1 Коэффициент растяжения матрицы

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} * \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} a_{11} * x_1 + a_{12} * x_2 \\ a_{21} * x_1 + a_{22} * x_2 \end{pmatrix}$$

$$MAX = \max_{x_1 \in \mathbb{R}, x_2 \in \mathbb{R}} \frac{\left\| \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} * \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \right\|_{\mathbb{R}^2}}{\left\| \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \right\|_{\mathbb{R}^2}} = \max_{0 \le \alpha \le 2\pi} \left\| \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} * \begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix} \right\|_{\mathbb{R}^2}$$

$$\begin{aligned} \left\| \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix} \right\|_{\mathbb{R}^2} &= \left\| \begin{pmatrix} a_{11}\cos\alpha + a_{12}\sin\alpha \\ a_{21}\cos\alpha + a_{22}\sin\alpha \end{pmatrix} \right\|_{\mathbb{R}^2} = \\ &= \sqrt{(a_{11}\cos\alpha + a_{12}\sin\alpha)^2 + (a_{21}\cos\alpha + a_{22}\sin\alpha)^2} = \\ &= \sqrt{(a_{11}^2 + a_{21}^2)\cos^2\alpha + (a_{12}^2 + a_{22}^2)\sin^2\alpha + (a_{11}a_{12} + a_{21}a_{22})2\sin\alpha\cos\alpha} \end{aligned} \quad (1)$$

$A_1 = a_{11}^2 + a_{21}^2$
$A_2 = a_{12}^2 + a_{22}^2$
$A_3 = a_{11}a_{12} + a_{21}a_{22}$
$A = A_1 - A_2$
$B = 2A_3$

$$F(\alpha) = A_1\cos^2\alpha + A_2\sin^2\alpha + A_3\sin 2\alpha$$

$(\cos^2\alpha)'_\alpha = -\sin 2\alpha$
$(\sin^2\alpha)'_\alpha = \sin 2\alpha$

$$F'_\alpha(\alpha) = (A_2 - A_1)\sin 2\alpha + 2A_3\cos 2\alpha = -A\sin 2\alpha + B\cos 2\alpha$$

IF $B > 0$

IF $A > 0$

$$MAX = \sqrt{F(\tfrac{1}{2}\operatorname{arctg}\tfrac{B}{A})} = \sqrt{F(\tfrac{1}{2}\operatorname{arctg}\tfrac{2(a_{11}a_{12}+a_{21}a_{22})}{(a_{11}^2+a_{21}^2)-(a_{12}^2+a_{22}^2)})}$$

ELSE IF $A < 0$

$$MAX = \sqrt{F(\tfrac{\pi}{4} + \tfrac{1}{2}\operatorname{arctg}\tfrac{A}{B})}$$

ELSE

$$MAX = \sqrt{F(\tfrac{\pi}{4})}$$

ENDIF

ELSEIF $B = 0$

IF $A > 0$

$$MAX = \sqrt{F(\pi)}$$

ELSEIF $A < 0$

$$MAX = \sqrt{F(\tfrac{\pi}{2})}$$

ELSE

$$MAX = \sqrt{a_{11}^2 + a_{21}^2}$$

ENDIF

ELSE

IF $A < 0$

$$MAX = \sqrt{F(\tfrac{\pi}{2} + \tfrac{1}{2}\operatorname{arctg}(\tfrac{B}{A}))}$$

ELSE IF $A > 0$

$$MAX = \sqrt{F(\tfrac{\pi}{4} - \tfrac{1}{2}\operatorname{arctg}\tfrac{A}{B})}$$

ELSE

$$MAX = \sqrt{F(\tfrac{3\pi}{4})}$$

ENDIF

ENDIF