Задача 6. Пресметнете определения интеграл $\displaystyle\int_0^{\frac{\pi}{2}} \left| \frac{1}{2} - \sin^2 x \right| \mathrm{d}\, x$.

Решение.

$$I = \int_0^{\frac{\pi}{2}} \left| \frac{1}{2} - \sin^2 x \right| \mathrm{d}\, x = \int_0^{\frac{\pi}{2}} \left| \frac{1 - 2\sin^2 x}{2} \right| \mathrm{d}\, x = \int_0^{\frac{\pi}{2}} \left| \frac{\cos^2 x - \sin^2 x}{2} \right| \mathrm{d}\, x =$$

$$= \frac{1}{2} \int_0^{\frac{\pi}{2}} |\cos 2x| \,\mathrm{d}\, x = \frac{1}{2} \int_0^{\frac{\pi}{4}} \cos 2x \,\mathrm{d}\, x - \frac{1}{2} \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos 2x \,\mathrm{d}\, x =$$

$$= \frac{1}{4} \int_0^{\frac{\pi}{4}} \cos 2x \,\mathrm{d}\, 2x - \frac{1}{4} \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos 2x \,\mathrm{d}\, x = \frac{1}{4} \sin 2x \Big|_0^{\frac{\pi}{4}} - \frac{1}{4} \sin 2x \Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}} =$$

$$= \frac{1}{4} - \frac{1}{4} \times 0 - \frac{1}{4} \times 0 + \frac{1}{4} = \frac{1}{2}.$$

$\square$