Recursive criterion of conjugation of finite-state binary tree's automorphisms

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In this paper conjugation problem in the group of finite-state automorphisms of rooted binary tree investigated.

Definition 1. *Define $FAutT_2$ as group of finite-state automorphisms of rooted binary tree.*

Definition 2. *Denote* **the marked tree** *for automorphism $f \in AutZ_2$ like this.*

- *The root of the tree note by automorphism f.*
- *If the vertex of the n-th level of the marked-type tree marked automorphism $a = (b, c) \circ \sigma$, then only one edge connects the n+1- th level with this vertex. Other vertex of this edge marked with automorphism $\pi_L(a) \circ \pi_R(a)$.*
- *If the vertex of the n-th level of the marked-type tree marked automorphism $a = (b, c)$, then two edges connect the n+1- th level with this vertex. Other vertex of one edge marked with automorphism $\pi_L(a)$ and another edge marked with $\pi_R(a)$.*

Automorphism that marked the vertex $t\ inD_f$ of marked-type tree denote as $D_f(t)$. The set of vertices of n-th level of tree D denote as $L_n(D)$.

Lemma 1. *Let*

$$a = (a_1, a_2) \circ \sigma, b = (b_1, b_2) \circ \sigma$$

$$a' = a_1 \circ a_2, b' = b_1 \circ b_2$$

If a' and b' conjugated in $FAutT_2$ then a and b conjugated in $FAutT_2$.

Theorem 1. *Automorphisms a and b conjugated in $FAutT_2$ if, and only if*

$$\forall t \in L_n(D_a), \exists x \in FAutT_2,\ D_a(t)^x = D_b(t * \alpha)$$

Corollary 1. *Automorphisms a and b conjugated in $FAutT_2$ if, and only if*

$$\exists n \in \mathbb{N}, \forall t \in L_n(D_a), \exists x \in FAutT_2\ D_a(t)^x = D_b(t * \alpha)$$

This techniques applied for finite-state conjugation problem solving of differentiable finite-state izometries of the ring of integer 2-adic numbers.

References

[1] Denis Morozov. Differentiable finite-state izometries and izometric polynomials of the ring of integer 2-adic numbers. 8th International Algebraic Conference, July 5-12 (2011), Lugansk, Ukraine.