ELEC 242 Test #3 04/18/18 **Sign the pledge**

2 hours; closed book; closed notes; no computers or calculators; show all work.
Three informative sheets allowed. The best 4 out of 5 problems will count.
All problems have equal weight. Use a new sheet of paper for each problem.

Problem [1]. The signal $y(t) = e^{-2t}\mathbb{I}(t)$, is the output of a causal system whose transfer function is

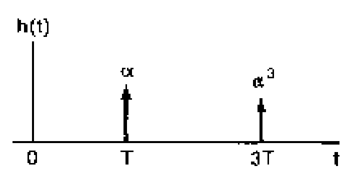
$$H(s) = \frac{s-1}{s+1}.$$

(a) Find and sketch all possible inputs $u(t)$ that could produce $y(t)$.
(b) Determine the input if it is known that $\int_{-\infty}^{\infty} |u(t)|\, dt < \infty$.
(c) Find the input $u(t)$ if it is known that a stable system exists that will have $u(t)$ as an output if $y(t)$ is the input. Find the impulse response g of this system and show by direct convolution that it has the required property, i.e. $u = g * y$. ■

Problem [2]. Given is a stable and causal system with a real impulse response $h(t)$ and transfer function $H(s)$. It is known that $H(s)$ is rational, one of its poles is $-1+j$, one of its zeros is $3+j$ and the difference between the degrees of the denominator and the numerator is 2. For each of the following statements determine whether it is true, false, or whether there is insufficient information to determine the statement's truth. Justify your answers.
(a) $h(t)e^{-3t}$ is absolutely integrable.
(b) The ROC of $H(s)$ is $\mathrm{Re}(s) > -1$.
(c) The differential equation relating the the input $u(t)$ and the output $y(t)$, may be written in a form having only real coefficients.
(d) $\lim_{s\to\infty} H(s) = 1$.
(e) $H(s)$ does not has fewer than four poles.
(f) $H(j\omega) = 0$, for at least one value of $\omega \in \mathbb{R}$.
(g) If the input to this system is $e^{3t}\sin t$, the output is $e^{3t}\cos t$. ■

Problem [3].

In long-distance communication, an echo is encountered due to the transmitted signal being reflected at the receiver, sent back down the line, reflected again at the transmitter and returned to the receiver. The impulse response for a system that models this effect is shown besides, where it is assumed that only one echo is received:



The parameter T corresponds to the one-way travel time along the communication channel, and the parameter α represents the attenuation in amplitude between transmitter and receiver.

(a) Determine the transfer function $H(s)$ and the associated region of convergence.

(b) Although $H(s)$ is not rational (i.e. does not consist of a ratio of polynomials) it is useful to find its poles and the zeros. The the zeros $\zeta \in \mathbb{C}$ satisfy $H(\zeta) = 0$, while the poles $p \in \mathbb{C}$ satisfy $\frac{1}{H(p)} = 0$. Find the poles and the zeros of $H(s)$, and sketch the associated pole-zero plot. ■

Problem [4]. The transfer function of a discrete-time, causal, LTI system $H(z)$ has two zeros at $z = 0$, a pole at $z = -\frac{1}{3}$ and a second pole at $z = \frac{1}{2}$. Furthermore it is known that $H(z) = 6$ when $z = 1$.

(a) Determine $H(z)$ and the associated impulse response $h[n]$.

(b) Determine the response of the system to the following inputs:

(i) $u[n] = \mathbb{I}[n] - \frac{1}{2}\mathbb{I}[n-1]$, and

(ii) the sequence $u[n]$ obtained from sampling the continuous-time signal

$$f(t) = 50 + 10\cos(20\pi t) + 30\cos(40\pi t),$$

at a sampling frequency $\Omega_s = 2\pi \cdot 40\, \frac{rad}{sec}$. ■

Problem [5]. Consider the LTI system with input $u[n]$ and output $y[n]$. When the input to the system is

$$u[n] = 5\,\frac{\sin(0.4\pi n)}{\pi n} + 10\cos(0.5\pi n),$$

the corresponding output is

$$y[n] = 10\,\frac{\sin[0.3\pi(n-10)]}{\pi(n-10)}.$$

Determine the frequency response $H(e^{j\omega})$ and the impulse response $h[n]$ of this system. ■