

1) Algorithm

// Evaluates a polynomial

// inputs: An array $P[0..n]$

// output: The value of the polynomial at x

$p \leftarrow P[n]$

for $i \leftarrow n-1$ down to 0 do

$p \leftarrow x * p + P[i]$

return p

Example

$$p(x) = 2x^4 - x^3 + 3x^2 + x - 5 \quad \text{at } x=3$$

coefficients	2	-1	3	1	-5
$x=3$	2	$3 \cdot 2 + (-1) = 5$	$3 \cdot 5 + 3 = 18$	$3 \cdot 18 + 1 = 55$	$3 \cdot 55 - 5 = 160$

$$P(3) = 160$$

$$P(x) = x(x(x(2x-1)+3)+1)-5 = p(x)$$

So, The number of multiplications and the number of additions are given by the same sum;

$$M(n) = A(n) = \sum_{i=0}^{n-1} 1 = n$$

2) # Algorithm

Search (A, i, j, key) {

 int mid = (i+j)/2

 if (A[mid] == key & & mid == key) then return mid;

 else if (A[mid] < key) then return Search(A, i, mid-1, key);

 else then return -1;

}

$$T(n) = 2T(n-1) + 1$$

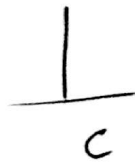
$$T(n) = O(\log n)$$

3) 1) Base case: If n is 1, the solution is trivial. Just move the disk

2) Otherwise: Move $(n-1)$ disks from peg A to peg C using Hanoi for $n-1$ disks

3) Move the left-over disk from peg A to peg B

4) Move $(n-1)$ disks from C to peg B using Hanoi $(n-1)$ disks



4) # pseudocode

FindMinMax(arr [l...r], minval, maxval)

finds the values of the smallest and largest elements in a given subarray

Input: A part of array arr [0...n-1] between indices l and r and $l < r$

Output: The values of the smallest and largest elements in arr [l...r], assigned to minval and maxval.

if $r == l$

minval \leftarrow arr[l];
maxval \leftarrow arr[r];

else if $r - l = 1$

if arr[l] \leq arr[r]
minval \leftarrow arr[l]
maxval \leftarrow arr[r]

else minval \leftarrow arr[r]
maxval \leftarrow arr[l]

$r - l > 1$

else

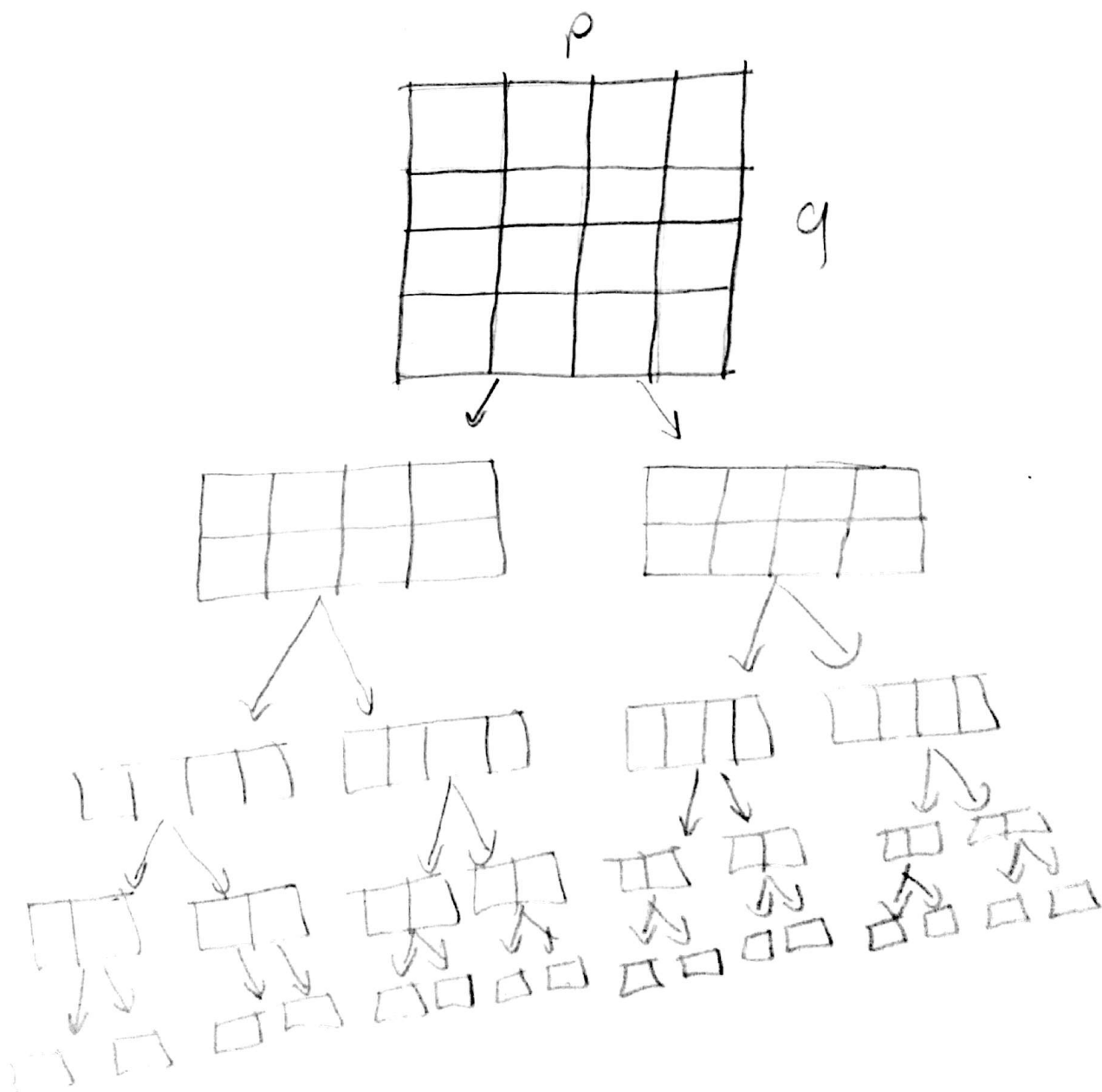
FindMinMax(arr [l... $\lfloor (l+r)/2 \rfloor$], minval, maxval)

FindMinMax(arr [$\lfloor (l+r)/2 \rfloor + 1$...r], minval/2, maxval/2)

if minval/2 $<$ minval
minval \leftarrow minval/2

if maxval/2 $<$ maxval
maxval \leftarrow maxval/2

- 5) 1) As small square, the unit square, cannot be cut into smaller pieces
- 2) All breaks have to be made completely along one axis
- 3) The total number of breaks cannot be more than n solution
- 4) p or q cannot equal 1
- $y \times$ pointed out in one of the answers that problems is easily solvable if one side has bars



6) b) Divide and conquer;

1) Does more work on the sub-problems and hence has more time consumption.

2) In divide-conquer the sub-problems are independent of each other.

Dynamic programming:

1) solve the sub-problems only once and then store it in the table.

2) In dynamic programming the sub-problems are not independent.

a) both of them divide problems into sub-problems.

2) a) Let $P(i, j)$ be the probability of A winning the series if A need i more games to win the series and B needs j more games to win the series. If team A wins the game, which happens with probability p , A will need $i-1$ more wins to win the series while B will still need j wins. If team A loses the game, which happens with probability $q = 1-p$, A will still need i wins while B will need $j-1$ wins to win the series. This leads to the recurrence

$$P(i, j) = pP(i-1, j) + qP(i, j-1) \quad \text{for } i, j > 0.$$

The initial conditions follow immediately from the definition of $P(i, j)$:

$$P(0, j) = 1 \quad \text{for } j > 0, \quad P(i, 0) = 0 \quad \text{for } i > 0.$$

b) Let $q = 1 - p$. First let us do a direct calculation.

$$\begin{aligned} P(A) &= P(A \text{ wins in 4 games}) + P(A \text{ wins in 5 games}) \\ &\quad + P(A \text{ wins in 6 games}) + P(A \text{ wins in 7 games}) \\ &= p^4 + \binom{4}{3} p^4 q + \binom{5}{3} p^4 q^2 + \binom{6}{3} p^4 q^3 \end{aligned}$$

To understand how these probabilities are calculated, note for example that

$$\begin{aligned} P(A \text{ wins in 5}) &= P(A \text{ wins 3 out of first 4}) \\ &\quad \times P(A \text{ wins 5th game} | A \text{ wins 3 out of first 4}) \\ &= \binom{4}{3} p^3 q p. \end{aligned}$$

so, for seven-game

$$P(A \text{ wins}) = \binom{7}{6} p^6 q + p^7$$

c) // Pseudocode

$q \leftarrow 1 \leftarrow p$

for $j \leftarrow 1$ to n do

$p[0, j] \leftarrow 1.0$

for $i \leftarrow 1$ to n do

$p[i, 0] \leftarrow 0.0$

for $j \leftarrow 1$ to n do

$p[i, j] \leftarrow p * p[i-1, j] + q * p[i, j-1]$

return $p(n, n)$

Both the time efficiency and the space efficiency are in $\Theta(n^2)$ because each entry of the $n \times n$ -by- n table is computed in $\Theta(1)$ time


```

8) def MaxSubSquare (arrA, row, cols):
    sub = Array.CreateInstance(row, cols)
    # copy the first row

```

```

    i = 0
    while i < row:
        sub[0][i] = arrA[0][i]
        i += 1

```

```

    # copy the first column

```

```

    i = 0
    while i < cols:
        sub[i][0] = arrA[i][0]
        i += 1

```

```

    # for rest of the matrix
    # check if arrA[i][j] == 1

```

```

    i = 1
    while i < row:

```

```

        j = 1
        while j < cols:

```

```

            if arrA[i][j] == 1:

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                sub[i][j] = Math.Min(
                    sub[i-1][j], Math.Min(
                        sub[i][j-1],
                        sub[i-1][j-1])) + 1

```

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            else:
                sub[i][j] = 0

```

```

            j += 1

```

```

        i += 1

```

```

    # Find the maximum entry and indexes of maximum entry
    # in sub[0][0]

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    int max-of-3 = sub[0][0]

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    int max-i = 0; max-j = 0;

```

```

    # continue of page in other page

```

i = 0

while i < row

j = 0

while j < cols

if max-of-s < sub[i][j]

max-of-s = sub[i][j]

max-i = i

max-j = j

++i ++j

Console.WriteLine("\n Maximum size submatrix is : \n")

i = max-i

while i > (max-i - max-of-s)

j = max-j

while j > (max-j - max-of-s)

Console.WriteLine(arrA[i][j])

--j

Console.WriteLine("\n")

--i

driver function to test above functions

def main()

arr ({ { 0, 1, 1, 0, 1 },
 { 1, 1, 0, 1, 0 },
 { 0, 1, 1, 1, 0 },
 { 1, 1, 1, 1, 0 },
 { 1, 1, 1, 1, 1 },
 { 0, 0, 0, 0, 0 } })

MaxSubSquare(arr, 6, 5)

Output :

1 1 1
1 1 1
1 1 1

3) def MatrixChainOrder(p, n):

For simplicity of the program, one extra row and one
extra column are allocated in m[][] . 0th row and
0th column of m[][] are not used

m = [[0 for x in range(n)] for x in range(n)]

m[i][j] = Minimum number of scalar multiplications needed
to compute the matrix A[i][j] A[i+1][j] ... A[j] = A[i...j] where
dimension of A[i] is p[i-1] x p[i]

cost is zero when multiplying one matrix.

for i in range(1, n):

m[i][i] = 0

L is chain length.

for L in range(2, n):

for i in range(1, n-L+1):

j = i+L-1

m[i][j] = sys.maxint

for k in range(i, j):

q = cost / scalar multiplications

q = m[i][k] + m[k+1][j] + p[i-1] * p[k] * p[j]

if q < m[i][j]:

m[i][j] = q

return m[1][n-1]

Driver program to test above function

arr = [1, 2, 3, 4]

size = len(arr)

print("Minimum number of multiplications " +
str(MatrixChainOrder(arr, size)))

Output: Minimum number of multiplications is 18