

Actividad 6 Estadística Aplicada GPO 51

Erika Denisse Cardona Padilla 1888872

1. a) $E(Y_t) = E(\varepsilon_t) = 0$

$$\begin{aligned} \text{b) } \text{Var}(Y_t) &= E(Y_t - \mu_{Y_t})^2 \\ &= E(\varepsilon_t - 0)^2 \\ &= E(\varepsilon_t^2) \\ &= \sigma^2_\varepsilon \end{aligned}$$

$$\begin{aligned} \text{c) } \text{Cov}(Y_t, Y_{t+k}) &= E(Y_t - \mu_{Y_t})(Y_{t+k} - \mu_{Y_{t+k}}) \\ &= E(\varepsilon_t) \cdot E(\varepsilon_{t+k}) \\ &= 0 \end{aligned}$$

d) El proceso no tiene memoria, no cuenta con medio, varianza constante \Rightarrow es proceso no estacionario.

2. a) converge exponencialmente a cero ya que converge solamente si su varianza lo hace, por sus variables aleatorias en el tiempo $t, t-1, t-2, \dots$

$$\begin{aligned} \text{b) Media} &= E(Y_t) = E(\varphi_1 Y_{t-1} + \varphi_2 Y_{t-2} + \varepsilon_t) \\ &= \varphi_1 E(Y_{t-1}) + \varphi_2 E(Y_{t-2}) + E(\varepsilon_t) \\ &= \varphi_1 Y_{t-1} + \varphi_2 Y_{t-2} \end{aligned}$$

$$\text{c) } \text{Var}(Y_t) = Y_0 = \frac{(1 - \varphi_2) \sigma^2_\varepsilon}{(1 + \varphi_2) [(1 - \varphi_2)^2 - \varphi_1^2]}$$

$$Y_1 = \frac{\varphi_1 Y_0}{1 - \varphi_2}$$

$$Y_2 = \frac{\varphi_2 (1 - \varphi_2) + \varphi_1^2 Y_0}{1 - \varphi_2}$$

$$\begin{aligned}
 d) \operatorname{Cov}(Y_t, Y_{t-1}) &= E(Y_t - \mu)(Y_{t-1} - \mu) \\
 &= E(\varphi_1 Y_{t-1}) \\
 &= \varphi_1 Y_1 + \varphi_2 Y_2
 \end{aligned}$$

$$\begin{aligned}
 e) \operatorname{Cov}(Y_t, Y_{t+1}) &= E(Y_t - \mu)(Y_{t+1} - \mu) \\
 &= \varphi_1 Y_0 + \varphi_2 Y_1
 \end{aligned}$$

$$\begin{aligned}
 f) \rho_1 &= \frac{\operatorname{Cov}(Y_t, Y_{t-1})}{\operatorname{Var}(Y_t)} \\
 &= \frac{E(Y_t, Y_{t-1})}{E(Y_t)^2} \\
 &= \frac{\varphi_1 Y_1 + \varphi_2 Y_0}{Y_0} \\
 &= \varphi_1 + \varphi_2
 \end{aligned}$$

3. a) $E(Y_t) = E(\theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \varepsilon_t) = 0$

$$\begin{aligned}
 b) \operatorname{Var}(Y_t) &= E(Y_t - \mu) = E(\theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \varepsilon_t) \\
 &= \theta_1 E(\varepsilon_{t-1}) + \theta_2 E(\varepsilon_{t-2}) + E(\varepsilon_t) \\
 &= \theta_1 \sigma_\varepsilon^2 + \theta_2 \sigma_\varepsilon^2
 \end{aligned}$$

$$c) \operatorname{Cov}(Y_t, Y_{t-k}) = (1 + \theta_1^2 + \theta_2^2) \sigma_\varepsilon^2$$

$$d) \rho_k = 0 \quad \forall k > 2$$

e) grafica autocorrelación proceso $\rho_1 \dots \rho_{10}$

