

2-adic numbers arithmetic

Practical Applications

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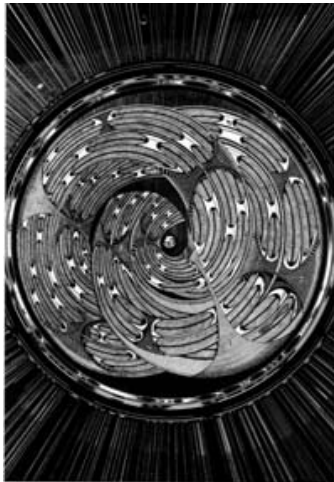


Figure: A.Fomenko, 2-adic solenoid

Outline

1 Objective

- Introduction
- Basic properties of ultrametrics

2 Description

- Key components of the course
- Possible directions for further study

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2-adic analysis is key to understanding the logic of the processor.

Not having deep understanding of the nature of the essences we work with, we have to strictly follow the rules that guarantee correct work results.

However, the rules limit the range of solvable tasks, while violation of them may lead to an unpredictable result. Yet, such violation might be appropriate, given that we are fully aware of what we are doing.

Clear understanding of the nature of a processor's arithmetic is necessary for building adequate computational models for problems encountered in the process of software development.

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Sufficiency

Consideration for the following reasons ultrametrics (non-Archimedean metric spaces) is natural:

- in the context of a real analysis the computer is a discrete system, but in terms of a 2-adic - continuous
- the modern computer is essentially an analog in terms of non Archimedean analysis

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Quick calculations

2-adic continuity of the processor's basic operations allows the creation of models that use floating point numbers, but all calculations are made in the set of integers.

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Theory

Ultrametrics

Basics of non Archimedean analysis. Ostrowski theorem.

Izometries

Izometrical polynomials of the ring \mathbb{Z}_2 . Inductive construction of isometries.

Hensel's lemma

Extend uniquely the root of polynomial in \mathbb{Z}_p to of the root of a polynomial in \mathbb{Z}_p .

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Practice

2-adic Arithmetics

Basic of arithmetics of the ring \mathbb{Z}_2 - addition, multiplication, division.

Presentation of real numbers in \mathbb{Z}_2

Rational numbers in \mathbb{Z}_2 . Some irrationals numbers in \mathbb{Z}_2 .

Fast Calculations

Using of 2-adic arithmetic for providing computational scheme close to the processor architecture.

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Links

This course can be used as the basic for the course "Groups' automata and the automorphisms' group of the regular rooted tree "

Bibliography I



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Denis Morozov

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