Book of abstracts of the

International Conference on Algebra

dedicated to 100th anniversary of S.M.Chernikov

August 20-26, 2012
Dragomanov National
Pedagogical University,
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Contents

Сергій Миколайович Черніков

(11.05.1912 – 23.02.1987)

Визначному вченому-алгебраїсту Сергію Миколайовичу Чернікову 11 травня 2012 р. виповнилось б 100 років. Він народився в м. Сергіїв Посад в родині священика. Дитинство Сергія Миколайовича пройшло в Україні. Після закінчення середньої школи кілька років викладав математику у школі в м. Саратові та водночас учився на заочному відділенні Саратовського педагогічного інституту. Після закінчення інституту працював в м. Свердловськ. В 1938 р. Сергій Миколайович захистив кандидатську, а в 1940 р. – докторську дисертації. В 1941 р. йому присвоєно звання професора. З 1967 р. С.М. Черніков – член-кореспондент АН УРСР.

С.М. Черніков завідував математичними кафедрами в Уральському політехнічному інституті (1939 – 1946 рр.), Уральському (1946 – 1951 рр.) та Пермському (1951 – 1961 рр.) університетах. В 1961 – 1964 рр. очолював відділ алгебри і геометрії Свердловської філії Математичного інституту АН СРСР, а з 1965 р. і до кінця життя – відділ алгебри Інституту математики АН України. Одночасно з 1965 р. викладав курс алгебри в Київському педагогічному інституті. Помер Сергій Миколайович Черніков 23 січня 1987 р.

С.М. Черніков – творець відомої алгебраїчної школи, з якої вийшли близько п'ятдесяти кандидатів та докторів фізико-математичних наук. Серед них – академік В.М. Глушков, член-кореспондент М.І. Каргаполов, член-кореспондент РАН І.І. Єрьомін, доктори фізико-математичних наук В.С. Чарін, Ю.М. Горчаков, В.П. Шунков, Д.І. Зайцев. Більшість з учнів С.М. Чернікова – українські вчені.

Сергія Миколайовича по праву можна вважати одним з основоположників сучасної теоріїї груп. Його наукова діяльність в галузі теорії груп охоплює період з кінця 30-х до другої половини 80-х років. В цілому тематику його досліджень можна охарактеризувати як вивчення груп із заданими властивостями підгруп або систем підгруп. Ним введено в теорію груп ряд дуже важливих понять: локальна розв'язність, локальна нільпотентність, шарова скінченність, локальна ступінчастість; висунуто нові плідні ідеї; поставлено важливі проблеми (деякі з них до цього часу не розв'язані); розроблено оригінальні підходи до дослідження. Сергію Миколайовичу належить багато фундаментальних результатів, які стали класичними. Під впливом його робіт ця тематика розвивалася і розвивається багатьма алгебраїстами (О.Ю. Шмідт, О.Г.Курош, А.І.Мальцев, В.М.Глушков, Р.Бер, Ф.Холл, Б.І.Плоткін, М.І. Каргаполов, О.Ю. Ольшанський, В.П.Шунков, О.Кегель, Б. Верфріц, Д. Робінсон, Г. Хейнекен, В.С. Чарін, М.С. Черніков, І.І. Єрьомін, Ю.М. Горчаков, Б. Хартлі, Ю.І. Мерзляков, Д.І. Зайцев, М. Томкінсон, Я.П. Сисак, О.Д. Артемович, А.В. Тушев та ін.). Цій тематиці присвячена відома монографія С.М.Чернікова "Группы с заданными свойствами системы подгрупп" (М.: Наука, 1980. – 384 с.).

С.М. Черніков – один з піонерів лінійного програмування і творець алгебраїчної теорії лінійних нерівностей. Займаючись розв'язуванням актуальних задач прикладного характеру в роки

Великої Вітчизняної війни, він установив так званий принцип граничних розв'зків, на основі якого в подальшому побудував всебічно розвинуту теорію лінійних нерівностей. Розроблений С.М. Черніковим метод згортки систем лінійних нерівностей поряд з іншими його результатами і методами є важливим теоретичним інструментом прикладних досліджень. Цей метод дозволив, зокрема, досліджувати суперечливі системи лінійних нерівностей та розробити методи оптимізації для задач з суперечливими системами обмежень, конструктивно описувати всю множину оптимальних розв'язків у задачах лінійного програмування та знайшов істотні застосування в теорії і методах розпізнавання образів (І.І. Єрьомін та його школа). С.М. Черніков виділив і дослідив важливий клас нескінченних систем лінійних нерівностей – поліедрально замкнені системи – і побудував для них теорію двоїстості. Цей клас систем ліг в основу теоретичного і чисельного аналізу нелінійних задач оптимізації через редукцію останніх до задач напівнескінченного та нескінченного лінійного програмування. Побудована С.М. Черніковим алгебраїчна теорія лінійних нерівностей викладена у його широко відомій монографії "Линейные неравенства" (М. : Наука, 1968. – 488 с.).

N. M. ADARCHENKO

A characterization of finite supersoluble groups

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By Ph. Hall's theorem, a finite group is soluble if all its maximal subgroups are of index a prime, or the square of a prime ([1], Theorem 10.5.7). By B. Huppert's theorem, a finite group is supersoluble if all its maximal subgroups are of index a prime ([1], Theorem 10.5.8).

Theorem 1. *Suppose that for every non-normal Sylow subgroup P of a finite group G the following condition is satisfied: if a maximal subgroup M of G contains $N_G(P)$, then $|G:M|$ is either a prime or the square of a prime. Then G contains a normal Hall $\{2,3\}'$-subgroup having a Sylow tower of supersoluble type.*

Using Theorem 1 we prove the following result.

Theorem 2. *Suppose that for every non-normal Sylow subgroup P of a finite group G the following condition is satisfied: if a maximal subgroup M of G contains $N_G(P)$, then $|G:M|$ is a prime. Then G is supersoluble.*

[1] Hall M. *The theory of groups.* New York: The Macmillan Company, 1959.

Neda AHANJIDEH

Note on the lengths of conjugacy classes of some finite simple groups

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Let G be a finite group and $Z(G)$ be its center. For $x \in G$, suppose that $cl_G(x)$ denotes the conjugacy class in G containing x and $C_G(x)$ denotes the centralizer of x in G. We will use $N(G)$ for the set $\{n : G$ has a conjugacy class of size $n\}$. This talk concerns the following open conjecture of J. G. Thompson which is Problem 12.38 in [3]:
Thompson's Conjecture: Let G be a finite group with $Z(G) = 1$. If S is a non-abelian finite simple group satisfying that $N(G) = N(S)$, then $G \cong S$.

This conjecture has received some attention in existing literature, which the most important one can be found in a paper of Chen [2]. The main result of this talk is:

Theorem 1. [1] *Let $n \geq 4$. If G is a finite group such that $Z(G) = 1$ and $N(G) = N(^2D_n(q))$, then $G \cong {}^2D_n(q)$.*

[1] Ahanjideh N., Ahanjideh M., On the validity of Thompson's conjecture for finite simple groups, *Com. Alg.*, in press.

[2] Chen G., On Thompson's conjecture, *J. Algebra*, **185** (1996) 184-193.

[3] Khukhro E. I., Mazurov V. D., *Unsolved problems in group theory: The Kourovka Notebook*, Sobolev Institute of Mathematics, Novosibirsk, 17th edition: 2010.

Hasan AL-EZEH, Moh'd ABU DAYEH

Some new properties of Hurwitz series ring

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Let R be a commutative ring with unity. A new very important differential ring were constructed from the ring R, called the Hurwitz series ring HR. This ring was extensively studied because it plays a central role in differential Algebra. In this paper some new properties of the ring HR are studied. An ideal I in a ring R is called pure if for any $a \in I$, there exists $b \in I$ such that $a \cdot b = a$. Pure ideals are interesting because they classify some types of important rings such as Von Neumann regular rings and PF-rings. In this paper we characterize pure ideals in the Hurwitz series ring HR. Quazi prime ideals were introduced in differential rings. They play a rule that resembles prime ideals in rings. Here, we completely characterize quazi prime ideals in the ring HR.

Bernhard AMBERG

Products of Chernikov groups and related problems

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The question whether every group G, which is the product of two abelian-by-finite subgroups A and B always contains a metabelian (or at least soluble) subgroup of finite index, is very difficult to answer. In particular, it is not known whether a product of two Chernikov subgroups is a Chernikov group. This has only been proved in special cases, for instance for groups G that are generalized soluble in some sense.

It is natural to consider these questions first in cases when the two subgroups A and B have abelian subgroups of small index, notably of index at most 2. In this talk we will discuss some new results for this situation that we have obtained recently in joint work with Ya. Sysak and L. Kazarin.

As an example we mention the following result (see Israel J. Math. 175 (2010), 363-389).

Theorem. Let the group $G = AB$ be the product of two Chernikov subgroups A and B, each of which contains an abelian subgroup A_0 resp. B_0 of index at most 2. If further one of the two subgroups, A say, is of dihedral type, i.e. it contains an involution τ that inverts every element of A_0, then G is a soluble Chernikov group.

The proof of this and similar results depend on delicate calculations with involutions. In particular extensive use is made of the fact that in any group two distinct involutions generate a dihedral group.

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A new characterization of $PGL(2,p)$ with its nse and order

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Let G be a finite group and $\pi_e(G)$ be the set of element orders of G. Let $k \in \pi_e(G)$ and m_k be the number of elements of order k in G. Set nse(G):= $\{m_k \mid k \in \pi_e(G)\}$. It is proved that if G is one of the simple K_4-groups or sporadic groups, then it can be uniquely determined by nse(G) and order of G. Also, if G is one of the groups A_4, A_5, A_6, A_7, A_8 and $PSL(2,q)$, for $q \in \{7,8,11,13\}$, then it can be uniquely determined only by nse(G). In this paper, as the main result, it is proved that if G is a group such that nse(G)=nse($PGL(2,p)$) and $|G| = |PGL(2,p)|$, where p is a prime number, then $G \cong PGL(2,p)$.

[1] Khalili Asboei A., Salehi Amiri S. S., Iranmanesh A., Tehranian A. *A new characterization of A_7, A_8*. Anale Stintifice ale Universitatii Ovidius constanta. (to appear).

[2] Khalili Asboei A., Salehi Amiri S. S., Iranmanesh A., Tehranian A. *A new characterization of sporadic simple group with its NSE and order.* Algebra and its Aplications. (to appear).

[3] Khatami M., Khosravi B., Akhlaghi Z. *A new characterization for some linear groups.* Monatsh Math, **163**, (2011), 39-50.

[4] Shao C. G., Shi W., Jiang Q. *Characterization of simple K_4-groups.* Front Math China, **3**, (2008), 355-370.

[5] Shen R., Shao C. G., Jiang Q., Shi W., Mazuro V. D. *A New Characterization of A_5.* Monatsh Math, (2010), 337-341.

Orest ARTEMOVYCH

Derivations of a ring and simplicity

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Throughout, a ring R means an associative ring with 1. An additive map $d : R \to R$ is called a derivation of R if $d(ab) = d(a)b + ad(b)$ for all $a, b \in R$. The set $\mathrm{Der}R$ of all derivations of R is a Lie ring under the operations of pointwise addition and Lie multiplication.

The various aspects of a simplicity of derivations has been studied many times: N. Jacobson (1937), I.N. Herstein (1955), S.A. Amitsur (1957), R.E. Block (1969), D.A. Jordan [1] and others.

Our aim is to present results on rings R with simple Lie ring $\mathrm{Der}R$ of derivations.

[1] Jordan D.A., On the simplicity of Lie algebras of commutative algebras, *J. Algebra* **228**, (2000), pp. 580-585.

Nahid ASHRAFI, Ebrahim NASIBI

Some results of r-clean rings

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Let R be an associative ring with unity. An element $a \in R$ is said to be r-clean if $a = e + r$, where e is an idempotent and r is a (von Neumann) regular in R. If every element of R is r-clean, then R is called an r-clean ring. In this paper, we prove that the concepts of clean ring and r-clean ring are equivalent for abelian rings. Further we prove that if 0 and 1 are the only idempotents in R, then an r-clean ring is an exchange ring. Also we show that center of an r-clean ring is not necessary r-clean, but if 0 and 1 are the only idempotents in R, then the center of an r-clean ring is r-clean. Finally we give some properties and examples of r-clean rings.

V. V. ATAMAS, O. O. RUSIN

The number of possible states of the $n \times n \times n$ Rubic's cube

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Investigation of Rubik's Cube generates a number of problems. The most popular of them is problem of finding the least number of rotations which enable to put the cube from any possible state to the solved state. The finding of number of possible states of Rubik's Cube is another aspect which have been studied. It is known [1] that for $3 \times 3 \times 3$ Rubik's Cube this number is:

$$8! \cdot 3^7 \cdot 12! \cdot 2^{10} \approx 4.33 \cdot 10^{19}.$$

In $n \times n \times n$ Rubik's Cube the number of possible states dependents on the parity of n. In particular the following theorem is proved.

Theorem 1. *The number of possible states of the $n \times n \times n$ Rubik's Cube is expressed by the next formula:*

$$P_n = \begin{cases} \frac{8! \cdot 3^7 \cdot 24!^{k^2-k}}{24 \cdot (4!^6)^{(k-1)^2}}, & \text{if } n = 2k, \\ \frac{8! \cdot 3^7 \cdot 12! \cdot 2^{10} \cdot 24!^{k^2-1}}{(4!^6)^{k^2-k}}, & \text{if } n = 2k+1. \end{cases}$$

The states of the $n \times n \times n$ Rubik's Cube form a group of order P_n.

[1] Kaloujnine L. A., Sushchansky V. I. *Transformations and permutations.* M.: Nauka, 1985.

V. BABYCH, N. GOLOVASHCHUK, S. OVSIENKO

Poincare group for bimodule problem

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Let $\mathcal{A} = (\mathsf{K}, \mathsf{V})$ be a faithful connected finite dimensional one-sided bimodule problem from the class $\mathcal{C}$ considered in [1] with a quasi multiplicative basis Σ. We associate a two-dimensional cell complex $\mathfrak{L}$ with $\mathcal{A}$ and construct its Poincare groupoid $\mathcal{G}_{\mathfrak{L}}$ ([2]). Poincare group of $\mathcal{G}_{\mathfrak{L}}$ is called Poincare group of bimodule problem $\mathcal{A}$.

Theorem 1. *The Poincare group of bimodule problem $\mathcal{A} \in \mathcal{C}$ is free.*

The proof of this statement uses the geometric techniques of diagrams, contracting closed walks in Σ ([3]). These methods are similar to those used in the geometric group theory.

[1] V. Babych, N. Golovashchuk, S. Ovsienko. *Generalized multiplicative bases for one-sided bimodule problems* // Algebra and Discrete Mathematics, Vol. **12** (2011), No. 2, 1 – 24.

[2] V. Babych, N. Golovashchuk. *Bimodule problems and cell complexes* // Algebra and Discrete Mathematics, 2006, No. 3, 16–28.

[3] Yu. Drozd, S. Ovsienko. *Coverings of tame boxes and algebras.* – Bonn, 2000. – 44 p. – Preprint of Max-Plank Institute, MPI-2000-26.

A.K. BAKHTIN[1], G.P. BAKHTINA[2], I.V. DENEGA[1]

Estimates of product of inner radii of mutually non-overlapping domains in multidimensional complex spaces

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In [1] was proposed a variant of the construction of a commutative an associative algebra in $\mathbb{C}^n$. We denote by Σ_m a set of systems of pairs $(\mathbb{B}_k, \Omega_k)$ of polycylindrical domains and points considered in theorem 3 [3]. A following theorem is proved (all definitions see in [2, 3]).

Theorem 1. *Let $n, m \in \mathbb{N}$, $m \geqslant 5$, $\gamma \in (0, \sqrt[3]{m}]$. Then for any elements of Σ_m, a following inequality is true*

$$R^{\gamma}(\mathbb{B}_0, \Omega_0) \prod_{k=1}^{m} R(\mathbb{B}_k, \Omega_k) \leqslant \left(\frac{4}{m}\right)^m \frac{\left(\frac{4\gamma}{m^2}\right)^{\frac{\gamma}{m}}}{\left(1-\frac{\gamma}{m^2}\right)^{m+\frac{\gamma}{m}}} \left(\frac{1-\frac{\sqrt{\gamma}}{m}}{1+\frac{\sqrt{\gamma}}{m}}\right)^{2\sqrt{\gamma}}.$$

[1] Bakhtin A.K. *A generalization of some results of the theory of schlicht functions on multidimensional complex spaces*// Reports NAS of Ukraine, 3, (2011), P. 7 – 11. (in Russian)

[2] Bakhtin A.K., Bakhtina G.P., Denega I.V. *The problem on a product of powers of generalized conformal radii for non-overlapping domains in $\mathbb{C}^n$*, Proceedings of the Institute of Mathematics of NAS of Ukraine, 2, (2010), P. 180 – 186. (in Russian)

[3] Zabolotnii Ya.V. *About one extremal problem of V.N. Dubinin*, Ukrainian Mathematical Journal, 1, (2012), P. 24 – 31. (in Ukrainian)

P. P. BARYSHOVETS

Groups with conjugacy classes of uncomplemented subgroups

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In paper [1] S.N.Chernikov has reised the general problem of studying the structure of groups with one system or another of complemented subgroups. A subgroup A of the group G is complemented in G, if G has a subgroup B such that $G = AB$ and $A \cap B = 1$. Ph.Hall [2] studied finite groups with complemented subgroups. Complete description any (both finite and infinite) groups with this property which has received the name completely factorizable groups, obtained N.V.Chernikova [3].

Theorem 1. *An infinite locally finite group G with no more than one class of conjugate uncomplemented subgroups is completely factorizable.*

It would be interesting to prove (or disprove) a similar statement for the general case: an infinite not completely factorizable group has nonfinite number of conjugacy classes of noncomplemented subgroups.

For nonperiodic groups this is obviously true.

[1] Chernikov S.N. *Groups with systems of complemented subgroups*// Mat. Sb. - 1954. - **35**. - P. 93–128.

[2] Hall Ph. *Complemented groups*// J. L. Math. Soc. - 1937. - **12**. - P. 201–204.

[3] Chernikova N.V. *Groups with systems of complemented subgroups*// Mat. Sb. - 1956. - **39**. - P. 273–292.

Yu. N. BELYAYEV

Representation of matrix functions by means of symmetric polynomials

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Let M be a matrix of order n; σ_j $(j = 1, \ldots, n)$ — sums of C_n^j principal minors of j-th order $\det M$, i.e. an elementary symmetric polynomials in the eigenvalues of matrix M. The recurrence formulae [1, p. 274]

$$\mathscr{B}_j = 0,\ j = 0, \ldots, n-2;\ \ \mathscr{B}_{n-1} = 1;\ \ \mathscr{B}_j = \sum_{i=1}^{n} (-1)^{i-1} \sigma_i \mathscr{B}_{j-i},\ j \geq n,$$

defines *symmetric polynomials of n-th order $\mathscr{B}_j$*.

Theorem 1. $M^j = \sum\limits_{l=0}^{n-1} M^l \sum\limits_{g=0}^{l} (-1)^{n-l+g-1} \sigma_{n-l+g} \mathscr{B}_{j-1-g}$,

$$\textit{where } j = \begin{cases} \textit{any integer}, \ \textit{if} \ \det M \neq 0, \\ \textit{integer, greater than or equal to } n, \ \textit{if} \ \det M = 0. \end{cases}$$

Theorem 2. *Let $f(\zeta) = \sum_{j=0}^{\infty} \alpha_j \zeta^j$ be an entire function, then*

$$f(M) = \sum_{l=0}^{n-1} M^l \Big(\alpha_l + \sum_{g=0}^{l} (-1)^{n-l+g-1} \sigma_{n-l+g} \sum_{j=n}^{\infty} \alpha_j \mathscr{B}_{j-1-g} \Big).$$

The main advantages of calculating the matrix functions by means of these formulae shown on a number of examples [2].

This work was supported by the Program "Scientific and scientific-pedagogical personnel of innovative Russia", state contr. 2.740.11.0618.

[1] Belyayev Yu. N. *Vector and tensor analysis* (in Russian). Syktyvkar: Syktyvkar University, 2010.

[2] Belyayev Yu. N. *The matrix approach of theory of waves in layered media* (in Russian). Saarbrucken: LAP, 2012.

N.V. BEZVERKHNII

The conjugacy membership problem for a cyclic subgroup of a $C(3)-T(6)$-group $G=(X,R)$

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Let $G=(X;R)$ be a group with $C(3)-T(6)$-small cancellation conditions. We construct an algorithm checking for any two words v,w in the alphabet X weather or not exists an integer n such that words v and w^n a conjugate in the group G.

Let M be a diagram over the group $G=(X;R)$. The region $D\subset M$ is calling *Dehn-region* if 1) $\partial D\cap\partial M$ is a subpath of a boundary cycles ∂D and ∂M; 2) the number of inner edges of the region D is denoted by $i(D)$ and $i(D)\in\{0,1\}$.

The strip in the diagram M is subdiagram $\Pi=\bigcup\limits_{i=1}^{k}D_i$ with next properties: 1) $\partial D_i\cap\partial M=p$ is a subpath of a boundary cycles ∂M and ∂D_i; 2) $\partial\Pi\cap\partial M$ is a subpath of a boundary cycles ∂M and $\partial\Pi$; 3) for $k=3$ $i(D_1)=i(D_2)=i(D_3)=2$, and adjacent regions have a common edge, and all three regions have a common vertex; for $k>3, k=2l+1$ $i(D_1)=i(D_2)=i(D_{2l})=i(D_{2l+1})=2$, $i(D_3)=i(D_5)=\ldots=i(D_{2l-3})=i(D_{2l-1})=3$, $i(D_4)=i(D_6)=i(D_{2l-4})=i(D_{2l-2})=2$; 4) the path $\partial D_i\cap\partial D_{i+1}$ is an edge $(i=1,\ldots,k-1)$.

We prove that in all reduced ring diagram M with boundary cycles σ,τ and boundary labels w^n and v over the $C(3)-T(6)$-group G without strips and Dehn-regions the number of regions with edges in σ is less then some lineal function of the number of regions with edges in τ and on the contrary.

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About adequate domain

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Let R be an associative ring with $1 \neq 0$. An element a of a ring R is called an adequate element if for any element b from R an element a can be represented as a product $a = rs$, where $rR + bR = R$, and for any non invertible divisor t of s we have $tR + bR \neq R$ [1]. A ring R is called a weakly continuous if the annihilator of each element is essential in a summand of R [2]. A ring R is ACS-ring if for a of R we have $aR = P + S$ where P is projective and S is singular as R-modules. A ring is uniform if any two nonzero ideals of R have nonzero intersection.

Theorem 1. *A commutative domain is an adequate if and only if for any non zero element a the factor-ring R/aR is a weakly continuous.*

Theorem 2. *A commutative Bezout domain is an adequate domain if and only if for any non zero element a the factor-ring R/aR is a ACS-ring.*

Theorem 3. *If R is a commutative Bezout domain if for every nonzero element a of a ring R a factor-ring R/aR is uniform, then R is an adequate domain.*

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Lucia BITCOVSCHI

Algebraic structure on the table ordered

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Let $E = \{e_1, e_2, ..., e_n\}$, be a lot of tabular records. By the table ordered on the lot $E = \{e_1, e_2, ..., e_n\}$, marked $T(e_{i_1}, e_{i_2}, ..., e_{i_k})$, we understand the ordered lot $\{e_{i_1}, e_{i_2}, ..., e_{i_k}\}$, where $e_{i_j} \in E, j = 1, 2, ..., k$, as k – is the number of elements in the table $T(e_{i_1}, e_{i_2}, ..., e_{i_k})$, and the element e_{i_k}– is named the last record.

The operation *"+"*, which adds in table $T(e_{i_1}, e_{i_2}, ..., e_{i_k})$ a record $e_{i_{k+1}}$, is the operation which transforms $T(e_{i_1}, e_{i_2}, ..., e_{i_k})$ in table $T(e_{i_1}, e_{i_2}, ..., e_{i_k}, e_{i_{k+1}})$ so that record $e_{i_{k+1}}$becomes the last record of the table, - $T(e_{i_1}, e_{i_2}, ..., e_{i_k}) + e_{i_{k+1}} = T(e_{i_1}, e_{i_2}, ..., e_{i_k}, e_{i_{k+1}})$.

Definition 1. *The operation "+", which concatenated table* $T_i = T(e_{i_1}, e_{i_2}, ..., e_{i_k})$ *with table* $T_j = T(e_{i_{k+1}}, e_{i_{k+2}}, ..., e_{i_{k+m}})$, *is operation* $(...((T(e_{i_1}, e_{i_2}, ..., e_{i_k}) + e_{i_{k+1}}) + e_{i_{k+2}}) + ...) + e_{i_{k+m}}$ *which results in table* $T(e_{i_1}, e_{i_2}, ..., e_{i_k}, e_{i_{k+1}}, e_{i_{k+2}}, ..., e_{i_{k+m}})$, *so that record* $e_{i_{k+m}}$ *becomes the last record of the table* $T_i + T_J$, *for other* $\mathrm{T_i}$, $\mathrm{T_j} \in \Re$.

So, the set of all tables on lot E with extended operation "add" - representing a couple which noted $(\Re, +)$.

Theorem 1. *Couple* $(\Re, +)$ *has identity record.*

Theorem 2. *Couple* $(\Re, +)$ *forms a semigroup.*

Theorem 3. *Couple* $(\Re, +)$ *forms a monoid.*

Theorem 4. *Couple* $(\Re, +)$ *forms a commutative monoid.*

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On finite generation of self-similar groups of finite type

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A self-similar group of finite type is the profinite group of all automorphisms of a regular rooted tree that locally around every vertex act as elements of a given finite group of allowed actions. R.I. Grigorchuk asked in [2, Problem 7.3(i)] under what conditions a self-similar group of finite type is topologically finitely generated. We address this question and establish certain criterion in the following theorem [1].

Theorem 1. *Let G be a level-transitive self-similar group of finite type given by patterns of depth d. The group G is finitely generated if and only if there exists $n \geq d$ such that the commutant of $St_G(d-1)|_{X^{[n]}}$ contains $St_G(n-1)|_{X^{[n]}}$.*

Using this criterion and GAP computations we show that for binary alphabet there are no infinite topologically finitely generated self-similar group given by patterns of depth ≤ 3, and there are 32 such groups for depth 4 (including the closures of the Grigorchuk group and the iterated monodromy group of $z^2 + i$ in the topology of the tree).

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On the representation type of elementary abelian p-groups with respect to the modules of constant Jordan type

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For a finite group G and a field k of characteristic $p > 0$, in [1] there was introduced kG-modules of constant Jordan type. For simplicity we assume k to be algebraic closed. We call a group G of cJ-tame (resp. cJ-wild) over k if the problem of classifying the kG-modules of constant Jordan type is tame (resp. wild); we call G of cJ-finite (resp. cJ-semiinfinite) type if there are, up to isomorphism, only finitely many (resp. infinitely many, but finitely many in every dimension) indecomposable modules of constant Jordan type.

We prove the following theorem.

Theorem 1. *An abelian p-group $G = (p, p, \dots, p)$ of order p^n is of cJ-finite type if $n = 1$ (for any p), of cJ-semiinfinite type if $n = 2, p = 2$, and cJ-wild if otherwise.*

The cases $n = 2, 3, p = 2$ are considered in [2] and the general case in [3]; the case $n = 1$ is trivial.

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Wreath products in the unit group

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Let $V(KG)$ be the group of normalized units of the group algebra KG of a locally finite p-group G over the field K of the positive characteristic p. It was proved in [3] that a wreath product $C_p \wr C_p$ of two cyclic groups of order p is involved in $V(KG)$. In [1, 2], all those locally finite p-groups were described for which $V(KG)$ does not contain a subgroup isomorphic to $C_p \wr C_p$.

We discuss the following conjecture of A. Shalev [4]:

• the group $V(KG)$ always possess a section isomorphic to the wreath product $C_p \wr G'$, where G' is the derived subgroup of G.

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Irina CHERNENKO

About unperiodic extensions with three involutions of quasicentral subgroups

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In works [1, 2] are descriptions of the groups that are extensions of quasicentral subgroups using quaternions group. This paper describes group G with three involutions which is an extension of quasicentral subgroups by quaternions group.

Theorem 1. *Group G has three involutions and it is an extension of the proper locally almost solvable subgroup F by Q_8, such that all subgroups $< u >\in F$ are normal in G, $|u| \in \{2^m, \infty\}$, if and only if it has the form $G = F \cdot H = L \cdot X$, where $F = L \times D$, L - group without involutions, D - Sylow 2-subgroup in F, which is not more than three involutions, $X = D \cdot H$, $H =< a, b >$, $|a| \in \{2^\alpha, \infty\}$, $\alpha > 1$, $|b| \in \{2^\beta, \infty\}$, $[a, b] = c \neq 1, |c| = 2^\gamma, \gamma > 0, F \cap H = Z$ - quasicentral subgroup of G, $f = a^2b^2 \in Z, [F, < c >] = 1, \omega(X)$ - group of type $(2, 2)$, $C_G(\omega(H)) \supset F$ and G is a group of one of the types described in* [3].

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N. S. CHERNIKOV

S. N. Chernikov and the theory of groups. The Kiev period

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I plan to give a survey of S.N. Chernikov's group-theoretic results relating to Kiev period of his life (1965-1987) and also to dwell on group-theoretic results (1965-2012) of various authors, solving S.N. Chernikov's questions and problems.

Ivan CHERVYAKOV

On minimax equivaivalent oversupercritical posets

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V. M. Bondarenko and M. V. Stepochkina [1, 2] proved that the Tits form of a poset S is positive (resp. nonnegative) definite if and only if any poset that is minimax equivalent to S has no critical (resp. supercritical) full subposets; the notion of the minimax equivalence was introduced by V. M. Bondarenko in [3].

We study some generalizations of critical and supercritical poset (so-called oversupercritical posets introduced by V. M. Bondarenko) up to minimax equivalence and weak isomorphism [4].

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About direct complement of fuzzy subgroups

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Let G be a group with a multiplicative binary operation. We recall that a fuzzy subset $\gamma : G \mapsto [0,1]$ is said to be a fuzzy group on G (see, for example, [1]), if it satisfies the following conditions: $\gamma(xy) \geq \gamma(x) \wedge \gamma(y)$, for all $x, y \in G$ and $\gamma(x^{-1}) \geq \gamma(x)$, for every $x \in G$.

Fuzzy group theory, as well as other fuzzy algebraic structures, was introduced very soon at the beginning of fuzzy set theory. Many new concepts were introduced and variety results were obtained but there are not too a lot of deep among the received results.

Let γ be a fuzzy group on G then γ is a direct product of fuzzy groups α and β if $\gamma(x) = (\alpha \cdot \beta)(x) = \bigvee_{u,v \in G, uv=x}(\alpha(u) \wedge \beta(v))$ for all $x \in G$ and $\alpha \cap \beta = \chi(e, \gamma(e))$. It was obtained following results:

Theorem 1. *Let G be a group, $H, K < G$, $G = HK$, $H \cap K =< e >$, γ is a fuzzy group on G and let $Supp(\gamma) = G$, then if $\bigvee\{\gamma(x) | x \in (H \cup K) \setminus \{e\}\} \geq \bigvee\{\gamma(x) | x \in G \setminus (H \cup K)\}$, then $\gamma = \gamma|_H \cdot \gamma|_K$.*

Corollary 1. *Let G be an elementary abelian group, γ be a fuzzy group on G and let $Supp(\gamma) = G$, then for any subgroup $H < G$ fuzzy group $\gamma|_H$ has a direct complement.*

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On locally soluble AFN–groups

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Antifinitary linear groups have been investigated in [1]. In this paper we consider the analogy of antifinitary linear groups in theory of modules over group rings.

Let A be an $\mathbf{R}G$–module where $\mathbf{R}$ is a ring, G is a group. We say that a group G is AFN–group if each proper subgroup H of G for which $A/C_A(H)$ is not a noetherian $\mathbf{R}$–module, is finitely generated.

Let $ND(G)$ be a set of all elements $x \in G$, such that $A/C_A(x)$ is a noetherian $\mathbf{R}$–module. $ND(G)$ is a normal subgroup of G. Later on it is considered $\mathbf{R}G$–module A such that $\mathbf{R}$ is a commutative ring, $C_G(A) = 1$.

The main results are theorems 1, 2.

Theorem 1. *Let A be an $\mathbf{R}G$–module, G be a locally soluble* AFN–*group. Then G has an ascending series of normal subgroups $\langle 1 \rangle = L_0 \leq L_1 \leq L_2 \leq \cdots \leq L_\gamma \leq \cdots \leq L_\delta = G$ such that each factor $L_{\gamma+1}/L_\gamma, \gamma < \delta$, is either abelian or hyperabelian.*

Theorem 2. *Let A be an $\mathbf{R}G$–module, G be a finitely generated soluble* AFN–*group. If $A/C_A(G)$ is not a noetherian $\mathbf{R}$–module, then the following conditions holds: (1) $A/C_A(ND(G))$ is a noetherian $\mathbf{R}$–module; (2) G has the series of normal subgroups $B \leq R \leq W \leq G$ such that B is abelian, R/B is locally nilpotent, W/R is nilpotent and G/W is a polycyclic group.*

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Filter regular sequence and generalized local cohomology with respect to a pair of ideals

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Let $(R, \mathfrak{m})$ be a Noetherian local ring. The two notions of filter regular sequence and generalized local cohomology module with respect to a pair of ideals are introduced, and their properties are studied. Some vanishing and non-vanishing theorems are given for this generalized version of generalized local cohomology module.

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Some properties of generalized local cohomology with respect to an ideal containing the irrelevant ideal

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Let R be a commutative Noetherian ring with non-zero identity and $\mathfrak{a}$ an ideal of R. Let M and N be finitely generated R-modules. We characterize the membership of the generalized local cohomology modules $H^i_{\mathfrak{a}}(M,N)$ in certain Serre subcategories of the category of modules from upper bounds. We also, using the above result, in certain graded situations, the tameness and asymptotically stability of the homogeneous components of $H^i_{\mathfrak{a}}(M,N)$ for some i's with a specified property is studied.

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A non-finitely based variety of centre-by-metabelian pointed-groups

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A pointed-group is a pair (G, g) consisting of a group G together with a distinguished element $g \in G$. A pointed-group may be regarded as a group with an extra nullary operation; so it is an algebra in the sense of universal algebra.

The "behavior"of the identities of pointed-groups in some situations is similar to the "behavior" of the identities of "ordinary" groups and in other situations these "behaviors" are different. There is a finite pointed-group without finite basis for its identities (Bryant, 1982) whereas the identities of every "ordinary" finite group have such a basis (Oates and Powell, 1964). On the other hand, every nilpotent, metabelian or nilpotent-by-(abelian of finite exponent) group has a finite basis for its identities (Higman, 1959; Cohen, 1967; Krasilnikov and Shmelkin, 1981) and the same is true for the identities of nilpotent, metabelian or nilpotent-by-(abelian of finite exponent) pointed-groups (Ali and Majeed, 1995; Ali and Majeed, 1998; Quick, 2003). We show that in the centre-by-metabelian case the "behavior" of the identities of pointed-groups is different from one of the identities of "ordinary" groups. Recall that every "ordinary" centre-by-metabelian group has a finite basis for its identities (McKay, 1972). Our result is as follows.

Theorem 1. *There exists a variety of centre-by-metabelian pointed-groups that does not admit any finite basis.*

O. DOMSHA

Reduction of matrices over commutative Bezout domain with stable range condition in localizations

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Let R be a commutative Bezout domain with $1 \neq 0$. Let $a \in R \setminus \{0\}$ and $S_a = \{b \in R \mid aR + bR = R\}$. We shall notice that S_a is saturated and multiplicative closed.

Let's denote $R_a = RS_a^{-1}$. A ring R is the ring of stable range 1 (in denotation $st.r.(R) = 1$) if for every elements $a, b \in R$ the condition $aR + bR = R$ implies that there exist such element $t \in R$ that $(a + bt)R = R$ [1]. A ring R is the ring of almost stable range 1 if for every non-zero $a \in R$ the the stable range of R/aR is equal 1 [2].

Theorem 1. *Let R is a commutative Bezout domain. If for every non-zero $a \in R$ $rad(R/aR = 0)$, then:*

1) $st.r.(R/aR) = 1$;
2) *R is the elementary divisor ring.*

Theorem 2. *If R is a commutative Bezout domain and for every non-zero $a \in R$ R_a is a domain of almost stable range 1, then R is elementary divisor ring.*

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Local automorphisms and derivations of nilpotent matrix algebras

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Recall that a linear map φ of an algebra A into A is said to be a *local derivation* (*a local automorphism*) if for every $\alpha \in A$ there exists a derivation (resp., an automorphism) φ_α of A such that $\varphi(\alpha) = \varphi_\alpha(\alpha)$. We study the local automorphisms and the local derivations of an algebra $R = NT(n, K)$ of all $n \times n$ strictly lower triangular matrices over K and of the Lie algebra $\Lambda(R)$. Its nontrivial examples are constructed (automorphisms and derivations of this algebras are well-known). When $n = 3$ the full descriptions of local derivations and local automorphisms for R and for its associated Lie algebra $\Lambda(R)$ and when $n = 4$ for the algebra R over a field K were established (see [1] and references 1, 2 on the author's articles ibid). The maps $\delta_{31,t} : \alpha \to \alpha + ta_{31}e_{31}$, $\delta_{42,t} : \alpha \to \alpha + ta_{42}e_{42}$ $(\alpha \in R)$, $t \in K$ are the local derivations of $\Lambda(R)$ over a field K for $n = 4$.

When K is a field and $n = 4$, local automorphisms and local derivations of the $\Lambda(R)$ are described. In particular,

Theorem 1. *Let K be a field and $R = NT(4, K)$. Then every local derivation of $\Lambda(R)$ is a sum of a local derivation of R, of a derivation of $\Lambda(R)$ and of the local derivations $\delta_{31,t}$, $\delta_{42,t}$ and ρ_k : $\alpha \to \alpha + a_{43}ke_{42}$ $(\alpha \in R)$ for $k \in K$.*

The abstract is supported by grants RFFI and Minobr. (the codes 12-01-00968 and 1.34.11).

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Betti series of the universal modules of second order derivations

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Let R be a coordinate ring of an affine irreducible curve represented by $\frac{k[x_1,x_2,\ldots,x_s]}{(f)}$ and m be a maximal ideal of R. In this article, Betti series of $\Omega_2(R_m)$ is studied. We proved that the Betti series of $\Omega_2(R_m)$ where $\Omega_2(R_m)$ denotes the universal module of second order derivations of R_m is a rational function under some conditions.

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Invariant operators of five-dimensional non-conjugate subalgebras of the Lie algebra of the Poincaré group $P(1,4)$

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Invariant operators (generalized Casimir operators) play an important role in the representation theory of Lie groups (Lie algebras), theory of special functions, theoretical and mathematical physics, theory of differential equations, etc. (see, for example, [2, 2, 3]).

The present report is devoted to the construction of invariant operators for all five-dimensional non-conjugate subalgebras of the Lie algebra of the Poincaré group $P(1,4)$.

At present, we have constructed invariant operators for all five-dimensional non-conjugate subalgebras of the Lie algebra of the group $P(1,4)$, using the results of classification of these subalgebras as well as the invariant operators for all real Lie algebras of dimension up to five, constructed in [4].

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Irreducible modules of Heisenberg algebra and their identities

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Let Φ be a field of zero characteristic. A linear algebra satisfying the identity $(xy)z \equiv (xz)y + x(yz)$ is called a Leibniz algebra. Denote the Heisenberg algebra by $H = \{a, b, c\}$ where $ba = -ab = c$ and another products are zero. Consider $T = \Phi[t]$ as a Lie algebra with zero multiplication. We turn T into a right H-module by setting $f(t)a = f'(t), f(t)b = tf(t), f(t)c = f(t)$.

Denote by $T \leftthreetimes H$ the direct sum of vector spaces T and H with a product $(f + x)(g + y) = f \cdot y + xy$, where $f, g \in T$, $x, y \in H$. Denote by $\widetilde{\mathbf{V}}_3$ the variety of almost polynomial growth, generated by the algebra $T \leftthreetimes H$.

Theorem 1. *Let W be an irreducible infinite - dimensional H-module. Then the Leibniz algebra $W \leftthreetimes H$ generates the variety $\widetilde{\boldsymbol{V}}_3$.*

The result is similar the one established for Lie algeras [1].

The authors were partially supported by RFBR grant 10-01-00209-a.

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Asymptotic behavior of colength of some varieties of Leibniz algebras

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A linear algebra satisfying the identity $(xy)z \equiv (xz)y + x(yz)$ is called a Leibniz algebra. Let $\mathbf{V}$ be a variety of Leibniz algebras over a field of zero characteristic. Denote by $P_n = P_n(\mathbf{V})$ the set of all multilinear polynomials degree n in the relatively free algebra of $\mathbf{V}$ with the free generators $x_1, x_2, \ldots, x_n$. Well known that the P_n is the module of the symmetric group S_n. Let χ_λ be the irreducible character of S_n corresponding to the partition $\lambda \vdash n$. Consider the decomposition of the character $\chi(P_n(\mathbf{V}))$ as a sum of irreducible components $\chi_n(\mathbf{V}) = \chi(P_n(\mathbf{V})) = \sum_{\lambda \vdash n} m_\lambda \chi_\lambda$. The total number of summands $l_n(\mathbf{V}) = \sum_{\lambda \vdash n} m_\lambda$ is called n-th colength of the variety $\mathbf{V}$.

A variety $\mathbf{V}$ is named the variety of associative type or the API-variety, if there exist such number m that $m_\lambda = 0$ for all partitions $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_k)$ with $\lambda_m > m$.

Theorem 1. *Let $\mathbf{M}$ be the API-variety of Leibniz algebras over a field of zero characteristic. Then colength of the variety $\mathbf{M}$ is bounded by polynomial function.*

The authors were partially supported by RFBR grant 10-01-00209-a.

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A method of constructing of orthogonal tuple of multiary operations

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Orthogonal hypercubes are used in affine and projective geometries, design of experiments, error-correcting and error-detecting coding theory, cryptology and theory of (t, m, s)-nets [2]. Every n-ary operation corresponds to n-dimensional hypercube.

k-ary operation g, which is formed by replacing $n-k$ variables in term $f(x_1, \ldots, x_n)$ with some elements from Q, is called *retract* of n-ary operation. Let $\upsilon_1, \ldots, \upsilon_n$ be partial injective transformations of $\overline{0,n}$. n-ary operations $f_1, \ldots, f_n$ will be called *orthogonal of the type* $\bar{\upsilon} := (\upsilon_1, \ldots, \upsilon_n)$ if

$$\{f_i(x_{\upsilon_i 1}, \ldots, x_{\upsilon_i n}) = b_i\}|_{i=1}^n$$

has a unique solution for all $b_1, \ldots, b_n \in Q$. A method of constructing of orthogonal tuple of operations using one quasigroup is given in the following theorem. Another method is described in [1].

Theorem 1. *Every n-tuple ${}^{\sigma_1}g_1, \ldots, {}^{\sigma_n}g_n$, where ${}^{\sigma_i}g_i$ is a parastrophe of i-ary retract of n-ary operation f, $i = 1, 2, \ldots, n$, are orthogonal of the type $\bar{\upsilon} := (\upsilon_1, \ldots, \upsilon_n)$, where*

$$\upsilon_i(k) := \begin{cases} \textit{is not defined, if } x_k = a_k, \\ \sigma_i(k) \textit{ in the other case.} \end{cases}$$

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Reduction of third-order matrices over the rings of stable range 2

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Let R be a nontrivial commutative ring. A ring R is to be a Hermite ring if every 1×2 matrix over R admits diagonal reduction. By a Bezout ring we mean a ring in which all finitely generated ideals are principal. It well know that for any matrix A with elements in a Hermite ring, we can find unimodular matrix U such that AU is triangular.

A ring R is called a ring of stable range 2, if for any elements $a, b, c \in R$ such that $aR + bR + R = R$ there exists elements $x, y \in R$ such that $(a + cx)R + (b + cy)R = R$.

Theorem 1. *Let R be a commutative Bezout ring of stable range 2. Then for any third-order matrix there exists matrices $P, Q \in GL_3(R)$ such that*

$$PAQ = \begin{pmatrix} \varepsilon_1 & 0 & 0 \\ 0 & \varepsilon_2 & 0 \\ a & b & c \end{pmatrix},$$

where $\varepsilon_1 | \varepsilon_2$ and $(a, b) = (\varepsilon_2, a, b, c)$.

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On superextensions of inverse semigroups

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In the talk we shall discuss the algebraic structure of various extensions of an inverse semigroup X.

Theorem 1. *For a semigroup X and its superextension $\lambda(X)$ the following conditions are equivalent:*

(1) *$\lambda(X)$ is a commutative Clifford semigroup;*

(2) *$\lambda(X)$ is an inverse semigroup;*

(3) *the idempotents of the semigroup $\lambda(X)$ commute and $\lambda(X)$ is sub-Clifford or regular in $N_2(X)$;*

(4) *X is a finite commutative Clifford semigroup, isomorphic to one of the following semigroups: C_2, C_3, C_4, $C_2 \times C_2$, $L_2 \times C_2$, $L_1 \sqcup C_2$, L_n, or $C_2 \sqcup L_n$ for some $n \in \omega$.*

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On finite simple groups with n-regular first prime graph component

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The prime graph of a finite group G is an undirected and simple graph which is defined as follows: the vertex set of this graph is the prime divisors of $|G|$ and two distinct vertices p and q are adjacent by an edge if G contains an element of order pq. Up to now, the concept of prime graph has been studied from both group theory and graph theory viewpoints. From the graph theory view, in [1–3], the simple groups which their first prime graph components are n-regular, where $0 \leq n \leq 2$, have been obtained. Here, we extend these results for an arbitrary n. In fact, we prove the following main theorem:

Theorem 1. *Let G be a finite nonabelian simple group. If the first prime graph component of G is n-regular, then G is a group with complete prime graph components.*

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Interassociates of the free commutative semigroup

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Let $(S, \cdot)$ and $(S, \circ)$ be semigroups defined on the same set. $(S, \circ)$ is called an interassociate of $(S, \cdot)$ if the next identities are satisfied: $a \cdot (b \circ c) = (a \cdot b) \circ c$ and $a \circ (b \cdot c) = (a \circ b) \cdot c$.

The term of an interassociativity was introdused by Zupnik in [3]. Gould, Linton and Nelson [2] described all interassociates of monogenic semigroups. Givens, Linton, Rosin and Dishman [1] solved the same problem for the free commutative semigroup on n generators.

Let X be a countable infinite set. We denote the free commutative semigroup on X by $FC(X)$. If $x \in FC(X)$, then a variant of $FC(X)$ is a semigroup on $FC(X)$ with multiplication $*_x$ defined by $a *_x b = axb$.

Theorem 1. *Every interassociate of $FC(X)$ coincides with $FC(X)$ or can be written as its variant.*

Theorem 2. $(FC(X), *_{x_1^{\alpha_1} \ldots x_n^{\alpha_n}}) \cong (FC(X), *_{y_1^{\beta_1} \ldots y_m^{\beta_m}})$ *if and only if $n = m$ and $\{\alpha_i\}_{i=1}^{n}$ is a permutation of $\{\beta_j\}_{j=1}^{m}$.*

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Action of a fuzzy semigroup on a fuzzy set

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Universal algebras having a semigroup (monoid or group) S of unary operations have always been of interest of mathematicians, specially to computer scientist and logicians. The algebraic structure so obtained is called S-act (or S-set). Here we are going to define the notion of the action of a fuzzy semigroup on a fuzzy set, $S^{(\mu)}$-set, and we give some characterization for fuzzy $S^{(\mu)}$-sets. Also we investigate some categorical properties of fuzzy $S^{(\mu)}$-sets.

Moreover extension principle, which enables us to extend every (universal) algebraic operation on an algebra A to an operation on the fuzzy subsets of A, is one of the most important tools in fuzzy set theory and fuzzy algebras. So we study the extension principle for the category of fuzzy S-acts.

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Quotient T-S-bi-acts based on congruence relations

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In this paper, S and T will denote monoids, that is, a semigroup with an identity element 1. Let S and M be non-empty sets and S a monoid. Then, M is called a *right S-act* and denoted by M_S, if there exists a mapping $M \times S \longrightarrow M$ written as $(m, s) \longrightarrow ms$ satisfying $m \cdot 1_S = m$ and $m(st) = (ms)t$ for all $m \in M$ and $s, t \in S$. Similarly, we can define *left S-acts*. Let X be a non-empty subset of M_S. If $xs \in X$ for all $s \in S$ and $x \in X$; alternatively, if $XS \subseteq X$, then X is called a *sub-act* of M_S. Similarly, we can define sub-acts for a left T-act ${}_TM$. In this paper, we consider the notion of congruence relation on T-S-bi-acts and prove some related results on sub-bi-acts. Specially, we prove isomorphism theorems on T-S-bi-acts and product of T-S-bi-acts based on congruence relations and discuss on some commutative diagrams of T-S-bi-acts.

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Groups with normalizer condition

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This talk will be a walk through the development of results on groups satisfying the normalizer condition.

Ali JAFARI

Introducing matrices defining elements for n-polytope and representing a number of Lie groups via them

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This paper introduces matrices representing elements for simplices and hypercubes. For each case, using the matrices, certain separate Lie group and its Lie Algebra have been defined. Then a Lie group and its dependent Lie Algebra have been defined for which the above-mentioned groups are subgroups of this Lie group. One of the characteristics of this Lie group is that it defines a collection of matrices in which non-integer powers are well-definable for them. Afterwards, Euler's polyhedron formula on the relation of a polytope's elements in 3-dimensional space was generalized into n-dimensional space. And then a Transformation Matrix was introduced which transforms the Simplex Matrix into Hypercube Matrix. By consecutive applying of the Transformation Matrix on successively resulting polytope matrices, a set of new polytope matrices were represented. Eventually by the use of the Transformation Matrix, a Lie group and its Lie Algebra were introduced. [1, 2].

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Introducing a new method for proving Goldbach's conjecture

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In this paper, a new model is represented in order to present Natural Numbers by their unique characteristics in Prime Number modules; and the uniqueness of each number's characteristic in a certain interval is proved. In this model "Congruence Matrices" are introduced and their specific characteristics are described. Then the intervals, through which the numbers' characteristics are periodically repeated, become introduced. In the mentioned model another new concept "Congruence Trees" are also introduced for every single number so as to group Congruent Numbers into Prime Numbers' modules. After that, the specific characteristics of Congruence Trees and their relation to Congruence Matrices are explained. Then on the basis of proposed model, the necessary and sufficient conditions of being Prime Number is explained and proved. According to this model, Goldbach's Conjecture is redefined and proved. [1, 2].

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Fractal properties of sets of real numbers represented by the alternating Lüroth series with a ban on combinations of symbols

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It is known [1] that any real number $x \in (0,1]$ can be represented in the form of alternating Lüroth series
$x = \frac{1}{a_1} + \sum\limits_{n=2}^{\infty} \frac{(-1)^{n-1}}{a_1(a_1+1)\dots a_{n-1}(a_{n-1}+1)a_n} \equiv \Delta^{\widetilde{L}}_{a_1\dots a_n\dots}, \; \forall a_n \in N.$

In the report we offer results of studying of sets of real numbers real numbers with different restrictions on usage of $\widetilde{L}$-symbols. In particular, we consider the set $D[\widetilde{L}, \overline{c_1c_2}]$. This is a set of real numbers represented by the alternating Lüroth series with a ban on a combination of given symbols c_1 and c_2 that

$$D[\widetilde{L}, \overline{c_1c_2}] = \{x : \; x = \Delta^{\widetilde{L}}_{a_1\dots a_n\dots}, \; a_k a_{k+1} \neq c_1c_2, \; \forall k \in N\}.$$

Theorem 1. *The set $D[\widetilde{L}, \overline{c_1c_2}]$ is a nowhere dense and N-self-similar set, moreover, its Hausdorff-Besicovitch dimension is a solution of equation:*
1. $\left(1 + \left(\frac{1}{c_1(c_1+1)}\right)^x\right) \sum\limits_{i \neq c_1} \left(\frac{1}{c_i(c_i+1)}\right)^x = 1$, *if* $c_1 = c_2$;
2. $\left(\frac{c_1(c_1+1)+c_2(c_2+1)-1}{c_1(c_1+1)c_2(c_2+1)}\right)^x + \sum\limits_{c_1 \neq i \neq c_2} \left(\frac{1}{c_i(c_i+1)}\right)^x = 1$, *if* $c_1 \neq c_2$.

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Denis KIRILYUK

n-ary analog of the theorem about central symmetry of line

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It should be noted, that S. A. Rusakov with the help of concepts of parallelogramme working in n-ary group, symmetry of points and vectors, has identified n-ary rs-group and has proved its existence and has built affine space W (G) with the method of fundamental sequences of vectors semiabelian rs-groups G [1].

The presented result has been received on basis of the technology of the research of the objects of the affine geometry on n-ary group developed by S. A. Rusakov and his apprentice Yu. I. Kulazhenko [2]. The received theorem is not only n-ary analog of the theorem of the central symmetry of a line, but it's also a criteria of semiabelian n-ary group G.

The main denominations and definitions which were used can be found in [1].

Theorem 1. *Let a,b,c,o be arbitrary points of G, t – number of $\mathbb{R}$. An n-ary group G is semiabelian if and only if for any point $d \in G$ such that $\overrightarrow{od} = \overrightarrow{oa} + t\overrightarrow{ab}$, the identity $\overrightarrow{oS_c(d)} = \overrightarrow{oS_c(a)} + t\overrightarrow{S_c(a)S_c(b)}$ is satisfied.*

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Groups containing an element with inert centralizer

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A subgroup H of a group G is called an inert subgroup of G, if for all $g \in G$, index $|H : H^g|$ is finite.

Theorem 1. *Let G be a group, $a \in G$, $\left|C_G(a) \cap a^G\right| < \infty$ and all subgroups $\langle a, a^g \rangle$ are nilpotent groups. If $C_G(a)$ is inert subgroup in G, then normal clousure $\left\langle a^G \right\rangle$ is a locally nilpotent group.*

Let G be a group and X — a conjugacy class of G. The commuting graph on X has vertex set X and an edge joining $x, y \in X$ whenever $xy = yx$. A conjugacy class X of a group G is connected if the commuting graph on X is connected.

Theorem 2. *Let G be a group G, $a \in G$, $a \neq 1$, $\left|C_G(a) \cap a^G\right| < \infty$ and all subgroups $\langle a, a^g \rangle$ are finite p-groups. If the conjugacy class a^G is connected, then the normal closure $H = \left\langle a^G \right\rangle$ is a locally finite p-group, containing a nonidentity abelian normal in H subgroup and factor-group $H/C_H(A)$ is a Fitting locally finite p-group.*

This work was supported by the Russian Foundation for Basic Research (project no. 09-01-00395-a, 10-01-00509-a).

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On topological, metric and fractal properties of a set of abnormal on the basis of 3 numbers

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From the beginning of the twentieth century investigations being conducted to study sets of real numbers defined in terms of frequency of their representation in a given scale. Well known are the sets of normal numbers. Properties of the sets were studied by Borel, Lebesgue, Besicovitch, Eggleston, Billingsley, Olsen, Pratsiovytyi, Torbin, Goncharenko etc.

Some analogue of frequency of number is its asymptotic average of sum of digits. We are interested in sets of numbers having prespecified asymptotic average of sum of digits and sets having not ones. It is a properties of last ones that the report is devoted. Namely, we regard a set

$$E = \left\{ x : \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \alpha_i(x) \text{ does not exist} \right\},$$

where $x \in [0;1]$, and $\{0,1,2\} \ni \alpha_i$ – is ternary digits of representation of number x.

Theorem 1. *The set E of numbers $[0;1]$ is continual, everywhere dense, everywhere discontinuous, superfractal set i.e. its Hausdorff-Besicovitch dimension $\alpha_0(E) = 1$. It has zero Lebesgue measure.*

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On prime radicals of $\mathcal{K}$-ordered algebras

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Let F be a (partially ordered) *po*-field [1] and $A = \langle A; +; \cdot \rangle$ is a linear algebra over a field F. An algebra A over a *po*-field F is called an algebra with the $\mathcal{K}$-order $\leqslant$ if the following conditions hold: $\langle A; +; \leqslant \rangle$ is a partially ordered group [1, 2]; if $a \leqslant b$, then $\lambda a \leqslant \lambda b$ for all $a, b \in A$ and $\lambda > 0$, $\lambda \in F$; from $0 \leqslant a$ it follows that $ab \leqslant a$ and $ba \leqslant a$ for any elements $a, b \in A$. An algebra A over a field F is called a *linearly $\mathcal{K}$-ordered (a lattice $\mathcal{K}$-ordered) algebra* if $\langle A; +; \leqslant \rangle$ is a linearly ordered (a lattice ordered) group. The *l-prime radical* of a lattice $\mathcal{K}$-ordered algebra A over a *po*-field F is the intersection of all l-ideals J of A such that $UV = \{z = \sum_{i=1}^{n=n(z)} x_i y_i \mid x_i \in U, y_i \in V\} \neq \{J\}$ for any nonzero l-ideals U and V in A/J.

Theorem 1. *The l-prime radical of a finite–dimensional linearly $\mathcal{K}$-ordered algebra over a linearly ordered field is equal to this algebra.*

Theorem 2. *The prime radical of a lattice $\mathcal{K}$-ordered associative algebra (a Lie algebra) over a partially ordered field contains the l-prime radical of this algebra.*

Theorem 3. *The l-prime radical and the prime radical of a finite–dimensional linearly $\mathcal{K}$-ordered associative algebra (a Lie algebra) are equal to this algebra. Moreover, the l-prime radical of a finite–dimensional linearly $\mathcal{K}$-ordered associative algebra is equal to the Jacobson's radical.*

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Rings on pure injective Abelian groups

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A ring, whose additive group is isomorphic to an Abelian group G is called a ring on G. An absolute ideal of an Abelian group G is a subgroup of G, which is an ideal in every ring on G. The principle absolute ideal of an Abelian group G, generated by an element g is the minimal among all absolute ideals of G containing g. We will call an Abelian group RAI-group if it admits a ring structure, in which every ideal is absolute. The problem of describing RAI-group was formulated in [1, problem 93]. It is easy to see that every fully invariant subgroup of an Abelian group is its absolute ideal. But the inverse is not true. An Abelian group is called afi-group if only its absolute ideals are fully invariant subgroups. The problem of describing afi-group was formulated in [3].

In this work, descriptions of principle absolute ideals, RAI-groups and afi-groups in the class of reduced algebraically compact Abelian groups are given.

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Induced representations of infinite-dimensional groups

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The *induced representations* were introduced and studied for a *finite groups* by Frobenius. The notion of induced representations was generalized for a *locally compact groups* by Mackey [1] in 1950. In 1962 Kirillov developed his *orbit methods* [2] for classification of all unitary irreducible representations of finite-dimensional nilpotent Lie groups. The *induced representation* $\mathrm{Ind}_H^G S$ of a locally compact group G is the unitary representation of the group G associated with unitary representation $S : H \to U(V)$ of a subgroup H of the group G.

Our aim is *to develop* the concept of induced representations *for infinite-dimensional groups* and *the orbit method*. The induced representations for infinite-dimensional groups in not unique, as in the case of a locally compact groups. It depends on two completions $\tilde{H}$ and $\tilde{G}$ of the subgroup H and the group G, on an extension $\tilde{S} : \tilde{H} \to U(V)$ of the representation $S : H \to U(V)$ and on a choice of the G-quasi-invariant measure μ on an appropriate completion $\tilde{X} = \tilde{H}\backslash\tilde{G}$ of the space $H\backslash G$. As the illustration we consider the “nilpotent” group $B_0^{\mathbb{Z}}$ of infinite in both directions upper triangular matrices and the induced representation corresponding to the so-called *generic orbits*.

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Finite groups with K-$\mathfrak{U}$-subnormal n-maximal subgroups

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All groups considered are finite.

Recall that a subgroup H of a group G is called a 2-maximal subgroup of G if there exists such maximal subgroup M of G that H is a maximal subgroup of M. Similarly we can define 3-maximal subgroups and so on. Let $\mathfrak{F}$ be a class of groups. Then a subgroup H of a group G is said to be $\mathfrak{F}$-subnormal (in sense Kegel [1]) or K-$\mathfrak{F}$-subnormal [2] in G if either $H = G$ or there exists a chain of subgroups $H = H_0 < H_1 < \ldots < H_t = G$ such that either H_{i-1} is normal in H_i or $H_i/(H_{i-1})_{H_i} \in \mathfrak{F}$ for all $i = 1, \ldots, t$.

We study groups in which every n-maximal subgroup is K-$\mathfrak{U}$-subnormal, where $\mathfrak{U}$ is the class of all supersoluble groups. In particular, a full discription of such groups G in the case when $|\pi(G)| \geq n+1$ have been found, and it is proved that in the case when $|\pi(G)| > n+1$, G is supersoluble.

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Classification of all parastrophic identities of the type (3;2) on quasigroups

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Let $(Q;\cdot)$ be a quasigroup. An operation $(\overset{\sigma}{\cdot})$ is called *parastrophic* to $(\cdot)$, if $x_{1\sigma} \overset{\sigma}{\cdot} x_{2\sigma} = x_{3\sigma} \;\Leftrightarrow\; x_1 \cdot x_2 = x_3$, where $\sigma \in S_3$. An identity $\omega = \upsilon$ is said to be *parastrophic*, if its operations are pairwise parastrophic. We say that an *identity has a type* $(3;2)$ if it has two variables with appearances 3 and 2. V.D. Belousov [1] classified these identities but without subterms like $z \overset{\sigma}{\cdot} z$. He proved that there exist seven parastrophic identities, namely (i)-(vii) (see below).

Theorem 1. *Fulfillment of a parastrophic identity of the type (3;2) in a quasigroup $(Q;\circ)$ means that one of its parastrophes $(Q;\cdot)$ satisfies at least one collection of the following identities:*

(i)	$x(x \cdot xy) = y,$	(ix)	$x^2 = x,\; yx \cdot y = x,$
(ii)	$x(y \cdot yx) = y,$	(x)	$yx^2 \cdot y = x,$
(iii)	$x \cdot xy = yx,$	(xi)	$x = y,$
(iv)	$xy \cdot x = y \cdot xy,$	(xii)	$x^2 = x,$
(v)	$xy \cdot yx = y,$	(xiii)	$ex = x,\; x^2 \cdot x = e,$
(vi)	$xy \cdot y = x \cdot xy,$	(xiv)	$ex = x,\; x \cdot x^2 = e,$
(vii)	$yx \cdot xy = y,$	(xv)	$x \cdot yx^2 = y,$
(viii)	$x^2 = x,\; xy = yx,$	(xvi)	$x \cdot x^2 y = y.$

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On integral p-adic representations of cyclic p-group

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Let G be a finite group, R be a commutative ring with identity and $GL(n,R)$ be a general linear group over the ring R, $n \in \mathbb{N}$. We say (see [1]), that matrix representations $\Gamma : G \to GL(n,R)$ and $\Delta : G \to GL(n,R)$ of the group G over the ring R are generally equivalent if there exists an automorphism φ of the group G and a matrix $C \in GL(n,R)$ such that $C^{-1}\Gamma(g)C = \Delta(\varphi(g))$ for all $g \in G$.

Using [2] we have obtained the following result.

Theorem 1. *Let G be a cyclic p-group of the order p^r, where $r \geq 2$ and $\mathbb{Z}_p$ be the ring p-adic integers. Let Λ_1, Λ_2, Λ_3 are the three irreducible pairwise nonequivalent matrix representations of the group G over the ring $\mathbb{Z}_p$. Two indecomposable matrix representation Γ and Δ of the group G over the ring $\mathbb{Z}_p$ which contains precisely three irreducible components Λ_1, Λ_2, Λ_3 are generally equivalent if and only if they are simply equivalent.*

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The additive group of a Lie nilpotent associative ring

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Let $\mathbb{Z}\langle X\rangle$ be the free unitary associative ring freely generated by an infinite countable set $X = \{x_1, x_2, \dots\}$. Define a left-normed commutator $[x_1, x_2, \dots, x_n]$ by $[a, b] = ab - ba$, $[a, b, c] = [[a, b], c]$. For $n \geq 2$, let $T^{(n)}$ be the ideal in $\mathbb{Z}\langle X\rangle$ generated by all commutators $[a_1, a_2, \dots, a_n]$ ($a_i \in \mathbb{Z}\langle X\rangle$). It can be easily seen that the additive group of the quotient ring $\mathbb{Z}\langle X\rangle/T^{(2)}$ is a free abelian group. Recently Bhupatiraju, Etingof, Jordan, Kuszmaul and Li [1] have noted that the additive group of $\mathbb{Z}\langle X\rangle/T^{(3)}$ is free abelian as well. In the present note we show that this is not the case for $\mathbb{Z}\langle X\rangle/T^{(4)}$. More precisely, let $T^{(3,2)}$ be the ideal in $\mathbb{Z}\langle X\rangle$ generated by $T^{(4)}$ together with all elements $[a_1, a_2, a_3][a_4, a_5]$ ($a_i \in \mathbb{Z}\langle X\rangle$). We prove that $T^{(3,2)}/T^{(4)}$ is a non-trivial elementary abelian 3-group and the additive group of $\mathbb{Z}\langle X\rangle/T^{(3,2)}$ is free abelian.

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Serial group rings of A_n and S_n over fields

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A module M over a ring R is said to be *uniserial*, if the lattice of submodules of M is a chain, and M is *serial*, if it is a direct sum of uniserial modules. A ring R is called *serial*, if it is serial as a right and left R-module. An equivalent condition is that there exists a complete system of orthogonal idempotents $e_1, \ldots, e_n \in R$ such that ever right module $e_i R$ is uniserial, and every left module Re_i is uniserial.

For instance, every semisimple artinian ring is serial, and the same is true for the localization $\mathbb{Z}_{(p)}$ of the ring of integers with respect to a prime ideal $p\mathbb{Z}$.

Let G be a finite group and let F be a field of characteristic p. The classical Maschke's theorem says that, if p does not divide the order of G, then $R = FG$ is a semisimple artinian ring, in particular it is serial.

In [1, p. 276, Quest. 7] the following problem was posed. Let F be an algebraically closed field. Are there infinitely many n such that FA_n or FS_n are non-semisimple serial rings?

The following result answers this question in negative for an arbitrary field F.

Theorem 1. *Let F be a field of characteristic p and $n \geq 2p$. Then none of the rings FA_n or FS_n is serial.*

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Y. I. KULAZHENKO

Self-returns with respect to the elements of the succession with an even number of points

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The results [1] and the investigations in [2] enabled the author to introduce the notion of self-return of elements of n-ary groups and to get a number of results in the field [3].

Let us consider an n-ary group G in the light of A. G. Kurosh's definition from [4].

We say that the point p from G self-returns with respect to the elements of the succession of the points $a_1, \dots, a_k$ from G if

$$S_{a_k}(\dots(S_{a_2}(S_{a_1}(p)))\dots) = p.$$

Theorem 1. *Let k be an arbitrary even natural number and*

$$a_1, a_2, \dots, a_k \qquad (*)$$

be a succession of points from G. Any point $p \in G$ self-returns with respect to the elements of the succession $(*)$ if and only if there is at least one point $l \in G$ so that l self-returns with respect to the elements of the succession $(*)$.

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On some generalized nilpotent fuzzy groups

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Recall that if γ, κ are the fuzzy groups on G and $\kappa \preccurlyeq \gamma$, then it is said that κ *is a normal fuzzy subgroup of* γ , if $\kappa(yxy^{-1}) \geq \kappa(x) \wedge \gamma(y)$ for every elements $x, y \in G$ [1, 1.4]. We denote this fact by $\kappa \trianglelefteq \gamma$. We need a following criteria of normality.

Proposition 1. *Let G be a group and γ, κ be the fuzzy groups on G. Suppose that $\kappa \preccurlyeq \gamma$. Then κ is a normal fuzzy subgroup of γ, if and only if*

$$\chi(x, \gamma(x)) \circledcirc \kappa \circledcirc \chi(x^{-1}, \gamma(x)) \preccurlyeq \kappa$$

for every elements $x \in G$.

Let γ, κ be the fuzzy groups on G and $\kappa \preccurlyeq \gamma$. We define a *normalizer $N_\gamma(\kappa)$ of κ in γ* as an union of all fuzzy points $\chi(x, a)$, where $a \leq \gamma(x)$, satisfying the following condition $\chi(x, a) \circledcirc \kappa \circledcirc \chi(x^{-1}, a) \preccurlyeq \kappa$.

Theorem 1. *Let G be a group and γ be a fuzzy group on G. If $N_\gamma(\kappa) \neq \kappa$ for each fuzzy subgroup κ of γ, then every finitely generated fuzzy subgroup of γ is nilpotent.*

This result is a fuzzy analogy of a Plotkin's theorem [2].

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On permutable fuzzy subgroups

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Our goal is to initiate a systematic study of algebraic properties of *an arbitrary fuzzy group* defined on a group G. *Permutability* is one of the main such properties. Let γ and δ two fuzzy groups on G. We define the operation $\odot$ on them by the following $(\gamma \odot \delta)(x) = \bigvee_{y,z \in G, yz=x} (\gamma(y) \wedge \delta(z))$.If $\delta \preceq \gamma$, we say that δ *is permutable in* γ, if $\lambda \odot \delta = \delta \odot \lambda$ for each $\lambda \preceq \gamma$. Let λ be a L-fuzzy group and $a \in L$. The subset $L_a(\lambda) = \{x \mid x \in X \text{ and } \lambda(x) \geq a\}$ is called *the a-level set* of λ. $L_a(\lambda)$ is a subgroup of G for every $a \leq \gamma(e)$. The level sets characterize the fuzzy subgroups as following: γ *is a fuzzy group on G if and only if every non-empty level of* γ *is a subgroup of G.* We obtained the following interesting result**:** *Let G be a group and* γ *be the fuzzy group on G. A fuzzy subgroup* δ *of* γ *is permutable in* γ *if and only if* $L_a(\delta)$ *is a permutable subgroup of* $L_a(\lambda)$ *for every* $a \leq \delta(e)$. This theorem is a significant generalization of the main results of the paper [1].

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Some non-periodic groups whose cyclic subgroups are self conjugate-permutable

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Let G be a group. A subgroup H of G is called *self conjugate-permutable* subgroup, if H satisfies the following condition: if $HH^g = H^gH$ then $H^g = H$ for each element $g \in G$ [1]. A group G is called a *generalized radical*, if G has an ascending series whose factors are locally nilpotent or locally finite.

Theorem 1. *Let G be a non-periodic locally generalized radical group. If every cyclic subgroup of G is self conjugate-permutable, then either G is abelian or $G = R\langle b\rangle$ where R is abelian, $b^2 \in R$, and $a^b = a^{-1}$ for each element $a \in R$. Moreover, in the second case, the following conditions hold:*

(i) if $b^2 = 1$, then the Sylow 2-subgroup D of R (if D is a non-trivial) is elementary abelian;

(ii) if $b^2 \neq 1$, then either subgroup D is elementary abelian or $D = E \times \langle v\rangle$ where E is elementary abelian and $\langle b, v\rangle$ is a quaternion group.

Conversely, if a group G satisfies the above conditions, then every cyclic subgroup of G is self conjugate-permutable.

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The non-periodic groups whose finitely generated subgroups are either permutable or pronormal

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In the papers [1–3] the groups, whose subgroups are either subnormal or pronormal, have been considered. In the paper [4] the authors described the locally finite groups, whose finitely generated subgroups are either permutable or pronormal. Here we consider some infinite groups whose finitely generated subgroups are either permutable or pronormal. The main result of this paper is the following

Theorem 1. *Let G be a locally generalized radical group whose finitely generated subgroups are either pronormal or permutable. If G is non-periodic then every subgroup of G is permutable.*

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Ivan KYRCHEI

Determinantal representations of the Drazin inverse matrix over a quaternion skew field

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Within the framework of theory of the column and row determinants over the quaternion skew field (see, e.g. [1]) we obtain determinantal representations of the Drazin inverse matrix.

Theorem 1. *If* $\mathbf{A} \in \mathrm{M}(n, \mathbb{H})$ *with* $Ind\mathbf{A} = m$ *and* $\mathrm{rank}\mathbf{A}^{m+1} = \mathrm{rank}\mathbf{A}^{m} = r$, *then the Drazin inverse* $\mathbf{A}^{D} = \left(a_{ij}^{D}\right) \in \mathbb{H}^{n\times n}$ *possess the following determinantal representations:*

$$a_{ij}^{D} = \frac{\sum\limits_{\beta \in J_{r,\,n}\{i\}} \mathrm{cdet}_i\left(\left(\mathbf{A}^{m+1}\right)_{.\,i}\left(\mathbf{a}_{.j}^{m}\right)\right)_{\beta}^{\beta}}{\sum\limits_{\beta \in J_{r,\,n}} \left|\left(\mathbf{A}^{m+1}\right)_{\beta}^{\beta}\right|} = \frac{\sum\limits_{\alpha \in I_{r,n}\{j\}} \mathrm{rdet}_j\left((\mathbf{A}^{m+1})_{j\,.}\,(\mathbf{a}_{i.}^{(m)})\right)_{\alpha}^{\alpha}}{\sum\limits_{\alpha \in I_{r,\,n}} |(\mathbf{A}^{m+1})_{\alpha}^{\alpha}|}.$$

We use the notations from [2].

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Lifting of minimal generating sets in wreath products

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We introduce the notion of *lifting condition* for permutational wreath product of two groups. We also prove that if such a product satisfies this condition then it has a minimal generating set. Examples of groups satisfying the lifting condition are given. Particularly, the group of all bijective automaton transformations and the group of all finite bijective automaton transformations over a fixed alphabet with at least two elements are groups with the lifting condition. Therefore these groups have minimal generating sets.

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Zero Divisors of Pentaquaternions

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In [1] pentaquaternion algebra $APQ(\mathbb{R}) = \langle \mathbb{R}^5, * \rangle$ was introduced. Here $*$ is defined with the following table:

	e_1	e_2	e_3	e_4
e_1	$-e_0$	e_3	0	$e_3 - e_0$
e_2	$-e_3$	$-e_0$	e_{12}	$e_{12} - e_0 - e_3$
e_3	0	$-e_{12}$	$-e_0$	$-e_0 - e_{12}$
e_4	$-e_0 - e_3$	$e_3 - e_0 - e_{12}$	$e_{12} - e_0$	$-4e_0$

and e_0 is an identity, $e_{12} := e_1 + e_2$. It was shown in [1] that for $a, b \in APQ(\mathbb{R}) \setminus \{0\}$ equations $a * x = b$ and $x * a = b$ has a solution. Nevertheless there is no contradiction with the dimension's classification of division $\mathbb{R}$-algebras, because $APQ(\mathbb{R})$ contains zero divisors. Below we explore them.

Consider $Z_y = \{x \mid x * y = 0\}$; Z_y is a vector space over $\mathbb{R}$.

Theorem 1. *There exists y such that $\dim Z_y = 0, 1, 2, 3$, but there is no y such that $\dim Z_y > 3$.*

Theorem 2. *If $\dim Z_y = 3$ then $y \in \langle (0, 1, 1, 1, -1) \rangle$.*

The proof relies on the structure of the Gröbner basis of ideal, generated by matrix' minors of the operator $x \mapsto x * y$.

Remark 1 *For $APQ(\mathbb{C})$ Theorem 1 also holds, but now $\dim Z_y = 3$ iff $y \in \langle (0, 1, 1, 1, -1) \rangle \cup \langle (0, 1, 1, \sqrt{2}i, 0) \rangle$.*

Remark 2 *Though $*$ is not commutative the results above also applies to ${}_xZ = \{y \mid x * y = 0\}$.*

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Triangular representations of self-similar groups

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Let p – prime and $Isom\, T_p$ be group of isometries of the regular (infinite) root tree T_p.

In [1] was found representation f_p of the group $Isom\, T_p$ by matrices from the unitriangular (infinite dimensional) group $UT(p)$. Let $M \in UT(p)$ – is bellow triangular matrix. Each diagonal M_i, $i \geq 0$ (bellow of the main diagonal, M_0 – main diagonal) of the matrix M is infinite sequence of elements from the ring $\mathbb{Z}_p$ (i.e. element from $\mathbb{Z}_p^\omega$).

Say (see, [2]), that sequence $\zeta \in \mathbb{Z}_p^\omega$ is p-automatic if it can be represented by Mahler equation.

Group $G \leq Isom\, T_p$ is called self-similar if this group can be generated by the set of finite automatons over alphabet of p elements.

Theorem 1. *Let $G \leq Isom\, T_p$ be self-similar group. Then, for any $g \in G$, matrix $M = f_p(g)$ satisfy p-automatic condition (Mahler equation) for any diagonal $M_i \in \mathbb{Z}_p^\omega$.*

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Yu. LESHCHENKO

On the Diameters of Commuting Graphs of Sylow p-subgroups of Symmetric Groups

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Given a non-abelian group G denote by $Z(G)$ the center of G. The *commuting graph* of G is the graph Γ_G with vertex set $G \setminus Z(G)$ such that two distinct vertices x and y are connected in Γ_G if and only if $xy = yx$. Commuting graphs associated with various classical finite groups were investigated by many authors (e.g. [1]).

This theses deals with the study of commuting graphs of Sylow p-subgroups of symmetric groups.

Let S_n be a symmetric group of degree n and $d(\Gamma)$ denotes the *diameter* of a graph Γ. Then we have the following

Theorem 1. *If P is a Sylow p-subgroup of S_n then*

1) *if $n < p^2$ then P is abelian;*

2) *if $p^2 \leq n < 2p^2$ then Γ_P is disconnected;*

3) *if $n = a_0 + a_1 p + p^k$, where $0 \leq a_0, a_1 < p$ and $k \geq 3$, then Γ_P is connected and $d(\Gamma_P) \leq 4$;*

4) *oterwise Γ_P is connected and $d(\Gamma_P) = 3$.*

Hence, if the commuting graph of the Sylow p-subgroup of S_n is connected, then its diameter is not greater than 4.

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Structural and enumerative questions for some groups and modules

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We consider relatively recent and new results on structural and enumerative questions for the Chevalley algebras and groups of Lie type and, also, for the projective spaces over local rings. See [1] – [4] etc.

The work is supported by the Russian Foundation for Basic Research (grant 12-01-00968).

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On norm of decomposable subgroups in locally finite groups

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Let Σ be the system of all subgroups of a group with some theoretical-group property. Let's remind that Σ-norm of group G is the intersection of normalizers of all subgroups of group G, included in Σ. Authors continue the study of different Σ-norms of group. In this article the connections between the norm N_G^d of decomposable subgroups and the norm N_G^A of Abelian non-cyclic subgroups of group G [1] are discovered in locally finite groups. The subgroup of a group G is named decomposable, if it can be presented as direct product of two proper subgroups [2].

Theorem 1. *The norm N_G^A of Abelian non-cyclic subgroups and the norm N_G^d of decomposable subgroups of a group coinside in the class of locally finite p-groups.*

Theorem 2. *For every locally nilpotent not primary group $N_G^A \supseteq N_G^d$, more over $N_G^A \neq N_G^d$ for finite groups as well as for infinite groups.*

Theorem 3. *Either $N_G^A \supseteq N_G^d$ or $N_G^A \subseteq N_G^d$ in the class of locally finite not locally nilpotent groups, more over in both cases there are subgroups, for which $N_G^A \neq N_G^d$.*

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D.V. LYTKINA[1], V.D. MAZUROV[2]

Groups with given properties of finite subgroups

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It was proven in [1] that a 2-group G is locally finite if every finite subgroup of G is nilpotent of class 2. Extending this result we prove the following

Theorem 1. *Let G be a group such that every finite subgroup of G generated by a pair of 2-elements is of exponent 4 or is nilpotent of class two. Then all 2-elements of G compose a locally finite subgroup.*

Supported by the Natural Science Foundation of China (grant no. 11171364), Russian Foundation of Basic Researches (grants no. 11-01-00456, 11-01-91158,12-01-90006), the Federal Target Grant "Scientific and educational personnel of innovation Russia" for 2009-2013 (government contract no. 14.740.11.0346) and the Program of Siberian Branch of RAS on Partnership for 2012-2014 (project no. 14).

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Homology of H_v-modules

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In this note, the notions of projective and injective H_v-modules in the category of H_v-modules over a commutative H_v-ring R are introduced. In this regards by considering the notion of H_v-modules, first we introduce projective and injective objects in $R_H - mod$, the category of H_v-modules over R and then we investigate the basic properties of these notions. In particular, we investigate the equivalence conditions for projectivity and injectivity according to exact sequences of H_v-modules and the functor Hom.

O. MAKARCHUK

Numerical structure associated with 2-4th image real numbers

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For any number $x \in [0;1]$ consider the sequence,

$$\alpha_n \in \{0,1,2,3\}, \forall n \in N$$

such, that

$$x = \sum_{j=1}^{\infty} \frac{\alpha_j}{2^j} = \Delta^2_{\alpha_1 \alpha_2 \ldots \alpha_n \ldots}.$$

The last equality is called 2-4th image real numbers x.

Let n be a positive integer, m be a non-negative integer does not exceeding $3 \cdot (2^n - 1)$, $||\alpha_{ij}||$ be a $k \times n$-matrix, whose elements belong to alphabet $A = \{0,1,2,3\}$, moreover, for each $i \in \{1,...,k\}$ the following condition holds:

$$\sum_{j=1}^{n} \frac{\alpha_{ij}}{2^j} = \frac{m}{2^n}.$$

For stochastic vector $\bar{p} = (p_0, p_1, p_2, p_3)$, let us define the value (number)

$$S^{\bar{p}}_{m,n} = \sum_{i=1}^{k} \prod_{j=1}^{n} p_{\alpha_{ij}},$$

which we will call the sum of all products.

Let $S^{\bar{p}}_{-1,n} = 0 = S^{\bar{p}}_{-2,n}$ for any positive integer n.

We study the properties of $S^{\bar{p}}_{m,n}$ and distribution of the random variable represented by binary fraction with two redundant digits 2 and 3.

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On centralizers of elements in the Lie algebra $W_2(K)$

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Let K be an algebraically closed field of characteristic zero and $K[x, y]$ be the polynomial ring in two variables over K. We denote by $W_2(K)$ the Lie algebra of all K-derivations of the polynomial ring $K[x, y]$. This Lie algebra carries very important information about the polynomial ring $K[x, y]$ and was studied by many authors from different points of view (see, for example, a survey in [1]). The structure of centralizers of elements and maximal abelian subalgebras in the subalgebra $sa_2(K) \subseteq W_2(K)$ was described in [2] in terms of closed polynomials ($sa_2(K)$ consists of all derivations of $K[x, y]$ with zero divergence). We give a description of centralizers of elements of the Lie algebra $W_2(K)$. For a given element $D \in W_2(K)$ the structure of its centralizer in $W_2(K)$ depends on the kernel $KerD$ which consists of all elements $f \in K[x, y]$ such that $D(f) = 0$.

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[2] Petravchuk A., Iena O., *On centralizers of elements in the Lie algebra of the special Cremona group $SA_2(K)$,* J. Lie Theory, **16**, no. 3 (2006), 561–567.

O.A. MALYUSHEVA

New example of variety of Lie algebras with fractional exponent

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We study numerical characteristic of Lie algebras varieties over a field of characteristic zero. Let $\mathbf{A}^2$ be a variety of Lie algebras determined by the identity $(x_1x_2)(x_3x_4) \equiv 0$ and $M = F_4(\mathbf{A}^2)$ be a relatively-free algebra of this variety over the set of free generators $\{z_1, z_2, z_3, z_4\}$. Consider linear transformation d of the four dimensional vector space $< z_1, z_2, z_3, z_4 >$ determined by the rule $z_1d = z_2$, $z_2d = z_3$, $z_3d = z_4$, $z_4d = z_1$. In this case d may be continued to a derivation of all algebra.

Let $D =< d >$ be a one dimensional Lie algebra with zero multiplication. So we may built a semidirect product $M \leftthreetimes D$ of algebras M and D.

In the paper [1] for first example of Lie algebra variety with fractional exponent was determined it value. Using similar ideas was proved new result.

Theorem 1. *For the Lie algebra variety $V = var(M \leftthreetimes D)$ over field of zero characteristic the following equality hold*

$$LEXP(\mathbf{V}) = HEXP(\mathbf{V}) \approx 3.83.$$

[1] Mishchenko S.P., Zaicev M.V. and Verevkin A.B. *On sufficient condition for existance of the exponent of the linear algebras variety.* M.: Vestnik Moskov. Univ. Ser. I Mat. Mekh., №**2**, (2011), pp. 36 - 39.

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The infinite series of Lie algebras variety with different fractional exponents

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Let A be an algebra over a field of characteristic 0. A natural and well established way of measuring the polynomial identities satisfied by A is through the study of the asymptotic behavior of it's sequence of codimensions $c_n(A)$, $n = 1, 2, \dots$. More precisely, if $F\{X\}$ is the free algebra on a countable set $X = \{x_1, x_2, \dots\}$ and P_n is the space of multilinear polynomials in the first n variables, $c_n(A)$ is the dimension of P_n modulo the polynomial identities satisfied by A.

For any algebra with sequence of codimensions $\{c_n(A)\}_{n\geq 1}$ let us define the PI-exponent of the algebra A as $\exp(A) = \lim_{n\to\infty} \sqrt[n]{c_n(A)}$ in case such limit exists.

Let $M_k = F_k(\mathbf{A}^2)$ be a relatively-free algebra of metabelian variety $\mathbf{A}^2$ over the set of free generators $\{z_1, z_2, \dots, z_k\}$ and d_k be a derivation of M_k determined by the rules $z_s d = z_{s+1}$, $s = 1, 2 \dots, k-1$, and $z_k d = z_1$.

We may built a semidirect product $L_k = M_k \leftthreetimes < d_k >$ of Lie algebra M_k and one dimensional Lie algebra $< d_k >$.

Theorem 1. *There exist a PI-exponent of the algebra L_k, $k \in \mathbb{N}$ and moreover the following strict inequalities hold*

$$3 < \exp(L_3) < \exp(L_4) < \dots < \exp(L_s) < \exp(L_{s+1}) < \dots < 4.$$

The authors were partially supported by RFBR grant 10-01-00209-a.

R. MARKOV

On characterization of regular semirings by Pierce chains

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Let BS be a ring of all central complemented idempotents (CCI) of semiring S, let $MaxBS$ be a space of maximal ideals in BS. A congruence ρ_M in semiring S called by **Pierce congruence** if for $a, b \in S$ then $a \equiv b(\rho_M) \Leftrightarrow ae^{\perp} = be^{\perp}$, where $e^{\perp}$ is the supplement for some CCI $e \in M$. Factor semiring S/ρ_M called by **Pierce stalk** of semiring S. A semiring S, which has no non-trivial direct summands is called by **indecomposable**. If S/ρ is indecomposable and for all congruences $\rho' < \rho$ factor semiring S/ρ' is decomposable, then S/ρ called by ***mi*-factor** of semiring S. A semiring S is called by **strong π-regular**, if for all $a \in S$ exists $x \in S$ and $n \in N$: $a^{n+1}x = xa^{n+1} = a^n$. If for all $a \in S$ the minimal n is limited, then S is called by **limited index semiring (LIS)**.

Theorem 1. *Let S be semiring with finite sets of all Pierce stalks and all mi-factors. Three conditions are equivalent:*

1. *S is strong π-regular LIS;*
2. *all Pierce stalks of S is strong π-regular LIS's;*
3. *all* mi-*factors of S is strong π-regular LIS's.*

[1] Pierce R. S., *Modules over commutative regular rings* // Mem. Amer. Math. Soc. — 1967. — Vol 70. — P. 1–112.

[2] Burgess W. D., Stephenson W., *An analogue of the Pearce sheaf for non-commutative rings*

[3] Chermnykh V. V., *Funktsionalnye predstavleniya polukolets.* – Kirov: VSHU, 2010. – 224 p.

Yuriy MATURIN

Filters in Different Modules

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All rings are considered to be associative with unit $1\neq 0$ and all modules are left unitary.

Let R be a ring and let H be an $R-$module. Consider a radical filter E in R. Set $\Psi_H(E) := \{L|L \leq H, \forall h \in H : \{x|x \in R, xh \in L\} \in E\}$.

Theorem 1. *$\Psi_H(E)$ is a radical filter in H. It is obvious that the function Ψ_H from the lattice of all radical filters in R to the lattice of all radical filters in H is isotone.*

Theorem 2. *Ψ_H is a meet-morphism of lattices. Moreover,*

$$\Psi_H(\inf_{i\in I}\{F_i\}) = \inf_{i\in I}\{\Psi_H(F_i)\}$$

for every non-empty $\{F_i|i \in I\}$ set of filters in R.

Theorem 3. *Let $H = \overset{\infty}{\underset{i=1}{\oplus}} M_i$, where $M_1, ..., M_q, ...$ are non-isomorphic simple $R-$modules. Consider a preradical t_n such that*

$$t_n(D) = \bigcap\{K|K \leq D, Tr_{D/K}(\{M_1, ..., M_n\}) = 0\},$$

where D is an $R-$module. Let $E_n = \{S|S \leq R, t_n(R/S) = R/S\}$. Then

$$\Psi_H(\sup_{n\in \mathrm{N}}\{E_n\}) \neq \sup_{n\in \mathrm{N}}\{\Psi_H(E_n)\}.$$

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On Lattices p-Composition Formations

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All groups considered are finite. All unexplained notations and terminologies are standard (see [1, 2]).

A non-empty formation $\mathfrak{F}$ is called a *p-composition formation* [3] if $c\mathrm{form}(\mathfrak{F}) \subseteq \mathfrak{N}_{p'}\mathfrak{F}$, where p is a prime number.

Let $\mathfrak{F}$ be a p-composition formation. The symbols $L_{c_p}(\mathfrak{F})$ and $L(\mathfrak{F})$ denote the lattice of all p-composition subformations of $\mathfrak{F}$ and lattice of all subformations of $\mathfrak{F}$ respectively.

We prove the following

Theorem 1. *Let $\mathfrak{F}$ be a p-composition formation, $\mathfrak{F} \neq (1)$. Then the following statements are equivalent:*

1) *every atom of $L_{c_p}(\mathfrak{F})$ is complemented in $L(\mathfrak{F})$;*

2) *for any group $G \in \mathfrak{F}$ we have*

$$G = A \times A_1 \times \ldots \times A_t,$$

where A is a nilpotent subgroup of G, $A_1, \ldots, A_t$ are simple non-abelian groups.

[1] Skiba A.N. *Algebra of Formations.* Minsk: Belaruskaya navuka, 1997.

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[3] Shemetkov L.A. Frattini extensions of finite groups and formations, *Communications in Algebra.* **25** (3), (1997), p. 955–964.

Ivanna MELNYK

On differentially prime submodules of differentially multiplication modules

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Associative differential rings with nonzero identity and finite set of pairwise commutative derivations are considered, all the modules are differential, left and unitary.

An R–module M is called *differentially multiplication* if for every differential submodule N of M there exists a differential ideal I of R such that $N = IM$. A differential module M is *differentially multiplication* if $Ann(M) = Ann(N)$ for every differential submodule $N \neq (0)$. A differential submodule N of M is *differentially prime* if M/N is differentially prime.

Theorem 1. *For a differentially multiplication R-module M and its proper submodule N the following conditions are equivalent:*

1. *N is a differentially prime submodule;*
2. *$Ann(M/N)$ is a differentially prime ideal of R (in the sense of Khadjiev);*
3. *$N = \mathfrak{P}M$ for some differentially prime ideal $\mathfrak{P}$ of R such that $AnnM \subseteq \mathfrak{P}$.*

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On the supersolvability of factorizable groups with $\mathbb{P}$-subnormal factors

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The Huppert theorem asserts that a finite group G is supersolvable if and only if every maximal subgroup of G has prime index. It follows that if H is a proper subgroup of a supersolvable group G, then there exists the chain of subgroups

$$H = H_0 \subset H_1 \subset \ldots \subset H_n = G \tag{1}$$

such that $|H_{i+1} : H_i|$ is prime for all i.

Let $\mathbb{P}$ be the set of all prime numbers. A subgroup H of a finite group G is called $\mathbb{P}$-subnormal in G whenever either $H = G$ or there is a chain of subgroups (1) such that $|H_i : H_{i-1}|$ is prime for all i, [1]. In [2] obtained the following result. Let $G = AB$, where A and B are solvable, and suppose that the indices $|G : A|$, $|G : B|$ are prime. Then G is solvable. From here the solvability of the product $G = AB$, where A and B are $\mathbb{P}$-subnormal solvable subgroups follows. Developing this observation we obtained the following theorem.

Theorem 1. *Let the finite group $G = AB$ be the product of the nilpotent subgroups A and B. If A and B are $\mathbb{P}$-subnormal in G, then G is supersolvable.*

[1] Vasilyev A. F., Vasilyeva T. I., Tyutyanov V. N. *Siberian Mathem. J.* **51 (6)**, (2010), p. 1004–1012.

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Denys MOROZOV

Conjugacy of piecewise-linear spheric-transitive automorphisms

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Theorem 1. *If* $x = (x_1, x_2) \circ \sigma,\ y = (y_1, y_2) \circ \sigma$, *then*

$$x \sim y \Leftrightarrow x_1 \circ x_2 \sim y_1 \circ y_2$$

Let us build a sequence of automorphisms $x^{(n)}$ on spheric-transitive automorphism x in the following way:

$$x^{(1)} = x, x^{(n)} = (x_1^{(n)}, x_2^{(n)}) \circ \sigma, x^{(n+1)} = x_1^{(n)} \circ x_2^{(n)}$$

Theorem 2.

$$x \sim y \Leftrightarrow \exists n \in \mathbb{N},\ x^{(n)} \sim y^{(n)}$$

Lemma 1. *If an automorphism* $a \in AutT_2$ *is piecewise-linear, then* $\exists N \in \mathbb{N}, \forall n \geqslant N$ *such, that the value of the function* $Lin^{(n)}(a)$ *is defined.*

Theorem 3. *Piecewise-linear functions* a *and* b *are conjugate in* $FAutT_2$ *if and only if*

$$\exists N \in \mathbb{N},\ Lin^{(N)}(a) = Lin^{(N)}(b)$$

Remark According to the theorem about differentiable finite-state automorphisms [1] and theorems 3 are the conjugacy criteria of differentiable finite-state automorphisms.

[1] Denis Morozov *Differentiable finite-state izometries and izometric polynomials of the ring of integer 2-adic numbers.* 8th International Algebraic Conference July 5 12 (2011), Lugansk, Ukraine.

Jose M. MUÑOZ-ESCOLANO

A finiteness condition for verbal conjugacy classes in a group

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This is a joint work with Pavel Shumyatsky.

A group G is Chernikov if it has a subgroup of finite index that is a direct product of finitely many groups of type C_{p^∞} for various primes p (quasicyclic p-groups).

On the other hand, let w be a word in n variables, and let G be a group. The verbal subgroup $w(G)$ of G determined by w is the subgroup generated by the set G_w consisting of all values $w(g_1, \ldots, g_n)$, where $g_1, \ldots, g_n$ are elements of G. The lower central words γ_k and the derived words δ_k are defined by the positions $\gamma_1 = \delta_0 = x$, $\gamma_{k+1} = [\gamma_k, \gamma_1]$ and $\delta_{k+1} = [\delta_k, \delta_k]$, respectively.

In this talk we present the following theorem.

Theorem 1. *Let k be a positive integer and let w be either the word γ_k or the word δ_k. Suppose that G is a group in which $\langle x^{G_w} \rangle$ is Chernikov for all $x \in G$. Then $\langle x^{w(G)} \rangle$ is Chernikov for all $x \in G$ as well.*

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On Supersolubility of Finite Products of Partially Conjugate-Permutable Subgroups

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All groups under consideration will be finite.

In 1997 T. Foguel [2] introduced the concept of conjugate-permutable subgroup. In this work we introduce a generalization of it.

Definition. *Let R be a subset of a group G. A subgroup H of G is called R-conjugate-permutable if $HH^x = H^xH$ for all x in R.*

Every conjugate-permutable subgroup is subnormal [2]. Let $F(G)$ be the Fitting subgroup of a group G. In the general case $F(G)$-conjugate-permutable subgroup need not to be subnormal (for example Sylow 2-subgroup in the symmetric group S_4).

It is well known ([1], p. 9, 128) that the product $G = AB$ of two normal supersoluble groups A and B is supersoluble if either commutator subgroup G' is nilpotent or $G' = A'B'$.

Theorem 1. *A group G is supersoluble if and only if $G = AB$ is the product of two supersoluble $F(G)$-conjugate-permutable subgroups A and B and either G' is nilpotent or $G' = A'B'$.*

[1] Foguel T. *Conjugate-Permutable Subgroups* Algebra J. 191 (1997), p. 235 - 239 .

[2] Weinstein M. and others. *Between nilpotent and soluble*, Polygonal Publishing House, Passaic, 1980.

Oksana MYKYTSEY

On antiisomorphisms between lattices of Scott continuous mappings

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Let L be a compact Hausdorff Lawson lattice. For a continuous semilattice S with bottom element the set $[S \to L]_0$ of all Scott-continuous mappings from S to L which preserve the bottom elements is a compact Hausdorff Lawson lattice with the Lawson topology.

For a poset X, by $X^{\triangle}$ we denote the *Lawson dual* of X which consists of the non-empty Scott open filters in X ordered by inclusion.

The poset $S^{\top}$ is a continuous semilattice with bottom elment such that its top element is isolated from below.

Then the Lawson dual $(S^{\top})^{\triangle}$ is a continuous semilattice with the top element $S^{\top}$ isolated from below, and with the bottom element $\{\top\}$. Consider a poset $S^{\wedge} = (S^{\top})^{\triangle} \setminus \{S^{\top}\}$.

Theorem 1. *For a compact Hausdorff Lawson lattice L and a continuous semilattice S with a bottom element, the posets $[S \to L]_0$ and $[S^{\wedge} \to \tilde{L}]_0$ are order antiisomorphic.*

The posets $M_{[L]}S^{op}$ and $M_{[\tilde{L}]}(S^{\wedge op})$ are also order antiisomorphic.

[1] Nykyforchyn O., Mykytsey O. *Conjugate measures on semilattices.* Visnyk LNU, Ser. mech.-mat, **72** (2010), 88-99.

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On a generalization of Vedernicov's classes of finite groups

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We consider only finite groups. In [1] V.A. Vedernikov introduced the classes of c-supersoluble, ca-supersoluble and ca-soluble groups. In [2] new properties and applications of these classes of groups were found.

Let J denotes a class (possibly empty) of simple groups. We say that the group G is J-group if the set of all composition factors of the group G is contained in J.

The group G is called:

1) Jca-soluble if every chief J-factor of G, which is abelian, central in G;

2) Jc-supersoluble if every chief J-factor of G is a simple group;

3) Jca-supersoluble if it Jc-supersoluble and every main J-factor of G, which is abelian, central in G;

We use these classes of groups to determine the structure of products of groups. Some of the previous results are generalized.

[1] Vedernikov V.A. *On some classes of finite groups.*. Dokl. Akad. Nauk BSSR, **32**:10, (1988), p. 872–875 (in Russian).

[2] Vasil'ev A.F., Vasilyeva T.I. *On finite groups whose principal factors are simple groups.* UnIsv. Vyssh. Uchebn. Zaved. Mat., **426**:11, (1997), p. 11–14.

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Constructions of RIP quasigroups

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Nuclear extensions are very natural generalizations of central extensions, they have been investigated by many authors and used for different constructions in loop theory.

We study systematically right nuclei of quasigroups obtained by an extension process in the category of quasigroups with right unit. The investigated extensions of quasigroups are defined by a slight modification of non-associative Schreier-type extensions of groups or loops (c.f. [1], [2]); they will be determined by a triple (L, K, f), where L is a loop, K is a quasigroup with right unit and $f : K \times K \to L$ is a function.

We characterize quasigroup extensions satisfying particular nuclear conditions and apply the results to the description of constructions of quasigroups with right inverse property, having a prescribed right nucleus.

We give also a characterization of quasigroups which are isomorphic to an f-extension of a right nuclear normal subgroup by the factor quasigroup.

[1] P. T. Nagy, K. Strambach, Schreier loops, *Czechoslovak Math. J.*, **133**, (2008), pp. 759-786.

[2] P. T. Nagy, I. Stuhl, Right nuclei of quasigroup extensions, *Communications in Algebra*, **40**, (2012), pp. 1893—1900.

[3] P. T. Nagy, I. Stuhl, Quasigroups arisen by right nuclear extensions, accepted in *Comment. Math. Univ. Carolin.*

M.I. NAUMIK

On the Identities on Semigroups of Linear Relations

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Let V be left n–dimensional space over the body F. $LR(V)$ be the semigroup of linear relations (see [1]). All another definitions and notations one can find in [2, 3]. Let $\Gamma \subseteq LR(V)$ be subsemigroup of $LR(V)$. Denote

$$ker\Gamma = \bigcap_{a\in\Gamma} kera, pr_2\Gamma = \sum_{a\in\Gamma} pr_2a, pr_1\Gamma = \bigcap_{a\in\Gamma} pr_1a,$$

$$Coker\Gamma = V/pr_2\Gamma, Coim\Gamma = pr_1\Gamma/ker\Gamma.$$

Theorem 1. *Let V_1 be vector space, $\Gamma \subseteq LR(V_1)$ is set and L is bimodule of V_2. Then ${}^t(\Gamma[V_2])$ and ${}^t(\Gamma[{}^tV_2])$ are canonically similar.*

Theorem 2. *Let $\Gamma \subseteq LR(V)$ be semigroup, W_1 and W_2 are nonempty words, and $x \notin C(W_1W_2)$. If $\Gamma \models W_1x = W_2x$ then $\Gamma[Coim\Gamma] \models W_1 = W_2$.*

Theorem 3. *Let $\Gamma \subseteq LR(V)$ be semigroup, W_1 and W_2 are nonempty words, and $x \notin C(W_1W_2)$. If $\Gamma \models xW_1 = xW_2$ then $\Gamma[pr_2\Gamma] \models W_1 = W_2$.*

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V. NESTERUK

On the Kolyvagin formula for elliptic curves over n-dimensional pseudolocal field

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Let E be an elliptic curve with good reduction over field K, m a positive integer, $(m, char(K)) = 1$, μ_m the group of m^{th} root of 1 in $\overline{K}$, $G_K = \mathrm{Gal}(\overline{K}/K)$ be the absolute Galois group of K, $\mathrm{E}_m(K)$ is group of m-torsion. For all $0 \le i \le 2$ the groups $\mathrm{H}^i(G_K, \mathrm{E}_m(\overline{K}))$ are finite. There are alternating, nondegenerate pairings

$$\mathrm{H}^i(G_K, \mathrm{E}_m(\overline{K})) \times \mathrm{H}^{2-i}(G_K, \mathrm{E}_m(\overline{K})) \to \mathbb{Z}/m\mathbb{Z}.$$

These pairing induce the nondegenerate pairing $\langle \cdot\,, \cdot \rangle\colon\ \mathrm{E}(K)/m\mathrm{E}(K) \times \mathrm{H}^1(G_K, \mathrm{E}(\overline{K}))_m \to \mathbb{Z}/m\mathbb{Z}$ [2], which is called the *Tate pairing*. We consider the relations of the Tate pairing and of the Weil pairing $\{\cdot\,, \cdot\}\colon \mathrm{E}(\overline{K})_m \times \mathrm{E}(\overline{K})_m \to \mu_m$. Let G be a finite group. Fix ξ a generator element of K^*/K^{*m}, which can be identified with primitive m^{th} root of 1, and choose ζ as follows $\zeta = \frac{\sigma(\xi^{1/m})}{\xi^{1/m}}$ (1). As in [1] we associate to elements $c_1 \in \mathrm{E}(K)/m\mathrm{E}(K)$ and $c_2 \in \mathrm{H}^1(G, \mathrm{E}(\overline{K}))_m$ the homomorphisms $\varphi_1\colon \mu_m \to \mathrm{E}_m(K)$, and $\varphi_2\colon \mu_m \to \mathrm{E}_m(K)$. We prove the Kolyvagin's formula $\zeta^{\langle c_1, c_2 \rangle} = \{e_1, e_2\}$ [1] for E with good reduction over n-dimensional pseudolocal field K.

Theorem 1. *Let ζ be primitive root of 1 in K that is chosen in a proper way (1) and $\varphi_1(\pi) = e_1$, $\varphi_2(\xi) = e_2 \in \mathrm{E}_m(\overline{K})$. Then $\zeta^{\langle c_1, c_2 \rangle} = \{e_1, e_2\}$.*

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NIKIFOROV R.[1], TORBIN G.[1,2]

Superfractality of sets of Q_∞-quasi-normal and Q_∞-non-normal numbers

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Let $\Delta^{Q_\infty}_{\alpha_1(x)\alpha_2(x)\ldots\alpha_n(x)\ldots}$ be the Q_∞-expansion of the number $x \in [0,1]$ ([1]), and let $N_i(x,n)$ be the number of digits "i" among the first n digits of the Q_∞-expansion of x. If the limit $\lim\limits_{n\to\infty} \frac{N_i(x,n)}{n}$ exists, then its value $\nu_i^{Q_\infty}(x)$ is said to be the asymptotic frequency of the digit "i" in the Q_∞-expansion of x.

The set

$$W(Q_\infty) = \left\{ x : \forall i \in \mathbb{N}_0, \lim_{n\to\infty} \frac{N_i(x,n)}{n} \text{exists} \wedge \exists i_0 : \nu_{i_0}(x) \neq q_{i_0} \right\}$$

is said to be the set of Q_∞-quasi-normal numbers.

Theorem 1.

$$\dim_H(W(Q_\infty)) = 1.$$

The set

$$D(Q_\infty) = \left\{ x : \exists i_0 : \overline{\lim_{n\to\infty}} \frac{N_{i_0}(x,n)}{n} > \underline{\lim_{n\to\infty}} \frac{N_{i_0}(x,n)}{n} \right\}$$

is said to be the set of Q_∞-non-normal numbers.

Theorem 2.

$$\dim_H(D(Q_\infty)) = 1.$$

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V.Ya. NIKITENKO, M.V. PRATSIOVYTYI

Topological, metric and fractal properties of the set of incomplete products of a given Cantor infinite product

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Let be given Cantor product $\prod\limits_{n=1}^{\infty}(1+\frac{1}{a_n})$.
The *subproduct* of Cantor product is
$\prod\limits_{n\in M\subset N}(1+\frac{1}{a_n})=\prod\limits_{n=1}^{\infty}(1+\frac{\varepsilon_n}{a_n})=\Delta_{\varepsilon_1\varepsilon_2\ldots\varepsilon_n\ldots}$,
where $\varepsilon_n=\begin{cases}1, n\in M,\\ 0, n\notin M.\end{cases}$

The value of the subproduct is called *incomplete product.*

The set of all incomplete products is
$P=\{x : x=\Delta_{\varepsilon_1\varepsilon_2\ldots\varepsilon_n\ldots}, \varepsilon_n\in\{0,1\}\}$.

Theorem 1. *The set of all incomplete products is: 1) nowhere dense; 2) perfect; 3) the set of null Lebesgue measure; 4) the set of null Hausdorff-Besikovich dimension.*

Theorem 2. *If (τ_n) is a sequence of independent random variables taking the values 0 and 1 and having distributions* $P\{\tau_n=i\}=p_{in}\geq 0$, $p_{on}+p_{1n}=1$, *then random variable*

$$\xi=\Delta_{\tau_1\tau_2\ldots\tau_n\ldots}$$

has either purely discrete distribution, when

$$M=\prod_{n=1}^{\infty}\max\{p_{0n},p_{1n}\}>0,$$

or singular of Cantor-type distribution, when $M=0$.

Tsunekazu Nishinaka

Primitivity group rings of one-relator groups with torsion

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Let R be a ring with the identity element. Then R is right primitive if and only if there exists a faithful irreducible right R-module M_R. If a group is finite or abelian, then the group ring of such groups can never be primitive. The first non-trivial example of primitive group ring was offered in 1972. After that, many examples which include the result for primitivity of group rings of free products of non-trivial groups(except $\mathbb{Z}_2 * \mathbb{Z}_2$) settled by Formanek were cnstructed. On the other hand, we do not know whether group rings of many important groups are primitive or not. The present author has recently given the primitivity of group rings of HNN extensions of free groups [2] and the primitivity of group rings of locally free groups [3].

In this talk, we introduce primitivity of group rings of one-relator groups with torsion.

Theorem 1. *Let K be a field, and G a non-cyclic one-relator group with torsion. Then KG is primitive.*

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B. NOVIKOV

Formal series over groups and semigroups

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An unified approach to the Malcev series over linearly ordered groups [1] and to the incidence algebras over arbitrary posets [2] is considered.

[1] L. Fuchs. Partially ordered algebraic systems. Pergamon Press, 1963.

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Andriy OLIYNYK

The Group of Finite State Automorphisms of p-Regular Rooted Tree

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Let p be an odd prime. Denote by $\mathcal{T}_p$ the p-regular rooted tree and by $\mathrm{Aut}\mathcal{T}_p$ the automorphism group of $\mathcal{T}_p$. Then $\mathrm{Aut}\mathcal{T}_p$ is isomorphic to the infinitely iterated wreath product of symmetric groups S_p of degree p. Each automorphism $g \in \mathrm{Aut}\mathcal{T}_p$ admits a unique decomposition of the form $(g_1, \ldots, g_p)\sigma$, where $g_1, \ldots, g_p \in \mathrm{Aut}\mathcal{T}_p$ and $\sigma \in S_p$. The automorphisms $g_1, g_2, \ldots, g_p$ are called the states of the first level of g. The states of the second level of g are the states of the first level of $g_1, g_2, \ldots, g_p$. In this way the states of the k-th level of g for each $k \geq 1$ are defined. An automorphism $g \in \mathrm{Aut}\mathcal{T}_p$ is called finite state automorphism if the set of all its states is finite. All finite state automorphisms form a subgroups $\mathrm{FAut}\mathcal{T}_p$ in $\mathrm{Aut}\mathcal{T}_o$ (for details, see [1]).

Theorem 1. *Let $k_1, \ldots, k_m$ ($m \geq 2$) be positive integers such that each prime divisor of their product is less or equal p. Then the group $\mathrm{FAut}\mathcal{T}_p$ contains a subgroup given by a presentation*

$$\langle x_1, \ldots, x_m | x_1^{k_1} = \ldots = x_m^{k_m} \rangle.$$

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Diagonal Limits of Cubes and Their Isometry groups

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Let τ be a supernatural number, $S(\tau)$ be the homogeneous symmetric group (for the definition of homogeneous symmetric groups see [1]). Each element of the wreath product $S(\tau) \wr C_2$ can be written as an infinite sequence $[\alpha; a_1, a_2, \ldots]$, where $\alpha \in S(\tau)$, $(a_1, a_2, \ldots) \in \{0,1\}^{\mathbb{N}}$. The group $S(\tau) \wr C_2$ contains the subgroup $S(\tau) \wr_\tau C_2$ of all sequences $[\alpha; a_1, a_2, \ldots]$ such that the sequence $(a_1, a_2, \ldots)$ is periodic and its period is a divisor of the supernatural number τ.

Inspired by Cameron-Tarzi space (for details, see [2]) for the supernatural number τ we introduce a countable metric spaces $B(\tau)$ that is the diagonal limit of scaled finite cubes. In the talk we characterize some properties of such spaces and their isometry groups.

Theorem 1. *The group $S(\tau) \wr C_2$ is isomorphic to some proper transitive subgroup $G(\tau)$ of the isometry group $IsomB(\tau)$. For any isometry $f \in IsomB(\tau)$ and point $x \in B(\tau)$ there exists an element $g_x \in G(\tau)$ such that the isometry f acts on x as the transformation g_x.*

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Anna OSINOVSKAYA, Irina SUPRUNENKO

Unipotent elements from subsystem subgroups of type A_3 in representations of the special linear group with locally small highest weights

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Let K be an algebraically closed field of characteristic $p \geq 11$; $G = A_r(K)$, $r > 3$; ω_i, $1 \leq i \leq r$, be the fundamental weights of G; $z \in G$ be a regular unipotent element from a subsystem subgroup of type A_3. For an irreducible representation φ of G with highest weight $a_1\omega_1 + \ldots + a_r\omega_r$ put

$$s(\varphi) = 1 + 3a_1 + 4(a_2 + \ldots + a_{r-1}) + 3a_r.$$

and denote by $J_\varphi(z)$ the set of Jordan block sizes of $\varphi(z)$ without their multiplicities. We describe $J_\varphi(z)$ in the case where a certain linear combination of three consecutive coefficients of the highest weight is smaller than p.

Theorem 1. *Let $p \geq 11$ and φ be a p-restricted irreducible representation of G with highest weight $\omega = a_1\omega_1 + \ldots + a_r\omega_r$. Assume that $3a_i + 4a_{i+1} + 3a_{i+2} < p$ for some $i < r - 1$.*

If $s(\varphi) \leq p$, then $|J_\varphi(z)| \geq s(\varphi) - 4$ and $J_\varphi(z)$ coincides with the corresponding set for the irreducible representation of $A_r(\mathbb{C})$ with the highest weight ω.

If $s(\varphi) > p$, then $|J_\varphi(z)| \geq p - 3$ and

$$\{1, \ldots, p\} \setminus \{2, p-3, p-2, p-1\} \subset J_\varphi(z).$$

This research has been supported by the Belarus Basic Research Foundation, project F10R-110.

Christos PALLIKAROS

Restricting the Weil representation to certain subgroups of the symplectic and unitary groups

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In this joint work with A. E. Zalesski we study the restriction of the Weil representations of symplectic and unitary groups to subgroups that are the centralizers of certain elements, and show that these are multiplicity free. We also obtain an explicit formula for the character of the restriction to the centralizer of a regular unipotent element.

Sergey PANOV

Some semifield planes over a field of odd order

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Using the coordinatized set W and the regular set R [1, 2], we construct some finite semifield planes. A projective plane π is set to be semifield, if W is a semifield.

It is well-known that a projective plane π is Desarguesian when regular set R is a field. It is proved

Theorem 1. *Up to isomorphism there are exactly two semifield planes of order 27, one of plane is Desarguesian and the other is non-Desarguesian.*

Also, for constructed non-Desarguesian plane from *Theorem 1* we satisfy the hypothesis about solvability of the group of collineations.

Theorem 2. *The full group of collineations of non-Desarguesian semifield plane of order 27 is solvable.*

The author were supported by the Russian Foundation for Basic Research (project 12-01-00968)

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Manoj Kumar Patel

$FI-$ Semi Projective Modules and their Endomorphism Rings

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In this paper we have studied the properties of FI-Semi projective module related with generalized Hopfian and variants of supplemented modules. Finally discuss the endomorphism ring of FI-Semi projective Modules.

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A journey into Fermat's equation

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Among the unexplored properties of the binomial expansion with relevant influences in limiting Fermat triples until an almost impossible condition of existence presented at the 5ecm 2008 ([1]) the most promising seemed the criterion of incompatible parities illustrated at the IACONU 2009 ([2]). Such achievement was improved in the talk at IACONU 2011 ([3]). As pointed out in the ICMSA 2011 ([4]) Fermat's equation is an amazing source of algebraic ideas all worthy to be illustrated like in a journey.

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J.V. PETECHUK

Minimal polynomials above the areas of integrity

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Let R_0 - is a subring of the commutative ring R, which is generated by the 1, p - is a prime natural number, $p \notin R^*$, I_p - is maximal ideal of the ring R, which contains pR, $R_p = {}^{R}/_{I_p}$ - is the field of characteristic p, n-th cyclotomic polynomial $\Phi_{\mathrm{n}}(x)$, $n \geq 1$ are determined by the equality
$x^n - 1 = \prod \Phi_d(x)$, d|n.

Polynomial of minimal positive degree of the ring $R_0\,[x]$, the root of which is algebraic above R_0 element of commutative expansion R' of the ring R is named minimal polynomial of this element. Let's notice that if R' - is the area of integrity, then minimal polynomials of algebraic above R_0 element is irreducible above R_0.

Theorem 1. *Let R - is the area of integrity, natural number $\mathrm{k} \neq 0$ in R, ε - is the primitive root of k degree from 1 in the field $\overline{\mathrm{Q(R)}}$, $d \neq 1$ - is deviser k, $f_d(x)$ - is any minimal polynomial of the element $\varepsilon^{\frac{\mathrm{k}}{\mathrm{d}}}$, a_d - is the leading coefficient of $f_d(x)$, P_d - is set of all simple devisors of numbers $0 < s < d$, $(s,d) = 1$. If all the elements P_d are irreversible in R and $\mathrm{a_d} \notin I_p$ for all $p \in P_d$, then $\Phi_d(x)$ - is minimal polynomial of the element $\varepsilon^{\frac{\mathrm{k}}{\mathrm{d}}}$ and $f_d(x) = a_d\Phi_d(x)$, $a_d \in R^*$.*

The statement of the theorem also takes place if $d = 1$.

Vasyl' PETRYCHKOVYCH, Nataliia DZHALIUK

The matrix Diophantine polynomial equations

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Consider the equation

$$A(\lambda)X(\lambda)+B(\lambda)Y(\lambda)=C(\lambda), \tag{1}$$

where $A(\lambda), B(\lambda)$, and $C(\lambda)$ are given, $X(\lambda)$, $Y(\lambda)$ are unknown $n\times n$ matrices over a polynomial ring $\mathcal{F}[\lambda]$, where $\mathcal{F}$ is a field. We investigate the solutions of equations (1).

The equation (1) is equivalent to the equation

$$T^A(\lambda)\widetilde{X}(\lambda)+T^B(\lambda)\widetilde{Y}(\lambda)=T^C(\lambda), \tag{2}$$

where $T^A(\lambda)=QA(\lambda)R^A(\lambda)$, $T^B(\lambda)=QB(\lambda)R^B(\lambda)$, $T^C(\lambda)=QC(\lambda)R^C(\lambda)$ are the triangular forms with invariant factors $\mu_i^A(\lambda)$, $\mu_i^B(\lambda), \mu_i^C(\lambda)$ of the matrices $A(\lambda), B(\lambda), C(\lambda)$ on the principal diagonals with respect to semiscalar equivalence [1].

Theorem 1. *Let the equation* (2) *be solvable and* $\deg\mu_i^C(\lambda)<\deg\mu_i^A(\lambda)+\deg\mu_i^B(\lambda)$. *Then the equation* (2) *have solutions* $\widetilde{X}(\lambda)=\|\widetilde{x}_{ij}(\lambda)\|_1^n$, $\widetilde{Y}(\lambda)=\|\widetilde{y}_{ij}(\lambda)\|_1^n$ *such that* $\deg\widetilde{x}_{ij}(\lambda)<\deg\mu_i^B(\lambda)$ *and* $\deg\widetilde{y}_{ij}(\lambda)<\deg\mu_i^A(\lambda)$. *Such solution is unique if and only if*

$$(\det T^A(\lambda), \det T^B(\lambda))=1.$$

Then
$X(\lambda)=R^A(\lambda)\widetilde{X}(\lambda)(R^C(\lambda))^{-1}$, $Y(\lambda)=R^B(\lambda)\widetilde{Y}(\lambda)(R^C(\lambda))^{-1}$ is the solutions of equation (1).

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Roman POPOVYCH

Large Order Elements in Artin-Schreier Extensions of Finite Fields

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It is well known that the multiplicative group of a finite field is cyclic. A generator of the group is called primitive element. The problem of constructing efficiently a primitive element for a given finite field is notoriously difficult in the computational theory of finite fields. That is why one considers less restrictive question: to find an element with high multiplicative order [1–3]. We are not required to compute the exact order of the element. It is sufficient in this case to obtain a lower bound on the order. High order elements are needed in cryptography, coding theory, pseudo random number generation and combinatorics.

For any prime number p, Artin-Shreier extension of a finite field F_p is the field F_{p^p}. It is known that the polynomial $x^p - x - a$ is irreducible over F_p for any non-zero element a in F_p, and we may take $F_{p^p} = F_p[x]/(x^p - x - a)$. Let $\theta = x(mod(x^p - x - a))$.

We show that for any non-zero element b in F_p the element $\theta + b$ has the multiplicative order at least 4^p.

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Mykola PRATSIOVYTYI

Geometry of real numbers in the representation systems with infinite alphabet is a basis of topological, metric, fractal, and probabilistic theories

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One can develop a theory of real numbers on the different bases, with essential use of the set of rational numbers or without one. One can construct the set of positive real numbers starting from the set or positive integers or nonnegative integers. Classic examples are models of real number by regular continued fraction as well as (positive or alternating) convergent series such that their terms are reciprocal to positive integers. Engel, Sylvester, Pierce, Ostrogradsky series et al. are among them.

Topological, metric, fractal, and probabilistic theories of real numbers in the different representation systems study topological, metric, fractal, and probabilistic properties of sets of real numbers defined by conditions on their representation respectively. The geometry of representation is a basis of these theories. Let us recall that geometry of numbers is a field of number theory studying number theory problems using geometrical notions and methods.

In the talk we study topological, metric, fractal, and probabilistic properties of sets of numbers defined by conditions on their representation in different systems with infinite alphabet. Systems with infinite alphabet generated by systems with finite alphabet and self-similar geometry are discussed in detail.

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On A_s-continued fraction expansion

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Let $A_s = \{\alpha_1, \alpha_2, \ldots, \alpha_s\}$ be a set of real numbers, where $0 < \alpha_1 < \ldots < \alpha_s$ and $s \geq 2$. Define α_1 and α_s from the condition $\alpha_1\alpha_s = \frac{(s-1)^2}{s}$ and let $\alpha_{i+1} = \alpha_i + (\beta_2 - \beta_1)$, $\forall i = \overline{1, s-2}$, where $\beta_1 = \frac{\sqrt{\alpha_1^2\alpha_s^2 + 4\alpha_1\alpha_s} - \alpha_1\alpha_s}{2\alpha_s}$, $\beta_2 = \frac{\sqrt{\alpha_1^2\alpha_s^2 + 4\alpha_1\alpha_s} - \alpha_1\alpha_s}{2\alpha_1}$.

Theorem 1. *The set L_{A_s} of all infinite continued fractions*

$$\cfrac{1}{a_1 + \cfrac{1}{a_2 + \ldots}} \equiv [a_1, a_2, \ldots, a_n, \ldots], \qquad (1)$$

where the elements a_n are in the set A_s, $n = 1, 2, \ldots$, is a closed interval $[\beta_1, \beta_2]$.

Continued fractions of the form (1) are said to be A_s*-continued fractions.*

Theorem 2. *A countable set of numbers from the interval $[\beta_1, \beta_2]$ has two A_s-continued fraction representations, and the rest of numbers of this interval has unique representation.*

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Fractal properties of functions defined in terms of Q-representation

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Let $3 < s$ be a fixed odd positive integer, $A = \{0, 1, \dots, s-1\}$,

$$\gamma(\alpha) = \begin{cases} 0, & \text{if } \alpha = 0, \\ 1, & \text{if } \alpha \in A \setminus \{0, s-1\}, \\ 2, & \text{if } \alpha = s-1. \end{cases}$$

For any sequence $(\alpha_n) \in L \equiv A^\infty = A \times A \times \dots$ define (c_k)

$$c_1 = 0, \qquad c_k = \begin{cases} c_{k-1}, & \text{if } \alpha_{k-1} \in A \setminus \{2, 4, \dots, s-3\}, \\ 1 - c_{k-1}, & \text{if } \alpha_{k-1} \notin A \setminus \{2, 4, \dots, s-3\}. \end{cases}$$

Let us define a function with argument represented by s-adic fraction $x = \frac{\alpha_1}{s} + \frac{\alpha_2}{s^2} + \dots + \frac{\alpha_k}{s^k} + \dots \equiv \Delta^s_{\alpha_1 \alpha_2 \cdots \alpha_k \cdots}$, $\alpha_k \in A$, and value of function has the following Q_3-representation

$$f(x) = \Delta^{Q_3}_{\beta_1 \beta_2 \cdots \beta_k \cdots} \equiv \psi_{\beta_1} + \sum_{i=2}^{\infty} \left[\psi_{\beta_i} \cdot \prod_{j=1}^{i-1} q'_{\beta_j} \right], \ \beta_k \in \{0, 1, 2\},$$

where $q'_i > 0$, $q'_0 + q'_1 + q'_2 = 1$, $\psi_0 = 0$, $\psi_k = \sum_{i=1}^{k-1} q'_i$,

$$\beta_k = \begin{cases} \gamma(\alpha_k), & \text{if } c_k = 0, \\ 2 - \gamma(\alpha_k), & \text{if } c_k \neq 0. \end{cases}$$

Theorem 1. *Function $f(x)$ is continuous and non-differentiable, its graph is a self-affine set, and the following equality holds $\int_0^1 = \frac{1}{2} q_0 + q_1$.*

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Normal form with respect to similarity of a matrix with minimal polynomial $m(\lambda) = (\lambda - \alpha)(\lambda - \beta)$, $(\alpha \neq \beta)$

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Let $M_{n,m}(\mathrm{R})$ be a set of $(n \times m)$-matrices over a principal ideal domain R with identity $e \neq 0$ ([1]); I_n is the $(n \times n)$ identity matrix and $0_{n,m}$ is the $(n \times m)$ zero matrix.

Theorem 1. *Let $A \in M_{n,n}(\mathrm{R})$ be a matrix with characteristic polynomial* $\det(I_n\lambda - A) = (\lambda - \alpha)^k(\lambda - \beta)^{n-k}$*, where $\alpha, \beta \in \mathrm{R}$, $\alpha \neq \beta$, $1 \leq k < n$. If $m(\lambda) = (\lambda - \alpha)(\lambda - \beta)$ – the minimal polynomial of the matrix A, then for matrix A there exists a matrix $T \in GL(n, \mathrm{R})$ such that*

$$TAT^{-1} = \begin{bmatrix} \alpha I_k & 0_{k,n-k} \\ C & \beta I_{n-k} \end{bmatrix},$$

where

$$C = \left[\begin{array}{ccccc|c} c_1 & 0 & \cdots & \cdots & 0 & \\ 0 & c_2 & 0 & \cdots & 0 & \\ \vdots & \vdots & \ddots & \vdots & \vdots & 0_{m,k-m} \\ 0 & \cdots & \cdots & 0 & c_m & \\ \hline \multicolumn{5}{c|}{0_{n-k-m,m}} & 0_{n-k-m,k-m} \end{array}\right] \in M_{n-k,k}(\mathrm{R}),$$

c_j belong to the complete system of residues modulo the ideal $(\alpha - \beta) = \mathrm{R}(\alpha - \beta)$ in which the zero class is represented by the zero of the domain R *and $c_1|c_2|\ldots|c_m$.*

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Algebraic Characterization of Behaviour of Generally Labelled Graphs

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The fundamental result in the finite automata theory is the Kleene's theorem, which states the equivalences between finite automata and regular languages. Extensions of this theorem to more general case of weighted automata are given in [1].

Both weighted automata and vertex-labeled graphs are a special case of the more general model of generally labelled graphs: oriented graphs in which both vertices and transitions are labelled with elements of two arbitrary sets and may model objects of different classes.

In recent years, these graphs have found much interest in theoretical computer science, as well as in several applied areas, such as software engineering, object-oriented modeling, description and verification of protocols, robotics [2].

In this work, we develop the theory of generally labelled graphs as a generalization of weighted automata theory, prove an analog of Kleene's theorem for these graphs and characterize their behaviour.

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On the representation of differentiations in functional rings and their applications in dynamical systems theory

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Take the ring $\mathcal{K} := \mathbb{R}\{\{x,t\}\}$, $(x,t) \in \mathbb{R}^2$, and consider the additional differentiation $D_t : \mathcal{K}\{u\} \to \mathcal{K}\{u\}$, depending on the functional variable u, which satisfies the Lie-algebraic commutator condition $[D_x, D_t] = (D_x u)D_x$ (*) for all $(x,t) \in \mathbb{R}^2$. Impose now on the differentiation (*) a new algebraic constraint $D_t^{N-1}u = (D_x\bar{z})^s, D_t\bar{z} = 0$ for $s, N \in \mathbb{Z}_+$, defining some smooth functional set (or "*manifold*") $\mathcal{M}^{(N)}$ of functions $u \in \mathbb{R}\{\{x,t\}\}$, and which allows to reduce naturally the initial ring $\mathcal{K}\{u\}$ to the basic ring $\mathcal{K}\{u\}|_{\mathcal{M}_{(N)}} \subset \mathbb{R}\{\{x,t\}\}$. Then the following natural problem of constructing the corresponding representation of differentiations above arises: *to find an equivalent linear representation of the reduced differentiation* $D_t|_{\mathcal{M}_{(N)}} : \mathbb{R}^{p(N)}\{\{x,t\}\} \to \mathbb{R}^{p(N)}\{\{x,t\}\}$ *in the functional vector space*

$$\mathbb{R}^{p(N)}\{\{x,t\}\}$$

for some specially chosen integer dimension $p(N) \in \mathbb{Z}_+$. As we will demonstrate, for the cases $s, N = 2, 3$ this problem proves to be completely analytically solvable, giving rise [1] to the corresponding Lax type integrability of the generalized Riemann type hydrodynamical system. Moreover, the same problem is also solvable for the more complicated constraint $D_t u + D_x^{-1}u = 0$, being the Lax type integrable nonlinear Ostrovsky-Vakhnenko dynamical system.

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On local nearrings of order 32

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The list of all local nearrings (Lnr) of order at most 31 can be extracted from the package "Sonata" (version 2.4) [2] of the Computer System Algebra GAP (version 4.4.12) [1]. Based on GAP, we constructed a general algorithm to classify all Lnr of order 32 and more. The Small Groups library in GAP classifies all groups of orders at most 512. Let $[n, i]$ be the i-th group of order n in this library. There are three Miller-Moreno groups of order 32 which are the additive groups of zero-symmetric Lnr, namely $[32, 2]$, $[32, 5]$ and $[32, 12]$ with structures $(C_4 \times C_2) \rtimes C_4$, $(C_8 \times C_2) \rtimes C_2$ and $C_4 \rtimes C_8$, respectively.

Theorem. *There are* 1396 *zero-symmetric Lnr on the group* $[32, 2]$ (368 *with multiplicative group* $[16, 3]$, 198 *with* $[16, 10]$, 764 *with* $[16, 11]$ *and* 66 *with* $[16, 14]$)*;* 1944 *on the group* $[32, 5]$ (548 *with multiplicative group* $[16, 3]$, 232 *with* $[16, 10]$, 1076 *with* $[16, 11]$ *and* 88 *with* $[16, 14]$) *and* 2406 *on the group* $[32, 12]$ (624 *with multiplicative group* $[16, 3]$, 312 *with* $[16, 10]$, 1350 *with* $[16, 11]$ *and* 120 *with* $[16, 14]$).

The library of these nearrings is arranged in 12 archived files each of which does not exceed 100 Kbit. They can be used to obtain any necessary information concerning such nearrings.

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Yu. RALKO

The metric theory of numbers represented by the Cantor series

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Let (d_n) be any sequence of positive integers greater than or equal to 2.

It is known that any number x belonging to $[0,1]$ can be represented by the Cantor series

$$x = \sum_{n=1}^{\infty} \frac{\varepsilon_n}{d_1 \dots d_n} = \Delta^{(d_n)}_{\varepsilon_1\varepsilon_2\dots\varepsilon_n\dots}, \text{ where } \varepsilon_i \in A_i = \{0,1,\dots,d_n-1\}.$$

Let $V_n \subset \{2,3,\dots,n,\dots\}$.

Theorem 1. *If the sequence (d_n) is bounded, i.e., $|V_n| < d_n$ for any n, then the set*

$$C\left[(d_n), V_n\right] = \{x : \varepsilon_n(x) \in V_n \neq \{0,1,\dots,d_n-1\}\}$$

is a nowhere dense set of zero Lebesgue measure, and its Hausdorff-Besicovitch dimension does not exceed

$$\log_d(d_n-1), \ \textit{where } d_n \le d.$$

Let $N_0(x,n)$ be a number of digits 0 in the representation of number x. The number

$$\nu_0(x) = \lim_{n\to\infty} \frac{N_0(x,n)}{n}$$

is called the frequency of digit 0 in the representation of x.

Theorem 2. *If the sequence (d_n) is bounded, then the set of all numbers $x \in [0,1]$ having no frequency of digit 0 is an everywhere dense set of zero Lebesgue measure, and its Hausdorff-Besicovitch dimension is equal to 1.*

S.M. RATSEEV

About Leibniz-Poisson algebras of polynomial growth

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Let $A(+,\cdot,\{,\},K)$ be a K-algebra with two binary multiplications $\cdot$ and $\{,\}$. Let the algebra $A(+,\cdot,K)$ with multiplication $\cdot$ be a commutative associative algebra with unit and let the algebra $A(+,\{,\},K)$ be a Leibniz algebra under the multiplication $\{,\}$. The letter means that $A(+,\{,\},K)$ satisfies the Leibniz identity

$$\{\{x,y\},z\} = \{\{x,z\},y\} + \{x,\{y,z\}\}.$$

Assume that these two operations are connected by the relations $\{a\cdot b,c\} = a\cdot\{b,c\}+\{a,c\}\cdot b$, $\{c,a\cdot b\} = a\cdot\{c,b\}+\{c,a\}\cdot b$, $a,b,c\in A$. Then the algebra $A(+,\cdot,\{,\},K)$ is called a Leibniz-Poisson algebra.

Denote by V_1 the Leibniz-Poisson variety defined by the identities $\{x_1,x_2\}\cdot\{x_3,x_4\} = 0$, $\{\{x_1,x_2\},\{x_3,x_4\}\} = 0$, $\{x,x\} = 0$. Denote by V_2 the Leibniz-Poisson variety defined by the identities $\{x_1,x_2\}\cdot\{x_3,x_4\} = 0$, $\{x_1,\{x_2,x_3\}\} = 0$.

Theorem 1. *In the case of characteristic zero of the base field, varieties V_1 and V_2 have almost polynomial growth.*

Theorem 2. *For a variety V of Leibniz-Poisson over a field of characteristic zero the following conditions are equivalent:*

(i) V has a polynomial growth of the codimension sequence;

(ii) $V_1 \not\subseteq V$, $V_2 \not\subseteq V$ and V satisfies the polynomial identity $\{x_1,y_1\}\cdot\ldots\cdot\{x_s,y_s\} = 0$ for some s;

(iii) nonzero submodules in $P_n(V)$ correspond to only those Young diagram in which the number of blocks outside the first row is bounded by a constant not depending on n.

Danila REVIN

The class of E_π-groups is nonradical

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We denote by $\pi(n)$ the set of prime divisors of an integer n. A subgroup H of a finite group G is called *a Hall π-subgroup* for a set π of primes if $\pi(|H|) \subseteq \pi$ and $\pi \cap \pi(|G : H|) = \varnothing$. According to [1], $G \in E_\pi$ if G possesses a Hall π-subgroup.

Recall that a class $\mathfrak{X}$ of finite groups is a *formation* if the following two conditions hold:
Frm1 If $G \in \mathfrak{X}$ and $N \trianglelefteq G$ then $G/N \in \mathfrak{X}$;
Frm2 If $M, N \trianglelefteq G$ and $G/M, G/N \in \mathfrak{X}$ then $G/(M \cap N) \in \mathfrak{X}$.

The next problem was possed by L.A.Semetkov [2, problem 18]: *Is the class E_π a formation?* The positive answer was obtained in [3, corollary 8].

In the talk we consider the problem dual to Shemetkov's problem: *Is E_π a Fitting class?* Recall that a class $\mathfrak{X}$ of finite groups is a *Fitting class* if the following two conditions hold:
Fit1 If $G \in \mathfrak{X}$ and $N \trianglelefteq G$ then $N \in \mathfrak{X}$;
Fit2 If $M, N \trianglelefteq G$ and $M, N \in \mathfrak{X}$ then $MN \in \mathfrak{X}$.

There exists a series of examples showing that the solution to the leter problem is negative, since the class E_π does not satisfy the condition **Fit2**, in general. This result was obtained by the author in collaboration with E.P.Vdovin.

The work is supported by RFBR (Grants 10-01-00391, 11-01-00456, and 12-01-90005).

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N.M. RUSIN

Linear 2-state ZC-automata

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Let Z be a countable alphabet, identified with the ring $\mathbb{Z}$ of integers. A permutational automaton $\mathcal{A} = \langle Z, Q, \varphi, \psi \rangle$ is called ZC-automaton, if in any inner state $q \in Q$ the output function ψ_q realizes a shift on some integer c_q:

$$\psi_q(z) = z + c_q, z \in Z.$$

In each inner state of a ZC-automaton a ZC-automaton transformation is defined ([1]). All ZC-automaton transformations form a group $GA_C(Z)$. The elements of the group $GA_C(Z)$ can be represented as infinite sequences

$$[g_1, g_2(x_1), g_3(x_1, x_2), \dots],$$

where $g_1 \in \mathbb{Z}$ and $g_i(x_1, \dots, x_{i-1}) : \mathbb{Z}^{i-1} \to \mathbb{Z}$, $i \geq 2$.

With respect to this representation of elements a few natural subgroups of the group $GA_C(Z)$ are determined. One of them is the group $LGA_C(Z)$ of linear ZC-automaton transformations. The coordinates of $u \in LGA_C(Z)$ are linear polynomials.

The group of a ZC-automaton is a subgroup of $GA_C(Z)$ generated by ZC-automaton transformations defined in its inner states. The following theorem is proved.

Theorem 1. *There is no 2-state linear ZC-automaton such that the group of this automaton is free of rank 2.*

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A.V. SADOVNICHENKO

On generalizations of G - invariant subspaces

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We consider some generalizations of G - invariant subspaces. Let F be a field and $G \leqslant GL(F, A)$. A subspace B of A is called nearly G - invariant, if $dim_F(BFG/B)$ is finite. This subspace is an analogue of a nearly normal subgroup. A subgroup H of a group G is called nearly normal if the index $|H^G : H|$ is finite. Such subgroups have been introduced by B.H. Neumann in his paper [1]. In the paper [2], the following type of subgroups was introduced. A subgroup H is called normal-by-finite, if the index $|H : Core_G(H)|$ is finite. A subspace B is called almost G - invariant, if $dim_F(B/Core_G(B))$ is finite.

Theorem 1. *Let F be a field, A be a vector space over F and G be a solvable periodic subgroup of $GL(F, A)$. If every subspace of A is either nearly G - invariant, or almost G - invariant, then G is an abelian - by - finite group of finite special rank.*

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I.O.SAVCHENKO

Fractal properties of a family linear sets

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Let $2 \le s$ be a fixed positive integer, λ is a real number, $\lambda \in (0,1)$, $A = \{0, 1, \dots, s-1\}$, $L = A^\infty$.

We consider two-parameter set

$$A_\lambda^s = \{x : x = \sum_{n=1}^{\infty} a_n \lambda^n, (a_n) \equiv \overline{a} \in L\}.$$

We study the topological, metric, and fractal properties of the set A_λ^s.

Theorem 1. *The set A_λ^s has the following properties*

1. it is a perfect set (closet set without isolated points);

2. if $\lambda \in (0, \frac{1}{s})$, then it is a nowhere dense, is of zero Lebesgue measure, Hausdorff-Besicovitch dimension is equal to

$$\alpha_0(A_\lambda^s) = -\log_\lambda s;$$

3. if $\lambda \in [\frac{1}{s}, 1)$, then it is a segment whose length is equal to $l = \frac{(s-1)\lambda}{1-\lambda}$.

Generalization of the set A_λ^s is set

$$B_\lambda^{\overline{s}} = \{x : x = \sum_{n=1}^{\infty} b_n \lambda^n, (b_n) \equiv \overline{b} \in M\},$$

where $M = \overline{s}^\infty$, $\overline{s} = \{b_0, b_1, \dots, b_{s-1}\}$, $0 \le b_0 < b_1 < \dots b_{s-1}$.

The report offers the results of the properties of the set $B_\lambda^{\overline{s}}$.

Natallia SAVELYEVA

On maximal Fitting subclasses of the class $\mathfrak{E}_\pi$ of all finite π-groups

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All groups considered are finite.

A normally hereditary class of groups $\mathfrak{F}$ is called a Fitting class if it is closed under the products of normal $\mathfrak{F}$-subgroups.

In the class $\mathfrak{S}$ of all soluble groups it is known [1] that every maximal (by inclusion) Fitting subclass of $\mathfrak{S}$ is normal in $\mathfrak{S}$. Recall that a Fitting class $\mathfrak{F} \neq \emptyset$ is called normal in a Fitting class $\mathfrak{H}$, if $\mathfrak{F} \subseteq \mathfrak{H}$ and for every $\mathfrak{H}$-group G a subgroup $G_\mathfrak{F}$ is $\mathfrak{F}$-maximal in G. The result of J. Cossey [1] was extended for the class $\mathfrak{S}_\pi$ of all soluble π-groups [2] (π is a non-empty set of primes). In this paper the question whether this property is held for the case of non-soluble π-groups is solved positively.

Let $\mathbb{P}$ be a set of all primes and $\emptyset \neq \pi \subseteq \mathbb{P}$. The symbol $\mathfrak{E}_\pi$ denotes the Fitting class of all π-groups. If a Fitting class $\mathfrak{F}$ is maximal (normal) in $\mathfrak{E}_\pi$ then we call $\mathfrak{F}$ π-maximal (π-normal).

Theorem 1. *Every π-maximal Fitting class is π-normal.*

Corollary 1. *(H. Laue, theorem 3.13 [3]) If a Fitting class $\mathfrak{F}$ is maximal in the class $\mathfrak{E}$ of all groups then $\mathfrak{F}$ is normal in $\mathfrak{E}$.*

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The structural properties of modal groups

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Study groups with the restriction on the lattice of subgroups of the papers [1–3]. In [3] put into consideration the variety of modal lattices γ_n determined by the condition $x \cap (y_1 + ... + y_n) \leq x \cap y_1 + ... + x \cap y_n$, where $n \geq 2$. We note that γ_2- variety of modular lattices and $\gamma_3 \cap \varepsilon$ covering γ_2, where ε-variety of distributive lattices. In [1] obtained a description of the modal groups for the parameter $n = 3$ and $n = 4$. The authors study the modal groups, for parameter $n \geq 5$.

Theorem 1. *Let A - any modal group is a lattice of its subgroups $RA \in \gamma_5$. Then, for all $x, y \in A$ are $x \cdot y^6 \cdot x^{-1} \in < y^6 >$ and the identity $[x^6, y^6] = 1$ of group A is true.*

Theorem 2. *Let A - non-Abelian periodic group. Given following conditions are equivalent: 1) A - modal group in which the identity $[x^2, y] = 1$ is true; 2) $A = B \times K$, where K - Abelian modal group, $B \in Q, Q^*, D, T_3$, $(B, K) = 1$.*

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N.N. (Jr.) SEMKO

Groups having many transitively normal subgroups

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Recently L.A.Kurdachenko and I.Ya.Subbotin introduced the concept of transitively normal subgroup. A subgroup H of a group G is said to be **transitively normal in G**, if H is normal in every subgroup $K \geq H$ such that H is subnormal in K. This property is connected with another important property. A group G is said to be a **T-group** if every subnormal subgroup of G is normal. In general not every subgroup of T-group is a T-group itself, so we come to the following type of groups. A group G is said to be a $\overline{T}$-group, if every subgroup of G is a T-group. We observed that $\overline{T}$-groups have many pronormal subgroups, every finitely generated subgroup of a $\overline{T}$-group is pronormal. Converse is also true. N.F.Kuzennyj and I.Ya.Subbotin showed that a locally soluble group G is a $\overline{T}$-group iff every finitely generated subgroup of G is pronormal. In this connection it is natural to consider an opposite situation: groups in which every not finitely generated subgroup is pronormal, or even more generally groups whose not finitely generated subgroups are transitively normal. The following result gives a description of locally nilpotent torsion-free groups with this property.

Theorem 1. *Let G be a locally nilpotent torsion-free group, whose not finitely generated subgroups are transitively normal. Then either G is an abelian group or G is finitely generated group.*

Vladimir SENASHOV

On the Characterization of an Almost Layer-Finite Groups

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Remind:

Definition 1. The group is called *layer-finite*, if the set of all its elements of any given order is finite.

This concept was first introduced by S.N. Chernikov in [1]

Definition 2. *The almost layer-finite group* is an extension of a layer-finite group by a finite group.

Definition 3. The group is called *Chernikov*, if it is either finite or a finite extension of a direct product of finite number of quasi-cyclic groups.

Definition 4. The group is called *Shunkov*, if for any its finite subgroup H, in the quotient group $N_G(H)/H$ any two conjugate elements of prime order generate a finite subgroup.

We prove the following proposition:

Theorem 1. *Let G be a Shunkov group, the centralizer of every involution of which (when $2 \in \pi(G)$) has a Chernikov periodic part. If in G the normalizer of any non-trivial finite subgroup has an almost layer-finite periodic part, then the group G has an almost layer-finite periodic part.*

This work was supported by RFBR (project 10-01-00509) and by Siberian Federal University (Project - an elite mathematical education at SFU).

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On Shunkov Groups

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Let q be a prime number. We remind the following two definitions.

Group G is called *conjugately q-biprimitively finite*, if for any its finite subgroup H in the quotient group $N_G(H)/H$, any pair of conjugate elements of order q generates a finite subgroup (V.P.Shunkov).

In particular, any periodic group is conjugately 2-biprimitively finite.

If a group G is conjugately q-biprimitively finite with respect to any prime number q, then G is called *conjugately biprimitively finite group* (V.P.Shunkov).

At the suggestion of V.D. Mazurov (2000) the conjugately biprimitively finite group is also called Shunkov group.

The report provides a review of the results on Shunkov groups. It will include the results of A.A Duzh, L. Gamudi, V.O. Gomer, M.N. Ivko, A.N. Izmaylov, Al.N. Ostylovskii, A.N.Ostylovskii, I.I. Pavlyuk, A.M. Popov, A.V. Rozhkov, A.G. Rubashkin, E.I. Sedova, V.I. Senashov, A.I.Sozutov, N.G.Suchkova, A.V.Timofeenko, K.A.Filippov, A.A. Cherep, N.S. Chernikov, A.A. Shafiro, A.K. Shlepkin, V.P. Shunkov.

[1] This work was supported by RFBR (project 10-01-00509) and by Siberian Federal University (Project - an elite mathematical education at SFU).

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Real numbers representation by the Cantor series

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Let (d_n) be a fixed sequence of positive integer numbers such that $d_n \geq 2$ for any $n \in \mathbb{N}$ and

$$\frac{1}{d_1}, \frac{1}{d_1 d_2}, ..., \frac{1}{d_1 d_2 ... d_n} \to 0, \quad n \to \infty.$$

Let we have a sequence of sets A_n, that $A_n \equiv \{0, 1, ..., \ d_n \ - \ 1\}$.

The sum

$$\frac{\alpha_1}{d_1} + \frac{\alpha_2}{d_1 d_2} + ... + \frac{\alpha_n}{d_1 d_2 ... d_n} + ..., \quad \alpha_n \in A_n \quad \forall n \in \mathbb{N} \tag{1}$$

is called *the Cantor serie.*

Denote by $\Delta^{(D)}_{\alpha_1 \alpha_2 ... \alpha_n ...}$ any number from $[0, 1]$, that has representation (1).

Theorem 1. *Any real number from* $[0, 1]$ *has unique or two representations by the Cantor series.*

Any real number, that has two representation (1) is *the D-rational number.*

Theorem 2. *Let* (d_n) *is a periodical sequence. A number from* $(0, 1)$ *is rational iff sequence* (α_n) *in (1) is periodical.*

Theorem 3. *A rational number* $\frac{p}{q}$ *from* $(0, 1)$ *is a D-rational iff exist* $n_0 \in \mathbb{N}$ *such that* $d_1 d_2 ... d_{n_0} \equiv 0 (mod \ q)$.

It is easy to see sequences (d_n) that any rational number from $(0, 1)$ has two representations by the Cantor serie are existing.

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Modular and neutral elements in the lattice of epigroup varieties

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A semigroup S is called an *epigroup* if for any element x of S there is a natural n such that x^n lies in some subgroup of S. An extensive information about epigroups may be found in [1, 2]. Every epigroup may be equipped by some naturally defined unary operation called *pseudoinversion* (see [1–3], for instanse). This allows to consider varieties of epigroups as algebras with the operations of multiplication and pseudoinversion. We denote by $\mathbf{T}$, $\mathbf{SL}$ and $\mathbf{ZM}$ the trivial variety, the variety of semilattices and the variety of semigroups with zero multiplication respectively.

Theorem 1. *If an epigroup variety* $\mathbf{V}$ *is a modular element of the lattice of all epigroup varieties then* $\mathbf{V} = \mathbf{M} \vee \mathbf{N}$ *where* $\mathbf{M} \in \{\mathbf{T}, \mathbf{SL}\}$*, while* $\mathbf{N}$ *is a nil-variety.*

Theorem 2. *The varieties* $\mathbf{T}, \mathbf{SL}, \mathbf{ZM}, \mathbf{SL} \vee \mathbf{ZM}$ *and only they are neutral elements of the lattice of all epigroup varieties.*

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On similarity of pairs of matrices, in which one matrix is nonderogatory

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Let $(M,\ N)$ be a pair of complex $n \times n$ matrices and let M be nonderogatory. The field of complex numbers C is assumed to be lexicographically ordered. The pair $(M,\ N)$ is similar to some pair $(A,\ B) = (\oplus_{i=1}^{m} J(\lambda_i),\ B)$, in which $J(\lambda_i)$, be a Jordan block with eigenvalue λ_i and $\lambda_1 < \ \ldots \ < \lambda_m$. Based on [1] we give at first the so-called near canonical form $(M,\ N)_{ncan}$ for similarity. Suppose that the pair $(A,\ B)$ is already near canonical pair, i.e. $(M,\ N)_{ncan} = (A,\ B)$. A row g_r is assigned to each entry b_{pq} of matrix $B = \|b_{pq}\|_1^n$. Consider now the matrix $G = \left\| \ g_1^t \quad g_2^t \quad \ldots \quad g_{n^2}^t \ \right\|^t$. Between the entries b_{pq} of matrix $B = \|b_{pq}\|_1^n$ of pair $(A,\ B)$ and the rows g_r of matrix G the one-to-one correspondence is established.

Theorem 1. *Let $g_{w_1},\ g_{w_2},\ \ldots,\ g_{w_h}$ denote the maximal system of the first linearly independent rows of matrix G, and*

$$b_{p_1,\, q_1},\ b_{p_2,\, q_2},\ \ldots,\ b_{p_h,\, q_h}$$

their corresponding entries of matrix B. The pair $(M,\ N)$ is similar to pair $(M,\ N)_{ncan} = (A,\ B)$, in which all entries in positions

$$(p_1,\ q_1),\ (p_2,\ q_2),\ \ldots,\ (p_h,\ q_h)$$

of matrix B are zero. This pair $(A,\ B)$ is uniquely determined.

[1] Vyacheslav Futorny, Roger A. Horn, Vladimir V. Sergeichuk. *A canonical form for nonderogatory matrices under unitary similarity // Linear Algebra and its Applications*, **435**, (2011), P. 830-841.

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On factorizability and associativity of matrices

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Consider $n \times n$ matrices A, B over a ring R. If $A = BC$ we say that B is a left divisor of A or A is a right multiples of B. If C is an invertible matrix, we say that A and B are right associates. Ernst Steinitz [1] showed that some extent properties of R are inherited by $M_n(R)$:

Theorem 1. *Let R be an algebraic integer ring, and let A, B be matrices over R which are right multiples of each other. Then A, B are right associates.*

I. Kaplansky extended this statement to any right Hermite domain and B. Zabavsky to a ring with stable rank 1. We got the following results:

Theorem 2. *Let R be an commutative elementary divisor domain. Let A, B be matrices over R and A is left divisor of B and B is right divisor of A. Then A, B are left and right associates.*

Theorem 3. *Let R be an commutative elementary divisor domain. Let A, B be matrices over R and B is left divisor of A and A is equivalent to B. Then A, B are right associates.*

[1] Steinitz E. *Rechteckige Systéme und Moduln in algebraischen Zahlkörpern, I // Math. Ann.* **71**, 1911, p. 328–354.

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On 2-transversals in quasigroups

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Binary groupoid $(Q, \circ)$ is called a quasigroup if for all ordered pairs $(a, b) \in Q^2$ there exist unique solutions $x, y \in Q$ to the equations $x \circ a = b$ and $a \circ y = b$.

For quasigroups it is possible to define the following kind of translation, namely, *middle translations*. If P_a is a middle translation of a quasigroup $(Q, \cdot)$, then $x \cdot P_a x = a$ for all $x \in Q$.

A k-transversal of a Cayley table of a quasigroup of order n ($k \leq n$) is a set of n cells, one in each row, one in each column, and such that maximum k cells contain different symbols.

Define graph of a pair of translations of a groupoid $(G, \cdot)$ in the following way. Any triplet is graph vertex. If a pair of triplets has at least one equal coordinate, then corresponding pair of vertex is connected with a common edge. We shall name this graph as a *2-translation graph.*

Theorem 1. *A quasigroup $(Q, \cdot)$ of order n does not have a 2-transversal (say, (a, b)-transversal) if and only if 2-translation graph $G(P_a, P_b)$ forms one cycle of the length $2n$.*

In other language the similar conditions are given in [1].

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L. A. SHEMETKOV

New criteria for the p-nilpotency of normal subgroups in finite groups

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All groups considered are finite, and G denotes a group. Let A and B be subgroups in G such that $G = AB$. Then B is called a supplement to A in G. Notations: $C_{p'}$ is the class of all groups with the only class of conjugate Hall p'-subgroups; $[B, R) = \{X \leq G \mid B \leq X < R\}$.

In [1] the following result is proved.

Theorem 1. *Let K be a normal subgroup of G. If a minimal supplement B to K in G contains a Sylow p-subgroup of K, then K is p-nilpotent.*

We use Theorem 1 for proving the following.

Theorem 2. *A normal subgroup K of G is p-nilpotent if and only if $K \in C_{p'}$ and for every supplement B to K in G the following condition is satisfied: if a Sylow p-subgroup B_p of B is a proper subgroup of a Sylow p-subgroup G_p of G, then there exists a subgroup $X \in [B_p, G_p)$ having a p-nilpotent supplement in G_pK.*

[1] Shemetkov L. A. *Ukrainian Math. J.* **23**, No. 5 (1971), p. 556–565.

O.L. SHEMETKOVA

Finite groups with SE-supplemented subgroups

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All groups considered are finite.

By [1], B_{seG} denotes a subgroup generated by all the subgroups of B which are S-quasinormally embedded in G. Following A. N. Skiba [1], a subgroup B is called SE-supplemented in G, if there is a subgroup H such that $G = BH$ and $B \cap H \leq B_{seG}$.

Theorem 1. *Let E be a normal subgroup in G, and p a prime divisor of $|E|$ such that $(p-1, |E|) = 1$. Let P be a Sylow p-subgroup in E. Assume that every maximal subgroup in P either is SE-supplemented in G or possesses a p-supersoluble supplement in G. Then E is p-nilpotent, and all its G-chief p-factors are cyclic.*

Theorem 2. *Let E be a normal subgroup in G, and p a prime divisor of $|E|$ such that $(p-1, |E|) = 1$. Let P be a Sylow p-subgroup in E. Assume that the following two conditions are satisfied:* 1) *every subgroup of order p in P is either $Q\mathfrak{U}$-central in G or SE-supplemented in G;* 2) *if $p = 2$, then every quaternion subgroup of order* 4 *in P is either Q-central in G or SE-supplemented in G. Then E is p-nilpotent, and all its G-chief p-factors are cyclic.*

[1] Skiba A. N. *Problems of Physics, Mathematics and Technics.* **4(5)**, (2010), p. 39–45.

E. SHIRSHOVA

On subgroups of interpolation groups

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Let G be a (partially ordered) *po*-group ([1,2]), $G^+ = \{x \in G | \, e \leqslant x\}$. In a *po*-group G, for $a \in G^+$ $(a \neq e)$, there is the convex directed subgroup $[a]$ whose positive cone G^+ consists of $g \in G$ such that $e \leqslant g \leqslant a^m$ for some integer $m > 0$. Elements a and $b \in G^+$ are called *almost orthogonal* if the inequalities $c \leqslant a$ and $c \leqslant b$ imply $c^n \leqslant a$ and $c^n \leqslant b$ for every integer $n > 0$ and for all $c \in G$. A *po*-group G is an i-group if whenever $a_1, a_2, b_1, b_2 \in G$ and $a_1, a_2 \leqslant b_1, b_2$, then there exists $c \in G$ such that $a_1, a_2 \leqslant c \leqslant b_1, b_2$.

Theorem 1. *Let a and b be some almost orthogonal elements of an i-group G. If $H(a,b) = \{g \in G | g = uv^{-1},\, u, v \in G^+, u, v \leqslant a, b\}$, then $H(a,b)$ is a convex directed subgroup which coincides with the set-theoretic intersection of the subgroups $[a]$ and $[b]$.*

Theorem 2. *If a and b are some almost orthogonal elements of an i-group G, then the following assertions hold: there is an o-homomorphism of the Riesz group $[a]$ onto the Riesz group $[a][b]/[b]$ with the kernel $H(a,b)$; the Riesz group $[a]/H(a,b)$ is o-isomorphic to the Riesz group $[a][b]/[b]$.*

Theorem 3. *If G is an i-group, and let H be the subgroup generated by the set-theoretic union of the subgroups $H(a_i, b_i)$ for all pairs of almost orthogonal elements a_i and b_i, then the Riesz group H is an o-ideal of G.*

[1] Fuchs L. *Partially Ordered Algebraic Systems.* – M.: Mir, 1965.

[2] Kopytov V.M. *Lattice-Ordered Groups.* – M.: Nayka, 1984.

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Periodic groups saturated by the groups $GL_2(3^n)$

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A group G is said to be saturated by set of groups $\Re$ if every finite subgroup of G is contained in a subgroup isomorphic to a group from $\Re$. We proved the following rezult:

Theorem 1. *Let infinite periodic shunkov group G is saturated by set: $\Re = \{GL_2(q)\}$, where q is degree of number 3. Then $G \simeq GL_2(Q)$, where Q is local finite field characteristic 3.*

[1] Shlepkin A. K., Rubashkin. A. G. *Siberian Math. J.* 45:6 (2004), p. 1140.

[2] Shunkov V. P. *Algebra and Logic* 11:4 (1972), p. 260.

Polina K. SHTUKKERT

Semifield planes and Latin squares

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We consider the issues associated with the construction of finite semifield planes, which are an extension of the class of desarguesian planes (see definitions in [1]). For building the semifield planes used their regular sets. Is used the computer program lists all the regular sets and it is been found: there are exactly 56 semifield planes of rank 2 over a field $GF(4)$, 6 of which are dezarguesian and 50 non-dezarguesian.

Theorem 1. *Up to isomorphism there is only desarguesian and the only non-desarguisian semifield planes of rank 2 order 16.*

A projective plane of order n exists if and only if there exists a complete system of $(n-1)$ mutually orthogonal Latin squares of order n [1, Theorem 5.8]. For each of the planes the complete systems of mutually orthogonal Latin squares of order 16 were built.

The solutions of V.V. Belyaev's questions recorded in [2] about enumerating of Latin squares of orders $n = 4, 5, 6$, of Latin $r \times 6$ rectangles as well as the number of $RC-, RCN-$ equivalent classes of Latin squares of orders $n = 4, 5$ (see definitions in [2]) was given.

The author were supported by the Russian Foundation for Basic Research (project 12-01-00968)

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[2] *Yenisei book (math questions and problems for the young researches).* First release. - Krasnoyarsk: KSU, 2004.- 32p.

M.P. SHUSHPANOV

Nonmodular lattices generated by modular elements

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An element a of the lattice $\langle L; \wedge, \vee\rangle$ is said to be *modular* if

$$\forall x, y \in L : x \leq y \to (a \vee x) \wedge y = (a \wedge y) \vee x.$$

In [1] it is announced that every lattice generated by three modular elements is modular. This gives rise to the following question (also formulated in [1]): is it true that every lattice generated by a set of its modular elements is modular? We answer this question in the negative.

Theorem 1. *For every $n \geq 4$, there exists a nonmodular lattice L generated by n elements each of which is modular in L.*

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Vadim Veniaminovich SIDOROV

Isomorphisms of lattices of subalgebras of semiring of continuous nonnegative functions

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Semiring is an algebraic system $\langle S, +, \cdot, 0\rangle$ such that $\langle S, +, 0\rangle$ is a commutative monoid, $\langle S, \cdot\rangle$ is semigroup, rules of distributivity of multiplication with respect to addition hold, and $0 \cdot s = s \cdot 0 = 0$.

Let X be a topological space, and let $\mathbb{R}^+$ be the set of all nonnegative real numbers. The set of all continuous nonnegative functions on X with pointwise operations of maximum $\vee$ and multiplication $\cdot$ of functions generates semiring $C^\vee(X)$.

Subalgebra in semiring $C^\vee(X)$ is its arbitrary subsemiring such that it can be multiplied on numbers from $\mathbb{R}^+$. Denote by $\mathbb{A}(C^\vee(X))$ the lattice of all subalgebras of semiring $C^\vee(X)$ with respect to inclusion $\subseteq$. Minimal subalgebra $A \in \mathbb{A}(C^+(X))$ that contains function $f \in C^\vee(X)$ is called unigenerated and denoted by $\langle f\rangle$.

Theorem 1. *Let X and Y be realcompact spaces, $|X| \neq 2$. If*

$$\alpha\colon A(C^\vee(X)) \to A(C^\vee(Y))$$

is a isomorphism, then there exist a homeomorphism $\mu\colon Y \to X$ and a number $t > 0$, such that

$$\alpha\colon \langle f\rangle \to \langle f^t \circ \mu\rangle$$

for all $f \in C^\vee(X)$.

[1] Vechtomov E.M., Sidorov V.V. *Isomorphisms of lattices of subalgebras of semirings of continuous nonnegative functions.* Journal of Mathematical Sciences. **177**(6), (2011), 817-846.

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On groups of DP- and PDP-transformations and their relations to Fractal Geometry

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Let $dim_H(E)$ and $dim_P(E)$ be the Hausdorff-Besicovitch dimension resp. packing dimension of a set $E \subset R^n$ (see, e.g., [2] for details). The Hausdorff-Besicovitch dimension is widely known as a "proper fractal dimension", which is monotone, shift-invariant and countable stable. An automorphism F of R^n is said to be DP-transformation if $\dim_H(E) = \dim_H(F(E)), \quad \forall E \subset R^n$. In [1] it has been shown that the set of all DP-transformations forms a group w.r.t. composition, and, therefore, the Fractal Geometry can be considered as a branch of mathematics studying invariants of the group of DP-transformations. We stude properties of DP-transformations w.r.t. other operations, which are almost unknown.

The packing dimension also has all desirable properties of dimension and much more popular in the applied sense. We show that the set of all transformations preserving the packing dimension (PDP-transformations) also forms a group and study some important subgroups of this group. Its algebraic properties will also be discussed.

[1] S. Albeverio, M. Pratsiovytyi, G. Torbin. Fractal probability distributions and transformations preserving the Hausdorff-Besicovitch dimension. *Ergodic Theory and Dynamical Systems*, **24**(2004), 1-16.

[2] K.J. Falconer. *Fractal geometry*, John Wiley & Sons, 1990.

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About classification of parastrophic distributivity identities within the class of loops

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Let $(Q;\cdot)$ be a quasigroup. An operation $(\overset{\sigma}{\cdot})$ is called *parastrophic* to $(\cdot)$, if $x_{1\sigma} \overset{\sigma}{\cdot} x_{2\sigma} = x_{3\sigma} \;\Leftrightarrow\; x_1 \cdot x_2 = x_3$, where $\sigma \in S_3$. If σ_1, σ_2, σ_3, τ, $\theta \in S_3$, then the identity

$$\left(x \overset{\sigma_1}{\cdot} y\right) \overset{\theta}{\cdot} \left(x \overset{\sigma_2}{\cdot} z\right) = x \overset{\sigma_3}{\cdot} \left(y \overset{\tau}{\cdot} z\right)$$

will be called a *parastrophic distributivity* and

$$\left(x \overset{\sigma_1 s}{\cdot} y\right) \overset{\theta s}{\cdot} \left(x \overset{\sigma_2 s}{\cdot} z\right) = x \overset{\sigma_3 s}{\cdot} \left(y \overset{\tau s}{\cdot} z\right)$$

will be called *dual identity* to above.

Theorem 1. *There are 192 identities of parastrophic distributivity, which define the class of commutative Moufang loops of degree three. The rest* $6^5 - 192 = 7584$ *identities define the trivial class of loops.*

For example, every of the identities $x(xy \cdot xz) = yz$, $(xz \cdot yz)x = yz$ defines the class of commutative Moufang loops of degree three. Recall, that a class of loops is called trivial, if every of its loop has one element only.

Problem. Is it true that a quasigroup is isotopic to a commutative Moufang loop if it satisfies a parastrophic distributive identity and its dual?

[1] Belousov V.D. Parastrophic-orthogonal quasigroups. Quasigroups and Related Systems 13 (2005), P. 25–72.

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On groups with systems of F-subgroups

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In this work was proved a nonsimplicity criteria of group with system F-subgroups (see [1]), that is an analogy of Frobenius groups.

Definition A group $G = F \leftthreetimes \langle a \rangle$ is called F-group with kernel F and complement $\langle a \rangle$, if $F \neq 1$ and the mappings $b \to a^{-1}b^{-1}ab$, $b \to a^{-2}b^{-1}a^2b$ are bijective on F.

It is clear, that $a^2 \neq 1$ and the bijection of mapping $b \to a^{-2}b^{-1}a^2b$ in definition has to do in case the order of element a is infinite or even.

Our main result is

Theorem 1. *Let G be a group, let H be a proper subgroup of G, $a \in H$ and $a^2 \neq 1$. If, for every element $g \in G \setminus H$, the subgroups $\langle a, a^g \rangle$ are F-groups with complement $\langle a \rangle$, then $G = F \leftthreetimes N_G(\langle a \rangle)$ and $F \leftthreetimes \langle a \rangle$, $F \cap H \leftthreetimes \langle a \rangle$ are F-groups with kernels F and $F \cap H$ accordingly.*

This work was supported by the Russian Foundation for Basic Research (project no. 09-01-00395-a, 10-01-00509-a).

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Quadratic Forms and Quadrics of Space over Local Rings

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Since the 70s the generalization of the main theorem of projective geometry and development of K-theory stimulated the transition from fields to more general rings of coefficients in researches of quadratic forms and projective space quadrics.

By [1], if all quadratic forms over a ring are diagonalizable, then in effect this is always a local ring $R = 2R$ of principal ideals. For $|R^* : R^{*2}| \leq 2$ along with enumeration of quadrics of projective space RP_{n-1} up to projective congruence combinatorial expression of number $N(n, s)$ its classes were found when maximal ideal J is nilpotent of degree s on R, [1, Theorem 3.2].

Applying the methods of integral representations of combinatorial sums, G. Egorychev and E. Zima have found simple formulas for the number $N(n, s)$ and posed the problem of finding an algebraic proof and an interpretation of the obtained formulas [2, Theorems 1 and 2]. The solution to this problem is given in [3]. More complicated problem of the enumeration of quadrics up to projective equivalence is solved.

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Yu.Yu. SUKHOLIT

Rationality of numbers in ternary-quinary numeral system

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Let $A = \{0, 1, 2, 3, 4\}$ be a set of digits (alphabet) of ternary system with two redundant digits.

Lemma 1. *For any number $x \in [0; 2]$ there exists a sequence β_k, $\beta_k \in A$, such that*

$$x = \sum_{k=1}^{\infty} 3^{-k}\beta_k \equiv \Delta_{\beta_1\beta_2\ldots\beta_k\ldots}.$$

Lemma 2. $\Delta_{\alpha_1\alpha_2\ldots\alpha_m} = \Delta_{\beta_1\beta_2\ldots\beta_m}$ *if and only if one can get the tuple* $(\beta_1\beta_2\ldots\beta_m)$ *from the tuple* $(\alpha_1\alpha_2\ldots\alpha_m)$ *replacing* (03) *with* (10), (04) *with* (11), (13) *with* (20), (14) *with* (21), (23) *with* (30), (24) *with* (31), (33) *with* (40), (34) *with* (41), *and vice versa.*

Theorem 1. *Every 3-5-rational number $x \in (0; 2)$ has a countable set of different 3-5-adic representations. Every 3-5-irrational number $x \in (0; 2)$ has a continuum set of different 3-5-adic representation.*

Theorem 2. *Real number $x \in (0; 2)$ is rational if and only if its canonical 3-5-adic representation is periodic.*

G.S. SULEIMANOVA

Large abelian unipotent subgroups in finite Lie-type groups

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Let G be a Lie-type group over a finite field K and let U be the maximal unipotent subgroup of G. Let $A(U)$ be a set of all large (i.e. of maximal order) abelian subgroups of U. In [1, Problem (1.6)] the problem of description of the set $A(U)$ for exceptional types was posed. In [6] the orders of subgroups from $A(U)$ were determined. In [2] it was proved that the set of all large normal abelian subgroups in U is coincides with the set of all normal subgroups of $A(U)$. In the present work classes of G-conjugate subgroups from the set $A(U)$ of exceptional type are listed. Partially the results were published in [3], [4], [5].

The author was supported by the Russian Foundation for Basic Research (project 12–01–00968)

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Properties of quadratic elements in modular representations of the symplectic group

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The behaviour of the images of quadratic unipotent elements in modular irreducible representations of the symplectic group is investigated. In what follows K is an algebraically closed field of characteristic $p > 2$, $\mathbb{N}_a$ is the set of all integers from 1 to a, $G = C_r(K)$, $r > 1$, $u \in G$ is a unipotent element with t blocks of size 2 and other blocks of size 1 in the standard G-module, ω_i, $1 \leq i \leq r$, are the fundamental weights of G, φ is a p-restricted irreducible representation of G with highest weight $\sum_{i=1}^{r} a_i\omega_i$, $J_\varphi(u)$ is the set of Jordan block sizes for an element $\varphi(u)$ (without multiplicities).

Theorem 1. *Let* $\sum_{i=1}^{r} a_i \geq p-1$. *Assume that* $t = 2q+1 > 1$, $r \geq 2q+4$, $a_{r-1} < p-1$ *and there exist* q *indices* $i_1, \ldots, i_q$ *such that* $a_{i_j} < p-1$, $1 \leq j \leq q$, $i_1 < \ldots < i_q < r-3$, $i_s - i_{s-1} > 1$. *Then* $J_\varphi(u) = \mathbb{N}_p$.

Theorem 2. *Let* $\sum_{i=1}^{r} a_i \geq p-1$. *Assume that* $t = 2q > 2$, $r > 2q+1$ *and there exist* q *indices* $i_1, \ldots, i_q$ *such that* $a_{i_j} < p-1$, $1 \leq j \leq q$, $i_1 < \ldots < i_q < r$, $i_s - i_{s-1} > 1$. *Then* $J_\varphi(u) = \mathbb{N}_p$.

Theorem 3. *Let* $\sum_{i=1}^{r} a_i < p-1$. *Assume that* $t > 2$ *and* $r > t+1$. *Set* $m = \min\{p, 1+a_1+2a_2+\ldots+(t-1)a_{t-1}+t(a_t+a_{t+1}+\ldots+a_r)\}$. *Then* $J_\varphi(u) = \mathbb{N}_m$.

Results of such kind can be used for solving recognition problems on representations and linear groups basing on the presence of specific unipotent matrices. This research has been supported by the Belarus Basic Research Foundation, project F10R-110.

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Uniserial structures and verbal subgroups in finite p-groups

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Let G be a finite p-group. We say that two families of subgroups $\{H_i\}_{i=1}^n$, $\{A_i\}_{i=1}^n$ of G define a uniserial-abelian structure in G if:

1. $H_n = G$, $H_0 = 1$
2. A_i is abelian for $i = 1, ..., n$
3. $H_i = H_{i-1} \ltimes A_i$ for $i = 1, ..., n$
4. The action of H_i on A_i is uniserial for $i = 1, 2, ..., n$,
5. For every $i = 2, 3, ..., n$ there exist an epimorphism $\varphi_i : A_i \longrightarrow A_{i-1}$ and a surjective map $\psi_i : H_i \longrightarrow H_{i-1}$, satisfying additional conditions.

In the talk we intend to discuss the properties of groups with uniserial-abelian structure and the verbal subgroups of such groups. In particular, we prove the following

Theorem 1. *Let G be a finite p - group with a uniserial - abelian structure. Then every verbal subgroup of G coincides with a term of the lower central series of G.*

This theorem generalizes results of [1, 2].

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A. Kh. TABAROV

Belousov problem for the class of quasigroup with balanced identity

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V.D. Belousov in [1] posed the problem: *What are the quasigroups or loops in which all congruences are normal? (Problem 20, p.221).* It is known that in the finite quasigroup every congruence is normal. There are classes of quasigroups and loops, in which every congruence is normal, for example JP-quasigroups, TS-quasigroups. V.A.Shcherbacov in [2] gave necessary and sufficient conditions normality congruence of a quasigroup in terms of subgroups of the multiplication group of a quasigroup. For the class of Bol quasigroup which is isotopic to a group this problem was solved in [3].

An identity $w_1 = w_2$ is called balanced if each propositional variable appears in w_1 and w_2 at most one time [4]. For details about irreducible balanced identity see [4].

Theorem 1. *Let $(Q, \cdot)$ be a quasigroup with irreducible balanced identity of the first kind of finite length. Then in $(Q, \cdot)$ all congruences are normal.*

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A.V. TIMOFEENKO

About systems of generating of Golod group

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Will be presented the new system of generators of finitely generated infinite p-groups of Golod type. It is supported by the Russian Foundation for Basic Research, grant № 10-01-00509-a.

A.V. TIMOFEENKO

The application of finite representations of crystallographic groups and finite groups of isometries of Euclidean space of dimension 3 and 4 in synthesis of polyhedrons with parquet sides

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Demonstrates the ATLAS of geometric representations of finite groups, necessary for the construction of convex polyhedra with parquet sides. Work performed under the project No 04/12 "'Teaching the New School for Masters'" on the program of strategic development of the Krasnoyarsk State Pedagogical University after V.P.Astaf'ev.

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Graded Cohen-Macaulay rings of wild representation type

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Let R be Cohen-Macaulay ring of Krull dimension d that is positively graded and generated in degree 1, i.e. $R = \bigoplus_{i=0}^{\infty} R_i$ and suppose that $R_0 = k$ is a field. Consider an R-sequence $(y) = (y_1, \ldots, y_d)$ with $\deg y_i = m$ and denote $\bar{R} = R/(y)R = \bigoplus_{i=0}^{\infty} \bar{R}_i$. Then $\dim_k \bar{R}_i$ does not depend on the choice of (y) and the next statement holds:

Theorem 1. *If there exists $c > d(m-1)+1$ such that $\dim_k \bar{R}_c > 2$, then R is of wild representation type.*

The proof of this theorem is based on generalization of Theorem A from [2]. For ACM projective schemes (i.e. with Cohen-Macaulay coordinate rings) we have the following corollaries:

Corollary 1. *Suppose that an ACM hypersurface $X \subset \mathbb{P}^n$ has degree $e \geq 4$. Then X is of wild representation type.*

Corollary 2. *Suppose that a complete intersection $X \subset \mathbb{P}^n$ has codimension k and is defined by polynomials of degrees $d_i > 1$. If $k \geq 3$ or $k = 2$ and $d_1, d_2 \geq 3$ then X is of wild representation type.*

As another consequence we also get the statement of Theorem 4.6 from [1] about wild representation type of Segre varieties.

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M.V. TSYBANYOV

About F-factorized modules

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While investigating of construction of a group G with F-factorized subgroups ([1], [2]) the structure of its infinite abelian normal subgroup was decisive. Upon that transition to consideration of such normal subgroup as G-modules was productive. On this way interesting examples of modules appeared and later they were named Chernikov modules.

A G-module A is named strongly F-factorized, if any subgroup of the additive group A is F-supplemented by some submodule from A.

Theorem 1. *G-module A is strongly F-factorized if and only if A contains such a finite submodule K, that the factor-module A/K is isomorphically embed in a direct sum $C \bigoplus V \bigoplus S$ of G-modules (at least one of which is non-zero) and upon that $C = \bigoplus_{i=1}^{n} C_i$, where C_i - irreducible Chernikov G-module, which complete additive subgroup has rang 1, V - direct sum of infinite set of simple order G-modules, $S = \bigoplus_{i=1}^{m} S_i$, where S_i - direct sum of infinite set of isomorphical irreducible finites of nonsimple order G-modules and a factor-group $G/C_G(S_i)$ - a finite cyclic group.*

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Anatoliy V. TUSHEV

On Induced Modules over Group Rings of Soluble Groups

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We consider induced modules developing technics and methods of [1–3]

Let Δ (G) be the set of all elements of a group G which have finite orbits under action of the group G by conjugations. It is easy to note that Δ (G) is a characteristic subgroup of G which is named the FC - center of G. We study relations between Δ (G) and induced modules over group rings of G.

Theorem 1. *Let G be a linear group of finite rank with trivial FC-center. Let k be a field of characteristic zero and let M be an irreducible kG-module, such that $C_G(M) = 1$. Then there exist a proper subgroup $S \leqslant G$ and a kS-submodule $U \leqslant M$ such that $M = U \otimes_{kS} kG$.*

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Classes of Kurosh-Chernikov loops and local theorem for them

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Researching of groups which possess subnormal infinite strings or normal infinite of certain types have brought at apparition of new classes of groups, called classes of Kurosh-Chernikov groups (see [1], [2] or [3]).

Naturally, in loops theorem appear analog classes, which we will call, also, Kurosh-Chernikov. In this work is showed that classes of RN- , RI- , Z-loops can be axiomatized with the help of some universal formulas, and classes of $\overline{RN}$, $\overline{RI}$, $\overline{Z}$-loops – with the help of some quasiuniversal formulas and is demonstrated local theorem for this loops, but also for ranked loops or free ranked loops. Combining properties RN, RI, etc. with condition of connectedness of subloops from theirs definition, we can obtain new local theorems for partial ranked loops.

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On the class of finite groups with K-$\mathfrak{F}$-subnormal Sylow subgroups

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We consider only finite groups. In [1] was initiated research of group structure, if all its Sylow subgroups are $\mathfrak{F}$-subnormal. In [2] have been continued research on this issue.

Let $\mathfrak{F}$ be a not-empty formation. The subgroup H of a group G is called K-$\mathfrak{F}$−subnormal (Kegel's subnormality) in G (which is denoted by H K-$\mathfrak{F}$-sn G) if there is a chain of subgroups $H = H_0 \subseteq H_1 \subseteq \cdots \subseteq H_m = G$ such that for every $i = 1, \ldots, m$ eiter H_{i-1} is subnormal in H_i or $H_i^{\mathfrak{F}} \subseteq H_{i-1}$.

Let $\widehat{w}\mathfrak{F} = (G \mid H \in \mathrm{Syl}(G)$ and H K-$\mathfrak{F}$-sn $G)$.

Theorem 1. *Let $\mathfrak{F}$ by a not-empty hereditary saturated formation and $\pi(\mathfrak{F}) = \mathbb{P}$. The following statements are equivalent:*

1. $\widehat{w}\mathfrak{F} = \mathfrak{F}$;

2. Every $\mathfrak{F}$-critical group G is either biprimary dispersive or $G/\Phi(G)$ is monolithic such that $\mathrm{Soc}(G/\Phi(G))$ is non-abelian and $G/\Phi(G)/\mathrm{Soc}(G/\Phi(G))$ is a group of prime power order.

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V. A. VASILYEV

On p-nilpotency of one class of finite groups

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Throughout this paper, all groups are finite.

We note that a modular subgroup is a modular element (in sense of Kurosh, [1, Chapter 2]) of the lattice of all subgroups of a group. For the first time the concept of modular subgroup was analyzed by R. Schmidt [2]. We use generalized modular subgroups to study p-nilpotent groups. Every subgroup H of a group G has the largest modular subgroup H_{mG} of G contained in H. We introduce the following concept

Definition. *A subgroup H of a group G is called m-supplemented in G if there exists a subgroup K of G such that $G = HK$ and $H \cap K \leq H_{mG}$.*

We obtained the following result.

Theorem 1. *Let G be a group, P a Sylow p-subgroup of G, where p is a prime divisor of $|G|$. Suppose that at least one of the following two assertions holds:*

(i) $(p-1, |G|) = 1$ *and every maximal subgroup of P not having a p-nilpotent supplement in G is m-supplemented in G.*

(ii) $(p-1, |G|) = 1$ *and every cyclic subgroup of P prime order or order 4 (if $p = 2$ and P is non-abelian) not having a p-nilpotent supplement in G is m-supplemented in G.*

Then G is p-nilpotent.

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I. VASIUNYK

Conditions, when abelian clean Bezout ring is an elementary divisors ring

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Let R be an an associative ring with $1 \neq 0$.

A ring R is called abelian ring if any idempotent element of this ring is a central. A ring R is called a clean ring if for every element x of the ring R there exist an invertible element $u \in R$ and an idempotent $e \in R$ such that $x = u + e$. A ring R is called an elementary divisors ring if any matrix A over R is equivalent to diagonal matrix $diag(\varepsilon_1, ..., \varepsilon_r, 0, ...0)$, where $R\varepsilon_{i+1}R \subseteq \varepsilon_i R \cap R\varepsilon_i, i = 1, ..., r-1$ [1]. A ring R is called a ring of stable range 2, if for any elements $a, b, c \in R$, where $aR + bR + cR = R$, there exists such element $x, y \in R$ that $(a + cx)R + (b + cy)R = R$ [2]. A ring R is called a projective-free ring if any finitely generated projective R-module is free [3].

Theorem 1. *Clean abelian Bezout ring R is elementary divisors ring if and only if R is duo-ring.*

Theorem 2. *A projective-free right (left) Bezout ring is Hermite ring if and only if stable range of R is equal 2.*

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Fibonacci representation of real numbers with infinite alphabet

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It is known that any real number $x \in [-1, \varphi]$ can be represented in the form

$$x = \sum_{n=1}^{\infty} \varepsilon_n \widehat{\varphi}^{n-1} = \varepsilon_1 \widehat{\varphi}^0 + \varepsilon_2 \widehat{\varphi}^1 + \cdots + \varepsilon_n \widehat{\varphi}^{n-1} + \cdots \ , \qquad (1)$$

where $\varphi = -\widehat{\varphi}^{-1}$, $\widehat{\varphi} = \frac{1-\sqrt{5}}{2}$, $\varepsilon_n \in \{0, 1\}$.

The Equality (1) is called the *$\widehat{\varphi}$-expansion* of this number. We denote symbolically the Equality (1) by $x = \Delta_{\varepsilon_1 \varepsilon_2 \ldots \varepsilon_n \ldots}$ and call it by the *$\widehat{\varphi}$-representation* of number x.

Theorem 1. *The following equality holds:*

$$\left[-1, -\widehat{\varphi}^{-1}\right] = \bigcup_{c \in \{0,1\}} \left(\Delta_{110} \cup \Delta_{c110} \cup \Delta_{c(1-c)110} \cup \Delta_{c(1-c)c110} \cup \ldots\right).$$

It is easy to show that for cylinders in the form

$$\Delta_{\underbrace{c(1-c)\ldots c(1-c)}_{2k}110} \quad \text{and} \quad \Delta_{\underbrace{c(1-c)\ldots(1-c)c}_{2k+1}110},$$

there exists partition analogous to partition in Theorem.

Using the above mentioned partitions one can prove that for any real number $x \in [-1, \varphi]$ there exists sequence $(\alpha_n) \in \mathbb{Z}$ such that $x = \Delta^{\infty}_{\alpha_1 \alpha_2 \ldots \alpha_n \ldots}$. In the talk we discuss topological and metric properties of numbers represented by the infinitesimal Fibonacci sequence with infinite alphabet. Moreover, we introduce an “addition” operation on the set of such representations and describe properties of the algebraic structure $((\alpha_n), \oplus)$.

E. M. VECHTOMOV, E. N. LYBIAGINA

Semirings of sc-functions

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Let X — a topological space, $\mathbf{I} = [0, 1]$ — a single numerical interval, considered with the usual operations of multiplication $\cdot$, max ($\vee$), min ($\wedge$) and with the standard topology. Through the $\mathbf{I}^X$ denotes the semiring of all functions $X \to \mathbf{I}$ with pointwise defined operations of addition $\vee$, and multiplication, and taking min functions and pointwise order relation.

Let $C(X, \mathbf{I})$ is fhe semiring of all continuous functions on a topological space X and taking values in a topological semiring $\mathbf{I}$, $\vee$ and the multiplication $\cdot$ on functions.

For a nonempty subset $M \subseteq \mathbf{I}^X$ denote by r_M the supremum of the set M in a complete lattice $\mathbf{I}^X$. The function $\varphi \in \mathbf{I}^X$ is called *sc-function* if $\varphi = r_M$ for some subset $M \subseteq C(X, \mathbf{I})$.

The set of all sc-functions on X denote by $SC(X, \mathbf{I})$. For any topological space X the set $SC(X, \mathbf{I})$ is a semiring under the operations $\vee$ and $\cdot$. With respect to pointwise order of $SC(X, \mathbf{I})$ is a complete distributive lattice and is a sublattice in $\mathbf{I}^X$.

If the homomorphism $\alpha : SC(X, \mathbf{I}) \to SC(Y, \mathbf{I})$ semirings preserves all least upper bounds and all constant functions then we call this homomorphism a *complete homomorphism.*

Theorem 1. *Category all semirings $SC(X, \mathbf{I})$ and their complete homomorphisms anti-equivalent (dual) category of all Tikhonov spaces and their continuous maps.*

E. M. VECHTOMOV, A. A. PETROV

Semirings similar to distributive lattices

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(Commutative) semiring is an algebraic structure $\langle S, +, \cdot \rangle$ such that both of reducts are commutative semigroups and satisfies the identity $x(y+z) = xy + xz$.

A semiring S is *idempotent* if $x + x = x = xx$ for all $x \in S$. A semiring is called *mono-semiring* if it satisfies an identity $x+y = xy$.

In what follows we mean by *semiring* an idempotent semiring with dual distributive law $x + yz = (x+y)(x+z)$.

A semiring S is called an *extension* of a family a semirings S_j ($j \in J$) by means of a semiring T if there is a congruence ρ on S such that $S/\rho \cong T$ and classes $[s]_\rho$ can be enumerated indexes j such that $[s_j]_\rho \cong S_j$ for all $j \in J$.

We denote by $\sim$ binary relation on semiring S such that $x \sim y \Leftrightarrow x+y = xy$. Transitive closure δ of a binary relation $\sim$ is a congruence on S and $\delta = \sim \circ \sim$.

A semiring is called *δ-semiring* if and only if $|S/\delta| = 1$.

A binary relation γ on semiring $S : x\gamma y \Leftrightarrow x+xy = x \wedge y+xy = y$ is a congruence.

Theorem 1. *A semiring S is extension of a family δ-semirings by means of a distributive lattice.*

Theorem 2. *A semiring S is extension of a family distributive lattices by means of a mono-semiring.*

Theorem 3. *Free semiring S is extension of a free distributive lattices by means of a free mono-semiring and S is subdirect product of a distributive lattice and a mono-semiring.*

Kseniia VERBININA

On a functional equation for the Euler series

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We study a convolution for formal Laurent series and its applications. One of these is a functional equation that completely characterizes the considered by Euler power series $f_a(z) = \sum\limits_{n=0}^{\infty} \frac{a_n n!}{z^{n+1}}$:

$$(a-b)f_a * f_b(z) = af_a(z) - bf_b(z).$$

A convolution representation of a differential operator of infinite order in the space of formal Laurent series is another application.

E.A. VITKO

On Strong Containment of Fitting Functors

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All groups considered in the paper are finite.

Let f and g be conjugate Fitting $\mathfrak{X}$-functors [1]. Then f is said to be strongly contained in g, denoted $f \ll g$, provided that for each $G \in \mathfrak{X}$, the following condition hold: if $X \in f(G)$, then there is $Y \in g(G)$ such that $X \leq Y$.

Let $\mathfrak{X}$ be a non-empty Fitting class, let f be a Fitting $\mathfrak{X}$-functor and let π be a set of primes. Define the class of groups $L_\pi(f)$ [1] as follows: $G \in L_\pi(f)$ if and only if $G \in \mathfrak{X}$ and the index $|G : X|$ is a π'-number for all $X \in f(G)$.

Let I be an index set, let π be a set of primes and $\Lambda = \{\pi_i : i \in I\}$ be a collection of pairwise disjoint non-empty set of primes such that $\mathbb{P} = \cup_{\pi_i \in \Lambda} \pi_i$ and $\pi' \subseteq \pi_i$ for some $\pi_i \in \Lambda$.

A π-soluble Fitting functor f is said Λ-normally embedded if f is π_i-normally embedded for each $\pi_i \in \Lambda$.

Theorem 1. *Let f and g be conjugate Λ-normally embedded π-soluble Fitting functors. Then $f \ll g$ if and only if $L_{\pi_i}(f) \subseteq L_{\pi_i}(g)$ for each $\pi_i \in \Lambda$.*

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Denys VOLOSHYN

Derived categories of noncommutative nodal curves

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A noncommutative curve is called *nodal* if all localizations of its structure sheaf are nodal in the sense of [3,7]. We describe the derived categories of coherent sheaves over noncommutative nodal curves of *string type* or *almost string types* [5,6]. Our description uses the technique of "matrix problems" more exactly representations of bunches of semi-chains [1]. Noncommutative nodal curves of string type are noncommutative analogues of projective configurations considered in [4]. The derived categories of coherent sheaves on projective configurations was described in the paper [2].

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[2] Burban I., Drozd Y. *Coherent sheaves on rational curves with simple double points and transversal intersections*, Duke Math. J., **21**, No. 2 (2004), 129–229.

[3] Burban I., Drozd Y. *Derived categories of nodal algebras*, J. Algebra, **272** (2004), 46–94.

[4] Drozd Y., Greuel G.-M. *Tame and wild projective curves and classification of vector bundles*, J. Algebra **246** (2001), 1–54.

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N.N. VOROB'EV

On Lockett Condition in the Class of All Finite π-Soluble Groups

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All groups considered are finite.

Let $\mathfrak{X}$ be a Fitting class and π a decomposition of a non-empty set of primes. Then:

(a) We call $\mathfrak{X}$ a $\vee_i$-class, for some $i \in I$, if there exists a Fitting class $\mathfrak{Y}$ such that $(\mathfrak{X}_*\mathfrak{E}_{\pi'(i)} \cap \mathfrak{X}) \vee \mathfrak{Y}\mathfrak{E}_{\pi(i)} = \mathfrak{X}$.

(b) In particular, if $\pi = \mathrm{Char}(\mathfrak{X})$, we call $\mathfrak{X}$ a $\vee_I$-class provided $\mathfrak{X}$ is a $\vee_i$-class for all $i \in I$.

A non-empty Fitting class $\mathfrak{F}$ is called a *Fitting class satisfying the Lockett condition in a Fitting class* $\mathfrak{H}$ (see [1]) if $\mathfrak{F} \subseteq \mathfrak{H}$ and $\mathfrak{F}_* = \mathfrak{F} \cap \mathfrak{H}_*$.

Let $\mathfrak{H}$ be a Fitting class such that every $\mathfrak{H}$-group has a nilpotent Hall π-subgroup, for some non-empty set of primes π. In the universe of all π-soluble groups we construct a continual set of Fitting classes $\mathfrak{F}$ with $\mathrm{Char}(\mathfrak{F}) = \pi$ satisfying the Lockett condition in the Fitting class $\mathfrak{H}$.

Theorem 1. *Every $\vee_I$-Fitting class $\mathfrak{F}$, that is contained in a Fitting class $\mathfrak{H}$, is a Fitting class satisfying the Lockett condition in $\mathfrak{H}$.*

[1] Savelyeva N.V., Vorob'ev N.T. Maximal subclasses of local Fitting classes, *Siberian Mathematical Journal* **49** (6), (2008), pp. 1124–1130.

N.T. VOROB'EV, A.V. TURKOVSKAYA

On π-normal lattice join of Fitting classes

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All groups considered are finite and soluble.

Let $\mathfrak{F}$ and $\mathfrak{H}$ be Fitting classes, then lattice join $\mathfrak{F}\vee\mathfrak{H}$ is the smallest of Fitting classes containing their union $\mathfrak{F}\cup\mathfrak{H}$. It is proved [1], that if $\mathfrak{F}\vee\mathfrak{H}$ is normal, then at least $\mathfrak{F}$ or $\mathfrak{H}$ is normal Fitting class. We recall that a Fitting class $\mathfrak{F}$ is said to be normal [2], if $G_{\mathfrak{F}}$ is maximal among the subgroups which lie in $\mathfrak{F}$ for every group $G \in \mathfrak{S}$.

Let π be a non-empty set of primes. A Fitting class $\mathfrak{F}$ is said to be π-normal or normal in a class of all f π-groups, if $\mathfrak{F} \subseteq \mathfrak{S}_\pi$ and $G_{\mathfrak{F}}$ is maximal among the subgroups which lie in $\mathfrak{F}$ for all $G \in \mathfrak{S}_\pi$.

If $\pi = P$, where P is a set of all primes, the Fitting class $\mathfrak{F}$ is normal [2]. We proved [3] that the product $\mathfrak{F}\mathfrak{H}$ of Fitting classes $\mathfrak{F}$ and $\mathfrak{H}$ is π-normal, if $\mathfrak{F}$ or $\mathfrak{H}$ is π-normal Fitting class. An analogous situation can arise with lattice join of Fitting classes.

Theorem 1. *The lattice join $\mathfrak{F}\vee\mathfrak{H}$ of Fitting classes $\mathfrak{F}$ and $\mathfrak{H}$ is π-normal, if $\mathfrak{F}$ or $\mathfrak{H}$ is π-normal Fitting class.*

[1] Cusack E. The join of two Fitting classes *Math.Z.* **167**, (1979), p. 37-47.

[2] Blessenohl D., Gaschütz W. Uber normale Schunk und Fittingklassen *Math.Z.* **148**, (1970), p. 1-8.

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S.N. VOROB'EV

On Fischer Sets of Finite Groups

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A *Fischer set* of G [1] is a Fitting set $\mathcal{F}$ of G which has the following property:

If $K \trianglelefteq L \in \mathcal{F}$ and if H/K is a nilpotent subgroup of L/K, then $H \in \mathcal{F}$.

We prove

Theorem 1. *Let $\mathcal{F}$ be a Fischer set of a group G.*

1) *If $\mathfrak{F}$ is a Fischer class and G a group, then the trace of $\mathfrak{F}$ in G is a Fischer set of G;*

2) *If $H \leq G$, then $\mathfrak{F}_H = \{S \leq H \mid S \in \mathcal{F}\}$ is a Fischer set of H;*

3) *If $H \trianglelefteq G$, $G_{\mathcal{F}} \leq N$, $G/G_{\mathcal{F}}$ is a $\sigma(\mathcal{F})$-soluble group and $\mathcal{F}_{G/N} = \{SN/N \mid S$ is an $\mathcal{F}$-injector of $SN\}$, then the quotient Fitting set $\mathcal{F}_{G/N}$ is a Fischer set of G/N.*

[1] Doerk K., Hawkes T. *Finite Soluble Groups. De Gruyter Expositions in Mathematics* **4**. Berlin-New York: Walter de Gruyter, 1992.

A.A. YADCHENKO

On degrees of certain irreducible linear groups

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Condition B. Let us say that the group $\Gamma = GA$ satisfies Condition B, if $G \lhd \Gamma$, $(|A|, |G|) = 1$, A is an odd-order group that is not normal in the group Γ, $C_G(a) = C_G(A) = C$ for each element $a \in A^{\#}$, and the group G has faithful irreducible complex character of degree n, which is a-invariant for at least one element $a \in A^{\#}$.

From the theorem, proved in the series of papers [1]- [3], it is obvious that if $n < 2|A|$ and A is of odd-order, then $n = |A| - 1$, $|A| + 1$, $2(|A| - 1)$ or $2|A| - 1$ and n is a degree of a certain prime number. Hence, n is divisible by the degree f of a certain prime number such that $f \equiv -1$ or $1(\mathrm{mod}|\mathrm{A}|)$. [3] hypothesizes the fairness of this statement for an arbitrary number n.

Theorem 1. *Assume the group Γ satisfies Condition B and Sylow 2-subgroup of the group G is Abelian. Then n will divide by the degree f of a certain prime number such that $f \equiv -1$ or 1(mod$|A|$).*

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B. ZABAVSKY

The stable range of some Bezout ring

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All rings will be commutative and have indentity; all modules will be unital. A ring R is a Bezout ring if every finitely generated ideal is principal. A ring R is said to have stable range 2 ($st.r(R) = 2$) if for each $a, b, c \in R$ such that $aR + bR + cR = R$ there exist such $x, y \in R$ that $(a + cx)R + (b + cy)R = R$. A ring R is said to have stable range 1 ($st.r(R) = 1$) if for each $a, b \in R$ such that $aR + bR = R$ there exist such $x \in R$ that $(a + bx)R = R$. A commutative ring R is PM-ring if each prime ideal is contained in only one maximal ideal. We denote $minR$ the space minimal prime ideals of R with the Zariski topology. We denote $J(R)$ the Jacobson radical.

Theorem 1. *Let R be a commutative Bezout ring of weak dimension at most one. If every cyclic flat R-modules is projective, then $st.r.(R) = 2$.*

Theorem 2. *Let R be a commutative Bezout PM-ring of weak dimension at most one. If every cyclic flat R-modules is projective, then $st.r.(R) = 1$.*

Theorem 3. *Let R be a commutative Bezout PM-ring such that $minR$ is compact, then $st.r.(R) = 1$.*

Theorem 4. *Let R be a commutative Bezout domain with nonzero Jacobson radical. If for every nonzero element a of R a factor-ring R/aR is reduced, then $st.r.(R) = 1$.*

M.V. ZADNIPRIANYI

On some normal properties of the S-representation of real numbers

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The expansion of an arbitrary number $x \in (0,1]$ into the Sylvester series
$x = \sum\limits_{i=1}^{\infty} \frac{1}{q_i} = \Delta_{q_1 q_2 \ldots q_n \ldots}$, where $q_1 \geq 2$, $q_{n+1} \geq q_n(q_n - 1) + 1$
can be rewritten in more convenient form:
$x = \overline{\Delta}^{S}_{g_1 g_2 \ldots g_n \ldots}$, де $g_1 = q_1 - 1$, $g_{k+1} = q_{k+1} - q_k(q_k - 1)$. The latter formal designation of the Sylvester expansion of x is called an *S-representation* and $g_k = g_k(x)$ is the *k-th symbol* of the S-representation. Since every number $x \in (0,1]$ can be uniquely expanded into the Sylvester series the given above definitions are correct.

A property of real numbers is called to be *normal* if it is valid for almost all (in the sense of Lebesgue measure) real numbers. That is, the set of real numbers which do not possess this property has Lebesgue measure zero.

Theorem 1. *The set of real numbers $x \in [0,1]$ whose symbols of S-representation are bounded is a continuous zero-set of Lebesgue.*

Corollary 1. *For almost all (in the sense of Lebesgue measure) $x \in [0,1]$ the equality $\lim\limits_{k\to\infty} \sup g_k(x) = \infty$ holds true.*

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On Lockett conjecture for Fitting classes that are not Lockett classes

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This paper deals only with finite groups.

In the seventies of the XX century within the solvable group theory a number of problems evolved that were connected with Fitting classes structural theory development. Among them the central place belonged to the common problem of Fitting class structure determination that is known within the group classes theory under the name "Lockett conjecture".

Locket conjecture ([1]). Each Fitting class $\mathfrak{F}$ coincides with the intersection of a certain normal Fitting class $\mathfrak{X}$ and $\mathfrak{F}^*$.

Fitting class $\mathfrak{F}$ that meets Lockett conjecture will be called $\mathfrak{L}$-class. For arbitrary local Fitting classes the indicated conjecture was proved within a solvable case in 1988 by N.T.Vorob'ev [2] and within an arbitrary case in 1996 by Gallego [3]. The following theorem is proved.

Theorem 1. *Let $\mathfrak{F} = (\mathfrak{S}_{p'})_*\mathfrak{Y}$ where $\mathfrak{Y}$ is such local Fitting class that $\mathfrak{S}_{p'} \cap \mathfrak{Y} = (1)$. Then $\mathfrak{F}$ is $\mathfrak{L}$-class that is not Lockett class.*

[1] Lockett P. *The Fitting class $\mathfrak{F}^*$* Math. Z. - 1974. - Vol.137, №2. - P. 131-136.

[2] Vorob'ev N.T. *On radical classes of finite groups with Lockett condition* Math. Z. - 1988. - Vol.43, №2. - P. 161-168.

[3] Gallego M.P. *Fitting pairs from direct limits and the Lockett conjecture* Comm. Algebra - 1996 - Vol.24, №6. - P. 2011-2023.

I.V. ZAMRIY

On a system of expansion of numbers with infinite alphabet

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Let $Q_3 = \{q_0, q_1, q_2\}$ be the set of positive real numbers defined by $q_0 + q_1 + q_2 = 1; \beta_0 = 0, \beta_1 = q_0, \beta_2 = q_0 + q_1$.

For a real number $x \in [0, 1]$, we let

$$x = \beta_{\alpha_1} + \sum_{k=2}^{\infty}[\beta_{\alpha_k} \prod_{j=1}^{k-1} q_{\alpha_j}] = \triangle^{Q_3}_{\alpha_1\alpha_2\ldots\alpha_n\ldots},$$

where $\alpha_n \in \{0, 1, 2\}$, denote the Q_3-expansion of x.

We investigated properties of sets of numbers defined by their modified Q_3-expansion: $x = \overline{\triangle}^{Q_3}_{c_1c_2c_3\ldots}$, where $\overline{\triangle}^{Q_3}_{c_1c_2c_3\ldots} =$
$= \triangle^{Q_3}_{\underbrace{0\ldots0}_{c_1}\underbrace{1\ldots1}_{c_2}\underbrace{2\ldots2}_{c_3}\underbrace{0\ldots0}_{c_4}\ldots}$.

A segment

$$\overline{\triangle}^{Q_3}_{c_1,\ldots,c_m} = [\sum_{n=1}^{m} c_n a_n; \sum_{n=1}^{m} c_n a_n + \sum_{n=m+1}^{\infty} \varepsilon_n a_n], \varepsilon_n \in \{0, 1, 2\}$$

is called a cylindrical segment of m-th rank with the base $c_1, \ldots, c_m$.

We obtain the following properties of the cylindrical segment:

1)$\overline{\triangle}^{Q_3}_{c_1\ldots c_m} = \overline{\triangle}^{Q_3}_{c_1\ldots c_m 0} \bigcup \overline{\triangle}^{Q_3}_{c_1\ldots c_m 1} \bigcup \overline{\triangle}^{Q_3}_{c_1\ldots c_m 2}$;

2) $\inf \overline{\triangle}^{Q_3}_{c_1\ldots c_m} = \inf \overline{\triangle}^{Q_3}_{c_1\ldots c_m 0}$; $\sup \overline{\triangle}^{Q_3}_{c_1\ldots c_m} = \sup \overline{\triangle}^{Q_3}_{c_1\ldots c_m 2}$;

3)$|\overline{\triangle}^{Q_3}_{c_1\ldots c_m}| = r_m \to 0, m \to \infty$;

4) $\bigcap_{m=1}^{\infty} \overline{\triangle}^{Q_3}_{c_1\ldots c_m} \equiv \overline{\triangle}^{Q_3}_{c_1\ldots c_m\ldots} = x \in [0, 1]$;

5)$\dfrac{|\overline{\triangle}^{Q_3}_{c_1\ldots c_m c}|}{|\overline{\triangle}^{Q_3}_{c_1\ldots c_m}|} = \dfrac{r_{m+1}}{a_{m+1} + r_{m+1}} = \dfrac{1}{\delta_{m+1} + 1}$, $\delta_{m+1} = \dfrac{a_{m+1}}{r_{m+1}}$;

6)$\overline{\triangle}^{Q_3}_{c_1\ldots c_m} = \overline{\triangle}^{Q_3}_{s_1\ldots s_k} \Leftrightarrow m = k \wedge \sum_{n=1}^{m}(c_n - s_n)a_n = 0$.

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On a class of polynomials of partitions

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Studying connections polynomials partitions [1], defined by some linear recurrent equations with triangular matrices parafunctions [2].

Theorem 1. *Let polynomials* $y_n(x_1, x_2, \ldots, x_n), n = 0, 1, \ldots$ *defined recurrent equations*

$$y_n = x_1 y_{n-1} - x_2 y_{n-2} + \ldots + (-1)^{n-2} x_{n-1} y_1 + (-1)^{n-1} a_n x_n y_0,$$

where $y_0 = 1$, *then just equality*

$$y_n = \left\langle \begin{array}{cccc} a_1 x_1 & & & \\ a_2 \frac{x_2}{x_1} & x_1 & & \\ \vdots & \ldots & \ddots & \\ a_n \frac{x_n}{x_{n-1}} & \ldots & \frac{x_2}{x_1} & x_1 \end{array} \right\rangle,$$

$$y_n =$$

$$= \sum_{\lambda_1 + 2\lambda_2 + \ldots + n\lambda_n = n} (-1)^{n-k} \left(\sum_{i=1}^{n} \lambda_i a_i \right) \frac{(k-1)!}{\lambda_1! \lambda_2! \cdot \ldots \cdot \lambda_n!} x_1^{\lambda_1} x_2^{\lambda_2} \cdot \ldots \cdot x_n^{\lambda_n},$$

where $k = \lambda_1 + \lambda_2 + \ldots + \lambda_n$.

Just as a similar theorem for recursive equations of the form

$$y_n = x_1 y_{n-1} + x_2 y_{n-2} + \ldots + x_{n-1} y_1 + a_n x_n y_0,$$

where $y_0 = 1$.

[1] *Bell E.T.* Partition polynomials. — Ann. Math. 29, 1927, p. 38 – 46.

[2] *Zatorsky R.A.* Calculus triangular matrices and its application. Ivano-Frankivs'k, Simyk, 2010 y., 508 p.

Andrei V. ZAVARNITSINE

Finite Groups whose Prime Graph has Five Connected Components

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The *prime graph* $\Gamma(G)$ of a finite group G is the graph whose vertex set is the set $\pi(G)$ of prime divisors of the order $|G|$ in which two distinct vertices $p, q \in \pi(G)$ are joined by an edge if and only if G contains an element of order pq. Let $s(G)$ denote the number of connected components of $\Gamma(G)$. From [1, 2] it follows that $s(G) \le 6$ for every group G. In fact, the sporadic group J_4 is the only finite group G such that $s(G) = 6$, see [3]. We classify the groups with $s(G) = 5$.

Theorem 1. *Let G be a finite group with $s(G) = 5$. Then G is simple. In particular, $G \cong E_8(q)$ with $q \equiv 0, \pm 1 \pmod 5$.*

In order to prove Theorem 1, we had to obtain some results about representations of simple groups. One such result stated below may be of independent interest.

Theorem 2. *Let $G = {}^3D_4(q)$ act on a vector space V over a field of characteristic not dividing q (possibly, zero). Then every element of G of order $q^4 - q^2 + 1$ fixes on V a nonzero vector.*

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[3] A. V. Zavarnitsine, On recognition of finite groups by the prime graph, *Algebra and Logic*, **45**, N 4 (2006), 220–231.

Vasyl ZEMBYK

Representations of nodal algebras of type A

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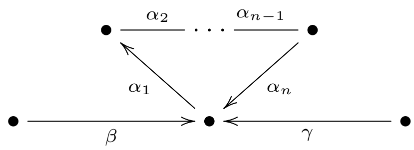
We consider finite dimensional algebras over an algebraically closed field $\mathbf{k}$.

An algebra Λ is called ***nodal*** if there is a hereditary algebra Γ such that $\Gamma \supset \Lambda \supset rad\,\Gamma = rad\,\Lambda$ and $length_\Lambda(\Gamma \otimes_\Lambda U) \leqslant 2$ for any simple A-module U. We say that Λ is of type A if $\Gamma \simeq \mathbf{k}Q$, where Q is a quiver of type A or $\tilde{A}$.

Theorem 1. *A nodal algebra Λ of type A is tame if and only if it is either skewed-gentle or is isomorphic to the algebra Λ_n given by the quiver*



with relations $\alpha_1\alpha_n = \alpha_1\gamma = 0$ or the opposite algebra Λ_n^{op}. Otherwise it is wild.

A complete description of skewed-gentle nodal algebras is given.

A.V. ZHUCHOK

Congruences on trioids

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J.-L. Loday and M.O. Ronco introduced the notions of a trialgebra and a trioid [1]. A set T equipped with three associative operations $\dashv$, $\vdash$ and $\perp$ satisfying the following axioms:

$$(x \dashv y) \dashv z = x \dashv (y \vdash z), (x \vdash y) \dashv z = x \vdash (y \dashv z),$$

$$(x \dashv y) \vdash z = x \vdash (y \vdash z), (x \dashv y) \dashv z = x \dashv (y \perp z),$$

$$(x \perp y) \dashv z = x \perp (y \dashv z), (x \dashv y) \perp z = x \perp (y \vdash z),$$

$$(x \vdash y) \perp z = x \vdash (y \perp z), (x \perp y) \vdash z = x \vdash (y \vdash z)$$

for all $x, y, z \in T$ is called a trioid. A trialgebra is a linear analogue of a trioid. If operations $\vdash$ and $\perp$ ($\dashv$, $\vdash$ and $\perp$) of a trioid coincide, then it becomes a dimonoid [2] (semigroup). For a general introduction and basic theory see [1].

The least idempotent congruence on a trioid with a commutative operation, the least semilattice congruence on a trioid with an idempotent operation and the least separative congruence on a trioid with a commutative operation were described in [3]. Here we characterize all semilattice congruences on an arbitrary trioid, the least group congruence on a trioid with an orthodox semigroup and the least group congruence on a trioid with an inverse semigroup.

[1] Loday J.-L., Ronco M. O. *Trialgebras and families of polytopes.* Contemp. Math., **346** (2004), P. 369–398.

[2] Zhuchok A.V. *Dimonoids.* Algebra i Logika, **50** (2011), no. 4, P. 471–496 (in Russian).

[3] Zhuchok A.V. *Some congruences on trioids.* Fundamentalnaya i Prikladnaya Mat., **17** (2011/2012), no. 3, P. 39-49 (in Russian).

Y.V. ZHUCHOK

The endomorphism monoid of an arbitrary semigroup

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An exact representation of the automorphism group of an arbitrary semigroup was described in [1] by means of a direct product of wreath products of groups. Natural in this direction is a problem of the description of the endomorphism monoid of an arbitrary semigroup. However the construction of a wreath product which was used in [1] is not suitable for description of an exact representation of the endomorphism monoid of a semigroup. Therefore here we use the construction of a wreath product of monoid with small category which was introduced by V.Fleischer [2] as a generalization of a wreath product of monoids and used in different cases (see, e.g., [3], [4]). In terms of this construction we describe a structure of the endomorphism monoid of an arbitrary semigroup.

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[4] Zhuchok Y.V., Kirtatdze L.V. *Endomorphisms of free product of finite semigroups.* Bulletin of Taras Shevchenko National University of Kyiv: Series Physics and Mathematics, **4** (2010), P. 27–30 (in Ukrainian).

Viktor ZHURAVLOV[1], Dmytro ZHURAVLOV[2]

Projective resolvent of irreducible module

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The algorithm for constructing projective resolvent of irreducible module of the tiled order was found. The program based on this algorithm was written to calculate the global dimension of the tiled order.

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Exponent matrices of admissible quiver

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Let Q be the admissible quiver. Denote by $\overline{Q}$ the subquiver of Q, formed by cycles (not necessarily by all) of weight 1, which are passing through all vertices of the quiver Q.

Theorem 1. *Admissible quiver Q has an infinite number of parameters, which are non equivalent to matrices if and only if there exists disconnected quiver $\overline{Q}$.*

Theorem 2. *Let P be a finite poset such that for any partition $P = P' \cup P''$, $P' \cap P'' = \emptyset$ there is a strict inclusion $P'_{\min} \cup P''_{\min} \cup P'_{\max} \cup P''_{\max} \supset P_{\min} \cup P_{\max}$. Then the quiver $\tilde{Q}(P)$ is not rigid.*

[1] M. Hazewinkel, N. Gubareni and V. V. Kirichenko, *Algebras, Rings and Modules*. Vol. 1, Series: Mathematics and Its Applications, **575**, Kluwer Acad. Publish., 2004. xii+380pp.

[2] M. Hazewinkel, N. Gubareni and V. V. Kirichenko, *Algebras, Rings and Modules*. Vol. 2, Series: Mathematics and Its Applications, **586**. Springer, Dordrecht, 2007. xii+400pp.

Olesya ZUBARUK

On ideals of Lie algebras associated with lower central series of p-Sylow subgroups

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We study Lie rings associated with lower central series of p-Sylow subgroups of the group of isometrics of the Bers space, and lattices of ideals of classes of Lie algebras associated with p-Sylow subgroups of symmetric groups.

Міжнародна алгебраїчна конференція
присвячена 100-річчю від дня народження С.М.Чернікова
(Київ, 20-26 серпня 2012 року)

Редакційна колегія:
Ю.Дрозд, А.Петравчук, М.Працьовитий

Комп'ютерний набір і верстка:
І.Раєвська, М.Раєвська

Відповідальні за випуск:
А.Петравчук, І.Раєвська, М.Раєвська

Підп. до друку 30.07.2012. Формат 60x84/16. Папір офс.
Офс. друк. Умов. друк. арк. 9,00. Фіз. друк. арк. 10,40.
Тираж 150 пр. Зам. №84

Інститут математики НАН України
01601, м. Київ - 4, вул. Терещенківська, 3.