Theorem 1. *for all $n \in Z^+$,*

$$Decep^*(s, \mathbf{b}, n) \geq Nash_{l^*}(s), \forall \mathbf{b} \tag{1}$$

Proof. We will prove by induction. By definition, when $n = 0$, $Decep^*(s, \mathbf{b}, 0) = Nash_{l^*}(s)$, $\forall \mathbf{b}$
Base case: when $n = 1$, (A is the joint action space of the attacker and defender team)

$$\begin{aligned}
Nash_{l^*}(s) &= \sum_{a \in A} \pi^*(s, a, s')T(s, a, s')(R^*(s, a) + \gamma Nash_{l^*}(s')) \\
Decep^*(s, \mathbf{b}, 1) &= \max_{a_d} \sum_{a_a \in A_a} y(a_a|\mathbf{b}, s)T(s, a, s')(R^*(s, a) + \gamma Decep^*(s', \mathbf{b}', 0)) \\
&= \max_{a_d} \sum_{a_a \in A_a} y(a_a|\mathbf{b}, s)T(s, a, s')(R^*(s, a) + \gamma Nash_{l^*}(s'))
\end{aligned}$$

Since we assume we know exactly what the attacker policy is, the optimal strategy for the defender team should be deterministic and any stochastic policy would not have a better payoff.

$$Decep^*(s, \mathbf{b}, 1) \geq \sum_{a_d \in A_d} \left[x^*_{l^*}(a_d|s) \sum_{a_a \in A_a} y(a_a|\mathbf{b}, s)T(s, a, s')(R^*(s, a) + \gamma Nash_{l^*}(s')) \right]$$

Given the definition of Nash equilibrium of a zero-sum game, when the defender team play its Nash strategy x_{l^*}, it gets minimum payoff when the attacker plays y_{l^*}. Any other attacker strategy will let the defender gain more payoff.

$$\begin{aligned}
Decep^*(s, \mathbf{b}, 1) &\geq \sum_{a_d \in A_d} \left[x^*_{l^*}(a_d|s) \sum_{a_a \in A_a} y^*_{l^*}(a_a|s)T(s, a, s')(R^*(s, a) + \gamma Nash_{l^*}(s')) \right] \\
&= Nash_{l^*}(s), \forall \mathbf{b}
\end{aligned}$$

Induction step: Let $k \in Z^+$ be given and suppose Equation (1) is true for $n = k$. Then for $n = k + 1$ we have

$$Decep^*(s, \mathbf{b}, k + 1) = \max_{a_d} \sum_{a_a \in A_a} y(a_a|\mathbf{b}, s)T(s, a, s')(R^*(s, a) + \gamma Decep^*(s', \mathbf{b}', k))$$

For $n = k+1$, we have $Decep^*(s', \mathbf{b}', k) \geq Nash_{l^*}(s')$. Thus similar with base step, we have $Decep^*(s, \mathbf{b}, k + 1) \geq Nash_{l^*}(s), \forall \mathbf{b}$

Conclusion: By the principle of induction, Equation (1) is true for all $n \in Z^+$.