Convolution with theoretical-number methods

Two dimensional convolutions of lengths 2^n

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Outline

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1. Motivation
   - The Basic Problem That We Studied
   - Previous Work
2. Results
   - Main Results
   - Basic Ideas for Implementation

Main Characteristics Of The Studied Algorithms

We study the convolution algorithms are well adapted for the processing of complex images

- pay special attention to ease of implementation and speed performance of these algorithms
- impose the condition of no computational errors in these algorithms

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Applications Of Two-Dimentional Number-Theoretical Convolutions

Integer FFT applied in various areas related to image processing and finding a correlation, such as tomography and processing of images taken by satellite. The motivation to use number-theoretical FFT is

- 32 bit integer algorithms have good specifications for optimizing on existing processors
- number-theoretical Fourier transform works well for convolutions with large kernels

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The Required Basic Information

Power of Algebra and Number Theory

We were studied classical works using algebraic methods for calculating the convolution. Different techniques, including the arithmetic of finite fields, linear algebra and group theory have been used.

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formulation of the problem Valentin Vovk, Mobile Lab 2

Implemented a two-dimensional convolution with algebraic methods for the size of $2^n \times 2^m$

- $0 \leqslant m, n \leqslant 10.$

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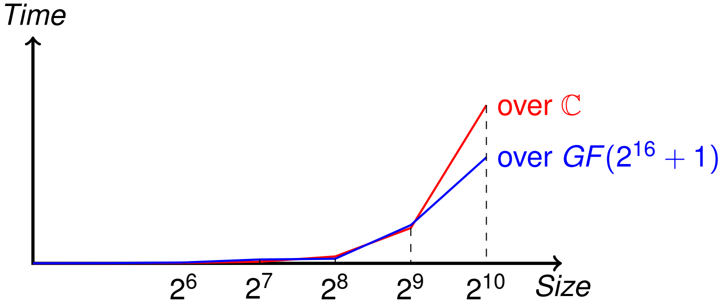
Plot Test Results

`tests conducted by Andrei Zavorotny, Mobile Lab 2`

As we see algebraic methods for convolution yield the best results on large sizes



Test Results

- Test results for large sizes

Size	1024	512	256
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- reducing the number of using modulo field's size
- using the FFT algoritm of length 32 based on symmetry of transform matrix
- transition from 64-bit to 32-bit arithmetic

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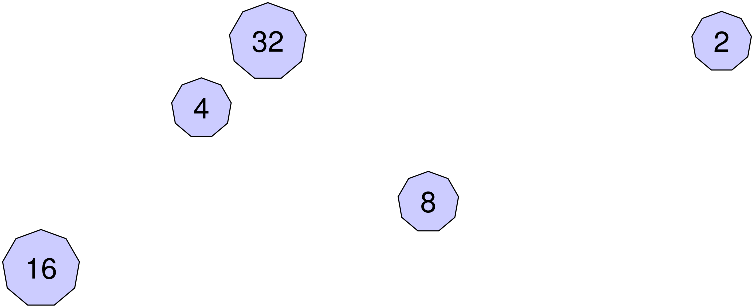
Cooley-Tukey scheme for for FFT



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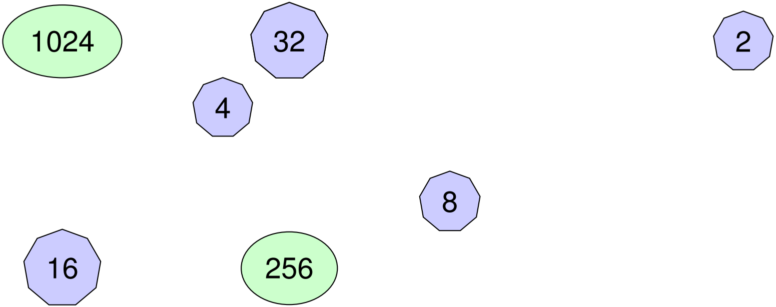

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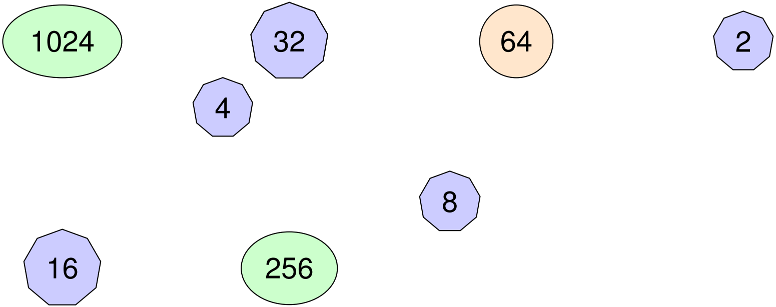

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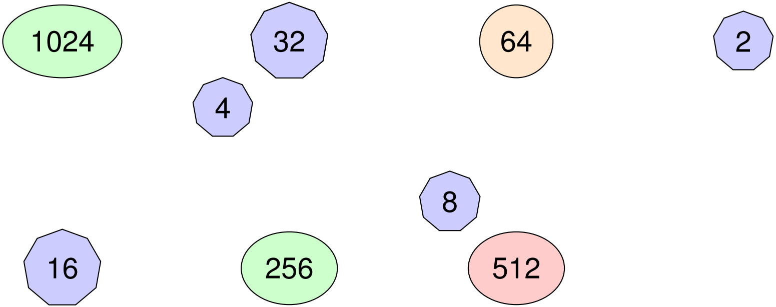

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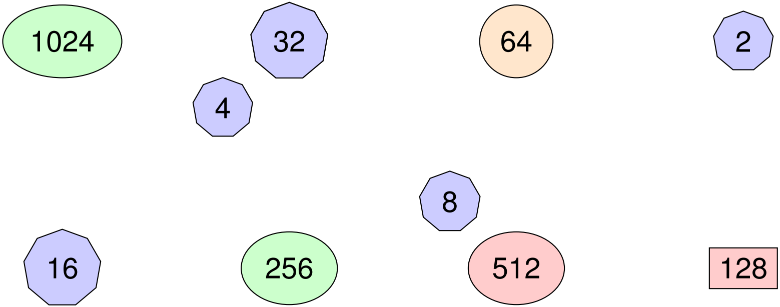

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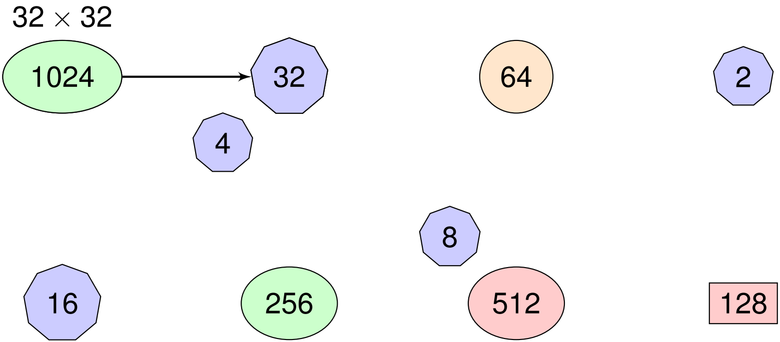

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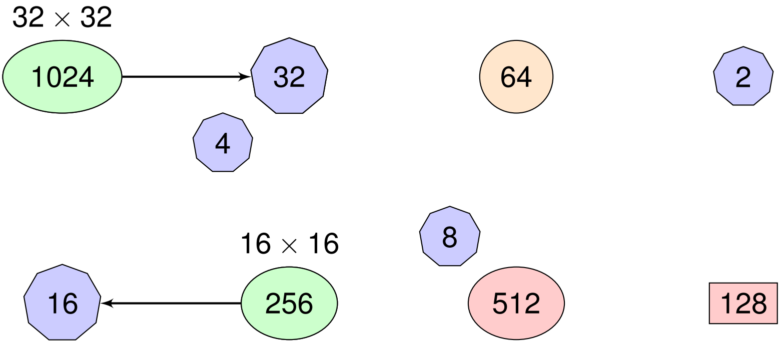

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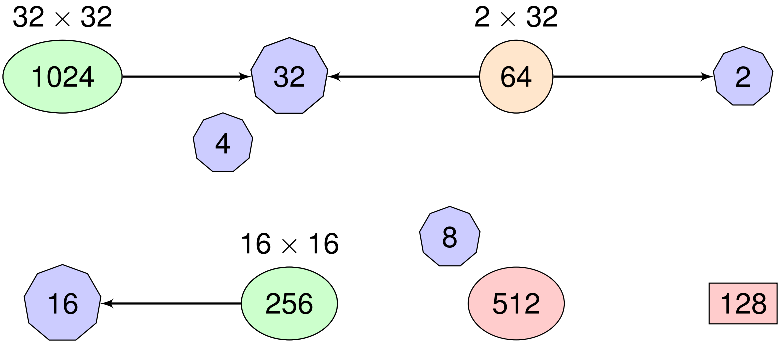

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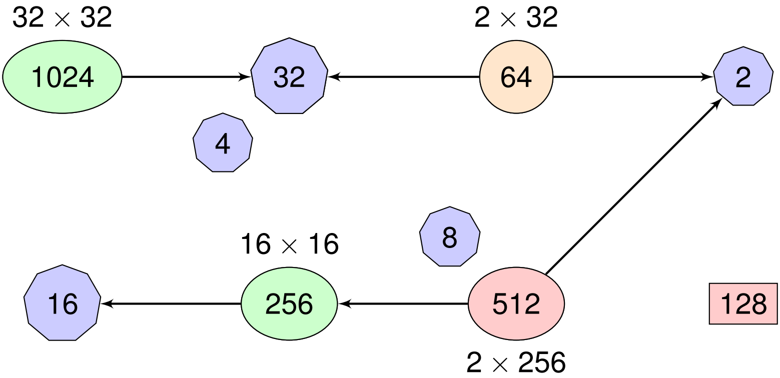

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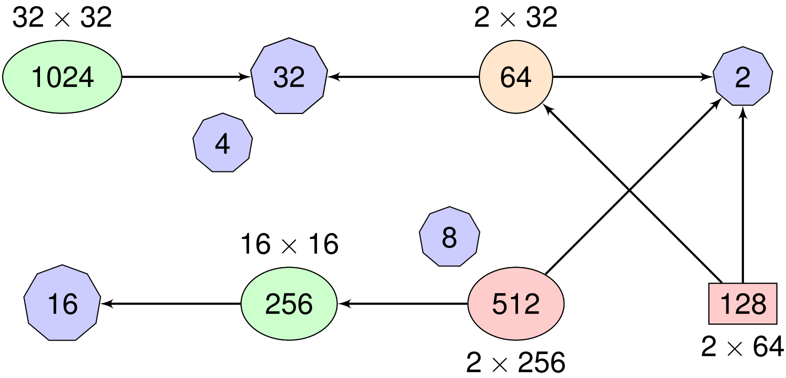

Summary

- In this work implemented a two-dimensional integer convolution
- Possible sizes are - 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024

- Outlook
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For Further Reading I

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