Differentiable finite-state izometries and izometric polynomials of the ring of integer 2-adic numbers.

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The aim of the report is to construct requirements which describe izometric polynomials of the ring of integer 2-adic numbers.

The results of this report continue investigations of 2-adic group automatous with the 2-adic izometric functions technique. Polynomials build the important class of izometric function that is why we investigate them.

In addition we investigate differentiable finite-state izometries. The class of finite-state izometries is very important class of izometries and is the object of investigation in many scientific researches.

Definition 1. *Define $S_n(x_1, x_2)$ as*

$$S_n(x_1, x_2) = \sum_{k=0}^{n-1} x_1^{n-k-1} \cdot x_2^k$$

Example 1. $S_1(x_1, x_2) = 1,\ S_2(x_1, x_2) = x_1 + x_2,\ S_3(x_1, x_2) = x_1^2 + x_1 \cdot x_2 + x_2^2$ *etc.*

Definition 2. *Define function $\mu(x) = \overline{x}$:*

$$\mu(x) = \begin{cases} 0, x \in 2Z_2 \\ 1, x \in Z_2^* \end{cases}$$

Definition 3.

$$D_f(x_1, x_2) = \frac{f(x_1) - f(x_2)}{x_1 - x_2}$$

Lemma 1. *Polynomial $f(x) \in Z_2[x]$ is isometry, if and only if*

$$\forall\ x_1, x_2 \in Z_2\ \overline{D_f}(x_1, x_2) = 1$$

Definition 4. *Let's define for the polynomial $f(x) = a_0 + a_1 x + a_2 x^2 + ... + a_n x^n$ values A_f and B_f:*

$$A_f = \mu\left(\sum_{k=1}^{[\frac{n+1}{2}]} a_{2k}\right), B_f = \mu\left(\sum_{k=2}^{[\frac{n+1}{2}]} a_{2k-1}\right)$$

Theorem 1. $\overline{a}_1 \oplus (A_f \cap (\overline{x}_1 \oplus \overline{x}_2)) \oplus (B_f \cap (\overline{x}_1 \cup \overline{x}_2))$ *is true if and only if* $\overline{a}_1 = 1,\ A_f = 0,\ B_f = 0$

Theorem 2. *According to theorem 1, polynom $f(x) = a_0 + a_1 x + a_2 x^2 + ... + a_n x^n$ is isometry if and only if when a_1 is invertible (odd) integer 2-adic number, the sum of coefficients with even numbers greater then 0 is even 2-adic number and the sum of coefficients with odd numbers greater then 1 is even 2-adic number.*

Theorem 3. *Finite-state izometry of the ring Z_2 is differentiable if and only if it's parted linear.*

References

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