

Outline

Objective

Introduction

Basic properties of ultrametrics

Description

Key components of the course

Possible directions for further study

Introduction

N-adic analysis is key to understanding the logic of the processor.

Not having deep understanding of the nature of the essences we work with, we have to strictly follow the rules that guarantee correct work results.

However, the rules limit the range of solvable tasks, while violation of them may lead to an unpredictable result. Yet, such violation might be appropriate, given that we are fully aware of what we are doing.

Clear understanding of the nature of a processor's arithmetic is necessary for building adequate computational models for problems encountered in the process of software development.

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Sufficiency

Consideration for the following reasons ultrametrics (non-Archimedean metric spaces) is natural:

- ▶ in the context of a real analysis of the computer is a discrete system, but in terms of a 2-adic - continuous
- ▶ the modern computer is essentially an analog in terms of non Archimedean analysis

Quick calculations

2-adic continuity of the basic operations of the processor allows the creation of models that use floating point numbers, and all calculations are made in the set of integers.

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Theory

Ultrametrics

Basic of non Archimedean analysis. Ostrowski theorem.

Isometries

Isometrical polynomials of the ring \mathbb{Z}_2 . Inductive construction of isometries.

Hensel's lemma

Extend uniquely the root of polynomial in \mathbb{Z}_p to of the root of a polynomial in \mathbb{Z}_p .

Practice

N-adic Arithmetics

Basic of arithmetics of the ring Z_2 - addition, multiplication, division.

Presentation of real numbers in Z_2

Rational numbers in Z_2 . Some irrationals numbers in Z_2 .

Fast Calculations

Using of 2-adic arithmetic for providing computational scheme close to the processor architecture.

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Applications

Using techniques discussed in the course is possible to build computational models that are close to the processor architecture.

As a result, these models have good performance characteristics. Moreover we have no calculation errors that appear in the calculation with floating point.

Next Course

This course can be used as the basic for the course "Groups' automata and the automorphisms' group of the regular rooted tree "

Bibliography I



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