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Improving student understanding of lock-in amplifiers

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The lock-in amplifier is a versatile instrument frequently used in physics research. We find that students tend to struggle with its basic operating principles, leading to a variety of difficulties. To improve their understanding, we have developed a research-based tutorial that makes use of a computer simulated lock-in amplifier. The tutorial allows students to realize their conceptual difficulties by comparing their predictions with the outcome of the simulations, hopefully leading them to a better understanding of the process of lock-in amplification.
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I. INTRODUCTION

The lock-in amplifier ("lock-in") is an instrument used extensively in laboratory research, especially in condensed matter physics.[1–6] In general, beginning researchers receive little to no formal training on the use of a lock-in. Often, their initial exposure is somewhat superficial (yet successful), leading them to believe that they have mastery over its function and use. According to professors interviewed during this investigation, improper or inefficient use of the lock-in, and misinterpretation of data obtained from such use, is anecdotally quite common. Additionally, this lack of understanding can result in a student's inability to troubleshoot and modify the experimental setup when faced with anomalous results.

Computer and web-based tools are becoming increasingly commonplace to aid in learning across many science and engineering fields.[7–18] For such tools to be maximally effective, it is important that they be developed using a research-based approach to ensure that they suit both the level and the prior experience of the students who will use them.[19,20] We have developed a lock-in amplifier tutorial to ease the transition of students who are just beginning their research in a laboratory setting, and to provide a firmer foundation for those who have already used lock-ins in their research. The lock-in tutorial focuses on helping students build a robust understanding of its fundamental operation, and aids students in developing an intuitive feel for many of the possible situations that they may encounter in experiments; it is deeply integrated with a computer-based model of a lock-in that is used for teaching concepts as well as testing student understanding.

Our goal in creating this research-based tutorial was to develop a tool that can instill an intuitive understanding of the basics of the lock-in functions, so that students who use such amplifiers in their research can more deeply understand how input signals, and other lock-in parameters, affect the output. By merging conceptual and mathematical aspects of the instrument, the tutorial strives to help students learn the relationship between the input parameters and expected outputs, so that they are able to troubleshoot unexpected outputs in their lab work.

In Secs. II–VII, we describe an idealized lock-in for which we will go into some detail about the mathematical foundation for lock-in amplification. The structure of the lock-in amplifier tutorial is then described, followed by an examination of the most prevalent student difficulties identified in our initial investigation. Finally, we summarize the results of the pretest and posttest scores to gauge the effectiveness of the tutorial.

II. THE IDEAL LOCK-IN AMPLIFIER

Throughout this paper (as in the tutorial), we work with an idealized version of a lock-in amplifier. We assume that the signal of interest is centered on a frequency f_S that is present in the input signal. If the amplitude can change, then the signal will not be a pure frequency since amplitude modulation leads to sidebands that surround the central frequency. In the case where there is no amplitude modulation we will treat the signal as pure. Tutorial questions involve both single-frequency sources as well as amplitude-modulated single-frequency sources. To separate the signal of interest from unwanted noise, a reference signal is defined. This reference has unit amplitude, and is dimensionless (for convenience).

The single-frequency input signal is first pre-amplified by a factor g to give

$$V_I = gA_S \cos(2\pi f_S t + \varphi). \tag{1}$$

Here, φ is the phase of the input signal of amplitude A_S and frequency f_S with respect to the reference signal. This amplified signal is then multiplied (or "mixed") by a reference for the x and y channels of the lock-in, given by

$$v_{RX} = \cos(2\pi f_R t) \tag{2}$$

and

$$v_{RY} = \sin(2\pi f_R t). \tag{3}$$

With these, the "unfiltered" x- and y-channel outputs of the lock-in will be

$$V_{MX}(t) = V_I(t) v_{RX}(t) \tag{4}$$

and

$$V_{MY}(t) = V_I(t) v_{RY}(t). \tag{5}$$

To understand the effect of the mixer, we rely on the trigonometric identities

$$\cos(a)\cos(b) = \frac{1}{2}\left[\cos(a+b) + \cos(a-b)\right], \tag{6}$$

and

$$\cos(a)\sin(b) = \frac{1}{2}\left[\sin(a+b) - \sin(a-b)\right], \tag{7}$$

the application of which yields

$$V_{MX} = \frac{1}{2} gA_S\{\cos[2\pi(f_S - f_R)t + \varphi] + \cos[2\pi(f_S + f_R)t + \varphi]\} \tag{8}$$

and

$$V_{MY} = \frac{1}{2} gA_S\left\{\sin[2\pi(f_S - f_R)t + \varphi] + \sin[2\pi(f_S + f_R)t + \varphi]\right\}. \tag{9}$$

In most experimental situations, the signal is close in frequency to the reference, so that $f_S - f_R \ll f_R$ and $f_S + f_R \approx 2f_R$. For the case when $f_S = f_R$, we have $f_S - f_R = 0$ and $f_S + f_R = 2f_R$, and the unfiltered x-channel and y-channel outputs contain a rapidly oscillating term superimposed on a time-independent one

$$V_{MX} = \frac{1}{2} gA_S\left\{\cos(\varphi) + \cos[2\pi(2f_R)t + \varphi]\right\} \tag{10}$$

and

$$V_{MY} = \frac{1}{2} gA_S\left\{\sin(\varphi) + \sin[2\pi(2f_R)t + \varphi]\right\}. \tag{11}$$

Finally, V_{MX} and V_{MY} are each fed through a low-pass filter with a time constant $\tau = 1/2\pi f_c$, where f_c is the cutoff or corner frequency of the filter. The filter roll-off is most commonly chosen to be one of four values (6 dB/octave, 12 dB/octave, 18 dB/octave, or 24 dB/octave). The values selected for both the time constant and the roll-off should be chosen carefully based upon the nature of the experiment. As a rule of thumb, the $6n\,\mathrm{dB/octave}$ filter ($n = 1, 2, 3, ...$) preserves signals with frequency $f \ll f_c$, while attenuating signals with $f \gg f_c$ according to a power law f^{-n} (e.g., $\propto f^{-2}$ for $12\,\mathrm{dB/octave}$ filters). The resulting filtered outputs are defined as V_{OutX} and V_{OutY} in the tutorial. In the idealized version where $f_R = f_S$, the time constant should be selected such that the low-pass filter attenuates the second-harmonic ($2f_R$) term from both V_{MX} and V_{MY}, resulting in a time-independent output signal. The relationship between V_{OutX} and V_{OutY} and the magnitude and phase of the input signal as initially defined is then given by

$$V_{OutX} = \frac{1}{2} gA_S \cos\varphi \tag{12}$$

and

$$V_{OutY} = \frac{1}{2} gA_S \sin\varphi, \tag{13}$$

which leads to

$$A_S = (2/g)\sqrt{V_{OutX}^2 + V_{OutY}^2} \tag{14}$$

and

$$\varphi = \tan^{-1}\left(\frac{V_{OutY}}{V_{OutX}}\right). \tag{15}$$

The most common use of a lock-in in laboratory research is to measure small signals that are synchronous with an external reference signal, often in the presence of large background signals (noise). For this class of measurements, the reference frequency is set equal to the signal frequency: $f_R = f_S$ (though it is worth noting that there are applications in which the two frequencies would not be the same.) Alternately, when changing a parameter that affects the signal, one is effectively modulating the signal in time. This type of "amplitude modulation" will produce sidebands around f_S that must be passed with acceptably low attenuation, while making sure that the signal is not flooded with background noise. While the lock-in is most commonly used in the ideal case ($f_R = f_S$ with frequency $2f_R$ strongly attenuated) with no other superimposed signals, there are many instances in which unwanted signals can interfere with measurement. For example, power-line noise (i.e., a 50-Hz or 60-Hz signal introduced by ac electrical power) can often superimpose large sinusoidal signals over the desired measurements. An understanding of the most common uses (and abuses) of lock-in amplifiers represents the primary focus of our lock-in tutorial.

III. METHODOLOGY

Before developing the lock-in tutorial, we interviewed five professors who commonly worked with students making use of lock-ins and ten graduate students working in condensed matter physics with varying degrees of experience with lock-ins (ranging from one month to three years of experience). The purpose of these interviews was to establish the most common uses of the lock-in and to ascertain what the most common difficulties are for new users. Alongside the development of the initial version of the tutorial, we developed a lock-in simulation, which was built into the structure of the tutorial. The simulation was built with the intention of allowing students to develop an understanding of the lock-in by giving them the opportunity to experience the device first-hand. We then interviewed graduate students while they made use of the tutorial using a think-aloud protocol[21] to better understand their difficulties and to fine-tune the tutorial as well as its associated pre-test, post-test, and simulation. The tutorial (along with the pre-test and post-test) was iteratively refined over 30 times based on feedback from graduate students and professors.

As the tutorial and its supplementary material underwent this process of final revision and fine-tuning based on think-aloud interview feedback, it was administered to 21 additional physics graduate students who had not been involved in its development. These students ranged in experience from those who had been introduced to the basics of the lock-in but had never made use of one themselves, to those with extensive experience with the lock-in and who either concurrently used a lock-in for their research or had made extensive use of one in the past. These students were administered the pre-test and post-test before and after the tutorial in order to assess its effectiveness. Despite the modifications that were made to the pre-test and post-test, a generalized grading rubric was developed that could be utilized in the scoring of all problems present in all versions of the pre-test and post-test (three researchers deliberated over a series of seven rubrics before reaching agreement on scoring performance). A more complete description of how the tutorial was developed as well as an in-depth discussion of the relevant rubrics will be given in a separate publication.

IV. TUTORIAL STRUCTURE

The tutorial begins with the pre-test that takes the form of a short quiz comprised of "puzzles" related to the operation of a lock-in for various concrete situations. Students are provided with supplementary information (in the form of a PowerPoint presentation) to guide them as they work through the pre-test. For each of the lock-in puzzles, the student is presented with a screenshot of the simulation interface with some of the inputs or outputs hidden, and they are asked to provide (or sketch) the missing information.

After students have completed the pre-test, they work through the tutorial. The tutorial begins with a brief comparative analysis of several measurement devices (the voltmeter, oscilloscope, spectrum analyzer, and the lock-in), which serves to motivate the value of the lock-in for making accurate measurements of the amplitude of a specific frequency within a given signal. The comparative analysis then leads into an in-depth examination of the dual-channel lock-in. This section contains two short narrated videos followed by a series of slides that provide a detailed explanation of a diagram of the dual-channel lock-in, including the basic function of the primary components.

After this basic treatment of the lock-in, students work with the simulation developed alongside the tutorial. The simulation allows the students to manipulate all of the settings commonly found on real lock-in, as well as to modify the characteristics of a variety of simulated input signals. The lock-in settings a student is able to manipulate include the reference frequency, time constant, and roll-off of the low-pass filter. The student can specify the frequency, amplitude, and phase of the primary input signal, as well as introducing a sinusoidal amplitude modulation to this signal. A secondary frequency can also be added to the input signal to simulate sources of interference. Finally, white (frequency-independent) noise can be introduced into the input signal.

The interactive portion of the tutorial is prefaced by a few examples that are similar in format to the questions that follow. Students are guided through an example of how to predict the output signal of the lock-in by first applying the equations derived earlier in the mathematical treatment of the mixer and then applying the rules of thumb regarding the low-pass filter. The students then work through a series of problems designed for use with the simulation, similar to the one in Fig. 1. For each problem, students are asked to predict the output signal when provided with a specific configuration of input signal and lock-in settings. After they have sketched their predictions, students enter their parameters into the

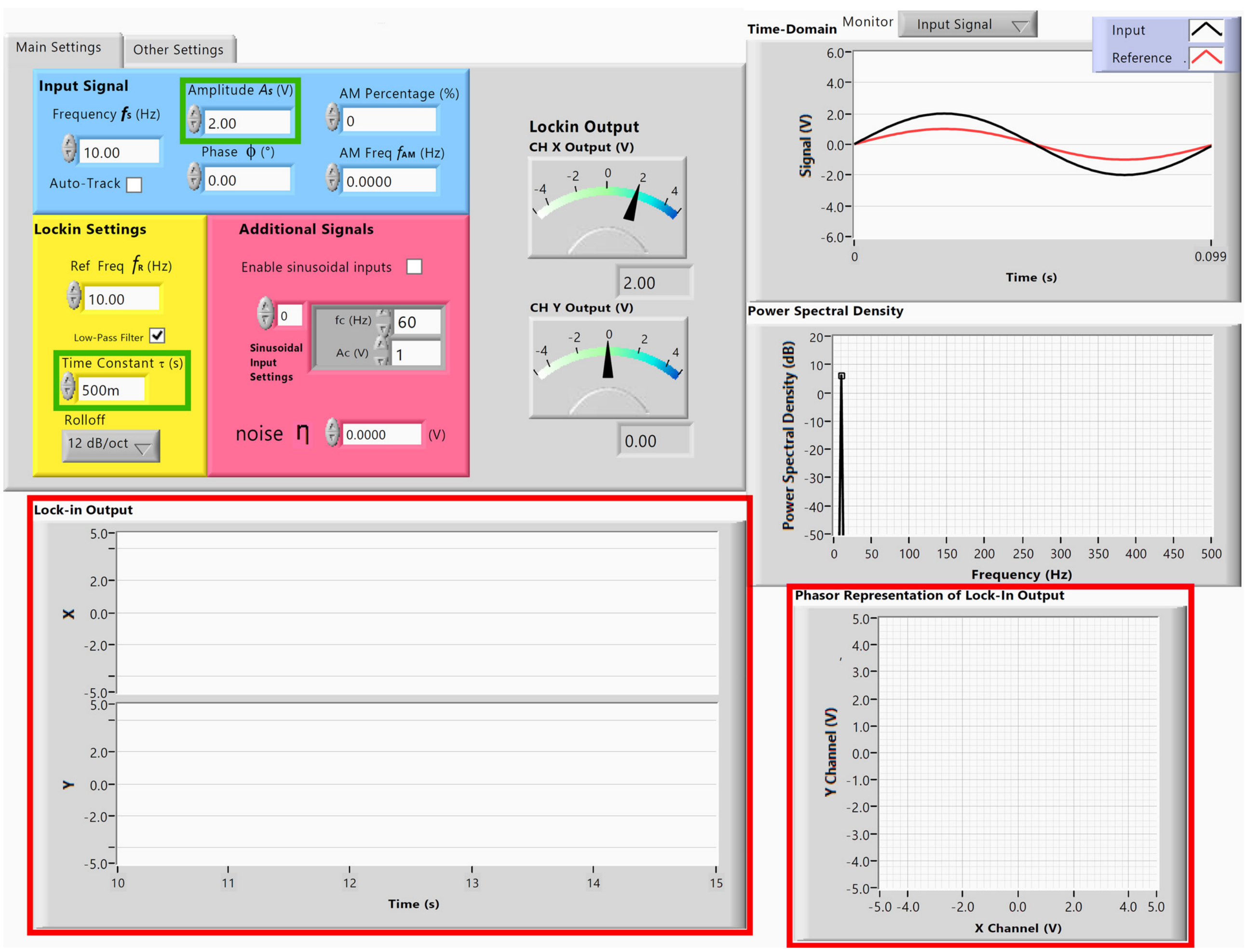

Fig. 1. Structure of a typical lock-in puzzle. The diagram shows the front panel of the lock-in simulator in a given state. Red bounding boxes indicate information that the student is expected to sketch or provide, consistent with the lock-in settings and/or output. Green bounding boxes highlight settings that were changed from a prior simulation.

lock-in simulator and compare the result of the simulation with their prediction. For each lock-in puzzle, two complementary explanations are given. One focuses on the mathematical signal processing, while the other attempts to provide a more conceptually intuitive explanation of the solution.

The post-test is structured similarly to the pre-test, consisting of a short quiz comprised of different puzzles that cover the same types of situations as in the pre-test. Students are again allowed access to the same supplementary material for the post-test as in the pre-test.

The tutorial, simulation, and the pre-test and post-test are available as the supplementary material;[22] updates to these materials will be posted on ComPADRE.[23]

V. STUDENT DIFFICULTIES

The interviews with students revealed a lack of coherent understanding of the fundamentals of a lock-in. For example, students often had a fuzzy conception of what the mixers in a lock-in do. Even for the most common and basic experimental arrangements, many students demonstrated only a superficial understanding of how a lock-in functions. The range and prevalence of these difficulties confirms that students are often using the lock-in as a "black box." Below we summarize the most common difficulties revealed by these students prior to working with the lock-in tutorial.

- **Difficulty in determining the most appropriate corner frequency (or time constant).** Interviewed students had great difficulty with the fact that the frequencies that will make it into the output signal can be estimated by making use of the time constant τ.
- **Difficulty understanding the function of the lock-in mixer.** Students often believe that the lock-in's output should have a frequency equal to that of the signal frequency. These students believe that the lock-in is providing an output similar to that which would be shown by an oscilloscope measuring the input signal.
- **Difficulty understanding the heterodyne case: $f_S \neq f_R$.** Students commonly believe a lock-in will completely filter out every signal not equal to the reference frequency. Students also commonly believe that the lock-in will produce a dc output even under the condition $f_S \neq f_R$.
- **Difficulty with the case in which $f_S \neq f_R$ and the two frequencies are out of phase ($\varphi \neq 0$).** A surprising difficulty that students expressed was the belief that, for the case where $\varphi \neq 0$ and $f_S \neq f_R$, the amplitude of the x-channel is $A_S \cos(\varphi)$ and the amplitude of the y-channel is $A_S \cos(\varphi)$. Students also showed weaker performance on problems in which $f_S = f_R$ and $\varphi \neq 0$, though this added difficulty did not result in any common incorrect answers.
- **Difficulty with the case in which $f_S = f_R$ and the $2f_R$ signal is not strongly attenuated.** Another situation that caused a considerable amount of confusion among students are situations in which $f_S = f_R$ and the $2f_R$ signal appears in the output because it is not strongly attenuated by the low-pass filter.
- **Difficulty with amplitude modulation of the input signal.** Students commonly believe that amplitude modulation necessarily affects both the x-channel and y-channel outputs equally. Students also believe that amplitude modulation should not be affected by the low-pass filter, or completely forget to consider the effects of the low-pass filter on this signal.
- **Difficulty with multiple frequencies present in the input signal.** One final aspect of the lock-in that practically all students had difficulty with are cases in which multiple frequencies are present in the input signal.

VI. IMPACT OF THE TUTORIAL ON STUDENT UNDERSTANDING

In addition to discussions with faculty members, the difficulties summarized above were determined by examining student's interactions with the pretest as well as the tutorial. To evaluate the effectiveness of the tutorial, a pre-test and post-test are given to all students who make use of the tutorial. In order to examine how well the tutorial addresses these difficulties, six matched pairs of lock-in puzzles were developed into pre-test/post-test questions. Table I summarizes the average pre-test and post-test scores from 21 physics graduate students who took the survey. The questions are sorted into those for which the students had to predict the output, and those for which the students had to predict the input. Students initially showed a limited ability to predict the output signal for the array of cases covered in the pre-test. Student performance on these questions before exposure to the tutorial is roughly 40% across all pre-test questions that asked the student to predict either the input or output. This average improved to over 80% across all corresponding post-test questions. Improved student performance on either of these two types of problems is likely to correlate with increased student ability to troubleshoot difficulties that may arise when making use of the lock-in.

VII. SUMMARY

We find that many physics graduate students who use lock-ins for their experimental research share common difficulties related to the basic operation of the instrument. The difficulty lies not with the (high-school level) mathematics involved, but rather with the manner in which they are introduced to the instrument's use, and whether students develop an integrated conceptual and quantitative understanding. The goal of this lock-in tutorial is to prepare students for both basic and more advanced (and trouble-free) usage of a lock-in

Table I. Summary of the average score, standard deviation, and number of instances of predicting input and output questions as well as the total average score in both the pre-test and post-test.

	Average pretest score (%)	Pretest standard deviation (%)	Number of instances in pretest	Average posttest score (%)	Posttest standard deviation (%)	Number of instances in posttest	p-value
Predicting output	38.1	42.6	135	87.7	27.7	151	<0.001
Predicting input	42.5	42.1	55	80.9	30.5	55	<0.001
Total score	39.4	42.5	190	85.9	28.7	206	<0.001

before they encounter these situations in real laboratory settings. The tool that we have developed and evaluated helps students of different prior backgrounds learn the basics of this instrument. We find that it also helps them make connections with the underlying mathematics that describes the operations of its major components. Finally, the tutorial develops the student's ability to predict output signals and input signals in a variety of cases likely to be encountered by students in laboratory setting.

Examination of the average student scores on the pre-tests and post-tests showed considerable improvement for all cases discussed in the tutorial. While there are many topics covered in this tutorial, it is straightforward to add additional topics (e.g., the treatment of broadband noise or the output response to sudden signal amplitude changes) by coupling a brief introduction with associated lock-in puzzles.

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[22]See supplementary material at http://dx.doi.org/10.1119/1.4934957 for the tutorial, simulation, pre- and post-test, and some additional reference materials.
[23]The most up-to-date materials are available on ComPADRE at <http://www.compadre.org/portal/items/detail.cfm?ID=13360>.

Small Lecture Table Galvanometer

This galvanometer, made by James W. Queen of Philadelphia at the end of the 19th century, did not have to be very large, for classes for the "new" subject of physics were small. Earlier, Natural Philosophy was placed in the junior year of college in the relatively fixed college curriculum that persisted through most of the century. It is of the standard D'Arsonval type and was probably as much designed to show the working mechanism of the galvanometer as a working instrument in the lecture hall and classroom. It is in the Greenslade Collection. (Notes and picture by Thomas B. Greenslade, Jr., Kenyon College)