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EE430 Project Part 2

Introduction

Spectrograms is a manner of frequency analysis which allow us to investigate the frequency representation of the time signal more accurate way. In this part of the project, by using spectrogram of the signals and the Doppler Effect, velocity and range of the source will be observed. Firstly, parameter choices and then test procedures and test results will be mentioned. After that by using 2 different approach (visual inspection and mathematical), “how the results are obtained” and “how the results can be improved” will be discussed in this report.

Introduction on MATLAB Code

MATLAB Code for this project is constructed as a function so that user can put input and get the results by changing inputs. But there are some ranges for the inputs and it will be discussed later in this report. In the function there will be 7 different input type and all ranges of these inputs are clarified in the code. User will pick signal type from 2 different options (sinusoidal wave and linear chirp) and then put some properties of the signal like Bandwidth, Duration, Frequency, Amplitude, Velocity of the source and Delay element. Total code will give the output for Estimated Velocity and Estimated Range and also figures for generated and transmitted signals and spectrogram of the transmitted signal like below

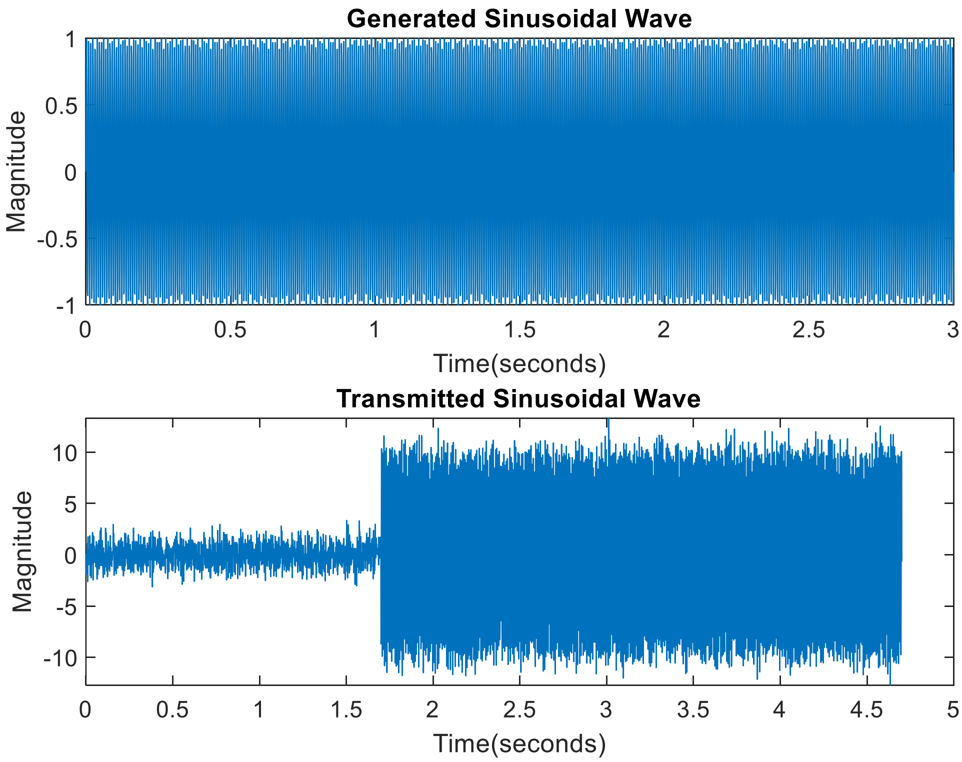

Figure 1-Example of Transmitted and Generated Wave

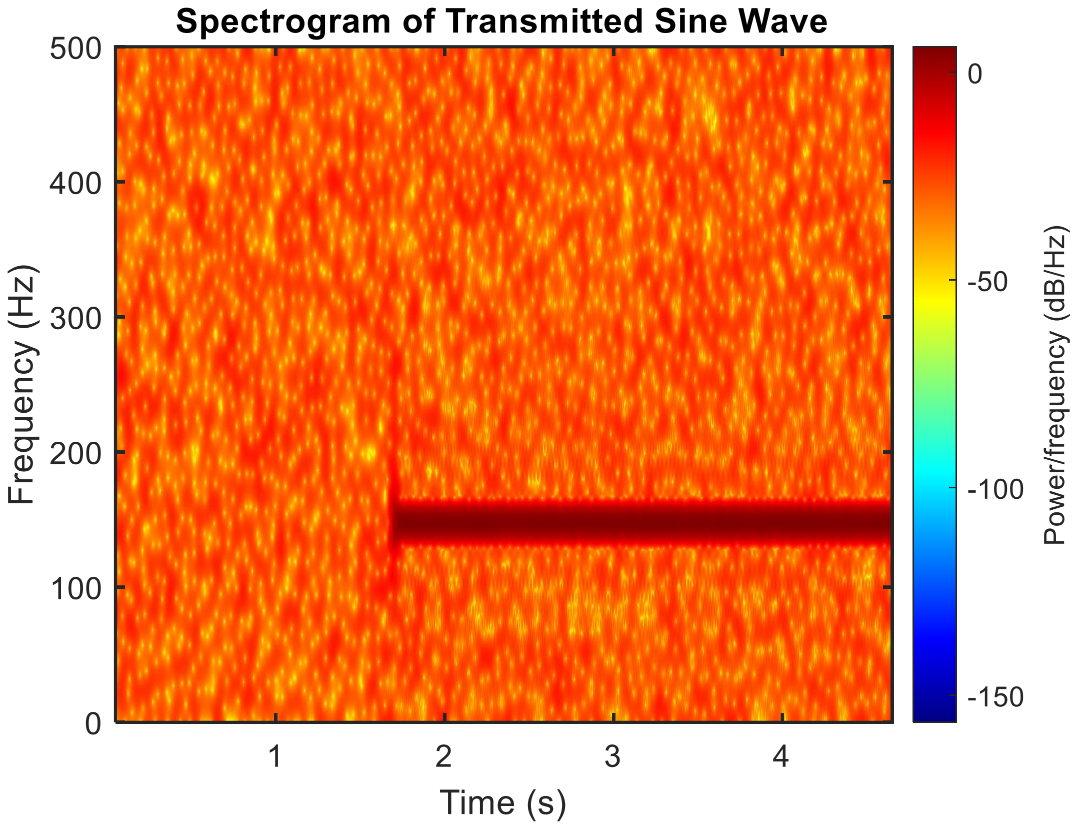

Figure 2-Example of Spectrogram Graph of a Wave

Parameters for Sinusoidal Signal

In generating sinusoidal signals, firstly the sampling frequency is decided as 1000 Hz and other parameters are chosen based on that decision. In Sinusoidal Signal parameters, there are 3 different parameters that can affect the result of our estimation which are Signal to Noise Ratio (depends on the amplitude of the sinusoidal signal), Velocity of the source and Frequency of the source. So, we have constructed some Monte Carlo trials, in order to pick some range for that values. Error signals were different for each trial and we changed only the one parameter to see the effect of that parameter. In the Figure 3, it can be seen that amplitude of the sine wave parameter has been encountered Monte Carlo trials (100 trials have been done for 10 different amplitude values) and it can be seen that when sine wave has low amplitude this means low SNR value also, estimation can have bigger errors but for the range of [1 10], estimation will have lower percentage errors so this range can be applicable (Upper range can be higher than “10”).

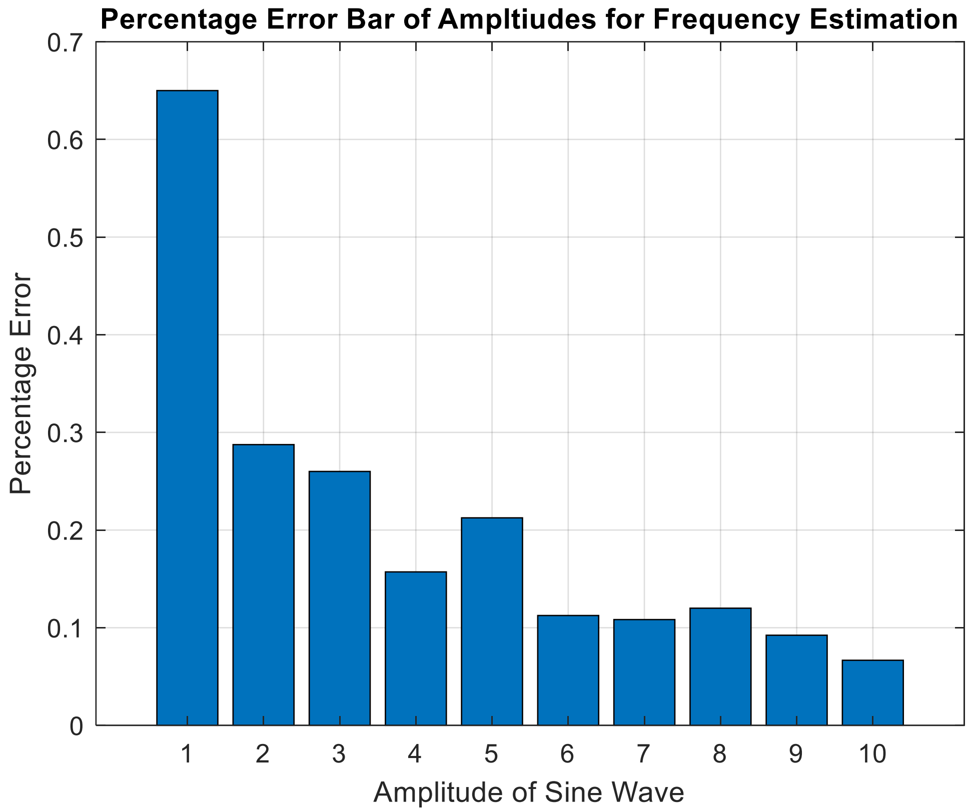

Figure 3-Error Histogram of Amplitude Change in Sine Wave

SNR value for Amplitude "10" = 15.38 dB

SNR value for Amplitude "1" = -7.47 dB

So, frequency estimation won't be affected when SNR is equal to -7.47 dB, but lower SNR values has a risk of affecting the estimation because spectrogram values of noisy components should be lower than the sine wave frequencies' spectrogram value and this is possible when noisy component is not much greater than sine wave components.

The other parameters to be chosen are Velocity of the Source and Frequency of the Signal. These two ranges depend on each other because our chosen sampling frequency prevents us to examine all possible frequency components at spectrogram. When sampling frequency is 1000 Hz and when we tried to examine the spectrograms of different signals, we have observed that when frequency is bigger than "500 Hz", Spectrogram observation is not like we expected and there are some huge errors in the results. We have observed these errors by again Monte Carlo trials by changing the Frequency of the sine wave and keep the other parameters same. We have chosen v=170, so transmitted sine wave should be two times greater than the frequency of the sine wave. In Figure 4, we can see the errors of the frequency estimation.

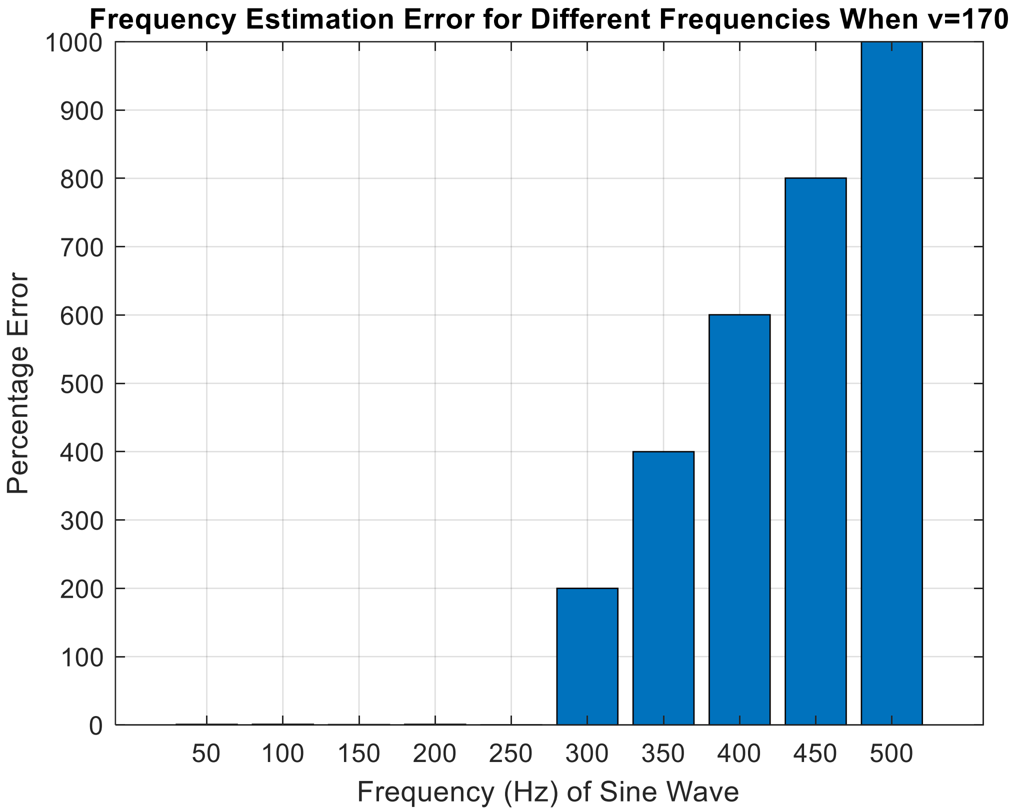

Figure 4-Error Histogram in Frequency Change in Sine Wave

As it can be seen from the figure when v=170 the transmitted frequency will be two times of the generated signal. And our method is getting errors when transmitted signals frequency is higher than 500 Hz so We have chosen the range that frequency of transmitted signal is not higher than 500 Hz.

Frequency Range chosen for generated sinusoidal signal = [0 250 Hz]

Velocity Range chosen for generated sinusoidal signal = [0 170 m/s]

So, by choosing these parameters, transmitted signal can not have a frequency value that is higher than 500 Hz and our estimation won't have bigger errors.

Estimation Procedure

In the estimation procedure, 2 different approaches will be used which are visual inspection of the spectrogram and automatic calculations using the STFT values. We will consider each approach in detail.

For visual inspection, spectrogram of a sinusoidal wave has an image that the darkest and bold color shows the maximum magnitude in our spectrogram image and for a sine wave, frequency of the transmitted signal is also a sine wave so its spectrogram will have a "line with darkest color" which shows the maximum magnitude in spectrogram. Gaussian distributed error won't have any effect on maximum spectrogram because its power spectrum is same for every frequency. Let's consider the visual inspection on a spectrogram result of a transmitted wave.

For v=170, frequency of generated signal is equal to 50 Hz and t0=1.8 Ts (Ts is sampling time)

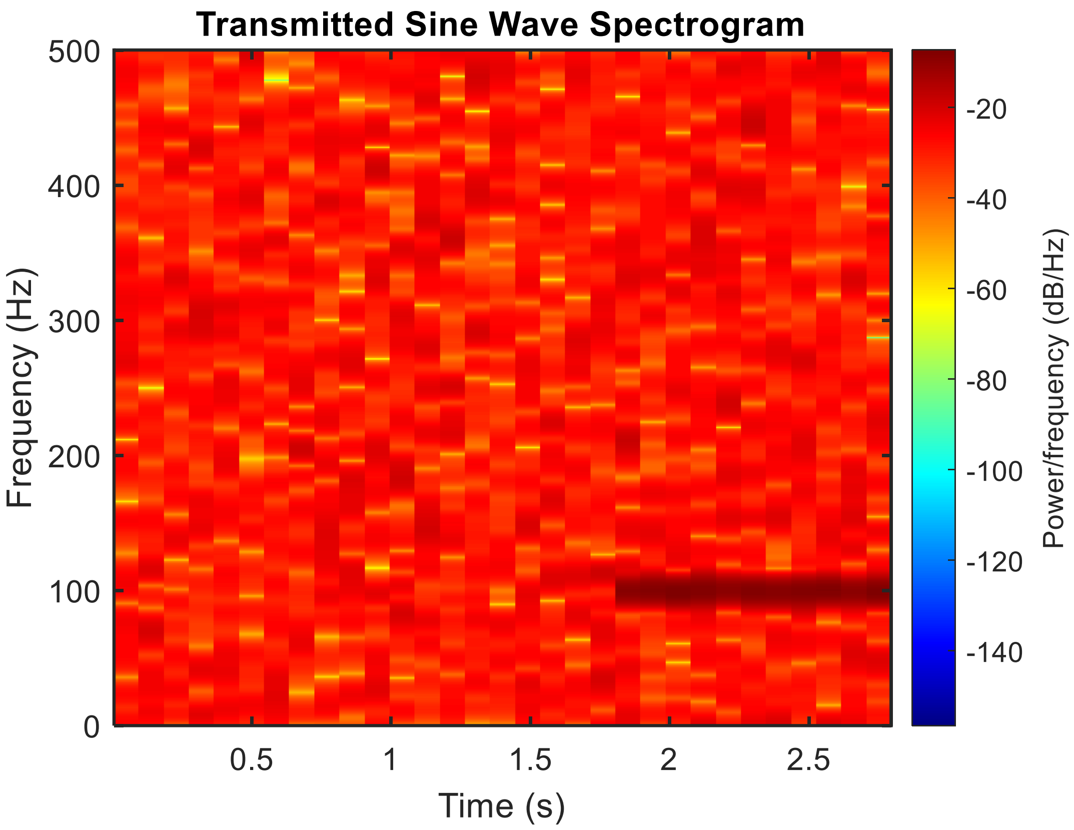

Figure 5-Spectrogram of Sine Wave for v=170, t0=1.8Ts, Frequency=50 Hz

It can be seen from the Figure 5 that transmitted signal has a frequency 100 Hz which is expected because from the Doppler Effect, received signal frequency is a multiplied version of generated signal frequency (multiplication factor is $\frac{c}{c-v}$ and in our case this is equal to "2"). And also, we can see from the spectrogram image that we can detect the starting time of the transmitted sine wave which corresponds to "t0" choice in our problem. So, we can detect the speed and range of the source by this two information.

We know the generated and transmitted frequency, and multiplication factor. From the multiplication factor, the observed equation is below

$$Estimated\_Velocity = c * (Estimated\_Frequency - F\_original)/ \\ Estimated\_Frequency \quad (2)$$

And also, we can find the range because arrival time can be observed from the visual inspection so

$$Estimated\_Range = c * Arrival\_Time \quad (1)$$

These two estimations can be observed from the spectrogram image but for the upper case for example, we have used the spectrogram with window length=100, window increase (overlapping windows) =10 and this will give us the correct arrival time value (in other words, time resolution and time arrival approximation are good enough) but frequency resolution is low and for numerical solution it can create a problem for us. So, we have to consider the effects of the window length and window overlap.

If we have higher amount of window overlap, frequency resolution is a bit better but it is again below the expectation. It can be seen from the Figure 6 (Number of overlaps has been increased to 99)

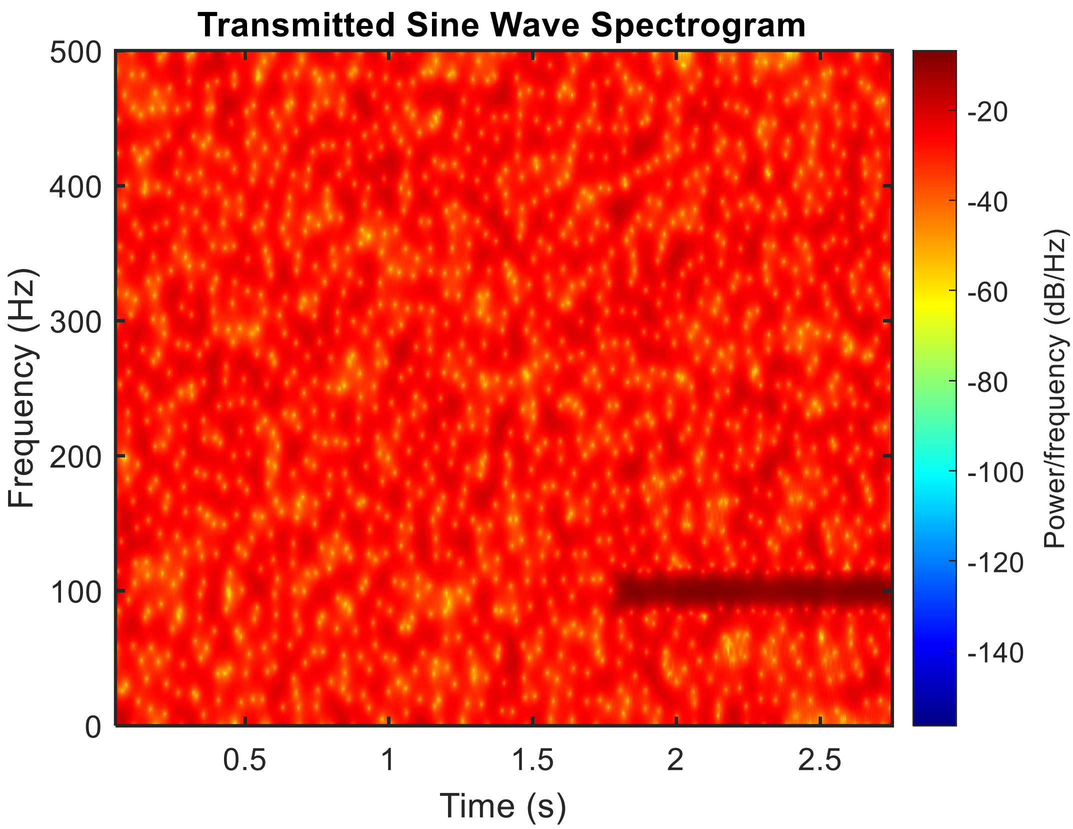

Figure 6-Spectrogram of Sine Wave when Overlap Percentage is increased

If we increase the window length and keep the window increase as “100” then we have much more great frequency resolution but time estimation for this scenario is not like we expected. In figure 7, t0=1.5 Ts, window length=1000, number of window overlap=100). So, window length has a negative effect on time estimation.

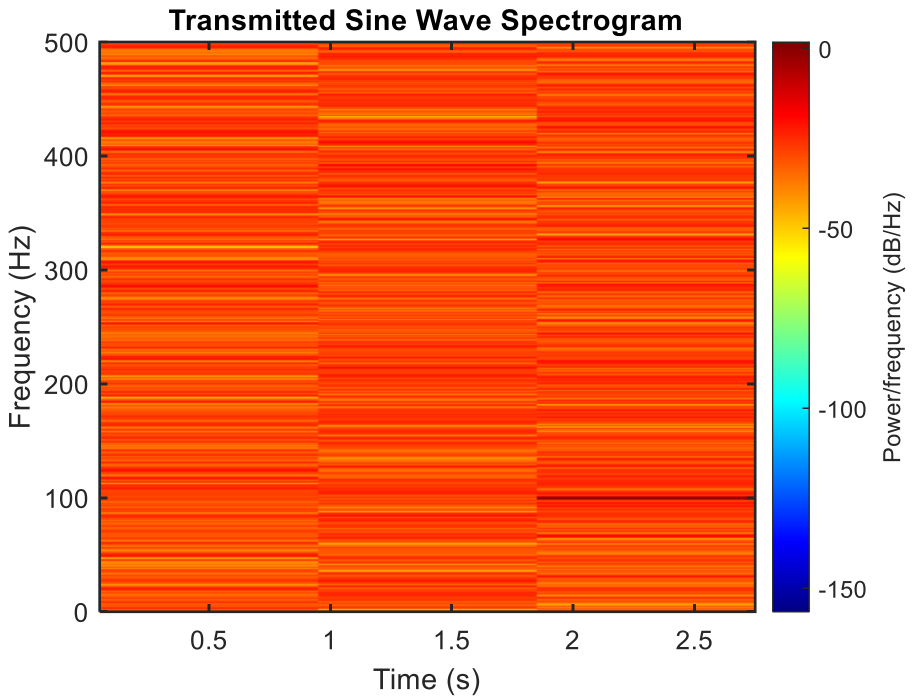

Figure 7-Spectrogram of Sine Wave when Window Length is increased

If we decrease the number of window length to “10” for window length “1000” then we will have a spectrogram image that won’t give an information that we need neither for frequency estimation nor time arrival so number of window overlap has a crucial effect on these estimations (it must be at least 10% percent of window length).

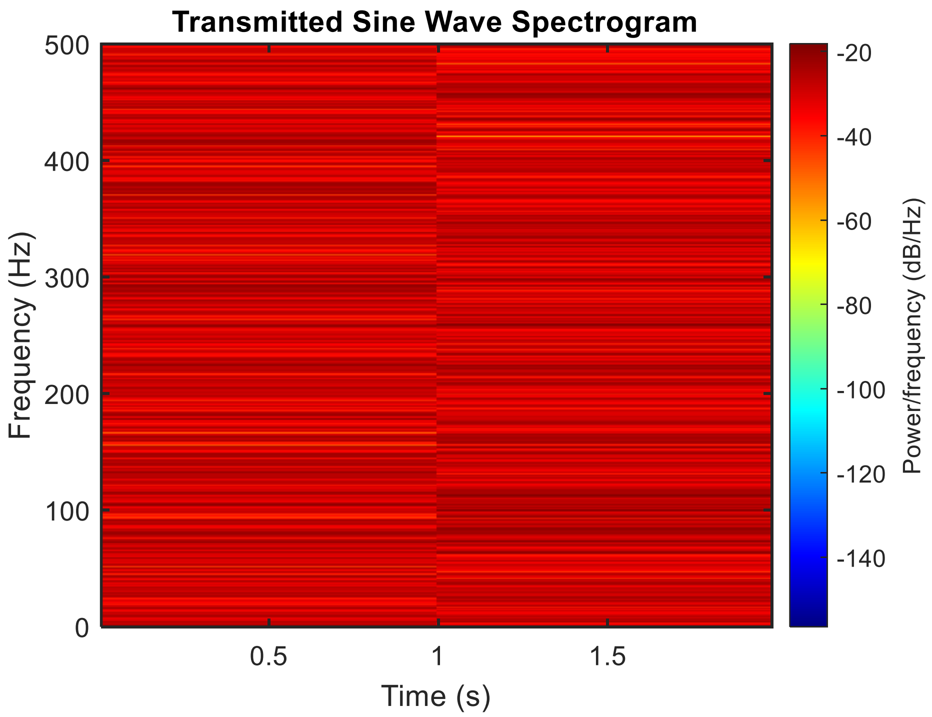

Figure 8-Spectrogram of Sine Wave when Window Overlap percentage is 1%

From all these cases, for better approximation of time arrival and frequency estimation, we have chosen window length=100 and number of window overlaps as “99”.

For Automatic Calculation part, we have extracted the power, time and index arrays for the spectrogram by the following code

```
[S,F,T,P]=spectrogram(data,100,99,10000,1000,"yaxis")
```

P array corresponds to power array and we have found the maximum powers of the array and find the indexes of the array. And by using these indexes, we have found the frequency values corresponds to these maximum powers (by using the F array) and chosen the end value of the frequency because transmitted signal has different maximum power in different time ranges and the end maximum powered frequency will definitely correspond to our estimated frequency. And for the arrival time we have used the following code part

```
I_time_arrival=find(abs(Maximum_Frequency_Array-Estimated_Frequency)<1,1,"first");
Estimated_Time_Arrival=T(I_time_arrival)
```

So, we can find the time arrival of the transmitted sine by comparing the maximum frequencies with the estimated frequency and if the time resolution is good enough, we can find the corresponded time arrival. In the end, we have estimated our frequency and time arrival so we can use the equations defined as (1) and (2) to estimate the range and velocity of the source.
For good estimation of frequency, window length and overlap must be higher because it will provide us to examine all the signal segments in a different way and maximum frequency value will be obtained with a great precision. In order to show that we made some trials with different window lengths to see the error in the estimation of the frequency.

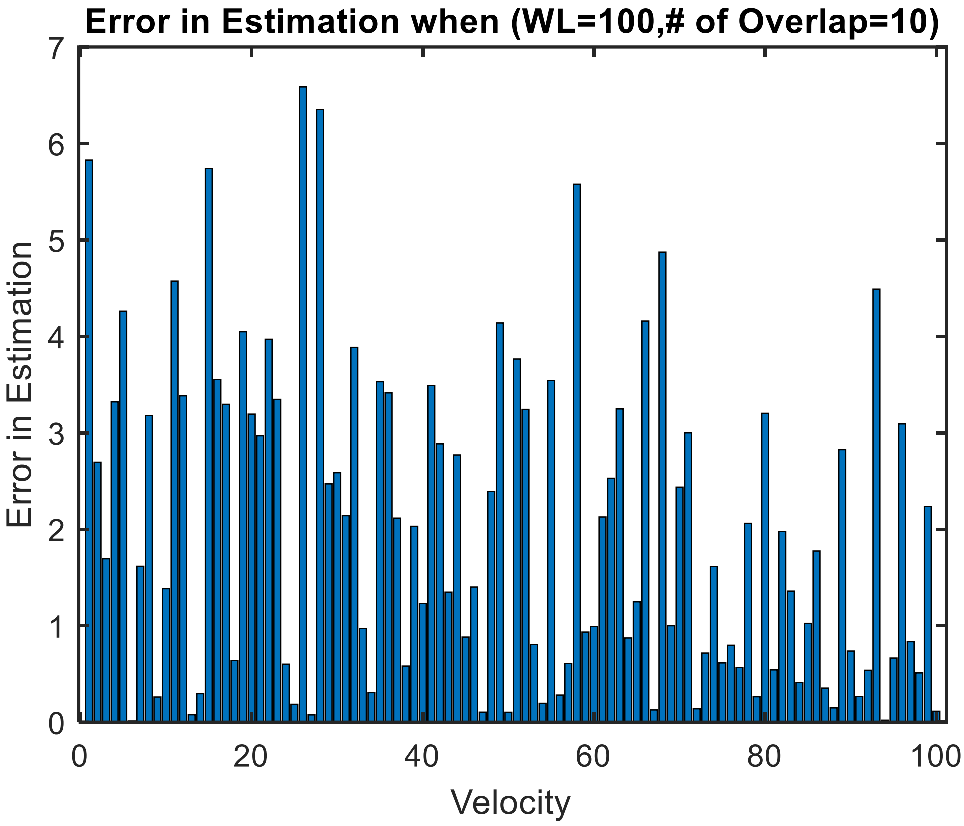

Figure 9-Error in Velocity Estimation when Window Length=100, Number of Overlap=10

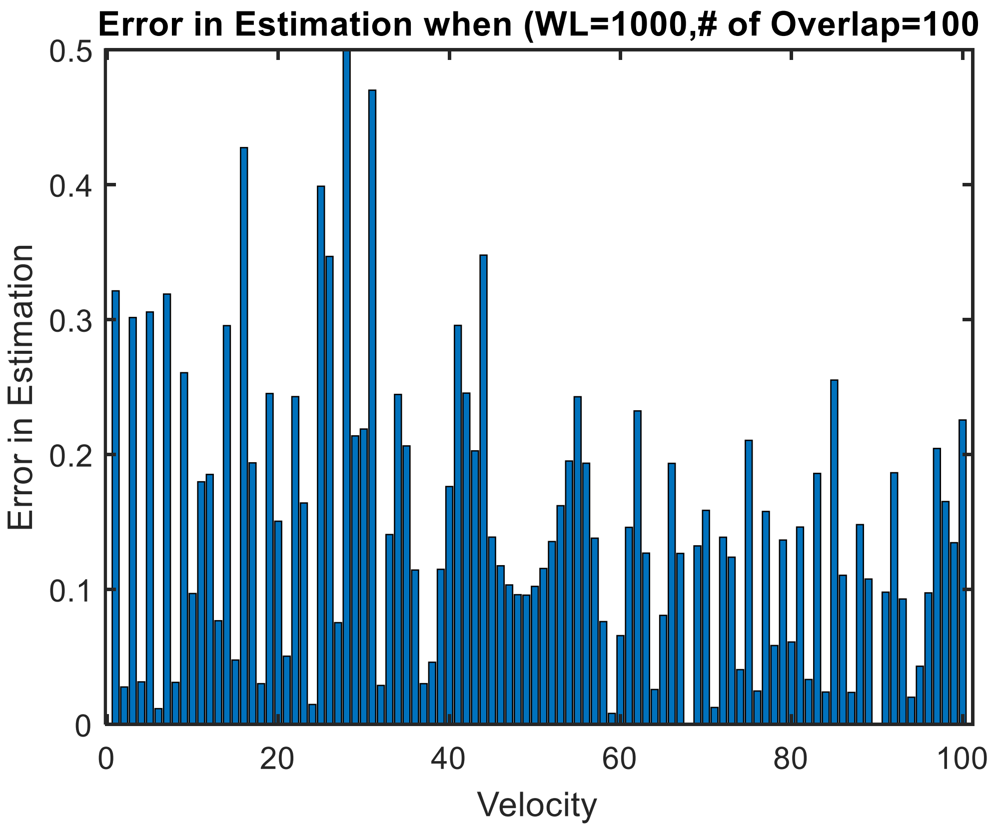

Figure 10-Error in Estimation when Window Length=1000, Number of Window Overlap=100

From Figure 9,10 it can be seen that for lower window lengths low velocity values can create a problem in estimation but for high window length the error of estimation is much smaller so we can get better estimation in frequency so that velocity estimation will be better.

Parameter Chosen for Chirp Data

In generating linear chirp data, again sampling frequency is chosen as “1000 Hz” and this will again affect the frequency range of the spectrogram. For linear chirp, it shows a linear change in the spectrogram so our gaussian error can affect the spectrogram value more we have to increase the signal to noise ratio to reduce the noisy effect more than sinusoidal signal. Let’s see different spectrogram images with the linear chirp amplitude

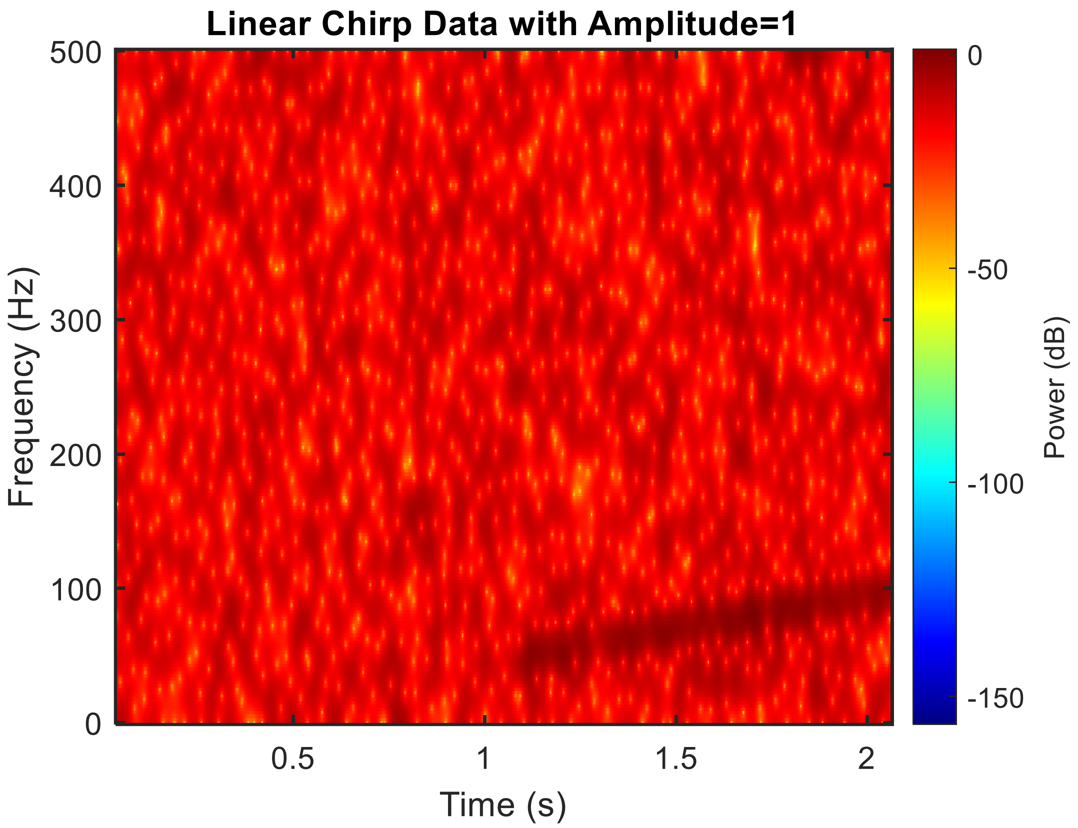

Figure 11-Example of Spectrogram of Linear Chirp with Low SNR value

SNR value for “1” amplitude chirp signal= -13.2204 dB

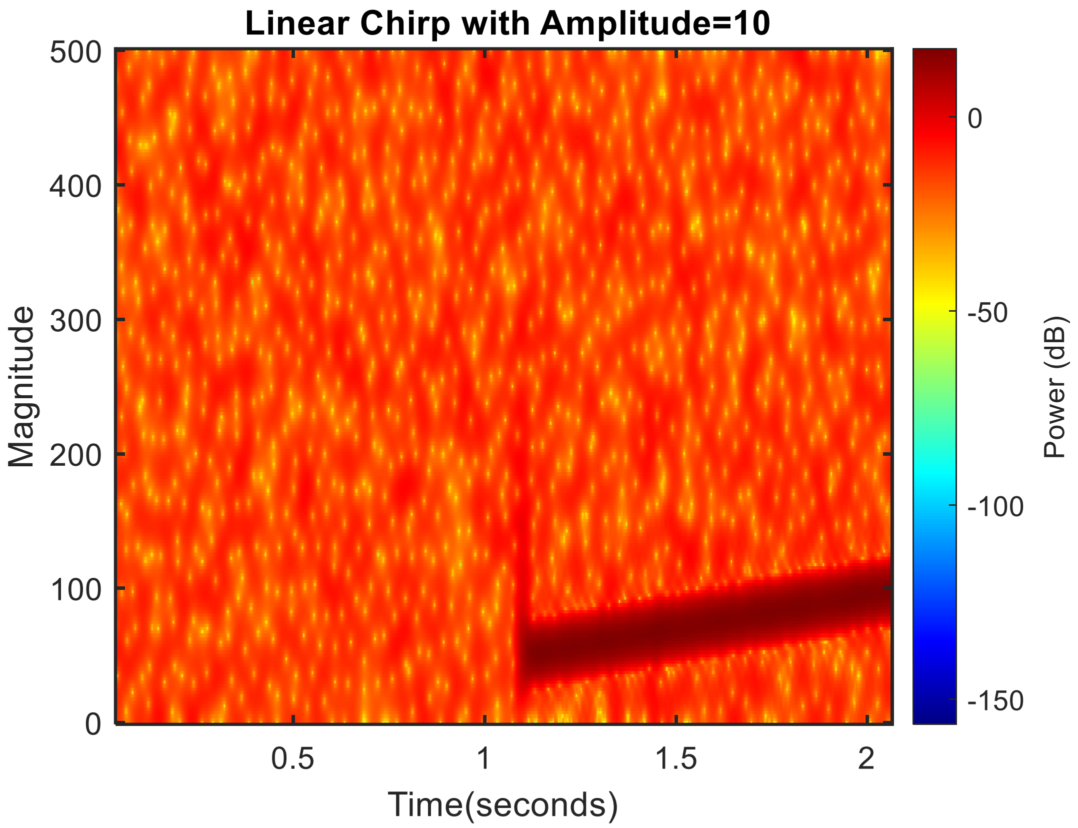

Figure 12-Spectrogram of Linear Chirp when SNR value is higher

SNR value for “10” amplitude chirp signal= 7.5221 dB

So, we can see from the Figure 11,12 that when amplitude of the linear chirp is “1” (also SNR is low valued) then visual inspection can not be made for that response but when amplitude is “10” then we can clearly see the linear chirp style response. So, amplitude range is chosen as [10, ∞]

Other parameters of linear chirp are Frequency, Duration, Bandwidth and Velocity. For these parameters, spectrogram range is again be considered because highest spectrogram range is “500 Hz” and if the signals frequency exceeds this range, spectrogram graph will have linearly decreasing shape and our technique will not work for this type of situation. So, we have chosen the frequency ranges so that it won’t exceed “500 Hz”. Initial Frequency range is chosen as the Sinusoidal Frequency range which is [0 250 Hz]. And the proportion of “m/bw” will determine the “linear increase in spectrogram” so it is chosen [0 50 Hz] and again the velocity range is chosen as [0 170 m/s]. We have determined the m/bw factor range but we need range of each of duration and bandwidth so bandwidth is chosen as [1 5] and duration range is chosen as [100 500]. So, our spectrogram cannot exceed “500 Hz” and it will have one linearly increasing or decreasing style of line.

Estimation Procedure

In the estimation procedure, 2 different approaches will be used which are visual inspection of the spectrogram and automatic calculations using the STFT values. We will consider each approach in detail.

For visual inspection, spectrogram of a linear chirp has a linearly increasing dark colored line shape. The end point of this increase depends on "m/bw" portion and initial frequency is f0 value. End point will be affected by the factor below

$$\left(\frac{c}{c-v}\right)^2$$

Initial frequency of the transmitted linear chirp depends on the factor below

$$\frac{c}{c-v}$$

Let's consider the visual inspection with an example with m=100, bw=1, f0=10, v=170 and amplitude=10 (t0=1.1 Ts also).

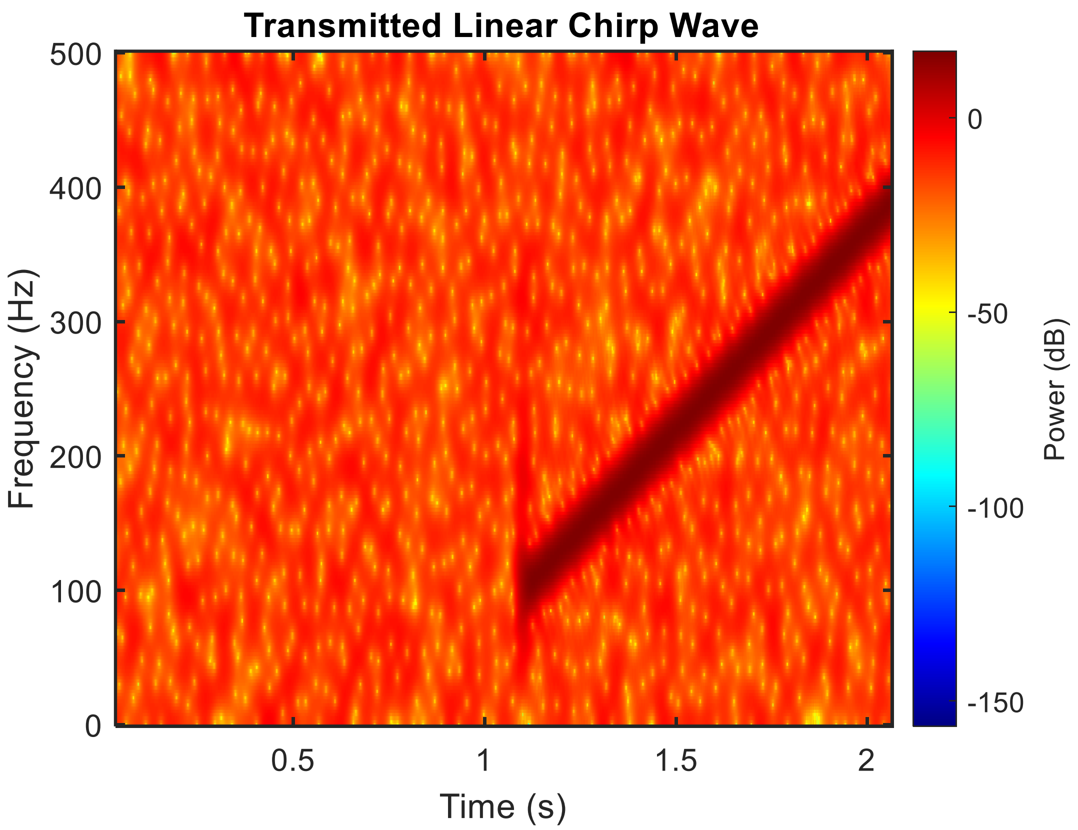

Figure 13-Spectrogram of Transmitted Linear Chirp

We can see from the Figure 13 that end frequency is "400 Hz", starting frequency is "100 Hz" as expected because we have used v=170 so our multiplication factors will be 4 and 2 respectively for 100 Hz and 50 Hz. We can see the arrival time correctly again 1.1 second. This result has been obtained when window overlap has a percentage of 99. Let's see the result when overlap percentage is 10%.

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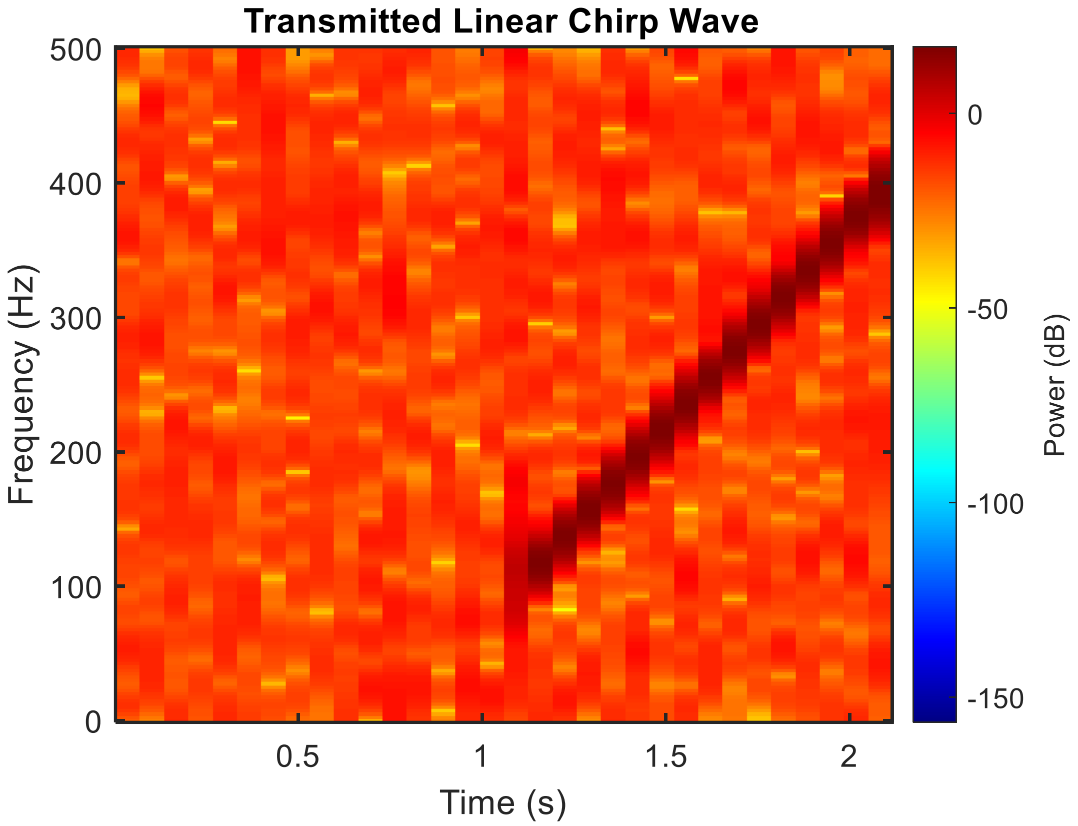

Figure 14-Spectrogram of Transmitted Linear Chirp when Window Overlap Percentage is low

Frequency resolution has rapidly changed when window overlap percentage is decreased and this can affect the frequency estimation in bad manner.

Numerical Estimation in Linear Chirp

For numerical approach, we have considered the end frequency and the start frequency. End frequency has a linear proportion with the second power of multiplication factor. Starting frequency has a linear proportion with multiplication factor.

$$Multiplication\ factor = \frac{c}{c-v}$$

Multiplication factor can be derived from the end frequency by the equation below

$$Multiplication\_Factor = sqrt(ifq(end)/m))$$

Ifq array shows the instantaneous frequency for each time arrival and the last instantaneous frequency is the end frequency. We will use initial frequency to obtain the arrival time by using the equation below

$$[val, ind] = (min(abs(ifq - Multiplication\_Factor * f0)))$$
$$Arrival\_Time = t(ind)$$

Let’s consider the error Figure 15 below

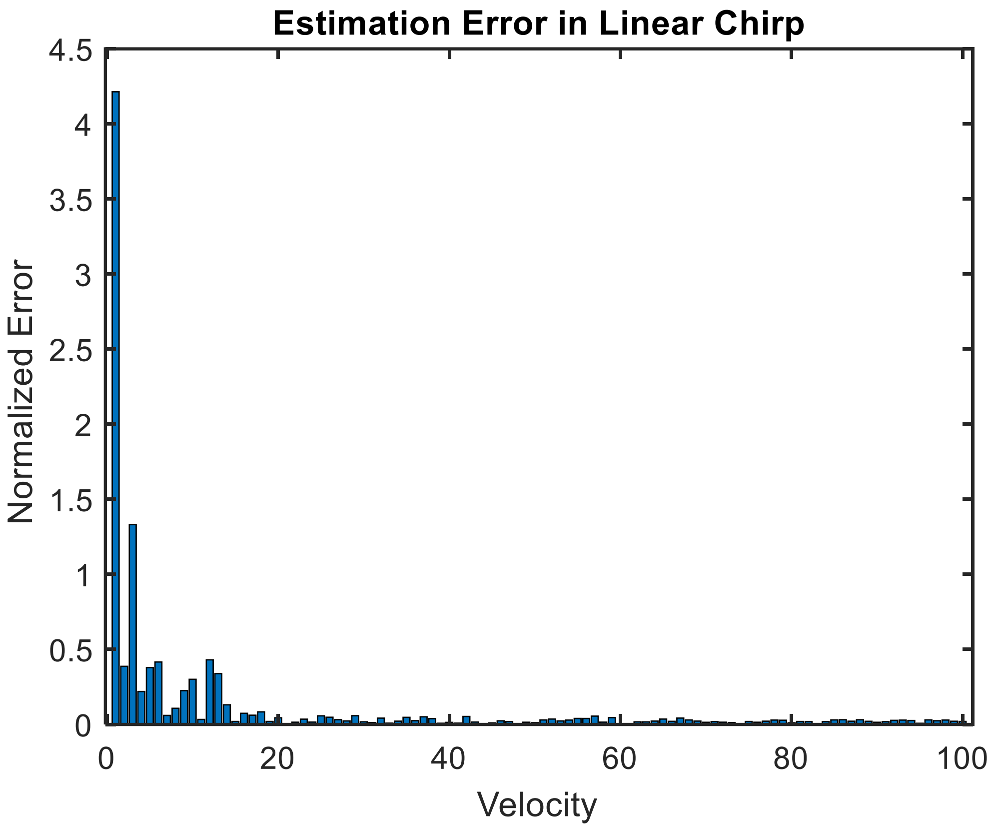

Figure 15-Estimation Error in Linear Chirp Estimator

We can see that in low velocity values the proportion is much smaller in the low velocity values so there are much bigger error percentages compared to high velocity values but error values can be considered as “small” and our approach is applicable.