Задача 7. Да се пресметне интегралът $\displaystyle\int_0^2 \ln(x^2+4)\mathrm{d}\,x$.

Решение.

$$I = \int_0^2 \ln(x^2+4)\mathrm{d}\,x \overset{\text{И.Ч.}}{=} x\ln(x^2+4)\Big|_0^2 - \int_0^2 x\,\mathrm{d}\ln(x^2+4) = 2\ln 8 - 0 - \int_0^2 x\ln'(x^2+4)\mathrm{d}\,x =$$

$$= 6\ln 2 - \int_0^2 x \times 2x \times \frac{1}{x^2+4}\,\mathrm{d}\,x = 6\ln 2 - 2\underbrace{\int_0^2 \frac{x^2}{x^2+4}\,\mathrm{d}\,x}_{J}.$$

$$J = \int_0^2 \frac{x^2}{x^2+4}\,\mathrm{d}\,x = \int_0^2 \frac{x^2+4-4}{x^2+4}\,\mathrm{d}\,x = \int_0^2 1 - \frac{4}{x^2+4}\,\mathrm{d}\,x = x\Big|_0^2 - 4\int_0^2 \frac{1}{x^2+4}\,\mathrm{d}\,x =$$

$$= 2 - 4\int_0^2 \frac{1}{x^2+2^2}\,\mathrm{d}\,x = 2 - 4\times\frac{1}{2}\arctan\frac{x}{2}\Big|_0^2 =$$

$$= 2 - 2\arctan 1 + 2\arctan 0 = 2 - 2\times\frac{\pi}{4} + 2\times 0 = 2 - \frac{\pi}{2}.$$

Следователно, $I = 6\ln 2 - 2J = 6\ln 2 + \pi - 4$.

□