Задача 8. Да се пресметне определеният интеграл

$$\int_1^2 x \log_2 \frac{1}{x} \,\mathrm{d}\, x$$

Решение.

$$\int_1^2 x \log_2 \frac{1}{x} \,\mathrm{d}\, x = \int_1^2 x \times \frac{\ln(x^{-1})}{\ln(2)} \,\mathrm{d}\, x = -\frac{1}{\ln(2)} \times \underbrace{\int_1^2 x \ln(x) \mathrm{d}\, x}_{I}.$$

$$I = \int_1^2 x \ln(x) \mathrm{d}\, x = \int_1^2 \ln(x) \mathrm{d}\, \frac{x^2}{2} \overset{\text{и.ч.}}{=} \frac{x^2 \ln(x)}{2} - \int_1^2 \frac{x^2}{2} \,\mathrm{d} \ln(x) =$$

$$= \frac{x^2 \ln(x)}{2} - \int_1^2 \frac{x^2}{2} \times \frac{\partial \ln(x)}{\partial x} \,\mathrm{d}\, x = \frac{x^2 \ln(x)}{2} - \int_1^2 \frac{x^2}{2} \times \frac{1}{x} \,\mathrm{d}\, x =$$

$$= \frac{x^2 \ln(x)}{2} - \frac{1}{2} \times \frac{x^2}{2} \bigg|_1^2 = \frac{4 \ln(2)}{2} - \frac{4}{4} - \frac{\ln(1)}{2} + \frac{1}{4} =$$

$$= 2 \ln(2) - 1 - 0 + \frac{1}{4} = 2 \ln(2) - \frac{3}{4}.$$

Окончателно,

$$\int_1^2 x \log_2 \frac{1}{x} \,\mathrm{d}\, x = -\frac{1}{\ln(2)} \left(2 \ln(2) - \frac{3}{4}\right) = -\frac{8 \ln(2) - 3}{4 \ln(2)} \approx -0.9179787193332774.$$