Задача. Да се построи краен детерминиран автомат, който разпознава всички думи от езика $L = \left\{\omega \in \{0, 1, 2\}^* \,|\, \omega \text{ е число в троична бройна система, което се дели на } 4\right\}$.

Решение. Нека с $\overline{\beta}$ означим число β в троичен запис, тоест $\overline{\beta} \in \{0, 1, 2\}^*$.

$$\begin{cases} \overline{\varepsilon} = 0 \\ \overline{0\beta} = \overline{\beta} \\ \overline{1\beta} = 1 \times 3^{|\beta|} + \overline{\beta} \\ \overline{2\beta} = 2 \times 3^{\beta} + \overline{\beta} \end{cases}$$

Обяснение:

$$\overline{d\beta} = \overline{d\underbrace{a_{n-1}a_{n-2}\ldots a_1a_0}_{n=|\beta|}} =$$

$$= d \times 3^n + a_{n-1} \times 3^{n-1} + \ldots + a_1 \times 3^1 + a_0 \times 3^0$$

Нека $L_0 = L = \left\{\beta \in \{0, 1, 2\}^* \,|\, \overline{\beta} \equiv 0 (\mod 4)\right\}$. За построяването на детерминиран автомат $\mathscr{A}$ с език $L(\mathscr{A}) = L_0 = L$ ще използваме **алгоритъма на Бжозовски**.

$$0^{-1}L_0 = \left\{\beta \,|\, \overline{0\beta} \equiv 0 (\mod 4)\right\} = \left\{\beta \,|\, \overline{\beta} \equiv 0 (\mod 4)\right\} = L_0$$
$$1^{-1}L_0 = \left\{\beta \,|\, \overline{1\beta} \equiv 0 (\mod 4)\right\} = \left\{\beta \,|\, 3^{|\beta|} + \overline{\beta} \equiv 0 (\mod 4)\right\} =: L_1$$
$$2^{-1}L_0 = \left\{\beta \,|\, \overline{2\beta} \equiv 0 (\mod 4)\right\} = \left\{\beta \,|\, 2 \times 3^{|\beta|} + \overline{\beta} \equiv 0 (\mod 4)\right\} =: L_2$$

Получихме два нови езика L_1 и L_2.

$$0^{-1}L_1 = \left\{\beta \,|\, \overline{0\beta} \in L_1\right\} = \left\{\beta \,|\, 3^{|0\beta|} + \overline{0\beta} \equiv 0 (\mod 4)\right\} = \left\{\beta \,|\, 3^{|\beta|+1} + \overline{\beta} \equiv 0 (\mod 4)\right\} =$$
$$= \left\{\beta \,|\, 3 \times 3^{|\beta|} + \overline{\beta} \equiv 0 (\mod 4)\right\} =: L_3$$

$$1^{-1}L_1 = \left\{\beta \,|\, \overline{1\beta} \in L_1\right\} = \left\{\beta \,|\, 3^{|1\beta|} + \overline{1\beta} \equiv 0 (\mod 4)\right\} = \left\{\beta \,|\, 3 \times 3^{|\beta|} + (3^{|\beta|} + \overline{\beta}) \equiv 0 (\mod 4)\right\} =$$
$$= \left\{\beta \,|\, 4 \times 3^{|\beta|} + \overline{\beta} \equiv 0 (\mod 4)\right\} = \left\{\beta \,|\, \overline{\beta} \equiv 0 (\mod 4)\right\} = L_0$$

$$2^{-1}L_1 = \left\{\beta \,|\, \overline{2\beta} \in L_1\right\} = \left\{\beta^{|2\beta|} + \overline{2\beta} \equiv 0 (\mod 4)\right\} = \left\{\beta \,|\, 3 \times 3^{|\beta|} + (2 \times 3^{|\beta|} + \overline{\beta}) \equiv 0 (\mod 4)\right\} =$$
$$= \left\{\beta \,|\, 5 \times 3^{|\beta|} + \overline{\beta} \equiv 0 (\mod 4)\} = \{\beta \,|\, 3^{|\beta|} + \overline{\beta} (\mod 4)\right\} = L_1$$

$$0^{-1}L_2 = \left\{\beta \,|\, \overline{0\beta} \in L_2\right\} = \left\{\beta \,|\, 2 \times 3^{|0\beta|} + \overline{0\beta} \equiv 0 (\mod 4)\right\} = \left\{\beta \,|\, 6 \times 3^{|\beta|} + \overline{\beta} \equiv 0 (\mod 4)\right\} =$$
$$= \left\{\beta \,|\, 2 \times 3^{|\beta|} + \overline{\beta} \equiv 0 (\mod 4)\right\} = L_2$$

$$1^{-1}L_2 = \left\{\beta \,|\, \overline{1\beta} \in L_2\right\} = \left\{\beta \,|\, 2 \times 3^{|1\beta|} + \overline{1\beta} \equiv 0 (\mod 4)\right\} = \left\{\beta \,|\, 6 \times 3^{|\beta|} + 3^{|\beta|} + \overline{\beta} \equiv 0 (\mod 4)\right\} =$$
$$= \left\{\beta \,|\, 3 \times 3^{|\beta|} + \overline{\beta} \equiv 0 (\mod 4)\right\} = L_3$$

$$2^{-1}L_2 = \left\{\overline{2\beta} \in L_2\right\} = \left\{\beta \,|\, 2 \times 3^{|2\beta|} + \overline{2\beta} \equiv 0 (\mod 4)\right\} = \left\{\beta \,|\, 6 \times 3^{|\beta|} + 2 \times 3^{|\beta|} + \overline{\beta} \equiv 0 (\mod 4)\right\}$$
$$= \left\{\beta \,|\, \overline{\beta} \equiv 0 (\mod 4)\right\} = L_0$$

Получихме един нов език L_3.

$$0^{-1}L_3 = \left\{\beta \,|\, \overline{0\beta} \in L_3\right\} = \left\{\beta \,|\, 3 \times 3^{|0\beta|} + \overline{0\beta} \equiv 0 (\mod 4)\right\} = \left\{\beta \,|\, 9 \times 3^{|\beta|} + \overline{\beta} \equiv 0 (\mod 4)\right\} =$$
$$= \left\{\beta \,|\, 3^{\beta} + \overline{\beta} \equiv 0 (\mod 4)\right\} = L_1$$

$1^{-1}L_3 = \left\{\beta \,|\, \overline{1\beta} \in L_3\right\} = \left\{\beta \,|\, 3 \times 3^{|1\beta|} + \overline{1\beta} \equiv 0 \pmod 4\right\} = \left\{\beta \,|\, 9 \times 3^{|\beta|} + (3^{|\beta|} + \overline{\beta}) \equiv 0 \pmod 4\right\} =$
$= \left\{\beta \,|\, 2 \times 3^{|\beta|} + \overline{\beta} \equiv 0 \pmod 4\right\} = L_2$

$2^{-1}L_3 = \left\{\beta \,|\, \overline{2\beta} \in L_3\right\} = \left\{\beta \,|\, 3 \times 3^{|2\beta|} + \overline{2\beta} \equiv 0 \pmod 4\right\} = \left\{\beta \,|\, 9 \times 3^{|\beta|} + (2 \times 3^{|\beta|} + \overline{\beta}) \equiv 0 \pmod 4\right\} =$
$= \left\{\beta \,|\, 3 \times 3^{|\beta|} + \overline{\beta} \equiv 0 \pmod 4\right\} = L_3$

език	δ	**0**	**1**	**2**
L_0	q_0	q_0	q_1	q_2
L_1	q_1	q_3	q_0	q_1
L_2	q_2	q_2	q_3	q_0
L_3	q_3	q_1	q_2	q_3

q_0 е начално състояние, тъй като $L_0 = L$ и отново q_0 е и финално състояние, тъй като $\varepsilon \in L_0$.