Middle East Technical University | Department of Computer Engineering

CENG 223

Discrete Computational Structures

Fall '2016-2017

Take Home Exam 2

Due date: 14 November 2015, 23:55

Question 1

Let $E = \prod_{i=1}^{n} E_i = E_1 \times E_2 \times ... \times E_n$ and $A_k \subset E_k$ for k=1,2,...,n. Define $f_k : E \to E_k$ by $f_k(x) = x_k$ for k=1,2,...,n where $x = (x_1, x_2, ..., x_n)$. That is, $f_1(x) = x_1$, $f_2(x) = x_2$, $f_3(x) = x_3$,..., and $f_n(x) = x_n$.

a. Prove or disprove that
$\prod_{k=1}^{n} A_k = \bigcap_{k=1}^{n} f_k^{-1}(A_k)$

b. Is f_2 1:1? Explain.

c. Is f_1 onto? Explain.

d. Prove that $\overline{f_k^{-1}(A_k)} = f_k^{-1}(\overline{A_k})$.

e. Prove or disprove that $\overline{A_1 \times \prod_{k=2}^{n} E_k} = \overline{A_1} \times \prod_{k=2}^{n} E_k$

Note that $f_k^{-1}(A_k) = \{x \in E \mid f_k(x) \in A_k\}$ is the inverse image of the set A_k, and $\overline{A_k} = E_k \setminus A_k$ (that is, the universal set of A_k is E_k)

Question 2

Define $f : Z \to N^+$ by

$$f(x) = \begin{cases} 2|x| & \text{if } x < 0 \\ 2x + 1 & \text{if } x \geq 0 \end{cases}$$

a. Show that f has inverse.

b. Find $f^{-1}(26)$.

Note that $N^+ = N \setminus \{0\}$

Question 3

Define $f(n) = 12(log_2(n) + n)(n + 3nlog_2 n) + 6n^2$ and $g(n) = n^2 log_2(n)$ for $n \geq 2$. Prove or disprove that f(n) is O(g(n)).

Question 4

If E is uncountable and $S \subseteq E$ is countable, Is $E \setminus S$ countable? Prove your solution.

Question 5

a. Let n be a positive integer. Prove the following.

   If $n \equiv 1(mod 3)$, then $n(n+1) \equiv 2(mod 3)$. Otherwise, $n(n+1) \equiv 0(mod 3)$

b. Use the Euclidean algorithm to find

   gcd(123,277)

c. Prove the following implication.

   If $p > 2$ is an even prime, then $p > 2^{100} + 1$

1 Regulations

1. You have to write your answers to the provided sections of the template answer file given. Other than that, you cannot change the provided template answer file. If a latex structure you want to use cannot be compiled with the included packages in the template file, that means you should not use it.

2. Do not write any other stuff, e.g. question definitions, to answers' sections. Only write your answers. Otherwise, you will get 0 from that question.

3. **Late Submission: Not allowed**

4. **Cheating: We have zero tolerance policy for cheating**. People involved in cheating will be punished according to the university regulations.

5. **Newsgroup:** You must follow the newsgroup (news.ceng.metu.edu.tr) for discussions and possible updates on a daily basis.

6. **Evaluation:** Your latex file will be converted to pdf and evaluated by course assistants. The .tex file will be checked for plagiarism automatically using "black-box" technique and manually by assistants, so make sure to obey the specifications.

2 Submission

Submission will be done via COW. Download the given template file, "the2.tex", when you finish your exam upload the .tex file with the same name to COW.
Note: You cannot submit any other files. Don't forget to make sure your .tex file is successfully compiled in Inek machines using the command below.

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$ pdflatex the2.tex
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