Introduction to N-adic numbers

Practical Applications

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Outline

1. Motivation
   - Objective
   - Basic Properties
2. Results
   - Main Results
   - Basic Ideas for Implementation

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Sufficiency

Consideration for the following reasons ultrametrics (non-Archimedean metric spaces) is natural:

- in the context of a real analysis of the computer is a discrete system, but in terms of a 2-adic - continuous
- the modern computer is essentially an analog in terms of non Archimedean analysis

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Quick calculations

2-adic continuity of the basic operations of the processor allows the creation of models that use floating point numbers, and all calculations are made in the set of integers.

- 32 bit integer algorithms have good specifications for optimizing on existing processors
- number-theoretical Fourier transform works well for convolutions with large kernels

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formulation of the problem Valentin Vovk, Mobile Lab 2

Implemented a two-dimensional convolution with algebraic methods for the size of $2^n \times 2^m$

- $0 \leqslant m, n \leqslant 10$.

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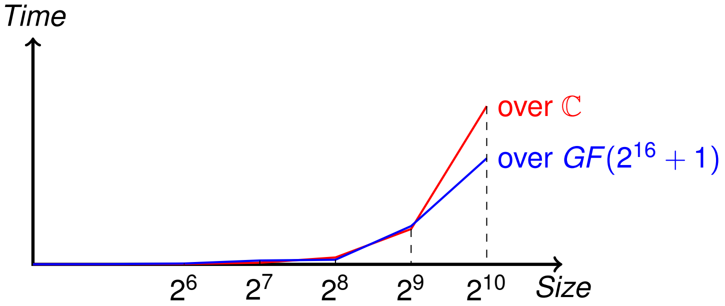
Plot Test Results

tests conducted by Andrei Zavorotny, Mobile Lab 2

As we see algebraic methods for convolution yield the best results on large sizes



Test Results

- Test results for large sizes

Size	1024	512	256
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Ferma	1402	506	61.12

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- reducing the number of using modulo field's size
- using the FFT algoritm of length 32 based on symmetry of transform matrix
- transition from 64-bit to 32-bit arithmetic

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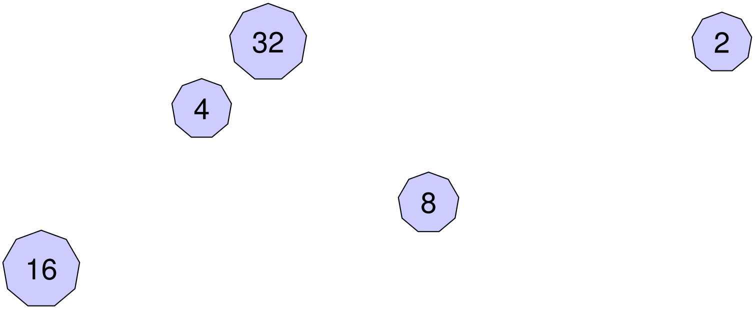
Cooley-Tukey scheme for for FFT



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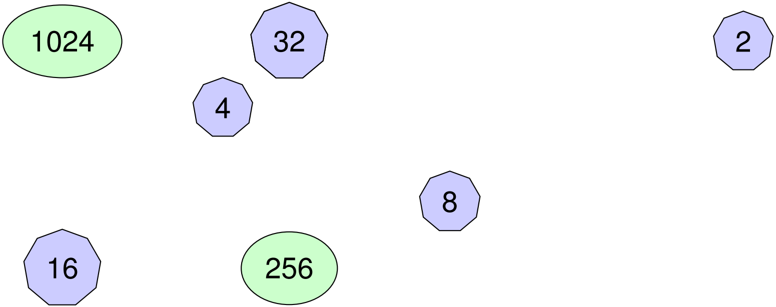

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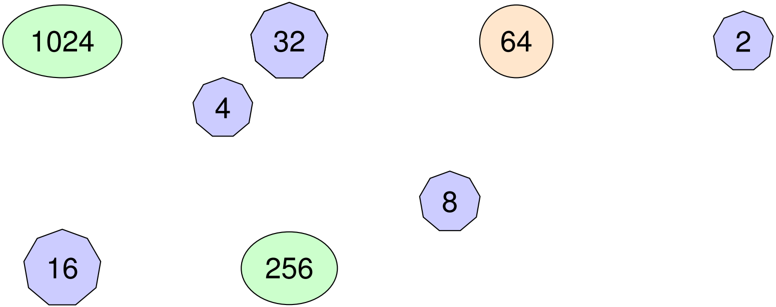

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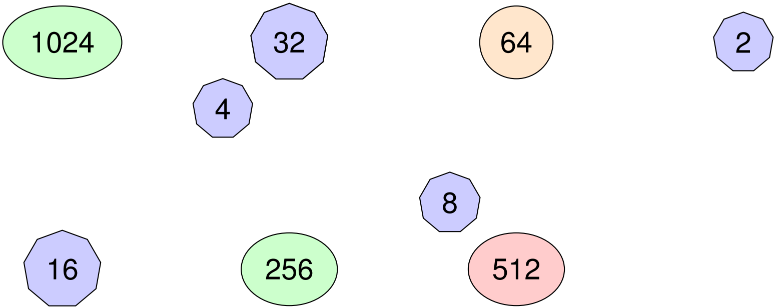

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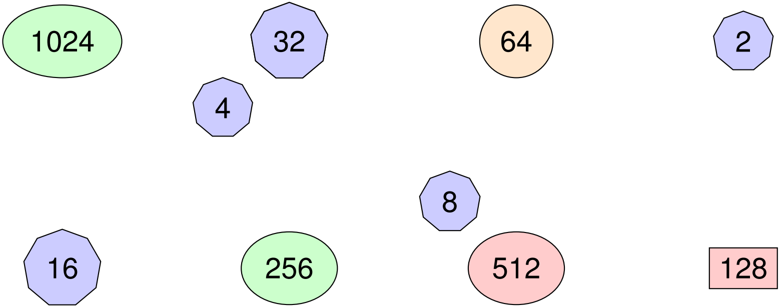

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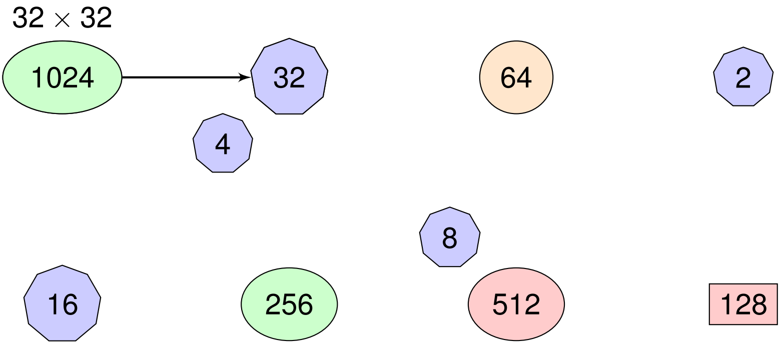

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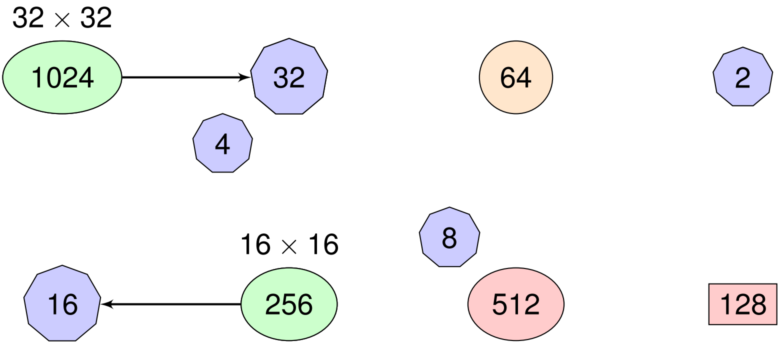

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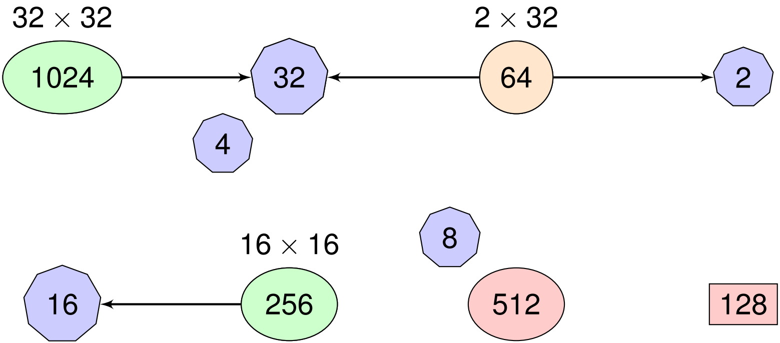

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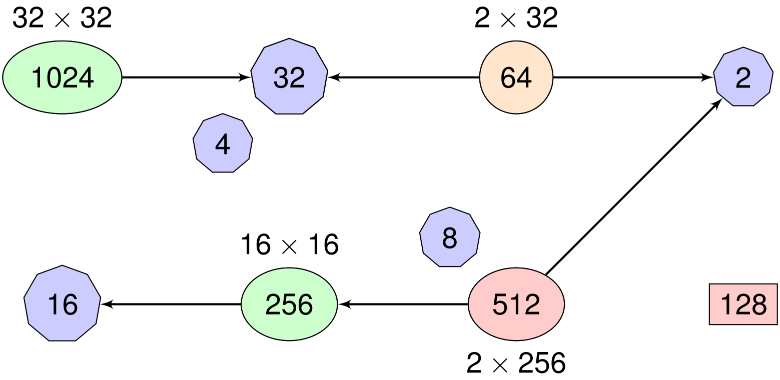

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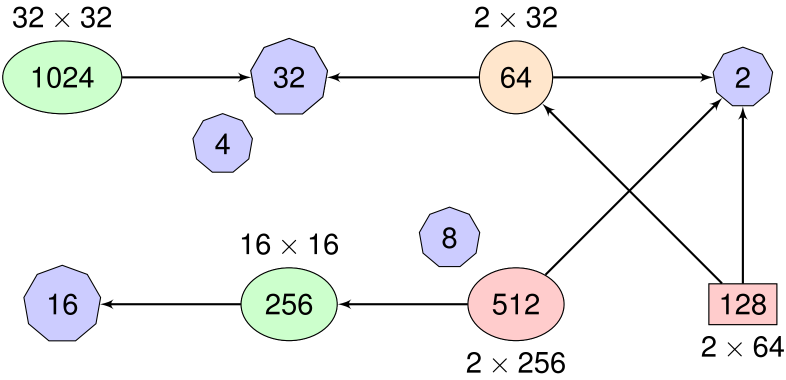

Summary

- In this work implemented a two-dimensional integer convolution
- Possible sizes are - 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024

- Outlook
  - Next task to investigate the possibility of constructing fast two-dimensional convolution for sizes that are not powers of two

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For Further Reading I

Fernando Q. GouvÃ■a
P-Adic Numbers: An Introduction .
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Denis Morozov
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