

# Convolution with theoretical-number methods

Two dimensional convolutions of lengths  $2^n$

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# Outline

- 1 Motivation
  - The Basic Problem That We Studied
  - Previous Work
- 2 Results
  - Main Results
  - Basic Ideas for Implementation

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We study the convolution algorithms are well adapted for the processing of complex images

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# Applications Of Two-Dimensional Number-Theoretical Convolutions

Integer FFT applied in various areas related to image processing and finding a correlation, such as tomography and processing of images taken by satellite. The motivation to use number-theoretical FFT is

- 32 bit integer algorithms have good specifications for optimizing on existing processors
- number-theoretical Fourier transform works well for convolutions with large kernels

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# The Required Basic Information

## Power of Algebra and Number Theory

We were studied classical works using algebraic methods for calculating the convolution. Different techniques, including the arithmetic of finite fields, linear algebra and group theory have been used.

They were adapting to the implementation of algorithms for discrete convolution in C++ language based processor architecture.

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formulation of the problem Valentin Vovk, Mobile Lab 2

Implemented a two-dimensional convolution with algebraic methods for the size of  $2^n \times 2^m$

- $0 \leq m, n \leq 10$ .

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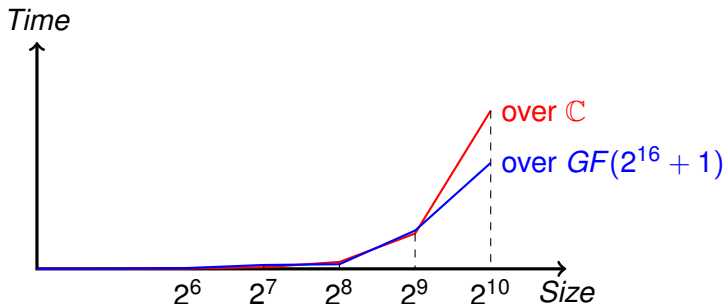
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# Plot Test Results

tests conducted by Andrei Zavorotny, Mobile Lab 2

As we see algebraic methods for convolution yield the best results on large sizes





# Test Results

- Test results for large sizes

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During construction of the algorithm we are

- reducing the number of using modulo field's size
- using the FFT algorithm of length 32 based on symmetry of transform matrix
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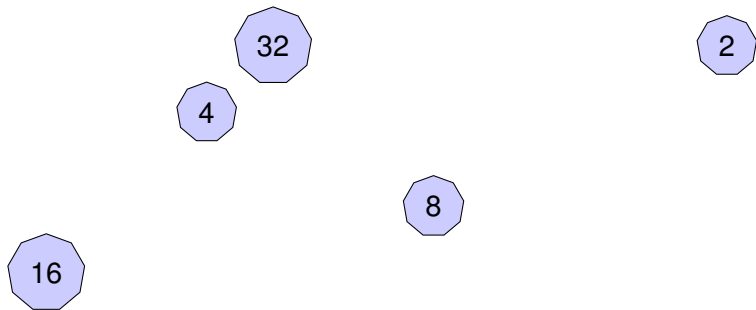
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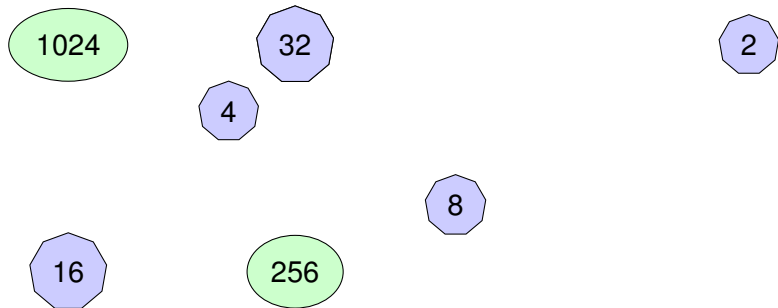
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# Cooley-Tukey scheme for for FFT

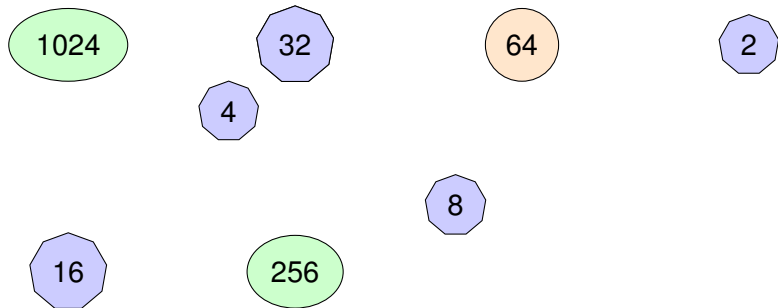


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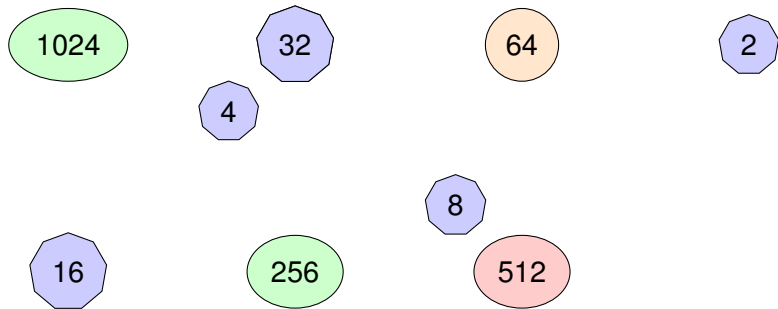




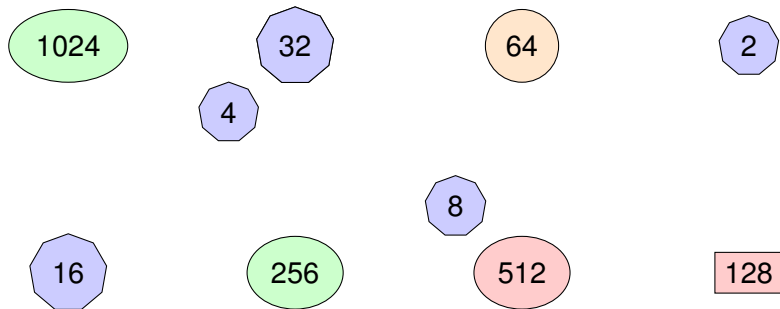
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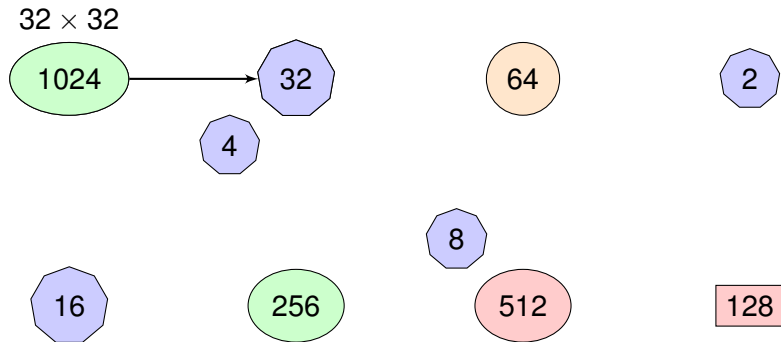
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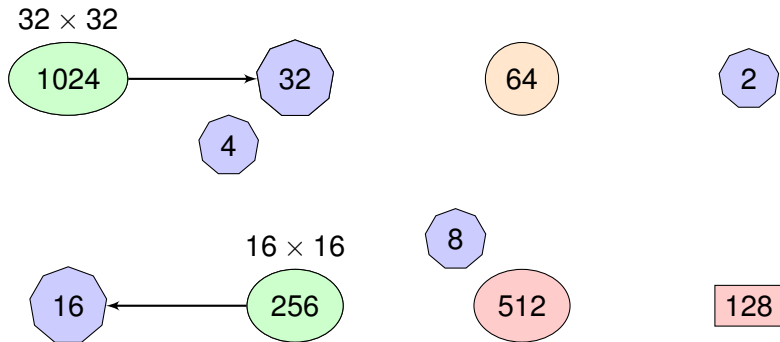
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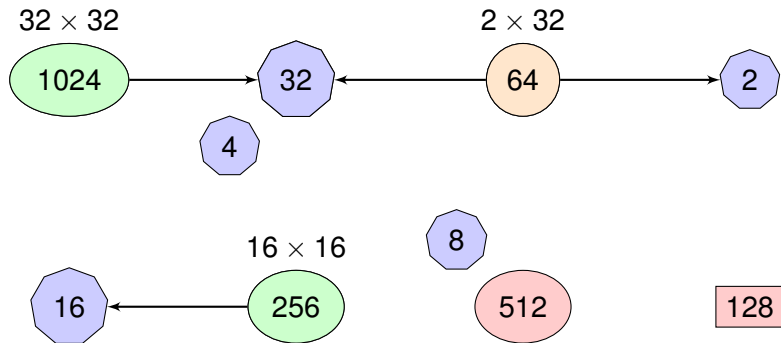
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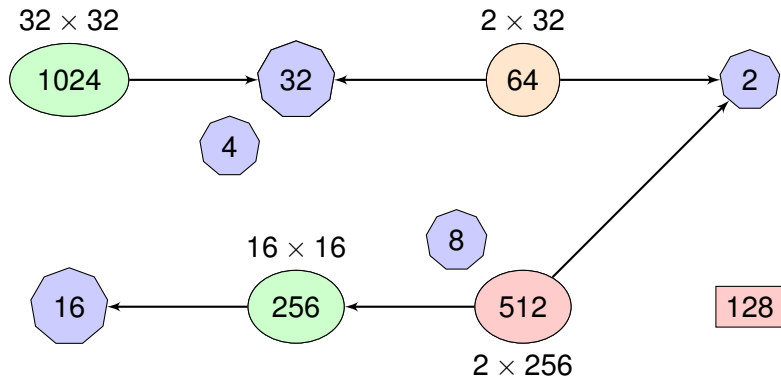
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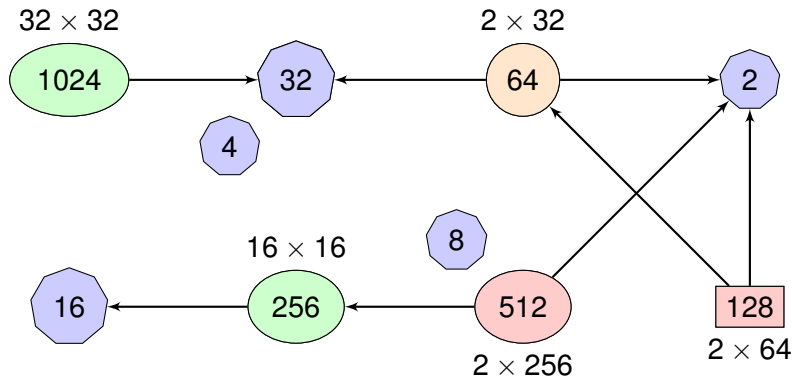
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- Possible sizes are - 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024
- Outlook
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# For Further Reading I



B. Bleihut

*Fast Algorithms for Digital Signal Processing [Russian translation].*

Mir, Moscow (1989)



D. Morozov.

The calculation of the convolution with the number-theoretical transforms

*Report for Mobile Lab 2, 8 pages, December 2011.*