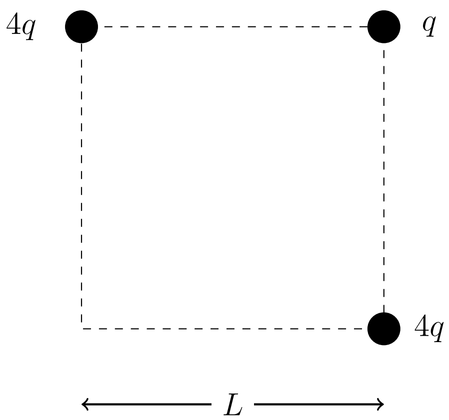
Çankaya University
Department of Computer Engineering
2009 - 2010 Spring Semester

PHYS 122 - General Physics II

First Midterm Examination

1) Three charges are fixed at the corners of a square of side length L. We want the net force on charge q to be zero. What is the charge we have to put on the fourth corner?



Answer:
Let's call the fourth charge Q. The force between q and Q is:

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\left[\frac{Qq}{(L\sqrt{2})^2}\left(\frac{\sqrt{2}}{2}\vec{i} + \frac{\sqrt{2}}{2}\vec{j}\right)\right] = 0$$

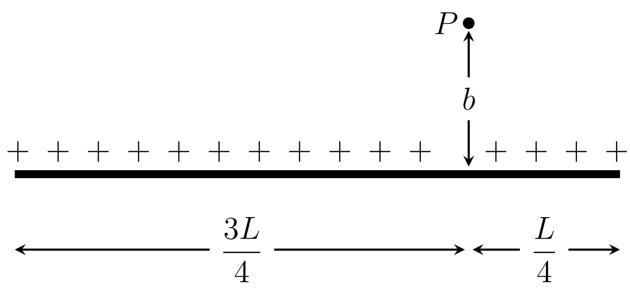
The net force on q must be zero:

$$\vec{F}_{net} = \frac{1}{4\pi\varepsilon_0}\left[\frac{4q^2}{L^2}\vec{i} + \frac{4q^2}{L^2}\vec{j} + \frac{Qq}{2L^2}\left(\frac{\sqrt{2}}{2}\vec{i} + \frac{\sqrt{2}}{2}\vec{j}\right)\right] = 0$$

$$\frac{4q^2}{L^2} + \frac{\sqrt{2}Qq}{4L^2} = 0$$

$$Q = -8\sqrt{2}q$$

2) A total charge Q is distributed uniformly on a rod of length L. Find the electric field at the point P in unit vector notation. (Do not evaluate the integrals)



Answer:

$$dq = \frac{Q}{L}\,dx$$

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{b^2 + \left(\frac{3L}{4} - x\right)^2}$$

$$dE_x = dE\sin\theta, \qquad dE_y = dE\cos\theta$$

$$E_x = \frac{Q}{4\pi\varepsilon_0 L}\int_0^L \frac{\frac{3L}{4} - x}{\left[b^2 + \left(\frac{3L}{4} - x\right)^2\right]^{3/2}}\,dx$$

$$E_y = \frac{Q}{4\pi\varepsilon_0 L}\int_0^L \frac{b}{\left[b^2 + \left(\frac{3L}{4} - x\right)^2\right]^{3/2}}\,dx$$

or alternatively

$$E_x = \frac{Q}{4\pi\varepsilon_0 L}\int_{-\frac{3L}{4}}^{\frac{L}{4}} \frac{x}{[b^2 + x^2]^{3/2}}\,dx$$

$$E_y = \frac{Q}{4\pi\varepsilon_0 L}\int_{-\frac{3L}{4}}^{\frac{L}{4}} \frac{b}{[b^2 + x^2]^{3/2}}\,dx$$

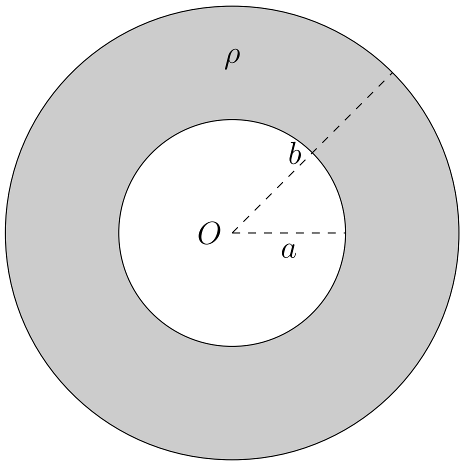
$$\overrightarrow{E} = E_x\,\overrightarrow{i} + E_y\,\overrightarrow{j}$$

3) A non-conducting spherical shell has inner radius a and outer radius b. It has uniform volume charge density ρ. Find the electric field for

a) $r < a$

b) $a < r < b$

c) $b < r$



Answer:

Using Gauss' Law:

a) $$4\pi r^2 E = \frac{q_{enc}}{\varepsilon_0}$$

$$E = 0$$

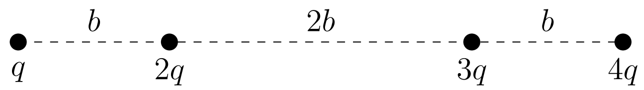
b) $$4\pi r^2 E = \frac{\left(\frac{4}{3}\pi r^3 - \frac{4}{3}\pi a^3\right)\rho}{\varepsilon_0}$$

$$E = \frac{(r^3 - a^3)\rho}{3r^2\varepsilon_0}$$

c) $$4\pi r^2 E = \frac{\left(\frac{4}{3}\pi b^3 - \frac{4}{3}\pi a^3\right)\rho}{\varepsilon_0}$$

$$E = \frac{(b^3 - a^3)\rho}{3r^2\varepsilon_0}$$

4) Find the work we have to do to assemble the system shown in figure, bringing each charge from infinity:



Answer:

$$U = \frac{1}{4\pi\varepsilon_0}\left(\frac{q\,2q}{b} + \frac{q\,3q}{3b} + \frac{q\,4q}{4b} + \frac{2q\,3q}{2b} + \frac{2q\,4q}{3b} + \frac{3q\,4q}{b}\right)$$

$$U = \frac{65}{3}\,\frac{q^2}{4\pi\varepsilon_0 b}$$

5) On a certain region of space, the electric potential is given by $V(x, y, z) = \sqrt{x^2 + y^2} - xyz^2$. Find the electric field.

Answer:

$$\overrightarrow{E} = -\frac{\partial V}{\partial x}\overrightarrow{i} - \frac{\partial V}{\partial y}\overrightarrow{j} - \frac{\partial V}{\partial z}\overrightarrow{k}$$

$$\overrightarrow{E} = \left(yz^2 - \frac{x}{\sqrt{x^2 + y^2}}\right)\overrightarrow{i} + \left(xz^2 - \frac{y}{\sqrt{x^2 + y^2}}\right)\overrightarrow{j} + 2xyz\,\overrightarrow{k}$$

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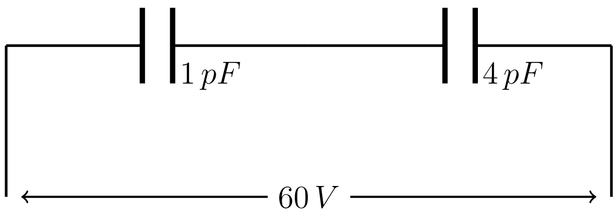
Second Midterm Examination

1) a) A capacitor stores an energy of $1\mu\, j$ when the potential difference between its plates is $3\,V$. Find its capacitance.

b) Find the charge on the $4\,pF$ capacitor in the figure.



Answer:

a)

$$U = \frac{1}{2}CV^2$$

$$C = \frac{2U}{V^2} = \frac{12\mu j}{(3V)^2}$$

$$C = 0.22\mu F = 2.2. \times 10^{-7}\,F$$

b)

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{1\,pF} + \frac{1}{4\,pF}$$

$$C_{eq} = 0.8\,pF$$

$$q = VC = 60V\,0.8pF = 48\,pC$$

$$q_1 = q_2 = 4.8 \times 10^{-11}\,C$$

2) A cylindrical wire made of Nichrome has radius $r = 1\,mm$ and length L. Nichrome's resistivity is $\rho = 5 \times 10^{-7}\,\Omega \cdot m$. When a potential difference of $100\,V$ is applied, the power dissipation is $2000\,W$. Find L.

Answer:

$$P = \frac{V^2}{R} = \frac{(100V)^2}{2000W} = 5\,\Omega$$

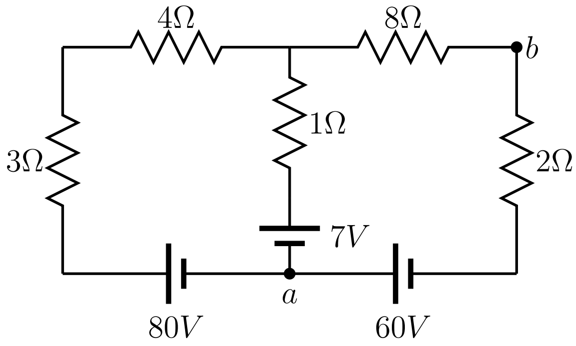
$$R = \frac{\rho L}{A} \quad \Rightarrow \quad L = \frac{AR}{\rho}$$

$$L = \frac{\pi(10^{-3}m)^2\,5\Omega}{5\,10^{-7}\,\Omega\,m} = 10\pi = 31.4\,m$$

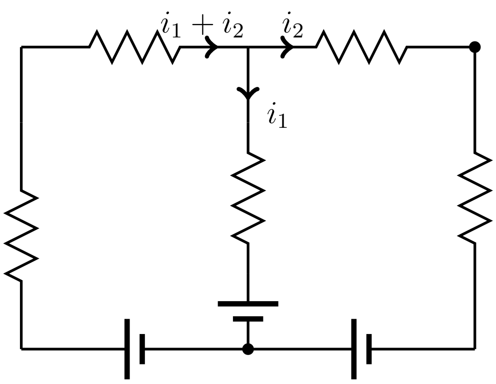
3) In the following circuit,

a) Find all currents

b) Find V_{ab}.



Answer:



$$
\begin{aligned}
80 - 7(i_1 + i_2) - i_1 - 7 &= 0 \\
7 + i_1 - 10 i_2 + 60 &= 0
\end{aligned}
$$

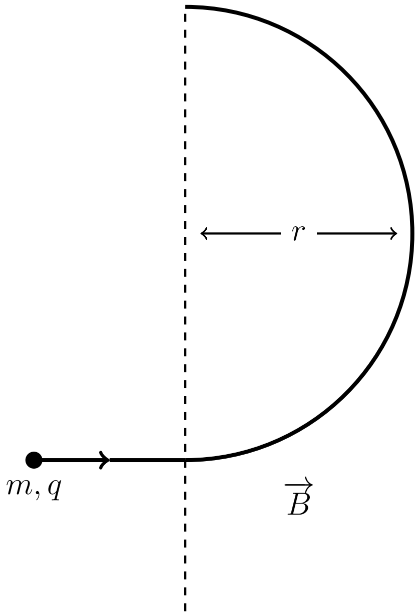
$$
\begin{aligned}
8 i_1 + 7 i_2 &= 73 \\
-i_1 + 10 i_2 &= 67
\end{aligned}
$$

The solution of the system of equations give:

$$i_2 = 7\,A, \quad i_1 = 3\,A, \quad i_1 + i_2 = 10\,A$$

$$V_{ab} = +7 + 3 - 56 = -46V$$

4) A charged particle with mass m, charge q, kinetic energy K enters a magnetic field and then follows a circle of radius r as seen in the figure. Find the magnitude and direction of the magnetic field $\overrightarrow{B}$.



Answer:

$$K = \frac{1}{2}mv^2 \quad \Rightarrow \quad v = \sqrt{2K/m}$$

$$r = \frac{mv}{qB}$$

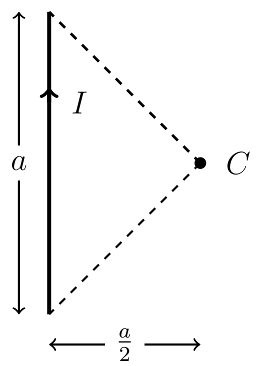
$$B = \frac{mv}{qr} = \frac{m\sqrt{2K/m}}{qr}$$

$$B = \frac{\sqrt{2Km}}{qr}$$

Using the right hand rule, we see that $\overrightarrow{B}$ is into the page.

5) A current of I flows around a square of side length a. Find the magnitude of the magnetic field at the center of the square.

Answer:



Using Biot-Savart law for one side of the square, we obtain the magnetic field at C as:

$$B = \int_{-\pi/4}^{\pi/4} \frac{\mu_0 I \cos\theta \, d\theta}{4\pi(a/2)}$$

$$B = \frac{\mu_0 I}{2\pi a} \sin\theta \Big|_{-\pi/4}^{\pi/4}$$

$$B = \frac{\mu_0 I \sqrt{2}}{2\pi a}$$

If we add the magnetic fields of all four sides:

$$B_{total} = 2\sqrt{2}\,\frac{\mu_0 I}{\pi a}$$

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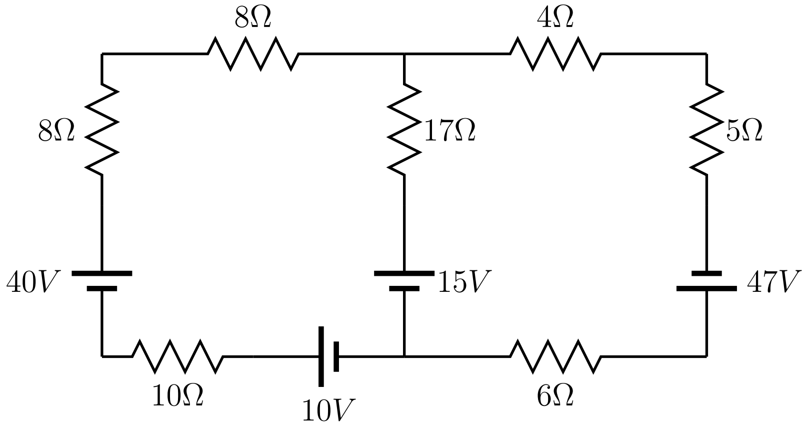
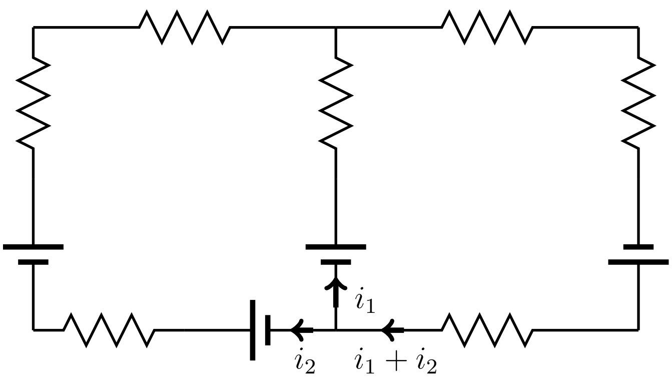
Final Examination

1) Find all currents in the following circuit:



Answer: Let's choose the currents as follows:



Now, using Kirchoff's loop rule, we obtain:

$$+10 - 10i_2 + 40 - 8i_2 - 8i_2 + 17i_1 - 15 = 0$$

$$+15 - 17i_1 - 4(i_1 + i_2) - 5(i_1 + i_2) + 47 - 6(i_1 + i_2) = 0$$

$$-17i_1 + 26i_2 = 35$$

$$32i_1 + 15i_2 = 62$$

The solution of this system of equations give:

$$i_1 = 1A, \quad i_2 = 2A, \quad i_1 + i_2 = 3A$$

2) a) The time constant of an RC circuit is $\tau = RC = 0.05\,s$. Find the time necessary for initially uncharged capacitor to be charged to 99% of its final charge.

Answer:

$$q = C\mathcal{E}(1 - e^{-t/RC})$$

Final charge is $C\mathcal{E}$ therefore $q = 0.99\,C\mathcal{E}$.

$$0.99 = 1 - e^{-t/\tau}$$

$$e^{-t/\tau} = 0.01$$

$$-t/\tau = \ln 0.01$$

$$t = 0.05s. \times 4.6 = 0.23s.$$

b) A particle of mass m, charge q travels in a circular path of radius R in a uniform magnetic field B. Find its period of revolution.

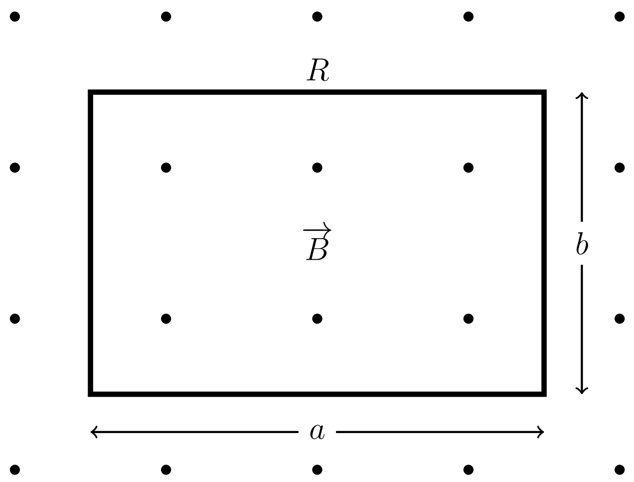
Answer:

$$r = \frac{mv}{qB}$$

$$T = \frac{2\pi r}{v}$$

$$T = \frac{2\pi m}{qB}$$

3) A variable magnetic field of magnitude $B = B_0(1 + t^3)$ is perpendicular to a rectangular loop. The loop has dimensions $a \times b$ and resistance R. Find the magnitude and direction of the induced current.



Answer:

$$\Phi_B = BA = Bab = B_0(1 + t^3)ab$$

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -B_0 3t^2 ab$$

$$I = \frac{\mathcal{E}}{R} = -\frac{3B_0 t^2 ab}{R}$$

Positive direction is counter-clockwise.
Therefore I is in clockwise direction.

4) a) In a series RLC circuit, $L = 90\,mH$, $C = 1.5\,\mu F$ and $f = 0.8\,kHz$. If the phase constant is 55°, what is the resistance of the coil?

Answer:

$$\omega = 2\pi f = 5027 s^{-1}$$

$$X_L = \omega L = 452\,\Omega$$

$$X_C = \frac{1}{\omega C} = 132\,\Omega$$

$$\frac{X_L - X_C}{R} = \tan 55 = 1.428$$

$$R = \frac{X_L - X_C}{\tan\phi} = 224\,\Omega$$

b) An oscillating LC circuit is made of a $4\,nF$ capacitor and a $9\,mH$ inductor. The maximum current is $3\,A$. What is the maximum charge on the capacitor?

$$\frac{1}{2} L I_{max}^2 = \frac{1}{2}\frac{q_{max}^2}{C}$$

$$q_{max} = \sqrt{LC}\, I$$

$$q_{max} = \sqrt{9 \cdot 10^{-3} \cdot 4 \cdot 10^{-9}}\; 3$$

$$q_{max} = 18\,\mu C$$

5) a) The displacement current through a rectangular loop of area $3.5\,m^2$ is $8\,A$. At what rate is the electric field through the loop changing?

$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 \frac{d(EA)}{dt} = \varepsilon_0 A \frac{dE}{dt}$$

$$\frac{dE}{dt} = \frac{i_d}{\varepsilon_0 A} = \frac{8}{8.85 \cdot 10^{-12} \cdot 3.5}$$

$$\frac{dE}{dt} = 2.58 \times 10^{11} V/m.s$$

b) The maximum electric field of an electromagnetic wave is $5\,V/m$. What is the intensity?

$$I = \frac{E_{rms}^2}{c\mu_0} = \frac{E_m^2}{2c\mu_0}$$

$$I = \frac{5^2}{2 \cdot 3 \cdot 10^8 \cdot 4\pi \cdot 10^{-7}}$$

$$I = 0.033\,W/m^2$$

Prepared by: Dr. Emre Sermutlu (2010)