

# CENG 223

## Discrete Computational Structures

Fall '2016-2017

### Take Home Exam 1

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Due date: October 30 2016, Sunday , 23:55

### Question 1

Determine if the following compound propositions are tautology or contradiction. Construct a truth table for each proposition.

a)

$$[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$$

b)

$$\neg((\neg p \wedge (p \vee q)) \rightarrow q)$$

### Question 2

Show that  $(p \rightarrow q) \wedge (p \rightarrow r)$  and  $(\neg q \vee \neg r) \rightarrow \neg p$  are logically equivalent by using logical equivalences. Use tables 6,7 and 8 given under the section '*Propositional Equivalences*' in your textbook and give the reference to the table and the law when you use it.

### Question 3

a) Let  $D(x, y)$  be “ $x$  drives  $y$ ” and  $F(x)$  be “ $x$  is a footballer”, where  $x$  represents people and  $y$  represents cars. Use quantifiers to express the following statements.

- a) There is a footballer who drives every car.
- b) Every car has at least one footballer who drives it.
- c) There is a car which no footballer drives.
- d) There is a car which is driven by exactly one *person*.
- e) There is at least one car such that only footballers drive it.

b) Use the predicates below to express the following statements in predicate logic.  
 $student(x) : x$  is a student,  $teacher(x) : x$  is a teacher,  $takes(x, y) : x$  takes course  $y$ ,  $teaches(x, y) : x$  teaches course  $y$ ,  $enjoys(x, y) : x$  enjoys course  $y$ .

- a) The teacher Ahmet Metin does not teach any course.
- b) Some teachers teach the courses they enjoy.
- c) Not all teachers teach all courses.
- d) Only students take a course.
- e) Every teacher teaches exactly 2 courses.

## Question 4

Prove the following claim by natural deduction. Use **only** the natural deduction rules  $\vee, \wedge, \rightarrow, \neg$  introduction and elimination. If you attempt to make use of a lemma or equivalence, you need to prove it by natural deduction too.

$$p, p \rightarrow (r \rightarrow q) \vdash \neg q \rightarrow \neg r$$

## Question 5

Prove the following claim by natural deduction. Use **only** the natural deduction rules  $\vee, \wedge, \rightarrow, \neg, \forall, \exists$  introduction and elimination. If you attempt to make use of a lemma or equivalence, you need to prove it by natural deduction too. Note that  $a$  is a constant in the formula below.

$$\exists x(p(x) \rightarrow q(a)) \vdash \forall y p(y) \rightarrow q(a)$$

## 1 Regulations

1. You have to write your answers to the provided sections of the template answer file given.
2. Do not write any extra stuff like question definitions to the answer file. Just give your solution to the question. Otherwise you will get 0 from that question.
3. **Late Submission:** Not allowed!
4. **Cheating: We have zero tolerance policy for cheating.** People involved in cheating will be punished according to the university regulations.
5. **Newsgroup:** You must follow the newsgroup (news.ceng.metu.edu.tr) for discussions and possible updates on a daily basis.
6. **Evaluation:** Your latex file will be converted to pdf and evaluated by course assistants. The .tex file will be checked for plagiarism automatically using "black-box" technique and manually by assistants, so make sure to obey the specifications.

## 2 Submission

Submission will be done via COW. Download the given template answer file "the1.tex". When you finish your exam upload the .tex file with the same name to COW.

**Note:** You cannot submit any other files. Don't forget to make sure your .tex file is successfully compiled in Inek machines using the command below.

```
$ pdflatex the1.tex
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