Student Information

Full Name : Deniz Rasim Uluğ
Id Number : 2172088

Answer 1

a)

Table 1: $(p \to q) \wedge (q \to r)] \to (p \to r)$

p	q	r	$p \to q$	$q \to r$	$(p \to q) \wedge (q \to r)$	$(p \to r)$	$[(p \to q) \wedge (q \to r)] \to (p \to r)$	
1	1	1	1	1	1	1	1	
1	0	1	0	1	0	1	1	
0	1	1	1	1	1	1	1	
0	0	1	1	1	1	1	1	
1	1	0	1	0	0	0	1	
1	0	0	0	1	0	0	1	
0	1	0	1	0	0	1	1	
0	0	0	1	1	1	1	1	

It's seen that the above formula is a tautology.
b)

Table 2: $\neg(\neg p \wedge (p \vee q) \to q)$

p	q	$\neg p$	$p\vee$ q	$\neg p \wedge (p \vee q)$	$\neg p \wedge (p \vee q) \to q$	$\neg(\neg p \wedge (p \vee q) \to q)$	
1	1	0	1	0	1	0	
1	0	0	1	0	1	0	
0	1	1	1	1	1	0	
0	0	1	0	0	1	0	

It's seen that the above formula is a contradiction.

Answer 2

$(p \to q) \wedge (p \to r)$	
$(\neg p \vee q) \wedge (p \to r)$	Table 7 Formula 1
$(\neg p \vee q) \wedge (\neg p \vee r)$	Table 7 Formula 1
$\neg p \vee (q \wedge r)$	Distributive Law
$(q \wedge r) \vee \neg p$	Commutative Law
$\neg(q \wedge r) \to \neg p$	Table 7 Formula 1
$(\neg q \vee \neg r) \to \neg p$	De Morgan's Law

Answer 3

A)

a) $\exists x \forall y(F(x) \land D(x,y))$
b) $\forall y \exists x(F(x) \land D(x,y))$
c) $\exists y \forall x((F(x) \to \neg D(x,y))$
d) $\exists y \exists x_1 \forall x(D(x_1,y) \land (D(x,y) \to x = x_1))$
e) $\exists y \forall x(\neg F(x) \to \neg D(x,y))$

B)

a) $\forall y \neg teaches(AhmetMetin, y)$
b) $\exists x \forall y(teacher(x) \land (teaches(x,y) \to enjoys(x,y)))$
c) $\neg(\forall x \forall y(teacher(x) \to teaches(x,y))$
c) $\exists x \exists y(teacher(x) \land \neg teaches(x,y))$ d) $\forall x \forall y(\neg student(x) \to \neg takes(x,y))$
e) $\forall x \exists y_1 \exists y_2(teacher(x) \to (teaches(x,y_1) \land teaches(x,y_2) \land y_1 \neg y_2 \land \forall y(teaches(x,y) \to (y = y_1 \lor y = y_2))))$

Answer 4

1	p	
2	$p \to (r \to q)$	
3	$r \to q$	$\Rightarrow$E, 1, 2
4	$\quad \neg q$	
5	$\quad\quad r$	
6	$\quad\quad q$	$\Rightarrow$E, 3, 5
7	$\quad\quad \bot$	$\neg$E, 4, 6
8	$\quad \neg r$	$\neg$I, 5–7
9	$\neg q \to \neg r$	$\Rightarrow$I, 4–8

Answer 5

First we will prove lemmas we will use.
Lemma1(LEM)

1	$\mid\ \mid\ \neg(p \lor \neg p)$	
2	$\mid\ \mid\ \mid\ p$	
3	$\mid\ \mid\ \mid\ p \lor \neg p$	$\lor$I, 2
4	$\mid\ \mid\ \mid\ \bot$	$\neg$E, 1, 3
5	$\mid\ \mid\ \neg p$	$\neg$I, 2–4
6	$\mid\ \mid\ \neg p \lor p$	$\lor$I, 5
7	$\mid\ \mid\ \bot$	$\neg$E, 1, 6
8	$\mid\ p \lor \neg p$	$\neg$I, 1–7

Lemma2

1	$\mid\ p \to q$	
2	$\mid\ \mid\ \neg(\neg p \lor q)$	
3	$\mid\ \mid\ \mid\ \neg p$	
4	$\mid\ \mid\ \mid\ \neg p \lor q$	$\lor$I, 3
5	$\mid\ \mid\ \mid\ \bot$	$\neg$E, 2, 4
6	$\mid\ \mid\ p$	$\neg$I, 3–5
7	$\mid\ \mid\ q$	$\Rightarrow$E, 1
8	$\mid\ \mid\ \neg p \lor q$	$\lor$I, 7
9	$\mid\ \mid\ \bot$	$\neg$E, 2, 8
10	$\mid\ \neg p \lor q$	$\neg$I, 2–9

Lemma3(De Morgan)

1	$\neg(p \vee q)$	
2	$\quad \neg(\neg p \wedge \neg q)$	
3	$\quad\quad p$	
4	$\quad\quad p \vee q$	$\vee$I, 3
5	$\quad\quad \bot$	$\neg$E, 1, 4
6	$\quad \neg p$	$\neg$I, 3–5
7	$\quad\quad q$	
8	$\quad\quad q \vee p$	$\vee$I, 7
9	$\quad\quad \bot$	$\neg$E, 1, 8
10	$\quad \neg q$	$\neg$I, 7–9
11	$\quad \neg q \wedge \neg p$	$\wedge$I, 6, 10
12	$\quad \bot$	$\neg$E, 2, 11
13	$\neg p \wedge \neg q$	$\neg$I, 2–12

Lemma4(Modus Tollens)

1	$p \to q$	
2	$\neg q$	
3	$\quad p$	
4	$\quad q$	$\Rightarrow$E, 1, 3
5	$\quad \bot$	$\neg$E, 2, 4
6	$\neg p$	$\neg$I, 3–5

Lemma5

1	$\exists x \neg p(x)$	
2	$\quad \neg p(b)$	
3	$\quad\quad \forall x p(x)$	
4	$\quad\quad p(b)$	$\forall$E, 3
5	$\quad\quad \bot$	$\neg$E, 2, 4
6	$\quad \neg \forall x p(x)$	$\neg$I, 3–5
7	$\neg \forall x p(x)$	$\exists$E, 1, 2–6

Lemma6

1	$\neg q \to \neg p$	
2	$\quad p$	
3	$\quad q$	Lemma4(Modus Tollens), 1, 2
4	$p \to q$	$\Rightarrow$I, 2–3

Now we can prove our main hypothesis. We will use or-elemination on $q(a) \lor \neg q(a)$ and will prove our argument for both case.

1	$\exists x(p(x) \to q(a))$	
2	$p(b) \to q(a)$	
3	$q(a) \lor \neg q(a)$	Lemma1(LEM)
4	$q(a)$	
5	$\neg(\forall y p(y) \to q(a))$	
6	$\neg(p(c) \to q(a))$	$\forall$E, 5
7	$\neg(\neg p(c) \lor q(a))$	Lemma2, 6
8	$p(c) \land \neg q(a)$	Lemma3(De Morgan), 7
9	$\neg q(a)$	$\land$E, 8
10	$\bot$	$\neg$E, 3, 8
11	$\forall y p(y) \to q(a)$	$\neg$I, 4–9
12	$\neg q(a)$	
13	$\neg q(a)$	
14	$\neg p(b)$	Lemma4(Modus Tollens), 2, 13
15	$\exists y \neg p(y)$	$\exists$I, 14
16	$\neg \forall y p(y)$	Lemma5, 15
17	$\neg q(a) \to \neg \forall y p(y)$	$\Rightarrow$I, 13–16
18	$\forall y p(y) \to q(a)$	Lemma6, 17
19	$\forall y p(y) \to q(a)$	$\lor$E, 3, 4–11, 12–18
20	$\forall y p(y) \to q(a)$	$\exists$E, 1, 2–19