Middle East Technical University Department of Computer Engineering

CENG 223

Discrete Computational Structures

Fall '2016-2017

Take Home Exam 3

Due date: December 9 2016, Friday , 23:55

Question 1

Use mathematical induction to prove that for all positive integers k and n,

$$\sum_{j=1}^{n} j(j+1)(j+2)\cdots(j+k-1) = \frac{n(n+1)(n+2)\cdots(n+k)}{(k+1)}$$

Question 2

Let $H_0 = 1$, $H_1 = 3$, $H_2 = 5$, and define

$$H_n = 5H_{n-1} + 5H_{n-2} + 63H_{n-3}$$

for $n \geq 3$. Show by strong induction that $H_n \leq 7^n$ for all $n \geq 0$.

Question 3

We have 5 *Discrete Mathematics* textbooks and 7 *Signals and Systems* textbooks at hand.

a) In how many ways can you make a collection of 4 books from these 12 textbooks with the condition that at least one *Discrete Mathematics* textbook must be in the collection.

b) In how many ways can you make a collection of 4 books from these 12 textbooks with the condition that at least one *Discrete Mathematics* textbook and at least one *Signals and Systems* textbook must be in the collection.

Question 4

Construct a recurrence relation for the number of different strings of length n composed of only 2's and 3's and having even number of 3's.

Question 5

Solve the following recurrence relation with the given initial conditions:

$$a_n = 4a_{n-1} + a_{n-2} - 4a_{n-3} \text{ for } n = 3, 4, 5, ...,$$

with $a_0 = 4$, $a_1 = 8$, $a_2 = 34$.

Question 6

Write a closed form expression for the generating function for the sequence $\{a_n\}$, where $a_n = \binom{10}{n+1}$ for $n = 0, 1, 2, ...$

1 Regulations

1. You have to write your answers to the provided sections of the template answer file given.
2. Do not write any extra stuff like question definitions to the answer file. Just give your solution to the question. Otherwise you will get 0 from that question.
3. **Late Submission:** Not allowed!
4. **Cheating: We have zero tolerance policy for cheating**. People involved in cheating will be punished according to the university regulations.
5. **Newsgroup:** You must follow the newsgroup (news.ceng.metu.edu.tr) for discussions and possible updates on a daily basis.
6. **Evaluation:** Your latex file will be converted to pdf and evaluated by course assistants. The .tex file will be checked for plagiarism automatically using "black-box" technique and manually by assistants, so make sure to obey the specifications.

2 Submission

Submission will be done via COW. Download the given template answer file "the3.tex". When you finish your exam upload the .tex file with the same name to COW.
Note: You cannot submit any other files. Don't forget to make sure your .tex file is successfully compiled in Inek machines using the command below.

```
$ pdflatex the3.tex
```