Carleton University

Department of Electronics

ELEC 4700 A

Assignment 2

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Date Submitted: February 23, 2020

Contents

1 Introduction

This assignment examined computationally solving Laplace's equation for various circumstances. Section 2 contains the solutions to question 1 which solved simple 1D and 2D circumstances with various boundary conditions. Section 3 contains the solutions to question 2 which introduces conductivity. In particular, a bottle neck of conductivity was added and the effects were examined.

Laplace's Equation is shown in Equation 1 below.

$$\nabla\left(\sigma_{x,y}\nabla V\right) = 0 \tag{1}$$

Where $\sigma_{x,y}$ is the conductivity function at any point and V is the voltage function at any point. When conductivity is constant, this equation simplifies to the one shown in Equation 2 below.

$$\frac{\delta^2 V}{\delta x^2} + \frac{\delta^2 V}{\delta y^2} = 0 \tag{2}$$

2 Question 1

This question solved simple cases of Laplace's equation, both in 1D in Section 2.1 and 2D in Section 2.2.

2.1 1a) 1D model

In this section Laplace's equation (see Equation 2) was solved in a 1D case. The voltage of a 3mm bar was set to 5V at one end and 0V at the other end. Solving Laplace's equation yielded the plot shown in Figure 1 below.

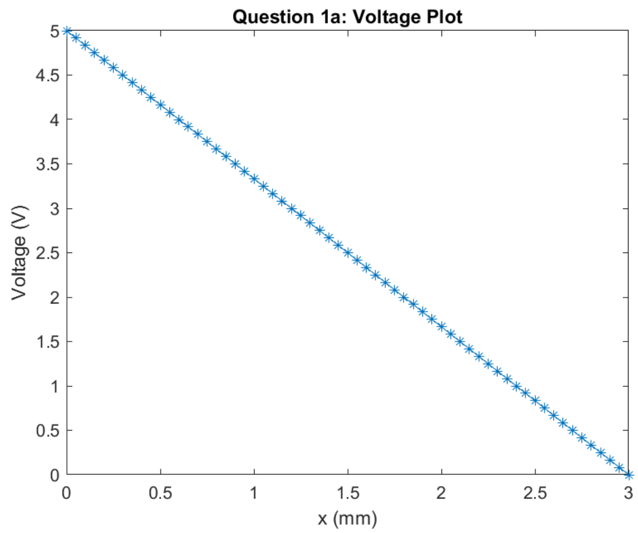

Figure 1: 1D Voltage Model

2.2 1b) 2D model

In this section Laplace's equation (see Equation 2) was solved in a 2D case. The voltage of a 3mm by 2mm box was set to 5V at either end and 0V on the sides. Solving Laplace's equation yielded the plot shown in Figure 2 below.

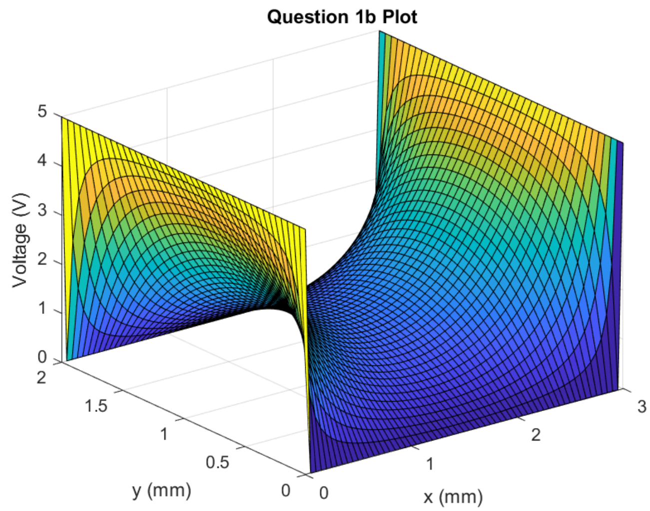

Figure 2: Simulated Model

The analytical solution for this case is shown below in Figure 3.

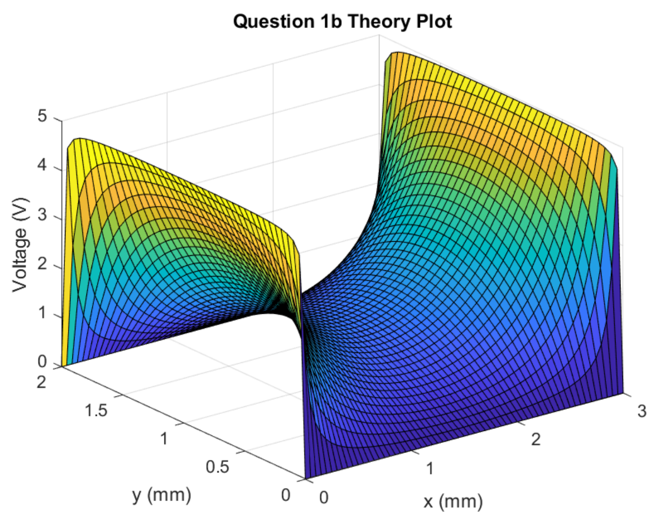

Figure 3: Analytical Model

One can see that both the analytical and the computed solution are very similar.

Increasing the mesh size when performing the simulation will increase the precision of the results significantly. However for 2 dimensional cases, the increase in run time will be quartic. Therefore, there is a very sharp limit to the precision that can be reasonably obtained.

On the other hand, analytical solutions are also very precise. For 2 dimensional cases, the increase in run time for increasing precision will be quadratic or cubic. In the case solved in this section, it was cubic. Since it is a complete order lower than the simulated algorithm, the precision that can be reasonably obtained will be much higher. One downside of analytical solutions is that they can be difficult to obtain, especially for complicated geometries. In that sense, they can be limiting compared to simulated solutions which can simulate most geometries.

3 Question 2

For this question, the voltage of the box was set to 5V at one end and 0V at the end. The other boundaries were left to float. The conductance of the box was also changed, 2 regions of very low conductance was added. Section 3.1 examines this situation. Sections 3.2, 3.3 and 3.4 examine the effects of mesh density, the bottle neck width and the average conductance respectfully on the the current through the box.

3.1 2a) Bottle Neck

In this section, the conductance of the box was set as per Figure 4.

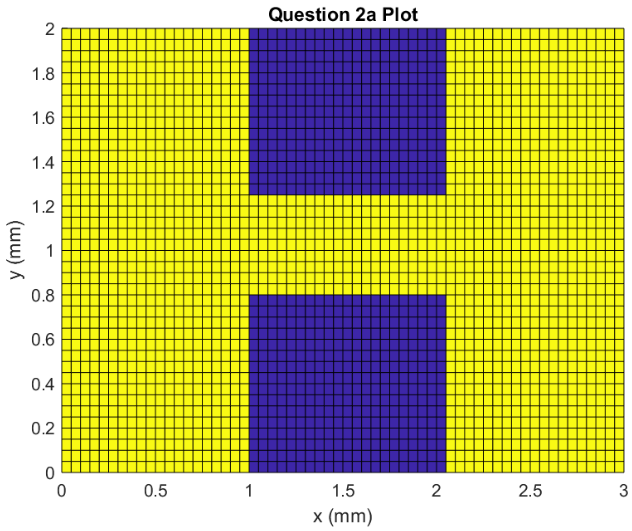

Figure 4: Conductivity Plot. Yellow regions have a conductance of 1S. Blue regions have a conductance of 10^{-2}S.

After simulating, the resulting voltage is shown in Figure 5 below.

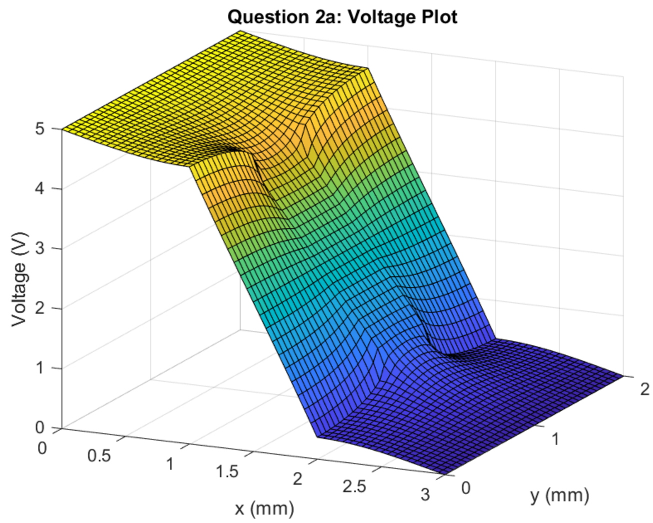

Figure 5: Voltage Plot

The electric field (in the x direction) for this situation is shown in Figure 6 below.

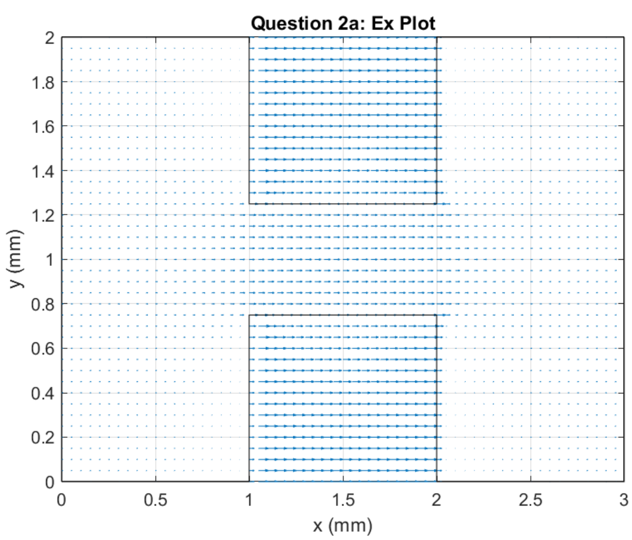

Figure 6: Electric Field (X Direction) Plot

The electric field (in the y direction) for this situation is shown in Figure 7 below.

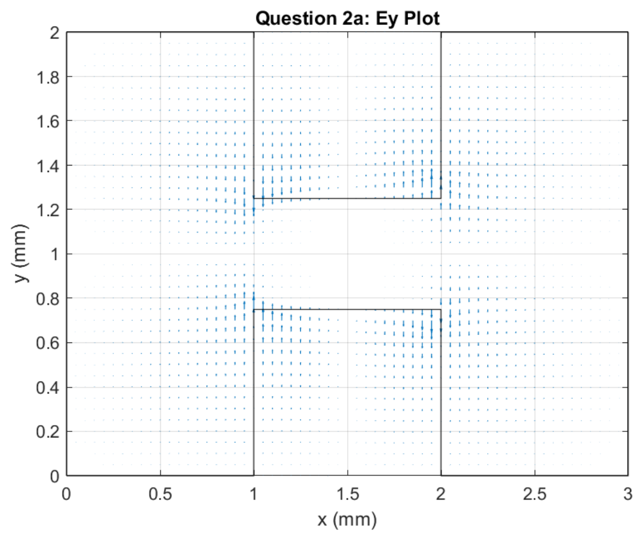

Figure 7: Electric Field (Y Direction) Plot

The total current density is shown in Figure 8 below.

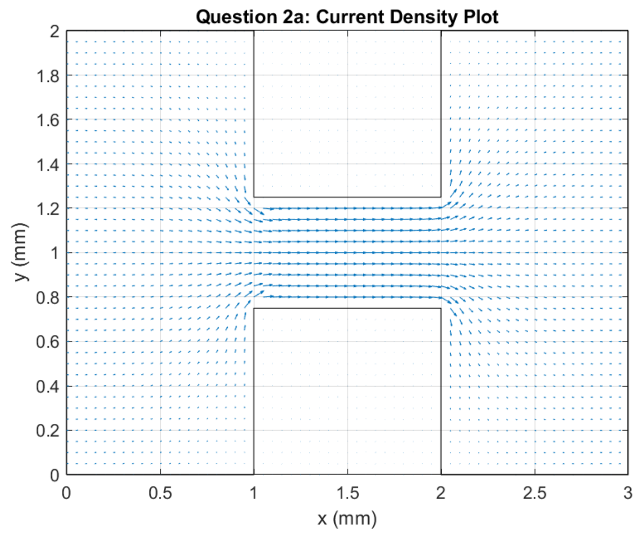

Figure 8: Current Density Plot

3.2 2b) Mesh Density

In this section the mesh size was varied and the effect of it on the current was examined. The mesh size is defined as the distance between points. Lower mesh sizes are therefore more detailed. This effect is shown in Figure 9 below.

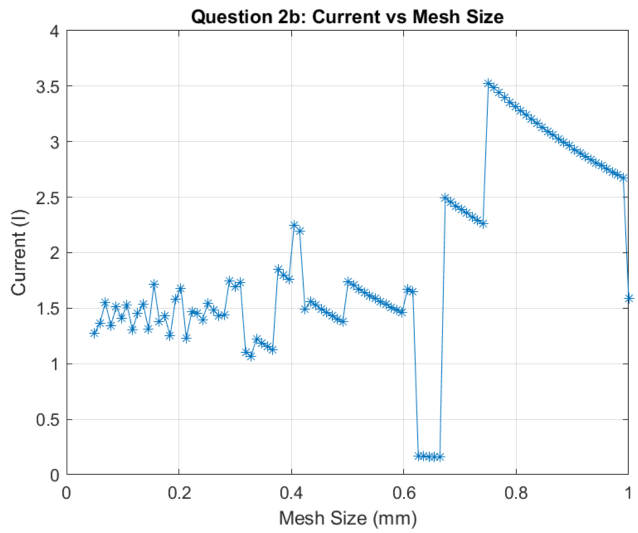

Figure 9: Current vs Mesh Density

As seen in the above figure, the conductance does change at different mesh sizes. However, at lower mesh sizes, the variability of the current is low. As the mesh size increases, the variability of the current increases. This demonstrates that at higher mesh sizes the results are very imprecise, but as the mesh size decreases, the precision of the results increases. Therefore lower mesh sizes tend to produce better results.

3.3 2c) Bottle Neck

In this section, the current through the box was examined as the bottle neck width was varied. The bottle neck width is defined as the distance between the two low conductivity regions on the box. The results of this investigation are shown in Figure 10 below.

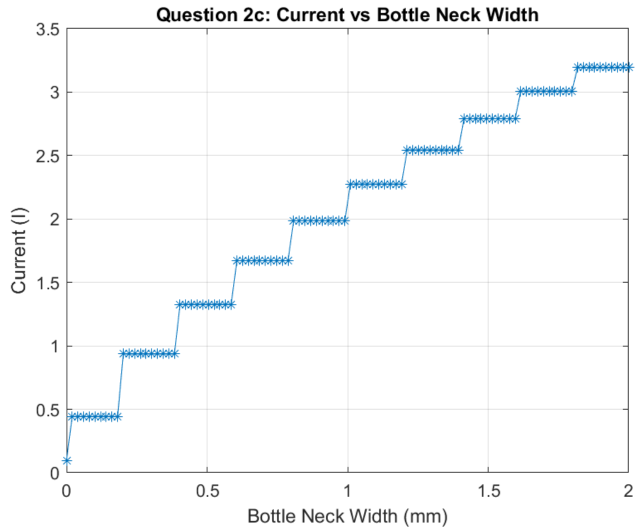

Figure 10: Current vs Bottle Neck Width

As the bottle neck width was increased, the current also increased. This is because as the bottle neck width is increased, there is an wider channel for electrons to pass through. The plot above shows that the current jumps in steps and does not increase smoothly. This is due to the meshing. As the bottle-neck width increases, there will only be effects when more grid points have their conductivity changed. This will occur discreetly, which results in the current jumping as seen in the above figure.

3.4 2d) Conductivity

In this section, the effect of the average conductivity on the current through the system were examined. To do so, a Monte-Carlo simulation was performed, where the conductivity of the low conductivity regions were varied between 0S and 1S, the bottle-neck width was varied between 0mm and the width of the box. The bottle-neck length was varied between 0mm and the box length. 10,000 trials were conducted. The average conductance and the current was calculated in each case. These results were plotted on Figure 11 below.

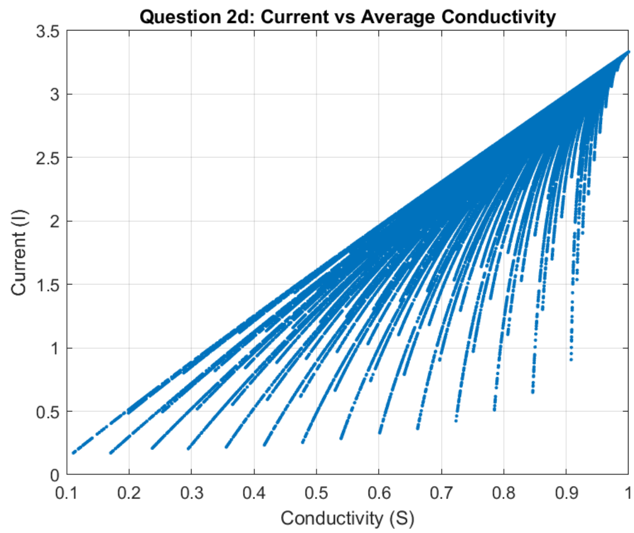

Figure 11: Current vs Conductivity

As seen in the plot above. There is an upper limit to the current that can pass through the system for each conductance. This limit is linear. A possible geometry that explains this maximum limit is when the bottle-neck is really wide, therefore not limiting the current that can pass through.

The current can be a lot lower depending on the geometry of the box. It also seems like there are certain branches that are visible on the plot. A possible reason for these branches are due to the discreetness that occurs from changing the size of the bottle-neck (see Section 3.3. for more detail).

References

[1] *Society for Neuroscience, Ottawa Chapter* [Online] Available FTP: http://cdn.sfn-ottawa.ca Directory: images/misc File: cu-logo-vertical.png?1415290081