Prediction of ship resistance using model tests

For all model tests, we have to choose a scale, λ, for the model. The size of the model will be restricted by the size of the tow tank facilities, available carriage speeds, and instrumentation. Generally, the bigger the model the better. In practice, a 6 meter long model is the norm for conventional displacement vessels in large test facilities.

The geometric scale is

$$\lambda = \frac{L_S}{L_M} \qquad (1)$$

so all the lengths scale by λ, surface areas scale by λ^2, and volumes and forces scale by λ^3. We should test at Froude and Reynolds numbers that are equal in both full scale and model scale, but as we have seen, this is not possible, as demonstrated again below. Note that gravity, fluid density, and fluid viscosity are not scaled here.

Constant R_n requires that

$$\begin{aligned} R_{n_M} &= R_{n_S} \\ \frac{V_M L_M}{\upsilon_M} &= \frac{V_S L_S}{\upsilon_S} \\ V_M &= V_S \lambda \end{aligned} \qquad (2)$$

Constant F_n requires that

$$\begin{aligned} F_{n_M} &= F_{n_S} \\ \frac{V_M}{\sqrt{g L_M}} &= \frac{V_S}{\sqrt{g L_S}} \\ V_M &= \frac{V_S}{\sqrt{\lambda}} \end{aligned} \qquad (3)$$

Clearly, it is impossible to satisfy both scaling requirements (eqns (2) & (3)) simultaneously. We have already seen that scaling according to R_n is practically impossible because the model speeds required to achieve equivalent R_n are just too high. As a consequence, we cannot properly model the viscous forces at model scale (recall the effects we have seen on phenomena like boundary layer thickness due to modeling at unequal R_n).

Scaling according to Froude number is practical, so we can model the gravity forces properly, and are thus able to model wave resistance phenomena. In practice, we scale model speed according to F_n so that the wave resistance is modeled correctly[1] and we choose a model size that, in conjunction with the model speed, should give Reynolds numbers that are high enough to ensure a turbulent flow regime. Ensuring turbulent flow at model scale does not constitute modeling according to the Reynolds scaling law, but it does mean that we can estimate viscous resistance effects without risking the errors associated with having different flow regimes at model scale (laminar) and full scale (turbulent).

[1] Note that Froude scaling implies a time scale of

$$\frac{T_S}{T_M} = \sqrt{\lambda}$$

so that something that takes place over, say, 10 seconds in ship scale will occur faster at model scale, depending on the scale factor. If the scale is 100, then 10 seconds at full scale should occur in 1 second at model scale.

In model tests, we measure the total resistance, not the separate components. We know that the total resistance is due primarily to two components: friction and waves. We also know that while the wave resistance is modeled correctly, the frictional resistance is not, which means that the total resistance is not. How do we proceed? We have to calculate frictional resistance for both the model and ship using empirically based equations. One such approach was established by the International Towing Tank Conference (ITTC-57 method), which is summarized below.

ITTC-57 method

Tow the model at constant speed over a range of speeds from low speeds to speeds exceeding the design, or trial speed. Measure total resistance.

1. From the test results, calculate C_{TM} at each speed:

$$C_{TM} = \frac{R_{TM}}{\frac{1}{2}\rho_M V_M^2 S_M}$$

where R_{TM} = total model resistance measured;
ρ_M = fresh water density (which is a function of temperature);
V_M = model speed;
S_M = model wetted surface.

2. Calculate the model frictional resistance coefficient C_{FM} using the ITTC-57 Model-Ship Correlation Line at each speed:

$$C_{FM} = \frac{0.075}{\left(\log_{10} R_{n_M} - 2\right)^2}$$

where

$$R_{n_M} = \frac{V_M L_M}{\upsilon_M}$$

and fluid viscosity is a function of the fresh water temperature. (Note that this is not truly a friction line. It is a model-ship correlation line, which embodies some contemporary modeling errors that persist to this day through its continued use.)

3. Calculate the residuary resistance coefficient C_R at each speed:

$$C_R = C_{TM} - C_{FM}$$

Note that the residuary resistance coefficient is the same at model and full scales.

4. Calculate the ship frictional resistance coefficient C_{FS} (for a smooth hull) using the ITTC-57 Model-Ship Correlation Line at each speed:

$$C_{FS} = \frac{0.075}{\left(\log_{10} R_{n_S} - 2\right)^2}$$

where

$$R_{n_S} = \frac{V_S L_S}{\upsilon_S}$$

and fluid viscosity is a function of the sea water temperature.

5. Calculate the total resistance coefficient for a smooth ship:

$$C_{TS} = C_{FS} + C_R$$

6. Add in an "incremental resistance coefficient" C_A to account for surface roughness of the ship. This value can vary, but C_A =0.0004 is a reasoanable default value.

$$C_{TS} = C_{FS} + C_R + C_A$$

7. Calculate the total ship resistance for each speed:

$$R_{TS} = C_{TS} \frac{1}{2} \rho_S V_S^2 S_S$$

This total resistance is for the naked hull. Drag due to appendages still needs to be added.

ITTC-78 method

The ITTC-57 method was superceded by the ITTC-78 method. The details of the work of the experimental naval architects and researchers who developed and are developing better model testing techniques are interesting and important. Here, we will stick to the outcome.

1. From the test results, calculate C_{TM} at each speed:

$$C_{TM} = \frac{R_{TM}}{\frac{1}{2}\rho_M V_M^2 S_M}$$

2. Calculate the model frictional resistance coefficient C_{FM} using the ITTC-57 Model-Ship Correlation Line at each speed:

$$C_{FM} = \frac{0.075}{\left(\log_{10} R_{n_M} - 2\right)^2}$$

3. Calculate the form factor k from the results of tests at (roughly) $0.12 < F_n < 0.2$. The ITTC has a method, but we'll use Prohaska's method (see handout and see commentary below):

$$\frac{C_T}{C_F} = (1+k) + y\frac{F_n^4}{C_F}$$

where k = 3-D form factor ($1+k$ is the intercept on the C_T/C_F axis);
y = coefficient (slope of the line);
C_F = calculated model frictional resistance coefficient from step 2;
C_T = total measured resistance coefficient from step 1.

4. Calculate the residuary resistance coefficient C_R at each speed:

$$C_R = C_{TM} - (1+k)C_{FM}$$

5. Calculate the ship frictional resistance coefficient C_{FS} (for a smooth hull) using the ITTC-57 Model-Ship Correlation Line at each speed:

$$C_{FS} = \frac{0.075}{\left(\log_{10} R_{n_S} - 2\right)^2}$$

6. Calculate the roughness allowance coefficient:

$$C_A = \left(105\left(\frac{k_s}{L_{WL}}\right)^{1/3} - 0.64\right)\times 10^{-3}$$

where k_s = measure of surface roughness (default value is 150×10^{-6} meters);
L_{WL} = length on the waterline.

7. Calculate the air resistance coefficient:

$$C_{AA} = \frac{A_{VT}}{1000S}$$

where A_{VT} = projected front area of the above water ship.

8. Calculate the total ship resistance coefficient:

$$C_{TS} = (1+k)C_{FS} + C_R + C_A + C_{AA} + C_{Appendages}$$

9. Calculate the total ship resistance & effective power for each speed:

$$R_{TS} = C_{TS}\frac{1}{2}\rho_S V_S^2 S_S$$

$$P_E = RV$$

Refer to a text book or the ITTC reports in order to get more details if and when you are responsible for model tests.

10. Note that it is conventional to give model test results as if the tests were done with 15°C water. The correction is made according to

$$C_{TM15} = C_{TM} - C_{FM} + C_{FM15}$$

where the terms subscripted by 15 should be evaluated as if the water temperature was 15°C.

Comments on Prohaska's method for determining form factor

In step 3 above, a method was described to find the form factor according to the equation:

$$\frac{C_T}{C_F} = (1+k) + y\frac{F_n^4}{C_F}$$

This is just another way of writing that the sum of total resistance is the sum of viscous and wavemaking effects (which are supposed to be independent of each other, as they were by Froude).

$$C_T = C_F(1+k) + C_W$$

where

$$C_W = yF_n^4$$

In other words, the wavemaking resistance coefficient increases with the fourth power of Froude number, or as Froude number is proportional to speed, we see that C_W increases with the fourth power of speed. We can go a step further and show that wavemaking resistance increases with V^6. This should be correct as long as interference effects are negligible

A second equation for surface roughness

Surface roughness coefficient is estimated above according to the equation

$$C_A = \left(105\left(\frac{k_s}{L_{WL}}\right)^{1/3} - 0.64\right)\times 10^{-3}$$

Watson reports Townsin's equation for roughness as

$$C_A = \left(44\left(\left(\frac{k_s}{L_{WL}}\right)^{1/3} - 10R_n^{-1/3}\right) + 0.125\right)\times 10^{-3}$$

How do these compare for a typical ship?

A few ways to estimate wetted surface area

In some circumstances, say in the absence of a detailed lines plan, it may be useful to be able to estimate the wetted surface area based on a few main dimensions. A few methods are shown below (see e.g. Watson's Practical Ship Design), all of which are for SI units and yield S in m^2.

Mumford's equation:

$$S = 1.7LT + C_B LB$$

Harvald's modification of Mumford's equation:

$$S = 1.025(1.7LT + C_B LB)$$

Taylor's equation:

$$S = C\sqrt{\Delta L}$$

where C is a constant that depends on form (see graph in Fig. 6.1, p.165 of Watson's book "Practical Ship Design").

Holtrop and Mennen's equation:

$$S = L(2T + B)C_m^{0.5}\left(0.453 + 0.4425C_B - 0.2862C_m - 0.003467\frac{B}{T} + 0.3696C_{wp} + 2.38\frac{A_{bt}}{C_B}\right)$$

where the term A_{bt} refers to the transverse sectional area of a bulb at the forward perpendicular.

Blockage and blockage correction

Blockage refers to the effects of boundaries on the flow around a body, specifically a model in a tank where…

- flow speeds are increased in restricted waters compared to the unrestricted case
- viscous wake effects change form drag
- boundaries (walls, bottom) effect waves

Blockage correction: correction of restricted flow caused by boundaries

- unnecessary if

$$B_{model} < \frac{B_{tank}}{10} \rightarrow \frac{B_{tank}}{15}$$

$$T_{model} < \frac{h_{tank}}{10} \rightarrow \frac{h_{tank}}{20}$$

An example of a blockage correction method is Conn's method:

$$\left(\frac{V_1}{V}\right)^3 \frac{F_n^2}{2} - \left(1 - \frac{a}{bh} + \frac{F_n^2}{2}\right)\frac{V_1}{V} + 1 = 0$$

where

$V_1 = V + \delta V$
V = carriage speed
δV = correction to speed for blockage effects
$F_n = \frac{V}{\sqrt{gh}}$ Froude number for depth
$a = \frac{\nabla}{L}$ mean cross sectional area of the submerged model
b = tank breadth
h = tank depth