Methods and Heuristics for Learning and Optimization

Exercise 6: Genetic Algorithms and Function Minimization

Assignment for 2 week; return no later than December 6, 2020

1 Introduction

In this exercise we will introduce the concept of Genetic Algorithms. The task we will focus on is the minimization of the following function:

$$f(x, y) = -\left|\frac{1}{2}x \sin\left(\sqrt{|x|}\right)\right| - \left|y \sin\left(30\sqrt{\left|\frac{x}{y}\right|}\right)\right|$$

where $x, y \in [10, 1000] \cap \mathbb{N}$.

2 Instructions for the Genetic Algorithm

2.1 Individuals and Fitness

An individual of the evolving population must represent a solution of the optimization problem to be solved. You should code a solution as a binary sequence made up of two halves, representing x and y, respectively. Indicate the size of the search space in this case.

To compute the fitness, you need a mapping function between binary and real numbers. Assuming binary sequences (x and y) are composed of $m = 10$ bits, a good candidate for the mapping formula is

$$map : x \mapsto \frac{x}{2^m}(b - a) + a$$

since it ensures that $map(x) \in [a, b] \cap \mathbb{R}$. Starting from the above formula, propose a mapping function to minimize $f(x, y)$, i.e., for $x, y \in [10, 1000] \cap \mathbb{N}$.

2.2 Selection

You can choose a population of size $N = 100$, and pick a person via the 5-tournament selection method (repeat it N times to keep the population size constant).

2.3 Crossover

You should use the `One Point Crossover` with a `Mid-Break` policy and a probability p_c [1] (repeat it $N/2$ times to keep the population size constant).

[1] Crossover probability p_c can be set to 0.6 for instance.

2.4 Mutation

You should implement a mutation operator that with a small probability (p_m) will swap each element of an individual.

3 Work to do

3.1 Function visualization

Visualize the function to minimize in a 3D space to get an idea on how difficult is to find its global minimum, and localize its global minimum.

3.2 Tests and evaluation

You should experiment with 2 different probabilities of mutation p_m:

- 0.01
- 0.1

and with the following crossover operators:

- without crossover
- `One Point Crossover` with a `Mid-Break` policy and a probability p_c

You should run the algorithm (in each of the above 4 cases) several times by changing the seed of a random number generator.

Measure and present:

1. The cumulative empirical probability to reach the following solution qualities (optimum, relative distance to optimum = 1.0%, relative distance to optimum = 2.5%) over the number of evaluations.
2. Report the best, average and standard deviation of fitness among the populations for $10^3, 10^4$, and 10^5 fitness evaluations. The number of fitness evaluations can be estimated by the product of the population size with the number of generations. These statistics should be computed over several runs of the genetic algorithm (e.g., 10 runs).

4 Report

The report should contain an introduction to the genetic algorithms, as well as a description and discussion about the methodology adopted, achieved experiments, and obtained results.

5 General conditions

The submitted work should be solely of your own. Both report and code, entitled *Surname Name TP number*, have to be uploaded on Moodle (TP6).