**Задача 8**. Да се пресметне определеният интеграл

$$\int_0^{\frac{\pi}{2}} x \cos^2 x \,\mathrm{d}\, x$$

**Решение**.

$$I = \int_0^{\frac{\pi}{2}} x \cos^2 x \,\mathrm{d}\, x$$

Имаме, че $\cos^2 x = \dfrac{2\cos^2 x}{2} = \dfrac{\cos^2 x + 1 - \sin^2 x}{2} = \dfrac{\cos^2 x - \sin^2 x}{2} + \dfrac{1}{2} = \dfrac{\cos 2x}{2} + \dfrac{1}{2}$.

Следователно,

$$\begin{aligned} I &= \int_0^{\frac{\pi}{2}} x \cos^2 x \,\mathrm{d}\, x = \frac{1}{2}\int_0^{\frac{\pi}{2}} x \cos 2x \,\mathrm{d}\, x + \frac{1}{2}\int_0^{\frac{\pi}{2}} x \,\mathrm{d}\, x = \\ &= \frac{1}{2}\int_0^{\frac{\pi}{2}} x \,\mathrm{d}\, \frac{\sin 2x}{2} + \frac{1}{2} \times \frac{x^2}{2}\bigg|_0^{\frac{\pi}{2}} = \frac{1}{4}\underbrace{\int_0^{\frac{\pi}{2}} x \,\mathrm{d} \sin 2x}_{J} + \frac{\pi^2}{16}. \end{aligned}$$

$$\begin{aligned} J &= \int_0^{\frac{\pi}{2}} x \,\mathrm{d} \sin 2x = x \sin 2x\bigg|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \sin 2x \,\mathrm{d}\, x = \\ &= \underbrace{\frac{\pi}{2}\sin\pi}_{=0} - \underbrace{0\sin 0}_{=0} - \frac{1}{2}\int_0^{\frac{\pi}{2}} \sin 2x \,\mathrm{d}\, 2x = -\frac{1}{2}(-\cos 2x)\bigg|_0^{\frac{\pi}{2}} = \\ &= \frac{1}{2}\cos\pi - \frac{1}{2}\cos 0 = -\frac{1}{2} - \frac{1}{2} = -1. \end{aligned}$$

Следователно,

$$I = \frac{1}{4} \times (-1) + \frac{\pi^2}{16} = \frac{\pi^2 - 4}{16} \approx 0.366850275068.$$

$\square$