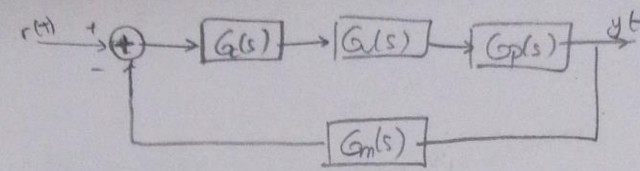
EE-407 - HW3

Question 1:



a) $$G_{OL}(s) = \frac{3 K_c \cdot e^{-2s}}{(60s+1)(5s+1)(3s+1)(2s+1)}$$

$$\approx \frac{3K_c e^{-2s} e^{-\theta_d s}}{(60s+1)} \qquad \theta_d = 5+3+2 = 10$$

↳ Choose this because it has highest time constant.

$$\approx \frac{3K_c e^{-12s}}{60s+1}$$

$K = 3K_c \quad \tau = 60 \quad \theta = 12$

b) use Pade' approximation;

$$e^{-12s} \approx \frac{1-6s}{1+6s}$$

$$\Rightarrow G_{OL}(s) \approx +K_c \cdot \frac{3(1-6s)}{(60s+1)(1+6s)}$$

$$q(s) = 1 + K_c \frac{3(1-6s)}{(60s+1)(1+6s)} = 0$$

$$\Rightarrow 360s^2 + 66s + 1 - 18K_c s + 3K_c = 0$$

$$\Rightarrow 360s^2 + (66 - 18K_c)s + 1 + 3K_c = 0$$

$$\begin{array}{c|cc} s^2 & 360 & 1+3K_c \\ s^1 & 66-18K_c & \\ s^0 & 1+3K_c & \end{array}$$

$66 - 18K_c > 0$

$\rightarrow K_c < \frac{66}{18} = \frac{11}{3}$

$$\boxed{\frac{-1}{3} < K_c < \frac{11}{3}}$$

$1 + 3K_c > 0$

$\rightarrow K_c > \frac{-1}{3}$

c) $360s^2+(66-18K_{cm})s+1+3K_{cm}=0$

$s=j\omega \Rightarrow -360\omega^2+j\omega(66-18K_{cm})+1+3K_{cm}=0$

$\Rightarrow \omega(66-18K_{cm})=0$

$\Rightarrow \cancel{\omega}=0$ or $\boxed{K_{cm}=\frac{11}{3}}$

$1+3K_{cm}-360\omega^2=0$

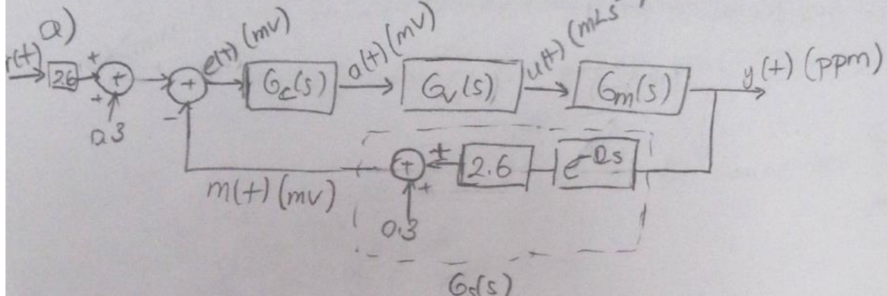
$\Rightarrow 360\omega^2=12$

$\Rightarrow \omega^2=\frac{1}{30}$

$\Rightarrow \omega=\frac{1}{100}$ rad/sec

Question 2!

a)



b) $G_c(s)=K_c+\frac{K_c}{\tau_I}\frac{1}{s}+K_c\tau_D s$ for a PID controller

$G_v(s)=\frac{0.6}{0.2s+1}$ $\qquad \tau_1\approx\frac{20}{5}=4\,sec$

$G_m(s)=\frac{0.8}{\tau_1 s+1}=\frac{0.8}{4s+1}$

For $G_s(s)$, $e^{-\theta s}$ term is the delay from the mixing process and measurement in sensor. Also, because we add 0.3 term to both feedback and input, we can ignore 0.3.

$\Rightarrow U_{water}=2000\,cm^3/s\,/\,5cm^2=400\,cm/s$

$\theta=\frac{290\,cm}{400\,cm/s}=0.725s$ $\qquad \Rightarrow G_s(s)=2.6e^{-0.725s}$

d)

$$G_{OL}(s) = \underbrace{\cancel{G_c(s)}}_{\downarrow} \cdot \frac{0.6}{0.2s+1} \cdot \frac{0.8}{4s+1} \, 2.6 e^{-0.725s}$$

Assume there is no controller for now

$$\Rightarrow \underset{\uparrow \atop \text{FOPDT}}{G_{OL}(s)} \approx \frac{1.25 \cdot e^{-0.2s} e^{-0.725s}}{(4s+1)} = 1.25 \frac{e^{-0.925s}}{(4s+1)}$$

It is seen from the graphs in the next page, the delay of FOPDT is greater than actual system. Also, FOPDT reaches to the steady-state slower.

e)

$$q(s) = 1 + K_c \cdot \frac{0.6}{0.2s+1} \cdot \frac{0.8}{4s+1} \cdot 2.6 e^{-0.725s}$$

$$\Rightarrow 0.8s^2 + 4.2s + 1 + 1.25 K_c e^{-0.725s} = 0$$

$s = jw$

$$\Rightarrow -0.8w^2 + 4.2jw + 1 + 1.25K_c e^{-j0.725w} = 0$$

$$\Rightarrow -0.8w^2 + j4.2w + 1 + 1.25 K_c (\cos(+0.725w) - j \sin(0.725w)) = 0$$

$$\Rightarrow \boxed{K_c \approx 6.55} \qquad \Rightarrow 4.2w - 1.25K_c \sin(0.725w) = 0$$

$w = 0$ or

f) For ITAE, I and D terms should be concelled. $\Rightarrow \tau_I = \infty, \tau_D = 0$

$$K_c = a = \frac{0.2}{K_p} \left(\frac{\tau_p}{\theta_p} \right)^{1.22} = 0.954$$

g)

$$T(s) = \frac{G_c(s) G_v(s) G_m(s)}{1 + G_c(s) G_v(s) G_m(s) G_s(s)}$$

$$\Rightarrow T + T G_c G_v G_m G_s = G_c G_v G_m$$

$$\Rightarrow T = G_c G_v G_m (1 - T G_s)$$

$$\Rightarrow G_c = \frac{1}{G_v G_m} \frac{T}{1 - T G_s}$$

$$q(s) = 1 + G_c G_v G_m G_s = 0$$

$$\Rightarrow 1 + \frac{1}{G_v G_m} \cdot \frac{T}{1 - T G_s} \cdot \cancel{G_v} G_m G_s = 0$$

$$\Rightarrow \frac{1 - \cancel{T G_s} + \cancel{T G_s}}{1 - T G_s} = 0$$

$$\Rightarrow \frac{1}{1 - T G_s} = 0$$

$$\Rightarrow \frac{1}{1 - T 2.6 e^{-0.725s}} = 0$$

Assume $T(s) = \dfrac{1}{1 + \tau_c s}$

$\tau_c = max(\tau_p, 8\theta_p)$ for moderate operation

$= max(4, 0)$

$= 4$

$$G_c(s) = \frac{1}{G_v G_m} \cdot \frac{\frac{1}{1+\tau_c s}}{1 - \frac{1}{1+\tau_c s} \cdot 2.6 e^{-0.725s}}$$

$$= \frac{(4s+1)(0.2s+1)}{0.6 \cdot 0.8} \cdot \frac{1}{1 + \tau_c s - 2.6 e^{-0.725s}} \qquad e^{-0.725s} \approx 1 - 0.725s$$

$$= \frac{1}{0.48} \cdot \frac{0.8s^2 + 4.2s + 1}{5.9s - 1.6} = \frac{1}{0.6} \frac{s^2 + 5.25s + 1.25}{5.9s - 1.6} = \frac{s^2 + 5.25s + 1.25}{3.54(s - 0.1)}$$

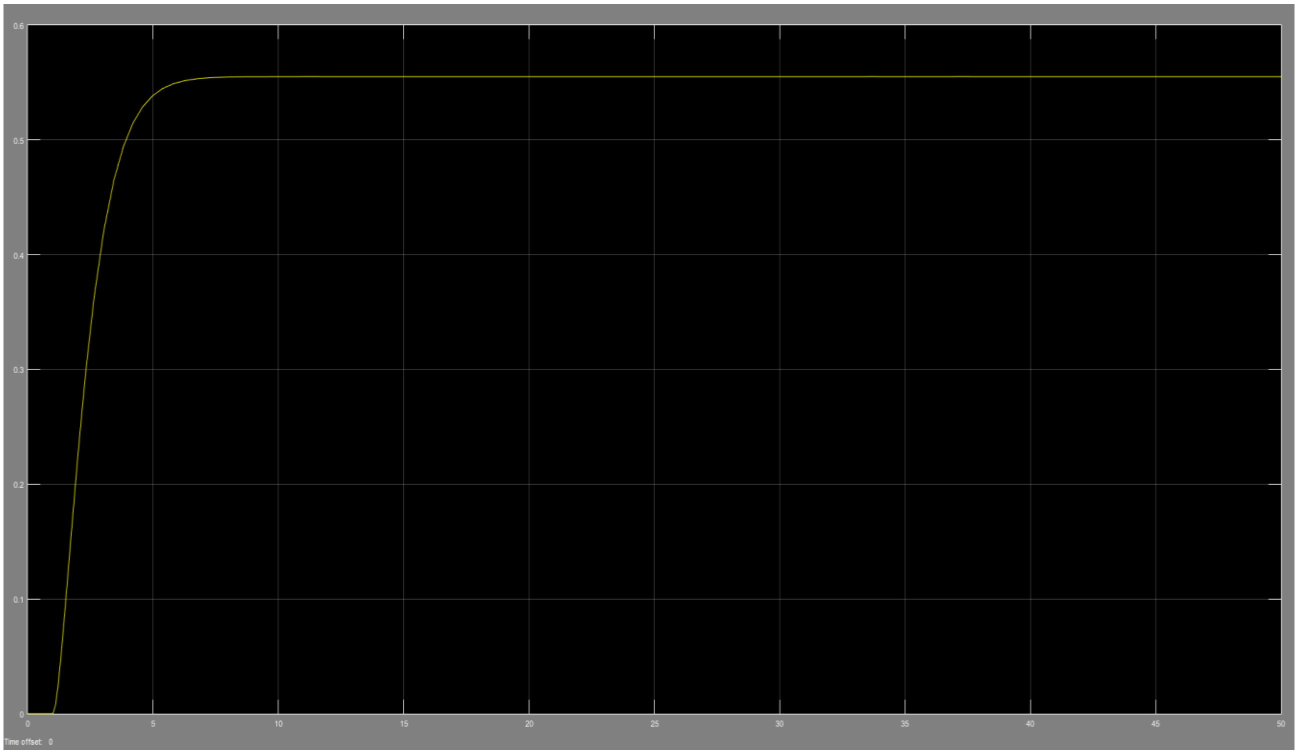


Figure 1: Question 2 part c, bump test result

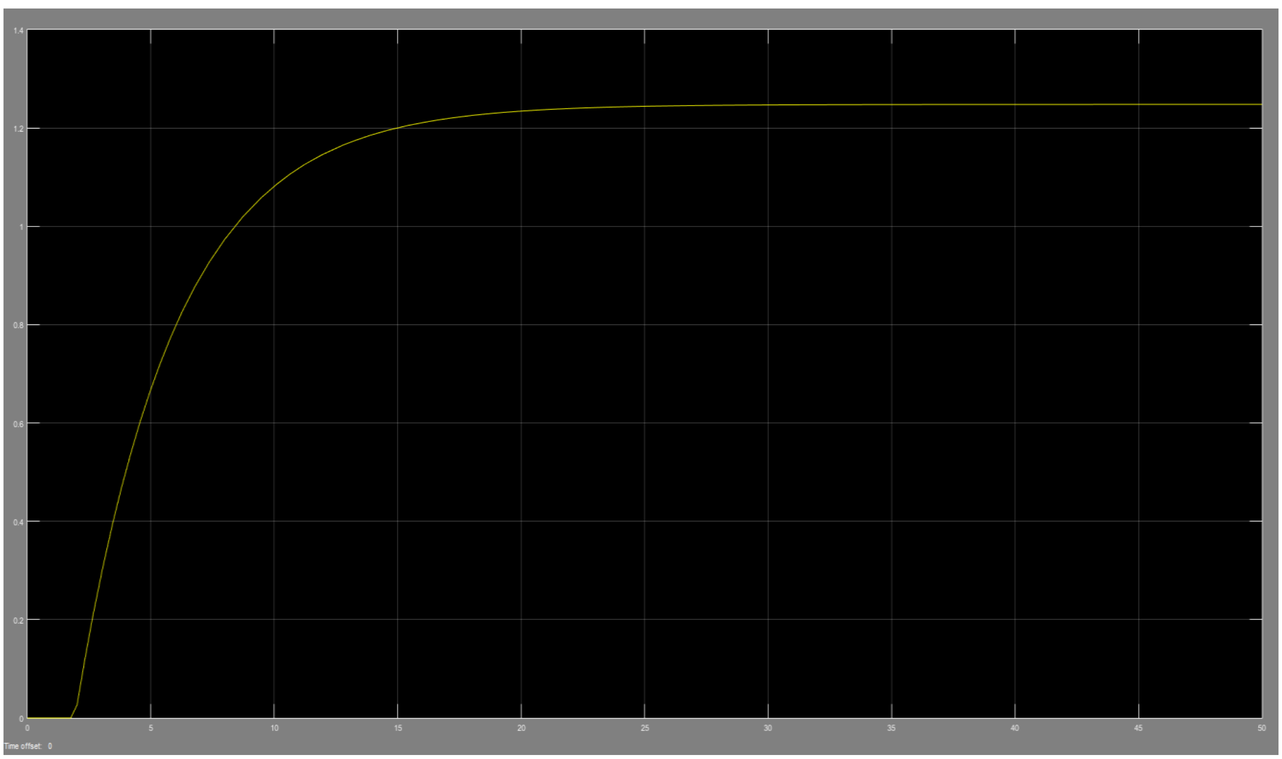


Figure 2: Question 2 part d, bump test result

$\tau_c = \max(4, 8 \cdot 0.925)$
$= 7.4$

h) $G_{02}(s) = 1.25 \cdot \dfrac{e^{-0.925s}}{4s+1}$

$$K_c = \frac{1}{K_p} \cdot \frac{(\tau_p + 0.5\theta_p)}{(\tau_c + 0.5\theta_p)} = \frac{1}{1.25} \cdot \frac{(4+0.5 \cdot 0.925)}{(7.4+0.5 \cdot 0.925)} = 0.45$$

$$\tau_I = \tau_p + 0.5\theta_p = 4 + 0.5 \cdot 0.925 = 4.46$$

$$\tau_D = \frac{\tau_p \theta_p}{2\tau_p + \theta_p} = 0.414$$

Question 3:

a)

$$-q_1+q_i=\dot h_1 A_1 \Rightarrow \dot h_1 A_1 = -\frac{h_1}{R_1}+q_i$$

$$-q_2+q_1=\dot h_2 A_2 \Rightarrow \dot h_2 A_2 = -\frac{h_2}{R_2}+\frac{h_1}{R_1}$$

$$-q_3+q_2=\dot h_3 A_3 \Rightarrow \dot h_3 A_3 = \frac{-h_3}{R_3}+\frac{h_2}{R_2}$$

$$\vdots$$

$$-q_n+q_{n-1}=\dot h_n A_n \Rightarrow \dot h_n A_n = \frac{-h_n}{R_n}+\frac{h_{n-1}}{R_{n-1}}$$

$$\dot h=\begin{bmatrix}\dot h_1\\ \dot h_2\\ \dot h_3\\ \vdots\\ \dot h_n\end{bmatrix}=\begin{bmatrix}\frac{-1}{A_1R_1} & 0 & 0 & 0 & \cdots\\ +\frac{1}{A_1R_1} & \frac{-1}{A_2R_2} & 0 & 0 & \cdots\\ 0 & +\frac{1}{A_2R_2} & \frac{-1}{A_3R_3} & 0 & \cdots\\ & & & \ddots & \\ 0 & \cdots & & +\frac{1}{A_{n-1}R_{n-1}} & \frac{-1}{A_nR_n}\end{bmatrix}h+\begin{bmatrix}+\frac{1}{A_1}\\ 0\\ 0\\ \vdots\\ 0\end{bmatrix}q_i$$

b)

$$sA_1H_1(s)=\frac{-1}{R_1}H_1(s)+Q_i(s) \qquad sA_2H_2(s)=\frac{-1}{R_2}H_2(s)+\frac{1}{R_1}H_1(s)$$

$$\Rightarrow\left(\frac{1}{R_1}+sA_1\right)H_1(s)=Q_i(s) \qquad \Rightarrow\left(\frac{1}{R_2}+sA_2\right)H_2(s)=\frac{1}{R_1}H_1(s)$$

$$\Rightarrow\frac{H_1(s)}{Q_i(s)}=\frac{1}{\frac{1}{R_1}+sA_1} \qquad \Rightarrow\frac{H_2(s)}{H_1(s)}=\frac{1/R_1}{1/R_2+sA_2}$$

$$\Rightarrow\frac{H_n(s)}{Q_i(s)}=\frac{H_1(s)}{Q_i(s)}\cdot\frac{H_2(s)}{H_1(s)}\cdots\frac{H_n(s)}{H_{n-1}(s)}=\frac{\frac{1}{R_1\cdot R_2\cdot R_3\cdots R_{n-1}}}{(1/R_1+sA_1)(1/R_2+sA_2)\cdots(1/R_n+sA_n)}=\frac{R_n}{(1+sA_1R_1)(1+sA_2R_2)\cdots(1+sA_nR_n)}$$

c)

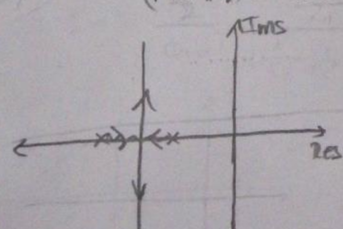
$$\frac{Q_n(s)}{Q_i(s)}=\frac{H_n(s)}{Q_i(s)}\cdot\frac{Q_n(s)}{H_n(s)}=\frac{H_n(s)}{Q_i(s)}\cdot\frac{1}{R_n}$$

$\Rightarrow$ DC gain would be 1.

This make sense because each tank regulates itself.

d)

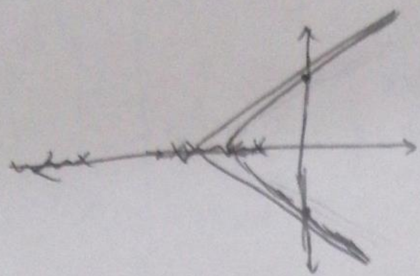
$$G(s)=\frac{R_2}{(1+sA_1R_1)(1+sA_2R_2)} \qquad s_1=-\frac{1}{A_1R_1},\ s_2=-\frac{1}{A_2R_2} \text{ are the poles}$$



Because the root locus does not cross the $j\omega$ axis for any K value, sustained oscillations cannot be obtained.

Phase of this system will not reach to $-180°$ because it will start at $0°$ and it will decrease to $180°$ because it has 2 pole. However, it will not reach $-180°$ until $w\to\infty$.

e) $G(s) = \dfrac{R_3}{(1+sA_1R_1)(1+sA_2R_2)(1+sA_3R_3)}$ $\quad s_1 = -\dfrac{1}{A_1R_1},\ s_2 = -\dfrac{1}{A_2R_2},\ s_3 = -\dfrac{1}{A_3R_3}$



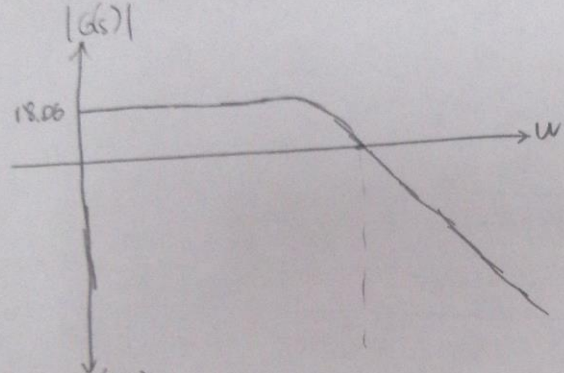
$$1+K_c \cdot \frac{\overset{\rightsquigarrow 1}{R_3}}{(1+s\underbrace{A_1R_1}_{1})(1+s\underbrace{A_2R_2}_{1})(1+s\underbrace{A_3R_3}_{1})} = 0$$

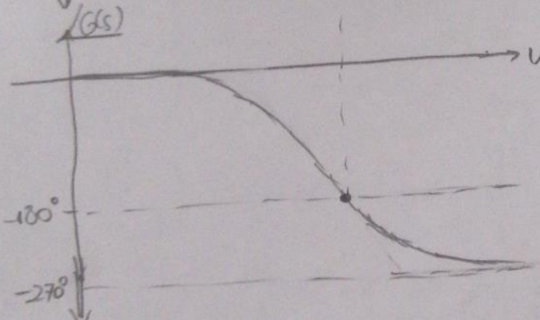
$\Rightarrow s = j\omega \Rightarrow s^3 + 3s^2 + 3s + 1 + K_c = 0$

$\Rightarrow -j\omega^3 - 3\omega^2 + j3\omega + K_c + 1 = 0$

$\Rightarrow \omega(3-\omega^2) = 0 \qquad K_c + 1 - 3\omega^2 = 0$

$\Rightarrow \cancel{\omega = 0}\ \boxed{\omega = \sqrt{3}}\ \cancel{\omega = -\sqrt{3}} \qquad \Rightarrow \boxed{K_c = 8}$





f) $P_u = \dfrac{2\pi}{\sqrt{3}} \cong 3.63$

$\Rightarrow$ For Decay ratio $= \dfrac{1}{4}$

$K_c = \dfrac{K_u}{1.7} = 4.7 \qquad \tau_I = \dfrac{P_u}{2} = 1.815 \qquad \tau_D = \dfrac{P_u}{8} = 0.454$

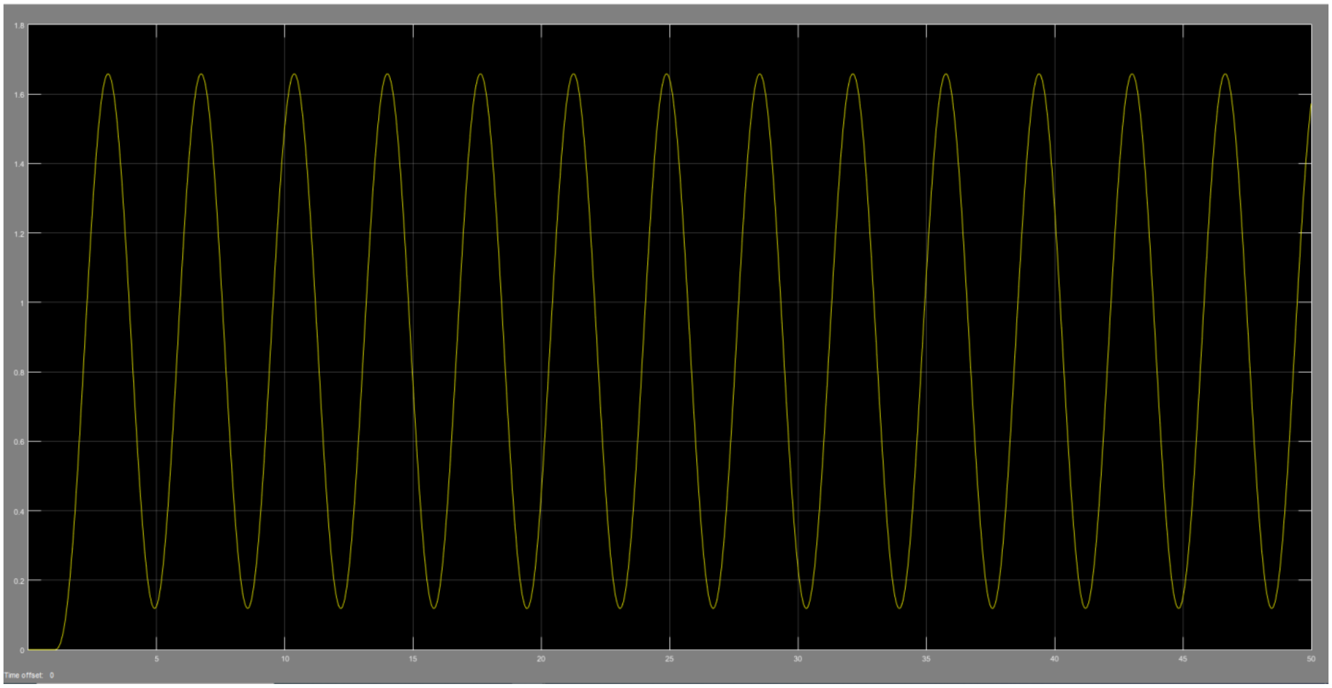


Figure 3: System response of Question 3 part e

5)

A_

5)

a- $y(t) = \left(1-e^{-\frac{t-t_0-\theta_p}{\tau_p}}\right) u(t-t_0-\theta_p)\Delta y + y_0$

$$\frac{y(t)-y_0}{\Delta y} = \left(1-e^{-\frac{t-t_0-\theta_p}{\tau_p}}\right)\underbrace{u(t-t_0-\theta_p)}_{=1 \text{ for } t>t_0+\theta_p}$$

$$e^{-\frac{t-t_0-\theta_p}{\tau_p}} = 1-\frac{y(t)-y_0}{\Delta y}$$

$$-\frac{t-t_0-\theta_p}{\tau_p} = \ln\left(1-\frac{y(t)-y_0}{\Delta y}\right)$$

$$t = -\ln\left(1-\frac{y(t)-y_0}{\Delta y}\right)\tau_p + t_0+\theta_p$$

$$t_{1/3} = -\ln\left(\underbrace{1-\frac{y_0+\frac{1}{3}\Delta y-y_0}{\Delta y}}_{2/3}\right)\tau_p - t_0+\theta_p \cong 0,405\,\tau_p + t_0+\theta_p$$

$$t_{2/3} = -\ln\left(\underbrace{1-\frac{y_0+\frac{2}{3}\Delta y-y_0}{\Delta y}}_{1/3}\right)\tau_p - t_0+\theta_p \cong 1,1\,\tau_p + t_0+\theta_p$$

$t_{2/3}-t_{1/3} \cong 0,7\,\tau_p$ $\qquad \Rightarrow t_0+\theta_p = t_{1/3} - 0,4\,\tau_p$

$\Rightarrow \tau_p \cong \frac{t_{2/3}-t_{1/3}}{0,7}$ $\qquad \Rightarrow \theta_p = t_{1/3} - t_0 - 0,4\,\tau_p$

B_

i. Because a dead band controller makes the process variable stay in a desired region rather than a desired point. Hence, the controller state changes a lot less frequently. An example could be an air conditioner. If a dead band is not used, the controller will constantly operate even if the temperature is very close to the desired value. If a dead band is introduced, then the controller will check if the temperature is in the desired range. If not, it will increase the temperature to upper boundary of the desired region when it is underneath the desired range and it will have fairly a lot of idle time before the temperature falls below the desired range again. Vice versa for the temperature above the desired range.

ii. Integrating system is a Type-I system and we know from our EE302 course that it steady-state error to unit-step is “0” as far as the closed-loop is stable. However, a self-regulating process cannot achieve this performance and it cannot prevent steady-state error.

iii. First case is a servo control and second case is regulatory control. Kc values for these cases are in the picture below.

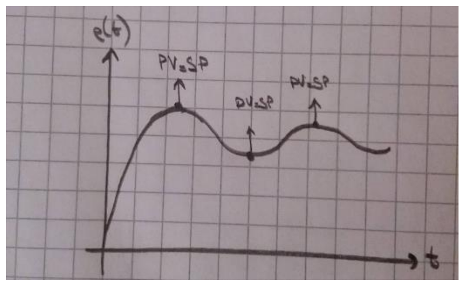
$$a\_\ K_c = \frac{0.202}{K_p} \cdot \left(\frac{\theta_p}{\tau_p}\right)^{-1.219}$$

$$b\_\ K_c = \frac{0.49}{K_p} \left(\frac{\theta_p}{\tau_p}\right)^{-1.084}$$

iv. IAE method punishes errors linearly. Hence, all erros will be punished at the same rate. ITAE method also pays attention to time factor. Hence, initial errors are expected to be punished softly. As time increases, punishment effect will also increase in ITAE. ISE method is linear with the square of the error, not with the error. Hence, punishment is multiplied by the square of error. This implies that it will tolerate small errors and punish big erros severely.

v. This will cause the system to work with positive feedback. If water level is lower than set value, controller will give a negative output and actuator output will be negative which is not likely here since it cannot pull water from the tank. Hence, it will be "0". The tank will keep emptying until it is fully empty. In the other case; if water level is higher than set value, the controller will give positive output this time and actuator will pour even more water on the tank and it will eventually overflow.

vi. First it will increase until PV=SP. Then it will start decreasing during overshoot and increase back when overshoot becomes undershoot. It will eventually settle at a certain value when PV settles at SP value. The graph is given in picture below.



vii. When we change the set-point of the system dramatically, it will take longer for process variable to reach set-point. In this duration, integral of error will keep accumulating for that long time. This is integral windup and it will keep winding until PV reaches and SP and gets bigger than SP where unwinding starts. It might cause the controller to saturate which can be problematic. "Velocity Controller Method" is one of the methods to overcome this problem.

viii. Decay ratio is kind of the speed it reaches steady-state. Hence, we can say that it is correlated to settling time. We can also say this for rise time and peak time. If a system reaches set point fast, it will reach the peak faster as well, intuitively.

ix. Derivative kick is the response that the system gives to a sudden change in set-point. e(t)=SP-PV. We know that PV changes continuously. When we change SP suddenly, error increases by a step function and controller tries to respond with an impulse. Usually this impulse will have negligible performance on the system. Yet, it might cause actuators to wear off or decrease their lifetime. To prevent this, we can apply derivative action not on e(t)=SP-PV, but only on -PV.

x. If noise is dominant on measurement, derivative action can be detrimental as amplified noise will be reflected on CO. This will not be good for the controller. It will be easier and effective to use low-pass filter to get rid of such noises.