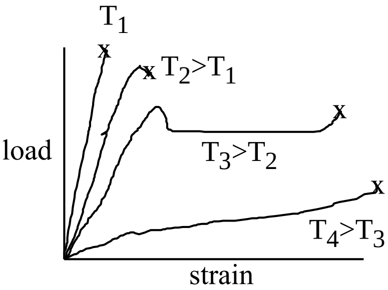
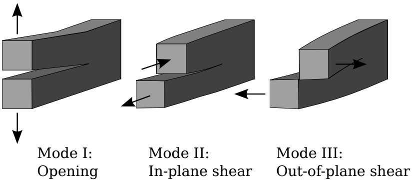
Typical generic temperature behavior:



T_1: Brittle behavior
T_2: Ductile behavior (yield before fracture)
T_3: cold drawing (stable neck)
T_4: uniform deformation

Fracture Modes



- Homogeneous material fracture: always mode I
- Mode II can be important for interfacial fracture (adhesion)
- Mode III is generally not important

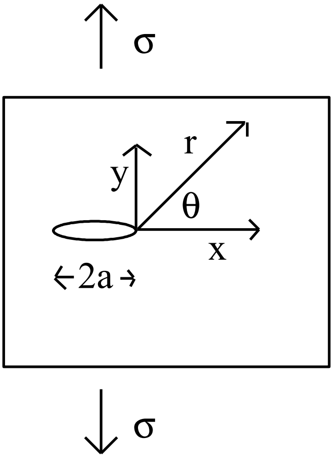
Brittle behavior (fracture mechanics)

http://en.wikipedia.org/wiki/Fracture_mechanics

Mode I crack (stress normal to crack)

2 equivalent approaches:

1. Irwin model (stress-based approach)
2. Griffith model (energy-based approach)



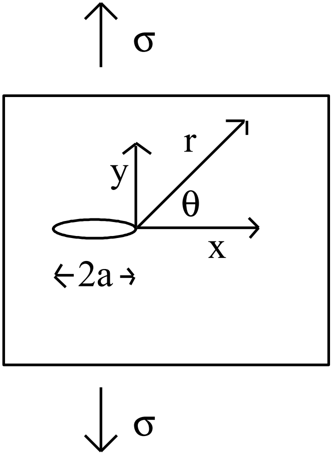
Assume ρ (crack tip radius of curvature) is very small:
For $r \ll a$

$$K_I = \sigma\sqrt{\pi a}$$

$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}} f(\theta)$$

different $f(\theta)$ for σ_{xx}, σ_{yy}, etc.

Limit for $r \rightarrow 0$



$$\sigma_{yy}\left(r=0\right)=2\sigma_0\sqrt{a/\rho}$$

Fracture condition

$$K_{Ic} = \sigma_f \sqrt{\pi a}$$

K_{Ic}=fracture toughness (critical value of K_I)
σ_f = fracture stress

$$\sigma_f = K_{Ic} / \sqrt{\pi a}$$

Assumptions:

- all strains in region of crack tip are elastic (very small plastic zone around crack tip).
- fracture occurs when the stress field defined by K_I reaches a critical value

Specificying the stress field is the same as specifying the stored elastic energy. One expects that an energy-based fracture criterion could also be derived. This leads us to the second approach:

Fracture occurs when available energy is sufficient to drive crack forward.

W = workd done on system by external stresses

U = elastically stored energy

(W-U) = energy available to drive crack forward.

$$\mathcal{G} = \frac{d}{dA}(W - U)$$

A= crack area

$\mathcal{G}$ = energy release rate

Fracture condition

$$\mathcal{G}_I = \mathcal{G}_{1c}$$

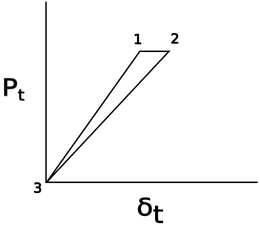
$\mathcal{G}_{Ic}$= critical energy rate (also referred to sometimes as fracture toughness - this is a material property

Lowest possible value of $\mathcal{G}_{Ic}$ is 2γ; γ is surface energy of the material

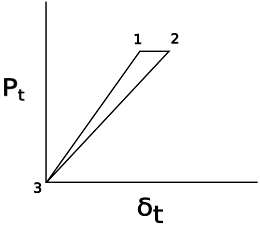
Some values of γ (1 mJ/m^2 = 1 erg/cm^2 = 1 dyne/cm)

- Polymers: 20-50 mJ/m^2 Van der Waals bonding between molecules
- Water: 72 mJ/m^2 Hydrogen bonding between molecules
- Metals:$\approx$1000 mJ/m^2 Metallic bonding

$P_t = -P$; $\delta_t = -\delta$ - tensile load and displacement



$P_t = -P$; $\delta_t = -\delta$ - tensile load and displacement

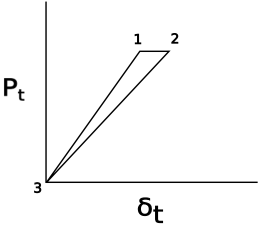


Compliance:

$$C = \frac{d\delta}{dP}\Big|_A$$

Calculating the Energy Release Rate

$P_t = -P$; $\delta_t = -\delta$ - tensile load and displacement

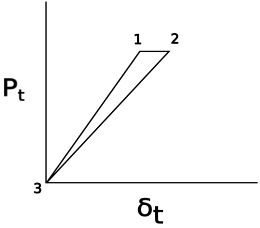


1. Apply tensile load at constant contact area, A

Compliance:

$$C = \left.\frac{d\delta}{dP}\right|_A$$

$P_t = -P$; $\delta_t = -\delta$ - tensile load and displacement

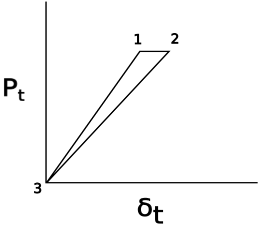


Compliance:

$$C = \left.\frac{d\delta}{dP}\right|_A$$

1. Apply tensile load at constant contact area, A
2. Reduce contact area by dA, compliance increases by dC, displacement increases by $d\delta_t$

$P_t = -P$; $\delta_t = -\delta$ - tensile load and displacement



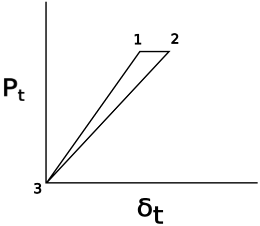
1. Apply tensile load at constant contact area, A
2. Reduce contact area by dA, compliance increases by dC, displacement increases by $d\delta_t$
3. Unload at new contact area

Compliance:

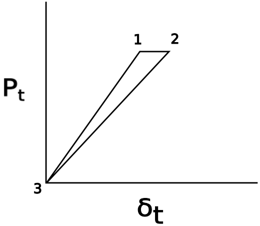
$$C = \left.\frac{d\delta}{dP}\right|_A$$

Energy release rate, $\mathcal{G}$

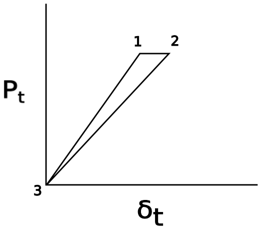
Energy release rate, $\mathcal{G}$



Energy release rate, $\mathcal{G}$

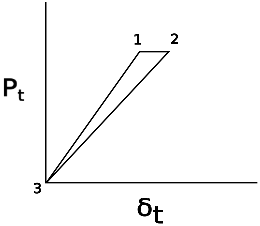


- $\mathcal{G} \equiv \frac{\text{energy input}}{\text{reduction in contact area}}$



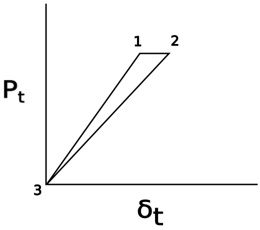
- $\mathcal{G} \equiv \frac{\text{energy input}}{\text{reduction in contact area}}$
- Energy input $= \frac{1}{2} P_t d\delta_t$

Energy release rate, $\mathcal{G}$



- $\mathcal{G} \equiv \frac{\text{energy input}}{\text{reduction in contact area}}$
- Energy input $= \frac{1}{2} P_t d\delta_t$
- $d\delta_t = P_t dC$

Energy release rate, $\mathcal{G}$

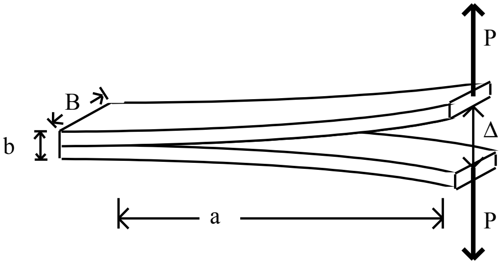


- $\mathcal{G} \equiv \dfrac{\text{energy input}}{\text{reduction in contact area}}$
- $\text{Energy input} = \frac{1}{2} P_t d\delta_t$
- $d\delta_t = P_t dC$

Combine expressions:

- $\mathcal{G} = -\frac{P_t^2}{2} \frac{dC}{dA}$

Double cantilever beam geometry:



$$\delta_t = \frac{64a^3}{EBb^3}P_t; \;\; C = \frac{\delta_t}{P_t} = \frac{64a^3}{EBb^3} \qquad (1)$$

$$\mathcal{G}_I = \frac{P_t^2}{2B}\frac{dC}{da} = \frac{96a^2P_t^2}{EB^2b^3}$$

At fixed load, $\mathcal{G}_I$ increases as the crack length increases - unstable geometry!

Displacement Control

Use Eq. 1 to substitute δ_t for P_t:

$$\mathcal{G}_I = \frac{3\delta_t^2 b^3 E}{128 a^4}$$

At a fixed displacement, crack will grow until $\mathcal{G}_I = \mathcal{G}_{Ic}$ and then stop. This is a better way to do the experiment.

General relationship between K_I and G_I

$$K_I = \sqrt{\mathcal{G}_I E^*}$$

- Plane stress (small B): $E^* = E$
- Plane strain (large B): $E^* = \frac{E}{1-\nu^2}$

$$K_I = \sqrt{\mathcal{G}_I E^*}$$

- Plane stress (small B): $E^* = E$
- Plane strain (large B): $E^* = \frac{E}{1-\nu^2}$

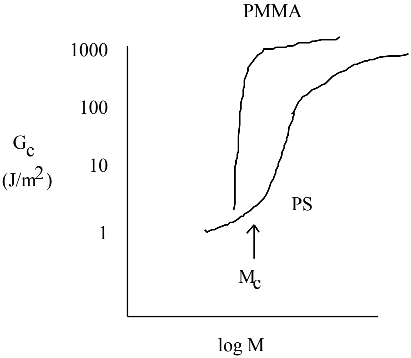
- Coresponding relationship exists between the materials properties, K_{Ic} and G_{Ic}:

$$K_I = \sqrt{\mathcal{G}_I E^*}$$

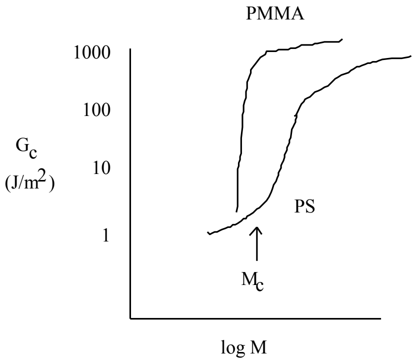
Typical values of G_{Ic}:

- $G_{Ic} = 2\gamma \ (\approx 0.1\,\mathrm{J/m^2})$ if only work during fracture is to break Van der Waals bonds
- $G_{Ic} \approx 1 - 2\,\mathrm{J/m^2}$ if only work during fracture is to break covalent bonds across interface
- $G_{Ic} \gg 1\,\mathrm{J/m^2}$ if fracture is accompanied by significant deformation of the sample

Fracture toughness of glassy polymers



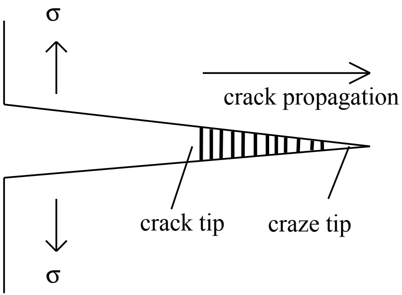
Fracture toughness of glassy polymers



Deformation is significant, but $\mathcal{G}_{Ic}$ is still small compared to other engineering materials

1. Shear bands due to strain softening (decrease in true stress after yield in shear)
2. Crazing - reqires net dilation of sample (fracture mechanism for PS and PMMA)
Crazes are load bearing - but they break down to form cracks - failure of specimen:

Crazing



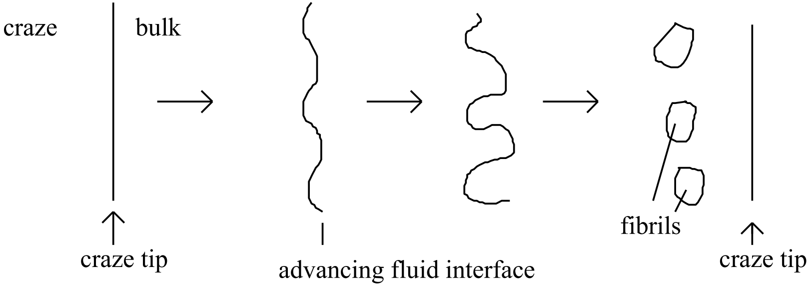
Fibrils are cold drawn polymer. Extension ratio remains constant as craze widens

Crack propagation:

1) new fibrils are created at the craze tip
2) fibrils break to form a true crack at the crack tip

(fibril formation at craze tip)

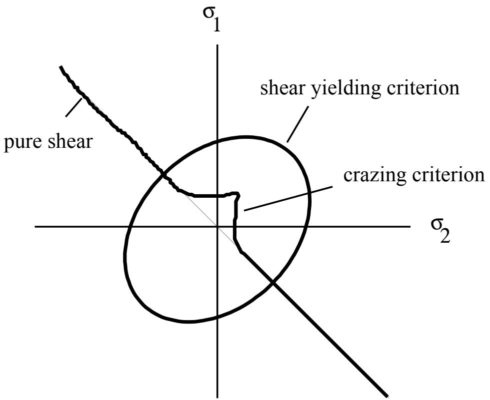
Material near the craze tip is strain softened, and can flow like a fluid between two plates.



Shear Deformation vs. Crazing

Shear deformation is preferable to crazing for producing high toughness.

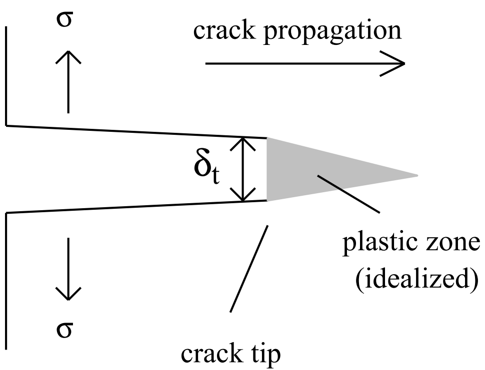
Plane stress - shear yielding and crazing criteria (for PMMA)



Crazing

1. Crazing requires dilitational stress field $\sigma_1 + \sigma_2 + \sigma_3 > 0$ (crazes have voids between fibrils)
2. Crazing occurs first for PMMA in uniaxial extension ($\sigma_2 = 0$)
3. $\mathcal{G}_{Ic}$ is determined by energy required to form a craze ($\approx 1000\,\mathrm{J/m^2}$)
4. Crazing requires strain hardening of fibrils - material must be entangled ($M > M_c$), M_c typically $\approx$30,000 g/mol.
5. In general, shear yielding competes with crazing at the crack tip

Suppose the material at the crack tip deforms by shear instead of crazing:



$\mathcal{G}_{Ic}$ determined by energy to create plastic zone

Assumptions:
1) Tensile stress througout plastic zone is yield stress σ_y
2) his stress acts to produce a crack opening displacement δ_t

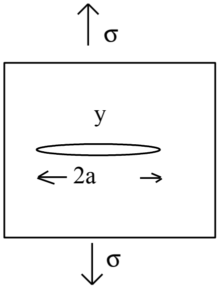
$$\mathcal{G}_{Ic} = \delta_t \sigma_y$$

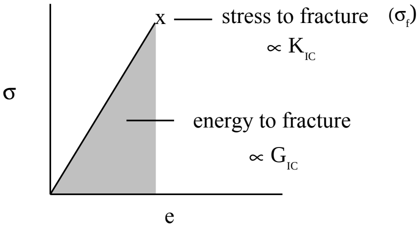
similar theory holds for crazing, with σ_c (crazing stress) replacing σ_y
σ_y and σ_c are similar, but δ_t isrestricted to smaller values for crazes

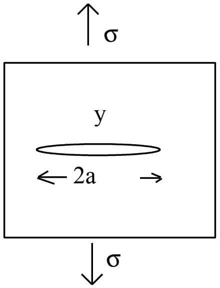
Toughened Polystyrene

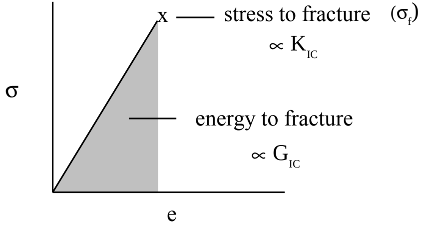
Polystyrene is a big business - how do we make it tougher?
Add “impact modifiers so there is more than one craze per crack
Show crazes, HIPS, ABS morphologies

Stress/strain behavior and fracture toughness

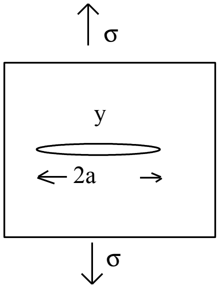


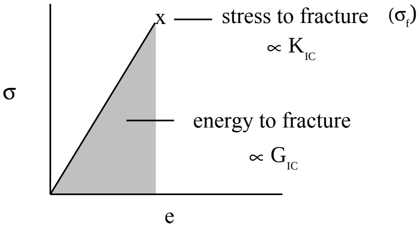




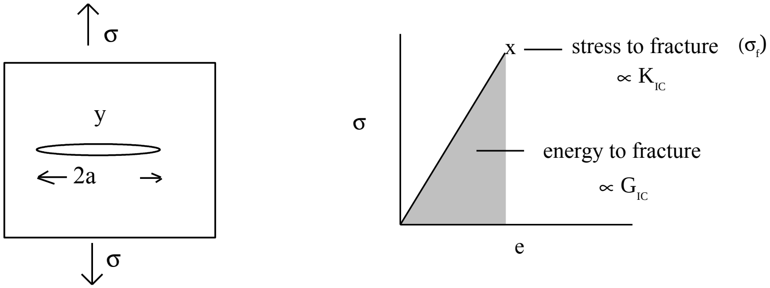


- Fracture stress depends on flaw size: for sharp crack $\sigma_f \propto \sqrt{a}$





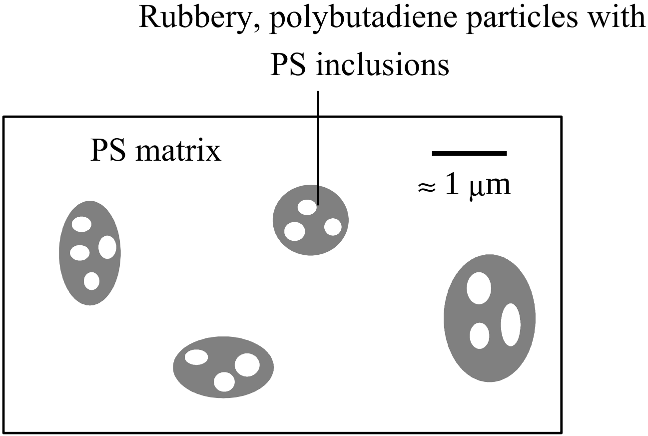
- Fracture stress depends on flaw size: for sharp crack $\sigma_f \propto \sqrt{a}$
- $K_{Ic} \propto \sigma_f$; $G_{Ic} \propto \sigma_f^2/E$



- Fracture stress depends on flaw size: for sharp crack $\sigma_f \propto \sqrt{a}$
- $K_{Ic} \propto \sigma_f$; $G_{Ic} \propto \sigma_f^2/E$
- valid only for brittle specimens: linear stress/strain curve up to fracture point

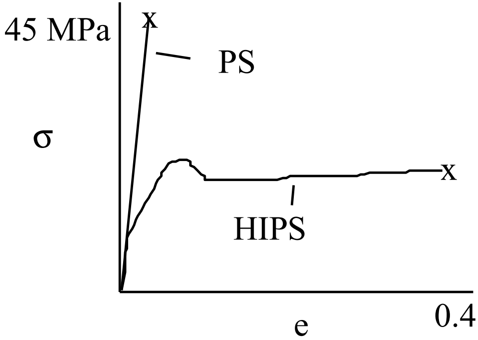
Comparison of PS and HIPS (high impact polystyrene)

HIPS morphology



Good adhesion between rubber particles and glassy matrix is required
(We want deformation, but not cracking)

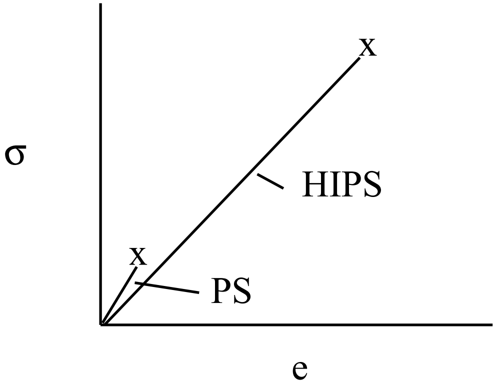
HIPS vs PS: No precrack



PS is brittle, E = 3gpa
HIPS is ductile, E=2.1 Gpa
large energy to fracture
deformation via crazing in vicinity of rubber particles (stress concentrators) throughout sample

HIPS vs PS: Samples with Precrack

(measurement of K_{IC} or G_{IC})

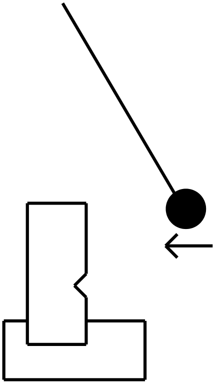


- Deformation limited to region around crack tip
- Much more deformation for HIPS - higher toughness

Some Characteristic Values

Material	E(GPa)	K_{IC} (MPa$\sqrt{\mathrm{m}}$	G_{IC}
Steel	200	50	12,000
Glass	70	0.7	7
High M polystyrene or PMMA	3	1.5	750
High Impact Polystyrene	2.1	5.8	16,000
Epoxy Resin	2.8	0.5	100
Rubber Toughned Epoxy	2.4	2.2	2,000
Glass Filled Epoxy Resin	7.5	1.4	300

Impact Tests



Decrease in pendulum velocity after breaking sample gives impact toughness
(This is Izod Charpy is similiar)
Fracture toughness is rate dependent, but same general features apply (HIPS is much tougher than PS at high strain rates as well, for the same general reasons).