Задача 7. Да се пресметне интегралът $\displaystyle\int_1^8 \frac{\mathrm{d}\,x}{\sqrt[3]{x} + x}$.

Решение.

$$I = \int_1^8 \frac{\mathrm{d}\,x}{\sqrt[3]{x} + x} \underset{t \in [1,2]}{\overset{x = t^3}{=\!=\!=}} \int_1^2 \frac{\mathrm{d}\,t^3}{t + t^3} = \int_1^2 \frac{3t^2}{t + t^3}\,\mathrm{d}\,t = 3\underbrace{\int_1^2 \frac{t}{t^2 + 1}\,\mathrm{d}\,t}_{J}.$$

$$J = \int_1^2 \frac{t}{t^2 + 1}\,\mathrm{d}\,t = \frac{1}{2}\int_1^2 \frac{\mathrm{d}\,t^2 + 1}{t^2 + 1} = \frac{1}{2}\ln(t^2 + 1)\Big|_1^2 = \frac{1}{2}(\ln 5 - \ln 2).$$

Следователно, $I = \dfrac{3}{2}(\ln 5 - \ln 2) = \dfrac{3}{2}\ln\dfrac{5}{2}$.

$\square$