CENG 222

Statistical Methods for Computer Engineering

Spring '2017-2018

Take Home Exam 1

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Student Information

Full Name : Deniz Rasim Uluğ
Id Number : 2172088

Answer 1

To find the answer with Monte Carlo simulation, first we will write a function in Matlab programming language that generates a number n from a Poisson distribution with $\lambda = 4 \times 5 = 20$. Then n will be the number of minions we have caught in 5 hours.
We will also write another function which generates numbers w and s from a distribution defined by its pdf as given in the question text. Then we will generate w and s values independently (from other minions) for each minion, and basically see if the more than 6 such minions' w and s values satisfy the given condition, and if so we have a success. This whole process is a single iteration. And the question actually asks the probability of "success".

We will iterate this process independently N times, and count how many of the iterations result in a success . The number of success divided by N will give us the desired value.

But first, let's calculate a number N, the total number of iterations we will do, which will satisfy the constraints given in Question 1, that with probability $(1 - \alpha) = 0.95$, our estimation of the "success probability" as described above will differ from the true value for more than $\epsilon = 0.005$. Keeping in mind we don't have an intelligent "guess" for the desired probability, so we bound $p \times (1 - p)$ by its maximum value 0.25 as described in p.115-116 where p is the probability of success, the formula for such an N is directly given in p.116 as;

$$N \geq 0.25(\frac{z_{\alpha/2}}{\epsilon})^2$$

Substituting, and with $z_{\alpha/2} = z_{0.025} = 1.96$ from table A4;

$$N \geq 0.25(\frac{1.96}{0.005})^2 = 38146$$

Which is large enough to justify the use of Normal Approximation used in the proof of the formula.

Now, to generate a Poisson variable, we used the Arbitrary Discrete Distribution method from p.106. In fact we directly used the code from p.107. As said we took lambda to be 20.

To generate numbers Weight and Speed, we used the rejection method. We took boundaries for both Weight and Speed to be between 0 and 20. 0 because it is given that both numbers are bigger than 0, and 20 because the probability for either of them to be 20 is very low, practically zero (around 10^{-8}). For $Y = f_{w,s}$ itself, we took the boundaries to be 0 and 0.14. 0 because Y is always positive, and 0.14 because the maximum value of Y is $\frac{1}{e^2}$ which is around 0.135

Now, running the described Monte Carlo process 39.000 times we found the probability of we catching more than 6 minions in 5 hours which satisfies the given condition to be 0.3049.

Answer 2

For this part we use a similar idea. Tough this time at each iteration,we find sum of the minions' weights and again sum this number across iterations, then divide by N, the number of experiments. We again take N to be 39000 in this part.

We used the same methods to generate Poisson and w,s variables as in section 1.
Doing this, we find the mean weight to be 42.001.

Answer 3

We again run a Monte Carlo simulation with number of iterations $N = 39000$.

At each iteration, we simply compute the desired expression. Then sum this value across iterations and finally divide by N to get the expectancy(ie. mean). Notice than the distribution $N(0, 1)$ is actually the Standard Normal distribution, so we used directly it.

To generate an Exponential variable, we used the inverse transform method. Let U be a standart uniformly distributed number, then $X_{exp} \sim Exp(2)4$ is;

$$X_{exp} = -(\frac{1}{\lambda})ln(U)$$

And to generate Standart Normal distribution, we again used rejection method with boundaries $[-10, 10]$ for X and $[0, 0.5]$ for Y. Again, $P(X)$ becomes practically zero outside this range and maximum possible value of Y is lower than 0.5.

Doing this, we find the mean of the expression to be 0.311.