Задача 7. Намерете неопределения интеграл $\int \arctan\sqrt{x}\,\mathrm{d}\,x,\ x > 0.$

Решение.

$$I = \int \arctan\sqrt{x}\,\mathrm{d}\,x = \underbrace{x\arctan\sqrt{x}}_{A} - \int x\,\mathrm{d}\arctan\sqrt{x} = A - \int x\arctan'\sqrt{x}\,\mathrm{d}\,x =$$

$$= A - \int x\,.\,\frac{1}{1+(\sqrt{x})^2}\,.\,(\sqrt{x})'\mathrm{d}\,x = A - \int \frac{x}{1+x}\,.\,\frac{1}{2}\,.\,x^{-\frac{1}{2}}\,\mathrm{d}\,x = A - \frac{1}{2}\underbrace{\int \frac{\sqrt{x}}{1+x}\,\mathrm{d}\,x}_{J}$$

$$J = \int \frac{\sqrt{x}}{1+x}\,\mathrm{d}\,x \underset{\mathrm{d}\,x = \mathrm{d}\,u^2 = 2u\,\mathrm{d}\,u}{\overset{u=\sqrt{x}}{=}} \int \frac{2u\,.\,u}{1+u^2}\,\mathrm{d}\,u = 2\int \frac{u^2}{1+u^2}\,\mathrm{d}\,u =$$
$$= 2\int \frac{u^2+1-1}{1+u^2}\,\mathrm{d}\,u = 2\int 1\,\mathrm{d}\,u - 2\int \frac{1}{1+u^2}\,\mathrm{d}\,u = 2u - 2\arctan(u) =$$
$$= 2\sqrt{x} - 2\arctan\sqrt{x}$$

Следователно,

$$I = A - \frac{1}{2}\int \frac{\sqrt{x}}{1+x}\,\mathrm{d}\,x = x\arctan\sqrt{x} - \frac{1}{2}\left(2\sqrt{x} - 2\arctan\sqrt{x}\right) + C =$$
$$= x\arctan\sqrt{X} + \arctan\sqrt{x} - \sqrt{x} + C = \underline{\underline{(x+1)\arctan\sqrt{x} - \sqrt{x} + C}}\,.$$

$\square$