1-a) - $f(n) \in O(g(n)) \Rightarrow f(n) \leq c \cdot g(n)$ $n > n_0$ and $c > 0$

$\lim_{n \to \infty} \frac{f(n)}{g(n)} = 0$ is find, Thus this statement is <u>true</u>, or

- $g(n) \in O(f(n)) \Rightarrow g(n) \leq c_0 f(n)$ $n > n_0$ and $c > 0$

$\lim_{n \to \infty} \frac{g(n)}{f(n)} = 0$ is find, Thus this statement is <u>true</u>

1-b) $f(n) \in O(g(n)) \Rightarrow f(n) \leq c.g(n)$ $n > n_0$ and $c > 0$

$\frac{f(n)}{h(n)} \in O\left(\frac{g(n)}{h(n)}\right) \Rightarrow \frac{f(n)}{g(n)} = c \frac{g(n)}{h(n)}$ $n > n_0$ and $c > 0$

$\lim_{n \to \infty} \frac{\frac{f(n)}{\cancel{h(n)}}}{\frac{g(n)}{\cancel{h(n)}}} = \frac{f(n)}{g(n)} = 0$ is find, Thus this statement is <u>true</u>

1-c) if k be a integer $f(n) \leq c.g(n)$ is true

$f(n)^k \in O(g(n)^k)$; $f(n)^k \leq c_1 g(n)^k$

$f(n)^k \leq c_1 g(n)^k \Rightarrow \lim_{n \to \infty} \frac{f(n)^k}{g(n)^k} = 0$ is find,

Thus this statement is true

2-) a-) $n^3 \in O(2^n)$ $n \geq n_0$ $c > 1$ $n^3 \leq c \cdot 2^n$

$$\lim_{n \to \infty} \frac{n^3}{2^n} = \lim_{n \to \infty} \frac{3n^2}{\cancel{n} \cdot 2^n \cdot \ln 2} = \lim_{n \to \infty} \frac{3}{n \cdot 2^n (\ln 2)^2} = 0$$ is true.

2-) b-) $2^n \in o(3^n) \Rightarrow 2^n < 3^n \cdot c$ $n > n_0$ $c > 0$

$$\lim_{n \to \infty} \frac{2^n}{3^n} = 0$$ is true

2- c) $n! \in O(100^n) \Rightarrow n! \leq c \cdot 100^n$ $n > n_0$ $c > 0$

$$\lim_{n \to \infty} \frac{n!}{100^n} \Rightarrow \underline{\text{stirling rules}} = \lim_{n \to \infty} \frac{\sqrt{2\pi n} \cdot \left(\frac{n}{e}\right)^k}{100^n} = \infty$$

is that false

3-) begin linear-Search(L[1:n], x)

```
for i=1 to n do
   if(L[i] = x) then
      return
   end if
end for
return 0
end
```

Worst case if $x = L(n)$, then worst case occurs

$w(n) = n \in O(n)$

Average case search is p where $0 \leq p \leq 1$ uniform distribution

$w(n) = n \Rightarrow A(n) = \sum_{i=1}^{n} i \cdot P_i$

Best case if $x = L[1]$ then best case occur $\Rightarrow \left(1 - \frac{p}{2}\right)n \in \underbrace{\Theta(n)}_{\text{linear Search Average case}}$

$c(n) = 1 \in O(1)$

4) -a) problem size n^2

4) -b) Main operation is comparison.

4) -c) This algorithm is looking to transpose

4) -d) Best case if first step is simetrik,

$B(n) = O(1)$

Worst case: $\sum_{i=0}^{n-2} \sum_{j=i+1}^{n-2} 1 = (n-2).(n- \; -i)$

$= w(n) = n^2$ is

$\theta(n^2)$

Average Case:

$$A(n) = \sum_{i=1}^{n} i P_i = \sum_{i=1}^{n^2} \frac{p}{n} \left(\frac{n(n+1).(2n+1)}{6} \right) + n(1-i)$$

$$= \frac{p(n+1)(2n+1)}{6} + n - np \Rightarrow \frac{2n^2p + 2np + np + p}{6}$$

$= \in \theta(n^2)$

15) -a) problem size is n

b) $T(0) = 1$

$T(1) = 1$

$T(n) = 1 + T(n/3) + T(n/3)$

$T(n/3) = 2 + 2T(n/3)$

$T(n/3) = 2(1 + T(n/3))$ times

addition operator make $\boxed{T(n/3)}$ time

c) show that above,

$T(n) = 2(1 + T(n/3))$ times, calculat main.

d) Asymtotic notation is $\Theta(n)$

6) Begin

```
    sumA1 =0
    sumA2=0
1   LengthA1=0     // Arrayin uzunlugu, iki arrayda esit uzunlukta olmasi icin
    LengthA2=0     // Arrayin uzunlugu
    j=0 , k=0
    for i=0 to n do
        if sumA1 > sumA2 and LengthA1 > LengthA2
            A2[j] = A[i]
    1       sumA2 += A[i]
n           ++LengthA1
            ++j
        else
            A1[k] = A[i]
    1       sumA1 = A[i]
            ++LengthA2
            ++k
        end elseif
    en for
end
```

* Hocam bu algoritmanin complext analysis ise

$T(n) = n$ bu yüzden ;

$T(n) \in \Theta(n)$ olur.

7)

```
begin
    count_zero (A[1:n])
    if n <= 1
        if A[0] = 0
            return 1
        else
            return 0
        end else if
    else
        return count_zero (A[1 : n/2]) + count_zero(A[n/2 : n])
    end else if
end
```

* Bu fonksiyon her eleman için yapıldığı için çok
  $T(n) = \Theta(n)$ 'dir.

* fakat recursive olduğu için $T(n) = \log n$

8-a) $T(n) = -4T(n-1) - 4T(n-2)$, $T(0) = 0$, $T(1) = 1$

$$\alpha^2 = -4\alpha - 4 \Rightarrow \alpha^2 + 4\alpha + 4 = 0$$

$$= (x+2)^2 \Rightarrow \alpha_1 = \alpha_2 = 2$$

$$\alpha(n) = c_1(-2)^2 + c_2(-2)^n . n$$

$$\alpha(0) = c_1 + c_2 = 0$$

$$T(n) = c_1 (-2)^2 + c_2 (-2)^n . n$$

$$T(0) = c_1 + c_2 . 0 \Rightarrow c_1 = 0$$

$$T(1) = 2c_1 - 2c_2 = 1 \quad c_2 = -\frac{1}{2}$$

$$T(n) = \left(-\frac{1}{2}\right) . (-2)^n . n \Rightarrow 2^n . n \in \Theta(2^n)$$

8-b) $T(n) = T(n-1) + 6T(n-2)$ $T(0) = 3$, $T(1) = 1$

$$\alpha^2 = \alpha + 6$$
$$x^2 - \alpha - 6$$
$$\alpha_1 = 3, \quad \alpha_2 = -2$$

$T(0) = 3$ $\quad c_1 + c_2$ $\quad T(1) = 3c_1 - 2c_2 = 6$

$c_1 = \frac{12}{5}$ $\quad c_2 = \frac{3}{5}$

$$T(n) = \frac{n}{5} 3^n + \frac{3}{5}(-2^n) \Rightarrow T(n) = \Theta(3^n)$$

9)-a) $T(n) = T(n-1) + (n^2+1)$, $T(0) = 3$

$= T(n-2) + (n-1)^2 + n^2 + 2$, $T(n-2) = T(n-3) + (n-2)^2 + 1$

$= T(n-3) + (n-2) + (n-1)^2 + n^2 + 3$

$= T(n-i) + \sum_{j}^{i-1} (n-i+1)^2 + \sum_{j=1}^{i} j$

$= T(0) + \sum_{j_0=0}^{n-1} j^2 + \sum_{j=1}^{n} j$

$= \frac{(n-1).(n).(2n-1)}{6} + \frac{n.n+1}{2} + 3 \Rightarrow T(n) \in \Theta(n^3)$

9-b) $T(n) = \sum_{i=1}^{n-1} T(i) + n^2$, $T(1) = 1$

$\Rightarrow T(n) - T(n-1) = \sum_{j=1}^{n-1} T(i) + n^2 - \sum_{i=1}^{n-2} T(i) - (n-1)^2$

$= T(n-1) + 2n-1$

$T(n) = 2T(n-1) + 2n-1$

$= 2(2T(n-2) + 2(n-1) - 1) + 2n-1$

$= 2(2(T(n-3) + 2(n-2) - 1) + 2(n-1) - 1) + 2n-1$

$T = n-1$ $\quad = 2^i T(n-i) + \sum_{j=0}^{i-1} 2^{j+1}(n-j) - \sum_{j=0}^{i-1} 2^j$

$= 2^{n-1} + \sum_{j=0}^{n-2} 2^{j+1} - \sum_{j=1}^{n-2} 2^{i-1}$

$= 2^{n-1} + \sum_{j=1}^{n-1} 2^{j-1} \left(\sum_{j=1}^{n-1} j + n-1 \right) - \sum_{j=1}^{n-2} 2^{j-1}$

$= 2^{n-1} + \frac{1-2^{n-1}}{1-2} \left(\frac{n.(n+1)}{2} + n-1 \right) - \frac{1-2^{n-2}}{1-2}$

$\Rightarrow 2^n . n^2 \Rightarrow T(n) = \Theta(2^n)$

9-c) $T(n) = 2T(n-1) - T(n-2) + n \qquad T(0)=0,\ T(1)=0$

$$T(n) = -T(n-2) + 2T(n-1) + n$$
$$= -T(n-2) + T(n-1) + T(n-1) + n$$
$$= -T(n-3) + \cancel{T(n-2)} + T(n-1) + n + n - \cancel{T(n-2)}$$
$$= -T(n-3) + T(n-1) + 2n$$
$$= -T(n-4) - T(n-3) + T(n-2) + 3n$$
$$= -T(n-5) - T(n-4) + T(n-3) + 4n$$

$$T(2) = 0 + 0 + 0$$
$$T(3) = T(1) + 2T(2) + n$$
$$T(4) = -T(2) + 2T(3)$$
$$T(n) = \frac{1}{6}\left(n^3 + 4n - 2n^2 - 4\right) \Rightarrow \Theta(n^3)$$

10) Let $T(n)$ be a monotonically increasing function that satisfies

$$T(n) = aT(n/b) + f(n)$$
$$T(1) = c$$

where $a \geq 1$, $b \geq 2$, $c > 0$. if $f(n) \in \Theta(n^d)$ where $d \geq 0$, then:

$$T(n) = \begin{cases} \Theta(n^d) & \text{if } a < b^d \\ \Theta(n^d \cdot \log n) & \text{if } a = b^d \\ \Theta(n^{\log_b a}) & \text{if } a > b^d \end{cases}$$

10-a) $f(n) = 3f(n/2) + n^2$, $f(1) = 4$

$a = 3 \geq 1$ $b = 2 \geq 2$ $c = 4 > 0$ and

$f(n) = n^2 \in \Theta(n^2)$, Thus $d = 2$

Since $3 < 2^2$, case 1 applies

$f(n) \in \Theta(n^d) = \Theta(n^2)$ found

10-b) $f(n) = 3f(n/2) + n^2 \log n$, $f(1) = 1$

$a = 3 \geq 1$ $b = 2 \geq 2$ $c = 1 > 0$

$f(n) = n^2 \log n = \Theta(n^2 \log n)$, Thus $d = 2$

since $3 < 2^2$, case 1 applies

$f(n) \in \Theta(n^d) = \Theta(n^2)$ found