Carleton University

Department of Electronics

ELEC 4700 A

# Assignment 4

*Author:*
Denis Shleifman
101001778

*Professor:*
Tom Smy

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# Contents

# 1 Introduction

# 2 Theory

The equations which governs the DC, AC and transient analysis of circuits with linear components is shown in Equations 1, 2 and 3 respectively below.

$$V = G^{-1} \cdot F \tag{1}$$

$$V = (G + j\omega C)^{-1} \cdot F(\omega) \tag{2}$$

$$V_n = \left(\frac{C}{\Delta t} + G\right)^{-1} \cdot \left(C\frac{V_{n-1}}{\Delta t} + F_t\right) \tag{3}$$

Where V is a matrix which represents the node voltages and currents. G is a matrix which represents the resistors, voltage and current sources in the circuit. C is a matrix which represents inductive and capacitive components of the circuit. F is a matrix which represents the value of the voltage and current sources. $\omega$ represents the frequency the circuit is being solved at, where $\omega = 2\pi f$. Finally, $\Delta t$ represents the time step for the transient simulation.

# 3 Circuit without noise

For this question the analysis of the circuit shown in Figure 1 was performed.

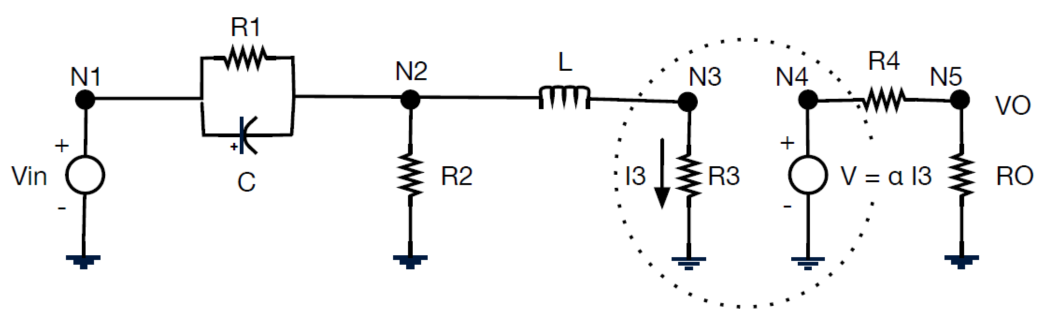


Figure 1: Circuit Without Noise Diagram

The G matrix for the circuit in Figure 1 is shown below.

| Eqn | I(vin) | I(r1) | I(c) | I(r2) | I(l) | I(r3) | I(b) | I(r4) | I(ro) | V(1) | V(2) | V(3) | V(4) | V(5) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | -1 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 1 | 1 | -1 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 | 1 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | -1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | -1 | 0 | 0 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 7 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 1 | 0 | 0 | 0 |
| 8 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 9 | 0 | 0 | 0 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 0 | 0 | 0 |
| 10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 1 | 0 | 0 |
| 11 | 0 | 0 | 0 | 0 | 0 | 43.89 | 0 | 0 | 0 | 0 | 0 | -1 | 0 | 0 |
| 12 | 0 | 0 | 0 | 0 | -100 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 13 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.10 | 0 | 0 | 0 | 0 | -1 | 1 |
| 14 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1000 | 0 | 0 | 0 | 0 | -1 |

The C matrix for the circuit in Figure 1 is shown below.

| Eqn | I(vin) | I(r1) | I(c) | I(r2) | I(l) | I(r3) | I(b) | I(r4) | I(ro) | V(1) | V(2) | V(3) | V(4) | V(5) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -0.25 | 0.25 | 0 | 0 | 0 |
| 9 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 10 | 0 | 0 | 0 | 0 | 0.20 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 11 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 12 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 13 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 14 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Performing a DC sweep of the input voltage yields the following chart shown in Figure 2 below. The plot shows the voltage at the 5th and 3rd nodes.

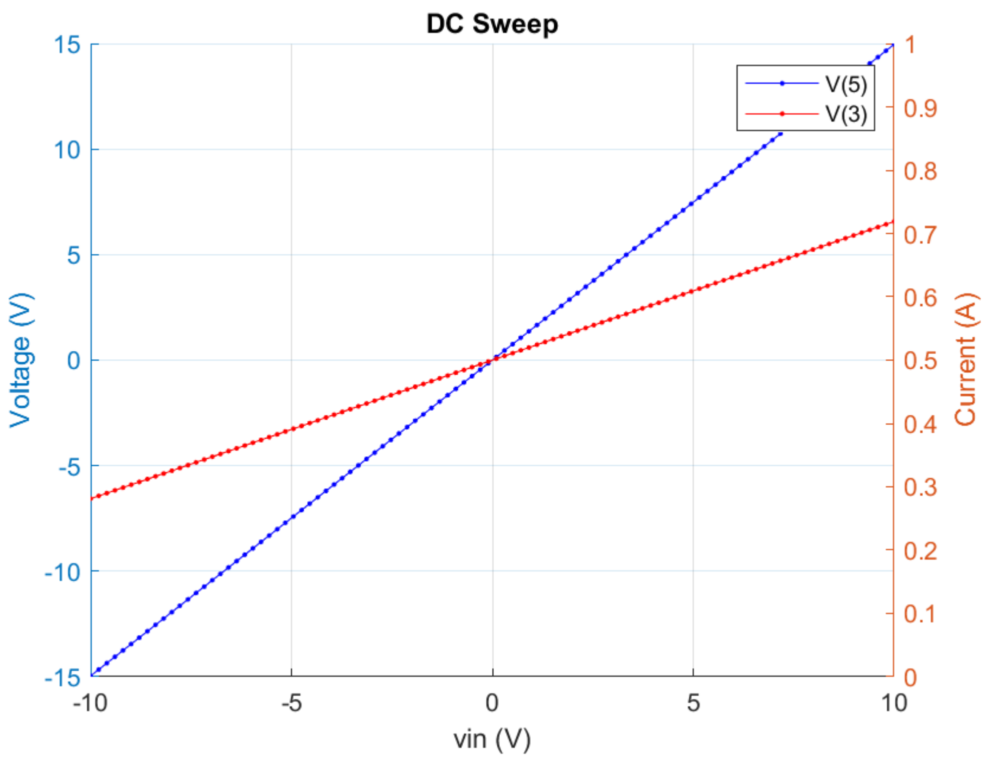


Figure 2: DC Sweep of Input Voltage

Performing a AC sweep of the frequency (while setting the input voltage to 1) yields the following chart shown in Figure 3 below. This plot shows the gain in decibels as well as the actual output voltage observed (with the phase information in the lower plots) at the output.

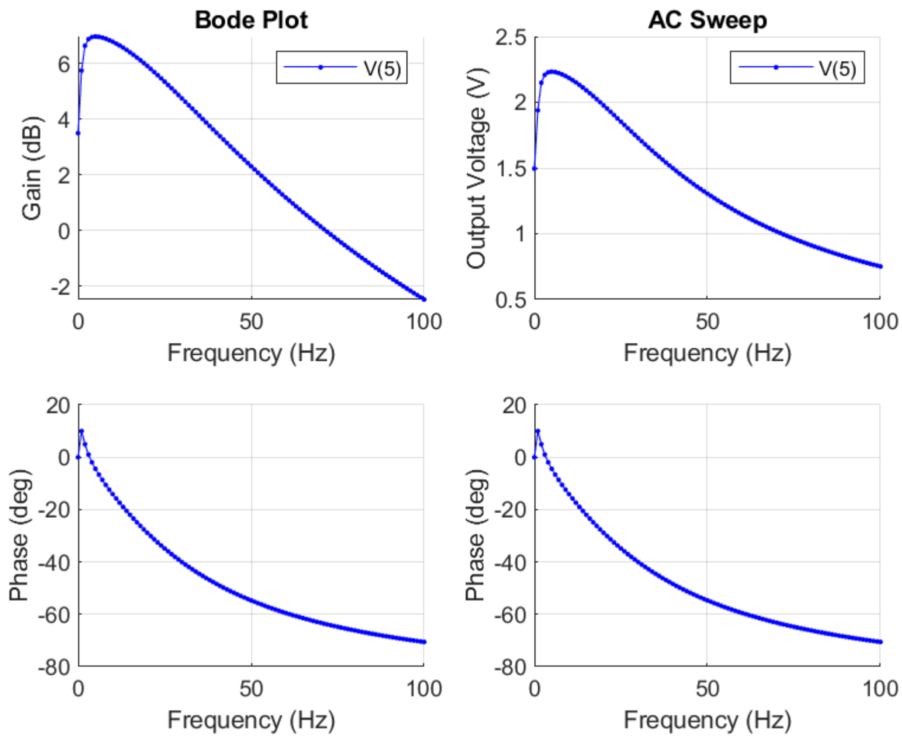


Figure 3: AC Sweep Frequency at Input Voltage 1V

Setting the frequency to 0.5 Hz, and varying the capacitance of the capacitor along a Gaussian distribution with a standard deviation of 0.05 yields the plots shown in Figure 4 below. The top plot shows the relationship between the gain and the capacitance, and the bottom plot shows a distribution of the gain that was produced.

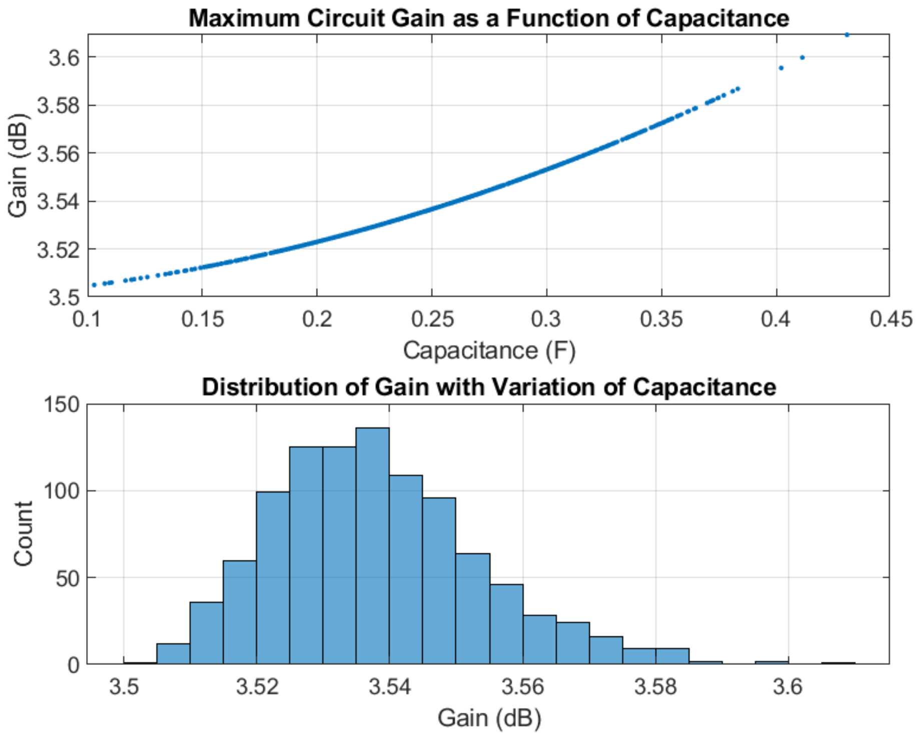


Figure 4: Impacts of Gaussian Variation of Capacitanc

For the next part, the circuit was simulated through transient simulation. Different inputs were used for the different simulations. The results of the step input, the sinusoidal input and the Gaussian pulse are shown in Figures 5, 6 and 7 below. The plots show both the input and output voltages in the time domain on the top plot, and in the frequency domain in the bottom plot.

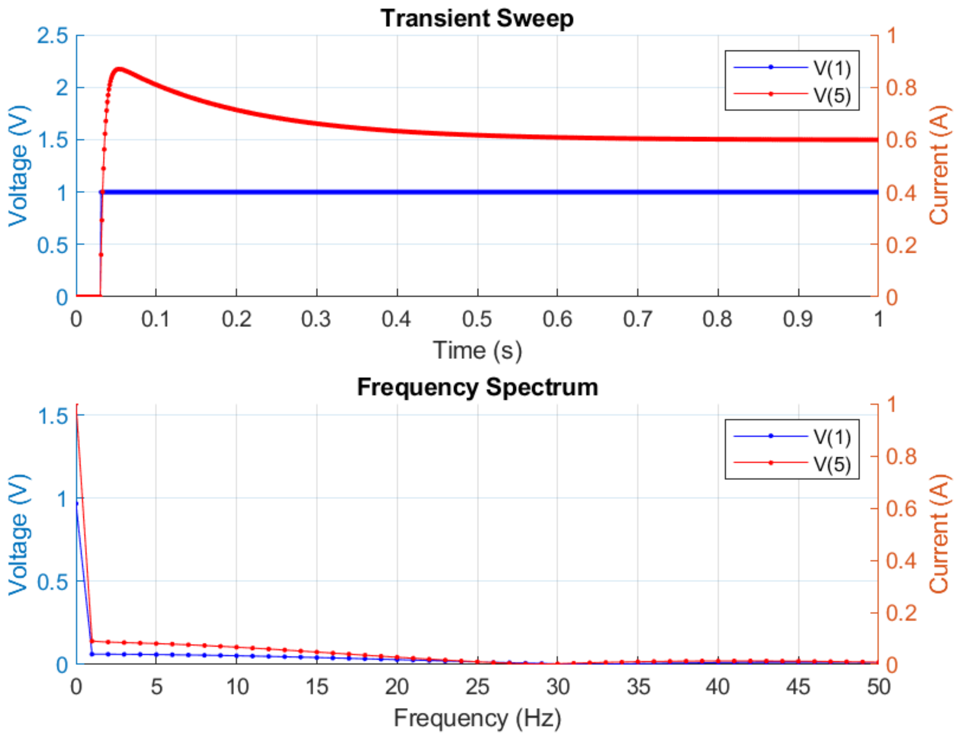


Figure 5: Step Input Transient Simulation

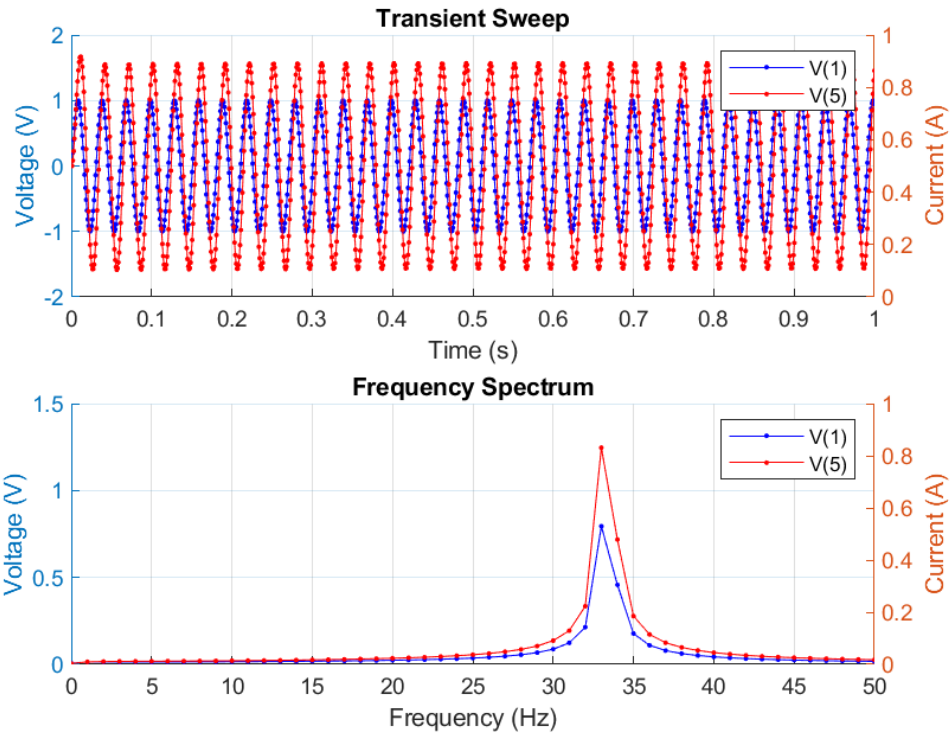


Figure 6: Sinusoidal Input Transient Simulation

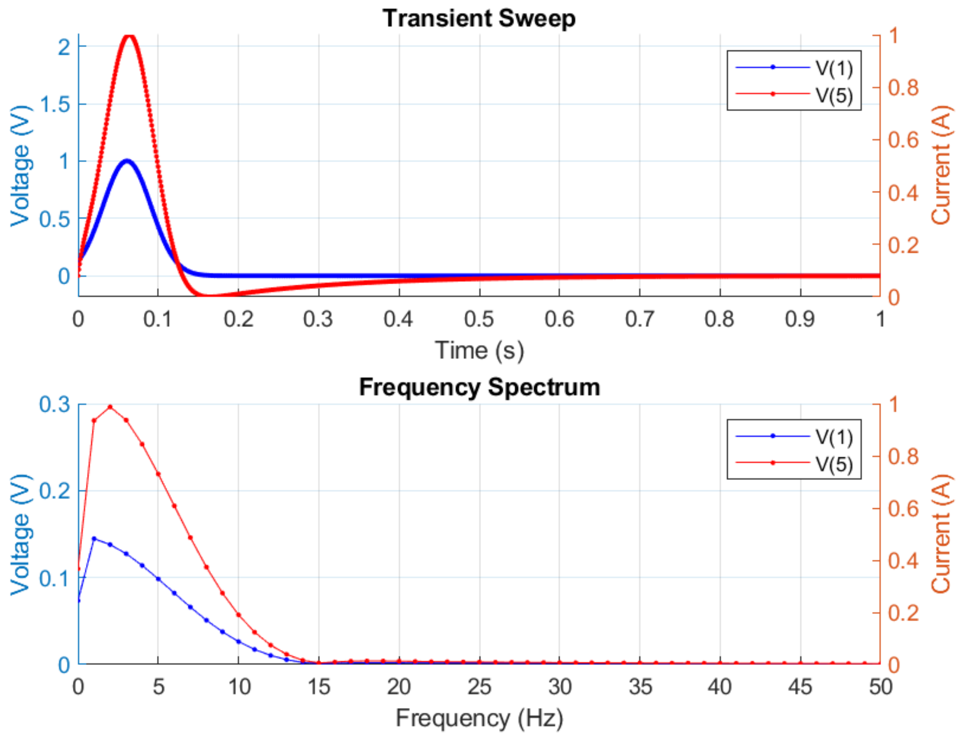


Figure 7: Gaussian Input Pulse Transient Simulation

# 4 Circuit with noise

For this section the circuit in Figure 1 was modified to inlcude noise. The updated circuit is shown in Figure 8 below.

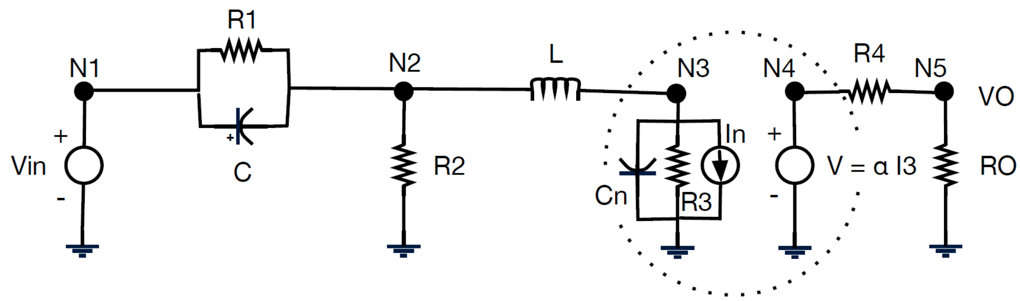


Figure 8: Circuit With Noise Diagram

The C matrix for the circuit in Figure 8 is shown below.

Using a Gaussian pulse as input, the resulting transient simulation is shown in Figure 9 below. The plot shows the input and outputs in both the time and frequency domains.

| Eqn | I(vin) | I(r1) | I(c) | I(r2) | I(l) | I(r3) | I(b) | I(r4) | I(ro) | I(in) | I(cn) | V(1) | V(2) | V(3) | V(4) | V(5) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -0.25 | 0.25 | 0 | 0 | 0 |
| 9 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 10 | 0 | 0 | 0 | 0 | 0.20 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 11 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 12 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 13 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 14 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 15 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 16 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1e-05 | 0 | 0 |

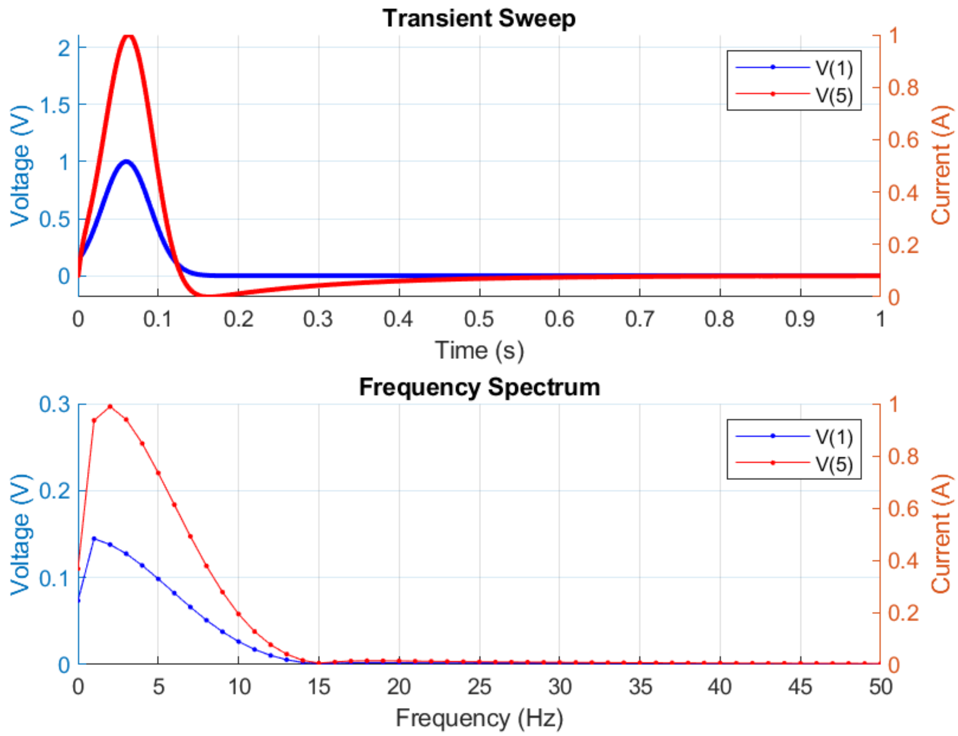


Figure 9: Gaussian Input Pulse Transient Simulation with noise source

The noise capacitor was varied to see its effect. In Figure 9 it was set to 0.01 mF. To see its effects, it was increased to 0.1mF, 1mF and 10mF. The resulting transient simulations are shown in Figures 10, 11 and 12 respectively.

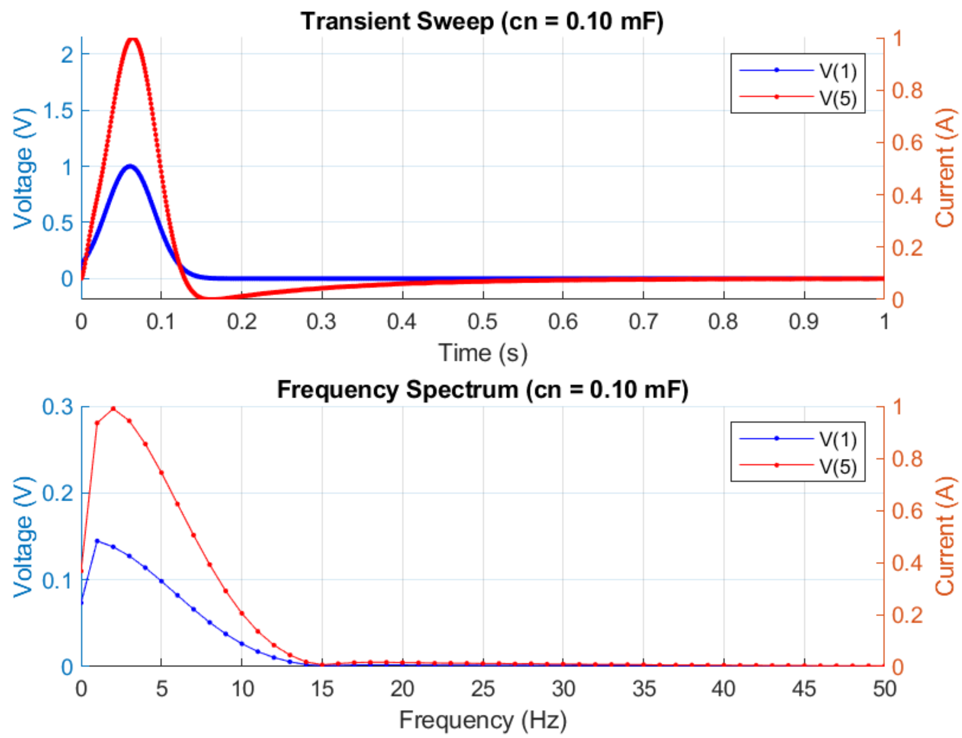


Figure 10: Gaussian Input Pulse Transient Simulation (cn = 0.10 mF)

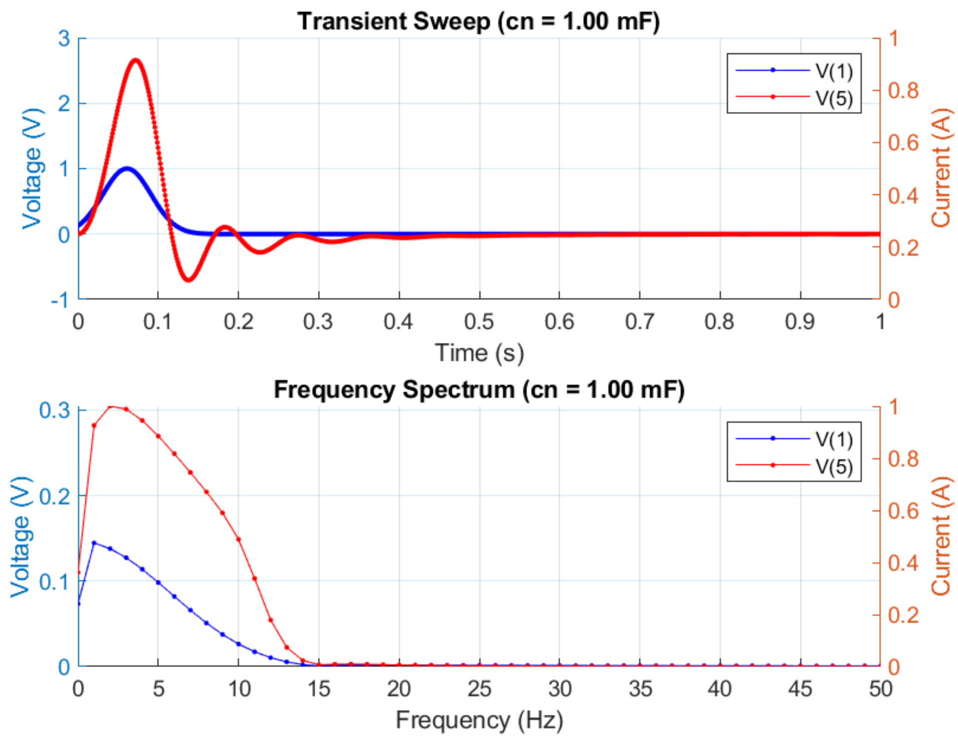


Figure 11: Gaussian Input Pulse Transient Simulation (cn = 1.00 mF)

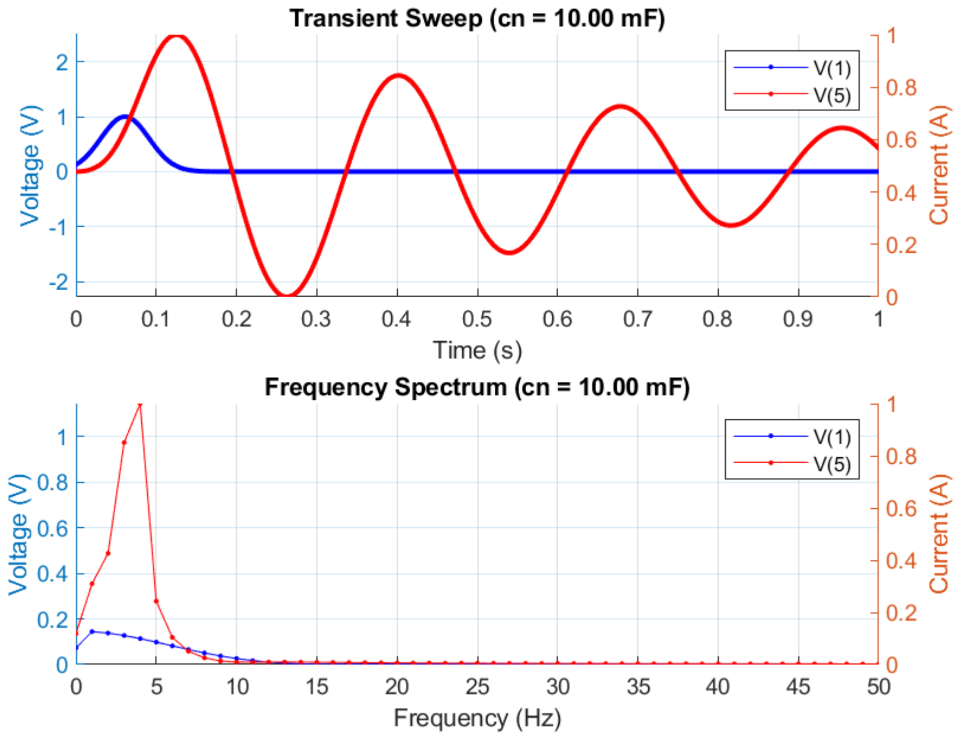


Figure 12: Gaussian Input Pulse Transient Simulation (cn = 10.00 mF)

The effects of changing the time step was also investigated. In Figure 9, the time step was set to 30.52 $\mu$ s. To see the effects of changing the time step, it was changed to and 0.98 ms and 31.25ms. The resulting transient simulations are shown in Figures 13 and 14 below.

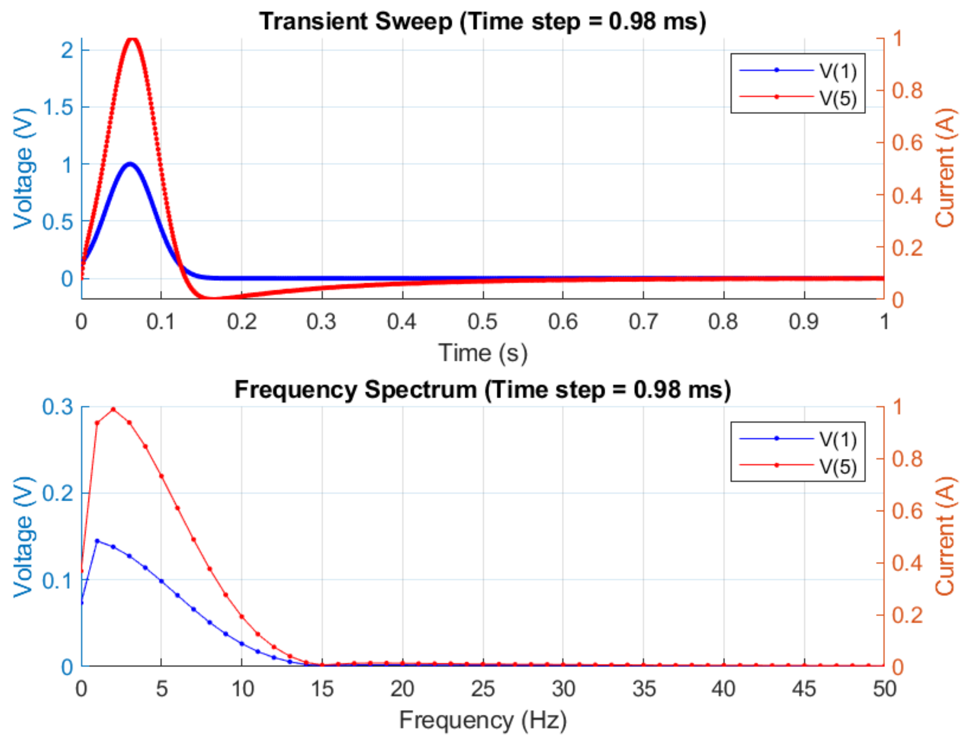


Figure 13: Gaussian Input Pulse Transient Simulation (Time step = 0.98 ms)

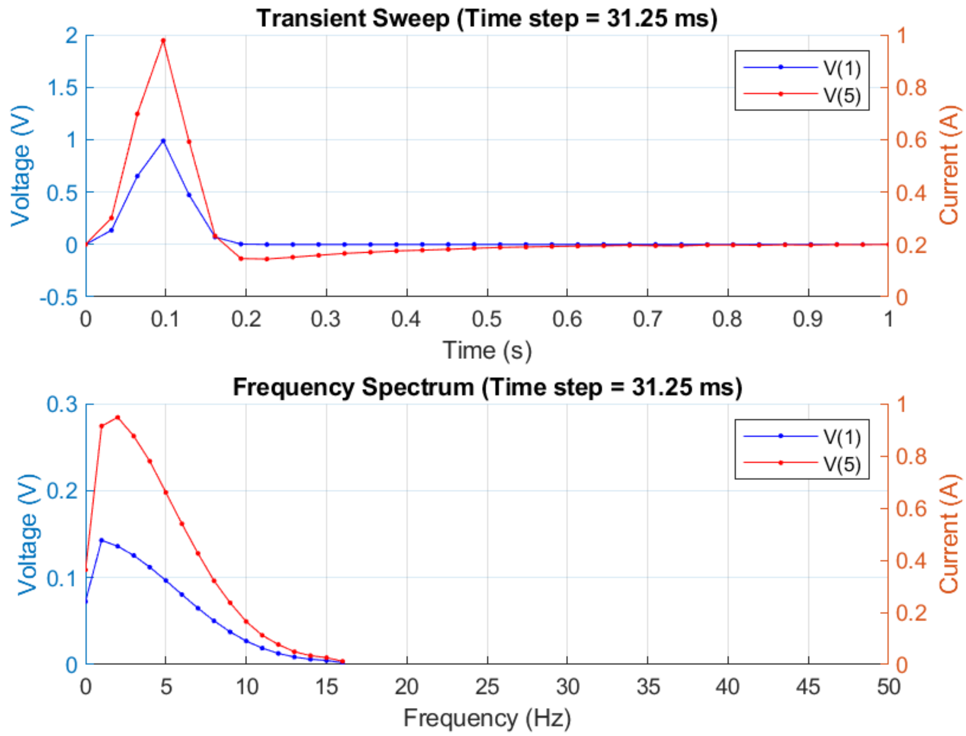


Figure 14: Gaussian Input Pulse Transient Simulation (Time step = 31.25 ms)

# 5 Non Linear Circuit Elements

For this section the voltage source on the output stage was changed to be modelled by the equation $V = \alpha I_3 + \beta I_3^2 + \gamma I_3^3$. This would mean that this was no longer a linear circuit element, meaning that it can not be solved through any of the methods described in Section 2.

Though the traditional method of solving circuits with non-linear elements is by using the newton-raphson method. This circuit in particular can be solved using a faster and simpler to implement method. In this case the circuit actually consists of two stages. The non-linear element is located in the second stage and is dependent on the current through the first stage. The simplification is able to be applied because there is no feedback loop, meaning that the current in the first stage has no dependency on the non-linear element. Because of this, the method of solving this is to first solve the first stage using methods as described in Section 2. Once the circuit is solved, and the current ($I_3$) is known, the voltage across the voltage source in the second stage is calculated. This is set as the input voltage of the second stage, which is then solved using the traditional methods as described in Section 2.

# References

[1] *Society for Neuroscience, Ottawa Chapter* [Online] Available FTP: http://cdn.sfn-ottawa.ca Directory: images/misc File: cu-logo-vertical.png?1415290081