G$ such that $K \leq \Phi(G)$ and $G/K \in \mathfrak{F}$. A formation $\mathfrak{F}$ is called ω'-saturated if $E_{\Phi_{\omega'}}\mathfrak{F} = \mathfrak{F}$ (see [2]). A group G is called ω-soluble if every non-abelian chief factor of G is an ω'-group.

A class $\mathfrak{F}^0$ was introduced by K. Doerk and T. Hawkes [1] as follows: $G \in \mathfrak{F}$ if and only if $(G \times G)/\{(g, g) \mid g \in Z(G)\} \in \mathfrak{F}$ (see [1, Proposition 3.11]).

Lemma 1 (see [1]). *Let $\mathfrak{F}$ be a non-empty formation. Then the following assertions hold:*

1) $\mathfrak{F}^0$ *is a formation;*

2) $\mathfrak{F}^0 \subseteq E_\Phi\mathfrak{F}$*;*

3) *if $G \in \mathfrak{F}^0$, then $[G^{\mathfrak{F}}, Aut(G)] = 1$.*

Lemma 2. *Let $\mathfrak{F} \neq \varnothing$ be a formation and $G = AB$, where A and B are normal in G and $A/A \cap G^{\mathfrak{F}}$ is ω-soluble. If $G^{\mathfrak{F}}$ is an ω-group, then $G^{\mathfrak{F}} = (G^{\mathfrak{F}} \cap A)(G^{\mathfrak{F}} \cap B)$.*

Lemma 3. *Let $G = A \times B$, D is normal in G and $D = (D \cap A)(D \cap B)$. Then $G/D \cong (A/D \cap A) \times (B/D \cap B)$.*

Lemma 4. *Let $\mathfrak{F} \neq \varnothing$ be a formation and $G = A \times B$, where $A/A \cap G^{\mathfrak{F}}$ is ω-soluble. Assume that $G^{\mathfrak{F}}$ is an ω-group. Then $G^{\mathfrak{F}} = A^{\mathfrak{F}}B^{\mathfrak{F}}$.*

Using Lemmas 1—4, we prove the following result.

Theorem. *Let ω be a set of primes, and let $\mathfrak{F}$ be a non-empty formation of ω-soluble groups. Then $\mathfrak{F}^0 \leq E_{\Phi_{\omega'}}\mathfrak{F}$.*

Corollary. *Let ω be a set of primes, and let $\mathfrak{F} \neq \varnothing$ be an ω'-saturated formation of ω-soluble groups. Then $(A \times B)^{\mathfrak{F}} = A^{\mathfrak{F}} \times B^{\mathfrak{F}}$ for any groups A and B.*

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8th International Algebraic Conference
July 5—12 **(2011)**, Lugansk, Ukraine SHORT COMMUNICATION

On partially ordered groups

E. Shirshova

Let G be a partially ordered group (po-group). G^+ denotes the positive cone of G, i.e., $\{x \in G | \, e \leqslant x\}$. Elements a and $b \in G^+$ are called *almost orthogonal* if the inequalities $c \leqslant a$ and $c \leqslant b$ imply $c^n \leqslant a$ and $c^n \leqslant b$ for every positive integer n and for all $c \in G$. A po-group G is said to be *($\mathcal{AO}$-group)* if each $g \in G$ has a representation $g = ab^{-1}$, where a and b are some almost orthogonal elements of G.

A po-group G is an *interpolation group* if whenever $a_1, a_2, b_1, b_2 \in G$ and $a_1, a_2 \leqslant b_1, b_2$, then there exists $c \in G$ such that $a_1, a_2 \leqslant c \leqslant b_1, b_2$. If G is a directed interpolation group, then G will be called a *Riesz-Fuchs group*. If G is an interpolation $\mathcal{AO}$-group, then G will be called a *pl-group*.

The usual terminology of partially ordered groups are used (see, for instance, [1,2]).

Theorem 1. *If φ is an o-isomorphism of an $\mathcal{AO}$-group A to a po-group B, then B is an $\mathcal{AO}$-group as well.*

A po-group G is called *a lex-extension* of a convex normal subgroup M by the po-group G/M if each strictly positive element in the group G/M consists entirely of positive elements of the group G.

Theorem 2. *Let G be a pl-group. If G is a lex-extension of a convex normal subgroup M, then M is a directed group.*

Theorem 3. *If a po-group G is a lex-extension of an $\mathcal{AO}$-group M, then G is an $\mathcal{AO}$-group if and only if the quotient group G/M is an $\mathcal{AO}$-group.*

Theorem 4. *Let a po-group G be a lex-extension of a po-group M. G is a pl-group if and only if G is a lex-extension of the pl-group M by the pl-group G/M.*

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Short communication

Two-elements generators systems of the meta-alternating groups of the infinite rank

V. Sikora

Let $\overline{n} =< n_1, n_2, \ldots >$ be an arbitrary non-finite sequence of the natural numbers ($n_i \in \mathbb{N}$, $n_i \geq 7, \forall\, i \in \mathbb{N}$). Let us consider the meta-alternating group

$$A(\overline{n}) = A_{n_1} \wr A_{n_2} \wr \ldots$$

of the infinite rank and meta-degree $(n_1, n_2, \ldots)$ (see [1]).

The following assertion are proved.

Theorem. *Let $h_1 \in A_{n_1}$, $h_1 \neq e$, $h_2(x_1)$ is the function from set $\{1, 2, \ldots, n_1\}$ to the group A_{n_2} and h_2 acts non-trivial at least in one point. Then we can construct non-finite table $v = [g_1, g_2(x_1), g_3(x_1, x_2), \ldots, g_k(x_1, x_2, \ldots, x_{k-1}), \ldots] \in A(\overline{n})$ for arbitrary table $u = [h_1, h_2(x_1), e, \ldots, e] \in A(\overline{n})$ and pair of u and v defines the generators system of the group $A(\overline{n})$ in the topological sense.*

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Groups and braces

L. V. Skaskiv

Let $(A,+)$ be an abelian group with a multiplication «·». As in [1] we call A a brace if A is right distributive, i.e.

- $(a+b)\cdot c=(a\cdot c)+(b\cdot c)$ for all $a,b,c\in A$, and
- A is a group with respect to circle operation " $\circ$ " defined by the rule

$$a\circ b=a+b+a\cdot b.$$

A group $(A,\circ)$ is called the adjoint group of a brace A and denoted by A°.

Let A be a brace, L a submodule of an A-module M, T a subgroup of A°. On the set of pairs

$$H(L,T)=\{(l,t)\mid l\in L, t\in T\}$$

we define a multiplication by the rule

$$(x,y)(u,v)=(xv+x+u, y\circ v)$$

for $x,u\in L$ and $y,v\in T$. Then $H(L,T)$ is a group. We prove the following

Theorem 1. *Let M be a module over a brace A, L a non-zero submodule of M, T a non-zero subgroup of A°. Then $H=H(L,T)=E\rtimes F$is a Frobenius group with a kernel E and a complement F, where E is isomorphic to the additive group L^+ of L and F is isomorphic to a subgroup T, if and only if the following hold:*

(i) *$L=Lh$ for every non-zero element $h\in T$,*

(ii) *$ann_T l=\{t\in T\mid lt=e\}=\{0\}$ for every non-zero element $l\in L$.*

Recall [1] that

$$A^{n+1}=A(A^n) \text{ and } A^{(n+1)}=(A^{(n)})A$$

for any positive integer n. A brace A is called right nilpotent (respectively left nilpotent) if $A^{(n)}=\{0\}$ for some positive integer n. In this way we obtain the following

Theorem 2. (1) *If A is a non-zero left nilpotent brace, then*

(i) *$H(A)$ is a nilpotent group;*

(ii) *$annA\neq\{0\}$.*

(2) *If A is a right nilpotent brace, then $H(A)$ is a solvable group.*

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Short communication

On laws of lattices of partially composition formations

A. N. Skiba, A. A. Tsarev, N. N. Vorobyov

All groups considered are finite. The subgroup $C^p(G)$ is the intersection of the centralizers of all the abelian p-chief factors of group G, with $C^p(G) = G$ if G has no abelian p-chief factors. For any set of groups $\mathfrak{X}$ we denote by $\mathrm{Com}^+(\mathfrak{X})$ the class of all simple abelian groups A such that $A \cong H/K$, where H/K is a composition factor of $G \in \mathfrak{X}$. The symbol $R_\omega(G)$ denotes the product of all soluble normal ω-subgroups of G. Let ω be a non-empty set of primes, $\omega' = \mathbb{P} \setminus \omega$. Let f be a function of the form

$$f : \omega \cup \{\omega'\} \to \{\text{formations of groups}\}. \tag{1}$$

Let

$$CF_\omega(f) = (G \mid G/R_\omega(G) \in f(\omega') \text{ and } G/C^p(G) \in f(p) \text{ for all } p \in \omega \cap \pi(\mathrm{Com}^+(G))).$$

If $\mathfrak{F}$ is a formation such that $\mathfrak{F} = CF_\omega(f)$ for some function f of the form (1), then $\mathfrak{F}$ is said to be ω-*composition* and f is said to be an ω-*composition satellite* of $\mathfrak{F}$ [1].

Every formation is 0-multiply ω-composition by definition. For $n > 0$, a formation $\mathfrak{F}$ is called n-*multiply* ω-*composition* if $\mathfrak{F} = CF_\omega(f)$ and all non-empty values of f are $(n-1)$-multiply ω-composition formations [1]. With respect to inclusion $\subseteq$ the set of all n-multiply ω-composition formations c_n^ω is a complete lattice.

The following theorems are proved.

Theorem 1. *Let $n \geqslant 1$. Then every law of the lattice of all formations c_0^ω is fulfilled in the lattice of all n-multiply ω-composition formations c_n^ω.*

Theorem 2. *Let $n \geqslant 1$. If ω is an infinite set, then the law system of the lattice c_0^ω coincides with the law system of the lattice c_n^ω.*

Corollary 1. *Let m and n be nonnegative integers. Then the law systems of the lattices c_m and c_n coincide.*

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SHORT COMMUNICATION

Вільна група континуального рангу, що породжується дійсними монотонними функціями

М. І. Сумарюк

У 1949 році Б. Нейман довів [2], що у групі всіх дійсних монотонних функцій над $\mathbb{R}$ існує вільна підгрупа континуального рангу. Доведення результату цієї статті є громіздким і технічно складним, яке не дає підходів побудувати конкретні зображення вільних груп континуального рангу дійсними монотонними функціями.

У даній роботі ми вказуємо конкретні аналітичні задання континуальної множини дійсних монотонних функцій, які вільно породжують вільну групу. Результат даної роботи грунтується на такому твердженні [1].

Теорема 1. *Нехай* $\Phi = \{\varphi_i\}_{i\in\mathbb{I}}$ *— система строго монотонних неперервних дійсних функцій над деякою областю* $\mathbb{D}$ *і виконуються наступні умови:*

1) *для довільних двох різних функцій* φ_1 *та* φ_2 *із* Φ *наступні суперпозиції* $\varphi_1 \circ \varphi_1$ *і* $\varphi_1^{(\varepsilon_1)} \circ \varphi_2^{(\varepsilon_2)}$, *де* $\varepsilon_1, \varepsilon_2 \in \{-1, 1\}$, *не здійснюють тотожного перетворення області* $\mathbb{D}$*;*

2) *існує точка* $x_0 \in \mathbb{D}$, *яка є спільною нерухомою точкою для всіх функцій* $\varphi \in \Phi$*;*

3) *кожна пара функцій* φ *та* φ^{-1}, *де* $\varphi \in \Phi$, *є двічі неперервно диференційовною в області* $\mathbb{D}$, *крім того, маємо* $\varphi'(x_0) = 1$ *та* $\varphi''(x_0) \neq 0$.

Тоді група $< \Phi >$, *що породжується системою* Φ, *є вільною групою з вільною базою* Φ.

Теорема 2. *Система непарних монотонних функцій*

$$f_\alpha(x) = \begin{cases} g_\alpha(x), & \text{якщо } x \geq 0, \\ -g_\alpha(|x|), & \text{якщо } x < 0, \end{cases}$$

де

$$g_\alpha(x) = x \exp\left(\frac{1}{2\alpha}(x^\alpha - 1)^2\right), x \geq 0, \alpha > 0,$$

є континуальною базою вільної групи.

Доведення теореми 2 випливає з теореми 1, якщо для даних функцій перевірити її умови у випадку області $\mathbb{D} = (0; +\infty)$ та нерухомої точки $x_0 = 1$.

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CONTRIBUTED ABSTRACT

Lie algebra associated with the Sylow p-subgroup of the automorphism group of a p-adic rooted tree

V. Sushchansky

Let T_p be a p-adic rooted tree, $Aut\,T_p$ be the automorphism group of T_p. The group $Aut\,T_p$ is profinite and hence all Sylow p-subgroups of this group are conjugated. In the talk we investigate a Sylow p-subgroup P of $Aut\,T_p$. In particular, the lower central series and the Lasard series of the pro-p-group P are described. This allows us to study the Lie algebra $L(P)$ over a p-element of field F_p assosiated with the group P. We show that $L(P)$ is isomorphic to a subalgebra of the inverse limit of maximal nilpotent subalgebras in simple classical algebras $A_n, n = 1, 2, \ldots$ over F_p, defined in [1]. For the finitary automorphism group of T_p a similar construction is proposed in [2]. The lower central series and the derived series of $L(P)$ are described.

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SHORT COMMUNICATION

On separated lattices of partially composition formations

A. A. Tsarev, N. N. Vorobyov

All groups considered are finite. All unexplained notations and terminologies are standard [1, 2]. In each group G we select a system of subgroups $\tau(G)$. We say that τ is a *subgroup functor* [1] if

1) $G \in \tau(G)$ for every group G;

2) for every epimorphism $\varphi : A \mapsto B$ and any $H \in \tau(A)$, $T \in \tau(B)$, we have $H^\varphi \in \tau(B)$ and $T^{\varphi^{-1}} \in \tau(A)$.

We consider only subgroup functors τ such that for any group G the set $\tau(G)$ consists of subnormal subgroups of G. A formation $\mathfrak{F}$ is called τ*-closed* if $\tau(G) \subseteq \mathfrak{F}$ for every group G of $\mathfrak{F}$.

Let Θ be a complete lattice of formations. The symbol $\Theta\mathrm{form}\mathfrak{X}$ denotes the intersection of all formations of Θ containing a collection of groups $\mathfrak{X}$. In particular, if $\mathfrak{X} = \{G\}$, we write $\Theta\mathrm{form}G$. Let $\{\mathfrak{F}_i \mid i \in I\}$ be an arbitrary collection of formations of Θ. We denote

$$\vee_\Theta(\mathfrak{F}_i \mid i \in I) = \Theta\mathrm{form}(\bigcup_{i\in I} \mathfrak{F}_i).$$

Let $\mathfrak{X}$ be a non-empty class of groups. A complete lattice of formations Θ is called $\mathfrak{X}$-*separated* [1] if for every term $\xi(x_1, \ldots, x_m)$ of the signature $\{\cap, \vee_\Theta\}$, every formations of Θ $\mathfrak{F}_1, \ldots, \mathfrak{F}_m$ and every group $A \in \mathfrak{X} \cap \xi(\mathfrak{F}_1, \ldots, \mathfrak{F}_m)$ there exist $\mathfrak{X}$-groups $A_1 \in \mathfrak{F}_1, \ldots, A_m \in \mathfrak{F}_m$ such that $A \in \xi(\Theta\mathrm{form}A_1, \ldots, \Theta\mathrm{form}A_m)$.

We prove:

Theorem 1. *The lattice of all τ-closed n-multiply ω-composition formations is $\mathfrak{G}$-separated.*

Corollary 1. *The lattice of all n-multiply ω-composition formations is $\mathfrak{G}$-separated.*

Corollary 2. *The lattice of all n-multiply formations is $\mathfrak{G}$-separated.*

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8th International Algebraic Conference
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On crystallographic groups of Lobachevski plane

N. V. Varekh, A. A. Dishlis, M. V. Tsybanov

Crystallographic groups are isometric groups of crystals. Strong definition of these groups may be given by means of concept of lattice of Euclidean space. At such approach these definitions may be transferred in plane and space of Lobachevski [1]. As yet in 1994 it was observed relation between crystallographic groups of Lobachevski space and quasicrystallographic groups of Euclidean space [2]. Quasicrystallographic group acts in variant on certain quasilattice of n-dimensional vector space Vn, i.e on linear span of m integer-valued linearly independent vector of Vn (m>n). In particular root lattices, constructed on roots of generalized root system [3] is quasilattices (root quasilattice). Groups of affine transformations of vector space, that bijective map certain its quasilattive on itself, is called symmetry group of this quasilattice [4].

Theorem 1. *Theorem 1. Symmetry group of any plane root quasilattice is isomorhic to some crystallographic group of Poincare model of Lobachevski plane.*

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8th International Algebraic Conference
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SHORT COMMUNICATION

Finite groups with m-supplemented maximal subgroups of Sylow subgroups

V. A. Vasilyev

Throughout this paper, all groups are finite.

Recall that a subgroup M of a group G is a modular subgroup in G, if the following conditions are true:

(1) $\langle X, M \cap Z\rangle = \langle X, M\rangle \cap Z$ for all $X \leq G, Z \leq G$ with $X \leq Z$;

(2) $\langle M, Y \cap Z\rangle = \langle M, Y\rangle \cap Z$ for all $Y \leq G, Z \leq G$ with $M \leq Z$.

We note that a modular subgroup is a modular element (in sense of Kurosh, [1, Chapter 2]) of the lattice of all subgroups of a group. For the first time the concept of modular subgroup was analyzed by R. Schmidt [2]. In the book of R. Schmidt [1, Chapter 5] modular subgroups were used to obtain new characterizations of various classes of groups. The subgroup which is generated by two its modular subgroups itself is a modular subgroup. Thus every subgroup H of a group G has the largest modular subgroup H_{mG} of G contained in H. We introduce the following concept

Definition. *A subgroup H of a group G is called m-supplemented in G if there exists a subgroup K of G such that $G = HK$ and $H \cap K \leq H_{mG}$.*

It's easy to see that every modular subgroup is m-supplemented and, at the same time, there exist groups in which the class of m-supplemented subgroups is wider than the class of all its modular subgroups.

Theorem. *Let E be a normal subgroup of a group G. If the maximal subgroups of every Sylow subgroup of E are m-supplemented in G, then each chief factor of G below E is cyclic.*

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Contributed abstract

On $\mathbb{P}$-accessible subgroups of finite groups

T. I. Vasilyeva, A. F. Vasilyev

We consider only finite groups. The concept of an $\mathfrak{F}$-accessible group was proposed by Kegel in [1].

Let $\mathfrak{F}$ be a non-empty formation. A subgroup H of a group G is called $\mathfrak{F}$-accessible in G [1, 2] if there exists a chain of subgroups $H = H_0 \subseteq H_1 \subseteq \cdots \subseteq H_{n-1} \subseteq H_n = G$ such that either H_{i-1} is normal in H_i or $H_i^{\mathfrak{F}} \subseteq H_{i-1}$ for all $i = 1, \ldots, n$. Here denote by $G^{\mathfrak{F}}$ the $\mathfrak{F}$-coradical of a group G, i.e., the smallest normal subgroup of G with $G/G^{\mathfrak{F}} \in \mathfrak{F}$.

Definition. *A subgroup H of a group G is called $\mathbb{P}$-accessible in G if there exists a chain of subgroups $H = H_0 \subseteq H_1 \subseteq \cdots \subseteq H_{n-1} \subseteq H_n = G$ such that either H_{i-1} is normal in H_i or $|H_i : H_{n-1}|$ is a prime for every $i = 1, \ldots, n$.*

Denote by $\mathfrak{U}$ the class of all supersoluble groups. Every $\mathfrak{U}$-subnormal subgroup of G is $\mathbb{P}$-accessible in G. In the general case the converse proposition it is false.

We find the properties of $\mathbb{P}$-accessible subgroups.

A formation $\mathfrak{F}$ is called π-saturated [3] whenever $G/\Phi(G) \cap O_\pi(G) \in \mathfrak{F}$, then $G \in \mathfrak{F}$.

Theorem. *Let $\mathfrak{F} = (G$ is a soluble group $\mid$ every Sylow p-subgroup for $p \in \pi$ is $\mathbb{P}$-accessible in $G)$. Then $\mathfrak{F}$ is a hereditary π-saturated formation.*

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8th International Algebraic Conference
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SHORT COMMUNICATION

About characterization of injectors of finite groups

I. V. Vatsuro, N. T. Vorobyov

All groups considered are finite.

In definitions and notations we follow [1].

Let $\pi = Supp(f) = \{p \in P | f(p) \neq \emptyset\}$ be a support of the function f and $SLR(f) = \cap_{p\in\pi} f(p)\mathfrak{E}_{\mathfrak{p}'}$. Then a Fitting class $\mathfrak{F}$ is called semilocal if there exists an H-function f such that $\mathfrak{F} = SLR(f)$.

We recall that if $\mathfrak{F}$ is class of finite groups, then an $\mathfrak{F}$-injector of a group G is a subgroup V of G with the property that $V \cap N$ is an $\mathfrak{F}$-maximal subgroup of N for all subnormal subgroups N of G.

Definition 1. *Let f be an H-function of class $\mathfrak{F}$ and $\pi = Supp(f)$. A subgroup G_f of the group G is called f-radical of G if $G_f = \prod_{p\in\pi} G_{f(p)}$.*

An H-function f is called full if $f(p)\mathfrak{N}_p = f(p)$ for all primes p and $\mathfrak{X}$-constant if there exists a Fitting class $\mathfrak{X}$ such that $f(p) = \mathfrak{X}$ for all $p \in Supp(f)$.

It is proved

Theorem. *Let $\mathfrak{F} = SLR(f)$ for some full $\mathfrak{X}$-constant H-function f with support π and a group G such that $G/G_{\mathfrak{X}}$ is soluble. A subgroup V is an $\mathfrak{F}$-injector of G if and only if V/G_f is a Hall π'-subgroup of G/G_f.*

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SHORT COMMUNICATION

On the structure of the Lockett section of Fitting functors

E .A. Vitko, N. T. Vorobyov

The notations used in this paper are standard [1, 2].

All groups considered in the paper are finite.

The Lockett section of a Fitting class contains the maximal and minimal elements. Besides the structure of the minimal element is described (see X.1.17 [1]). In [3] the problem about the structure of the minimal element of the Lockett section of a Fitting functor in the class $\mathfrak{S}$ of finite soluble groups was posed (problem 8.7 [2]). Statement of the problem generally in the class $\mathfrak{S}^\pi$ of finite π-soluble groups is natural.

Definition 1. *Let $\mathfrak{X}$ be a non-empty Fitting class. A Fitting $\mathfrak{X}$-functor is a mapping f which assigns to each $G \in \mathfrak{X}$ a non-empty set $f(G)$ of subgroups of G such that the following conditions are satisfied: (i) if α is an isomorphism of G onto $\alpha(G)$, then $f(\alpha(G)) = \{\alpha(X) : X \in f(G)\}$; (ii) if $N \trianglelefteq G$, then $f(N) = \{X \cap N : X \in f(G)\}$.*

We call a Fitting $\mathfrak{X}$-functor f 1) π-soluble if $\mathfrak{X} = \mathfrak{S}^\pi$; 2) hereditary if class $\mathfrak{X}$ is hereditary; 3) conjugate if $f(G)$ is a conjugacy class of subgroups of G for all $G \in \mathfrak{X}$.

Denote by Hall_π the π-soluble functor such that $\mathrm{Hall}_\pi(G) = \{G_\pi : G_\pi$ is a Hall π-subgroup of $G\}$. Let f and g be hereditary Fitting $\mathfrak{X}$-functors, then the set $(f \circ g)(G) = \{X : X \in f(Y)$ for some $Y \in g(G)\}$ is called [2] their product. Let f and g be conjugate Fitting $\mathfrak{X}$-functors. Then f is said to be strongly contained in g, denoted $f \ll g$, provided that the following condition hold: if $X \in f(G)$, then there is a $Y \in g(G)$ such that $X \leq Y$. Let f be a Fitting $\mathfrak{X}$-functor. Define f^* by $f^*(G) = \{\pi_1(T) : T \in f(G^2)\}$ for each $G \in \mathfrak{X}$ where π_1 is the projection T onto the first component.

Definition 2. *Let $\mathfrak{X}$ be a non-empty Fitting class, f be a conjugate Fitting $\mathfrak{X}$-functor. By the Lockett section of f is meant $\mathrm{Locksec}(f) = \{g : g$ is a conjugate Fitting $\mathfrak{X}$-functor and $f^* = g^*\}$.*

Let $\mathfrak{X}$ be a non-empty Fitting class, f be a conjugate Fitting $\mathfrak{X}$-functor. Then denote by f_* a conjugate Fitting $\mathfrak{X}$-functor such that $f_* \in \mathrm{Locksec}(f)$ and $f_* \ll g$ for all $g \in \mathrm{Locksec}(f)$.

The following theorem is the solution of the specified problem.

Theorem. *If π is a non-empty set of primes and the π-soluble Fitting functor $f = Hall_\pi$, then $f_* = f \circ Rad_{(\mathfrak{S}^\pi)_*}$.*

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8th International Algebraic Conference
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SHORT COMMUNICATION

On closure operation on classes of finite groups

S. N. Vorobyov

All groups considered are finite soluble. All unexplained notations and terminologies are standard (see [1]). A class map C is called a closure operation if for all classes $\mathfrak{X}$ and $\mathfrak{Y}$ the following conditions are satisfied:

1) $\mathfrak{X} \subseteq \mathrm{C}\mathfrak{X}$;

2) $\mathrm{C}\mathfrak{X} = \mathrm{C}(\mathrm{C}\mathfrak{X})$;

3) if $\mathfrak{X} \subseteq \mathfrak{Y}$, then $\mathrm{C}\mathfrak{X} \subseteq \mathrm{C}\mathfrak{Y}$.

A class $\mathfrak{X}$ is said to be C-closed if $\mathfrak{X} = \mathrm{C}\mathfrak{X}$.

Let $\mathfrak{N}^k$ be the class of groups of nilpotent length at most k. For a class of groups $\mathfrak{X}$ we define:

$$\mathrm{S}_{\mathrm{F}\mathfrak{N}^k}\mathfrak{X} = (H : H \leq G \in \mathfrak{X} \text{ and } H^{\mathfrak{N}^k} \lhd\lhd G).$$

In [2] it was proved that $\mathrm{S}_{\mathrm{F}\mathfrak{N}^k}$ is a closure operation. If $\mathrm{S}_{\mathrm{F}\mathfrak{N}^k}\mathfrak{X} = \mathfrak{X}$, we call $\mathfrak{X}$ $\mathrm{S}_{\mathrm{F}\mathfrak{N}^k}$-closed.

We prove:

Theorem. *A class $\mathfrak{X}$ is a $\mathrm{S}_{\mathrm{F}\mathfrak{N}^k}$-closed if and only if $1 \neq G, G \in \mathfrak{X}, M < \cdot\, G$ and $H^{\mathfrak{N}^{k+1}} \leq M$ implies $M \in \mathfrak{X}$.*

A Fitting class $\mathfrak{F}$ is called a Fischer class (see [1]) if

1) $\mathfrak{F} = \mathrm{N}_0\mathfrak{F} \neq \varnothing$, and

2) if $K \lhd G \in \mathfrak{F}$ and H/K is a nilpotent subgroup of G/K, then $H \in \mathfrak{F}$.

If $k = 1$ we have:

Corollary. *A Fitting class $\mathfrak{X}$ is a Fischer class if and only if for non-identity group $G \in \mathfrak{X}$ and a maximal subgroup M of G such that M contains the nilpotent residual of G, M is an $\mathfrak{X}$-group.*

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Short communication

On the characterization of products of π-normal Fitting classes

N. T. Vorobyov, A. V. Turkouskaya

All groups considered are finite and soluble. The notation used in this paper are standard [1].

Let π be a non-empty set of primes. A Fitting class $\mathfrak{F}$ is said to be π-normal or normal in a class of all finite soluble π-groups $\mathfrak{S}_\pi$, if $\mathfrak{F} \subseteq \mathfrak{S}_\pi$ and $G_\mathfrak{F}$ is $\mathfrak{F}$-maximal in G for all $G \in \mathfrak{S}_\pi$.

Note that if $\pi = P$, where P is a set of all primes, the Fitting class $\mathfrak{F}$ is normal [2].

If $\mathfrak{F}$ and $\mathfrak{H}$ are Fitting classes, then a class of groups $\mathfrak{F} \circ \mathfrak{H} = (G : G/G_\mathfrak{F} \in \mathfrak{H})$ is a Fitting product of $\mathfrak{F}$ with $\mathfrak{H}$. We can easily show that if either $\mathfrak{F}$ or $\mathfrak{H}$ is π-normal, then $\mathfrak{F} \circ \mathfrak{H}$ is π-normal. For characterization of π-normal Fitting classes we will use the Lockett's star operation [3]. Let $\mathfrak{F}$ be an arbitrary Fitting class, then the class $\mathfrak{F}^*$ is the smallest Fitting class containing $\mathfrak{F}$ and for any groups G and H we have $(G \times H)_{\mathfrak{F}^*} = G_{\mathfrak{F}^*} \times H_{\mathfrak{F}^*}$ [3].

It is proved

Theorem 1. *Let π be a non-empty set of primes, $\mathfrak{F}$ and $\mathfrak{H}$ are π-normal Fitting classes. Any two of the following statements are equivalent:*

(a) $\mathfrak{F} \circ \mathfrak{H}$ *is normal in* $\mathfrak{S}_\pi$*;*

(b) $\mathfrak{F} \circ \mathfrak{H}^*$ *is normal in* $\mathfrak{S}_\pi$*;*

(c) $\mathfrak{F}^* \circ \mathfrak{H}$ *is normal in* $\mathfrak{S}_\pi$*;*

(d) $\mathfrak{F}^* \circ \mathfrak{H}^* = \mathfrak{S}_\pi$*;*

(e) *There exists a set $\sigma \subseteq \pi$ of primes such that $\mathfrak{F}^* \circ \mathfrak{S}_\sigma = \mathfrak{F}^*$ and $\mathfrak{S}_\sigma \circ \mathfrak{H}^* = \mathfrak{S}_\pi$;*

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SHORT COMMUNICATION

О реализации транзитивных групп подстановок в качестве групп изометрий

М. В. Зельдич

Проблемы реализации групп и групп подстановок автоморфизмами разных типов дискретных структур, в частности, графов и метрических пространств с разными ограничениями хорошо известны ещё с тридцатых годов прошлого столетия (см., например, обзорную статью [1]).

Первая из них, т.е. проблема реализации абстрактных групп для большинства классов рассматриваемых дискретных структур, решается положительно. Например, известно, что каждая конечная абстрактная группа реализуется как полная группа автоморфизмов некоторого графа, некоторого частичного порядка на множестве или как полная группа изометрий конечного метрического пространства [1]. Проблема же конкретной реализации, т.е. реализации групп подстановок в виде полных групп автоморфизмов дискретных структур из того или иного класса, значительно труднее и полное её решение неизвестно для большинства классов дискретных структур. Например, хорошо известно, что не каждая группа подстановок может быть реализована, как полная группа автоморфизмов графа или полная группа изометрий метрического пространства; однако вопрос, какие именно группы подстановок могут так реализоваться - это известные трудные проблемы [1].

Настоящий доклад посвящён изучению вопроса о возможности реализации транзитивных групп подстановок в качестве полных групп изометрий.

Вводится ряд новых понятий, в терминах которых устанавливаются некоторые результаты о свойствах и структуре групп изометрий конечных метрических пространств, в частности, с метрикой, инвариантной относительно транзитивного действия на метрическом пространстве заданной группы подстановок (например, регулярного действия конечной метризованной группы на себе).

В качестве следствий при $n > 2$ получены элементарные доказательства невозможности реализации знакопеременной группы чётных подстановок An (в её естественном представлении подстановками) в качестве группы изометрий [2], а также, напротив, возможности такой реализации для группы диэдра Dn в её регулярном подстановочном представлении.

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СВЕДЕНИЯ ОБ АВТОРАХ

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Topical section VI

Representations and Linear Algebra

8th International Algebraic Conference
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Quasi multiplicative bases for bimodule problems

V. Babych, N. Golovashchuk, S. Ovsienko

Let $\Bbbk$ be an algebraically closed field. A pair $\mathcal{A} = (\mathsf{K}, \mathsf{V})$ consisting of the regular category K over $\Bbbk$ and K-bimodule V is called a *normal bimodule problem* over K. We assume that K acts sincerely on V. For a locally finite dimensional normal bimodule problem $\mathcal{A} = (\mathsf{K}, \mathsf{V})$ let $\Sigma_0 = \mathrm{Ob}\,\mathsf{K}$, and for any $X, Y \in \mathrm{Ob}\,\mathsf{K}$ let $\Sigma_1^0(X, Y)$ be a basis of $\mathsf{V}(X, Y)$ and let $\Sigma_1^1(X, Y)$ be a basis of $\mathrm{Rad}\,\mathsf{K}(X, Y)$. The bigraph $\Sigma = (\Sigma_0, \Sigma_1)$ is called a *basis* of the bimodule problem $\mathcal{A}$.

Let T be the ternary relation on Σ_1 such that $(a, b, c) \in \mathrm{T}$, if $cb = \sum_{x \in \Sigma_1} \lambda_x x$, $\lambda_x \in \Bbbk$, and $\lambda_a \neq 0$. Such a triple $(a, b, c) \in \mathrm{T}$ we call a triangle. Denote by T_0 the subset of all $(a, b, c) \in \mathrm{T}$ with $a, b \in \Sigma_1^0$, $\varphi \in \Sigma_1^1$, and by T_1 the subset of all $(a, b, c) \in \mathrm{T}$ with $a, b, c \in \Sigma_1^1$. The 2-dimensional complex $\mathbf{\Sigma}$ on the bigraph Σ is defined by setting $\mathbf{\Sigma}_2 = \{\mathsf{C}_t \mid t \in \mathrm{T}\}$ and $\partial(\mathsf{C}_t) = \langle c_t b_t a_t^{-1} \rangle$, where $t = (a_t, b_t, c_t) \in \mathrm{T}$.

A normal bimodule problem $\mathcal{A} = (\mathsf{K}, \mathsf{V})$ is called *admitted*, provided that $\mathrm{Ob}\,\mathsf{K}$ can be decomposed as $\mathrm{Ob}\,\mathsf{K} = \mathrm{Ob}\,\mathsf{K}^+ \sqcup \mathrm{Ob}\,\mathsf{K}^-$ such that condition $\mathsf{V}(X, Y) \neq 0$ implies $X \in \mathrm{Ob}\,\mathsf{K}^-$, $Y \in \mathrm{Ob}\,\mathsf{K}^+$, and $\mathrm{Rad}\,\mathsf{K}(X, Y) \neq 0$ implies $X, Y \in \mathrm{Ob}\,\mathsf{K}^+$.

Theorem. *Let $\mathcal{A}$ be a finite dimensional admitted normal bimodule problem which does not contain any subproblem of strictly unbounded type. Then there exist a basis Σ of $\mathcal{A}$ and an associated complex $\mathbf{\Sigma}$ such that:*

1) every $\varphi \in \mathbf{\Sigma}_1^1$ belongs either to one or two triangles from T_0;

2) if $\varphi \in \mathbf{\Sigma}_1^1$ belongs to two triangles $(a_1, b_1, \varphi), (a_2, b_2, \varphi) \in \mathrm{T}_0$, then $a_1 \neq a_2, b_1 \neq b_2$;

3) for two solid arrows $a \neq b$ with a common starting vertex, the pair (a, b) belongs to at most two triangles from T_0;

4) in the latter case, if there are $(a, b, \varphi_1), (a, b, \varphi_2) \in \mathrm{T}_0$ with $\varphi_1 \neq \varphi_2$, then there exist two other triangles $(a_1, b_1, \varphi_1), (a_2, b_2, \varphi_2) \in \mathrm{T}_0$ such that $a_1 \neq a_2, b_1 \neq b_2$.

Such a basis we call *quasi multiplicative*.

Remark. *There exists a list $\mathcal{A}^i = (\mathsf{K}^i, \mathsf{V}^i)$, $i = 1, 2, 3, 4$, of admitted bimodule problems of strictly unbounded type with $|\,\mathrm{Ob}\,\mathsf{K}| \leqslant 7$ such that the condition of the theorem is enough to check for the problems of this list.*

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$8^{\rm th}$ International Algebraic Conference
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SHORT COMMUNICATION

On indecomposable projective representations of direct products of finite groups over a field of characteristic p

Leonid F. Barannyk

Let K be a field of characteristic $p > 0$, K^* the multiplicative group of K and $G = G_p \times B$ a finite group, where G_p is a p-group and B is a p'-group. We assume that $|G_p| \neq 1$ and $|B| \neq 1$. Denote by l the product of all pairwise distinct prime divisors of $|B|$. We also assume that if G_p is non-abelian, then $[K(\varepsilon) : K]$ is not divisible by p, where ε is a primitive $l^{\rm th}$ root of 1. Let $K^\lambda G$ be a twisted group algebra of G over K with a 2-cocycle $\lambda \in Z^2(G, K^*)$. We have $K^\lambda G \cong K^\mu G_p \otimes_K K^\nu B$, where μ is the restriction of λ to $G_p \times G_p$ and ν is the restriction of λ to $B \times B$.

If every indecomposable $K^\lambda G$-module is isomorphic to the outer tensor product $V\#W$, where V is an indecomposable $K^\mu G_p$-module and W is an irreducible $K^\nu B$-module, then we say that the algebra $K^\lambda G$ is of OTP representation type. A group $G = G_p \times B$ is defined to be of OTP projective K-representation type if there exists a cocycle $\lambda \in Z^2(G, K^*)$ such that $K^\lambda G$ is of OTP representation type. We say that a group G is of purely OTP projective K-representation type if $K^\lambda G$ is of OTP representation type for any $\lambda \in Z^2(G, K^*)$.

We prove the following theorems.

Theorem 1. *Let D be the subgroup of G_p such that $G'_p \subset D$ and $D/G'_p = \mathrm{soc}(G_p/G'_p)$. Assume that if $K^\mu G_p$ is not a uniserial algebra, $|G'_p| = p$ and $\dim_K(K^\mu G_p/\mathrm{rad}K^\mu G_p) = p^{-2}\cdot |G_p|$, then $|D : Z(D)| \leq p^2$, where $Z(D)$ is the center of D. Then the algebra $K^\mu G_p \otimes_K K^\nu B$ is of OTP representation type if and only if $K^\mu G_p$ is a uniserial algebra or K is a splitting field for $K^\nu B$.*

Theorem 2. *Let $G = G_p \times B$ be an abelian group and s the number of invariants of G_p. The group G is of OTP projective K-representation type if and only if $p^{s-1} \leq [K : K^p]$ or B has a subgroup H such that $B/H \cong C \times C$ and K contains a primitive $m^{\rm th}$ root of 1, where $m = \max\{\exp(B/H), \exp H\}$.*

Theorem 3. *A nilpotent group $G = G_p \times B$ is of purely OTP projective K-representation type if and only if G_p is cyclic or $K = K^q$ and K contains a primitive $q^{\rm th}$ root of 1, for each prime $q \mid |B|$.*

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8th International Algebraic Conference
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PLENARY TALK

Castelnuovo-Mumford regularity of algebras of SL_2-invariants

L. Bedratyuk

Let K be a field, $\operatorname{char} K = 0$. Let V_d be $d+1$-dimensional SL_2-module of binary forms of degree d and let $V_{\boldsymbol{d}} = V_{d_1} \oplus V_{d_2} \oplus \cdots \oplus V_{d_n}$, $\boldsymbol{d} = (d_1, d_2, \ldots, d_n)$. Denote by $K[V_{\boldsymbol{d}}]^{SL_2}$ the algebra of polynomial SL_2-invariant functions on $V_{\boldsymbol{d}}$. It is well known that the algebra $\mathcal{I} := K[V_{\boldsymbol{d}}]^{SL_2}$ is finitely generated and let $f_1, f_2, \ldots, f_m$ be its minimal generating set.The measure of the intricacy of this algebra is the length of their chains of syzygies, called homological(or proective) dimension $\operatorname{hd}\mathcal{I}$. In [1] Brouwer and Popoviciu gave a classification of the cases in which $\operatorname{hd}\mathcal{I} \leq 15$. Let $R := K[x_1, \ldots, x_m]$ be positively graded by $\deg(x_i) = \deg(f_i)$, $i = 1, \ldots, m$. Recall that a finite graded free resolution of $\mathcal{I}$ of length $l = \operatorname{hd}\mathcal{I}$ is an exact sequence of R-modules

$$0 \longrightarrow F_l \longrightarrow F_{l-1} \longrightarrow \cdots \longrightarrow F_1 \longrightarrow F_0 \longrightarrow \mathcal{I} \longrightarrow 0,$$

where $F_i = \oplus_j R(-a_{i,j})^{\beta_{i,j}}$, $F_0 = R$ are finitely generated graded free R-modules and $R(-a_{i,j})$ denotes a free R-module generated by homogeneous polynomial of degree $a_{i,j}$. The image F_i is called the i-th module of syzygies of $\mathcal{I}$. Since the algebra $\mathcal{I}$ is Cohen-Macaulay then the Auslander-Buchsbaum theorem implies that $\operatorname{hd}\mathcal{I} = \dim \mathcal{I} - \operatorname{tr}\deg_K \mathcal{I}$, see also [2]. The Castelnuovo-Mumford regularity $\operatorname{reg}\mathcal{I}$ of the algebra $\mathcal{I}$ is called the maximum of the numbers $a_{i,j} - i$. For example, if $\mathcal{I}$ is free(i.e. $\operatorname{hd}\mathcal{I} = 0$), the regularity of $\mathcal{I}$ is the maximum of the degrees of a set of homogeneous minimal generators of $\mathcal{I}$.

We present here the results of calculations of the Castelnuovo-Mumford regularity for the algebras of SL_2-invariants $\mathcal{I}$ in the cases $\operatorname{hd}\mathcal{I} \leq 8$:

$\boldsymbol{d}$	2	3	4	$(1,1)$	$(1,2)$	$(2,2)$	(1^3)	5	6	(1,3)	(1,4)
$\operatorname{reg}\mathcal{I}$	2	4	3	2	3	2	2	35	29	11	17
$\operatorname{hd}\mathcal{I}$	0	0	0	0	0	0	0	1	1	1	1

$\boldsymbol{d}$	(2,3)	(2,4)	$(4,4)$	$(1,1,2)$	$(1,2,2)$	(2^3)	(1^4)	$(3,3)$	8
$\operatorname{reg}\mathcal{I}$	13	11	11	5	7	5	3	18	42
$\operatorname{hd}\mathcal{I}$	1	1	1	1	1	1	1	2	3

$\boldsymbol{d}$	(1^5)	$(1^3,2)$	$(1,2^3)$	(2^4)	$(1^2,2^2)$	(1^6)	$(1,1,3)$
$\operatorname{reg}\mathcal{I}$	7	13	20	15	14	12	42
$\operatorname{hd}\mathcal{I}$	3	4	5	5	6	6	8

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CONTRIBUTED ABSTRACT

On the variety of commuting triples of matrices

V. V. Beniash-Kryvets

Let $C(3,n) = \{(A,B,C) \in M_n(F)^3 \mid AB = BA, AC = CA, BC = CB\}$ denote the variety of triples of commuting $(n \times n)$ matrices over an algebraically closed field F of characteristic zero. Gerstenhaber [1] proposed the question for which natural numbers n this variety is irreducible. Guralnick proved in [2] that the answer is positive for $n = 3$ and negative for $n \geq 32$. Using the results of Neubauer and Sethuraman [3], Guralnick and Sethuraman [4] proved that $C(3,n)$ is irreducible for $n = 4$. In [5] Holbrook and Omladič proved that $C(3,n)$ is irreducible for $n = 5$. Extending the dimension argument in [2] they also proved that $C(3,n)$ is reducible for $n \geq 30$. Using perturbation by generic triples Omladič [6], Han [7], and Šivic [8] proved that $C(3,n)$ is irreducible for $n = 6$, $n = 7$, and $n = 8$ respectively. Using some modification of arguments in [2] and [5] we prove the following theorem.

Theorem 1. *The variety $C(3,n)$ is reducible for $n = 29$.*

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Contributed abstract

Про групу невироджених ортогональних операторів на алгебрі Лі L

С. В. Білун

Розглядаються алгебри Лі L (не обов'язково скінченновимірні) над алгебраїчно замкненим полем характеристики 0, на яких визначено лінійний оператор $T(\neq \pm E)$ такий, що $[T(x), T(y)] = [x, y]$ для довільних $x, y \in L$.

Для зручності лінійний оператор $T : L \longrightarrow L$ будемо називати лієво ортогональним, якщо $[T(x), T(y)] = [x, y]$ для довільних $x, y \in L$. Неважко переконатися, що множина невироджених лієво ортогональних операторів на алгебрі Лі L утворює групу, яку ми будемо позначати через $O(L)$ (її можна розглядати як аналог ортогональної групи евклідового простору). Ми вказуємо деякі інваріантні відносно $O(L)$ підмножини алгебри Лі L.

Твердження 1. *Нехай L — довільна алгебра Лі над полем $\mathbb{K}$, I — ідеал алгебри L такий, що центр фактор-алгебри L/I нульовий. Тоді ідеал I інваріантний відносно лієво ортогональної групи $O(L)$, тобто $T(I) \subseteq I$ для довільного $T \in O(L)$.*

Теорема 1. *Нехай L – скінченновимірна алгебра Лі над алгебраїчно замкненим полем полем $\mathbb{K}$ характеристики 0. Тоді розв'язний радикал $S(L)$ є інваріантним відносно дії групи $O(L)$.*

Лема 1. *Нехай L — довільна алгебра Лі над полем $\mathbb{K}$, $Z(L)$ – центр алгебри L. Тоді $Z(L)$ є інваріантним відносно дії групи $O(L)$.*

Теорема 2. *Нехай L — довільна нільпотентна алгебра Лі над полем $\mathbb{K}$. Тоді верхній центральний ряд алгебри L*

$$0 = Z_0(L) \subset Z_1(L) \subset Z_2(L) \subset ... \subset Z_n(L) = L$$

є інваріантним відносно дії групи $O(L)$.

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8^{th} International Algebraic Conference
July 5—12 **(2011)**, Lugansk, Ukraine PLENARY TALK

Simple modules with a torsion over the Lie algebra of unitriangular polynomial derivations in two variables

Yu. Bodnarchuk

Let u_2 be a Lie algebra of polynomial derivations of the form

$$c_1\frac{\partial}{\partial x_1}+c_2(x_1)\frac{\partial}{\partial x_2},$$

where $c_1\in\mathbb{F}, c_2(x_1)\in\mathbb{F}[x_1]$. There is a natural basis of u_2 as a $\mathbb{F}-$ vector space:

$$X=\frac{\partial}{\partial x_1}, Y_i=x_1^i\frac{\partial}{\partial x_2},\ i=0,1,2,\ldots$$

with relations $[X,Y_{i+1}]=(i+1)Y_i,\ \ [Y_i,Y_j]=0$.

Let $\widetilde{W}=\widetilde{W}(u_2)$ be an universal envelope algebra of u_2.

Theorem 1. *$\widetilde{W}$ is a generalized Weil algebra (in the sense [1]) over $D=\mathbb{F}[H,Y_0,z_1,z_2,\ldots]$, i.e*

$$\widetilde{W}=\mathbb{F}[z_1,z_2,\ldots][H,Y_0]<X,Y_1>,$$

where $H=XY_1,\ \ Y_1X=H-Y_0,\ z_0=Y_0,\ \sigma:H\to H+Y_0,Y_0\to Y_0,z_i\to z_i,i=1,2,\ldots$ is an automorphism of the center of $\widetilde{W}$, which is a polynomial ring $\mathbb{F}[H,Y_0,z_1,z_2,\ldots]$ in countable number commutative variables $z_i=Y_i+\sum_{j=1}^{i-1}\alpha_iY_jY_1^{i-j}$. The coefficients α_i are uniquely determined by the conditions $Hz_i=z_iH$.

The torsion of a left module M over $\widetilde{W}$ is defined as a submodule $Tor(M)=\{m|\exists a\in D(a\neq 0)\ am=0\}$ We say that M is a module with a torsion if $Tor(M)\neq 0$.

By arguments similar to [1] one can get

Theorem 2. *1. All simple $\widetilde{W}-$ modules with a torsion are weight modules which can be identified with the quotient*

$$\widetilde{W}/\Big(\widetilde{W}(H-\lambda)+\widetilde{W}(Y_0-\gamma_0)+\widetilde{W}(z_1-\gamma_1)+\ldots\Big)$$

with the left multiplication as an action.

2. Classes of isomorphic simple modules above can be parametrized by a sequence $\{\gamma_i\}$ and a set one of the forms $\lambda+\mathbb{Z}(\lambda\neq 0)$ or $\mathbb{Z}^{>0},\ \mathbb{Z}^{<0}$.

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8th International Algebraic Conference
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Plenary talk

Critical posets and posets with nonnegative Tits form

V. M. Bondarenko, M. V. Styopochkina

A poset S is called P-critical (resp. NP-critical or P-supercritical) if the Tits form of any proper subset of it is positive (resp. nonnegative), but the Tits form of S itself does not possess this property.

All P-critical posets are described by the authors in [1]; there are 75 such posets, up to duality. In the same paper the authors also describe all finite posets with positive Tits form; we have here three infinite series of such posets and 108 non-series posets (up to duality). In [2] the authors describe all P-supercritical posets; there are 115 such posets (up to duality).

We continue study P-critical and P-supercritical posets, and posets with positive and nonnegative Tits form, paying special attention to the $\mathbb{Z}$-equivalence of the quadratic Tits forms among themselves and with the quadratic Tits forms of quivers.

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8th International Algebraic Conference
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SECTION TALK

On the existence of faithful finite-dimensional representations of semigroups generated by idempotents with partial null multiplication

V. M. Bondarenko, O. M. Tertychna

Let I be a (finite or infinite) set without 0 and J a subset in $I \times I$ without elements of the form (i, i). We define $S(I, J)$ to be the semigroup (having a zero element) with generators e_i, where $i \in I \cup 0$, and the following relations:

1) $e_0 = 0$ ($e_0 e_i = e_i e_0 = 0$ for any $i \in I$);
2) $e_i^2 = e_i$ for any $i \in I$;
3) $e_i e_j = 0$ for any pair $(i, j) \in J$.

This semigroup may be both finite and infinite (a criterion for $S \in \mathcal{I}$ to be infinite is indicated in [1]).

The set of all semigroups of the form $S(I, J)$ is denoted by $\mathcal{I}$.

We study finite-dimensional representations of such semigroups over any field k. In particular, we prove the following theorem.

Theorem 1. *Each semigroup from $\mathcal{I}$ has a faithful finite-dimensional representation over k.*

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CONTRIBUTED ABSTRACT

On monomial matrices and representations

V. V. Bondarenko

Monomial representations play an important role in representation theory and its applications (see e. g. [1, 2]). We thoroughly study monomial, quasi-monomial and block-monomial matrices and matrix representations of different objects over an arbitrary field. At the same time, we pay special attention to equivalences of such matrices and representations.

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8th International Algebraic Conference
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PLENARY TALK

Tilting for noncommutative curves

I. Burban, Yu. Drozd

We introduce a new class of noncommutative projective curves and show that in certain cases the derived category of coherent sheaves on them has a tilting complex. In particular, we prove that the right bounded derived category of coherent sheaves on a reduced rational projective curve with only simple nodes and cusps as singularities can be fully faithfully embedded into the right bounded derived category of the finite dimensional representations of a certain finite dimensional algebra of global dimension two. As an application of our approach we show that the dimension of the bounded derived category of coherent sheaves on a rational projective curve with only nodal or cuspidal singularities is at most two. In the case of the Kodaira cycles of projective lines, the corresponding tilted algebras belong to a well-known class of gentle algebras.

For details and proofs see [1].

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[1] I. Burban, Y. Drozd. Tilting on non-commutative rational projective curves. arXiv: 0905.1231 [math.AG].

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8th International Algebraic Conference
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PLENARY TALK

Miniversal deformations of forms

A. Dmytryshyn, V. Futorny, V. Sergeichuk

The reduction of a matrix to its Jordan form is an unstable operation: both the Jordan form and the reduction transformations depend discontinuously on the entries of the original matrix. Therefore, if the entries of a matrix are known only approximately, then it is unwise to reduce it to Jordan form. For these reasons V.I. Arnold [1] constructed miniversal deformations of matrices under similarity; that is, a simple normal form to which not only a given square matrix A but all matrices B close to it can be reduced by similarity transformations that smoothly depend on the entries of B.

We give miniversal deformations of matrices of bilinear forms and sesquilinear forms; that is, of matrices with respect to *congruence transformations* $A \mapsto S^T AS$ and **congruence transformations* $A \mapsto S^* AS$, in which S is nonsingular.

We use the canonical matrices for congruence and *congruence that were constructed by Horn and Sergeichuk [2]. Our miniversal deformations have the form $A_{\text{can}} + D$ in which A_{can} is a canonical matrix for congruence or *congruence and D is a matrix whose entries are zeros or independent parameters. The independent parameters are arbitrary complex numbers for matrices under congruence; they are arbitrary complex, or real, or pure imaginary numbers for matrices under *congruence.

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8th International Algebraic Conference
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SECTION TALK

Genera and cancellation in stable homotopy category

Yu. Drozd, P. Kolesnyk

Let $\mathcal{S}$ be the stable homotopy category of polyhedra (finite cell complexes) [1], $\mathcal{S}_p$ be its localization with respect to a prime number p. Two polyhedra X, Y are said to be *in the same genus* (denoted by $X \sim Y$) if their images in every category $\mathcal{S}_p$ are isomorphic. Denote by $\operatorname{add} X$ the set of all polyhedra which are direct summands (with respect to wedge as direct sum) of X in $\mathcal{S}$, and by $B(X)$ the wedge of all spheres S^n such that the stable homotopy group $\pi_n^S(X)$ is infinite. We prove the following results.

Theorem 1. *$X \sim Y$ if and only if $X \vee B(X) \simeq Y \vee B(X)$ (in the category $\mathcal{S}$).*

Theorem 2. *If $X \vee Z \simeq Y \vee Z$ and $Z \in \operatorname{add} X$, then $X \simeq Y$ (in $\mathcal{S}$).*

The proofs are based upon relations between genera of polyhedra and genera of lattices over orders (as considered in the theory of integral representations [2]).

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8th International Algebraic Conference
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PLENARY TALK

Factorizations in rings of block triangular matrices

N. Dzhaliuk, V. Petrychkovych

Let R be an integral domain of finitely generated principal ideals with diagonal reduction of matrices. We will denote by $M(n,R)$ a ring of all $n \times n$-matrices over R, by $BT(n_1,\dots,n_k,R)$ a subring of block upper triangular matrices $A = \operatorname{triang}(A_1,\dots,A_k)$, $A_i \in M(n_i,R)$, $i = 1,\dots,k$. Let d_m^A be the m-th determinantal divisor of the matrix $A \in M(n,R)$ and D^A be the canonical diagonal form of matrix A, i.e. $D^A = UAV = \operatorname{diag}(\mu_1^A,\dots,\mu_n^A)$, $\mu_i^A \mid \mu_{i+1}^A$, $i = 1,\dots,n-1$, $U,V \in GL(n,R)$.

Suppose that the nonsingular matrix $A \in BT(n_1,\dots,n_k,R)$ has a factorization

$$A = BC, \quad B, C \in BT(n_1,\dots,n_k,R), \tag{1}$$

i.e. $B = \operatorname{triang}(B_1,\dots,B_k)$, $C = \operatorname{triang}(C_1,\dots,C_k)$, $B_i, C_i \in M(n_i,R)$, $i = 1,\dots,k$. Then the determinants $\Delta_i = \det A_i$ and canonical diagonal forms $D_i^A = \operatorname{diag}(\mu_1^{A_i},\dots,\mu_{n_i}^{A_i})$ of their diagonal blocks A_i have factorizations $\Delta_i = \varphi_i\psi_i$, where $\varphi_i = \det B_i$, $\psi_i = \det C_i$ and $D_i^A = \Phi_i\Psi_i = \operatorname{diag}(\varphi_{i1},\dots,\varphi_{in_i})\operatorname{diag}(\psi_{i1},\dots,\psi_{in_i})$, $\varphi_{il} \mid \varphi_{i,l+1}$, $l = 1,\dots,n_i-1$, $\Phi_i = D^{B_i}$, $i = 1,\dots,k$. The factorization (1) of the matrix A is called Δ_i*-parallel factorization*, if $\det B_i = \varphi_i$, $\det C_i = \psi_i$ and is called D_i*-parallel factorization*, if B_i, C_i are equivalent to matrices Φ_i, Ψ_i, $i = 1,\dots,k$, respectively.

We describe the factorizations of matrices in the rings $BT(n_1,\dots,n_k,R)$. The conditions, under which the factorizations of matrices $A \in BT(n_1,\dots,n_k,R)$ are the same block triangular form up to the association, i.e. they are the factorizations in the ring $BT(n_1,\dots,n_k,R)$, are received [1]. We establish the criterions of uniqueness of such factorizations.

Theorem 1. *Let $A \in BT(n_1,\dots,n_k,R)$ be a nonsingular matrix. Then there exist unique up to the association factorizations of matrix A:*

i) Δ_i-parallel factorization of matrix A if and only if $(\varphi_s, \psi_{s+t}) = 1$ for all $s = 1,\dots,k-1$, $t = 1,\dots,k-s$ and $((\varphi_i,\psi_i), d_{n_i-1}^{A_i}) = 1$, $i = 1,\dots,k$;

ii) D_i-parallel factorization of matrix A if and only if $(\det\Phi_s, \det\Psi_{s+t}) = 1$ for all $s = 1,\dots,k-1$, $t = 1,\dots,k-s$ and Ψ_i is d-matrix, i.e. $\psi_{il} \mid \psi_{i,l+1}$, $i = 1,\dots,k$.

We remark that the problem of description up to the association of the factorizations of matrices over a principal ideal domains was formulated by Z.I.Borevich [2].

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8th International Algebraic Conference
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SECTION TALK

Classification of the five-dimensional non-conjugate subalgebras of the Lie algebra of the Poincaré group $P(1,4)$

Vasyl M. Fedorchuk, Volodymyr I. Fedorchuk

It is well known that non-conjugate subalgebras of Lie algebras of Lie groups of the point transformations play an important role in solving different tasks of the theoretical and mathematical physics, mechanics, gas dynamics etc. (see, for example, [1, 2]). It turned out that the possibilities of the above mentioned applications, as well as the results obtained essentially depend on structural properties of the non-conjugate subalgebras of Lie algebras. Therefore, the investigation of structural properties of the non-conjugate subalgebras of the Lie algebras is important from different points of view. One way to study the structural properties of non-conjugate subalgebras of the Lie algebras consists in classifying these subalgebras into isomorphism classes.

The present report is devoted to the classification of the five-dimensional non-conjugate subalgebras of the Lie algebra of Poincaré group $P(1,4)$ into isomorphism classes. The group $P(1,4)$ is a group of rotations and translations of the five-dimensional Minkowski space $M(1,4)$. Some applications of this group in the theoretical and mathematical physics can be found in [3, 4]. By now, using the Mubarakzyanov's classification [5, 6] of the real Lie algebras with dimensions from one to five, we have obtained the complete classification of all non-conjugate subalgebras with dimensions from one to five of the Lie algebra of the group $P(1,4)$ into isomorphism classes.

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8th International Algebraic Conference
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PLENARY TALK

Representations of Lie algebra of vector fields on a torus

V. Futorny, Yuly Billig

We will discuss free field realizations of Kac-Moody algebras and their applications. In particular, recent joint results with Y. Billig on the representations of the Lie algebra of vector fields on N-dimensional torus will be considered.

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8th International Algebraic Conference
July 5–12 **(2011)**, Lugansk, Ukraine

Short communication

A criterion for unitary similarity of upper triangular matrices in general position

Tatiana G. Gerasimova, Nadya Shvai

This is joint work with Vladimir V. Sergeichuk, Vyacheslav Futorny, and Douglas Farenick.

Each square complex matrix is unitarily similar to an upper triangular matrix with diagonal entries in any prescribed order. Let $A = [a_{ij}]$ and $B = [b_{ij}]$ be upper triangular $n \times n$ matrices that

- are not similar to direct sums of matrices of smaller sizes, or
- are in general position and have the same main diagonal.

We prove in [1] that A and B are unitarily similar if and only if

$$\|h(A_k)\| = \|h(B_k)\| \qquad \text{for all } h \in \mathbb{C}[x] \text{ and } k = 1, \dots, n, \tag{1}$$

where $A_k := [a_{ij}]_{i,j=1}^k$ and $B_k := [b_{ij}]_{i,j=1}^k$ are the leading principal $k \times k$ submatrices of A and B, and $\| \cdot \|$ is the Frobenius norm. The criterion (1) with the operator norm instead of the Frobenius norm was studied in [2].

The work was started while we were visiting Professor Douglas Farenick in September–December 2009 supported by the University of Regina Visiting Graduate Student Research Program.

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Short communication

Almost split sequences and irreducible morphisms of categories of complexes

Hernán Giraldo

We study irreducible morphisms of categories of complexes and the existence of the almost split sequences in some subcategories of categories of complexes.

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8th International Algebraic Conference
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Горенштейновы матрицы

Юлия Хоменко

Пусть $M_n(\mathbb{Z})$ кольцо кольцо всех $n \times n$ — матриц над кольцом целых чисел $\mathbb{Z}$ и U_n и $E_n \in M_n(\mathbb{Z})$ где $U_n = \begin{pmatrix} 1 & \dots & 1 \\ \vdots & & \vdots \\ 1 & \dots & 1 \end{pmatrix}$ и E_n — единичная матрица.

Матрица $\varepsilon = (\alpha_{ij}) \in M_n(\mathbb{Z})$ называется матрицей показателей, если она удовлетворяет таким условиям:

$$\alpha_{ii} = 0 \text{ и } \alpha_{ij} + \alpha_{jk} \geq \alpha_{ik} \text{ для } i, j, k = 1, ..., n \tag{1}$$

Матрица показателей называется приведенной, если она не имеет симметрических нулей.

Приведенная матрица показателей $\varepsilon(\alpha_{ij})$ называется горенштейновой, если существует подстановка σ без неподвижных элементов такая, что $\alpha_{ij} + \alpha_{j\sigma(i)} = \alpha_{i\sigma(i)}$ для всех i, j

Обозначим через ε^T — матрицу, транспонированную к матрице ε,

$sU_n = \begin{pmatrix} s & \dots & s \\ \vdots & & \vdots \\ s & \dots & s \end{pmatrix}$

Теорема. *Пусть $E_n \in M_n(\mathbb{Z})$ — матрица показателей. Тогда матрица* $G_{2n}(s,t) = \left(\begin{array}{c|c} \varepsilon & sU_n - \varepsilon^T \\ \hline tU_n - \varepsilon^T & \varepsilon \end{array}\right)$ *является горнештейновой с подстановкой* $\sigma = \begin{pmatrix} 1 & \dots & n & n+1 & \dots & 2n \\ n+1 & \dots & 2n & 1 & \dots & n \end{pmatrix}$

СВЕДЕНИЯ ОБ АВТОРАХ

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8th International Algebraic Conference
July 5—12 **(2011)**, Lugansk, Ukraine

Section talk

Induced representations and hypercomplex numbers

Vladimir V. Kisil

We review the construction of induced representations of the group $G = \mathrm{SL}_2(\mathbb{R})$. Firstly we note that G-action on the homogeneous space G/H, where H is any one-dimensional subgroup of $\mathrm{SL}_2(\mathbb{R})$, is a linear-fractional transformation on hypercomplex numbers. Thus we investigate various hypercomplex characters of subgroups H. The correspondence between the structure of the group $\mathrm{SL}_2(\mathbb{R})$ and hypercomplex numbers can be illustrated in many other situations as well. We give examples of induced representations of $\mathrm{SL}_2(\mathbb{R})$ on spaces of hypercomplex valued functions, which are unitary in some sense. Raising/lowering operators for various subgroup prompt hypercomplex coefficients as well.

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8th International Algebraic Conference
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Колчаны действительных матриц

В. Кириченко, М. Хибина

Мы будем использовать результаты и терминологию монографий [1] и [2]. Напомним, что действительная матрица A называется $(0,1)$-матрицей, если ее элементами являются действительные числа 0 и 1. Ясно, что $(0,1)$-матрица является неотрицательной. Колчан без кратных стрелок и кратных петель называется простым колчаном. Очевидно, колчан Q является простым колчаном тогда и только тогда, когда его матрица смежности $[Q]$ является $(0,1)$-матрицей.

Пусть $A=(a_{ij})\in M_n(\mathbb{R})$ — действительная квадратная матрица порядка n. Построим простой колчан $Q(A)$ матрицы A следующим образом:

(1) множеством вершин $VQ(A)$ колчана $Q(A)$ является множество $\{1,2,\dots,n\}$;

(2) множество стрелок $AQ(A)$ задается так: существует одна стрелка из i в j тогда и только тогда, когда $a_{ij}\neq 0$. Таким образом, если $a_{ij}=0$, то нет стрелки из i в j.

Пусть $A=(a_{ij})\in M_n(\mathbb{R})$ — неотрицательная матрица. Матрица называется перестановочно приводимой, если существует матрица подстановки B_σ, такая, что $B_\sigma^T A B_\sigma = \begin{pmatrix} A_1 & A_{12}\\ 0 & A_2\end{pmatrix}$, где A_1 и A_2 квадратные матрицы порядка меньшего, чем n. В противном случае, матрица A называется перестановочно неприводимой.

Теорема 1. *Неотрицательная матрица $A=(a_{ij})\in M_n(\mathbb{R})$ перестановочно неприводима тогда и только тогда, когда ее колчан $Q(A)$ сильно связен.*

Следствие. *Ненулевая неотрицательная матрица A подобна стохастической тогда и только тогда, когда A имеет положительный собственный вектор $\vec{z}^T=(z_1,\dots,z_n)^T$ с собственным значением 1.*

Определение. *Пусть $P=(p_{ij})\in M_n(\mathbb{R})$ является стохастической матрицей. Колчаном марковской цепи, соответствующей матрице P, называется колчан $Q(P)$.*

Теорема 2 ([2, теорема 7.5.10]). *Колчан Q является регулярным тогда и только тогда, когда он сильно связен и наибольший общий делитель длин всех его ориентированных циклов равен единице.*

Теорема 3. *Марковская цепь является регулярной тогда и только тогда, когда ее колчан сильно связен и наибольший общий делитель длин всех его ориентированных циклов равен единице.*

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Contributed abstract

Some properties of determinants of the spatial matrices

R. V. Kolyada, O. M. Melnyk

In [1] the spatial matrices are studied. In this report we study the polynomial spatial matrices over a field.

Let

$$A(x) = \|A_{ijk}\|, (i, j, k = 1, 2, ..., n), \quad (1)$$

be a spatial matrix of order n, the elements of which are polynomials of degree not exceeding 2 over the field. The matrix $A(x)$ is given as

$$A(x) = A^0_{ijk}x^2 + A^1_{ijk}x + A^2_{ijk}, \quad (2)$$

where A^m_{ijk}, $m = 0, 1, 2$ are the numerical cubical matrices.

The totality of elements of the cubical matrix with the fixed index i is called a cut of the orientation, wich we denote by (i). By analogy we define the cuts of the orientation by (j) and (k).

For all the transversales of the matrix A_{ijk} the sum

$$\left|A_{\overset{+}{i}\overset{\pm}{j}\overset{\pm}{k}}\right| = \sum (-1)^{I_j+I_k} A_{i(1)j(1)k(1)} A_{i(2)j(2)k(2)} \cdots A_{i(n)j(n)k(n)},$$

where the sum is taken over all possible combinations of any permutation $j^{(1)}, j^{(2)}, \ldots, j^{(n)}$ with any permutation $k^{(1)}, k^{(2)}, \ldots k^{(n)}$ where the permutation $i^{(1)}, i^{(2)}, \ldots, i^{(n)}$ is fixed, is called a cubical determinant of $n-$th order with the signature $(\overset{+}{i}\ \overset{\pm}{j}\ \overset{\pm}{k})$. The totality of $(n!)^2$ members

$$A_{i(1)j(1)k(1)} A_{i(2)j(2)k(2)} \cdots A_{i(n)j(n)k(n)}$$

defines a cubical determinant with the signature $(\overset{+}{i}\ \overset{\pm}{j}\ \overset{\pm}{k})$. For each transversal i (j or k) we give the determinant as a polynomial

$$p(x) = p^i_0 x^{2n} + p^i_1 x^{2n-1} + \cdots + p^i_{2n}. \quad (3)$$

Theorem 1. *For a cubical polynomial matrix (2) and a determinant (3) the following holds*

$$p^i_1 = tr A^1_{ijk}, \quad p^i_{2n} = \det A^2_{ijk}.$$

The theorem follows Viete theorem when $i, j, k = 1$.

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CONTRIBUTED ABSTRACT

Matrix representations of small semigroups

E. M. Kostyshyn

We study finite-dimensional matrix representations of small semigroups over any field k as well as their direct products, semidirect products, etc. [1]. We consider both specific and abstract semigroups, find minimal systems of generators and the corresponding defining relations, and define their representation type.

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Short communication

On the equivalence of two A_∞-structures on $\mathrm{Ext}^*_A(M, M)$

Oleksandr Kravets

This is a joint work with Serge Ovsienko.

Let $\Bbbk$ be a field, A an associative $\Bbbk$-algebra (not necessarily unital) and ${}_AM$ a left module over A. Our talk is dedicated to the proof that two different A_∞-structures on $\mathrm{Ext}^\bullet_A({}_AM, {}_AM)$ are isomorphic.

We will consider the projective resolution $P_\bullet \xrightarrow{\varepsilon} A$ in the category of A-A-bimodules and the associated projective resolution $Q_\bullet = P_\bullet \underset{A}{\otimes} M$ of ${}_AM$ in the category of left A-modules. It's known that $\mathrm{Ext}^\bullet_A({}_AM, {}_AM)$ coincides both with the homologies of $\mathrm{Hom}^\bullet_A({}_AQ, {}_AM)$ and with the homologies of $\mathrm{Hom}^\bullet_A({}_AQ, {}_AQ)$. The first way to define an A_∞-structure on $\mathrm{Ext}^\bullet(M, M)$ is through the A_∞-structure on $\mathrm{Hom}^\bullet(Q, M)$. This way is described in [1]. The second way is through the **dg**-algebra $\mathrm{Hom}^\bullet(Q, Q)$ considered as an A_∞-algebra as in [2].

Theorem 1. *Let $f\colon \mathrm{Hom}^\bullet(Q, Q) \to \mathrm{Hom}^\bullet(Q, M)$ is the canonical quasi-isomorphism of complexes defined as the composition with the mapping $\varepsilon \otimes \mathbf{1}_M\colon Q_\bullet = P_\bullet \otimes M \to M$. Then f can be extended to the A_∞-quasi-isomorphism of these complexes regarded as A_∞-algebras.*

Remark. *The proof is made for the more general case when $\Bbbk$ is not necessarily a field but only a commutative ring with unity. The only other change in assumptions is that ${}_AM$ is projective if considered as $\Bbbk$-module.*

As an almost immediate consequence the following theorem is proved.

Theorem 2. *Two A_∞-structures on $\mathrm{Ext}^\bullet_A({}_AM, {}_AM)$ described previously are in fact A_∞-isomorphic.*

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CONTRIBUTED ABSTRACT

The von Neumann regular of a factorization of symmetric polynomial matrices

M. Kuchma

Let $R = P[x]$ be a ring of polynomials in x over an arbitrary field P, $M_n(R)$ be a ring of $n \times n$ matrices over R. In [1] involution $^\nabla$ is defined on the ring of matrices $M_n(R)$ as

$$A(x)^\nabla = (a_{ij}(x))^\nabla = (a_{ji}(x)^\nabla).$$

A matrix $A(x) \in M_n(R)$ is called $^\nabla-$ symmetric if $A(x)^\nabla = A(x)$. An $m \times n$ matrix $A(x)$ over R is said to be von Neumann regular over R if there exists an $n \times m$ matrix $A(x)^{(1)}$ over R such that $A(x)A(x)^{(1)}A(x) = A(x)$.

The existence and construction of von Neumann and the Moore-Penrose inverses of integer and polynomial matrices were studied in [2-4].

Theorem 1. If symmetric matrix $A(x) \in M_n(R)$ has a factorization

$$A(x) = B(x)B(x)^\nabla \tag{1}$$

where $rankA(x) = rankB(x) = r$ and the matrix $B(x)$ is von Neumann regular then

$$A(x) = P(x)\begin{pmatrix} H(x) & 0 \\ 0 & 0 \end{pmatrix}P(x)^\nabla \tag{2}$$

where $P(x) \in GL_n(R)$, matrix $H(x)^\nabla = H(x)$ and $rankH(x) = r$.

In particular, if matrix $H(x) \in GL_n(R)$ then matrix $A(x)$ is von Neumann regular with inverse

$$A(x)^{(1)} = P(x)^\nabla \begin{pmatrix} (H(x))^{-1} & 0 \\ 0 & L \end{pmatrix} P(x)$$

for some $(n-r) \times (n-r)$ integer matrix L and matrix $P(x)$ satistying (2).

Theorem 2. If for symmetric matrix $A(x) \in M_n(R)$ exists von Neumann inverse $A(x)^{(1)}$ and $A(x)$ has a factorization (1) then matrix $B(x)^{(1)} = B(x)^\nabla A(x)^{(1)}$ is von Neumann inverse of $B(x)$.

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8th International Algebraic Conference
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SHORT COMMUNICATION

Explicit representation formulas for the least squares solution of the quaternion matrix equation AXB = C

I. Kyrchei

By $\mathbb{H}^{m\times n}$ denote the set of all $m\times n$ matrices over the quaternion skew field $\mathbb{H}$ and by $\mathbb{H}_r^{m\times n}$ denote its subset of matrices of rank r. Let

$$\mathbf{AXB}=\mathbf{D}, \tag{1}$$

where $\mathbf{A}\in\mathbb{H}^{m\times n}$, $\mathbf{B}\in\mathbb{H}_{r_2}^{p\times q}$, $\mathbf{D}\in\mathbb{H}^{m\times q}$ are given, $\mathbf{X}\in\mathbb{H}^{n\times p}$ is unknown. Using determinantal representations of the Moore-Penrose inverse over the quaternion skew field [1], we obtained explicit representation formulas for the least squares solution with minimum (Euclid) norm of the matrix equation (1) in the following theorem.

Theorem 1. *If* $\operatorname{rank}\mathbf{A}=r_1<n$ *and* $\operatorname{rank}\mathbf{B}=r_2<p$*, then the least squares solution* $\mathbf{X}_{LS}=(x_{ij})\in\mathbb{H}^{n\times p}$ *of (1) possesses the following determinantal representations*

$$x_{ij}=\frac{\sum\limits_{\beta\in J_{r_1,\,n}\{i\}}\operatorname{cdet}_i\left((\mathbf{A}^*\mathbf{A})_{.\,i}\left(\mathbf{d}^{\mathbf{B}}_{.j}\right)\right)^{\beta}_{\beta}}{\sum\limits_{\beta\in J_{r_1,\,n}}\left|(\mathbf{A}^*\mathbf{A})^{\beta}_{\beta}\right|\sum\limits_{\alpha\in I_{r_2,\,p}}\left|(\mathbf{BB}^*)^{\alpha}_{\alpha}\right|}=\frac{\sum\limits_{\alpha\in I_{r_2,\,p}\{j\}}\operatorname{rdet}_j\left((\mathbf{BB}^*)_{j\,.}\left(\mathbf{d}^{\mathbf{A}}_{i\,.}\right)\right)^{\alpha}_{\alpha}}{\sum\limits_{\beta\in J_{r_1,n}}\left|(\mathbf{A}^*\mathbf{A})^{\beta}_{\beta}\right|\sum\limits_{\alpha\in I_{r_2,p}}\left|(\mathbf{BB}^*)^{\alpha}_{\alpha}\right|},$$

where

$$\mathbf{d}^{\mathbf{B}}_{.\,j}=\left(\sum_{\alpha\in I_{r_2,\,p}\{j\}}\operatorname{rdet}_j\left((\mathbf{BB}^*)_{j\,.}\left(\tilde{\mathbf{d}}_{1\,.}\right)\right)^{\alpha}_{\alpha},\ldots,\sum_{\alpha\in I_{r_2,\,p}\{j\}}\operatorname{rdet}_j\left((\mathbf{BB}^*)_{j\,.}\left(\tilde{\mathbf{d}}_{n\,.}\right)\right)^{\alpha}_{\alpha}\right)^T,$$

$$\mathbf{d}^{\mathbf{A}}_{i\,.}=\left(\sum_{\beta\in J_{r_1,\,n}\{i\}}\operatorname{cdet}_i\left((\mathbf{A}^*\mathbf{A})_{.\,i}\left(\tilde{\mathbf{d}}_{.\,1}\right)\right)^{\beta}_{\beta},\ldots,\sum_{\beta\in J_{r_1,\,n}\{i\}}\operatorname{cdet}_i\left((\mathbf{A}^*\mathbf{A})_{.\,i}\left(\tilde{\mathbf{d}}_{.\,p}\right)\right)^{\beta}_{\beta}\right),$$

are the column vector and the row vector, respectively. $\tilde{\mathbf{d}}_{i.}$ *and* $\tilde{\mathbf{d}}_{.j}$ *are the ith row and the jth column of* $\widetilde{\mathbf{D}}=\mathbf{A}^*\mathbf{DB}^*$ *for all* $i=\overline{1,n}$, $j=\overline{1,p}$.

If $\{\mathbf{A},\mathbf{B},\mathbf{D}\}\subset\mathbb{H}^{n\times n}_n$, then the solution of (1) has obtained in [2] within the framework of the theory of column-row determinants as well.

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8th International Algebraic Conference
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PLENARY TALK

Bideterminants for Schur superalgebras

F. Marko, A. N. Zubkov

We will review classical results regarding bideterminants for Schur algebras, the structure of their simple modules and the process of modular reduction. Afterwards, we will define Schur superalgebra $S(m|n,r)$ and its $\mathbb{Z}$-form $S(m|n,r)_{\mathbb{Z}}$, and discuss bideterminants for Schur superalgebras over a field of characteristic zero.

Then we will solve a problem of Muir [2] and describe a $\mathbb{Z}$-form of a simple $S(m|n,r)$-module $D_{\lambda,\mathbb{Q}}$ over the field $\mathbb{Q}$ of rational numbers, under the action of $S(m|n,r)_{\mathbb{Z}}$. This $\mathbb{Z}$-form is the $\mathbb{Z}$-span of modified bideterminants $[T_\ell : T_i]$. Finally, we will prove that each $[T_\ell : T_i]$ is a $\mathbb{Z}$-linear combination of modified bideterminants corresponding to $(m|n)$-semistandard tableaux T_i.

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8^{th} International Algebraic Conference
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Plenary talk

Approximation, duality, and stability

Roberto Martínez Villa, Alex Martsinkovsky

This goal of this lecture is to propose a general framework that would allow to extend a duality (or equivalence) between two categories to a duality (or equivalence) between "larger" categories. More precisely, suppose we are given a functor between two categories that becomes a duality when restricted to appropriate subcategories. Look at all objects that can be approximated, in a suitable sense, by the objects in the subcategories. Usually, such approximations arise as morphisms, with or without lifting/extending properties. Now pass to the quotient categories, where the difference between an object and its approximation vanishes. If the original functor respects the approximations, we should get a functor between the quotient categories, and, under favorable conditions, this new functor will again be a duality.

I will illustrate this approach by going over the main result of [1], which provides a far-reaching generalization of the Bernstein-Gelfand-Gelfand correspondence. Here, the duality is the Koszul duality between the categories of Koszul (hence, graded) modules. The approximations are given by truncation of modules in large enough degrees and, dually, by passing to a high syzygy module. The former approximation gives an example of an approximating morphism without a lifting/extending property, whereas the latter does not even provide, in general, a morphism between an object and its approximation. To maintain control over the morphisms, we formally invert the radical and the syzygy endofunctor. In the end, we have a duality between the projective stable homotopy theory of modules over a finite-dimensional Koszul algebra with a noetherian dual, and the category of tails over the Koszul-dual algebra. For example, the Koszul-dual of an exterior algebra is the symmetric algebra, and we recover the BGG correspondence between the stable category of finite modules over an exterior algebra and the bounded derived category of coherent sheaves on the corresponding projective space.

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8th International Algebraic Conference
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SHORT COMMUNICATION

Z-valued characters for Janko group J_4

Mohammad Shafii Mousavi, Ali Moghani

Let G be an arbitrary finite group and $h_1, h_2 \in G$ we say h_1 and h_2 are Q-conjugate if $t \in G$ exists such that $t^{-1} < h_1 > t =< h_2 >$ which is an equivalence relation on group G. Suppose H be a cyclic subgroup of order n of a finite group G. Then, the maturity discriminant of H denoted by m(H), is an integer number delineated by $|N_G(H) : C_G|$ in addition, the dominant class of $K \cap H$ in the normalizer $N_G(H)$ is the union of $t = \frac{\varphi(n)}{m(H)}$ conjugacy classes of G where $\varphi(n)$ is Euler function, i.e. the maturity of G is clearly defined by examining how a dominant class corresponding to H contains conjugacy classes. The group G should be matured group if $t = 1$, but if $t \geq 2$, the group G is an unmatured concerning subgroup.

According to the main result of W. Feit and G. M. Seitz (see, Illinois J. Math. 33 (1), 103-131, 1988), the Janko group J_4 is an unmatured group.In this paper, all the dominant classes and Z- valued characters for J_4 are derived and we prove the following theorem: The Janko group J_4 has fifteen unmatured dominant classes of order 7, 14, 20, 21, 24, 28, 31, 33, 35, 37, 40, 42, 43, 66 with maturity of 2 and 3.

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Contributed abstract

On P-numbers of quadratic Tits forms of posets

Yu. M. Pereguda

We consider the quadratic forms over the field of real numbers $\mathbb{R}$

$$f(z) = f(z_1, \dots, z_n) = \sum_{i=1}^{n} z_i^2 + \sum_{i<j} f_{ij} z_i z_j.$$

Let $s \in \{1, \dots, n\}$. The s-deformation of $f(z)$ is the form

$$f^{(s)}(z, a) = f^{(s)}(z_1, \dots, z_n, a) = a z_s^2 + \sum_{i \neq s} z_i^2 + \sum_{i<j} f_{ij} z_i z_j,$$

where a is a parameter. Denote by $F_+^{(s)}$ the set of all $b \in \mathbb{R}$ such that the form $f^{(s)}(z, b)$ is positive definite, and put $F_-^{(s)} = \mathbb{R} \setminus F_+^{(s)}$. Further, put $m_f^{(s)} = \sup \mathrm{F}_-^{(s)} \in \mathbb{R} \cup \infty$; $m_f^{(s)}$ is called the s-th P-number of $f(z)$.

The author (together with V. M. Bondarenko) determines all P-number for the positive quadratic Tits forms of posets.

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8th International Algebraic Conference
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SHORT COMMUNICATION

On sectorial matrices

Andrey Popov

A complex $n \times n$ matrix $\mathbf{A}$ is said to be sectorial if the values of the corresponding quadratic form $\mathbf{x}^*\mathbf{A}\mathbf{x} = \sum_{i,j=1}^{n} a_{ij}x_j\bar{x}_i$ belong to the sector $\{z : |\arg z| \le \alpha\}$ of the complex plane with $\alpha \in [0, \pi/2)$. The number α will be called a semiangle of sectorial matrix and the smallest possible semiangle will be called the index of sectoriality. For $\alpha = 0$ the notion of sectorial matrix coincides with the notion of positive semidefinite matrix [1]. The necessary and sufficient condition for the matrix $\mathbf{A}$ to be positive semidefinite is well known [1].

We give necessary and sufficient conditions for a given matrix or a block-matrix to be sectorial.

Let $\mathbf{A}^* = \|\bar{a}_{ji}\|_{i,j=1}^{n}$ be the adjoint matrix of $\mathbf{A}$ and $\mathbf{A}_R = \frac{\mathbf{A}+\mathbf{A}^*}{2}$, $\mathbf{A}_I = \frac{\mathbf{A}-\mathbf{A}^*}{2i}$ be the Hermitian components of $\mathbf{A}$.

Theorem 1. *In order for a square matrix $\mathbf{A}$ to be sectorial it is necessary and sufficient that the following two conditions are fulfilled: 1) $\mathbf{A}_R$ is positive semidefinite; 2) $\operatorname{rank} \mathbf{A} \le \operatorname{rank} \mathbf{A}_R$. Moreover, for the index of sectoriality of $\mathbf{A}$ the following identity holds*

$$\alpha_{\mathbf{A}} = \max\Big\{\arctan|\lambda| : \lambda \quad \textit{an eigenvalue of} \quad \mathbf{A}_I\mathbf{A}_R^{[-1]}\Big\},$$

where $\mathbf{A}_R^{[-1]}$ is the pseudoinverse [2] to $\mathbf{A}_R$.

Theorem 2. *The square matrix $\mathbf{A}$ is sectorial if and only if the following conditions are fulfilled: 1) all principal minors of the matrix $\mathbf{A}_R$ are nonnegative; 2) the corresponding principal minors of the matrices $\mathbf{A}$ and $\mathbf{A}_R$ are vanish simultaneously.*

Let $\mathbf{S} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix}$ be a square block matrix with square blocks $\mathbf{A}$ and $\mathbf{D}$. Suppose that $\mathbf{A}$ is nonsingular.

Theorem 3. *The matrix $\mathbf{S}$ is sectorial if and only if the following conditions are fulfilled: 1) the matrix $\mathbf{A}$ is sectorial; 2) the Schur complement $\mathbf{D}_R - (\mathbf{C}+\mathbf{B}^*)\mathbf{A}_R^{-1}(\mathbf{C}^*+\mathbf{B})/4$ of the matrix $\mathbf{S}_R$ is positive semidefinite matrix;*
3) $\operatorname{rank}\big(\mathbf{D} - \mathbf{C}\mathbf{A}^{-1}\mathbf{B}\big) = \operatorname{rank}\big(\mathbf{D}_R - (\mathbf{C}+\mathbf{B}^*)\mathbf{A}_R^{-1}(\mathbf{C}^*+\mathbf{B})/4\big)$.

A parametrization of all block sectorial matrices $\mathbf{S}$ with given entries $\mathbf{A}$ and $\mathbf{C}$ is obtained.

This is a joint work with Yu. M. Arlinskiĭ.

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Section talk

Про триангуляризацію матриць над областю головних ідеалів

В. М. Прокіп

Нехай $M_n(\mathrm{R})$ — кільце $(n \times n)$-матриць над областю головних ідеалів R з одиницею $e \neq 0$, $0_{n,n}$ — нульова $(n\times n)$-матриця. Надалі через $\left[A, B\right]$ будемо позначати комутатор матриць $A, B \in M_n(\mathrm{R})$, тобто $\left[A, B\right] = AB - BA$.

Кажуть, що пара матриць $A, B \in M_n(\mathrm{R})$ триангуляризуються, якщо вона перетворенням подібності зводиться до нижнього трикутного вигляду, тобто для A і B існує матриця $U \in GL(n, \mathrm{R})$ така, що

$$UAU^{-1} = \left[\alpha_{ij}\right] = T_A \qquad \text{і} \qquad UBU^{-1} = \left[\beta_{ij}\right] = T_B$$

— нижні трикутні матриці, тобто $\alpha_{ij} = 0$ і $\beta_{ij} = 0$ для всіх $i < j$. Наша мета — вказати умови, за яких пара матриць $A, B \in M_n(\mathrm{R})$ триангуляризується. Наведені результати справедливі для матриць над ID-кільцями [1], тобто над комутативними областями з одиницею, над якими ідемпотентна матриця діагоналізується.

Теорема 1. *Ідемпотентні матриці $A, B \in M_n(\mathrm{R})$ триангуляризуються тоді і тільки тоді, коли комутатор $\left[A, B\right]$ нільпотентна матриця.*

Теорема 2. *Нехай $A, B \in M_n(\mathrm{R})$ — матриці з мінімальними многочленами $m_A(\lambda) = (\lambda-\alpha_1)(\lambda-\alpha_2)$ та $m_B(\lambda) = (\lambda-\beta_1)(\lambda-\beta_2)$ відповідно, де $\alpha_i, \beta_i \in \mathrm{R}$; $\alpha_2-\alpha_1 \neq 0$; $\beta_2-\beta_1 \neq 0$. Нехай, далі, $A - I_n\alpha_1 = 0_{n,n} \ (\operatorname{mod} (\alpha_1 - \alpha_2))$ і $B - I_n\beta_1 = 0_{n,n} \ (\operatorname{mod} (\beta_1 - \beta_2))$. Тоді:*

(1) *Матриці A і B триангуляризуються тоді і тільки тоді, коли комутатор $\left[A, B\right]$ нільпотентна матриця.*

(2) *Якщо комутатор $[A, B]$ нільпотентна матриця, то для матриць A і B існує матриця $V \in GL(n, \mathrm{R})$ така, що*

$$VAV^{-1} = \operatorname{diag}(\alpha_{11}, \alpha_{22}, \ldots, \alpha_{nn}) \text{ і } VBV^{-1} = \begin{bmatrix} \beta_{11} & 0 & 0 & \ldots & 0 \\ \beta_{21} & \beta_{22} & 0 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ \beta_{n1} & \beta_{n2} & \ldots & \beta_{n,n-1} & \beta_{nn} \end{bmatrix},$$

де $\alpha_{ii} \in \{\alpha_1, \alpha_2\}$, $\beta_{jj} \in \{\beta_1, \beta_2\}$.

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CONTRIBUTED ABSTRACT

On monic divisors of polynomial matrices

A. Romaniv

Let $A(x)$ be $n \times n$ nonsingular matrix polynomial over field F. There exist such invertible matrices $P(x), Q(x)$, that

$$P(x)A(x)Q(x) = \Psi(x) = \operatorname{diag}\left(\varepsilon_1(x), \ldots, \varepsilon_n(x)\right), \varepsilon_i | \varepsilon_{i+1}, i = 1, \ldots, n-1,$$

where $\Psi(x)$ is a canonical diagonal form (c.d.f.) of matrix $A(x)$. Let us write the matrix $\Psi(x)$ as the product

$$\Psi(x) = \Phi(x)\Delta(x),$$

where

$$\Phi(x) = \operatorname{diag}\left(\varphi_1(x), \ldots, \varphi_n(x)\right), \varphi_i | \varphi_{i+1}, i = 1, \ldots, n-1, \deg\det\Phi(x) = nr.$$

The necessary and sufficient conditions of existence of a monic factor of a matrix polynomial $A(x)$ with prescribed c.d.f. $\Phi(x)$ over a field of complex numbers were established by P.S. Kazimirs'kii. In this paper we consider this problem over infinite field F.

Let us consider a determinant matrix introduced by P.S. Kazimirs'kii:

$$V\left(\Psi, \Phi\right) = \left\| \begin{array}{ccccc} 1 & 0 & \ldots & 0 & 0 \\ \frac{\varphi_2}{(\varphi_2,\ \varepsilon_1)} k_{21} & 1 & \ldots & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ \frac{\varphi_n}{(\varphi_n,\ \varepsilon_1)} k_{n1} & \frac{\varphi_n}{(\varphi_n,\ \varepsilon_2)} k_{n2} & \ldots & \frac{\varphi_n}{(\varphi_n,\ \varepsilon_{n-1})} k_{n,\,n-1} & 1 \end{array} \right\|,$$

where

$$k_{ij} = \begin{cases} 0, & (\varphi_i, \varepsilon_j) = \varphi_j; \\ k_{ij0} + k_{ij1}x + \ldots + k_{ijh_{ij}}x^{h_{ij}}, & (\varphi_i, \varepsilon_j) \neq \varphi_j, \end{cases}$$

$$h_{ij} = deg\frac{(\varphi_i, \varepsilon_j)}{\varphi_j} - 1, i = 2, \ldots, n, \ \ j = 1, \ldots, n-1, \ \ i > j,$$

where k_{ijl} - are quantities adjoined to field F. Let $F(k)[x]$ - transcendental extension field F by joining all the parameters k_{ijl}.

Theorem. *Let F be an infinite field, $A(x)$ be a nonsingular matrix polynomial, which has the form*

$$A(x) = P^{-1}(x)\Psi(x)Q^{-1}(x), \Psi(x) = \operatorname{diag}\left(\varepsilon_1(x), \ldots, \varepsilon_n(x)\right), \varepsilon_i(x) | \varepsilon_{i+1}(x), i = 1, \ldots, n-1,$$

The matrix $A(x)$ has a left monic divisor with c.d.f. $\Phi(x)$, $degdet\Phi(x) = nr$ if and only if there exist such invertible matrix $U(x)$ that $\left(V\left(\Psi, \Phi\right) P(x)\right)^{-1}\Phi(x)U(x)$ is regular over $F(k)[x]$.

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Contributed abstract

Аффинная классификация точек трехмерных подмногообразий

В. М. Савельев

Пусть $U \in GL(n,R)$ и $V \in GL(m,R)$ — аффинные преобразования в касательном T_OF^n и нормальном N_OF^n пространствах в точке O к подмногообразию $F^n \subset E^{n+m}$ [1], а $A^\alpha(x,x)$ $(\alpha = 1,\dots,m)$ − компоненты вектора второй квадратичной формы H подмногообразия F^n в точке O, $\{A^1(x,x), A^2(x,x),\dots,A^m(x,x)\}$, A^α — симметрическая вещественная $n\times n$-матрица. Согласно работе [2], аффинными классами точек подмногообразия $F^n \subset E^{n+m}$ являются классы эквивалентности пространства $\mathcal{H} = \{H\}$ всевозможных векторных вторых квадратичных форм (т.е. m-мерных векторов симметрических $n\times n$-матриц) по действию группы $G = GL(n,R)\times GL(m,R)$ определенному формулой $\Lambda H = V\begin{pmatrix} U^*A^1V \\ \vdots \\ U^*A^mV\end{pmatrix}$.

В случае трехмерного подмногообразия $F^3 \subset E^8$ имеется пять линейно независимых квадратичных форм $\varphi^{i_1} = L^{i_1}_{ij}x^ix^j$, $(i_1 = 1,\dots,5)$. В этом случае аффинная классификация точек подмногообразия сводится к классификации точек гиперповерхности $F^3 \subset E^4$. Уравнение пучка вторых квадратичных форм можно привести к одному из следующих 5 видов:

1. $\varphi = \lambda_1((x^1)^2-(x^3)^2)+\lambda_2((x^2)^2-(x^3)^2)+2\lambda_3x^1x^2+2\lambda_4x^1x^3+2\lambda_5x^2x^3.$
2. $\varphi = \lambda_1((x^2)^2+(x^3)^2)+\lambda_2((x^1)^2-(x^2)^2)+2\lambda_3x^1x^2+2\lambda_4x^1x^3+2\lambda_5x^1x^3.$
3. $\varphi = \lambda_1((x^1)^2+(x^2)^2)+\lambda_2(x^3)^2+2\lambda_3x^1x^2+2\lambda_4x^1x^3+2\lambda_4x^2x^3.$
4. $\varphi = \lambda_1((x^1)^2-(x^2)^2)+\lambda_2(x^3)^2+2\lambda_3x^1x^2+2\lambda_4x^1x^3+2\lambda_5x^2x^3.$
5. $\varphi = \lambda_1(x^2)^2+\lambda_2(x^3)^2+2\lambda_3x^1x^2+2\lambda_4x^1x^3+2\lambda_5x^2x^3.$

В случае подмногобразия $F^3 \subset E^7$ пучок вторых квадратичных форм приводится к одному из следующих 10 видов:

1. $\varphi = \lambda_1((x^1)^2+(x^2)^2-(x^3)^2)+2\lambda_2x^1x^2+2\lambda_3x^1x^3+2\lambda_4x^2x^3.$
2. $\varphi = \lambda_1((x^1)^2+(x^2)^2+(x^3)^2)+2\lambda_2x^1x^2+2\lambda_3x^1x^3+2\lambda_4x^2x^3.$
3. $\varphi = \lambda_1((x^1)^2-(x^2)^2)+2\lambda_2x^1x^2+2\lambda_3x^1x^3+2\lambda_4x^2x^3.$
4. $\varphi = \lambda_1((x^1)^2+(x^2)^2)+2\lambda_2x^1x^2+2\lambda_3x^1x^3+2\lambda_4x^2x^3.$
5. $\varphi = \lambda_1((x^2)^2+(x^3)^2)+2\lambda_2x^1x^2+2\lambda_3x^1x^3+2\lambda_4(-2(x^1)^2+(x^2)^2+x^2x^3).$
6. $\varphi = \lambda_1(x^1)^2+2\lambda_2((x^2)^2-(x^3)^2+x^1x^2)+2\lambda_3x^1x^3+2\lambda_4x^2x^3.$
7. $\varphi = \lambda_1(x^1)^2+2\lambda_2((x^2)^2+(x^3)^2+x^1x^2)+2\lambda_3x^1x^3+2\lambda_4x^2x^3$
8. $\varphi = \lambda_1(x^1)^2+2\lambda_2((x^3)^2+x^1x^2)+2\lambda_3x^1x^3+2\lambda_4x^2x^3.$
9. $\varphi = \lambda_1(x^1)^2+2\lambda_2(-(x^2)^2+x^2x^3)+\lambda_3(x^3)^2+2\lambda_4x^1x^2.$
10. $\varphi = \lambda_1(x^1)^2+2\lambda_2(x^2)^2+\lambda_3(x^3)^2+2\lambda_4x^1x^2.$

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Short communication

On classification of Chernikov p-groups

I. Shapochka

Let M be an arbitrary divisible abelian p-group with minimality condition and H be a some finite p-group. In papers [1, 2] the Chernikov p-groups $G(M, H, \Gamma)$, which are the extensions of the group M by the group H and which are defined by some matrix representation Γ of the group H over the ring $\mathbb{Z}_p$ of p-adic integers, has been studied using the theory of integral p-adic representation of finite groups. In particular all Chernikov p-groups of type $G(M, H, \Gamma)$ with the fixed p-group H were classified up to isomorphism in the case if H is the cyclic p-group of order p^s ($s \leq 2$). It's also has been shown that the problem of the description up to isomorphism of all Chernikov groups of type $G(M, H, \Gamma)$ with the fixed group H is wild if one of the following conditions holds:

1) H is a noncyclic p-group and $p \neq 2$;
2) H is a noncyclic 2-group of order $|H| > 4$;
3) H is a cyclic p-group of order p^s ($s > 2$ if $p \neq 2$, $s > 3$ if $p = 2$).

Recently we have obtained the classification up to isomorphism of some Chernikov 2-groups of type $G(M, H, \Gamma)$ in the case if H is the Klein four-group (see also [3]).

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8th International Algebraic Conference
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Canonical forms for matrices under semiscalar equivalence

B. Shavarovskii

In the report, some classes of polynomial matrices are singled out for which canonical forms with respect to semiscalar equivalence are indicated. These forms enable one to solve the classification problem for families of numerical matrices up to similarity. This seems to be a rather complicated problem in the general case.

Let F be the arbitrary field. Two polynomial matrices $M(x),\ N(x) \in M_n(F[x])$ are said to be semiscalarly equivalent if there exists a nonsingular matrix $Q \in M_n(F)$ and an invertible matrix $R(x) \in M_n(F[x])$ such that $M(x) = QN(x)R(x)$ (see [1]). Assume that the matrix $N(x)$ has the following Smith form: $\operatorname{diag}(1,\ x^l,\ \dots,\ x^l)$.

Proposition 1. *A polynomial matrix $N(x)$ can be reduced by semiscalarly equivalent transformations to the form*

$$A(x) = \left\| \begin{matrix} 1 & & & \\ a_1(x) & x^l & & \\ \dots & & \ddots & \\ a_{r-1}(x) & & & x^l \end{matrix} \right\| \oplus x^l E_{n-r}, \tag{1}$$

where E_{n-r} is the identity matrix of order $n-r$, $1 < r \le n$, and

$$a_i(x) = x^{l_i} + a_{i1}x^{l_i+1} + \ \dots\ + a_{i,\,l-l_i-1}x^{l-1}, i = 1,\ \dots,\ r-1, 0 < l_1 < \ \dots\ < l_{r-1} < l,$$

$$a_{i,\,l_{i+1}-l_i} = a_{i,\,l_{i+2}-l_i} = \ \dots\ = a_{i,\,l_{r-1}-l_i} = 0.$$

Theorem 1. *If $2l_1 \ge l$, then the matrix $A(x)$ of the form* (1) *is uniquely defined.*

Futher, consider the case in which $l_1 + l_{t-1} < l$, $l_1 + l_t \ge l$, $1 < t \le r$.

Theorem 2. *Let $q_h (h = 1,\ \dots,\ t-1)$ is the least number such that $l_h + l_{q_h} \notin M = \{l_1,\ \dots,\ l_{r-1}\}$ and $l_h + l_{q_h} < l$. Then, in the class $\{QN(x)R(x)\}$ of semiscalarly equivalent matrices, there exists a unique matrix of the form* (1) *in which element $a_{q_h}(x)$ does not have a monomial of degree $l_h + l_{q_h}$.*

Thus, under the assumptions of theorems 1 or 2 the matrix $A(x)$ can be regarded as canonical with respect to semiscalarly equivalent transformations. Let us assume that the conditions of theorems 1 or 2 are not satisfied. Then there exists a number $l_t \in M$ such that $l_t + l_i \in M$ or $l_t + l_i \ge l$ for $i = 2,\ \dots,\ r-1$. In that case the canonical form of matrix $N(x)$ with respect to the above transformations is also indicated.

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CONTRIBUTED ABSTRACT

Колчаны полусовершенных колец

К. В. Усенко

Пусть A — полусовершенное кольцо, R — его радикал Джекобсона. Обозначим через $Q(A)$ колчан Габриеля кольца A, $PQ(A)$ — первичный колчан A, $\Gamma(A)$ — колчан Пирса кольца A [1].

Определение 1. *Кольцо A называется слабопервичным, если произведение любых двух ненулевых идеалов, не содержащихся в радикале Джекобсона R, отлично от нуля.*

Колчан Q будем называть простым, если он не содержит кратных стрелок.

Пусть $N_n = \{1, ..., n\}$ множество вершин колчана $\Gamma(A)$. Колчан $\Gamma(A)$ называется полным, если для любых двух вершин $i, j \in N_n$ $i \neq j$ существует стрелка из i в j.

Теорема 1. *Колчан $\Gamma(A)$ слабопервичного полусовершенного кольца является простым полным колчаном.*

Теорема 2. *Пусть A полусовершенное полудистрибутивное кольцо и $Q(A) = \Gamma(A)$. Тогда A — артиново кольцо и $R^2 = 0$.*

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Topical section VII

Rings and Modules

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PLENARY TALK

On the Tate-Poitou exact sequence for finite modules over pseudoglobal fields

V. Andriychuk, L. Zdoms'ka

Let K be a pseudoglobal field (that is an algebraic number field in one variable with a pseudofinite [1] constant field). We prove that there exist the nine-term exact sequence relating the Galois cohomology groups $H^i(G_S, M)$ and $P^i_S(G_S, M)$, $i = 0, 1, 2$, where S is a nonempty set of primes of K, M is a finite G_S-module whose order is not divisible by char K, G_S is the Galois group of the maximal extension of K ramified only at primes in S, and $P^i_S(K, M) = \prod'_{v\in S} H^i(K_v, M)$ with appropriate topologies. That sequence is known long time ago in the case of global ground field [2].

Next, we draw the usual consequences. In particular:

- the duality between the groups $Ш^1_S(K, M) = \mathrm{Ker}\,(H^1(G_S, M) \to P^1_S(K, M))$ and $Ш^2_S(K, M^D) = \mathrm{Ker}\,(H^1(G_S, M^D) \to P^1_S(K, M^D))$,where M^D is the Cartier dual of M;
- the triviality of the Euler-Poincare characteristic;
- the properties of the Tate-Cassels pairing in abelian varieties over pseudoglobal fields and their applications to the study of Brauer groups.

The proofs follow the outlines used in the case of global ground field [2], and the class field theory for pseudoglobal fields [3].

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8th International Algebraic Conference
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Finite-dimensional subalgebras in polynomial Lie algebras of rank one

I. Arzhantsev, E. Makedonskii and A. Petravchuk

Let $\mathbb{K}$ be an algebraically closed field of characteristic zero and $\mathbb{K}[X] = \mathbb{K}[x_1, \dots, x_n]$ the polynomial algebra over $\mathbb{K}$. Recall that a *derivation* of $\mathbb{K}[X]$ is a linear operator $\mathbb{K}[X] \to \mathbb{K}[X]$ such that $D(fg) = D(f)g + fD(g)$ for all $f, g \in \mathbb{K}[X]$. Denote by $W_n(\mathbb{K})$ the Lie algebra of all derivations of $\mathbb{K}[X]$ with respect to the standard commutator. The study of the structure of the Lie algebra $W_n(\mathbb{K})$ and of its subalgebras is an important problem appearing in various contexts (note that in case $\mathbb{K} = \mathbb{R}$ or $\mathbb{K} = \mathbb{C}$ we have the Lie algebra $W_n(\mathbb{K})$ of all vector fields with polynomial coefficients on $\mathbb{R}^n$ or $\mathbb{C}^n$). Since $W_n(\mathbb{K})$ is a free $\mathbb{K}[X]$-module (with the basis $\frac{\partial}{\partial x_1}, \dots, \frac{\partial}{\partial x_n}$), it is natural to consider the subalgebras $L \subseteq W_n(\mathbb{K})$ which are $\mathbb{K}[X]$-submodules. Following the paper of V.M. Buchstaber and D.V. Leykin [1], we call such subalgebras the *polynomial Lie algebras*. In [1], the polynomial Lie algebras of maximal rank were considered. Earlier, D.A. Jordan studied subalgebras of the Lie algebra $Der(R)$ for a commutative ring R which are R-submodules in the R-module $Der(R)$ (see [2]).

We study at first the centralizers of elements in a polynomial Lie algebra of rank one.

Proposition 1. *Let L be a subalgebra of the Lie algebra $W_n(\mathbb{K})$. Assume that L is a submodule of rank one in the $\mathbb{K}[X]$-module $W_n(\mathbb{K})$. Then the centralizer of any nonzero element in L is abelian.*

Using this statement we give a characterization of finite dimensional subalgebras of polynomial Lie algebras L of rank one.

Theorem 1. *Let L be a polynomial Lie algebra of rank one in $W_n(\mathbb{K})$, where $\mathbb{K}$ is an algebraically closed field of characteristic zero, and $F \subset L$ a finite-dimensional subalgebra. Then one of the following conditions holds.*

(1) *F is abelian;*

(2) *$F \cong A \rtimes \langle b \rangle$, where $A \subset F$ is an abelian ideal and $[b, a] = a$ for every $a \in A$;*

(3) *F is a three-dimensional simple Lie algebra, i.e., $F \simeq sl_2(\mathbb{K})$.*

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Short communication

f-(g(x)-clean) rings

N. Ashrafi, Z. Ahmadi

An associative ring R with identity is called a "clean" ring if every element of R is the sum of a unit and an idempotent. Let $C(R)$ denote the center of a ring R and $g(x)$ be a polynomial of ring $C(R)[x]$. An element $r \in R$ is called "$g(x)$-clean" if $r = s + u$ where $g(s) = 0$ and u is a unit of R and R is $g(x)$-clean if every element is $g(x)$-clean. In this paper, we introduce the concept of f-($g(x)$-clean) rings and we study various properties of f-($g(x)$-clean) rings.

Definition 1. *An element $x \in R$ is said to be a full element if there exist $s, t \in R$ such that $sxt = 1$. The set of all full elements of a ring R will be denoted by $K(R)$.*

Obviously, invertible elements and one-sided invertible elements are all in $K(R)$. So it may be more natural to work with full elements rather than with units. Therefore we define:

Definition 2. *Let $C(R)$ denote the center of a ring R and $g(x)$ be a polynomial of ring $C(R)[x]$. An element in R is said to be f-($g(x)$-clean) if it can be written as the sum a root of $g(x)$ and a full element. A ring R is called a f-($g(x)$-clean) ring if each element in R is a f-($g(x)$-clean) element.*

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PLENARY TALK

An analogue of the Conjecture of Dixmier is true for the algebra of polynomial integro-differential operators

V. V. Bavula

In 1968, Dixmier posed six problems for the algebra of polynomial differential operators, i.e. the Weyl algebra. In 1975, Joseph solved the third and sixth problems and, in 2005, I solved the fifth problem and gave a positive solution to the fourth problem but only for homogeneous differential operators. The remaining three problems are still open. The first problem/conjecture of Dixmier (which is equivalent to the Jacobian Conjecture as was shown in 2005-07 by Tsuchimito, Belov and Kontsevich) claims that the Weyl algebra 'behaves' like a finite field extension.

The first problem/conjecture of Dixmier: *is true that an algebra endomorphism of the Weyl algebra an automorphism*?

In 2010 [1], I proved that this question has an affirmative answer for the algebra of polynomial integro-differential operators. In my talk, I will explain the main ideas, the structure of the proof and recent progress on the first problem/conjecture of Dixmier.

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Short communication

Rad-supplemented lattices

Çigdem Biçer, Ali Pancar

Throughout this study, we assume that all lattices are complete modular bounded. Let L be such a lattice. An element $a \in L$ is called *small* in L if $a \vee b \neq 1$ holds for every $b \neq 1$. The intersection of all the maximal ($\neq 1$) elements in L is called *the radical* of L, written $r(L)$. In a compactly generated lattice L, $r(L)$ is the join of all small elements in L. An element $a \in L$ is called a *supplement* of b in L if $b \vee a = 1$ and $b \wedge a$ is small in $\frac{a}{0}$. L is called *supplemented* if every element has a supplement in L, and it is called *amply supplemented* in case $b \vee a = 1$ implies that b has a supplement $a' \leq a$.

As a generalization of supplement elements in a lattice L, we call an element $a \in L$ Rad-*supplement* if $b \vee a = 1$ and $b \wedge a \leq r(\frac{a}{0})$. A lattice L is Rad-*supplemented* if every element has a Rad-supplement in L. We obtain various properties of Rad-supplements and Rad-supplemented lattices. We show that if an element $a \in L$ is a Rad-supplement in L, we can write $r(\frac{a}{0}) = a \wedge r(L)$. We prove that in compactly generated lattice if $a \vee b = 1$ and $\frac{a}{0}$, $\frac{b}{0}$ are Rad-supplemented, then L is Rad-supplemented. We study also amply Rad-supplemented lattices.

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SHORT COMMUNICATION

Maximal submodules of direct products of infinite collection of simple modules over Dedekind domain

Y. Bilyak

Throughout R will be commutative associative ring with $1 \neq 0$ and all modules are unitary.

Definition 1. *Domain R is Dedekind domain if every nonzero ideal is product of finite number of prime ideals.*

Let A be submodule of module M.

Definition 2. *We will say that A is maximal submodule of M, if for every submodule B of module M if from $B \supset A$ we have $B = M$. A is maximal submodule of M, if M/A is simple $R-$module.*

Consider some set I and $D-$ some collection of subsets of I .

Definition 3. *The family of subsets D is said to be filter over I if*
1) $\emptyset \notin D$;
2) for $\forall A, B \in D$ we have $A \cap B \in D$;
3) for $\forall A \in D, \forall B \subseteq I$ if $B \supseteq A$ then $B \in D$.
Maximal filter is said to be ultrafilter.

The study of properties for direct products of simple modules started in [1]. Our result give us description of all maximal submodules of such product.

Theorem 1. *Let R be Dedekind domain and $M = \prod_{i \in I} S_i$, where $\forall i \in I$ module S_i is simple. Then there exists bijection between maximal submodules of module M and ultrafilters over set I.*

As a corollary we get one of results from [2].

Corollary 1. *There exists bijection between maximal ideals of direct products of collection of fields and ultrafilters over index set of this collection.*

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Contributed abstract

О кусочно нетеровых полусовершенных полудистрибутивных кольцах

С. В. Билык

Пусть A - полусовершенное кольцо, $A_A = P_1^{n_1} \oplus \ldots \oplus P_s^{n_s}$ — разложение правого регулярного A-модуля в прямую сумму неразложимых проективных модулей. Кольцо A называется кусочно нетеровым, справа, если $End_A P_i$ нётеровы для $i = 1, \ldots, s$. Аналогично определяется кусочно нётеровы слева кольца A.

Предложение. *Кусочно нётерово справа полусовершенное полудистрибутивное кольцо является кусочно нётеровым слева.*

Теорема. *Первичный радикал кусочно нётерова полудистрибутивного кольца нильпотентен.*

Сведения об авторах

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8th International Algebraic Conference
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О числе образующих идеалов SPSD-колец

С. В. Билык, З. Д. Пащенко

Пусть A — ассоциативное кольцо с $1 \neq 0$. Обозначим через $\mu_r^*(A) = \max\limits_{I \subseteq A} \mu_r(I)$, где $\mu_r(I)$ — минимальное число образующих правого идеала I кольца A. Аналогично определяется $\mu_l^*(A)$. Термин SPSD-кольцо A означает, что кольцо A является полусовершенным и полудистрибутивным.

Теорема. *Пусть A — SPSD-кольцо. Тогда факторкольцо A/R^2 артиново с двух сторон (R — радикал Джекобсона кольца A).*

Следствие. *Для любого SPSD-кольца A определен колчан $Q(A)$, состоящий из s вершин, где s — число попарно неизоморфных простых A-модулей.*

Теорема. *Следующие условия равносильны для приведенного SPSD-кольца A:*

$$\mu_r^*(A) = \mu_l^*(A) = s.$$

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Short communication

On the proper class injectively generated by simple modules

Engin Büyükaşik

The notion of *cofinite submodule* is introduced in [1]. Namely, a submodule N of a module M is said to *cofinite* if M/N is finitely generated. A submodule K of M is called *small* in M (denoted by $K \ll M$) if $M \neq K + T$ for every proper submodule T of M. Given submodules $K \leq L \leq M$, the inclusion $K \leq L$ is called *cosmall in* M if $L/K \ll M/K$. A submodule $L \leq M$ is called *coclosed in* M if L has *no* proper submodule K for which the inclusion $K \leq L$ is cosmall in M. A submodule $N \leq M$ is said to be *cofinitely coclosed* in M if there is *no* proper *cofinite* submodule K of N, the inclusion $K \leq N$ is cosmall in M. Let *CFC* be the class of all short exact sequences

$$0 \longrightarrow A \xrightarrow{\alpha} B \xrightarrow{\beta} C \longrightarrow 0$$

where *Im*(α) is cofinitely coclosed in B. The class CFC is a proper class and therefore it is natural to ask about the homological objects, such as, injective, projective, coinjective and coprojective objects of this class. Some of the results related to cofinitely coclosed submodules and the class CFC are as follows.

Theorem 1. *The proper class injectively generated by all* simple *modules equals the proper class of all short exact sequences* $0 \to A \to B \to C \to 0$ *of modules such that the image of the monomorphism* $A \to B$ *is* cofinitely coclosed *in* B.

Theorem 2. *The following statements are equivalent for a ring* R *and* R*-module* A.

(1) R *is perfect ring.*

(2) A *is CFC-coprojective module if and only if* A *is projective module.*

Theorem 3. *A module* C *is* CFC-*coprojective if and only if the character module* C^* *is neat coinjective.*

Theorem 4. *Flat modules are CFC-coprojective modules.*

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8th International Algebraic Conference
July 5—12 **(2011)**, Lugansk, Ukraine CONTRIBUTED ABSTRACT

Миноры полусовершенных колец

В. Н. Дармосюк

Основные определения, используемые в тезисе, можно найти в монографиях [1] и [2].

Пусть A — кольцо, — конечнопорожденный проективный -модуль, который представляется в виде прямой суммы n неразложимых A модулей. Кольцо эндоморфизмов $B = End_A(P)$ называется минором порядка n кольца A.

Многие свойства колец справедливы и для миноров этих колец.

Пусть A наследственное кольцо, e — ненулевой идемпотент кольца A, $e \neq 1$, тогда все кольца eAe тоже наследственные. Обратное утверждение неверно.

Для фробениусовых колец ситуация выглядит противоположным образом. Если A фробениусово кольцо, то не всегда кольцо eAe фробениусово. Но если все кольца eAe фробениусовы, то и кольцо A фробениусово.

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8th International Algebraic Conference
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Fujita's Algebra for the strongly connected quivers with $p = 2, 3, 4$

I. Dudchenko

The strongly connected quivers with $p = 2, 3, 4$ and their Fujita's algebra are considered. For this Fujita's algebra to established the frobenius algebras.

In recent year the Japanese mathematician Fujita entered to consideration of A-full matrix algebra. These algebra play an important role in research of different classes of rings, in particular the tiled orders [1]. In particular in the article [2] Frobenius full matrix algebras are studied.

For the strongly connected quivers without the loops of Q we have Fujita's algebra, such that $Q[A] = Q$.

Sort out all quivers from [3]

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$socP_1 = U_2$, $socP_2 = U_1$

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$socP_1 = U_3$, $socP_2 = U_1$, $socP_3 = U_2$.

$$\begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix}$$

$socP_1 = U_3$, $socP_2 = U_4$, $socP_3 = U_1$, $socP_4 = U_2$.

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$socP_1 = U_4$, $socP_2 = U_1$, $socP_3 = U_2$, $socP_4 = U_3$. Only for these quivers there is Fujita's algebra that is Frobenius algebras.

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SHORT COMMUNICATION

The proper class of small supplement modules

Yilmaz Durğun

Small module was firstly introduced by [3]. We introduced the notion of *small supplement* by using small modules. We investigate the class SS of short exact sequence of R-modules determined by *small supplement* modules. SS is the class of all short exact sequences $0 \longrightarrow A \xrightarrow{\alpha} B \xrightarrow{\beta} C \longrightarrow 0$ where $Im(\alpha)$ has(is) a small supplement in B, that is, there is a submodule K of B such that $\operatorname{Im} f + K = B$ and $\operatorname{Im} f \cap K \ll I$ where I injective module.WS is the class of all short exact sequences $0 \longrightarrow A \xrightarrow{\alpha} B \xrightarrow{\beta} C \longrightarrow 0$ where $Im(\alpha)$ has(is) a weak supplement in B and the class $\overline{WS}$ which is the least proper class contain WS over a hereditary ring R defined in [1].

To prove that SS is a proper class we will use the result of [2] that states that a class $\mathcal{P}$ of short exact sequences is proper if $Ext_{\mathcal{P}}(C, A)$ is a subfunctor of $Ext_R(C, A)$, then $Ext_{\mathcal{P}}(C, A)$ is a subgroup of $Ext_R(C, A)$ for every R-modules A, C and the composition of two $\mathcal{P}$-monomorphism (epimorphism) is a $\mathcal{P}$-monomorphism(epimorphism).Some of the results related to the class SS are as follows.

Lemma 1. *For every homomorphism* $f : A \to A'$, $f_* : \operatorname{Ext}(C,\, A) \longrightarrow \operatorname{Ext}(C,\, A')$ *preserves short exact sequences from* SS.

Lemma 2. *For every homomorphism* $g : C' \to C$, $g^* : \operatorname{Ext}(C,\, A) \longrightarrow \operatorname{Ext}(C',\, A)$ *preserves short exact sequences from* SS.

Corollary 1. $\operatorname{Ext}_{\mathrm{SS}}(C,\, A)$ *is a subgroup of* $\operatorname{Ext}(C,\, A)$ *for every modules* C *and* A.

Theorem 1. *Over a hereditary ring* R, SS *is a proper class.*

Corollary 2. *If* R *is hereditary ring, then* $\mathrm{SS} = \overline{\mathrm{WS}}$.

Corollary 3. *Let* $\mathcal{S}m$ *be the class of all small modules. Then,* $\underline{k}(\mathcal{S}m) = \mathrm{SS}$.

Corollary 4. *Global dimension of* SS *is less than or equal to 1.*

Theorem 2. P *is* SS*-projective if and only if* $\operatorname{Ext}(P,\, T) = 0$ *and* $\operatorname{Ext}(P,\, S/T) = 0$ *for every small module* S *and its torsion bounded submodule* T *over DVR.*

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Section talk

An example of non-nilpotent Leibniz algebra with Engel condition

Yu. Frolova

A linear algebra with multiplication satisfying the identity $(xy)z \equiv (xz)y + x(yz)$ is called a Leibniz algebra. Any Lie algebra is a Leibniz algebra. For necessary definitions see the monograph [1].

We will omit the brackets and use the left-normed notation $abc = (ab)c$. Any element of a Leibniz algebra can be written as a linear combination of left-normed elements. For example, $x(yz) \equiv xyz - xzy$. Let capital letters denote the operators of the respective left multiplication. For example, $xz = xZ$.

In the paper [2] the author has constructed a non-nilpotent metabelian Lie algebra satisfying the identity $xY^{p+1} \equiv 0$ over any field $\mathbb{Z}_p$ where p is a prime.

Theorem 1. *Let* $\mathbf{V}$ *be a variety of Leibniz algebras over* $\mathbb{Z}_p$ *determined by the identities* $xY^p \equiv 0$ *and* $x(yz) \equiv 0$. *Then* $\mathbf{V}$ *is not nilpotent.*

We follow the ideas of the paper [2]. We construct a non-nilpotent Leibniz algebra over $\mathbb{Z}_p$ which satisfies the identities $xY^p \equiv 0$ and $x(yz) \equiv 0$.

Let F be a set of functions of natural argument with values in $\mathbb{Z}_p$ and W be the vector space over $\mathbb{Z}_p$ with the basis $\{e_f | \; f \in F\}$. Consider W as an abelian Lie algebra. Let δ_m be an endomorphism of the vector space W determined by $e_f\delta_m = e_{\overline{f}}$, where

$$\overline{f}(i) = \begin{cases} f(i), & \text{if } i \neq m, \\ f(i) + 1, & \text{if } i = m. \end{cases}$$

Let $L = \langle x_i | i \in \mathbb{N}\rangle_{\mathbf{Z}_p}$, where $x_i = \delta_i - \varepsilon, \; i = 1, 2, \ldots,$ be abelian Lie algebra and $M = W \oplus L$ be direct sum of vector spaces W and L. We define the product as $(w_1 + l_1)(w_2 + l_2) = w_1 l_2$, where $w_1, w_2 \in W, \; l_1, l_2 \in L$, and denote this algebra by the same letter M.

The algebra M is not nilpotent but $M \in \mathbf{V}$. Remark also that if A is a Lie algebra and $A \in \mathbf{V}$ then A is nilpotent.

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PLENARY TALK

Bezout rings with finite Krull dimension

A. Gatalevych

Let R be a nontrivial commutative ring. Recall that the Krull dimension of R is the maximal length n of a chain $P_0 \subset P_1 \subset \cdots \subset P_n$ of prime ideals inside R. By convention, a ring R has Krull dimension -1 if and only if it is trivial (i.e., $1_R = 0_R$) [1]. By a Bezout ring we mean a ring in which all finitely generated ideals are principal. An n by m matrix $A = (a_{ij})$ is said to be diagonal if $a_{ij} = 0$ for all $i \neq j$. We say that a matrix A of dimension n by m admits a diagonal reduction if there exist invertible matrices $P \in GL_n(R), Q \in GL_m(R)$ such that PAQ is a diagonal matrix. We say that two matrices A and B over a ring R are equivalent if there exist invertible matrices P, Q such that $B = PAQ$. Following Kaplansky [2], we say that if every matrix over R is equivalent to a diagonal matrix (d_{ii}) with the property that every (d_{ii}) is a divisor of $d_{i+1,i+1}$, then R is an elementary divisor ring. A ring R is to be a Hermite ring if every 1×2 matrix over R admits diagonal reduction. A row $(a_1; a_2; \ldots; a_n)$ over a ring R is called unimodular if $a_1R + a_2R \cdots + a_nR = R$. If $(a_1; a_2; \ldots; a_n)$ is a unimodular n-row over a ring R then we say that $(a_1; a_2; \ldots; a_n)$ is reducible if there exists $(n-1)$-row $(b_1; b_2; \ldots; b_{n-1})$ such that the $(n-1)$-row $(a_1 + a_nb_1; a_2 + a_nb_2; \ldots; a_{n-1} + a_nb_{n-1})$ is unimodular. A ring R is said to have stable range n if n is the least positive integer such that every unimodular $(n+1)$-row is reducible. A commutative Bezout ring R with identity is said to be adequate if it satisfies such conditions: for every $a, b \in R$, with $a \neq 0$, there exist $a_i, d \in R$ such that (i) $a = a_id$, (ii) $(a_i, b) = (1)$, and (iii) for every nonunit divisor d' of d, we have $(d', b) \neq (1)$. [3]

In theorems 1, 2 we obtain the generalizations of the results in [4, 5].

Theorem 1. If R is commutative Bezout ring of Krull dimension one, with stable range 2, then R is an elementary divisor ring. In fact, it is adequate.

Theorem 2. If R is commutative semihereditary Bezout ring of Krull dimension two, with stable range 2, then R is an elementary divisor ring.

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8th International Algebraic Conference
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Solution of some problems for certain class of unit-central rings

A. Gatalevych, B. Zabavsky

In [1] there were established commutativity theorems for certain classes of rings in which every invertible element is central. In this paper the follows open problems were formulated:

[1, Question 3.1] Is every unit-central right duo ring commutative?

[1, Question 3.4] Is every unit-central ring with stable rang 1 commutative?

We say that an associative unital ring R is unit-central if $U(R) \subseteq Z(R)$, i.e. if every invertible element of the ring lies in the center[1]. A rectangular matrix A with elements in a ring R has a diagonal reduction if there exist invertible matrices P, Q of appropriate sizes such that PAQ is a diagonal matrix (d_{ii}) with the property $Rd_{i+1,i+1}R \subseteq d_{ii}R \cap Rd_{ii}$. A ring over which every matrix has a diagonal reduction is called an elementary divisor ring [2]. A ring is called a right (left) Bezout ring if every finitely generated right (left) ideal is principal. A ring R is said to have stable range 1 if for all $a, b \in R$ such that $aR + bR = R$, there exists $y \in R$ such that $a + by$ is a unit [3].

Theorem. A unit-central simple Bezout domain R with stable range 1 is an elementary divisor ring if and only if R is a field.

Example. In the article [4] it was proved that if the skew polynomial ring $R[x;\sigma]$ is left or right duo, then $R[x;\sigma]$ is commutative. For the construction of negative answer to question 3.1 we will use the rings from [5].

Let $R = Z[\sqrt{-7}]$ and $K = Q(\sqrt{-7})$ be the quotient field of the ring R.

Will consider an automorphism $G : R \to R$ such that $G(a+b\sqrt{-7}) = a - b\sqrt{-7}$, $a, b \in Z$. A skew polynomial ring $K[x;\sigma]$ is to be a duo-ring [11]. We will consider the subring S of the ring $K[x;\sigma]$ which consists of polynomials the free member of which belongs R. The only units in the ring S are 1 and -1, therefore consequently the ring S is a unit-central ring but not commutative.

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SHORT COMMUNICATION

Induced comodules over the category

O. Gnatyuk

The notion of the box was defined in the works [1], [2] as the formalization of matrix problems. In this work we suggest interpretation of representations of the box as comodules with naturally defined morphisms between them.

By definition of the box $\mathcal{A} = (A, V, \mu_V, \varepsilon)$, A is the category, V is coalgebra over A with counit $\varepsilon_V : V \to A$ and comultiplication $\mu_V : V \to V \underset{A}{\otimes} V$. For the box $\mathcal{A}$ we define the category $Rep(\mathcal{A})$ of representations. The objects of this category are the modules over A, morphisms are defined by the formula: $Hom_{\mathcal{A}}(M, N) = Hom_A(V \underset{A}{\otimes} M, N)$, the composition of morphisms $\psi : V \underset{A}{\otimes} N \to L$ and $\varphi : V \underset{A}{\otimes} M \to N$ is defined as superposition: $V \underset{A}{\otimes} M \underset{\mu\otimes id_M}{\longrightarrow} V \underset{A}{\otimes} V \underset{A}{\otimes} M \underset{id\otimes\varphi}{\longrightarrow} V \underset{A}{\otimes} N \underset{\psi}{\longrightarrow} L$.

Let V be coalgebra over $\Bbbk$-linear category A and X — A-module. Then V-comodule $X_V = V \underset{A}{\otimes} X$ with comultiplication $\mu_{X_V} = \mu_V \otimes 1_X : X_V \longrightarrow V \underset{A}{\otimes} X_V$ is called induced V-comodule. We denote $V - comod_{ind}$ the category of induced comodules. Objects of this category are induced comodules, morphisms between two objects are morphisms between the corresponding induced comodules. Then the next proposition holds:

The category $Rep(\mathcal{A})$ of representations of the box and the category $V - comod_{ind}$ of induced comodules are equivalent (see [3]).

Theorem 1. *Let A, B be the categories, $\varphi : A \longrightarrow B$ the functor, $\varphi^* : B - Mod \longrightarrow A - Mod$, $\varphi^*_{comod} : V^{\varphi}_{comod} \longrightarrow V_{comod}$ the induced functors. Then there exists the functor*

$$\varphi^A_D : V^{\varphi} \underset{B}{\otimes} M \longrightarrow V \underset{A}{\otimes} \varphi^*(M),$$

that is full and faithful.

We obtain the list of exact shurian (see [4]) comodules having the positive quadratical Tits form.

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8th International Algebraic Conference
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Plenary talk

On characteristic submodules and operations over them

A. Kashu

Let R-pr be the „big lattice" of preradicals of the category of left R-modules R-Mod. For $M \in R$-Mod we denote by $\mathbb{L}^{ch}(M)$ the lattice off all *characteristic* submodules of M $\left(N \in \mathbb{L}^{ch}(M) \Leftrightarrow f(N) \subseteq N \quad \forall f : M \to M\right)$. Every submodule $N \subseteq M$ defines two preradicals α_N^M and ω_N^M by the rules: $\alpha_N^M(X) = \sum_{f:M\to X}(N)$, $\omega_N^M(X) = \bigcap_{f:X\to M} f^{-1}(N)$ [1]. So we obtain two mappings: $\alpha^M : \mathbb{L}^{ch}(M) \to R\text{-}pr \quad \left(N \rightsquigarrow \alpha_N^M\right)$, $\omega^M : \mathbb{L}^{ch}(M) \to R\text{-}pr$ $\left(N \rightsquigarrow \omega_N^M\right)$. We denote: $\boldsymbol{A}^M = Im\left(\alpha^M\right)$, $\boldsymbol{\Omega}^M = Im\left(\omega^M\right)$.

Theorem 1. *For every module $M \in R$-Mod the following lattices are isomorphic: $\mathbb{L}^{ch}(M)$, $\boldsymbol{A}^M$, $\boldsymbol{\Omega}^M$ and $\boldsymbol{I}^M = R\text{-}pr/\cong_M$, where $r \cong_M s \Leftrightarrow r(M) = s(M)$ [2].*

Using the preradicals α_N^M and ω_N^M, as well as the operations of product and coproduct in R-pr, in the lattice $\mathbb{L}^{ch}(M)$ four operations are defined: *α-product* $K \cdot N = \alpha_K^M \alpha_N^M(M) = \alpha_K^M(N)$, *$\omega$-product* $K \odot N = \omega_K^M \omega_N^M(M) = \omega_K^M(N)$, *$\alpha$-coproduct* $N : K = \left(\alpha_N^M : \alpha_K^M\right)(M)$ and *ω-coproduct* $N \odot K = \left(\omega_N^M : \omega_K^M\right)(M)$. Some properties of these operations are studied and some relations between them and lattice operations of $\mathbb{L}^{ch}(M)$ are established. In particular, the following laws of distributivity are proved.

Theorem 2. $(K_1 + K_2) \cdot N = (K_1 \cdot N) + (K_2 \cdot N)$;
$(K_1 \cap K_2) \odot N = (K_1 \odot N) \cap (K_2 \odot N)$;
$(N : (K_1 + K_2)) = (N : K_1) + (N : K_2)$;
$N \odot (K_1 \cap K_2) = \left(N \odot K_1\right) \cap \left(N \odot K_2\right)$ [2].

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8th International Algebraic Conference
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SHORT COMMUNICATION

On QF-rings

M. V. Kasyanuk

Let $A = P_1^{n_1} \bigoplus \ldots \bigoplus P_s^{n_s}$ be a decomposition of quasi- Frobenius ring into a direct sum of indecomposable projective A- modules.

Proposition. *Let $\varphi : P_i \mapsto P_j$ be a homomorphism of indecomposable projective modules over quasi- Frobenius ring. Then* $\operatorname{Ker} \varphi \neq 0$.

Proposition. *Let A be a semichain ring and all homomorphisms of indecomposable projective modules have nonzero kernel. Then A is a quasi- Frobenius ring.*

Theorem. *Let A be a hereditary quasi- Frobenius ring. Then it is a semisimple Artian ring. On the contrary, a semisimple Artian ring is quasi- Frobenius and hereditary.*

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Сагайдаки кускових областей

Наталія Кайдан

Означення. *Скінченне кільце A називається кусковою областю, якщо будь-який ненульовий гомоморфізм нерозкладних проективних $A-$ модулів є мономорфізмом.*

Теорема. *Сагайдак $Q(A)$ кускової області A не містить орієнтовних циклів і з кожної вершини в іншу йде не більше однієї стрілки. Навпаки, якщо є скінченний орієнтовний граф, який задовольняє вказаним умовам, то існує кускова напівдосконала область A, така, що $Q(A) = \Gamma$.*

Обернене твердження отримується за допомогою розгляду алгебри шляхів скінченного орієнтованного графа Γ, який задовольняє умовам теореми. В силу [2] така алгебра буде скінченно вимірною та спадковою, тобто кусковою областю.

Скінченний орієнтований граф Γ, який задовольняє умовам теореми 1, називається ациклічним графом (сагайдаком). Нагадаємо, що точка сагайдака Γ називається джерелом (стоком), якщо в неї не входить (не виходить) стрілка.

Твердження. *В ациклічном сагайдаку існують джерела і стоки.*

Література

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SHORT COMMUNICATION

Derivations of finitary incidence rings

N. Khripchenko

Let $\mathcal{C}$ be a partially ordered category (pocategory) [2], $FI(\mathcal{C})$ its finitary incidence ring [2]. We investigate the Lie ring $\mathrm{ODer}\, FI(\mathcal{C})$ of outer derivations of $FI(\mathcal{C})$. For this we introduce the notion of a derivation of a pocategory $\mathcal{C}$. The Lie ring of outer derivations of $\mathcal{C}$ is denoted by $\mathrm{ODer}\,\mathcal{C}$.

Theorem 1. $\mathrm{ODer}\, FI(\mathcal{C})$ *is isomorphic to* $\mathrm{ODer}\,\mathcal{C}$.

In particular, if $P(\preccurlyeq)$ is a quasiordered set, R an associative unital ring, then there is a pocategory $\mathcal{C}(P,R)$, associated with P and R [2]. The finitary incidence ring of $\mathcal{C}(P,R)$ is denoted by $FI(P,R)$ and called a finitary incidence ring of P over R. It turns out that $\mathrm{ODer}\,\mathcal{C}(P,R)$ can be described. As a consequence, we obtain the description of $\mathrm{ODer}\, FI(P,R)$.

Theorem 2. $\mathrm{ODer}\, FI(P,R)$ *is isomorphic to* $H^1(\overline{P}, C(R)) \rtimes \prod_{i\in I} \mathrm{ODer}\, R$.

Here $H^1(\overline{P}, C(R))$ is the first cohomology group of the order complex of the associated poset $\overline{P} = P/_\sim$ with the values in the additive group of the center of R, considered as an abelian Lie ring, $\mathrm{ODer}\, R$ is the Lie ring of outer derivations of R, and i runs over the set of the connected components of P.

Furthermore, let A be a unital algebra over a ring K. Then $FI(P,A)$ is also a K-algebra. Therefore, $\mathrm{ODer}\, FI(P,A)$ becomes a Lie algebra over K. One can prove that the semidirect product from the theorem above is actually the product of Lie algebras in this case. Using this description we study the subalgebra $K\text{-}\mathrm{ODer}\, FI(P,A) \subset \mathrm{ODer}\, FI(P,A)$ of outer K-derivations of $FI(P,A)$.

Theorem 3. $K\text{-}\mathrm{ODer}\, FI(P,A)$ *is isomorphic to* $H^1(\overline{P}, C(A)) \rtimes \prod_{i\in I} K\text{-}\mathrm{ODer}\, A$.

Corollary 1. *Let R be commutative unital ring. Then* $R\text{-}\mathrm{ODer}\, FI(P,R) \cong H^1(\overline{P}, R)$. *In particular,* $R\text{-}\mathrm{ODer}\, FI(P,R)$ *is abelian.*

This corollary generalizes [1, Theorem 2] and Theorems 7.1.4, 7.1.9 as well as Propositions 7.1.6, 7.1.8 from [3].

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Short communication

Superminimal exponent matrices

Volodymyr Kirichenko, Makar Plakhotnyk

Here we continue the consideration of the problem of description of exponent matrices [1], i.e. of those nonzero integer n-dimensional matrices $A = (\alpha_{ij})$ that for all possible indices i, j, k inequalities $\alpha_{ij}+\alpha_{jk} \geqslant \alpha_{ik}$ take place together with the condition for diagonal elements to be equal to 0.

There is well known notion of equivalent exponent matrices. Two exponent matrices are equivalent if one can be obtained from another by compositions of the following transformations: (1) by transposing of two lines and two columns with the same numbers and (2) by adding some integer to all elements of some line with simultaneously subtracting this integer from all elements of the column with the same number.

It is known that applying these equivalent transformations one can come to the matrix whose the first line is equal to 0. It is easy to show that in this case all elements of the matrix are non negative. That is why considering of non negative exponent matrices is actual.

Such exponent matrices form a semigroup. We call non negative exponent matrix minimal if it is generator of mentioned semigroup, i.e. it can not be presented as sum of two other non negative exponent matrix.

We call non negative exponent matrix super minimal it is not unstrictly greater then some another non negative exponent matrix.

We have proved the following theorem.

Theorem 1. *All superminimal n-dimensional exponents matrices are all matrices which are equivalent in the cense of part (1) of definition to some block matrix* $A_{n,k} = \left(\begin{array}{c|c} \mathcal{O}_k & U \\ \hline \mathcal{O} & \mathcal{O}_{n-k} \end{array}\right)$ *such that $\mathcal{O}_k$ together with $\mathcal{O}_{n-k}$ consist with zeros and have dimension k and $n-k$ correspondingly, matrix $\mathcal{O}$ consist with zeros and matrix U consist with ones, where $1 \leqslant k < n$.*

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CONTRIBUTED ABSTRACT

On graded Lie algebras of characteristic two with component L_0 containing a Heizenberg ideal

I. S. Kirillov, M. I. Kuznetsov

According to [1] the classification of simple Lie algebras of absolute toral rank two over an algebraically closed field of characteristic $p = 2$ may be obtained from the classification of simple Lie algebras $\mathfrak{g}$ with solvable maximal subalgebra $\mathfrak{g}_0$ such that $\mathfrak{g}/\mathfrak{g}_0$ is an irreducible $\mathfrak{g}_0$-module. If the nilradical of the adjoint representation of $\mathfrak{g}_0$ on $\mathfrak{g}$ is nontrivial, then the problem is reduced to the classification of 1-graded transitive irreducible Lie algebras $L = L_{-1} + L_0 + l_1 + \ldots + L_r$ with solvable subalgebra L_0. The classification of such graded Lie algebras for $p > 2$ was found in [2]. Moreover, in [3] all 1-graded Lie algebras such that L_0 contains noncentral radical were described. The result depends on whether the radical of L_0 contains a Heizenberg subalgebra as an ideal or not. The authors have found all 1-graded Lie algebras for $p = 2$ for the case when L_0 contains a Heizenberg ideal. The list of algebras L consists of algebras of derivations of rank 2 classical simple Lie algebras and Lie algebras of Cartan type.

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8th International Algebraic Conference
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On quasi-prime differential preradicals

M. Komarnytskyj, I. Melnyk

Let (R, Δ) be an associative differential ring with nonzero identity, and let (M, D) be a differential module over (R, Δ), where $\Delta = \{\delta_1, \dots, \delta_n\}$ is the set of pairwise commutative ring derivations, $D = \{d_1, \dots, d_n\}$ is the set of module derivations consistent with the corresponding ring derivations δ_i.

A *differential preradical* over the differential ring R is a subfunctor $\sigma\colon R - \mathrm{DMod} \to R - \mathrm{DMod}$ of the identity functor on $R - \mathrm{DMod}$.

A nonempty collection $\mathcal{F}$ of left differential ideals of the differential ring R is said to be a *differential preradical HK-filter* of R (see [3]) if the following conditions hold:

HK1. If $I \in \mathcal{F}$ and $I \subseteq J$, where J is a left differential ideal of R, then $J \in \mathcal{F}$;

HK2. If $I \in \mathcal{F}$ and $I \in \mathcal{F}$, then $I \bigcap J \in \mathcal{F}$;

HK3. If $I \in \mathcal{F}$, then $(I : a^{(\infty)}) \in \mathcal{F}$ for each $a \in R$.

If a differential preradical filter $\mathcal{F}$ satisfies an extra condition

HK4. If $I \subseteq J$ with $J \in \mathcal{F}$ and $(I : a^{(\infty)}) \in \mathcal{F}$ for all $a \in J$, then $I \in \mathcal{F}$,

then the filter $\mathcal{F}$ is called a *differential radical HK-filter*.

Quasi-primary HK-torsion theory σ is a *HK*-torsion theory such that the intersection $\sqrt{\sigma}$ of all quasi-prime *HK*-torsion theories τ such that $\sigma \leq \tau$, is a quasi-prime *HK*-torsion theory.

Theorem 1. *Every HK-torsion theory of the differential noetherian completely bounded ring is a finite irreducible intersection of quasi-primary HK-torsion theories.*

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Contributed abstract

Идемпотентные идеалы полусовершенных колец

И. В. Кулаковская

Пусть A — полусовершенное кольцо, R — его радикал Джекобсона. Будем говорить, что кольцо A удовлетворяет слабому условию Накаямы, если из равенства $R^2 = R$ следует, что $R = 0$. Идеал I (двусторонний) кольца A называется идемпотентным, если $I^2 = I$.

Теорема 1. *Пусть A — полусовершенное кольцо, удовлетворяющее слабому условию Накаямы, в котором каждый двусторонний идеал является идемпотентным. Тогда A полупростое артиново кольцо. Наоборот, в полупростом артиновом кольце каждый двусторонний идеал является идемпотентным.*

Сведения об авторах

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Contributed abstract

On one-sided almost nilpotent ideals of associative algebras over fields

V. Luchko, A. Petravchuk

It is well-known that if I is a one-sided nilpotent ideal of an associative ring R then I is contained in a two-sided nilpotent ideal of R. One-sided ideals of associative rings were studied by many authors (see, for example [1], [2]) and the above mentioned property of nilpotent ideals cannot be transferred in general on other ideals. As it was shown in [2] one-sided Lie nilpotent ideals do not always lie in two-sided Lie nilpotent ideals, the same is true for almost nilpotent ideals (an associative algebra R over a field will be called almost nilpotent if R contains a nilpotent ideal of finite codimension). But we can give a characterization of such one-sided ideals modulo nilpotent ideals.

Theorem 1. *Let R be an associative algebra over an arbitrary field and I a right (left) almost nilpotent ideal of R. Then R contains a nilpotent ideal T such that the subalgebra $I+T/T$ of the quotient algebra R/T is a finite dimensional extension of an ideal J/T with property $(J/T)^2 = 0$.*

Corollary 1. *Every nonzero almost nilpotent ideal I of a semiprime associative algebra R over a field contains a subalgebra S such that $S^2 = 0$, $\dim S = \infty$ and $dim I/S < \infty$.*

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8th International Algebraic Conference
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О полусовершенных кольцах с ненулевым цоколем

Л. З. Мащенко

Пусть A ассоциативное кольцо с $1 \neq 0$, M — унитарный A — модуль. Напомним, что цоколем $M(socM)$ называется сумма всех простых подмодулей модуля M.

Пусть $A_A = P_1^{n_1} \oplus ... \oplus P_s^{n_s}$ (соответственно ${}_AA = Q_1^{n_1} \oplus ... \oplus Q_s^{n_s}$) разложение полусовершенного кольца A в прямую сумму правых (левых) попарно неизоморфных неразложимых проективных A — модулей, R — радикал Джекобсона кольца A.

Пусть M правый — A - модуль и N — левый A - модуль. Мы полагаем $topM = M/MR$ и $topN = N/RN$.

Хорошо известно, что модулями $U_i = P_i/P_iR\ V_i = Q_i/RQ_i$ $(i = 1, ..., s)$ исчерпываюся все попарно неизоморфные правые (левые) простые A — модули.

Предположим, что $socP_i$ — прост для всех $i = 1, ..., s$. Пусть $socP_i = U_{\sigma(i)}$.

В этом случае определено отображение $i \longrightarrow \sigma(i)$ множества $\{1, ..., s\} = N_s$ в себя.

Изучение этого отображения важно для случая артиновых колец. Если кольцо A квазифробениусово, то σ является подстановкой.

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Contributed abstract

Spectrum of prime fuzzy subhypermodules

R. Mahjoob, R. Ameri

Let R be a commutative hyperring with identity and M be an unitary R-hypermodule. We introduce and characterize the prime fuzzy subhypermodules of M. We investigate the Zariski topology on $FHspec(M)$, the prime fuzzy spectrum of M, the collection of all prime fuzzy subhypermodules of M.

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Short communication

Some properties of quasi multiplication modules over noncommutative rings

M. Maloid

Let R-associative left invariant ring. An R-module M is called weak multiplication if $specM = \emptyset$ or for every prime submodule N of M, $N = IM$ where I is an ideal of R. Recall that every left R-module M is called multiplication module, if for every submodule N of M there exist an ideal B of R such that $N = BM$. Main facts of multiplication modules are defined in [1]. Left ideal P of R is called left prime ideal, if $aRb \subseteq P$ implies $a \in P$ or $b \in P$. Ring R is called prime ring, if zero ideal of this ring is prime ideal. Ideal P of R is called weak-prime ideal, if $aRb \neq 0 \subseteq P$ implies $a \in P$ or $b \in P$. A proper submodule $N \subseteq M$ over a ring R is said to be weakly prime submodule if whenever $0 \neq rRm \in N$, for some $r \in R$ and $m \in M$, then or $m \in N$ or $rM \subseteq N$. A ring R is called a local ring if it has a unique maximal ideal.

An R-module M is called quasi multiplication module if for every weakly prime submodule N of M,we have $N = IM$, where I is an ideal of R. Lets give torsion theory T or category $R - Mod$ like $T(M) = \{m \in M : rRm = 0\}$.

Proposition 1. *Every weak prime T-semisimple submodule of module M will be prime submodule.*

Proposition 2. *Let M be a module over a quasi local ring R with maximal ideal P and $PM = 0$. Then every proper submodule of M is weakly prime.*

Lemma 1. *Let M be an R-module. Assume that N and K are submodules of M such that $K \subseteq N$ with $N \neq M$. Then the following hold:*

(1) *If N is a weakly prime submodule of M, then N/K is a weakly prime submodule of M/K.*

(2) *If K and N/K are weakly prime submodules, then N is weakly prime.*

Definition 1. *An R-module M is called a secondary module provided that for every element $r \in R$, the R-endomorphism of M produced by multiplication by r is either surjective or nilpotent.*

Theorem 1. *Let M be a secondary R-module and N a non-zero weakly prime R-submodule of M. Then N is secondary.*

Recall that the similar results for commutative case were done in [2].

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Short communication

An example of Leibniz algebra variety with the fractional exponent

O. A. Malyusheva, S. P. Mishchenko

We study numerical characteristics of Leibniz algebra varieties over a field F of characteristic zero. A Leibniz algebra is a non-associative algebra with a bilinear multiplication, satisfying the Leibniz identity:

$$(xy)z \equiv (xz)y + x(yz).$$

In particular any Lie algebra is a Leibniz algebra. Probably, this class of algebras has been determined for the first time in [1]. Necessary others definitions see for instance in the monograph [2].

Let ${}_2\mathbf{N}$ be a variety of Leibniz algebras determined by the identity $x(yz) \equiv 0$ and $M = F_3({}_2\mathbf{N})$ be a relatively-free algebra of this variety over the set of free generators $\{z_1, z_2, z_3\}$. Consider linear transformation d of the three dimensional vector space $< z_1, z_2, z_3 >$ determined by the rule $z_1 d = z_2$, $z_2 d = z_3$, $z_3 d = z_1$. In this case d may be continued to a derivation of all algebra M.

Let $D =< d >$ be a one dimensional vector space. So we may built a direct product $L = M \oplus D$ of vector spaces M and D, in which multiplication is determined as for each $m_1, m_2 \in M$ and $\alpha, \beta \in F$

$$(m_1 + \alpha d)(m_2 + \beta d) = m_1 m_2 + \beta m_1 d.$$

More then ten years ago in the paper [3] was given the example of Lie algebra variety $\mathbf{V}$ with fractional exponent, i.e. the variety, for which both lower exponent $LEXP(\mathbf{V})$ and upper exponent $HEXP(\mathbf{V})$ of the variety $\mathbf{V}$ exist, but their values are not integer. In the case of Leibniz algebras the similar result also was proved.

Theorem 1. *For the Leibniz algebra variety $\mathbf{W} = var(L)$ over a field of zero characteristic F the following strict inequalities hold*

$$3 < LEXP(\mathbf{W}) \leq HEXP(\mathbf{W}) < 4.$$

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SHORT COMMUNICATION

Radical and preradical filters in semisimple modules

Yu. P. Maturin

Let R be an associative ring with unit $1\neq 0$. All modules are left unitary (see [1-3]).

A non-empty collection E of submodules of a left R-module M satisfying (C1), (C2), (C3) [(C1), (C2), (C4)] is called a preradical [radical] filter of M (see [4—5]).

Let $Rf(M)$ [$Pf(M)$] be the set of all radical [preradical] filters of the module M.

Theorem 1. *If M is a semisimple R-module, then $Rf(M) = Pf(M)$.*

Theorem 2. *Let M be a semisimple module, then the lattice $(Rf(M), \subseteq)$ $[= (Pf(M), \subseteq)]$ is a chain if and only if M has no non-trivial fully invariant submodules.*

Theorem 3. *Let M be a non-zero semisimple module. $Card(Rf(M)) = 2$ if and only if M is a finitely generated module that has no non-trivial fully invariant submodules.*

Theorem 4. *Let M be a semisimple module. $Card(Rf(M)) = 3$ if and only if M is a sum of a countable number of minimal submodules belonging to the same isomorphic class.*

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Section talk

Слабопервичные полудистрибутивные артиновые кольца

И. А. Михайлова, К. В. Усенко

Напомним, что кольцо A называется слабопервичным, если произведение любых двух ненулевых двусторонних идеалов, которые не содержатся в радикале Джекобсона R кольца A, отлично от нуля [2]- глава 6.

Определение полудистрибутивного кольца см. в [1]- глава 14.

Напомним, что колчан Q называется сильно связным, если существует путь из любой его вершины в другую (возможно совпадающую с исходной).

Стрелка, начало и конец которой совпадают, называется петлей.

Колчан Q называется простым, если он не содержит кратних стрелок и кратных петель.

Теорема 1. *Для любого сильносвязного колчана Q без петель существует артиново слабопервичное полудистрибутивное кольцо A такое, что $Q(A) = Q$.*

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8th International Algebraic Conference
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The description of finite rings with cyclic additive group

Volodymyr Plakhotnyk, Alexander Vlasiuk

At the algebra textbook by D.K. Faddeev and I.S. Sominski, at p. 16 there is a problem on description of those rings which have pq elements. A natural question on the description of rings whose additive group is cyclic appeared. It turned out that this problem can be solved by 2-course university math methods but using the Dirichlet theorem was necessary. A.S. Oliynyk noted that this problem is mentioned somewhere at the Internet. We have found the cite www.uni.illinois.edu/~wbuck/thesis.pdf but proving there is wrong.

Nevertheless the proposition from the thesis title is not so known and should be mentioned here.

Let Λ be a ring whose additive group is cyclic of order n. Then $\Lambda = \{0, a, 2a, \dots, (n-1)a |\, a^2 = ma\}$, where a is a generator of this group, m is integer such that $0 \leqslant m < n$. The formula $a \cdot a = ma$ uniquely determinates the multiplication in the ring Λ, as the distributivity gives that

$$\alpha a \cdot \beta a = (\underbrace{a + \ldots + a}_{\alpha \text{ times}}) \cdot (\underbrace{a + \ldots + a}_{\beta \text{ times}}) = (\underbrace{a + \ldots + a}_{\alpha\beta \text{ times}}) = \alpha\beta a^2 = \alpha\beta m a.$$

In this case we will denote the ring Λ as Λ_m.

It is obvious that each Λ_m is commutative. It is remaining to establish which of rings of the form Λ_m are isomorphic.

Lemma 1. *Let m be divisor of n. Then the ring Λ_m contains exactly m elements, such that product of each of them by an arbitrary element of Λ_m is 0.*

Really let x be such number that $1 \leqslant x \leqslant n$ and $xa \cdot \alpha a = 0$ for an arbitrary $\alpha a \in \Lambda_m$. This condition is equivalent to $xa \cdot a = 0$, i.e. $xma = 0$. The last equality is equivalent to fact that x is belongs to the set $\left\{\frac{n}{m}, \frac{2n}{m}, \dots, \frac{mn}{m}\right\}$ which is necessary.

Corollary 1. *If m_1 and m_2 are distinct divisors of n, then rings Λ_{m_1} and Λ_{m_2} are not isomorphic.*

Lemma 2. *Let $d = (m, n)$, i.e. d is GCF of numbers m and n. Then rings Λ_d and Λ_m are isomorphic.*

To construct the automorphism $\varphi:\ \Lambda_d \to \Lambda_m$ it is necessary to find such automorphism of cyclic order n group which would be rings homomorphism. It is known that such automorphism is given by formula $\varphi(a) = xa$, where x is coprime with n. The condition for φ to be a ring homomorphism is $\varphi(a \cdot a) = \varphi(a) \circ \varphi(a)$, i.e. $\varphi(da) = xa \circ xa$ which is the same as $dxa = x^2 ma$.

Taking under attention the condition $(x, n) = 1$ obtain the equivalent condition $d - xm \equiv 0(\mod n)$. This means that d can be written in the form $d = xm + yn$ for some integers x, y with additional condition $(x, n) = 1$. As there exist integers x_0, y_0 such that $d = x_0 m + y_0 n$ then $x_0 \frac{m}{d} + y_0 \frac{n}{d} = 1$. This implies that integers x_0 and $\frac{n}{d}$ are coprime.

From the university algebra course it is known that all pairs (x, y) of numbers such that $x\frac{m}{d} + y\frac{n}{d} = 1$ are given by formulas $x = x_0 + k\frac{n}{d}$, $y = y_0 - k\frac{m}{d}$ where k is arbitrary integer and so it is necessary to prove that there is such integer k that $x_0 + k\frac{n}{d}$ is coprime with d. We may use Dirichlet theorem on prime numbers in the AP $x_0 + k\frac{n}{d}$. As integers x_0 and $\frac{n}{d}$ are coprime then AP contains infinitely many prime numbers. Taking such of them which is no divisor of d obtain the necessary k. So we have proven the following theorem

Theorem. *Let the additive group of the ring Λ be cyclic group of order n. Then there is one to one correspondence between the set of all such non isomorphic rings and the set of all different divisors of n i.e. each such ring is isomorphic to some and Λ_m which is unique and in this Λ_m for the generator a of additive group the condition $a^2 = ma$ for the n-divisor m takes place.*

This theorem follows from lemmas 1 and 2.

A question on possibility to proving this theorem without Dirichlet is natural. Second author was succeed to prove the main result without using the Dirichlet theorem.

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SHORT COMMUNICATION

The variety of Jordan algebras generated algebra of upper triangular matrix $UT_2(F)^{(+)}$ has almost polynomial growth

A. Popov

We give here the new example of the variety of Jordan algebra with almost polynomial growth. Earlier we had given first example of the variety of Jordan algebra with such property [1, 2]. Necessary definitions see for instance in the books [3] and [4].

Let F field of characteristic zero and $\mathbf{V}$ be the variety of linear algebras over F. Let $P_n(\mathbf{V})$ be the vector space of the multilinear polynomials in the first n variables of relatively free algebra. The sequence of dimensions $c_n(\mathbf{V}) = \dim P_n(\mathbf{V})$ defines the growth of the variety $\mathbf{V}$. The varieties with the following extremal properties are of special interest: each proper subvariety is of polynomial growth, while the growth of the variety itself is not polynomial. In this case, one says that the variety has almost polynomial growth. We proved following theorem.

Theorem 1. *Let $\mathbf{V}$ be the variety generated by the algebra $UT_2(F)^{(+)}$. Then*

1) $\mathbf{V}$ is the variety of Jordan algebras defined by the polynomial identity

$$2\left(\left(x^2y\right)y + \left(y^2x\right)x - \left((yx)\,x\right)y - \left((xy)\,y\right)x\right) \equiv x^2y^2 - (xy)^2;$$

2) codimension sequence $c_n(\mathbf{V}) = (n-2)\,2^{(n-1)} + 1$;

3) $\mathbf{V}$ is the variety of the almost polynomial growth.

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Short communication

Local near-rings on metacyclic Miller-Moreno p-groups

I. Raevska

In this work necessary and sufficient conditions of existence of local nearrings on a metacyclic Miller-Moreno p-group are given.

Recall that a (left) nearring R with identity 1 is called local if the set L of all non-invertible elements of R forms a subgroup of the additive group R^+ of R. It is well-known that the additive group of every finite local nearring is a p-group for a prime p [1].

A finite group is called a Miller-Moreno group if it is nonabelian but all its proper subgroups are abelian.

Each metacyclic Miller-Moreno p-group is either a quaternion group Q_8 or a group of the form $G =< a > \rtimes < b >$ with $a^{p^m} = b^{p^n} = 1$ and $b^{-1}ab = a^{1+p^{m-1}}$ for some $m \geq 2$ and $n \geq 1$.

As it is known, the group Q_8 cannot be the additive group of a local nearring [2].

Let R be a local nearring whose additive group R^+ is a metacyclic Miller-Moreno p-group of order p^{m+n} with $m \geq n$ written additively as $R^+ =< e_1 > + < e_2 >$ with generators e_1 and e_2 and relations $e_1p^m = e_2p^n = 0, e_1 + e_2 = e_2 + e_1(1 + p^{m-1})$.

Lemma 1. *The set L of all non-invertible elements of R is a subgroup of index p in R^+.*

Clearly every element $x \in R$ can uniquely be written in the form $x = e_1x_1+e_2x_2$, where $0 \leq x_1 \leq p^m - 1$ and $0 \leq x_2 \leq p^n - 1$. Since the subgroup L is of index p in R^+ by Lemma 1, we can chose $e_2 \in L$ and e_1 as an identity element of R so that $L =< e_1p > + < e_2 >$. Then $xe_1 = x = e_1x_1 + e_2x_2$ and $xe_2 = e_1\alpha(x) + e_2\beta(x)$ for some uniquely determined elements $\alpha(x) \in Z_{p^m}$ and $\beta(x) \in Z_{p^n}$. In particular, each element $x \in R$ determines two mappings $\alpha(x) : R \to Z_{p^m}$ and $\beta(x) : R \to Z_{p^n}$.

Lemma 2. *If $x, y \in R$, then $xy = e_1(x_1y_1 + \begin{pmatrix} y_1 \\ 2 \end{pmatrix} x_1x_2p^{m-1} + y_2\alpha(x)) + e_2(x_2y_1 + y_2\beta(x))$, where the mappings $\alpha(x) : R \to Z_{p^m}$ and $\beta(x) : R \to Z_{p^n}$ satisfy the following conditions:*

1. $\beta(x) = 1 \Leftrightarrow x_1 \neq 0 (mod\, p^n)$,
2. $\beta(xy) = \beta(x)\beta(y)$,
3. $\alpha(xy) = x_1\alpha(y) + \alpha(x)\beta(y)$.

As an application of Lemma 2, we have the following result.

Theorem. *Each local nearring R whose additive group is a metacyclic Miller-Moreno p-group of size p^{m+n} with $m \geq n$ is determined by the mappings $\alpha(x) : R \to Z_{p^m}$ and $\beta(x) : R \to Z_{p^n}$ satisfying conditions 1 - 3 of lemma 2.*

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SHORT COMMUNICATION

On local nearrings with Miller-Moreno group of units

M. Raevska

In this report local nearrings with Miller-Moreno group of units are considered. Using the computer programm GAP4.4, we found a full list of groups of orders 16 and 32 which can be the additive groups of local nearrings with Miller-Moreno group of units.

Definition ([1]). *A set* $(R, +, \cdot)$ *with to binary operations, addition and multiplication, is called a (left) nearring if*

1. $(R, +)$ *is a (not necessarily abelian) group called the additive group of* R *and denoted by* R^+,
2. $(R, \cdot)$ *is a semigroup,*
3. *the multiplication satisfies the left distributive law with respect to the addition, i.e.* $x(y + z) = xy + xz$ *for all elements* $x, y, z \in R$.

A nearring R is called local if $(R, \cdot)$ is a monoid with an identity element 1 and the set L of all non-invertible elements of $(R, \cdot)$ is a subgroup in the additive group R^+. The group of all invertible elements of $(R, \cdot)$ is called the group of units of R and denoted by R^*.

Clearly if the nearring R is local, then $R = L \cup R^*$. In particular, if the subgroup L is of index 2 in R^+, then $R^* = 1 + L$.

Recall that a finite group is called a Miller-Moreno group if it is nonabelian and all its proper subgroups are abelian.

Theorem. *Let* R *be a finite local nearring whose group of units* R^* *is a Miller-Moreno 2-group and let* $Z(R^*)$ *be the center of* R^**. Then the following statements hold:*

1. $|R^+ : L| = 2$ *and so* $R^* = 1 + L$;
2. *if* $-1 \in Z(R^*)$, *then the subgroup* L *is abelian;*
3. *if* $-1 \notin Z(R^*)$, *then there exists an abelian subgroup* M *of* L *such that* $|L : M| = 2$ *and* M *is normal in* R^+.

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CONTRIBUTED ABSTRACT

Elementary reduction of matrices over a commutative Bezout ring with stable range 1

O. M. Romaniv

Throughout this paper R will denote an commutative ring with $1 \neq 0$.

A ring R is called a *Bezout ring* (see [1]) if any finitely generated ideal in R is principal.

If whenever $aR + bR = R$ there is an $t \in R$ such that $(a + bt)R = R$, then we say ring R has *stable range 1* (see [2]).

Recall that a ring is said to be a *ring with elementary reduction of matrices* (see [3]) if every matrix can be reduced to a diagonal form by using only elementary transformations.

Theorem 1. *Let R be a commutative Bezout ring with stable range 1. Then R is a ring with elementary reduction of matrices.*

Theorem 2. *All singular matrices over a commutative Bezout ring with stable range 1 are products of idempotent matrices.*

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PLENARY TALK

On elementary divisor rings and properties of its elements

V. Shchedryk

The notion of elementary divisor rings (e.d.r.) was introduced by I. Kaplansky [1]. Recent research shows that the methods of algebraic K-theory can be successfully used in study of such rings. A very promising approach to such study is based on the notion of stable range of ring which is an important invariant of K-theory [2-6]. This notion is founded on analysis of some relations between elements of ring. Observe that the first criterion whether a Bezout ring is an e.d.r. was suggested by I. Kaplansky [1] by the notion of the great common divisor of elements of the ring. In this connection, we single out results of N. Dubrovin [7] and B. Zabavsky [5]. This paper is devoted to study of the properties of elements e.d.r.

Theorem 1. *Let R be a commutative Bezout domain. The following conditions are equivalent:*

1. R is an elementary divisor domain.

2. Let a, b, c be elements in R. If $(a, b, c) = 1$ there exist α, β, m, n such that

$$a(m\alpha) + b(\alpha\beta) + c(\beta n) = 1.$$

3. Let a, b, c be elements in R. Then there exists a matrix of the form

$$\begin{Vmatrix} a & b & c \\ 0 & * & * \\ * & * & 0 \end{Vmatrix}$$

with $det A = (a, b, c)$.

4. Let a_1, a_2, b_1, b_2 be elements in R such that

$$(a_1, a_2) = (b_1, b_2) = 1.$$

There exists $r \in R$ such that $b_1 + rb_2 = \alpha\beta$, where

$$(\alpha, \beta) = (a_1, \alpha) = (a_2, \beta) = 1.$$

Theorem 2. *Let*

$$a(m\alpha) + b(\alpha\beta) + c(\beta n) = 1.$$

Then the matrices $\begin{Vmatrix} \beta & m \\ -\alpha a & \alpha b + cn \end{Vmatrix}, \begin{Vmatrix} \alpha & c\beta \\ n & -\beta b - am \end{Vmatrix}$ *are invertible and*

$$\begin{Vmatrix} \beta & m \\ -\alpha a & \alpha b + cn \end{Vmatrix} \begin{Vmatrix} b & c \\ a & 0 \end{Vmatrix} \begin{Vmatrix} \alpha & c\beta \\ n & -\beta b - am \end{Vmatrix} = \begin{Vmatrix} 1 & 0 \\ 0 & ac \end{Vmatrix}.$$

Definition. *Let $a, b \neq 0$. An element b is stabilized by an element a if there exists a positive integer n such that*

$$(a^n, b) = (a^{n+1}, b).$$

Theorem 3. *Let $a, b, c \neq 0$. If the element b is stabilized by either a or c then*

$$\left\| \begin{matrix} b & c \\ a & 0 \end{matrix} \right\| \sim \left\| \begin{matrix} 1 & 0 \\ 0 & ac \end{matrix} \right\|.$$

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SECTION TALK

A new property of the Leibniz algebra variety with the identity $x(y(zt)) \equiv 0$

T. Skoraya

The characteristic of basic field will be equal to zero. We study the variety of Leibniz algebras ${}_3\mathbf{N}$ determined by the identity

$$x(y(zt)) \equiv 0.$$

The algebras of this variety are left nilpotent of the class not more than 3. Some properties of this variety was described in the paper [1]. In particular, in this paper was proved that the variety ${}_3\mathbf{N}$ has a over exponential growth. Recently was proved that there exist only two subvarieties of almost polynomial growth [2]. Here we give the information about the growth of subvarieties of the variety ${}_3\mathbf{N}$.

Remind that an algebra is called Leibniz algebra, if the operator of the multiplication of the right side is a derivation of this algebra. Others necessary definitions see for instance in the book [3].

Let V be a variety of Leibniz algebras and $P_n(V)$ bi the multilinear polynomials of degree n in $x_1, \dots, x_n$ in the relatively free algebra in V, then $c_n(V)$ is the n^{th} codimension of V.

If the sequence $\sqrt[n]{c_n(V)}$ is bounded, then there are exist upper and lower limits. In the case, when limits are coincide, then we denote $\lim_{n\to\infty} \sqrt[n]{c_n(V)}$ by $Exp(V)$.

Theorem. *Let* $\mathbf{V} \subset {}_3\mathbf{N}$ *be the proper subvariety of the variety* ${}_3\mathbf{N}$. *Then the exponent* $EXP(\mathbf{V})$ *is a non-negative integer.*

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Section talk

Напівланцюгові кільця з розмірністю Круля

В'ячеслав В. Швиров

Означення. *Нехай M — правий R-модуль. Розмірність Круля модуля M будемо позначати через $Kdim(M)$ і визначимо наступним чином:*

якщо $M = 0$, тоді $Kdim(M) = -1$;

якщо α — ординал і $Kdim(M) \not< \alpha$, тоді $Kdim(M) = \alpha$, якщо не існує нескінченного убиваючого ланцюга $M = M_0 \supseteq M_1 \supseteq M_2 \supseteq \ldots$ підмодулів M_i, таких, що для $i = 1, 2, \ldots$, $Kdim(M_i/M_{i+1}) \not< \alpha$.

Якщо не існує ординалу α, такого, що $Kdim(M) = \alpha$, то будемо говорити, що M не має розмірності Круля.

Означення. *Правою розмірністю Круля кільця R називається розмірність Круля правого регулярного модуля R_R і будемо позначати її через $Kdim(R)$.*

Твердження. *Нехай A — кусково нетерове кільце, таке, що всі eAe для будь-якого локального ідемпотенту e є дискретно нормовані кільця. Тоді A є несінгулярне напівспадкове кільце.*

Теорема 1. *Нехай R напівланцюгове нетерове справа базисне кільце. Якщо сагайдак кільця R є цикл, то розмірність Круля кільця R дорівнює 1. Якщо сагайдак R є ланцюг, тоді розмірність Круля кільця R дорівнює 0.*

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SHORT COMMUNICATION

Generalized Frattini subalgebras of Lie algebras of finite length

A. V. Syrokvashin, A. F. Vasilyev

Throughout L will denote a Lie algebra of finite lenght over associative and commutative ring Λ with a unit (briefly, a Lie Λ-algebra). For all definitions and notations we refer to Bakhturin' book [1].

Giovanni Frattini introduced his subgroup, now called the Frattini subgroup, in nineteenth century. The Lie algebra analogue, the Frattini subalgebra, appeared in [2]. The main properties of the Frattini subalgebra of the Lie Λ-algebras of finite length can be found in [1]. The concept of Frattini subgroup has been generalized in various ways, for example, see [3]. In this paper we propose a generalizations of some well-known results of the Frattini theory of Lie Λ-algebras.

Let θ be a functions mapping each Lie Λ-algebra L onto a certain nonempty system $\theta(L)$ of its subalgebras. We say that θ is a m-functor in class Lie Λ-algebras, if for any Lie Λ-algebra L the set $\theta(L)$ contains the algebra L itself and some of its maximal subalgebras and the following condition is satisfied: $(\theta(L))^{\varphi} = \theta(L^{\varphi})$ for every epimorphism φ of every Lie Λ-algebra L.

Definition. *Let θ be a m-functor in class of Lie Λ-algebras. The subalgebra $\Phi_\theta(L) = \cap M$, where $M \in \theta(L)$ is called a Frattini θ-subalgebra of Lie Λ-algebra L. A maximal ideal of the Lie Λ-algebra L contained in the subalgebra $\Phi_\theta(L)$ is called the Frattini θ-ideal of Lie Λ-algebra L and denoted by $\phi_\theta(L)$.*

Theorem. *Let θ be a m-functor in the class of Lie Λ-algebras such that for any Lie Λ-algebra L set $\theta(L)$ contains all maximal subalgebras in L, not being ideals in L. Let M and N be ideals of Lie Λ-algebra L such that M/N is nilpotent and $N \subseteq \Phi_\theta(L)$. Then the ideal M is nilpotent.*

Corollary. *Let θ be a m-functor in the class of Lie Λ-algebras such that for any Lie Λ-algebra L set $\theta(L)$ contains all maximal subalgebras in L, not being ideals in L. Then the Frattini θ-ideal $\phi_\theta(L)$ of any Lie algebra L is nilpotent.*

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Contributed abstract

Глобальна розмірність черепичних порядків в $M_5(D)$

Ірина Циганівська

Нехай $\mathcal{O}$ — дискретно нормоване кільце з простим елементом π, $\pi\mathcal{O} = \mathcal{O}\pi$ — єдиний максимальний ідеал кільця $\mathcal{O}$. Кільце вигляду

$$\Lambda = \begin{pmatrix} \mathcal{O} & \pi^{\alpha_{12}}\mathcal{O} & \dots & \pi^{\alpha_{1n}}\mathcal{O} \\ \pi^{\alpha_{21}}\mathcal{O} & \mathcal{O} & \dots & \pi^{\alpha_{2n}}\mathcal{O} \\ \dots & \dots & \dots & \dots \\ \pi^{\alpha_{n1}}\mathcal{O} & \pi^{\alpha_{n2}}\mathcal{O} & \dots & \mathcal{O} \end{pmatrix},$$

де $\alpha_{ij} \in \mathbb{Z}$, $\alpha_{ii} = 0$ для всіх i та $\alpha_{ij} + \alpha_{jk} \geq \alpha_{ik}$ для всіх i, j, k, називається черепичним порядком. Це первинне нетерове напівдосконале напівдистрибутивне кільце з ненульовим радикалом Джекобсона [1].

Черепичний порядок Λ має класичне кільце часток $M_n(D)$, де D — тіло часток кільця $\mathcal{O}$.

Ми описали з точністю до ізоморфізму всі черепичні порядки в $M_5(D)$. Таких порядків 41.

Максимальне скінченна глобальна розмірність черепичних порядків в $M_5(D)$ дорівнює 4.

При цьому глобальну розмірність 1 має 1 порядок ширини 1; глобальну розмірність 2 мають 7 порядків ширини 2, 7 порядків ширини 3, 1 порядок ширини 4; глобальну розмірність 3 мають 8 порядків ширини 2, 11 порядків ширини 3; глобальну розмірність 4 мають 3 порядки ширини 2 та 3 порядки ширини 3.

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SHORT COMMUNICATION

Radical semiperfect modules

Burcu Nişancı Türkmen, Ali Pancar

Throughout the whole text, all rings are to be associative, identity and all modules are left unitary. Let M be such a module. We shall write $N \leq M$ ($N \ll M$) if N is a submodule of M (small in M). An epimorphism $f : P \longrightarrow M$ is called a *small cover* if $Ker(f) << M$. By $Rad(M)$ we denote the radical of M. Let $U, V \leq M$. V is called a *supplement* of U in M if it is minimal with respect to $M = U+V$. V is a supplement of U in M if and only if $M = U+V$ and $U \cap V \ll V$ (see [6]). Following [6], M is called *supplemented* (*weakly supplemented* in [4]) if every submodule of M has a supplement in M. As a proper generalization of supplemented modules, Zöschinger introduced a notion of modules whose radical has supplements called *radical supplemented* and the author determined the structure of the modules over dedekind domains in [7].

Recall a projective module P with a small cover $f : P \longrightarrow M$ is called a *projective cover*. A module M is called *semiperfect* if every factor module of M has a projective cover, and it is called *perfect* if , for any index set I, $M^{(I)}$ is semiperfect.

Motivated by Zöschinger, we introduce the concept of radical semiperfect modules as a proper generalization of semiperfect modules. We call a module M *radical semiperfect* if every factor module of M by submodule containing $Rad(M)$ has a projective cover.

Some properties of radical semiperfect modules

We shall mention the following main results.

Theorem 1. *Let M be a module with small radical. If M is radical semiperfect, then it is semiperfect.*

Corollary 1. *Every projective radical semiperfect modules is semiperfect.*

Proposition 1. *Every factor module of a radical semiperfect module M is radical semiperfect. In particular, every direct summand of M is radical semiperfect.*

Proposition 2. *Let M be a radical semiperfect module. If $f : P \longrightarrow M$ is a small cover, P is radical semiperfect.*

Theorem 2. *Let R be any ring with identity. R is (semi-)perfect if and only if every (finitely generated) left R-module is radical semiperfect.*

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Short communication

On Rad-$\oplus$-supplemented modules

Ergül Türkmen

Throughout, R is an associative ring with identity and all modules are unital left R-modules. Let M be an R-module. By $N \subseteq M$, we mean that N is a submodule of M. A submodule $L \subseteq M$ is said to be *essential* in M, denoted as $L \trianglelefteq M$, if $L \cap N \neq 0$ for every nonzero submodule $N \subseteq M$. A submodule S of M is called *small (in M)*, denoted as $S \ll M$, if $M \neq S + L$ for every proper submodule L of M. By $\mathrm{Rad}(M)$ we denote the sum of all small submodules of M or, equivalently the intersection of all maximal submodules of M. A module M is called *supplemented* (see [12]), if every submodule N of M has a *supplement*, i.e. a submodule K minimal with respect to $N + K = M$. K is a supplement of N in M if and only if $N + K = M$ and $N \cap K \ll K$ (see [12]). A module M is called *supplemented* (*weakly supplemented* in [5]) if every submodule of M has a supplement in M, and it is called $\oplus$-*supplemented* if every submodule of M has a supplement that is a direct summand of M.

In [10], another generalization of supplement submodules was called as Rad-*supplement* (in [13], generalized supplement). For modules $N, K \subseteq M$, K is called a Rad-*supplement* of N in M if $M = N + K$ and $N \cap K \subseteq Rad(K)$. Adapting the concept of supplemented modules, one calls a module M Rad-*supplemented* if every submodule has a Rad-supplement in M, and the module Rad-$\oplus$-*supplemented* (or *generalized* $\oplus$-*supplemented*) if every submodule has a Rad-supplement that is a direct summand of M. Clearly every ($\oplus$-) supplemented module is Rad-($\oplus$-) supplemented.

We investigate some properties of Rad-$\oplus$-supplemented modules and it is proved that a projective Rad-$\oplus$-supplemented module is completely Rad-$\oplus$-supplemented. Moreover, a noetherian ring R is artinian if and only if every free left R-module is Rad-$\oplus$-supplemented. In addition, we give a condition for an infinite direct sum of Rad-$\oplus$-supplemented module to be Rad-$\oplus$-supplemented module and over dedekind domains the structure of Rad-$\oplus$-supplemented module is completely determined.

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SHORT COMMUNICATION

On the irreducible representations of soluble groups of finite rank over a locally finite field

A. V. Tushev

We recall that a group G has finite (Prufer) rank if there is an integer r such that each finitely generated subgroup of G can be generated by r elements; its rank $r(G)$ is then the least integer r with this property. A group G is said to be polycyclic if it has a finite series in which each factor is cyclic. A group G is said to have finite torsion-free rank if it has a finite series in which each factor is either infinite cyclic or locally finite; its torsion-free rank $r_0(G)$ is then defined to be the number of infinite cyclic factors in such a series.

It follows from [1] that if a polycyclic group G has faithful irreducible representation over a locally finite field k then the group G is finite. However, as it was shown in [2], infinite locally polycyclic groups of finite rank may have faithful irreducible representations over a locally finite field k. Moreover, in [2] we found necessary and sufficient conditions for existence of faithful irreducible representations of locally polycyclic soluble groups of finite rank over a locally finite field k.

In the presented paper we are searching necessary and sufficient conditions for existence of faithful irreducible representations of soluble groups of finite rank over a locally finite field k.

The subgroup $Soc(G)$ of a group G generated by all its minima normal subgroups is said to be the socle of the group G (if the group G has no minimal normal subgroups then $Soc(G) = 1$). The subgroup $abSoc(G)$ of a group G generated by all its minima normal abelian subgroups is said to be the abelian socle of the group G (if the group G has no minimal abelian normal subgroups then $abSoc(G) = 1$) .

An abelian group is said to be minimax if it has a finite series each of whose factor is either cyclic or quasi-cyclic. If B is an abelian minimax group then the spectrum $Sp(B)$ of the group B is the set of prime numbers p such that the group B has an infinite p-section. It is easy to note that the set $Sp(B)$ is finite.

Let G be an infinite group, we say that an infinite normal subgroup A of the group G is G-just-infinite if $|A : B| < \infty$ for any proper G-invariant subgroup B from A.

It is not difficult to note that if a soluble group G of finite rank has a torsion-free normal subgroup then it has a torsion-free minimax abelian G-just-infinite subgroup.

Proposition 1. *Let G be a soluble group of finite rank which has a torsion-free normal subgroup. Then the group G has a torsion-free minimax abelian normal subgroup $jiSoc(G) \neq 1$ such that $jiSoc(G)$ is a direct product of finitely many of G -just-infinite subgroups and $jiSoc(G) \cap B \neq 1$ for any non-trivial torsion-free normal subgroup B of the group G.*

Theorem 2. *Let G be a soluble group of finite rank and let k be a locally finite field of characteristic p . If the group G has a faithful irreducible representation over the field k then the abelian socle $abSoc(G)$ of the group G is a locally cyclic $\mathbb{Z}G$ -module, where the group G acts on $abSoc(G)$ by conjugations, and $char k \notin \pi(abSoc(G))$.*

We also obtained a sufficient condition for existence of faithful irreducible representations of soluble groups of finite rank over a locally finite field.

Theorem 3. *Let* G *be a soluble group of finite rank and let* k *be a locally finite field of characteristic* p *. Suppose that the abelian socle* $abSoc(G)$ *of the group* G *is a locally cyclic* $\mathbb{Z}G$ *-module, where the group* G *acts on* $abSoc(G)$ *by conjugations, such that* $chark \notin \pi(abSoc(G))$ *and* $chark \notin Sp(jiSoc(G))$ *. Then the group* G *has a faithful irreducible representation over the field* k *.*

We should note that the above theorem does not give a necessary condition for existence of faithful irreducible representations of soluble groups of finite rank over a locally finite field. It follows from a result by Wherfritz [3] in which a simple kG-module was constructed, where k is a field of order p, G is a soluble group of rank 2 and $jiSoc(G)$ is a p-divisible group.

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SECTION TALK

Reduction a pair of matrices to special triangular form over Bezout domain of almost stable range 1

I. S. Vasyunyk, S. I. Bilavska

Let R be a ring of an almost stable range 1, with $1 \neq 0$. Remaind, that R is called a ring of almost stable range 1 if for any non zero and non invertible element $a \in R$ the stable range of factor-ring R/aR is 1 [1-2]. Following Kaplansky [3] we say that it every matrice over R admits a diagonal reduction then R is elementary divisor ring. Remaind, that ring of almost stable range 1 is an elementary divisor ring [1].

Theorem 1. *A commutative Bezout domain R is domain of almost stable range 1 if and only if for any elements a, b, c from $R\backslash\{0\}$, besides $a, b, c \notin U(R)$ and $aR + bR + cR = R$ exists $r \in R$ such that $(a + rb)R + cR = R$.*

Theorem 2. *Let matrice $A_i(i = 1, 2)$ size $m \times k$ over domain of almost stable range 1, besides at least one of this matrices are not zero divisor. Then exists invertible matrices $P, Q(i = 1, 2)$ correspond size with elements from domain R such that*

$$PA_iQ_i = \begin{pmatrix} \varepsilon_1^i & 0 & \dots & \dots & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ * & \dots & \varepsilon_r^i & \vdots & \vdots \\ 0 & 0 & 0 & \ddots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 0 \end{pmatrix}$$

where $\varepsilon_j^i(j = 1, 2, \dots r), r \leq m$ are elementary divisors of matrice A_i

Theorem 3. *Let $A = B \cdot C$, where B,C are matrices over domain of almost stable range 1 and matrice A is not zero divisor. Then elementary divisor of matrice A divided on correspond elementary divisors of matrices B and C.*

Theorem 4. *Let for matrice A over ring $R[x]$, where R is a commutative regular ring perform correlation $A = B \cdot C$. Then elementary divisors of matrice A divided on correspond elementary divisors of matrices B and C.*

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SHORT COMMUNICATION

Semifields with finite set of kernels

E. M. Vechtomov, A. V. Cheraneva

A *semifield* is an algebraic structure $\langle U, +, \rangle$ which is an additive commutative semigroup and a multiplicative group at the same time, and a multiplication is distributive with respect to addition from both sides. Class of unit 1 of any congruence on semifield is called *kernel* of semifield. Let $\mathrm{Con}U$ be the lattice of all kernels (congruencies) of semifield U. The least kernel of semifield U connected element $u \in U$ is called a *principal kernel* and denoted (u).

This work is extension of the report [1]. The theory of semifields and its kernels is explained in articles [2] and [3].

Semifield U is called:

biregular if every its principal kernel has a complement in lattice $\mathrm{Con}U$;

prime if $\mathrm{Con}U$ has two elements;

Gelfand if for each of its different maximal kernels A and B there exist principal kernels $(u) \subseteq A \setminus B$ and $(v) \subseteq B \setminus A$ such that $(u) \cap (v) = \{1\}$;

Rickart if the pseudocomplement of every its principal kernel has a complement in lattice $\mathrm{Con}U$.

Theorem 1. *All kernels of an arbitrary semifield with finite set of kernels are principal.*

Corollary 1. *Let U be a semifield with finite set of kernels. The following statements are equivalent:*

1) *U is biregular;*
2) *U is isomorphic to direct product of a finite number of prime semifields;*
3) *lattice $\mathrm{Con}U$ is Boolean.*

Theorem 2. *An arbitrary semifield U with finite set Con U has following properties:*

1. *The lattice $\mathrm{Con}U$ is distributive.*
2. *U is direct product of nondecomposable semifields.*
3. *If U is Gelfand then U is direct product of a finite number of semifields with unique maximal kernel.*
4. *If U is Rickart then U is direct product of a finite number of semifields with unique minimal nontrivial kernel.*

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8th International Algebraic Conference
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PLENARY TALK

Fractionally clean Bezout rings

B. Zabavsky

Throughout this notes R is assumed to be a commutative ring with $1 \neq 0$. Let P be a ring property. Following Vamos [1] a ring R is fractionally P provided that the classical quotient ring $Q(R/J)$ of a ring R/J satisfies P for every ideal J of R. A ring R is fractionally clean if for every nonzero and nonunit element $a \in R$ the classical quotient ring $Q(R/J(aR))$ is clean, where $J(aR)$ is the Jacobson radical of the ring R/aR. Obviously, the fractionally regular ring is the fractionally clean ring [2]. A ring is called clean if every element is the sum of a unit and an idempotent. A PM^* ring is a ring such each nonzero prime ideal is contained in a unique maximal ideal [3]. A ring R is neat ring if R/aR is a clean ring for every nonzero and noninvertible element $a \in R$ [3].

A n by m matrix $A = (a_{ij})$ is said to be diagonal if $a_{ij} = 0$ for all $i \neq j$. We say that the matrix A of dimension n by m admits a diagonal reduction if there exist invertible matrices $P \in GL_n(R)$, $Q \in GL_m(R)$ such that PAQ is a diagonal matrix. Following the Kaplansky [4] we say that if every matrix over R admits a diagonal reduction then R is elementary divisors ring.

Theorem 1. *Let R be a fractionally clean PM^* Bezout domain. Then:*
1) R — elementary divisor domain;
2) R — neat domain.

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8th International Algebraic Conference
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Short communication

Zero adequate ring is an exchange ring

B. V. Zabavsky, S. I. Bilavska

Let R be a commutative ring with $1 \neq 0$.

Definition 1. *An element$a$ of a ring R is called an adequate element if for any b from R an element a can be represented as a product $a = r \cdot s$, where $rR + bR = R$ and for any non invertible divisor $s\prime$ of s we can obtain $s\prime R + bR \neq R$ [1,2].*

A commutative Bezout ring in which any non zero element is adequate is called an adequate ring [3]. A Bezout ring which any element(even zero) is adequate is called an adequate ring [1,4].

Theorem 1. *Let a be an adequate element of a commutative Bezout ring. Then $\bar{0}$ is an adequate element of the factor-ring R/aR.*

Theorem 2. *A zero adequate ring is an exchange ring.*

Theorem 3. *Let R de a commutative Bezout ring and let a be an adequate element of R. Then the following statements hold:*
1). R/aR is an exchange ring;
2). R/aR is a clean ring;
3). R/aR is a ring of idempotent stable range 1;
4). R/aR is a PM-ring;

The set of all minimal prime ideals of a ring R is denoted by $minR$.

Theorem 4. *Let R be a commutative Bezout ring in which zero is an adequate element and $minR$ is a finite set. Then R is a zero adequate ring.*

Theorem 5. *Let R be a commutative Bezout domain in which for any non invertible and non zero element a the factor-ring R/aR is a ring in which zero is an adequate element. Then R is an adequate domain.*

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8th International Algebraic Conference
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Reduction of matrices over ring with stable range 1 in localization

B. Zabavsky, O. Domsha

All rings we consider will be commutative and have identity. A ring is a Bezout ring if every finitely generated ideal is principal. Following Kaplansky [1] a ring R is said to be an elementary divisor ring if every matrix over R is equivalent to a diagonal matrix. A ring R is said to be adequate if R is Bezout and for $a, b \in R$ with $a \neq 0$ there exists $r, s \in R$ such as $a = rs$, $rR + bR = R$ and if a nonunit $s\prime$ divider s, then $s\prime R + bR \neq R$. A ring R is a ring with stable range 1 if for every $a, b \in R$ with $aR + bR = R$ there exist $t \in R$ such $(a + bt)R = R$.

Let R be a commutative Bezout domain, a — nonzero and noninvertible element of domain R. Let's denote the set S_a by

$$S_a = \{b | b \in R, aR + bR = R\}.$$

Its proved that the set S_a is saturated and multiplicatively closed [2].

Let a — nonzero and non invertible element of domain R. Let's denote $R_a = RS_a^{-1}$.

Theorem 1. *Let R be an adequate domain. Then for any nonzero and non invertible element $a \in R$ the set R_a is a commutative Bezout domain of stable range 1.*

Theorem 2. *Let R be such commutative Bezout domain for any nonzero element $a \in R$ stable range of (R_a) is equal 1. Then R is an elementary divisor ring.*

Theorem 3. *Let R is such a commutative Bezout domain for any nonzero and noninvertible element $a \in R$ the localization R_a is an adequate ring. Then R is an elementary divisor ring.*

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On matrix equations over polynomial ring with involution

V. Zelisko

Let involution ∇ be defined in a polynomial ring $\mathbb{C}[x]$ in one of the possible ways [1]:

$$(\alpha) \quad (\sum_{i=1}^{m} a_i x^i)^\nabla = \sum_{i=1}^{m} \overline{a_i}(-x)^i,$$

$$(\beta) \quad (\sum_{i=1}^{m} a_i x^i)^\nabla = \sum_{i=1}^{m} a_i(-x)^i,$$

$$(\gamma) \quad (\sum_{i=1}^{m} a_i x^i)^\nabla = \sum_{i=1}^{m} a_i(x)^i.$$

The involution ∇ transfers onto the matrix ring $M_n\big(\mathbb{C}[x]\big)$ as follows:

$$A(x)^\nabla = \|a_{ij}(x)\|^\nabla = \|a_{ji}(x)^\nabla\|.$$

Consider the question of the uniqueness of solutions of the matrix equation

$$A(x)X(x) - Y(x)A(x)^\nabla = C(x), \tag{1}$$

where elements of matrix $A(x)$ and $C(x)$ belong to the polynomial ring $\mathbb{C}[x]$ with involution ∇, defined in one of the ways (α), (β) or (γ), $X(x)$ and $Y(x)$ are unknown matrices of order n over $\mathbb{C}[x]$.

Theorem. Matrix equation (1) , where $A(x)$ is a unital matrix of non-zero degree over $\mathbb{C}[x]$, has a unique solution if and only if the roots of $\det A(x)$ do not belong to the imaginary axis of the complex plane in the case of involution (α) or do not equal zero for involution (β). In the case of identity involution (γ) the equation (1) does not have a unique solution.

Conditions of the existence of a unique solution of a matrix equation

$$A(x)X(x)A(x)^\nabla - Y(x) = C(x), \tag{2}$$

where $A(x), C(x) \in M_n\big(\mathbb{C}[x]\big)$ are similar, and involution ∇ is defined in one of the ways (α), (β) or (γ).

Solving the equations (1) and (2) leads to linear matrix equations over the field $\mathbb{C}$ with involution.

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8th International Algebraic Conference
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Модулі над черепичними порядками

Віктор Журавльов

Нехай Λ — зведений черепичний порядок над дискретно нормованим кільцем $\mathcal{O}$, M — незвідний Λ - модуль. Для довільного $m \in M$ і для довільного цілочислового вектора $\vec{f} = (f_1, \ldots, f_t) \in \mathbb{Z}^t$ покладемо $m\vec{f} = (f_1 m, \ldots, f_t m)$. Тоді для незвідних Λ - модулів $M_1, \ldots, M_s$ формальна сума $M_1\vec{f_1} + \cdots + M_s\vec{f_s}$ визначає модуль, елементами якого є $m_1\vec{f_1} + \cdots + m_s\vec{f_s}$, де $m_i \in M_i$.

Теорема 1. *Нехай Λ — зведений черепичний порядок. Для довільного незвідного Λ - модуля M існує проективна резольвента цього модуля, всі ядра якої мають вигляд*

$$K_i = \sum_{j=1}^{m_i} M_j^{(i)} \vec{f}_j^{(i)},$$

де $M_j^{(i)}$ — незвідні Λ - модулі, $\vec{f}_j^{(i)}$ — цілочисельні вектори.

Теорема 2. *Нехай Λ — зведений черепичний порядок, $M_1, \ldots, M_s$ — незвідні Λ - модулі, K — ядро епіморфізму $\varphi\colon \bigoplus\limits_{i=1}^{s} M_i \to \sum\limits_{i=1}^{s} M_i$, що діє за правилом $\varphi(m_1, \ldots, m_s) = m_1 + \cdots + m_s$, де $s = 3, 4$. Ядро K розкладається в пряму суму модулів тоді і тільки тоді, коли існує набір $i_1, \ldots, i_s$ попарно різних індексів таких, що $M_{i_1} \cap M_{i_k} \subset M_{i_s}$ для всіх $k \neq 1, s$.*

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Contributed abstract

Черепичні порядки в $M_6(D)$ скінченної глобальної розмірності

Віктор Журавльов, Дмитро Журавльов

Ми описуємо з точністю до ізоморфізму всі черепичні порядки в $M_6(D)$ скінченної глобальної розмірності. Глобальна розмірність таких порядків не перевищує 6. Існує 3 неізоморфних черепичних порядка в $M_6(D)$ глобальної розмірності 6.

Відомості про авторів

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Про одну точну послідовність модулів

Тетяна Журавльова

Нехай Λ — зведений черепичний порядок над дискретно нормованим кільцем $\mathcal{O}$, тобто нетерове первинне напівдосконале напівдистрибутивне кільце з ненульовим радикалом Джекобсона, $Q = M_n(D)$ — кільце часток черепичного порядку Λ, де D — тіло часток дискретно нормованого кільця $\mathcal{O}$.

Означення 1. *Λ - модуль M, що є підмодулем простого Q - модуля, називається незвідним.*

Нехай $M_1, \dots, M_s$ — незвідні модулі. Покладемо $M_{i_1 \cdots i_k} = M_{i_1} \cap \cdots \cap M_{i_k}$, де $i_1 < \cdots < i_k$.

Теорема 1. *Нехай Λ — черепичний порядок, $M_1, \dots, M_s$ — незвідні Λ - модулі. Наступна послідовність модулів*

$$0 \to M_{1 \cdots s} \to \bigoplus_{i_1 \cdots i_{s-1}} M_{i_1 \cdots i_{s-1}} \to \cdots \to \bigoplus_{i_1 i_2} M_{i_1 i_2} \to \bigoplus_{i=1}^{s} M_i \to \sum_{i=1}^{s} M_i \to 0$$

є точною.

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TOPICAL SECTION VIII

SEMIGROUPS AND ALGEBRAIC SYSTEMS

8th International Algebraic Conference
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Contributed abstract

Multipliers and extensions of semigroups

A. Mamoon Ahmed

Let H be a normal subsemigroup of a group G, then a multiplier on H can be extended to a multiplier on G. In this paper we present entension and dialation theorems for multipliers, and we prove a more general dialation theorem than the one proved by Laca and Raeburn [1].

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CONTRIBUTED ABSTRACT

Diagonal acts over semigroups of isotone transformations

T. V. Apraksina

A *right act* over a semigroup S (see [1]) is defined as a set X with a mapping $X \times S \to X$, $(x,s) \mapsto xs$ and the axiom $(xs)t = x(st)$ for $x \in X; s,t \in S$. A *left act* is defined analogously. Obviously, the set $S \times S$ is a right act over the semigroup S if the action is defined by this way: $(x,y)s = (xs, ys)$, and the left act relating to the rule $s(x,y) = (sx, sy)$. We call this act *diagonal*. The set $S^n = \underbrace{S \times \cdots \times S}_{n}$, for any natural number n, is also a right and left act over the semigroup S under the rules $(s_1, \ldots, s_n)s = (s_1 s, \ldots, s_n s)$ and $s(s_1, \ldots, s_n) = (ss_1, \ldots, ss_n)$ respectively. In [2] it was considered for what semigroup S the diagonal act is cyclic, i. e. generated by one element. If (s_0, t_0) is a generating element then $\forall s,t\, \exists u : (s_0 u = s \,\&\, t_0 u = t)$.

Remark. *If $(S{\times}S)_S$ is a cyclic diagonal right act, then diagonal right acts $(\underbrace{S \times \cdots \times S}_{n})_S, \forall n \in \mathbb{N}$ are also cyclic.*

It was proved in [2] that for an infinite set X diagonal acts over semigroups $T(X), P(X)$ of full and partial transformations on X are cyclic.

Let $O(X)$ denote a semigroup of *isotone* (order preserving) transformations $\alpha : X \to X$ of a partially ordered set X. Further, $PO(X)$ is a set of all $\alpha \in P(X)$ such that $\forall x, y \in dom\, \alpha\ (x \le y \implies x\alpha \le y\alpha)$

Main results:

Theorem. *For any linearly ordered set Γ consisting of more than one element the diagonal right and left acts $S \times S$ over the semigroup $S = O(\Gamma)$ are not cyclic.*

Theorem. *Necessary and sufficient condition for a diagonal right act $PO(X) \times PO(X)$ to be cyclic is the existence of two isomorphic to X subsets X_1 and X_2, such that $\forall x \in X_1\, \forall y \in X_2 (x \not\le y \,\&\, y \not\le x)$.*

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SHORT COMMUNICATION

About quasigroups with distinct conjugates

G. B. Belyavskaya, T. V. Popovich

A quasigroup is an ordered pair (Q, A) where Q is a set and A is a binary operation defined on Q such that each of the equations $A(a, y) = b$ and $A(x, a) = b$ is uniquely solvable for any pair of elements a, b in Q. With any quasigroup (Q, A) the system $\Sigma(A)$ of six (not necessarily distinct) and the set $\overline{\Sigma}(A)$ of its conjugates (parastrophes) are connected: $\Sigma(A) = (A,\ {}^rA,\ {}^lA,\ {}^{lr}A,\ {}^{rl}A,\ {}^sA)$, where ${}^1A = A$, ${}^rA = A^{-1}$, ${}^lA = {}^{-1}A$, ${}^{lr}A = {}^{-1}(A^{-1})$, ${}^{rl}A = ({}^{-1}A)^{-1}$, ${}^sA = A^*$. It is known of [1] that $|\ \overline{\Sigma}(A)\ | = 1,2,3$ or 6.

`Definition 1.` A quasigroup is called a distinct conjugate quasigroup or, shortly, a DC-quasigroup, if all its conjugates are pairwise distinct, that is $|\ \overline{\Sigma}\ | = 6$.

Let $\overline{T} = \{A(x, A(x, y)) = y, A(A(y, x), x) = y, A(x, y) = A(y, x), A(A(x, y), x) = y\}$.

`Theorem 1.` A quasigroup (Q, A) is a DC-quasigroup if and only if $A \neq {}^rA, {}^lA, {}^sA, {}^{lr}A$. A quasigroup (Q, A) is a DC-quasigroup if and only if it satisfies none of four identities of the set $\overline{T}$.

Let V_1 be the class of quasigroups satisfying all identities of $\overline{T}$; V_2 (V_3^1, V_3^2, V_3^3) be the class of quasigroups satisfying exactly the identity $A(A(x, y), x) = y$ (exactly the identity the identity $A(x, A(x, y)) = y$, $A(A(y, x), x) = y$, $A(x, y) = A(y, x)$, respectively) of $\overline{T}$; V_6 be the class of quasigroups which satisfy none of four identities of $\overline{T}$.

`Theorem 2.` All quasigroups can be divided on six disjoint classes $V_1, V_2, V_3^1, V_3^2, V_3^3, V_6$. The direct product of two quasigroups of the same class is contained in this class. The direct product of a quasigroup from V_1 and a quasigroup of V_2, V_3^1, V_3^2, V_3^3 or V_6 is contained in V_2, V_3^1, V_3^2, V_3^3 or V_6 respectively. The direct product of two quasigroups from distinct classes of $V_2, V_3^1, V_3^2, V_3^3, V_6$ is contained in V_6.

`Proposition 1.` All abelian groups of exponent 2 are contained in the class V_1, the rest abelian groups are contained in the class V_3^3. Nonabelian groups are DC-groups.

`Proposition 2.` Any DC-quasigroup is noncommutative. Any conjugate of a DC-quasigroup is a DC-quasigroup. Any quasigroup containing a DC-subquasigroup is a DC-quasigroup. The direct product of two DC-quasigroups is a DC-quasigroup. The direct product of two quasigroups from distinct classes of $V_2, V_3^1, V_3^2, V_3^3, V_6$ is a DC-quasigroup.

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8th International Algebraic Conference
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SHORT COMMUNICATION

The monoid of strong endomorphisms of hypergraphs

E. Bondar

The monoid of strong endomorphisms for undirected graphs without multiple edges was considerate in [1] by means of the wreath product of the monoid with the small category. In present paper we study the strong endomorphism monoid for a certain class of hypergraphs. Undefined notions can be find, for example, in [2], [3].

A hypergraph H is a pair $H = (V, \mathcal{E})$ where V is a set of elements, called vertices, and $\mathcal{E}$ is a family of non-empty subsets of V called edges.

A transformation $\alpha : V \to V$ of hypergraph H is called a hypergraph strong endomorphism if for any $A \subseteq V$ condition $A \in \mathcal{E}$ holds if and only if $A\alpha \in \mathcal{E}$. Strong endomorphisms of hypergraph H form a monoid under transformation composition. This monoid is denoted by $SEndH$.

By C we denote a class of hypergraphs H without multiply edges such that

$$\text{condition } |e| - |e'| = 1 \text{ implies } |e| = 2,\ |e'| = 1 \text{ for all } e, e' \in \mathcal{E}.$$

Suppose, $H \in C$, $x \in V$ and

$$N(x) = \{A \subseteq V|\ (|A| = 1\ \&\ A \cup \{x\} \in \mathcal{E})\ \vee\ (|A| \geq 2\ \&\ A \cup \{x\} \in \mathcal{E}\ \&\ x \notin A)\}.$$

We will say that $N(x)$ is a neighborhood of x. Further, we define the relation of equivalence ν as follows

$$x\nu y \Leftrightarrow N(x) = N(y).$$

Let x_ν be the equivalence class such that x is contained in x_ν. By H/ν we denote the canonical strong factor hypergraph of H, where equivalence classes $x_\nu, x \in V$ forms the set of vertices, and the family of edges $\mathcal{E}(H/\nu)$ contains $\{x_\nu, ..., y_\nu\}$ iff for any $a \in x_\nu, ..., b \in y_\nu$ condition $\{a, ..., b\} \in \mathcal{E}$ holds.

A hypergraf $U[(Y_u)_{u\in U}]$ is called the generalized lexicographic product of hypergraph U with the graphs $(Y_u)_{u\in U}$ if

$$V(U[(Y_u)_{u\in U}]) = \{(u, y_u)|u \in U, y_u \in Y_u\},$$

and an arbitrary subset $\{(u_1, y_{u_1}), ..., (u_k, y_{u_k})\} \subseteq V(U[(Y_u)_{u\in U}])$ is contained in $\mathcal{E}(U[(Y_u)_{u\in U}])$ if and only if

(i) $\{u_1, , ..., u_k\} \in \mathcal{E}(U)$;

(ii) $u_1 = ... = u_k$, $\{u_1\} \notin \mathcal{E}(U)$ и $\{y_{u_1}, ..., y_{u_k}\} \in \mathcal{E}(Y_{u_1})$.

We define a small category $\mathcal{K}$ by putting the objects set $Ob\mathcal{K} = \{Y_u|u \in U\}$, where $U = H/\nu$ and $Y_u(u \in U)$ are the sets such that $|Y_u| = |u|$. Let $MorK = \bigcup_{u,v\in U} Map(Y_u, Y_v)$ be the set of morphisms, where $Map(Y_u, Y_v)$ is a set of all maps from Y_u to Y_v.

By $AutU\ wr\ \mathcal{K}$ we denote the wreath product of the automorphism group $AutU$ of hypergraph U with the small category $\mathcal{K}$ [4].

Theorem. *Let $H = U[(Y_u)_{u\in U}]$ be a finite hypergraph of class C, $U = H/\nu$, $\mathcal{K}$ be the small category defined as above. Then*

$$SEndH \cong AutU\ wr\ \mathcal{K}.$$

This theorem generalizes the main result of [1].

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8th International Algebraic Conference
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CONTRIBUTED ABSTRACT

On semigroup of partial connected homeomorphisms of the interval with the fixed initial point

Ivan Chuchman

Let $I = [0, 1]$ be the unit interval with the usual topology. A partial map $\varphi\colon I \rightharpoonup I$ is called *connected* if the domain and the range of φ are connected subsets of I. By $\mathfrak{I}_{\mathsf{c}}(I, 0)$ we denote a semigroup of all connected partial homeomorphisms φ of I such that $\varphi(0) = 0$ with the operation of composition.

We describe algebraic properties of the semigroup $\mathfrak{I}_{\mathsf{c}}(I, 0)$ and discuss on the semigroup (inverse) topologies on $\mathfrak{I}_{\mathsf{c}}(I, 0)$ which are natural with the respect to algebraic structure of the semigroup $\mathfrak{I}_{\mathsf{c}}(I, 0)$.

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Full list of a finite commutative semigroups for which inverse monoid of local automorphisms is permutable

V. Derech

We say that a semigroup S is a permutable semigroup if the congruences of S commute with each other, that is, $\alpha \circ \beta = \beta \circ \alpha$ is satisfied for all congruences α and β of S. Let S be a semigroup. By a local automorphism of S we mean an isomorphism between two subsemigroups of S. The set of all local automorphisms forms an inverse monoid under composition.

We define one class of nilpotent semigroups as follows. Let H ($|H| \geq 4$)be a finite set. Let 0 and z be two different fixed elements from H. We define an operation $*$ on H as follows:

$a)$ for arbitrary $x \in H$ $0 * x = x * 0 = 0$;

$b)$ for any x from H $x * x = 0$;

$c)$ if $x \neq y$ and $\{x, y\} \cap \{0, z\} = \emptyset$, then $x * y = y * x = z$;

$d)$ for arbitrary $x \in H$ $x * z = z * x = 0$.

It is easy to verify that $(H, *)$ is commutative nilpotent semigroup. Let $\mathcal{N}$ denote the class of nilpotent semigroups defined above.

Theorem. *Full list of a finite commutative semigroups for which inverse monoid of local automorphisms is permutable:*

(1) *chain;*

(2) *primitive semilattice;*

(3) *elementary abelian p-group;*

(4) *null semigroup;*

(5) *nilpotent semigroup from the class $\mathcal{N}$.*

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8th International Algebraic Conference
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SHORT COMMUNICATION

Subtraction Menger algebras

W. A. Dudek, V. S. Trokhimenko

The set of partial mappings from A^n into A closed with the composition

$$f[g_1 \dots g_n](x_1, \dots, x_n) = f(g(x_1, \dots, x_n), \dots, g_n(x_1, \dots, x_n))$$

is called a *Menger algebra of n-place functions* [1]. This algebra satisfies the identity

$$(f[g_1 \dots g_n])[h_1 \dots h_n] = f[g_1[h_1 \dots h_n] \dots g_n[h_1 \dots h_n]].$$

Any n-place function is a subset of the Cartesian product A^{n+1}. Thus the set-theoretic operations on n-place functions have a natural interpretation. As is well known, the set-theoretic inclusion $\subset$ and the operations $\cap$, $\cup$ can be expressed by the set-theoretic difference (subtraction) in the following way:

$$f \subset g \longleftrightarrow f \setminus g = \emptyset, \;\; f \cap g = f \setminus (f \setminus g), \;\; f \cup g = h \setminus ((h \setminus f) \cap (h \setminus g)).$$

where f, g, h are arbitrary n-place functions such that $f \subset h$ and $g \subset h$.

We present abstract characterizations of Menger algebras of n-place functions closed with respect to the set-theoretic difference (subtraction) of functions. The proofs of our results which are a generalizations of results obtained by B. M. Schein for semigroups (cf. [3]), one can find in [2].

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CONTRIBUTED ABSTRACT

The semigroup $\upsilon(X)$ of upfamilies on a semilattice X

V. Gavrylkiv

Given a semilattice X we study the algebraic properties of the semigroup $\upsilon(X)$ of upfamilies on X. The semigroup $\upsilon(X)$ contains the Stone-Čech extension $\beta(X)$, the superextension $\lambda(X)$, and the space of filters $\varphi(X)$ on X as closed subsemigroups. We prove that $\upsilon(X)$ is a semilattice iff $\lambda(X)$ is a semilattice iff $\varphi(X)$ is a semilattice iff the semilattice X is finite and linearly ordered. We prove that the semigroup $\beta(X)$ is a band if and only if X has no infinite antichains, and the semigroup $\lambda(X)$ is commutative if and only if X is a bush with finite branches.

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8th International Algebraic Conference
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SHORT COMMUNICATION

Варианты свободной полугруппы и прямоугольные связки

А. Горбатков

Пусть S — произвольная полугруппа и $x \in S$ — фиксированный элемент. Определим на S бинарную операцию $*_x$ следующим образом:

$$a *_x b = axb \quad (a, b \in S).$$

Тогда $(S, *_x)$ является полугруппой, которая называется вариантом полугруппы S.

Преобразование $\tau : S \to S$ называется полуретракцией [1], если для всех $a, b \in S$ выполняется условие

$$\tau(ab) = \tau(\tau(a)\tau(b)).$$

Известно, что каждая полуретракция моноида определяет на нем конгруэнцию [1], а именно: если τ — полуретракция, то ее отношение равнозначности является конгруэнцией. Аналогично, каждая идемпотентная полуретракция полугруппы определяет на ней конгруэнцию [2].

Если ρ — конгруэнция на полугруппе S такая, что S/ρ — полугруппа левых нулей (соответственно полугруппа правых нулей, прямоугольная связка), то ρ будем называть $\mathcal{LZ}$-конгруэнцией (соответственно $\mathcal{RZ}$-конгруэнцией, $\mathcal{RB}$-конгруэнцией).

Зафиксируем непустое не более чем счетное множество X. Пусть X^+ — свободная полугруппа над алфавитом X, X^* — свободный моноид над X с пустым словом θ, $u \in X^+$. Вариант $(X^+, *_u)$ полугруппы X^+ обозначим через X_u^+ и положим $X_\theta^+ = X^+$.

Для всех $w \in X^+$ через $\overline{w}$ обозначим слово, полученное из w записью букв в обратном порядке.

Пусть $w \in X^*$. Напомним, что слово $f \in X^*$ называется префиксом слова w, если $w = fv$ для некоторого $v \in X^*$.

Для всех $w \in X_u^+$ $(u \in X^*)$ определим $h_u(w)$ как самый короткий непустой префикс слова w такой, что $h_u(w) \notin X^+u$ и $wu = h_u(w)uf$ для некоторого $f \in X^*$. Ясно, что h_u — преобразование полугруппы X_u^+. Далее определим преобразования $\overline{h_u}$ и r_u полугруппы X_u^+, полагая

$$\overline{h_u}(w) = \overline{h_{\overline{u}}(\overline{w})}, \quad r_u(w) = h_u(w) *_u \overline{h_u}(w)$$

для всех $w \in X_u^+$.

Теорема. *Преобразования h_u, $\overline{h_u}$ и r_u являются полуретракциями, которые определяют на X_u^+ наименьшие $\mathcal{LZ}$-, $\mathcal{RZ}$- и $\mathcal{RB}$-конгруэнции соответственно.*

В случае когда $u = \theta$, из предыдущей теоремы получаем описание [3] наименьших $\mathcal{LZ}$-, $\mathcal{RZ}$- и $\mathcal{RB}$-конгруэнций на свободной полугруппе X^+.

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СВЕДЕНИЯ ОБ АВТОРАХ

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8th International Algebraic Conference
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Plenary talk

On semigroups of (almost) monotone injective partial selfmaps of integers with cofinite domains and images

Oleg Gutik

We discuss on the structure of the semigroup $\mathscr{I}_{\infty}^{\nearrow}(\mathbb{Z})$ of monotone injective partial selfmaps of the set of integers having cofinite domain and image.

We describe the Green relations on $\mathscr{I}_{\infty}^{\nearrow}(\mathbb{Z})$, the structures of the band and maximal subgroups of $\mathscr{I}_{\infty}^{\nearrow}(\mathbb{Z})$. We show that $\mathscr{I}_{\infty}^{\nearrow}(\mathbb{Z})$ is bisimple and all of its non-trivial semigroup homomorphisms are either isomorphisms or group homomorphisms.

We also prove that every Baire topology τ on $\mathscr{I}_{\infty}^{\nearrow}(\mathbb{Z})$ such that $(\mathscr{I}_{\infty}^{\nearrow}(\mathbb{Z}),\tau)$ is a Hausdorff semitopological semigroup is discrete and we construct a non-discrete Hausdorff semigroup inverse topology τ_W on $\mathscr{I}_{\infty}^{\nearrow}(\mathbb{Z})$. We show that the discrete semigroup $\mathscr{I}_{\infty}^{\nearrow}(\mathbb{Z})$ does not embed in some classes of compact-like topological semigroups and that its remainder under the closure in a topological semigroup S is an ideal in S. We also describe the Bohr compactification of the semigroup $(\mathscr{I}_{\infty}^{\nearrow}(\mathbb{Z}),\tau)$ for some semigroup topologies τ on $\mathscr{I}_{\infty}^{\nearrow}(\mathbb{Z})$.

Next we present similar results for the semigroup $\mathscr{I}_{\infty}^{\rightleftharpoons}(\mathbb{Z})$ of almost monotone injective partial selfmaps of the set of integers having cofinite domain and image.

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8th International Algebraic Conference
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PLENARY TALK

Congruences on Γ-semigroups

H. Hedayati

A semigroup is an algebraic structure consisting of a non-empty set S together with an associative binary operation. The formal study of semigroups began in the early 20th century. Semigroups are important in many areas of mathematics, for example, coding and language theory, automata theory, combinatorics and mathematical analysis. In 1986, Sen and Saha defined the notion of a Γ-semigroup as a generalization of a semigroup. One can see that Γ-semigroups are generalizations of semigroups. Many classical notions of semigroups have been extended to Γ-semigroups and a lot of results on Γ-semigroups are published by a lot of mathematicians. Let $\mathcal{S}$ and Γ be two non-empty sets. $\mathcal{S}$ is called a Γ-*semigroup* if there exists a mapping $\mathcal{S} \times \Gamma \times \mathcal{S} \longrightarrow \mathcal{S}$ written as $(x, \gamma, y) \mapsto x\gamma y$ satisfying $(x\gamma y)\beta z = x\gamma(y\beta z)$ for all $x, y, z \in \mathcal{S}$ and $\gamma, \beta \in \Gamma$. Let $\mathcal{S}_1$ be a Γ_1-semigroup and $\mathcal{S}_2$ a Γ_2-semigroup. Then $(f, g) : (\mathcal{S}_1, \Gamma_1) \longrightarrow (\mathcal{S}_2, \Gamma_2)$ is called a *homomorphism* if $f : \mathcal{S}_1 \longrightarrow \mathcal{S}_2$ and $g : \Gamma_1 \longrightarrow \Gamma_2$ are functions and $f(x\gamma y) = f(x)g(\gamma)f(y)$ for all $x, y \in \mathcal{S}_1$ and $\gamma \in \Gamma_1$.

In this paper, we prove some results on ideals, quotient of Γ-semigroups and their corresponding theorems. Also, by considering the notion of congruence relation, we construct a new Γ-semigroup and discuss the formation of ideals of Γ-semigroup. Finally, by using the congruence relations induced by homomorphisms, we establish some isomorphism theorems and investigate the commutativity of some diagrams of Γ-semigroups.

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8th International Algebraic Conference
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Cubical sets and trace monoids

A. Husainov

We build the adjoint functors between the category of pointed sets with the actions of trace monoids and the category of cubical sets. This allows us to construct adjoint functors between the category of asynchronous systems with weak morphisms and the category of pointed higher dimensional automata.

Let E be a set, $I \subseteq E \times E$ an irreflexive symmetric relation. A *trace monoid* $M(E, I)$ is given by the set of generators E and relations $ab = ba$, for all $(a, b) \in I$. A homomorphism $f : M(E, I) \to M(E', I')$ is called *basic* if $f(E) \subseteq E' \cup \{1\}$. The homomorphism $f : M(E, I) \to M(E', I')$ is called *independence preserving* if it is basic and for all $(a, b) \in I$ it is true that $f(a) = 1 \vee f(b) = 1 \vee (f(a), f(b)) \in I'$.

Denote by FPCM the category of trace monoids and basic homomorphisms, Set_* the category of pointed sets. Let $(\mathrm{FPCM}, \mathrm{Set}_*)$ be the category of pairs $(M(E, I), X)$ consisting of a trace monoid with a pointed right $M(E, I)$-set X. A morphism $(M(E, I), X) \to (M(E', I'), X')$ is a pair (f, σ) consisting of a basic homomorphism $f : M(E, I) \to M(E', I')$ and a pointed map $\sigma : X \to X'$ which satisfy to the condition $\sigma(x\mu) = \sigma(x)f(\mu)$ for all $\mu \in M(E, I)$, $x \in X$.

Proposition 1. *The category* $(\mathrm{FPCM}, \mathrm{Set}_*)$ *is cocomplete.*

Let $\square$ be a category of partially ordered sets $\mathbb{I}^n = \{0, 1\}^n$ and compositions of

$$\delta_i^{n,\nu}(x_1, \cdots, x_n) = (x_1, \cdots, x_{i-1}, \nu, x_i, \cdots, x_n),$$
$$\varepsilon_i^n(x_1, \cdots, x_n) = (x_1, \cdots, x_{i-1}, x_{i+1}, \cdots x_n).$$

A *cubical set* is an arbitrary functor $Q : \square^{op} \to \mathrm{Set}$. For $n \geq 0$, n*-cubes* are elements of $Q_n = Q(\mathbb{I}^n)$. Let Cube be a category of cubical sets and natural transformations. There is a functor $U : (\mathrm{FPCM}, \mathrm{Set}_*) \to \mathrm{Cube}$ defined by $U(M(E, I), X)_n =$

$$\{(x, e_1, \cdots, e_n) : x \in X \ \&\ \{e_1, \cdots, e_n\} \subseteq E \cup \{1\} \ \&\ e_j e_k = e_k e_j \text{ for } 1 \leq j < k \leq n\}.$$

The standard way we can construct a left adjoint F for the functor U.

Consider the subcategory $(\mathrm{FPCM}^{\parallel}, \mathrm{Set}_*) \subset (\mathrm{FPCM}, \mathrm{Set}_*)$ consisting of morphisms (f, σ) where f are independence preserving. Denote by $\mathrm{Cube}^{\parallel}$ a category of cubical sets and morphisms g for which $F(g)$ are independence preserving.

Theorem 1. *The restrictions F and U give the adjoint functors* $U^{\parallel} : (\mathrm{FPCM}^{\parallel}, \mathrm{Set}_*) \to \mathrm{Cube}^{\parallel}$ *and* $F^{\parallel} : \mathrm{Cube}^{\parallel} \to (\mathrm{FPCM}^{\parallel}, \mathrm{Set}_*)$.

Let pt be the cubical set with $\mathrm{pt}_0 = \{p\}$ and $\mathrm{pt}_n = \emptyset$, for $n > 0$. Then $F(\mathrm{pt}) = \mathrm{pt}_*$ is the pointed set {p, *} with the action of 1. An *asynchronous system* can be defined as an object of the comma-category $\mathrm{pt}_*{\downarrow}(FPCM, \mathrm{Set}_*)$. The morphisms of the category $\mathrm{AS}^b = \mathrm{pt}_*{\downarrow}(FPCM, \mathrm{Set}_*)$ are called *weak*. Usual morphisms of asynchronous systems are given in pairs (f, σ) for which $\sigma^{-1}(*) = *$. Let $U_{\mathrm{pt}}^{\parallel} : pt_*{\downarrow}(FPCM^{\parallel}, \mathrm{Set}_*) \to \mathrm{pt}{\downarrow}Cube^{\parallel}$ be a functor induced by $U^{\parallel}$.

Theorem 2. *The functor* $U_{\mathrm{pt}}^{\parallel} : \mathrm{AS}^b \to \mathrm{pt}{\downarrow}\mathrm{Cube}^{\parallel}$ *has a left adjoint functor.*

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SHORT COMMUNICATION

Gluskin theorem for partial transformation semigroups

A. A. Klyushin

In 1961 L. M. Gluskin proved the theorem: two quasiordered sets are isomorphic or antiisomorphic if and only if their transformation semigroups are isomorphic. After that some other kinds of transformation semigroups are studied. In this connection the question has appeared: may this theorem be extended to partial transformation semigroups? The answer proved to be positive.

Next we will introduce some definitions.

Let X be the set. The semigroup of full transformations of X is denoted by $T(X)$. If $x \in X$ and $f, g \in T(X)$ we write $(x)fg$. The semigroup $T(X)$ is a subsemigroup of $PT(X)-$ partial transformation semigroup. If $f \in PT(X)$, then $f : domf \to X$, where $domf \subset X$. The binary relation on X is said to be quasiorder, if it is reflexive and transitive. Let ρ be a binary relation on X. The (partial) transformation f is ρ-preserving, if for every $(x, y) \in \rho$ and $x, y \in domf$ we have $((x)f, (y)f) \in \rho$. The semigroup of all ρ-preserving (partial) transformations is denoted be $T_\rho(X) \quad (PT_\rho(X) \quad)$. Let ρ_1, ρ_2 are the binary relations on the sets X_1, X_2 correspondingly. If there exists a bijective mapping $f : X_1 \to X_2$ with a property: $(x, y) \in \rho_1 \Leftrightarrow ((x)f, (y)f) \in \rho_2$ then we say the structures $\{\rho_1\}$ and $\{\rho_2\}$ are isomorphic. Let S be a semigroup with 0. The set $ndr(S) = \{f \in S | \forall g \in S \quad g \neq 0 \Rightarrow gf \neq 0\}$ is said to be a right nondivisors of a zero. This set is a subsemigroup of S. Let $x \in X$. The transformation $c_x \in T(X)$ such as $\forall y \in S \quad c_x(y) = x$ is named a constant. The set of all constants is denoted by $C(X)$. If ρ is reflexive, $C(X) \subset T_\rho(X)$. The subsemigroup $C(X)$ is a set of all right zeroes of $T_\rho(X)$.

Proposition 1. $ndr(PT_\rho(X)) = T_\rho(X)$.

Proposition 2. *Let $\varphi : PT_{\rho_1}(X_1) \to PT_{\rho_2}(X_2) -$ be an isomorphism of semigroups and $\alpha \in PT_{\rho_1}(X_1)$. Then φ induces the isomorphisms $\varphi : C(X_1) \to C(X_2)$. Let $f : X_1 \to X_2$, $f(x) = y$ if $\varphi(c_x) = c_y$. Then $(dom\,\alpha)f = dom\,(\alpha)\varphi$.*

Theorem. *Let $\rho_1 -$ nontrivial quasiorder relation on the set X_1, $\rho_2 -$ reflexive relation on the set X_2. The semigroups $PT_{\rho_1}(X_1)$ and $PT_{\rho_2}(X_2)$ are isomorphic if and only if the structure $\{\rho_2\}$ is isomorphic to $\{\rho_1\}$ or $\{\rho_1^{-1}\}$.*

Every isomorphism $\varphi : PT_{\rho_1}(X_1) \to PT_{\rho_2}(X_2)$ may be written as $(\alpha)\varphi = f^{-1}\alpha f$, for all $\alpha \in PT_{\rho_1}(X_1)$, where $f : X_1 \to X_2 -$ is isomorphism of the structure $\{\rho_1\}$ or $\{\rho_1^{-1}\}$ onto the $\{\rho_2\}$.

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Short communication

On maximal nilpotent subsemigroups of partial wreath product of inverse semigroups

E. Kochubinska

Semigroup S is called nilpotent if for some $k \in \mathbb{N}$ $S^k = \{0\}$. We distinguish two classes of nilpotent subsemigroups. First class includes nilpotent subsemigroup containing semigroup zero 0. In this case we have $T^k = \{0\}$ for some $k > 0$. Second class includes subsemigroups of S, which are nilpotent as semigroups, but their zero element differs from 0. In this case we have $T^k = \{e\}$ for some idempotent $e \neq \mathrm{id}$ and some $k \geq 1$. We will call these subsemigroups proper nilpotent subsemigroups. A nilpotent subsemigroup $T \subset S$ is called maximal nilpotent subsemigroup, if it is not contained in any other nilpotent subsemigroup $T' \subset S, T \neq T'$.

We consider partial wreath product of inverse semigroups, which is isomorphic to the semigroup $\mathrm{PAut}T$ of partial automorphisms of regular rooted tree.

Theorem 1. *Let S be a maximal nilpotent subsemigroup of the semigroup $\mathcal{IS}_n$. Then subsemigroup $P \wr_p S$ is a maximal nilpotent subsemigroup of the semigroup $P \wr_p \mathcal{IS}_n$. Moreover, every maximal nilpotent subsemigroup of semigroup $P \wr_p \mathcal{IS}_n$ is of this form.*

Let S' be a proper nilpotent subsemigroup of $\mathrm{PAut}T$ with a zero $e \in E(\mathrm{PAut}T)$. Denote by T_x the maximal subtree of T such that its root is $x \in VT$ and none of the edge of T_x is in $\mathrm{dom}(e)$.

Theorem 2. *Proper maximal subsemigroup of $\mathrm{PAut}T$ is (canonically) isomorphic to*

$$\prod_{x \in \mathrm{dom}(e)} Nilp_x,$$

where $Nilp_x$ is a maximal nilpotent subsemigroup of $\mathrm{PAut}T_x$.

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PLENARY TALK

On ultraproducts of the Noetherian V-monoids with zero

M. Komarnytskyj, H. Zelisko

Let S be a monoid with zero.

A monoid S is said to be right Noetherian if S satisfies ascending chain condition on right ideals [1].

A right S-act A_S is the set A with a multiplication $A \times S \to A$ given by $(a, s) \mapsto as$ such that $a(st) = (as)t$ for all $a \in A$ and for all $s, t \in S$ and $a \cdot 1 = a$ for all $a \in A$.

An S-act A_S is injective if for every S-monomorphism $f : M_S \to N_S$ and S-homomorphism $g : M_S \to A_S$ there is an S-homomorphism $h : N_S \to A_S$ satisfying $h \circ f = g$.

A monoid S is called V-monoid if every totally irreducible right S-act is injective.

Theorem 1. The ultraproduct $S = (\prod\limits_{i\in\mathbb{N}} S_i)/\mathfrak{D}$ of the family of a right Noetherian V-monoids $\{S_i\}_{i\in\mathbb{N}}$ over the nonprincipal ultrafilter $\mathfrak{D}$ is a V-monoid.

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CONTRIBUTED ABSTRACT

On the second 0-cohomology group of completely 0-simple semigroups

A. Kostin

0-Cohomology groups of completely 0-simple semigroups $\mathcal{M}^0(G; I, \Lambda; P)$ was studied by B. V. Novikov in [1]. The result of that paper was $H_0^n(S, A) \cong H^n(G, A)$ for all $n > 2$. In the case when $n = 2$ the exact sequence was given only.

The second 0-cohomology group $H_0^2(S, A)$ was calculated in [2] when A is a trivial 0-module. The aim of this theorem is a generalization of this result to an arbitrary 0-module.

Let S be a completely 0-simple semigroup $\mathcal{M}^0(G; I, \Lambda; P)$ and Γ be a bipartite graph that corresponds to the sandwich-matrix P: $I,\ \Lambda$ are partitions and (λ, j) is an edge if $p_{\lambda j} \neq 0$. Let T be a maximal acyclic subgraph of Γ and

$$\begin{aligned} \tilde{Z_T^2} &= \{x_{i1}\rho(\lambda, j) \mid \rho : E_\Gamma \longrightarrow A,\ \rho(\lambda, j) = 0 \text{ if } (\lambda, j) \in T\}, \\ B_T^2 &= \{x_{i1}\left[e_{1\lambda}a_j + b_\lambda + \hat{\alpha}(p_{\lambda j})\right] \mid a_j,\ b_\lambda \in A,\ \hat{\alpha} \in Z^1(G, A),\ (\lambda, j) \in E_\Gamma\}, \end{aligned}$$

where E_Γ is the edges of Γ.

Theorem. *Let $S = \mathcal{M}^0(G; I, \Lambda; P)$ and A be a 0-module under S, Γ — a bipartite graph that corresponds to sandwich-matrix P, T — a maximal acyclic subgraph and ν — cyclomatic number of the graph Γ. Than the following sequence*

$$0 \longrightarrow A^\nu/(B_T^2 \cap \tilde{Z_T^2}) \longrightarrow H_0^2(S, A) \longrightarrow H^2(G, A) \longrightarrow 0$$

is exact.

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SHORT COMMUNICATION

On the connections of acts with biacts

I. B. Kozhukhov, M. Yu. Maksimovskiy

Let S be a semigroup and X be a set. We say that X is an act over S (or an S-act) if a mapping $X \times S \to X$, $(x, s) \mapsto xs$ is defined such that $x(ss') = (xs)s'$ for all $x \in X$, $s, s' \in S$ (see [1]).

A set X is called a biact over the semigroups S, T (or an (S, T)-biact) if X is simultaneously an S- and a T-act, and $x(st) = (xs)t$ for all $x \in X$, $s \in S$, $t \in T$.

It is clear that any (S, T)-biact is an act over the direct product $S \times T$; indeed, we may put $x \cdot (s, t) = (xs)t$. Conversely, it was proved in the work [2] that in the case when S and T are monoids, any $(S \times T)$-act is also the (S, T)-biact. However it is not true for arbitrary semigroups (non-monoids) as the following example shows.

Example. *Let $X = \{1, 2, 3, 4, 5\}$, and $S = \{l_1, l_2\}$ be a left zero semigroup, and $T = \{r_1, r_2\}$ be a right zero semigroup. Define the action of $S \times T$ on X by the rule*

$$(l_1, r_1) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 2 & 1 & 2 \end{pmatrix}, \quad (l_1, r_2) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 3 & 1 & 3 \end{pmatrix},$$

$$(l_2, r_1) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 2 & 2 & 1 \end{pmatrix}, \quad (l_2, r_2) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 3 & 3 & 1 \end{pmatrix}.$$

Then X is an (S, T)-biact but it is impossinle to define the multiplications $X \times S \to X$ and $X \times T \to X$ so that $(xs)t = (xt)s = x \cdot (s, t)$ for all $x \in X$, $s \in S$, $t \in T$.

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SECTION TALK

Morita equivalence of semigroups

V. Laan

Two monoids are called *Morita equivalent* if the categories of acts over them are equivalent. This notion is more general than the notion of isomorphism of monoids. If the categories of acts over two semigroups are equivalent then these semigroups are isomorphic. So two semigroups are called *Morita equivalent* if certain subcategories of the categories of acts over them are equivalent.

It turns out that Morita equivalence of two semigroups with local units can be described in several different ways. For example, such semigroups are Morita equivalent if they have equivalent Cauchy completions, if they have a joint enlargement, if they are contained in a unitary Morita context with surjective mappings, or if one of them is a strict local isomorphic image of a Rees matrix semigroup over the other. Although Morita equivalent semigroups need not be isomorphic, they share a number of properties (such properties are called *Morita invariants*). For example, if they have "sufficiently good" local units then their congruence lattices are isomorphic.

In our talk we give an overview of recent results about Morita equivalent semigroups. The talk is based on joint research with László Márki.

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SHORT COMMUNICATION

On idempotent semigroups of linear relations

M. I. Naumik

Let V be a left n-dimensional vector space over a skew field F. A binary relation between the elements of a set V is called a linear relation, if it is a subspace of V. Recall that a set $LR(V)$ of all linear relations on the space V is a semigroup. All the rest definitions and notifications can be taken from [1,2].

Theorem 1. *A set of idempotents $J \subseteq LR(V)$ is a left-singular (right-singular) semigroup if and only if $pr_2a = pr_2b$, $socketa = socketb$ ($pr_1a = pr_1b$; $kera = kerb$) for all idempotents $a, b \in J$.*

Let J be an arbitrary idempotent semigroup of linear relations. A relation D such that for all $a, b \in J$ we have $(a, b) \in D \Leftrightarrow aba = a \wedge bab = b$ is a congruence on J. A quotient semigroup J/D is a semilattice and classes of congruences are maximal rectangle subsemigroups in J.

Theorem 2. *If an idempotent semigroup $J \subseteq LR(V)$ has an order n, then the chain length in J/D is not greater than n.*

Theorem 3. *Every idempotent semigroup of linear relations is a finite semilattice of rectangle semigroups.*

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SECTION TALK

On self-induced metric on finite groupoids

M. N. Nazarov

Let us remind that groupoid is a set with binary operation. We will call a metric ρ on groupoid D self-induced if it can be unambiguously derived from operation on D. The most interesting will be to find self-induced metric that is equivalent to groupoid's operation. Such metrics are of great importance for special biological models of morphogenesis which use groupoids as an adjustable parameters. In practice such models may even use imperfect pseudo metric for which triangle inequality holds for majority elements and is slightly broken for some exceptional elements.

To derive a metrics from operation on groupoid we use 3 step algorithm.

Step 1: The number of elements in every special set $C_L\left(a,b\right) = \{c|\,\{ca,cb\} = \{a,b\}\}$ and $C_R\left(a,b\right) = \{c|\,\{ac,bc\} = \{a,b\}\}$ is computed.

Step 2: From the system of linear equations the intermediate delta values $\{\delta_R(a,b),\delta_L(a,b)\}_{a,b\in D}$ are computed.

$$\delta_R(a,a) = \delta_L(a,a) = 0\,\forall a$$

$$\delta_R(a,b) = \left(1 + \sum_{\substack{c\in D\setminus C_R(a,b)\\ ac=d_1;\,bc=d_2}} \delta_R(d_1,d_2)\right)\cdot\Big(|C_R(a,b)|+1\Big)$$

$$\delta_L(a,b) = \left(1 + \sum_{\substack{c\in D\setminus C_L(a,b)\\ ca=d_1;\,cb=d_2}} \delta_L(d_1,d_2)\right)\cdot\Big(|C_L(a,b)|+1\Big)$$

Step 3: Three types of self-induced pseudo metrics are calculated.

$$\rho_R(a,b) = |\delta_R(a,b)| + \sum_{c\in D} |\delta_R(a,c) - \delta_R(b,c)|$$

$$\rho_L(a,b) = |\delta_L(a,b)| + \sum_{c\in D} |\delta_L(a,c) - \delta_L(b,c)|$$

$$\rho(a,b) = \rho_R(a,b) + \rho_L(a,b)$$

For all this pseudo metrics the hypothesis of being full metric is proposed (triangle inequality), and for combination of $\rho_R(a,b)$ and $\rho_L(a,b)$ the hypothesis of operation equivalence is added.

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CONTRIBUTED ABSTRACT

Partial actions and automata

B. Novikov, G. Zholtkevich

The notion of the partial action, proposed in [2] for groups, was extended later to monoids [3]. Replacing the full action of the free monoid by a partial action, we get

Definition 1. *Let X be a set, Σ^* a free monoid on an alphabet Σ. A triple (Σ, X, δ^*) is* **a preautomaton** *if $\delta^* : X \times \Sigma^* \dashrightarrow X$ is a partial mapping such that*
1) $\forall x \in X \;\; \delta^*(x, \varepsilon) = x$,
2) if $\delta^*(x, u) \neq \emptyset$ and $\delta^*(\delta^*(x, u), v) \neq \emptyset$, then $\delta^*(x, uv) \neq \emptyset$ and

$$\delta^*(x, uv) = \delta^*(\delta^*(x, u), v) \tag{1}$$

3) if $\delta^*(x, u) \neq \emptyset$ and $\delta^*(x, uv) \neq \emptyset$, then $\delta^*(\delta^*(x, u), v) \neq \emptyset$ and (1) holds.

A language $L \subset \Sigma^*$ is called **prerecognizable** if there are a preautomaton (Σ, X, δ^*), an element $x_0 \in X$ and a subset $T \subset X$, such that $|X| < \emptyset$ and $L = \{w \in \Sigma^* \mid \emptyset \neq x_0 w \in T\}$. An algebraic characterization of such languages is given by

Theorem 1. *A language $L \subset \Sigma^*$ is prerecognized iff L is an union of a finite number of classes of a right congruence on Σ^*.*

For example, the language $L = \{a^n b^n | n > 0\}$ is prerecognizable since it is a class of its right syntactic congruence.

Definition 2. *We call a language $L \in \Sigma^*$ by* **a** pd**-language** *if it is a disjoint union of a finite number of languages of the kind HC^*, where H and C are prefix codes.*

The following assertion generalizes Theorem IV.4.1 from [1] for languages, recognized by preautomata.

Theorem 2. *If a language L is prerecognized then it is a pd-language.*

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Prime preradicals in the category $S-$Act

M. Komarnytskyj, R. Oliynyk

Let S be a semigroup with 0 and 1. Each left $S-$act A is assumed to be unitary (i. e., $1A = A$) and centered (i.e., $0a = s0 = 0$ where $0 \in A$ for all $s \in S$ and $a \in A$). We consider category of left $S-$acts and their homomorphisms and denote it by $S - Act$. The terminology and necessary definitions can be found in [1,2,3].

A preradical ([2]) in the category $S - Act$ is a functor $\sigma : S - Act \to S - Act$ such that:

1. $\sigma(M)$ is subact in M for each $M \in S - Act$;

2. For each homomorphism $f : M \to N$, the diagram

$$\begin{tikzcd} \sigma(M) \arrow[r, hook] \arrow[d] & M \arrow[d, "f"] \\ \sigma(N) \arrow[r, hook] & N \end{tikzcd}$$

is commutative.

Notice that the preradicals are the subfunctors of the identity functor in the category $S-$Act.

Denote by $S-pr$ the complete big lattice of all preradicals in $S-Act$. There is a natural partial ordering in $S-pr$ give by $\sigma \preceq \tau$ if $\sigma(M) \subseteq \tau(M)$ for each $M \in S-Act$. The smallest and the largest elements are denoted respectively by 0 and 1.

For fully invariant submonoid N of M (see [4]), the preradical ω_N^M are defined as follows: $\omega_N^M(K) = \cap\{f^{-1}(N) | f \in Hom_S(K, N)\}$ for $K \in S - Act$.

Let $\sigma \in S - pr$. The preradical σ is prime in $S - pr$ if $\sigma \neq 1$ and for any $\tau, \eta \in S - pr$ such that $\tau\eta \preceq \sigma$ implies that $\tau \preceq \sigma$ or $\eta \preceq \sigma$.

Theorem. *Let $M \in S - Act$ and N is fully invariant submonoid N of M. The following conditions are equivalent:*

1. N is prime in M([2]).

2. ω_N^M is a prime preradical.

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8th International Algebraic Conference
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SHORT COMMUNICATION

On the homology of the free product of semigroups

Lyudmyla Yu. Polyakova

It was shown in [1, 2] that 0-homology can be used to compute the Eilenberg-McLane homology of semigroups. With the help of such technique we compute the homology groups of the free product of semigroups.

Let S and T be semigroups and A be an $S * T$-module. Let $A(S-1)$ (respectively $A(T-1)$) be the subgroup in A, generated by the elements $as-a$, where $a \in A, s \in S$ (respectively $at-a$, where $a \in A, t \in T$). Let $A_1 = A(S-1) \cap A(T-1)$.

Proposition. *The group $H_1(S * T, A)$ is an extension of $H_1(S, A) \oplus H_1(T, A)$ by A_1.*

Theorem. *Let $S = \prod^*_{\lambda \in \Lambda} S_\lambda$ be the free product of semigroups S_λ. Then*

$$H_n(S, A) \cong \bigoplus_{\lambda \in \Lambda} H_n(S_\lambda, A)$$

for every S-module A and $n > 1$.

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SHORT COMMUNICATION

On sets of conjugates of quasigroups

T. V. Popovich

A quasigroup is an ordered pair (Q, A) where Q is a set and A is a binary operation defined on Q such that each of the equations $A(a, y) = b$ and $A(x, a) = b$ is uniquely solvable for any pair of elements a, b in Q. With any quasigroup (Q, A) the system $\Sigma(A)$ of six (not necessarily distinct) and the set $\overline{\Sigma}(A)$ of conjugates (parastrophes) are connected: $\Sigma(A) = (A,\ {}^rA,\ {}^lA,\ {}^{lr}A,\ {}^{rl}A,\ {}^sA)$, where ${}^rA = A^{-1},\ {}^lA = {}^{-1}A,\ {}^{lr}A = {}^{-1}(A^{-1}), {}^{rl}A = ({}^{-1}A)^{-1},\ {}^sA = A^*$. It is known of [1] that $|\ \overline{\Sigma}(A)\ | = 1,2,3$ or 6.

Theorem 1. *The following conjugate sets of a quasigroup (Q, A) are only possible: $\overline{\Sigma}_1(A) = \{A\}$, $\overline{\Sigma}_2(A) = \{A, {}^sA\} = \{A = {}^{lr}A = {}^{rl}A,\ {}^lA = {}^rA = {}^sA\}$; $\overline{\Sigma}_6(A) = \{A,\ {}^rA,\ {}^lA,\ {}^{lr}A,\ {}^{rl}A,\ {}^sA\}$; $\overline{\Sigma}_3(A) = \{A, {}^{lr}A, {}^{rl}A\}$ and three cases are possible: $\overline{\Sigma}^1_3(A) = \{A = {}^rA,\ {}^lA = {}^{lr}A,\ {}^{rl}A = {}^sA\}$, $\overline{\Sigma}^2_3(A) = \{A = {}^lA,\ {}^rA = {}^{rl}A,\ {}^{lr}A = {}^sA\}$, $\overline{\Sigma}^3_3(A) = \{A = {}^sA,\ {}^rA = {}^{lr}A,\ {}^lA = {}^{rl}A\}$.*

It is proved that the set $\overline{T} = \{A(x, A(x, y)) = y,\ A(A(y, x), x) = y,\ A(x, y) = A(y, x),\ A(A(x, y), x) = y\}$ of four identities is connected with conjugates of a quasigroup (Q, A) any two identities of which imply the rest two ones.

Theorem 2. *Let (Q, A) be a quasigroup, then $\overline{\Sigma}(A) = \overline{\Sigma}_1(A)$ if and only if it satisfies any two identities of $\overline{T}$; $\overline{\Sigma}(A) = \overline{\Sigma}_2(A)$ if and only if it satisfies exactly the identity $A(A(x, y), x) = y$ of $\overline{T}$; $\overline{\Sigma}(A) = \overline{\Sigma}^1_3(A)$ if and only if it satisfies exactly the identity $A(x, A(x, y)) = y$ of $\overline{T}$; $\overline{\Sigma}(A) = \overline{\Sigma}^2_3(A)$ if and only if it satisfies exactly the identity $A(A(y, x), x) = y$ of $\overline{T}$; $\overline{\Sigma}(A) = \overline{\Sigma}^3_3(A)$ if and only if it satisfies exactly the identity $A(x, y) = A(y, x)$ of $\overline{T}$; $\overline{\Sigma}(A) = \overline{\Sigma}_6(A)$ if and only if (Q, A) satisfies none of four identities of $\overline{T}$.*

The necessary and sufficient conditions for coincidence of a T-quasigroup ([2]) with a conjugate are defined and some examples of quasi-groups with distinct conjugate sets are given.

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SHORT COMMUNICATION

On the endomorphism semigroup of some infinite monounary algebras

I. Pozdnyakova

A monounary algebra is a pair $\langle A; f\rangle$, where A is a set and f is an unary algebraic operation. A monounary algebra is called *connected* if an intersection of any two its monogenic subalgebras is not empty. The monogenic subalgebra $[\alpha]$ of $\langle A; f\rangle$ is called *a subalgebra of a maximal height* if the equality $f^m(\alpha) = f^n(\beta)$ implies that $m \geq n$ for every $\beta \in A$ and for all $m, n \in N_0$.

Let $\mathfrak{F}$ be a class of all connected monounary algebras $\langle A; f\rangle$ such that following conditions holds:

(i) *$\langle A; f\rangle$ does not contain cyclic and metamonogenic subalgebras;*

(ii) *$\langle A; f\rangle$ contain at least one monogenic subalgebra of a maximal height.*

Denote by $G(A)$ the endomorphism semigroup of $\langle A; f\rangle$. We call the idempotent e from $G(A)$ *a minimal* if the equality $ee' = e'$ implies that $e = e'$ for all idempotents $e' \in G(A)$.

Lemma. *Let $\langle A; f\rangle \in \mathfrak{F}$ and $e \in G(A)$ is a minimal idempotent. Then*

(i) *the set $F = G(A)e$ is a two-sided ideal in the endomorphism semigroup $G(A)$;*

(ii) *F does not depend from the choice of the minimal idempotent e;*

(iii) *element g lies in F if and only if $g \in G(A)$ and $g(A) = [\gamma]$ for the some $\gamma \in A$;*

(iv) *there exists the bijection θ from A to F such that $\theta(\gamma)(A) = [\gamma]$.*

Theorem. *The algebra $\langle A; f\rangle$ is isomorphic to the algebra $\langle F; f'\rangle$, where $f'(g) = g(fe)$ for all $g \in F$.*

Theorem. *Let $\langle A; f\rangle$ and $\langle A'; f'\rangle$ are arbitrary monounary algebras of the class $\mathfrak{F}$. The endomorphism semigroup $G(A)$ is isomorphic to $G(A')$ if and only if the algebras $\langle A; f\rangle$ and $\langle A'; f'\rangle$ are isomorphic. For every isomorphism ψ from $G(A)$ to $G(A')$ there exists the isomorphism φ from $\langle A; f\rangle$ to $\langle A'; f'\rangle$ such that $\psi(g) = \varphi g \varphi^{-1}$ for all $g \in G(A)$.*

Corollary. *Let $\langle A; f\rangle$ be an arbitrary monounary algebra of the class $\mathfrak{F}$. Every automorphism of the endomorphism semigroup $G(A)$ is inner.*

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8th International Algebraic Conference
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Congruences of the semigroup $\mathcal{IO}_{\mathbb{N}}$

V. O. Pyekhtyeryev, K. S. Tretyak

Let $\mathbb{N}$ be a set of positive integers with nature linear order. Denote by $\mathcal{IO}_{\mathbb{N}}$ the semigroup of all order-preserving partial injections $a : \mathbb{N} \to \mathbb{N}$. This semigroup is very interesting object for investigations, because it is intersection of symmetric inverse semigroup $\mathcal{IS}_{\mathbb{N}}$ and semigroup of all monotone transformations of the set $\mathbb{N}$. The semigroup $\mathcal{IS}_X$ has a central role in the semigroup theory, so there is a lot of papers dedicated to its studying. Semigroups of monotone transformations were explored in [1], [2], [3], [4].

The present report is dedicated to congruences of the semigroup $\mathcal{IO}_{\mathbb{N}}$. In particular, we prove that this semigroup contains only one non-Rees congruence.

Let ξ be a cardinal number. Two transformations a and b of the semigroup $\mathcal{IO}_{\mathbb{N}}$ have *difference of rank* ξ, if $\max(\operatorname{rank}(a \setminus (a \cap b)), \operatorname{rank}(b \setminus (a \cap b))) = \xi$. Denote by $\widetilde{\rho}$ binary relation on the semigroup $\mathcal{IO}_{\mathbb{N}}$, defined by the rule:

$$a\widetilde{\rho}b \textit{ if and only if } a \textit{ and } b \textit{ have difference of a finite rank.}$$

Theorem. 1. *The binary relation $\widetilde{\rho}$ is non-Rees congruence of the semigroup $\mathcal{IO}_{\mathbb{N}}$.*

2. *The semigroup $\mathcal{IO}_{\mathbb{N}}$ has only one non-Rees congruence $\widetilde{\rho}$.*

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8th International Algebraic Conference
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Duality for ternary operations

A. V. Reshetnikov

Let f be a binary associative operation. The operation g is called *dual* to f if $g(x, y) = f(y, x)$. An operation, which is dual to an associative operation, is also associative. The duality principle is often used in mathematics. If we have something proven for an associative operation, we usually can construct a dual assertion which is correct for the dual operation. An example is the following. Let X be a set, f be an operation on X defined as $f(x, y) = x$ for all $x, y \in X$. It is well-known that f is associative, so (X, f) is a semigroup. The operation $g(x, y) = y$ is the dual to f. According to the dual principle, g is also associative, and (X, g) is also a semigroup.

In the example above, (X, f) is called *a left zero semigroup*, and (X, g) is called *a right zero semigroup*.

In the beginning of the XX-th century the concept of a group was generalized to the concept of an n*-ary group* (one of the well-known papers on this topic is [**?**]). Different definitions appeared, but they are equal and use the following idea. A group can be defined as a set G with an associative operation f such that the equations $f(x, a) = b$ and $f(a, y) = b$ are solvable for all $a, b \in G$. If the associativity definition and the equations be generalized to the case of n-arity, we get a definition of the n-ary group.

The following definition appeared (let us watch the ternary case only for ease). Let X be a set, f be an operation such that $f(f(a, b, c), d, e) = f(a, f(b, c, d), e) = f(a, b, f(c, d, e))$ for all $a, b, c, d, e \in X$. Then f is called *associative* (see [**?**]). But it is not good to define a ternary semigroup as a set with a ternary associative operation since the duality principle becomes not generalized. For example, the operations $f(x, y, z) = x$ and $h(x, y, z) = z$ are associative, but the operation $g(x, y, z) = y$ is not. It allows to consider the ternary left and right zeroes semigroups, but middle zeroes don't form a ternary semigroup, if we accept such definition.

A better definition of the ternary semigroup is suggested. Let X be a set, f and g be ternary operations on X. We say that f and g are *dual* if there exists a permutation σ on $\{1, 2, 3\}$ such that $f(x_1, x_2, x_3) = g(x_{\sigma(1)}, x_{\sigma(2)}, x_{\sigma(3)})$. So, for a given ternary operation there exist 5 dual operations. We say that f is *new-associative* if

$$\forall a, b, x, y, z \in X \quad f(f(x, a, b), y, z) = f(x, f(b, y, a), z) = f(x, y, f(a, b, z)).$$

Theorem. *Let f and g be dual ternary operations. If f is new-associative, then g is also new-associative.*

If a ternary new-associative operation f is defined on a set X, let us say that (X, f) is a *new-ternary semigroup*. It follows from the theorem that (X, g) is also a new-ternary semigroup if f and g are dual. *This fact is the duality for new-ternary operations.* According to the new definition, left, midle and right zeroes form new-ternary semigroups.

Duality can also be generalized to the case of n-ary operations.

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Short communication

On endomorphism semigroups of the some relational systems

E. A. Romanenko

Let $\langle A;\circ\rangle$ be an arbitrary groupoid, $R_\circ$ is a ternary relation corresponding to $\circ$ operation. Then $(a,b,c)\in R_\circ$ if and only if $a\circ b=c$ for all $a,b,c\in A$. Define on the set $R_\circ$ equivalence relations $\varepsilon_1,\varepsilon_2,\varepsilon_3$ such that

$$((x_1,x_2,x_3),(y_1,y_2,y_3))\in\varepsilon_i \Leftrightarrow x_i=y_i \quad (i=1,2,3).$$

Denote by $\mathrm{End}(R_\circ)$ endomorphism semigroup of the relational system $\langle R_\circ;\varepsilon_1,\varepsilon_2,\varepsilon_3\rangle$. And let $\mathfrak{A}$ be the class of groupoids such that corresponding relational system satisfies the following condition:

$$(\varepsilon_1\cap\varepsilon_3)\vee(\varepsilon_2\cap\varepsilon_3)=\varepsilon_3.$$

For example, class $\mathfrak{A}$ contains all transformation semigroups of a finite set that have rank less then the power of this set.

Recall that [1], an ordered triple $(\varphi_1,\varphi_2,\varphi_3)$ of the transformations of the set A is called *an endotopism* of the groupoid $\langle A;\circ\rangle$, if the equality $\varphi_3(a\circ b)=\varphi_1(a)\circ\varphi_2(b)$ holds for all $a,b,c\in A$. Denote by $G(A)$ endotopism semigroup [1] of the groupoid $\langle A;\circ\rangle$.

Theorem. *Let $\langle A;\circ\rangle$ and $\langle A';*\rangle$ be groupoids of the class $\mathfrak{A}$, $\langle R_\circ;\varepsilon_1,\varepsilon_2,\varepsilon_3\rangle$ and $\langle R_*;\varepsilon'_1,\varepsilon'_2,\varepsilon'_3\rangle$ — corresponding relational systems. Semigroups $\mathrm{End}(R_\circ)$ and $\mathrm{End}(R_*)$ are isomorphic if and only if, the system $\langle R_\circ;\varepsilon_1,\varepsilon_2,\varepsilon_3\rangle$ is isomorphic to the system $\langle R_*;\varepsilon'_1,\varepsilon'_2,\varepsilon'_3\rangle$ or to the system $\langle R_*;\varepsilon'_2,\varepsilon'_1,\varepsilon'_3\rangle$.*

From the previous Theorem, using results obtained in [1], we get the following

Theorem. *Let $\langle A;\circ\rangle$ and $\langle A';*\rangle$ are groupoids of the class $\mathfrak{A}$. Endotopism semigroups $G(A)$ and $G(A')$ are isomorphic if and only if, the groupoids $\langle A;\circ\rangle$ and $\langle A';*\rangle$ are isotopic.*

From the Bruck's Theorem [2] and the previous Theorem we obtain the next

Theorem. *Let $\langle A;\circ\rangle$ and $\langle A';*\rangle$ are groupoids of the class $\mathfrak{A}$, and $\langle A;\circ\rangle$ — semigroup with identity. Endotopism semigroups $G(A)$ and $G(A')$ are isomorphic, if and only if semigroup $\langle A;\circ\rangle$ is isomorphic to groupoid $\langle A';*\rangle$, and, therefore, $\langle A';*\rangle$ is the semigroup with identity.*

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SHORT COMMUNICATION

Normality of nuclei in (α, β, γ)-inverse loops

V. A. Shcherbacov

Basic definitions are in [1].

Definition 1. *A quasigroup $(Q, \circ)$ is an $(\alpha; \beta; \gamma)$-inverse quasigroup if there exist permutations α, β, γ of the set Q such that*

$$\alpha(x \circ y) \circ \beta x = \gamma y$$

for all $x, y \in Q$ [2].

Theorem 1 ([3]).

1. *If $\alpha = \varepsilon$, then in $(\varepsilon; \beta; \gamma)$-inverse loop $(Q, \circ)$ $N_l = N_r = N_m \trianglelefteq Q$.*
2. *If $\gamma = \varepsilon$, then in $(\alpha; \beta; \varepsilon)$-inverse loop $(Q, \circ)$ $N_l = N_r = N_m \trianglelefteq Q$.*
3. *If $\beta = \alpha^{-1}$, then in $(\alpha; \alpha^{-1}; \gamma)$-inverse loop $(Q, \circ)$ $N_l = N_r = N_m \trianglelefteq Q$.*
4. *If $\gamma = \beta^{-1}$, then in $(\alpha; \beta; \beta^{-1})$-inverse loop $(Q, \circ)$ $N_l = N_r = N_m \trianglelefteq Q$.*

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CONTRIBUTED ABSTRACT

About one approach of finding quasigroup identities from some variety of loops

Abdullo Tabarov

The report is devoted to one approach of finding identities in the class of quasigroups isotopic to known classes of loops from some variety of loops. It is assumed that the class of loops from some variety of loops are given by an identity or system of identities. For this, the notion a derived identity is introduced and it is proved that for any identity from some of variety of quasigroups (loops) there is derivative identity. The introduced notion of derived identities allows to find an arbitrary identity for the class of quasigroups which are isotopic not only to groups but also to loops from some variety of loops and generalizes the method of A.A. Gvaramiya [1], where for the class of quasigroups are isotopic to a groups, it is possible to obtain any quasigroup identity from the group identities. Also by this way it is easy to obtain V.D. Belousov's identities which characterized the class of quasigroups is isotopic to a group (an abelian group)[2]. As an illustration we show that the class of quasigroups which is isotopic to the nilpotent group is characterized by identity. It should be noted that the notion of a derived identity in terms of free quasigroup and the theory of automats also can be found in [1]. However, our proposed approach does not require using free objects, in particular a free quasigroups. Sufficiently confined by the methods of theory of quasigroups.

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SHORT COMMUNICATION

On complementation in the lattice of subalgebras of a Boolean algebra

M. T. Tarashchanskii

Let $\mathfrak{A}$ be a Boolean algebra. If $\mathfrak{D} \subset \mathfrak{A}$, then by $\langle\mathfrak{D}\rangle$ is denoted the subalgebra generated by $\mathfrak{D}$. Let $Sub(\mathfrak{A})$ be the lattice of all subalgebras of the algebra $\mathfrak{A}$. We say that $\mathfrak{B} \in Sub(\mathfrak{A})$ has a complement with respect to $\mathfrak{A}$ if there exists $\mathfrak{C} \in Sub(\mathfrak{A})$ such that $\mathfrak{B} \cap \mathfrak{C} = \{0,1\}$ and $\langle\mathfrak{B} \vee \mathfrak{C}\rangle = \mathfrak{A}$. K.P.S. Bhaskara Rao and M. Bhaskara Rao in [1] have shown that there is a complement for any finite $\mathfrak{B} \in Sub(\mathfrak{A})$. They also established conditions for the existence of a complement of the algebra generated by an ideal in $\mathfrak{A}$. If the Boolean algebra $\mathfrak{A}$ is countable, then every $\mathfrak{B} \in Sub(\mathfrak{A})$ has a complement in $\mathfrak{A}$ [?], [?]. On the other hand L. Heindorf proved in [3] that Boolean algebra $\mathfrak{A}$ is a countable set if and only if any $\mathfrak{B} \in Sub(\mathfrak{A})$ is continuously complemented in natural topology of $Sub(\mathfrak{A})$.

In this talk we will give a new sufficient condition on $\mathfrak{B} \in Sub(\mathfrak{A})$ ensuring the existence of a complement to $\mathfrak{B}$.

For $\mathfrak{B} \in Sub(\mathfrak{A})$ we denote by $\mathfrak{C}(\mathfrak{B})$ the set of all elements $C \in \mathfrak{A}, C \notin \mathfrak{B}$ satisfying the following condition: if $B \subset C$ and $B \in \mathfrak{B}$, then $B = 0$. We will say that an algebra $\mathfrak{B} \in Sub(\mathfrak{A})$ is absolutely non-dense in algebra $\mathfrak{A}$ if $\langle\mathfrak{B} \vee \mathfrak{C}(\mathfrak{B})\rangle = \mathfrak{A}$.

Theorem. *If $\mathfrak{B} \in Sub(\mathfrak{A})$ is absolutely non-dense in the algebra $\mathfrak{A}$, then $\mathfrak{B}$ has complement with respect to $\mathfrak{A}$.*

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8th International Algebraic Conference
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SECTION TALK

The lattice of the ideals at the semigroup of correspondences of the finite group

Tetyana Turka

Let G be the universal algebra. If the subalgebra from $G \times G$ is considered as a binary relation on G, then the set $S(G)$ of all the subalgebra from $G \times G$ is the semigroup to the products of relations. The semigroup $S(G)$ is called *a semigroup of correspondence* of algebra G.

The problem of the study of the semigroups of correspondence has been set by Kurosh ([1]). We study the structure of the lattice of the ideals of the semigroups of correspondence of the finite group G.

Let $\mathcal{F}_G = \{F_1, F_2, ..., F_k\}$ be the set of all nonisomorphic factors of the finite group G. Let $F_i \preceq F_j$ if and only if, when F_i is a factorgroup F_j. This relation $\preceq$ on the set $\mathcal{F}_G$ is a partial order.

Each subgroup of $G \times G$ has the form $\{(g_1, g_2) | \varphi(g_1 H_1) = g_2 H_2\}$, where $H_1 \lhd G_1$, $H_2 \lhd G_2$, but $\varphi : G_1/H_1 \to G_2/H_2$ is isomorphism factorgroup. If $G_1/H_1 \simeq G_2/H_2 \simeq F$, then we denote such subgroup $G_1/H_1 \times_F G_2/H_2$.

Lemma 1. *Let* $(G_1'/H_1' \times_{F'} G_2'/H_2') \cdot (G_1''/H_1'' \times_{F''} G_2''/H_2'') = (G_1/H_1 \times_F G_2/H_2)$. *Then* $F \preceq F'$, $F \preceq F''$.

Theorem 1. *Element* $G_1/H_1 \times_{F'} G_2/H_2$ *belongs to the main ideal* J_F *if and only if, when it is* $F' \preceq F$.

Corollary 1. $J_{F_1} \subseteq J_{F_2} \Leftrightarrow F_1 \preceq F_2$.

For the partially ordered set $(M, \leq)$ via $\mathrm{Ach}(M)$ we set the sets of all the antichains M. For the arbitrary antichains $L_1 = \{a_1, a_2, \ldots, a_m\}$ and $L_2 = \{b_1, b_2 \ldots, b_n\}$ let $L_1 \preceq L_2$ if and only if, when it is for each a_i is found b_j, and that $a_i \leq b_j$. The relation $\preceq$ on the set $\mathrm{Ach}(M)$ is the relation of the partial order.

Each antichain $L = \{F_{i_1}, ..., F_{i_m}\}$ from $\mathrm{Ach}(\mathcal{F}_G)$ we put ideal $J_L = J_{F_{i_1}} \cup ... \cup J_{F_{i_m}}$ in correspondence.

Theorem 2. *The reflection* $L \mapsto J_L$ *is isomorphism of the lattice* $\mathrm{Ach}(\mathcal{F}_G)$ *on the lattice of all the ideals of the semigroup* $S(G)$.

Theorem 3. *If the ideals of the semigroups of correspondences* $S(G)$ *of the finite group* G *form the chain, then either* G *is the cyclic* p*—group, or* G *is the group of exponent* p.

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CONTRIBUTED ABSTRACT

О гомоморфизмах A-лупы на конечные лупы

В. И. Урсу

Лупа L называется *разрешимой* (соотв., *нильпотетной*), если она содержит конечную цепочку нормальных делителей $L \supset L_1 \supset ... \supset L_k = \{e\}$ с коммутативными (соотв., центральными) факторами L_{i-1}/L_i, $i = 1, 2, ..., k$.

Условимся говорить, что подлупа в лупе L *финитно отделима от элемента* $a \notin H$, если существует гомоморфизм φ лупы L в некоторую конечную лупу, при котором $\varphi(a) \notin \varphi(H)$. Лупа L, каждая подлупа которой отделима от всех не входящих в нее элементов, будет называться *финитно отделимой* лупой. При достаточных общих преположениях легко показать, что финитная отделимость является более сильным свойством, чем *финитная аппроксимируемость*: для любого элемента $a \neq e$ из L существует гомоморфизм φ лупы L в некоторую конечную лупу, при котором $\varphi(a) \approx e$.

Лупа в которой любая внутренная подстановка является автоморфизмом называется *A-лупой*. Теория A-луп была создана Браком и Пэйджа [1], которые доказали следующий важный результат: если A-лупа L диассоциативна, т. е. любые два ее элемента порождают ассоциативную подлупу, то в L любые три элемента связаных ассоциативным законом порождают ассоциативную подлупу (теорема Муфанг). Более того [2], если L еще и коммутативна, то L является лупой Муфанг. Наконец, недавно в работе автора и А. Ковальского [3] для нильпотентных A-луп получены следующие результаты: (i) подлупы конечно порожденной нильпотентной A-лупы выполняет условие максимальности; (ii) конечно порожденная нильпотентна A-лупа финитно аппроксимируема; (iii) все тождества истинных в нильпотетной A-лупы имеют конечный базис. В настоящей заметке подробно изучается строение разрешимых A-луп, обладающих свойством финитной отделимости. В частности, оказывается что разрешимые A-лупы с обрывом возрастающих цепочек подлуп финитно отделимы, что усиливает цитированный выше результат о финитной аппроксимируемости конечно порожденных нильпотентных A-луп. Тем самым доказывается существование алгоритма для решения задачи о вхождения элемента в подлупу для нильпотентных A-луп данной ступени нильпотентности.

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СВЕДЕНИЯ ОБ АВТОРАХ

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8th International Algebraic Conference
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Near-filters and quasi-ideals of semigroups of partial transformations

V. Velichko

In the structural theory of semigroups, has become increasingly used the concept of quasi-ideal, which generalizes the notion of an ideal semigroup. In the monograph [1] is summarized a certain stage of learning and the use of quasi-ideals that to describe different properties of semigroups and rings.

The main concepts that were used in work [2] for description of one-sided ideals of some semigroups is the notion of near-filter and co-near-filter which were identified for the lattice subsets of some set.

Sets of $Eq(X)$ and $Bool(X)$ are researched with a native order of their elements. Let

$$(\mathcal{U}, \leq) = (Eq(X), \subseteq) \times (Bool(X), \subseteq)$$

- straight R-product of partially ordered sets $Eq(X)$ and $Bool(X)$.

Let $\mathcal{NF}(\mathcal{U}, \leq)$ - the set of all near-filters set $(\mathcal{U}, \leq)$, $F \in \mathcal{NF}(\mathcal{U}, \leq)$. Put

$$G(F) = \left\{\varphi \in S | (\pi^{\varphi}_{dom\varphi}; Im\,\varphi) \in F\right\}.$$

Lemma. *The set $Q = g(F)$ is either empty or is quasi-ideal of semigroup $\mathcal{PT}(X)$.*

Let Q — quasi-ideal of semigroup $\mathcal{PT}(X)$. Put

$$F(Q) = \left\{(\pi^{\varphi}_{dom\varphi}; Im\,\varphi) | \varphi \in Q\right\}.$$

Lemma. *The set $F = f(Q)$ is near-filter partially ordered set $(\mathcal{U}; \leq)$.*

Let $Q(S)$ — the set of all quasi-ideals of semigroup of partial transformations $\mathcal{PT}(X)$, $\mathcal{NF}^*(\mathcal{U}, \leq)$ — the set all near-filters F of $(\mathcal{U}; \leq)$ for which $G(F) \neq \emptyset$. Have already been defined the mappings

$$G : \mathcal{NF}^*(\mathcal{U}, \leq) \to Q(\mathcal{PT}(X)) : F \mapsto g(F),$$

$$F : Q(\mathcal{PT}(X)) \to \mathcal{NF}^*(\mathcal{U}, \leq) : Q \mapsto f(Q).$$

Theorem. *Mapping f and g bijection.*

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SHORT COMMUNICATION

On quasi-ordered sets whose semigroups of isotone partial maps are regular

V. A. Yaroshevich

Let us denote by $T(X)$ the semigroup of all transformations $\alpha : X \to X$ of a set X. Let $PT(X)$ be the set of all partial maps, i.e. the maps $\beta : X_1 \to X$ where $X_1 \subseteq X$ is the domain of the map β. If σ is a binary relation on X then a mapping $\alpha : X \to X$ is called *isotone* when

$$\forall x, y \in X \quad (x, y) \in \sigma \;\Rightarrow\; (x\alpha, y\alpha) \in \sigma. \tag{1}$$

We denote by $T_\sigma(X)$ the set of all isotone transformations $\alpha : X \to X$. It is easy to prove the inclusion $\alpha \in T_\sigma(X)$ is equivalent to the following:

$$\sigma\alpha \subseteq \alpha\sigma. \tag{2}$$

Now we want to produce a definition of *isotone partial* transformation. There are several ways for it. Let us demand for every full map α which is partial also, the new definition to become (1). In particular we can take (2) as the definition. We denote the semigroup of all partial maps satisfying (2) by $\widetilde{PT}_\sigma(X)$.

The problem of regularity for $T_\leqslant(X)$ where X is a poset was solved in [1, 2] completely. Conditions of regularity for $\widetilde{PT}_\leqslant(X)$ when X is a quasi-ordered set but not a chain were considered in [3]. Now the author pays his attention to chain-case referring to [1]. Before we can give the theorem, let us introduce some definitions.

For a nonzero cardinal μ, let G_μ be the poset of height 1 having μ maximal elements and one minimal element, in which the minimal element is lying below all maximal elements, and no further ordering.

For a chain X, a subset of X is *co-final* if it is not bounded above in X. If X has a co-final subset, then the *co-finality* of X is the least cardinal isomorphic to a co-final subset of X. Analogously, a subset of X is *co-initial* if it is not bounded below in X, and the *co-initiality* of X is the least cardinal α such that α^* is isomorphic to a co-initial subset.

A *Dedekind cut* of a chain X is a pair (A, B) of disjoint intervals of X whose union is X, each element of A preceding every element of B. (Note that A or B may be void.) A Dedekind cut (A, B) is said to be a *gap* if A has no last element and B has no first element. Associated with every gap (A, B) is a pair (α, β) such that α is the co-finality of A and β is the co-initiality of B. (If A or B is void, then α or β is 0, respectively.) A chain X is said to be *quasi-complete*, if for every gap (A, B), $\alpha + 1$ is not embeddable in A and $(\beta + 1)^*$ is not embeddable in B, where (α, β) is the pair associated with (A, B).

Theorem. *Let $(X, \leqslant)$ be a quasi-ordered set. Then the semigroup $\widetilde{PT}_\leqslant(X)$ is regular iff at least one of the following conditions holds: (1) X is a quasi-complete chain; (2) X is an antichain; (3) $X \cong G_\mu$ for any nonzero μ; (4) $x \leqslant y$ for all $x, y \in X$.*

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8th International Algebraic Conference
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Напівгрупи перетворень рісівських напівгруп

А. І. Закусило

Одним з найперспективніших напрямків структурної теорії напівгруп є побудова загальної теорії напівгрупових конструкцій. Важливе місце в ряду таких конструкцій посідає конструкція напівгрупи Ріса матричного типу.

В [1] визначено конструкцію двобічного напівпрямого добутку напівгруп, яка є узагальненням відомої конструкції подвійного напівпрямого добутку напівгруп. Конструкція двобічного напівпрямого добутку дозволяє побудувати конструкцію вінцевого голоморфу, яка була використана в [4] для описання будови напівгрупи ендоморфізмів цілком 0-простих напівгруп.

В термінах вінцевих голоморфів в [2] описано будову напівгруп рестриктивних ендоморфізмів квазірегулярних рісівських напівгруп матричного типу над довільним моноїдом з нулем, а в [3] описано будову оболонок зсувів таких напівгруп. Це є узагальненням та доповненням визнаних результатів В.М. Усенка та інших відомих у світі математиків.

У цій роботі ми продовжуємо вивчати напівгрупи ендоморфізмів квазірегулярних рісівських напівгруп матричного типу над моноїдом з нулем та їх структурні властивості.

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8th International Algebraic Conference
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Free rectangular dimonoids

A. Zhuchok

The notions of a dialgebra and a dimonoid were introduced by J.-L. Loday [1] and investigated in many papers (see, for example, [1]-[4]). Examples of dialgebras and dimonoids were given in [1]-[4]. For further details and background see J.-L. Loday [1].

A set D equipped with two binary operations $\prec$ and $\succ$ satisfying the following axioms:

$$(x \prec y) \prec z = x \prec (y \prec z), \ (x \prec y) \prec z = x \prec (y \succ z),$$
$$(x \succ y) \prec z = x \succ (y \prec z), \ (x \prec y) \succ z = x \succ (y \succ z),$$
$$(x \succ y) \succ z = x \succ (y \succ z)$$

for all $x, y, z \in D$, is called a dimonoid. A dimonoid $(D, \prec, \succ)$ will be called rectangular, if both semigroups $(D, \prec)$ and $(D, \succ)$ are rectangular bands.

Now we give examples of rectangular dimonoids and construct a free rectangular dimonoid.

Lemma 1. *Let $(D, \prec)$ be a rectangular band and $(D, \succ)$ be a right zero semigroup. Then $(D, \prec, \succ)$ is a rectangular dimonoid.*

Lemma 2. *Let $(D, \prec)$ be a left zero semigroup and $(D, \succ)$ be a rectangular band. Then $(D, \prec, \succ)$ is a rectangular dimonoid.*

Let $I_n = \{1, 2, ..., n\}$, $n > 1$ and let $\{X_i\}_{i \in I_n}$ be a family of arbitrary nonempty sets X_i, $i \in I_n$. Define the operations $\dashv$ and $\vdash$ on $\prod_{i \in I_n} X_i$ by

$$(x_1, ..., x_n) \dashv (y_1, ..., y_n) = (x_1, ..., x_{n-1}, y_n),$$
$$(x_1, ..., x_n) \vdash (y_1, ..., y_n) = (x_1, y_2, ..., y_n)$$

for all $(x_1, ..., x_n), \ (y_1, ..., y_n) \in \prod_{i \in I_n} X_i$.

Lemma 3. *For any $n > 1$, $(\prod_{i \in I_n} X_i, \dashv, \vdash)$ is a rectangular dimonoid.*

Let X be an arbitrary nonempty set. Then $(X^2, \dashv, \vdash)$ can be considered as the free rectangular band. Other examples of rectangular dimonoids can be found in [3] and [4].

We denote the dimonoid $(X^3, \dashv, \vdash)$ by $FRct(X)$.

Theorem 1. *$FRct(X)$ is a free rectangular dimonoid.*

In addition, we describe the structure of free rectangular dimonoids, characterize some least congruences on free rectangular dimonoids and discuss the connections between rectangular dimonoids and restrictive bisemigroups [5].

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SECTION TALK

Endomorphism semigroups of free products

Yu. Zhuchok

In many cases endomorphism semigroup of algebraic system carries a substantial information about the assumed system and gives a new, convenient enough language on the basis of which it is possible to study a structure of this system [1], [2]. In the work we study endomorphism semigroup of free product of semigroups from some class in terms of a wreath product of transformation semigroup and small category.

Let K be an abstract class of semigroups, $D = \{S_\alpha | \alpha \in Y\}$ be an arbitrary disjoint family of semigroups of this class and $D' = \{S_\alpha | \alpha \in Y'\}$ be an any disjoint family of semigroups such that $Y \subseteq Y'$. We call the class K a wreath class (or shortly a w-class) if for every homomorphism $\varphi : Fr[S_\alpha]_{\alpha \in Y} \to Fr[S_\alpha]_{\alpha \in Y'}$ of free products $Fr[S_\alpha]_{\alpha \in Y}$ and $Fr[S_\alpha]_{\alpha \in Y'}$ and any $\alpha \in Y$ there exists $\beta \in Y'$ such that $S_\alpha \varphi \subseteq S_\beta$.

For instance, the class of all idempotent semigroups, the class of all finite semigroups and the class of all semigroups with zero are w-classes.

Let further W be an union of all w-classes of semigroups. It is easy to see that W is a maximal w-class in the class of all semigroups.

If g is a homomorphism of free product $Fr[S_\alpha]_{\alpha \in Y}$, then for a restriction $g|_{S_\alpha}$ we use the designation g_α. For arbitrary semigroups S_i, S_j by H_{ij} we denote the set of all homomorphisms of S_i to S_j and for each $i \in Y$ we put $H_{i*} = \bigcup_{j \in Y} H_{ij}$.

Lemma. *A transformation φ of free product F of semigroups $S_\alpha, \alpha \in Y$ from a class W is an endomorphism if and only if for any $w = a_1 a_2 ... a_k \in F$, $a_i \in S_{j_i}$*

$$w\varphi = a_1\varphi_{j_1} a_2\varphi_{j_2} ... a_k\varphi_{j_k}, \text{ where } \varphi_{j_i} \in H_{j_i *}, 1 \leq i \leq k.$$

Let $Fr[S_\alpha]_{\alpha \in Y}$ be a free product of semigroups S_α, $\alpha \in Y$ from the class W, C be a small category such that $ObC = \{S_\alpha | \alpha \in Y\}$ and for all S_α, $S_\beta \in ObC$ let

$$Mor_C(S_\alpha; S_\beta) = H_{\alpha\beta}.$$

We denote by $T(Y)$ the semigroup of all transformations ξ of a set Y such that $H_{\alpha(\alpha\xi)} \neq \varnothing$ for all $\alpha \in Y$. Then objects ObC of the category set left $T(Y)$-act, that is a semigroup of transformations $T(Y)$ acts on the left on ObC by the rule: $\varphi S_\alpha = S_{\alpha\varphi}$.

Thus, appears a construction of a wreath product of monoid and small category (see, e.g., [3])

$$T(Y)\, wr\, C = \{(\varphi; f) | \varphi \in T(Y), f \in Map(ObC; MorC), S_\alpha f \in Mor_C(S_\alpha; \varphi S_\alpha)\},$$

where $MorC = \bigcup_{S_\alpha, S_\beta \in ObC} Mor(S_\alpha; S_\beta)$ and operation on the $T(Y)\, wr\, C$ is defined by the next rule:

$$(\varphi; f), (\psi; g) = (\varphi\psi; f_\psi g).$$

Here $S_\alpha(f_\psi g) = (\psi S_\alpha) f\, S_\alpha g$ for all $S_\alpha \in ObC$ and $(\psi S_\alpha) f\, S_\alpha g$ is a composition of morphisms in the category C.

The main result of the paper is the following theorem.

Theorem. *Endomorphism semigroup* $End\,Fr[S_\alpha]_{\alpha\in Y}$ *of free product* $Fr[S_\alpha]_{\alpha\in Y}$ *of semigroups* $S_\alpha, \alpha \in Y$ *from the class* W *and the wreath product* $T(Y)\,wr\,C$ *of transformation semigroup* $T(Y)$ *with the small category* C *are isomorphic.*

Besides other exact representations of endomorphism semigroup of free product of semigroups from the class W are described and problem of definability of free products of semigroups from a maximal w-class by their endomorphism semigroups is studied.

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SECTION TALK

On generating systems of some transformation semigroups of the boolean

T. Zhukovska

Let M be a poset with a partial order $\leq$. A transformation $\alpha : M \to M$ is called *order-decreasing* if $\alpha(x) \leq x$ for all $x \in \mathrm{dom}(\alpha)$. The set of such transformations is denoted by $\mathcal{F}(\mathcal{M})$. A transformation α is called *order-preserving* if for every $x, y \in \mathrm{dom}(\alpha)$, $x \leq y$ implies $\alpha(x) \leq \alpha(y)$. The set of such transformations is denoted by $\mathcal{O}(\mathcal{M})$. We consider a subset $\mathcal{C}(\mathcal{M}) = \mathcal{F}(\mathcal{M}) \cap \mathcal{O}(\mathcal{M})$. The sets $\mathcal{F}(\mathcal{M})$, $\mathcal{O}(\mathcal{M})$ and $\mathcal{C}(\mathcal{M})$ are semigroups with respect to the composition of transformations.

Many authors (see [1] and references therein) studied the semigroups $\mathcal{F}(\mathcal{M})$, $\mathcal{O}(\mathcal{M})$ and $\mathcal{C}(\mathcal{M})$ in the case where the order $\leq$ on M is linear. We deal with the set of all subsets of a $n-$element set $N = \{1, 2, \ldots, n\}$ naturally ordered by inclusion, that is, with the boolean $\mathcal{B}_n$.

We focus on three classic semigroups: the symmetric semigroup of all transformations of the set $\mathcal{B}_n$; the semigroup of all partial transformations of the set $\mathcal{B}_n$ and the symmetric inverse semigroup of all partial injective transformations of $\mathcal{B}_n$. Thus, we have nine semigroups of order-consistent transformations of the set $\mathcal{B}_n$: $\mathcal{F}(\mathcal{B}_n)$, $\mathcal{PF}(\mathcal{B}_n)$, $\mathcal{IF}(\mathcal{B}_n)$, $\mathcal{O}(\mathcal{B}_n)$, $\mathcal{PO}(\mathcal{B}_n)$, $\mathcal{IO}(\mathcal{B}_n)$, $\mathcal{C}(\mathcal{B}_n)$, $\mathcal{PC}(\mathcal{B}_n)$, $\mathcal{IC}(\mathcal{B}_n)$.

Denote by $J(S)$ the set of idempotents of defect 1 of a transformation semigroup S. Let ϵ be an identity transformation of semigroup S.

Our main results are the following:

Theorem 1. *The symmetric semigroup of order-decreasing transformations $\mathcal{F}(\mathcal{B}_n)$ is generated by the set $J(\mathcal{F}(\mathcal{B}_n)) \bigcup \{\epsilon\}$, where $|J(\mathcal{F}(\mathcal{B}_n))| = 3^n - 2^n$. The semigroup of all partial order-decreasing transformations $\mathcal{PF}(\mathcal{B}_n)$ is generated by the set $J(\mathcal{PF}(\mathcal{B}_n)) \bigcup \{\epsilon\}$, where $|J(\mathcal{PF}(\mathcal{B}_n))| = 3^n$.*

Theorem 2. *The semigroups $\mathcal{IF}(\mathcal{B}_n)$, $\mathcal{O}(\mathcal{B}_n)$ (for $n \geq 2$), $\mathcal{PO}(\mathcal{B}_n)$ (for $n \geq 2$), $\mathcal{IO}(\mathcal{B}_n)$, $\mathcal{C}(\mathcal{B}_n)$ (for $n \geq 3$), $\mathcal{PC}(\mathcal{B}_n)$ (for $n \geq 3$), $\mathcal{IC}(\mathcal{B}_n)$ are not generated by the idempotents.*

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Index of participants

Наукове видання

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