Introduction to N-adic numbers

Practical Applications

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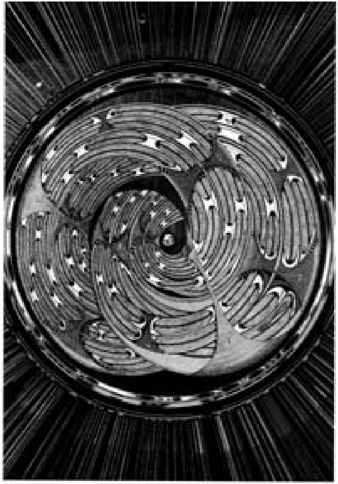

Figure : A.Fomenko, 2-adic solenoid

Outline

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N-adic analysis is key to understanding the logic of the processor.

Not having deep understanding of the nature of the essences we work with, we have to strictly follow the rules that guarantee correct work results.

However, the rules limit the range of solvable tasks, while violation of them may lead to an unpredictable result. Yet, such violation might be appropriate, given that we are fully aware of what we are doing.

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Sufficiency

Consideration for the following reasons ultrametrics (non-Archimedean metric spaces) is natural:

- in the context of a real analysis the computer is a discrete system, but in terms of a 2-adic - continuous
- the modern computer is essentially an analog in terms of non Archimedean analysis

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Quick calculations

2-adic continuity of the processor's basic operations allows the creation of models that use floating point numbers, but all calculations are made in the set of integers.

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Theory

Ultrametrics

Basics of non Archimedean analysis. Ostrowski theorem.

Izometries

Izometrical polynomials of the ring Z_2. Inductive construction of isometries.

Hensel's lemma

Extend uniquely the root of polynomial in $\mathbb{Z}_p$ to of the root of a polynomial in Z_p.

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Practice

N-adic Arithmetics

Basic of arithmetics of the ring Z_2 - addition, multiplication, division.

Presentation of real numbers in Z_2

Rational numbers in Z_2. Some irrationals numbers in Z_2.

Fast Calculations

Using of 2-adic arithmetic for providing computational scheme close to the processor architecture.

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Links

This course can be used as the basic for the course "Groups' automatous and the automorphisms' group of the regular rooted tree "

Fernando Q. Gouvêa
P-Adic Numbers: An Introduction .
Springer (1997)

Nil Koblitz
P-adic Numbers, p-adic Analysis, and Zeta-Functions .
Mir (1981)

Denis Morozov
Differentiable finite-state izometries and izometric polynomials of the ring of integer 2-adic numbers.
8th Int. Algebraic Conf. in Ukraine: Abstr.
Lugansk, July 2011