Lock-In Amplifiers

Principles and Operation

- or -

A Field Survival Guide

Matt Kowitt, Stanford Research Systems

Before starting: Q- what is a lock-in amplifier?

first answer:

A lock-in amplifier is an electronic laboratory instrument used for making sensitive, narrow-band AC voltage (or current) measurements, especially in the presence of significant noise.

... we will come back to this question

Outline of this talk

1. History and overview of lock-in amplifiers
2. Basic terms and settings
3. Time constants and output filters
4. Phase
5. Lock-in amplifiers in the frequency domain
6. Common mistakes
7. Advanced topics

Early papers ...

SEPTEMBER, 1941 R. S. I. VOLUME 12

A Pentode Lock-In Amplifier of High Frequency Selectivity

WALTER C. MICHELS AND NORMA L. CURTIS*
Bryn Mawr College, Bryn Mawr, Pennsylvania

(Received May 16, 1941)

An amplifier which is based on the scheme of heterodyning the signal to be measured with a locking-in voltage of the same frequency, as proposed originally by Cosens, is described and its characteristics are discussed. The particular design used, with the locking-in voltage applied to the screen grids of two pentodes, possesses a distinct advantage over earlier instruments because the tubes operate on a linear portion of their transfer characteristics, reducing harmonic response, and also because the main power supply is direct current, so that there is little drain on the control supply. It is found that the response band can be given a width of the order of one cycle per second without undue inconvenience or expense.

om P. Forman, in
nstruments of Science: an
istorical Encyclopedia (1998)

RADIATION LABORATORY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
CAMBRIDGE MASSACHUSETTS

REPORT
787

R. H. Dicke
July 16, 1945

THE MEASUREMENT OF THERMAL RADIATION AT MICROWAVE FREQUENCIES

PER AUTHORITY OF "SECDEF MEMO OF 2 AUGUST 1950" THIS DOCUMENT IS
UNCLASSIFIED

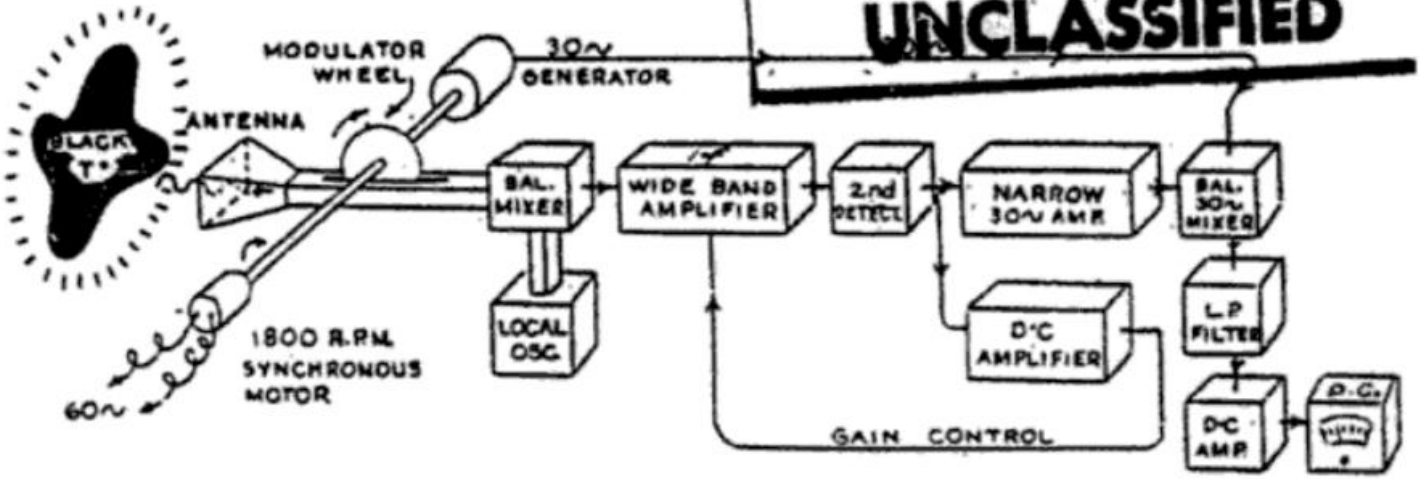


Fig. 3 - Microwave Radiometer

This figure also appears in
Dicke (1946), RSI 17, 268-275

In Fig. 4 there is plotted the average power against frequency for the output of the second detector of Fig. 3.

Research on students' understanding of Lock-Ins

from DeVore, Seth, et al. (2016) Am. J. Physics 84(1), 52-56

Improving student understanding of lock-in amplifiers

Seth DeVore, Alexandre Gauthier, Jeremy Levy, and Chandralekha Singh
Department of Physics and Astronomy, University of Pittsburgh, Pittsburgh, Pennsylvania 15260

(Received 9 November 2014; accepted 16 October 2015)

The lock-in amplifier is a versatile instrument frequently used in physics research. We find that students tend to struggle with its basic operating principles, leading to a variety of difficulties. To improve their understanding, we have developed a research-based tutorial that makes use of a computer simulated lock-in amplifier. The tutorial allows students to realize their conceptual difficulties by comparing their predictions with the outcome of the simulations, hopefully leading them to a better understanding of the process of lock-in amplification. © 2016 American Association of Physics Teachers.
[http://dx.doi.org/10.1119/1.4934957]

I. INTRODUCTION

The lock-in amplifier ("lock-in") is an instrument used extensively in laboratory research, especially in condensed matter physics.[1–6] In general, beginning researchers receive little to no formal training on the use of a lock-in. Often, their initial exposure is somewhat superficial (yet successful), leading them to believe that they have mastery over its function and use. According to professors interviewed during this investigation, improper or inefficient use of the lock-in, and misinterpretation of data obtained from such use, is anecdotally quite common. Additionally, this lack of understanding can result in a student's inability to troubleshoot and modify the experimental setup when faced with anomalous results.

Computer and web-based tools are becoming increasingly commonplace to aid in learning across many science and engineering fields.[7–18] For such tools to be maximally effec- the pretest and posttest scores to gauge the effectiveness of the tutorial.

II. THE IDEAL LOCK-IN AMPLIFIER

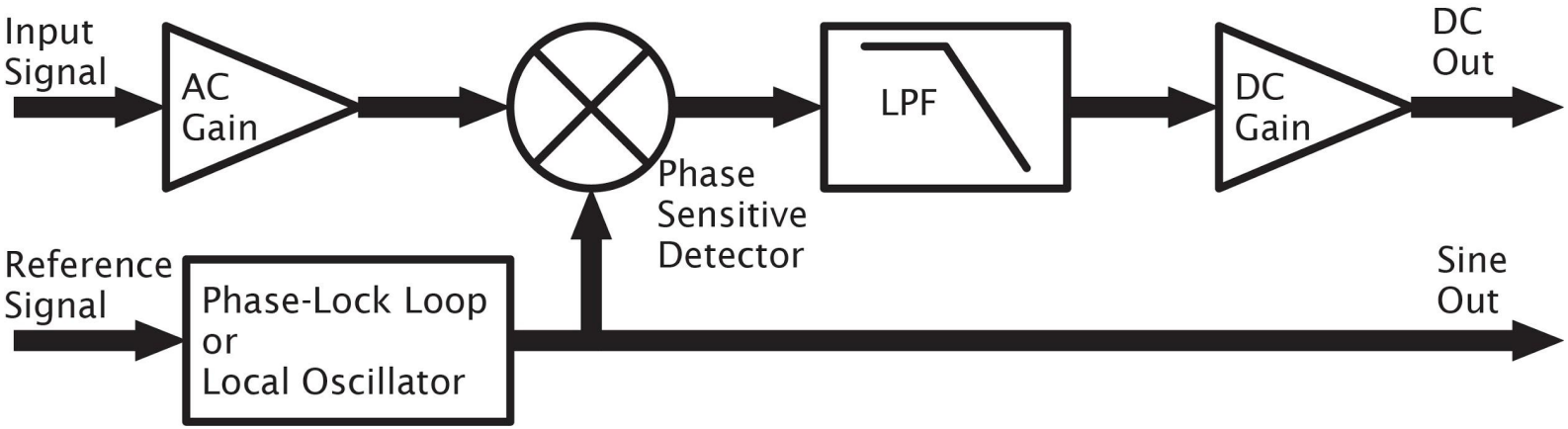
Throughout this paper (as in the tutorial), we work with an idealized version of a lock-in amplifier. We assume that the signal of interest is centered on a frequency f_S that is present in the input signal. If the amplitude can change, then the signal will not be a pure frequency since amplitude modulation leads to sidebands that surround the central frequency. In the case where there is no amplitude modulation we will treat the signal as pure. Tutorial questions involve both single-frequency sources as well as amplitude-modulated single-frequency sources. To separate the signal of interest from unwanted noise, a reference signal is defined. This reference has unit amplitude, and is dimensionless (for convenience).

Most common difficulties:

- Relationship between time constant and output BW
- Understanding the function of the lock-in mixer
- Heterodyne case: $f_S \neq f_R$
- Phase shifted signals
- Case when $2f$ not strongly attenuated
- Amplitude modulation
- Multiple frequencies present

What is a lock-in amplifier?

What is a lock-in amplifier?



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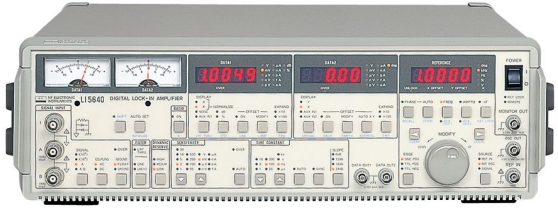

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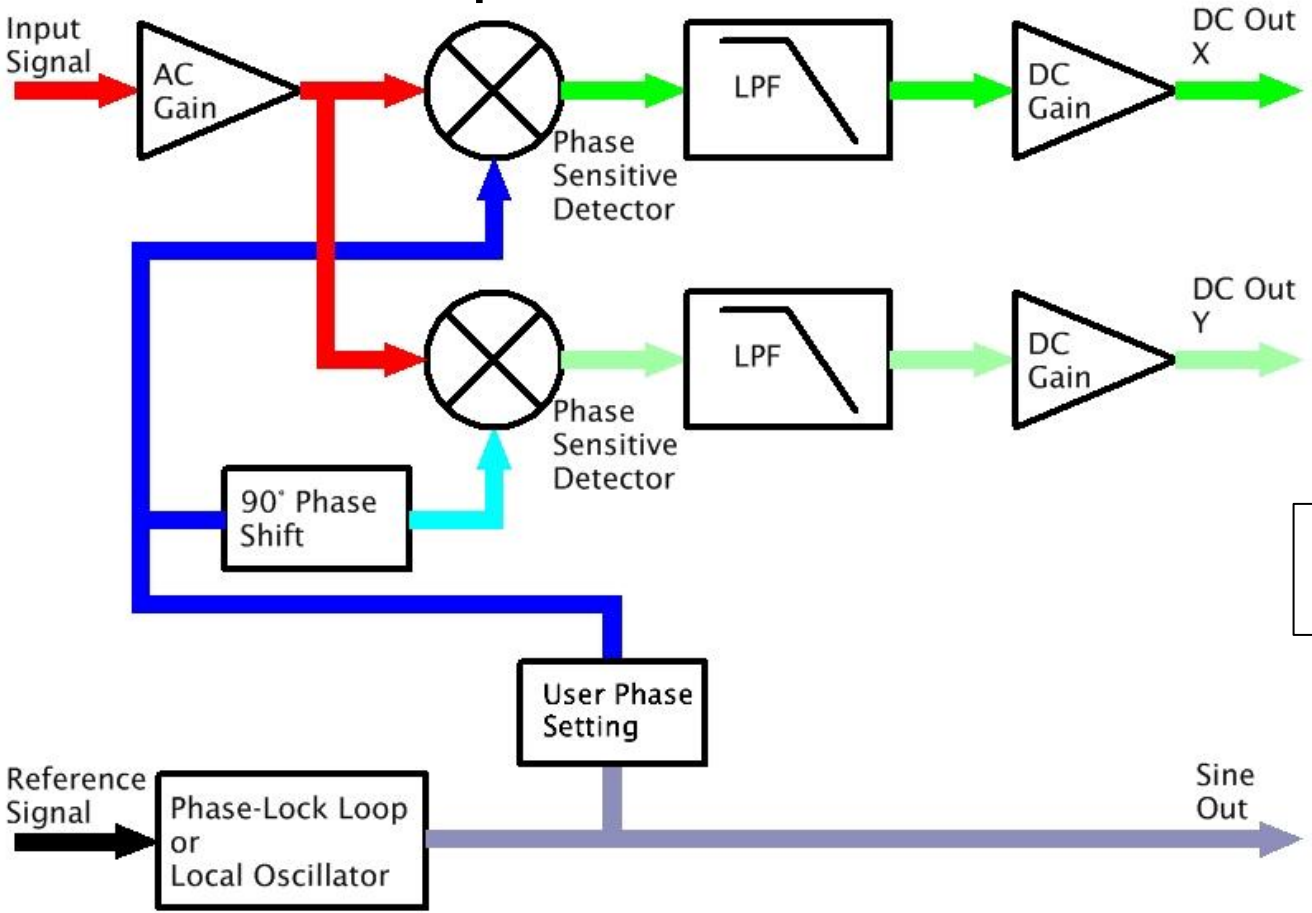

$R = \sqrt{(X^2+Y^2)}$

$\theta = \tan^{-1}(Y/X)$

Trigonometry

$$\sin(a)\sin(b) = [\cos(a-b) - \cos(a+b)]/2$$

$$\sin(a)\cos(b) = [\sin(a-b) + \sin(a+b)]/2$$

input signal $s(t)=\sqrt{2}\,V_{sig}\sin(2\pi f_{sig}t + \theta_{sig})$ where V_{sig} is the *RMS* signal

ref. oscillators $r_X(t)=\sin(2\pi f_{ref}t+\theta_{ref})$; $r_Y(t)=\sin(2\pi f_{ref}t+\theta_{ref}+\pi/2) = \cos(2\pi f_{ref}t+\theta_{ref})$

PSD outputs: $V_{psd\text{-}X} = s(t)\,r_X(t)$; $V_{psd\text{-}Y} = s(t)\,r_Y(t)$

$$V_{psd\text{-}X} = (V_{sig}/\sqrt{2})\left\{\cos(2\pi[f_{sig}-f_{ref}]t + \theta_{sig} - \theta_{ref}) - \cos(2\pi[f_{sig}+f_{ref}]t + \theta_{sig} + \theta_{ref})\right\}$$

$$V_{psd\text{-}Y} = (V_{sig}/\sqrt{2})\left\{\sin(2\pi[f_{sig}-f_{ref}]t + \theta_{sig} - \theta_{ref}) + \sin(2\pi[f_{sig}+f_{ref}]t + \theta_{sig} + \theta_{ref})\right\}$$

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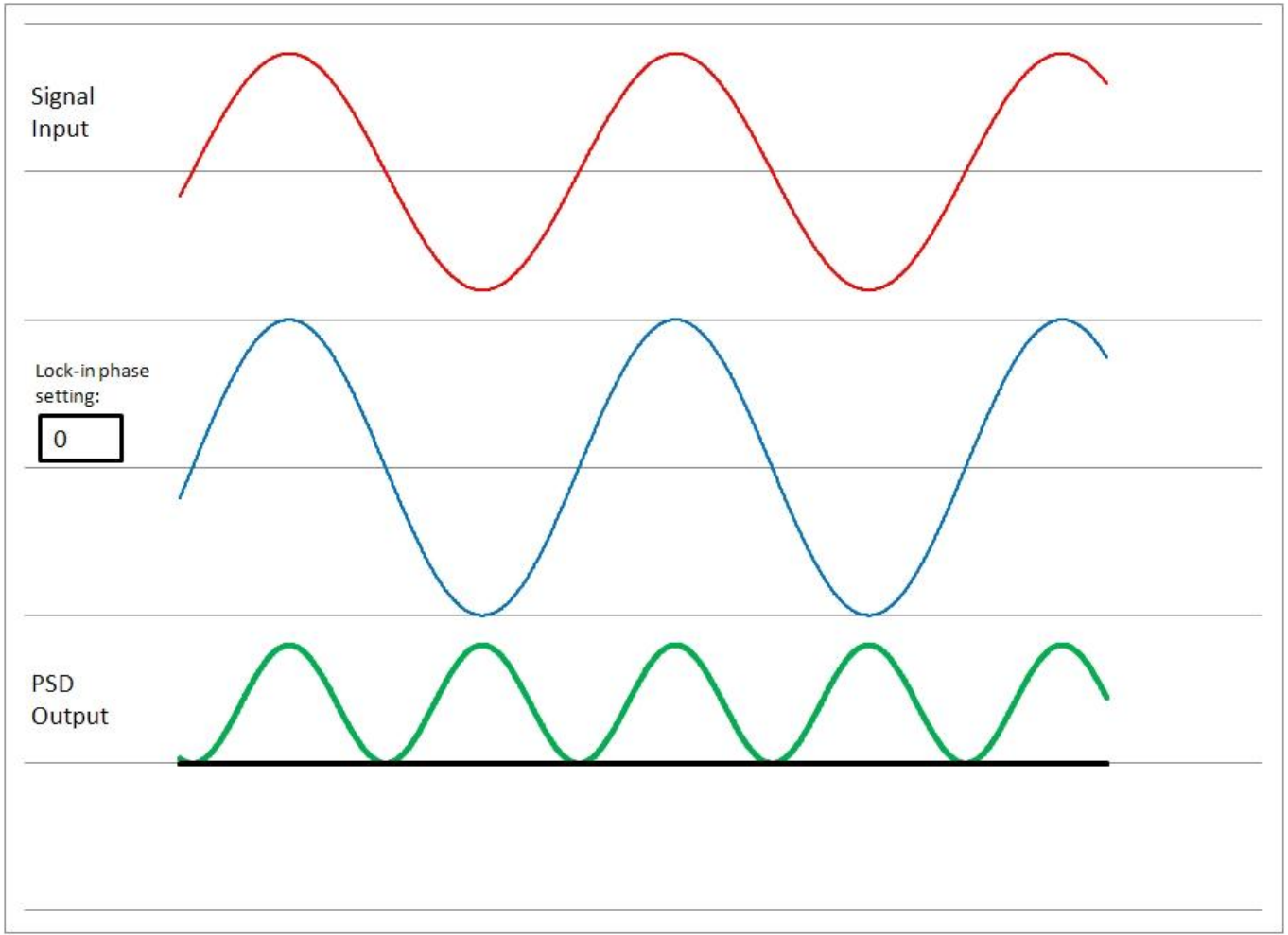
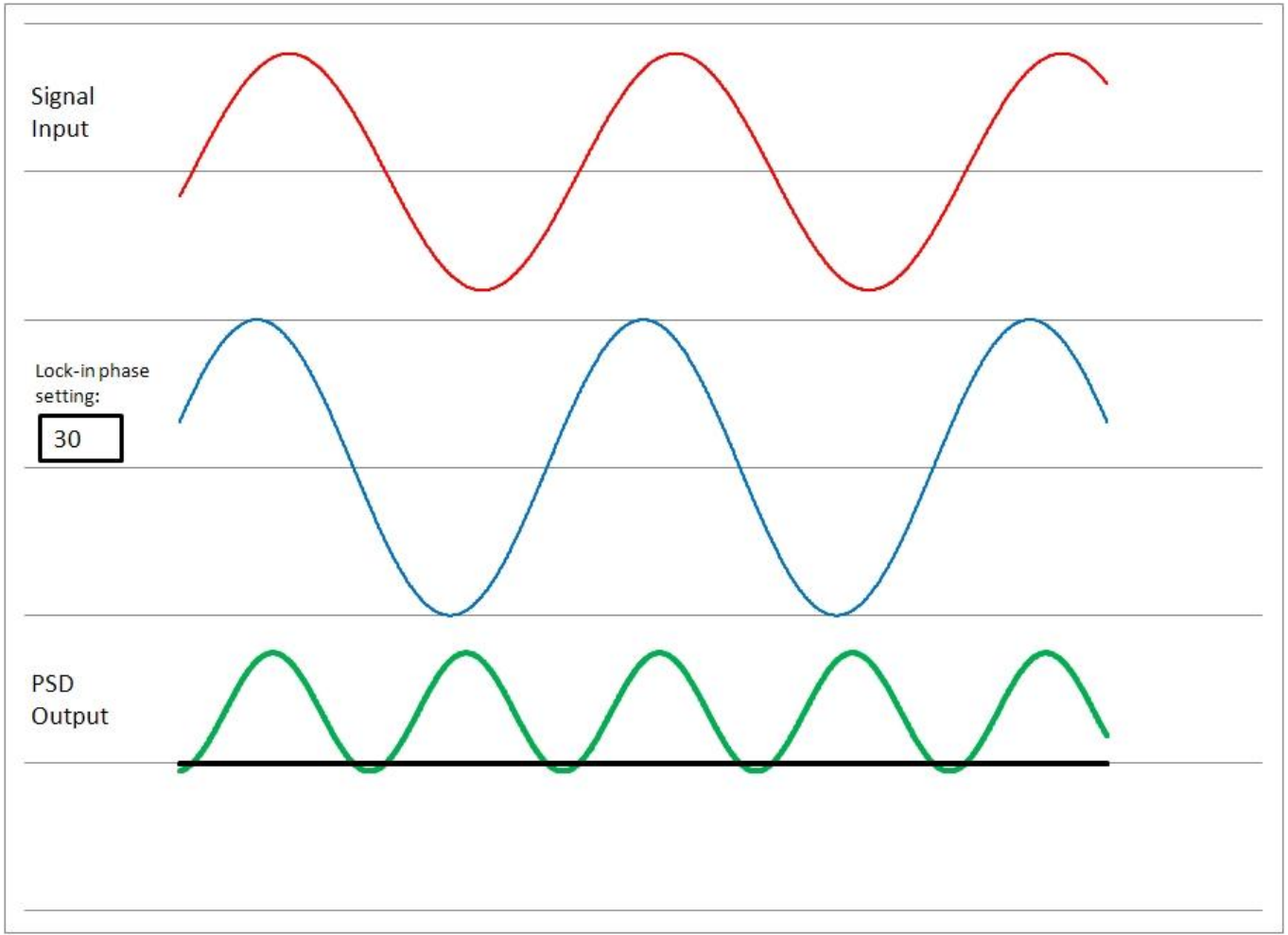
The "DC" term

The "2f" term

PSD Output



PSD Output



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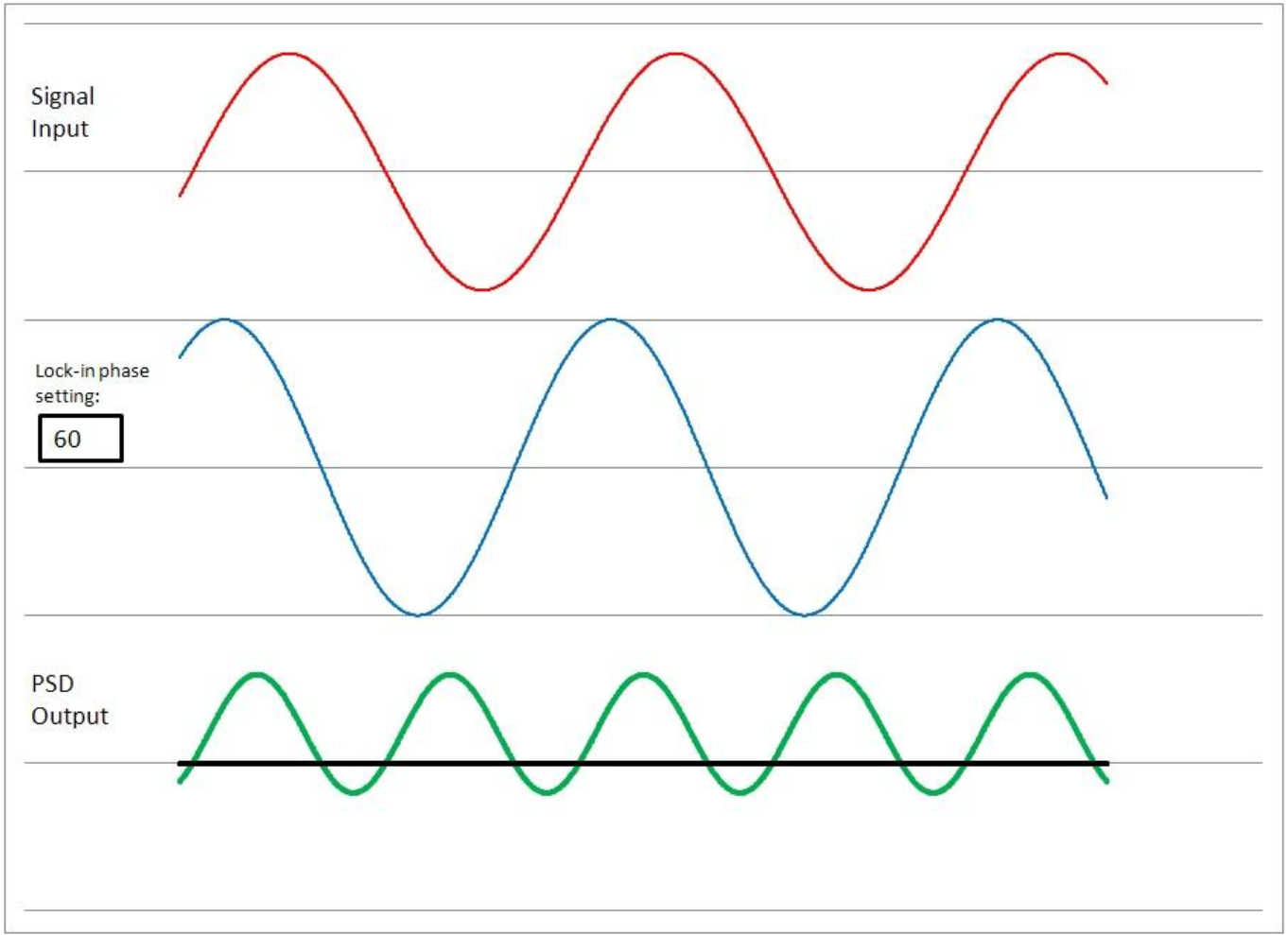

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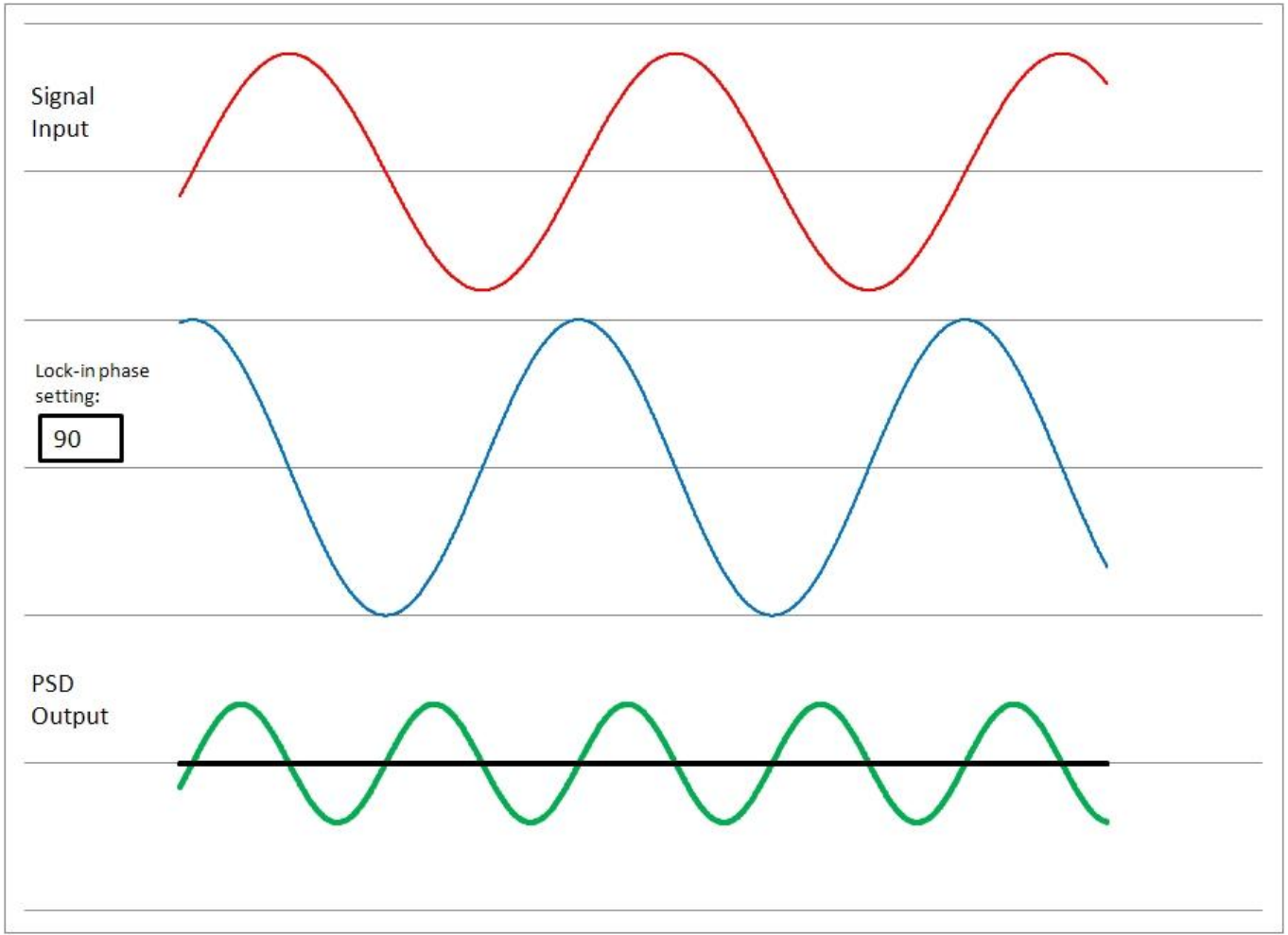

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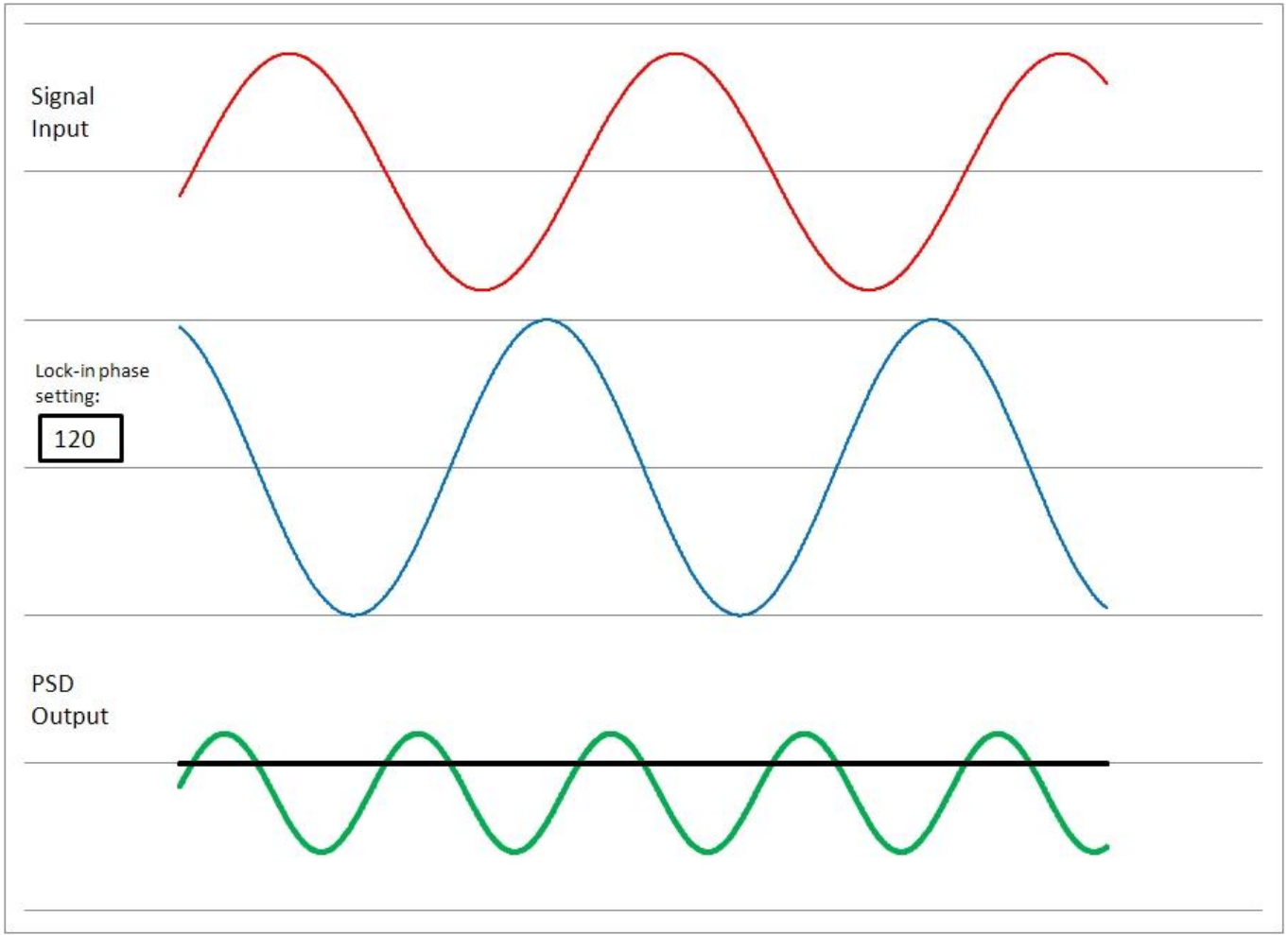

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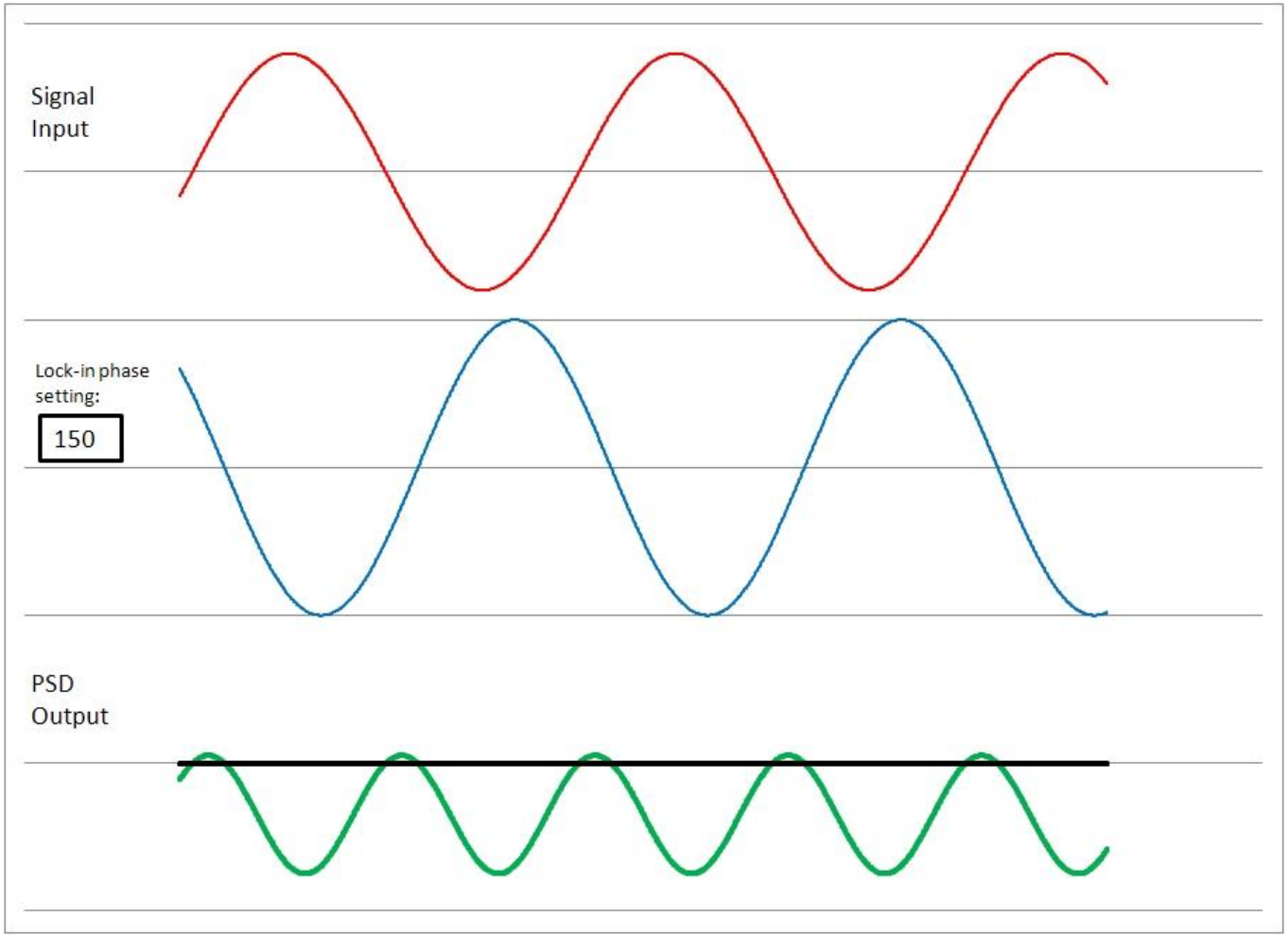

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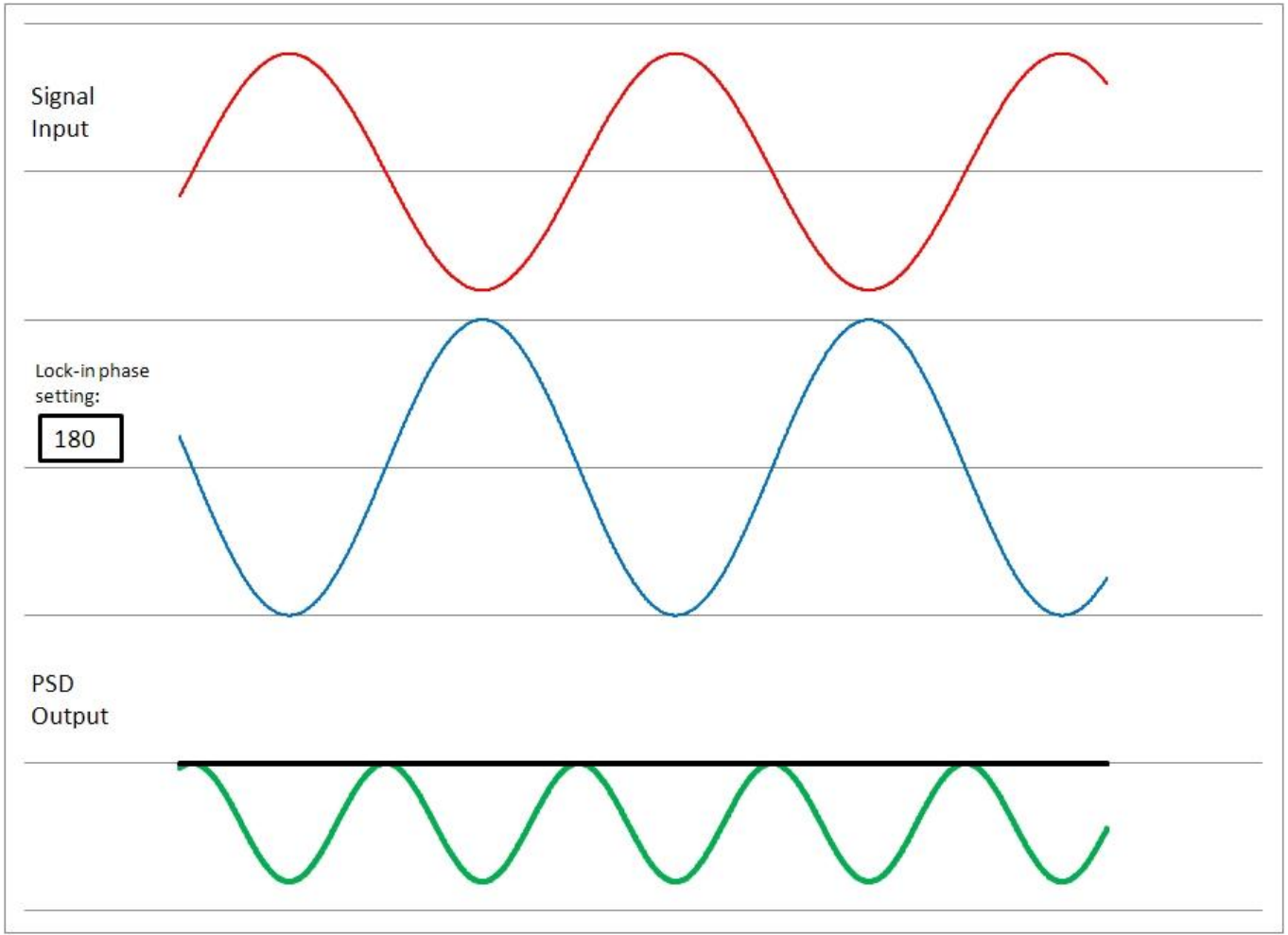

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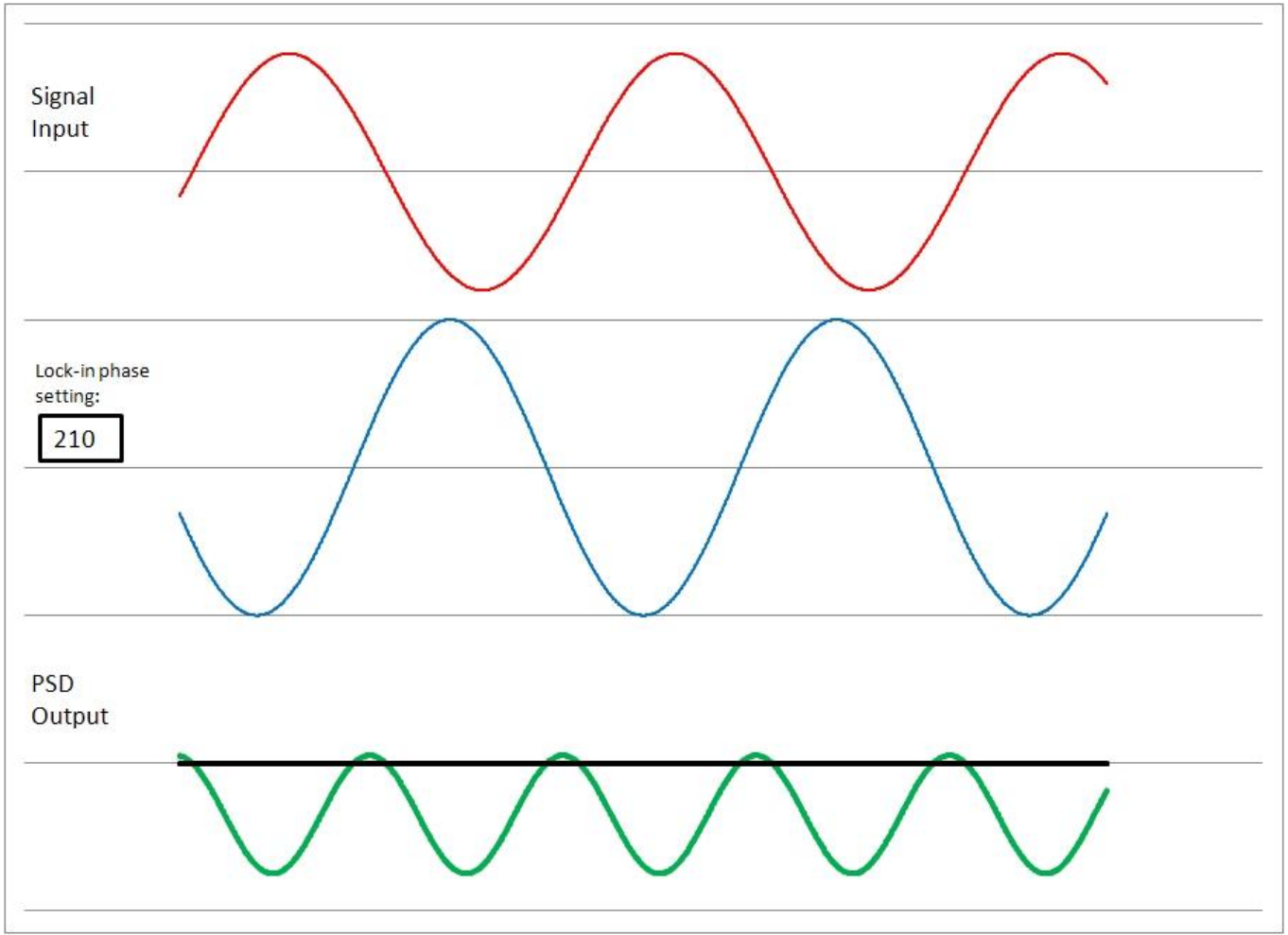

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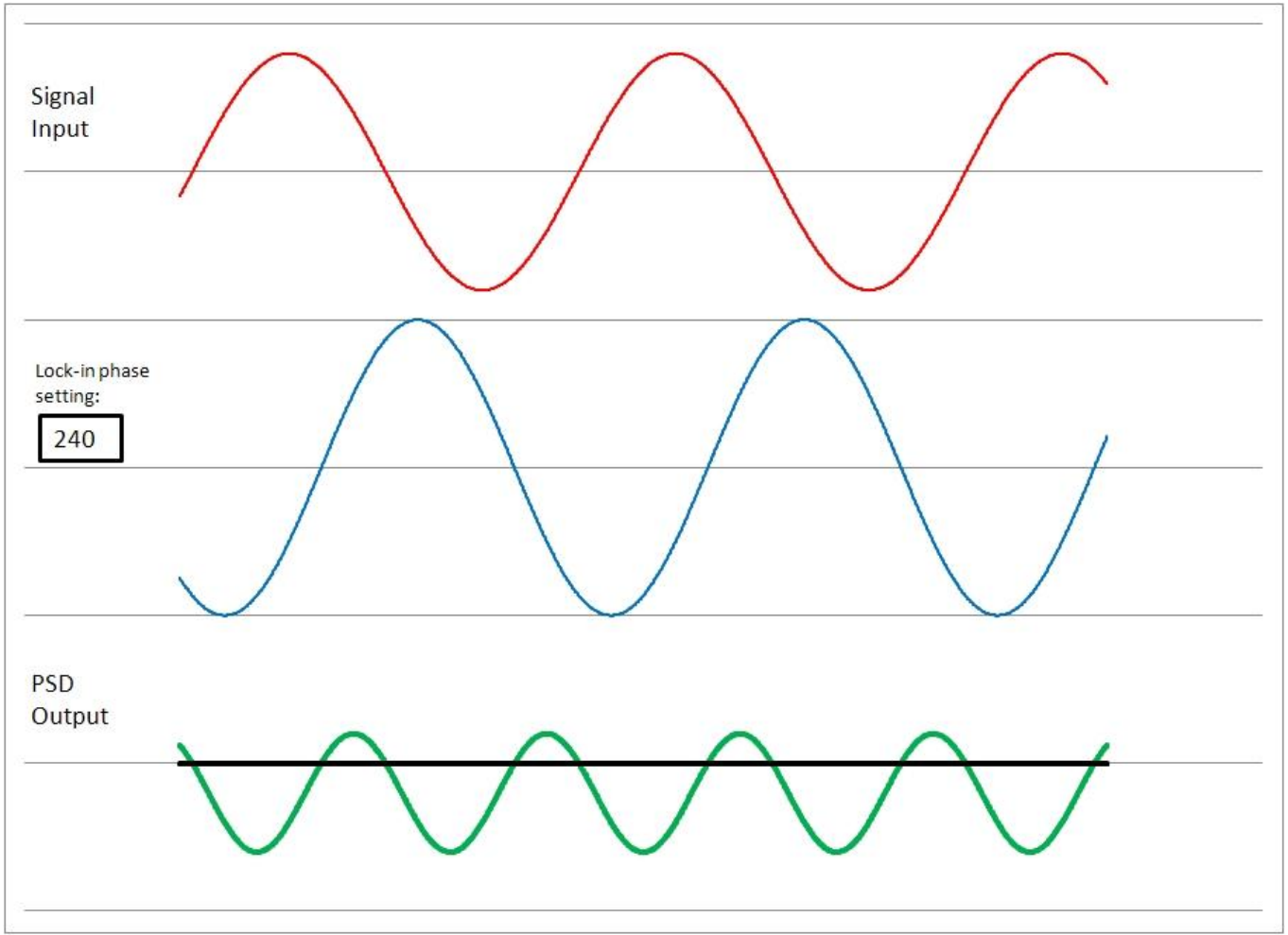

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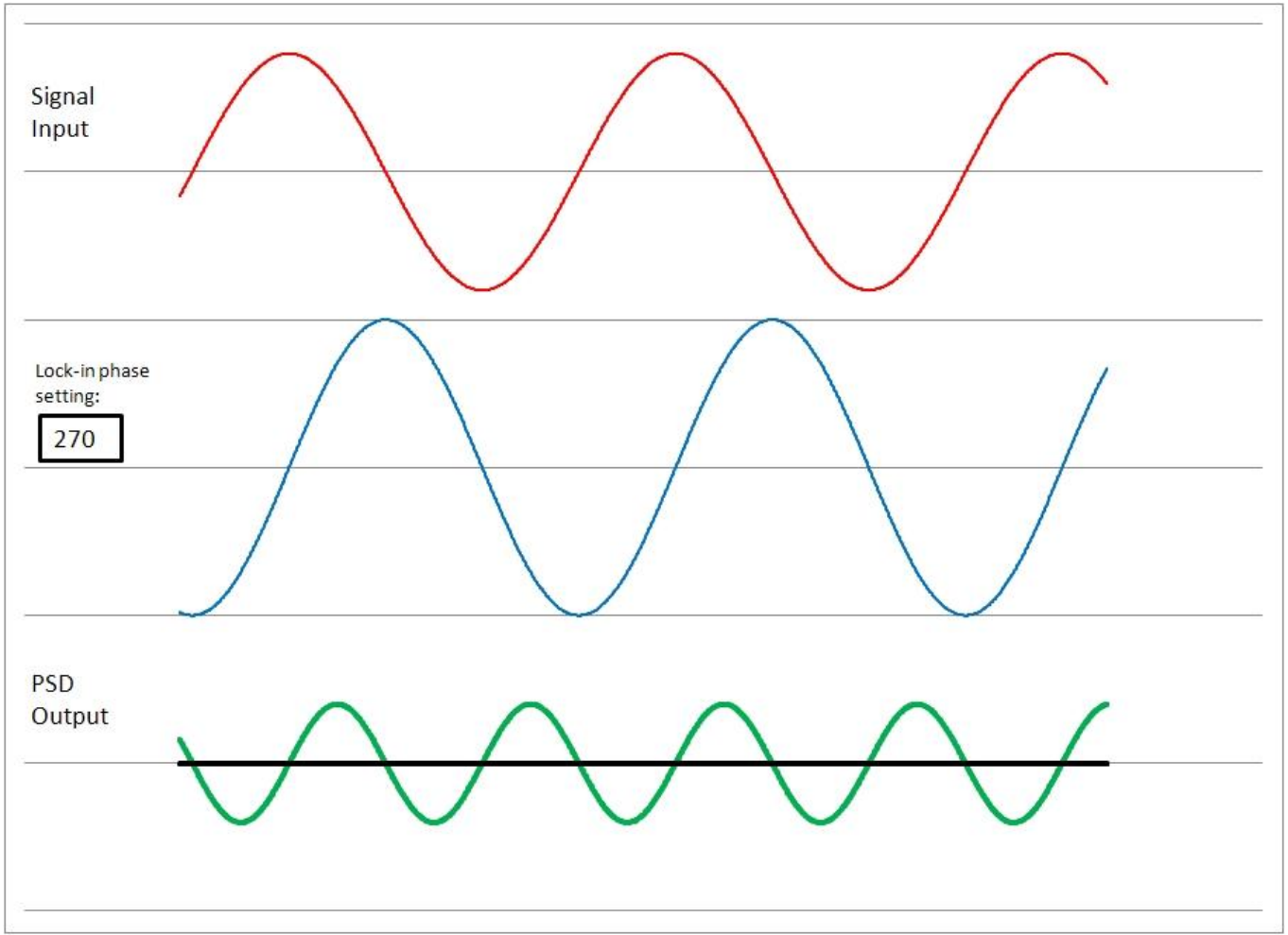

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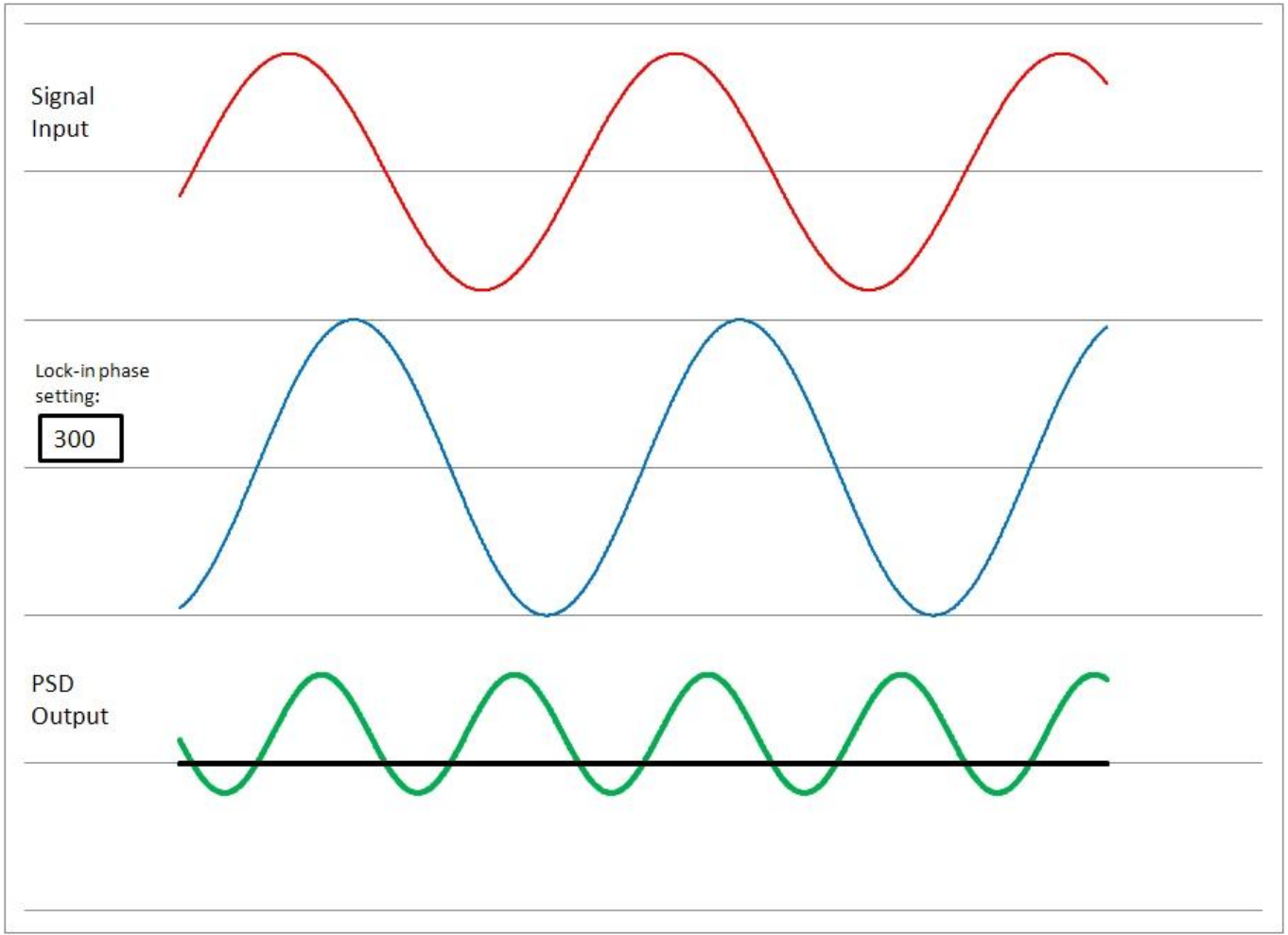

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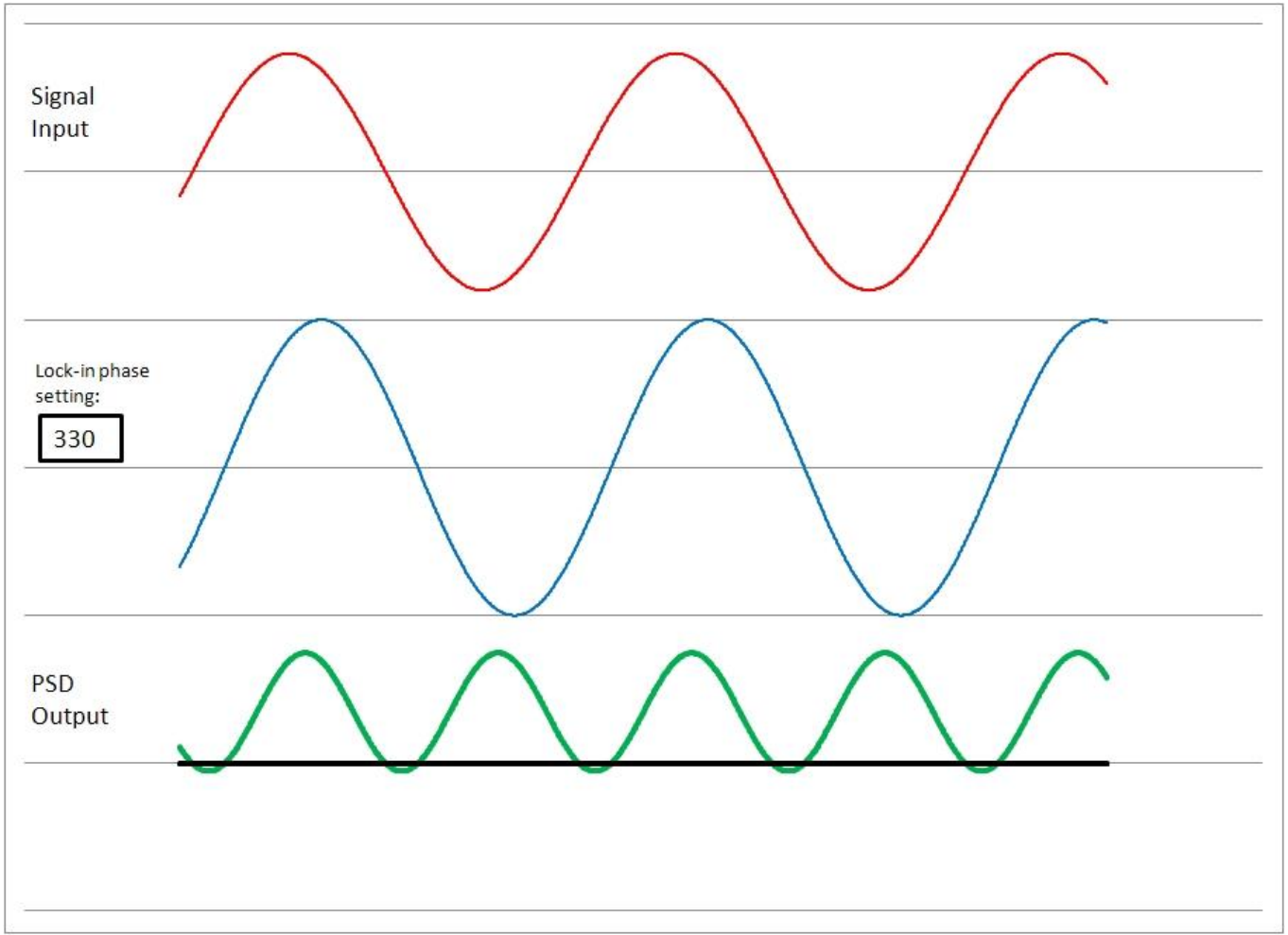

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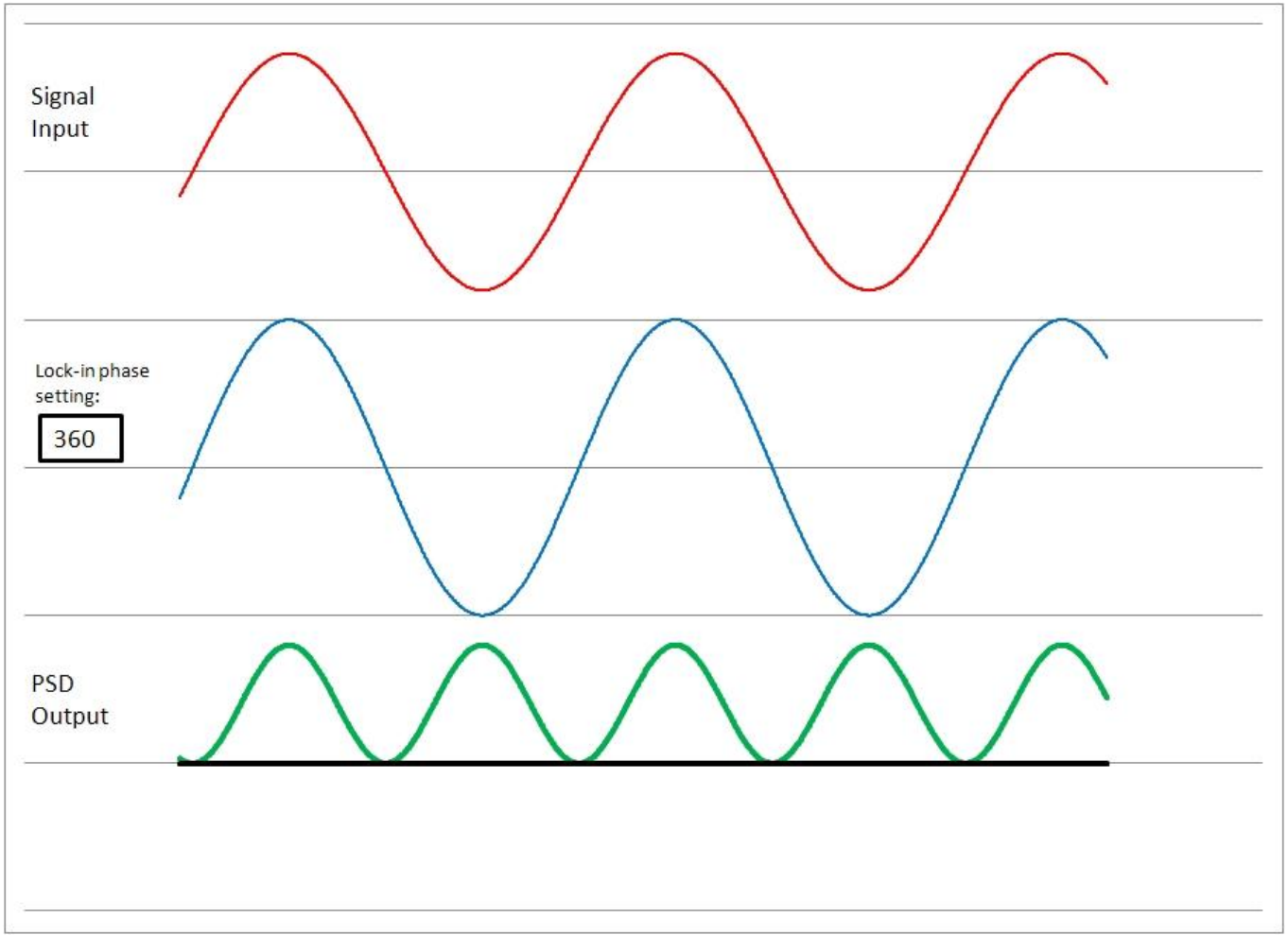


Trigonometry

Typically, we have $f_{sig} = f_{ref}$, and we filter out the 2 f term. The measured phase, θ, is the difference $\boldsymbol{\theta = \theta_{sig} - \theta_{ref}}$

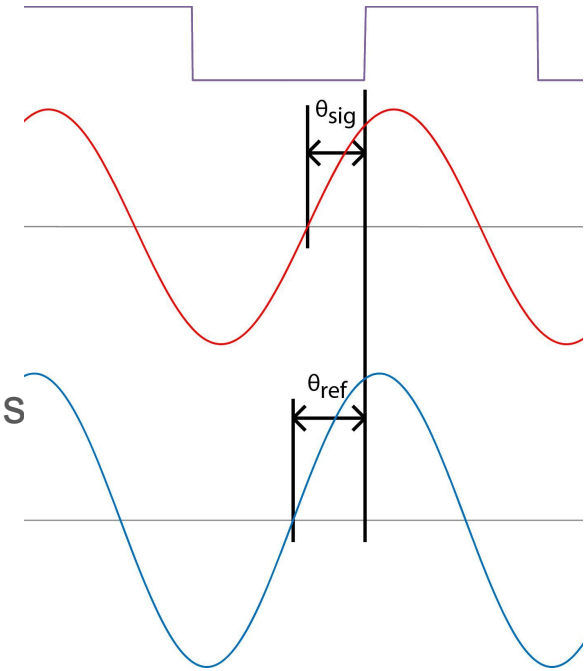
Then we have the relationships

$$\mathbf{V_{psd\text{-}X} = (V_{sig}/\sqrt{2})\cos(\theta) \quad ; V_{psd\text{-}Y} = (V_{sig}/\sqrt{2})\sin(\theta)}$$

The lock-in displays the input signal in Volts RMS, referenced to input. This means the lock-in **R** corresponds to $\mathbf{V_{sig}}$. We finally have:

$$\mathbf{X = R\cos(\theta) \quad ; \quad Y = R\sin(\theta)}$$

$$\mathbf{R = \sqrt{(X^2 + Y^2)} \quad ; \quad \theta = \tan^{-1}(Y / X)}$$



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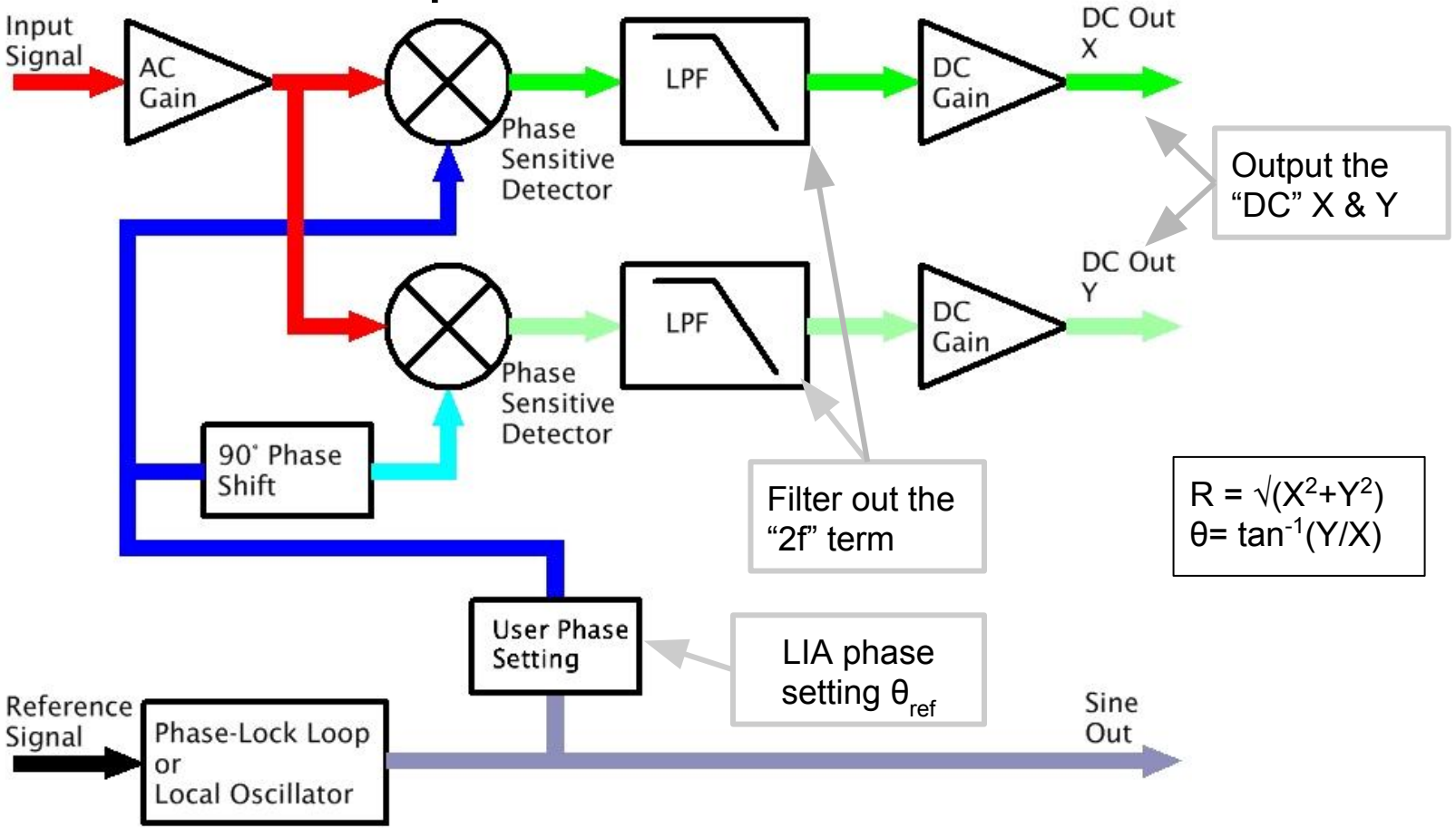


Why use a lock-in?

- Measurements at frequencies less affected by low-frequency noise sources
- Challenging S:N environments
- Phase-sensitive measurements

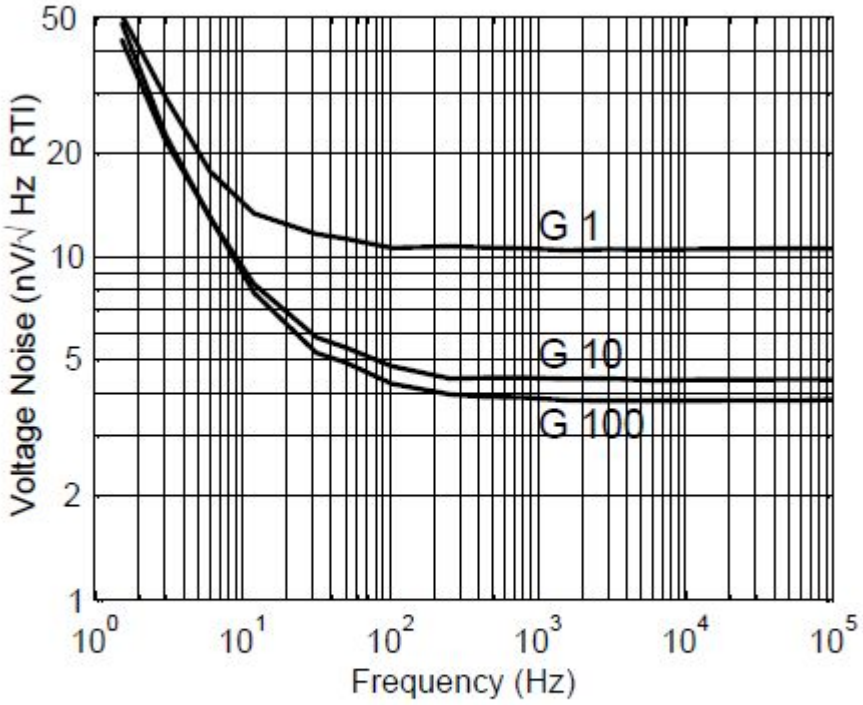
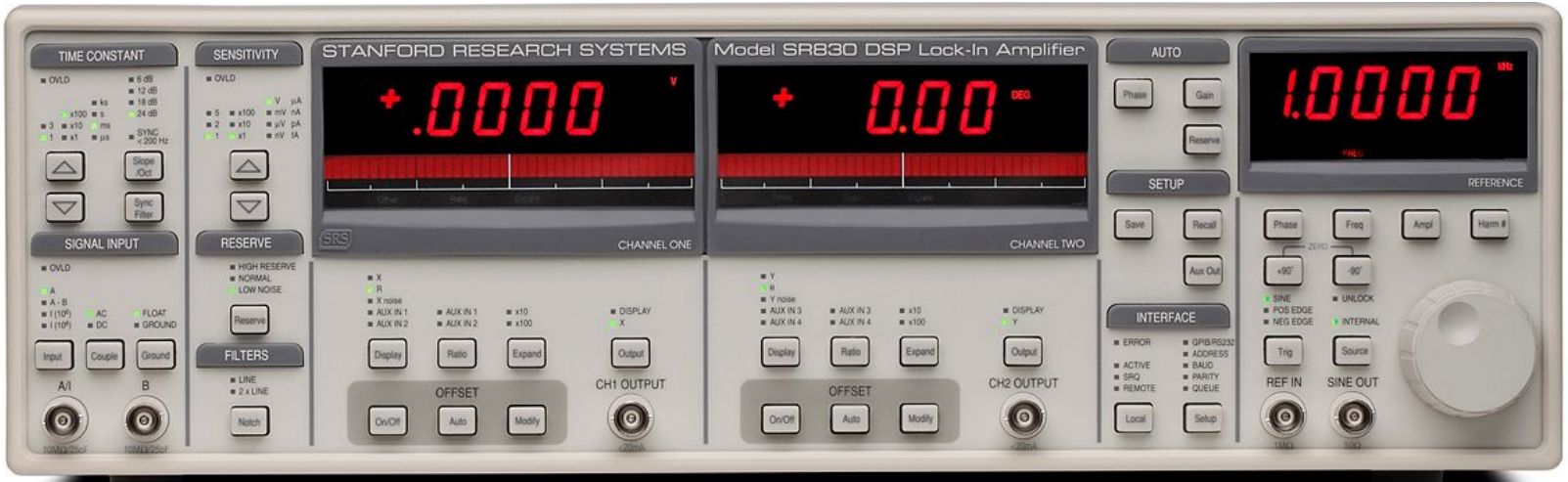


Figure 3.1: The SIM910 noise density versus frequency.

Standard settings and terms

- Sensitivity
- Time Constant
- Dynamic Reserve
- Offset
- Expand



Standard settings and terms

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Sensitivity ∝ (1/gain)
= input that gives FS output

Example: Sensitivity = 500 nV, (Lock-In FS = 10 V)
Input signal = 200 nV*rms*

then Output = (200nV / 500nV)*10V = 4 V*dc*

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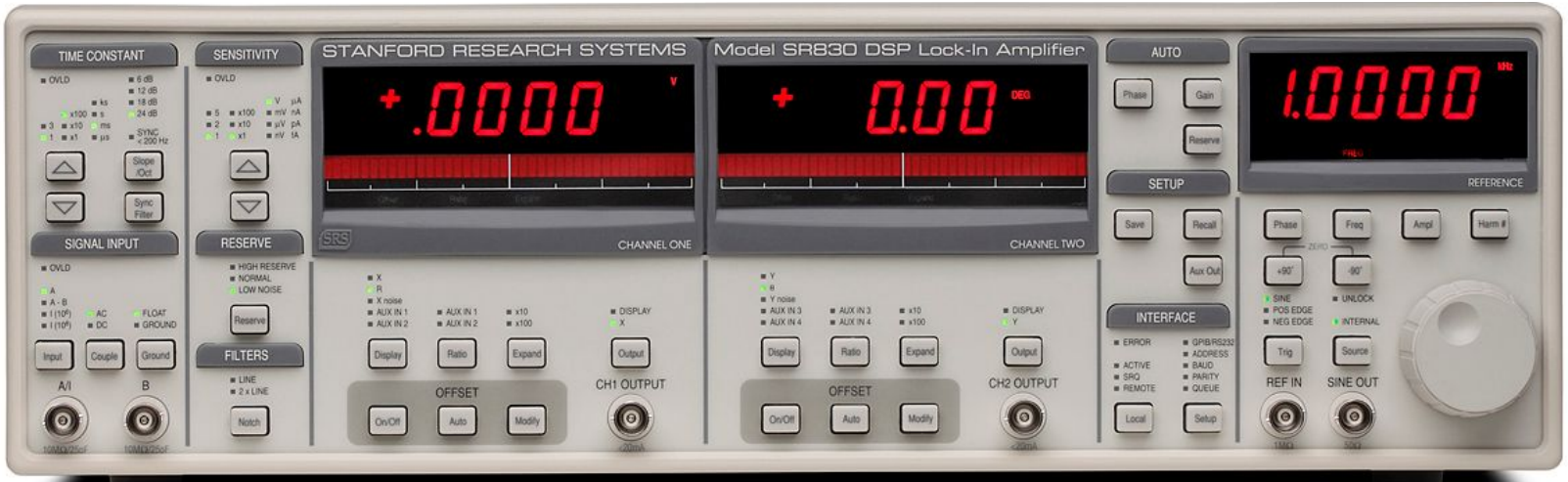

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Time Constant for Low Pass filter

Classic filter is simple “RC”, cascade of 1 to 4 poles

Frequency response $|H(f)| = 1 / [\sqrt{(1+(2\pi f \times TC)^2)}]^n$

WHERE “f ” is frequency in the PSD *OUTPUT*

Determines: Output bandwidth

Settling time

Possible “2f” ripple

Equivalent Noise Bandwidth (ENBW)

Standard settings and terms

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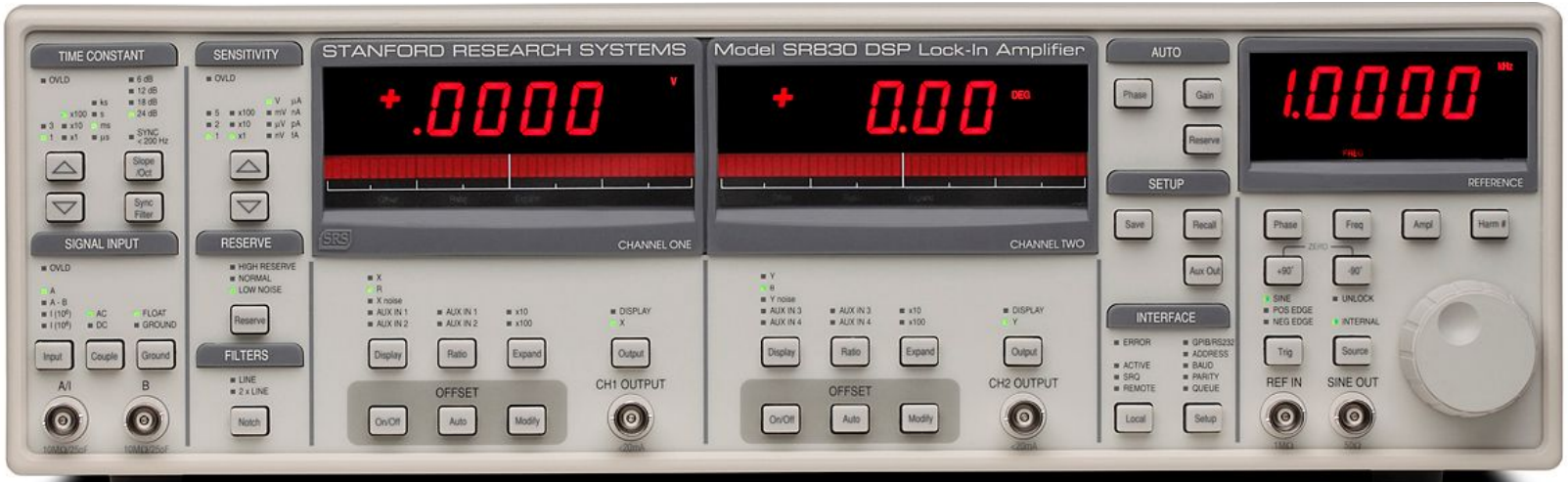

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Sensitivity ⇒ *overall* gain for lock-in, but
Dynamic Reserve ⇒ *allocation* between AC and DC gain.

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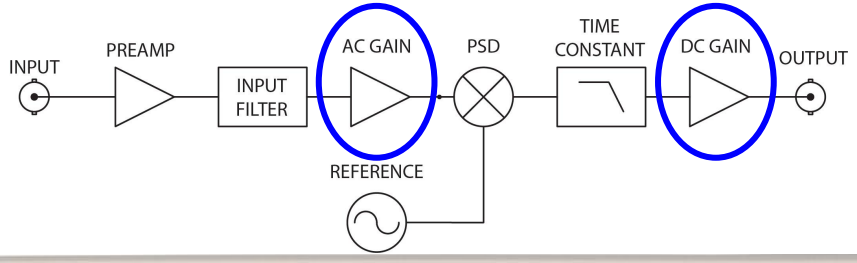
(nominal for DSP lock-in; actual gain for analog lock-in)



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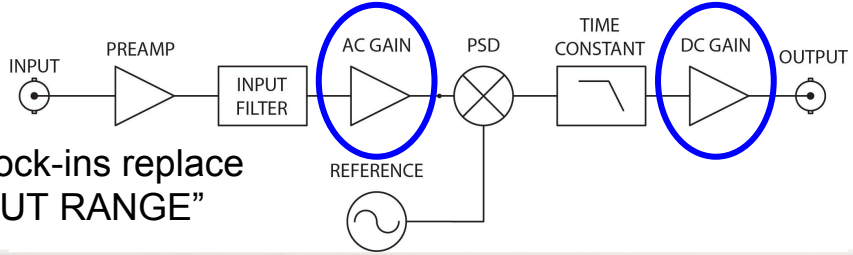
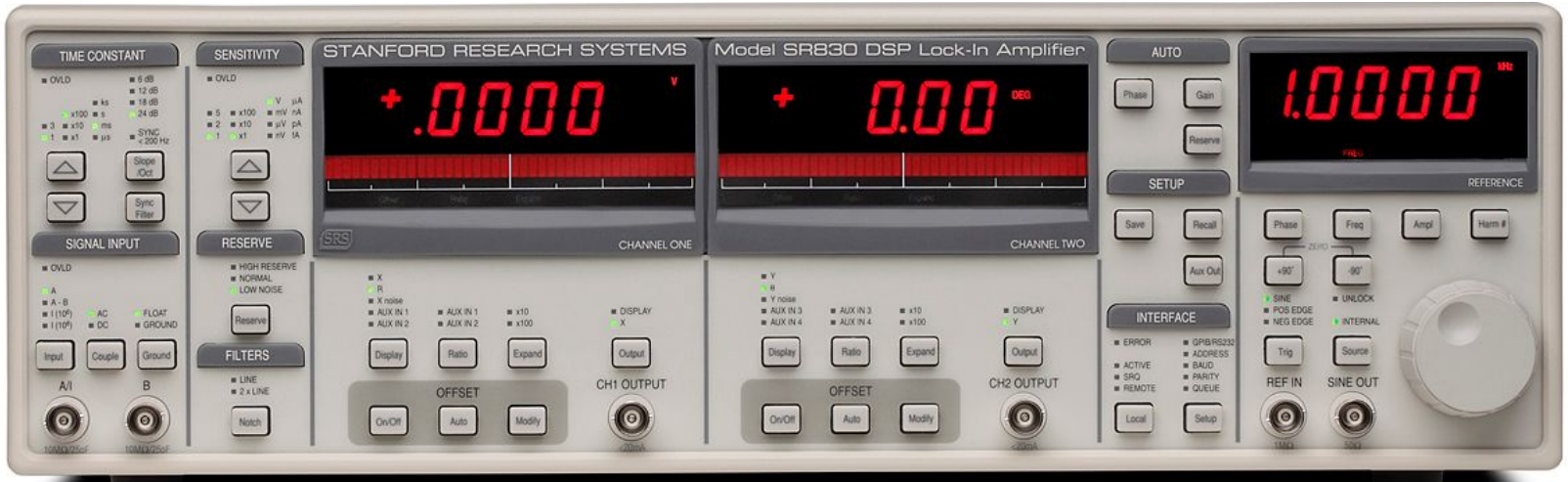


Newer DSP lock-ins replace this with “INPUT RANGE”

Standard settings and terms

- Sensitivity
- Time Constant
- Dynamic Reserve
- Offset
- Expand



Standard settings and terms

- Sensitivity
- Time Constant
- Dynamic Reserve
- Offset
- Expand

Offset operates on X, Y, and / or R

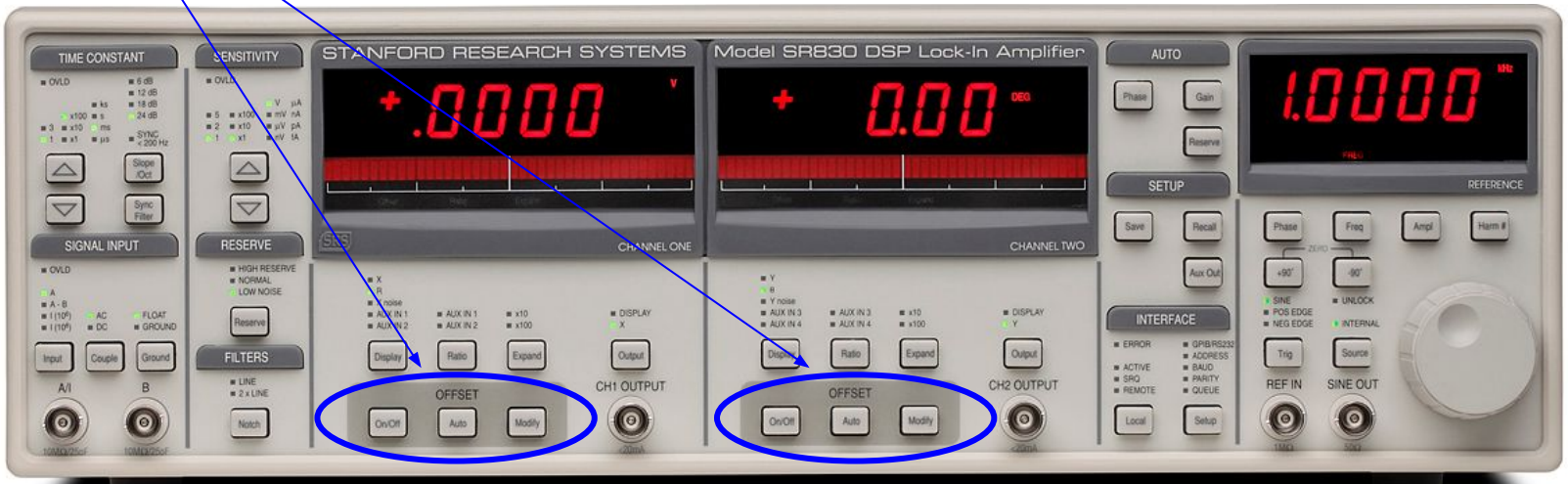
Expressed as % (of full scale)

"Auto" offset nulls the output *at the moment pressed*

Offset is an output modifier

Offsets of X or Y do not affect R or θ

Commonly used together with Expand ...



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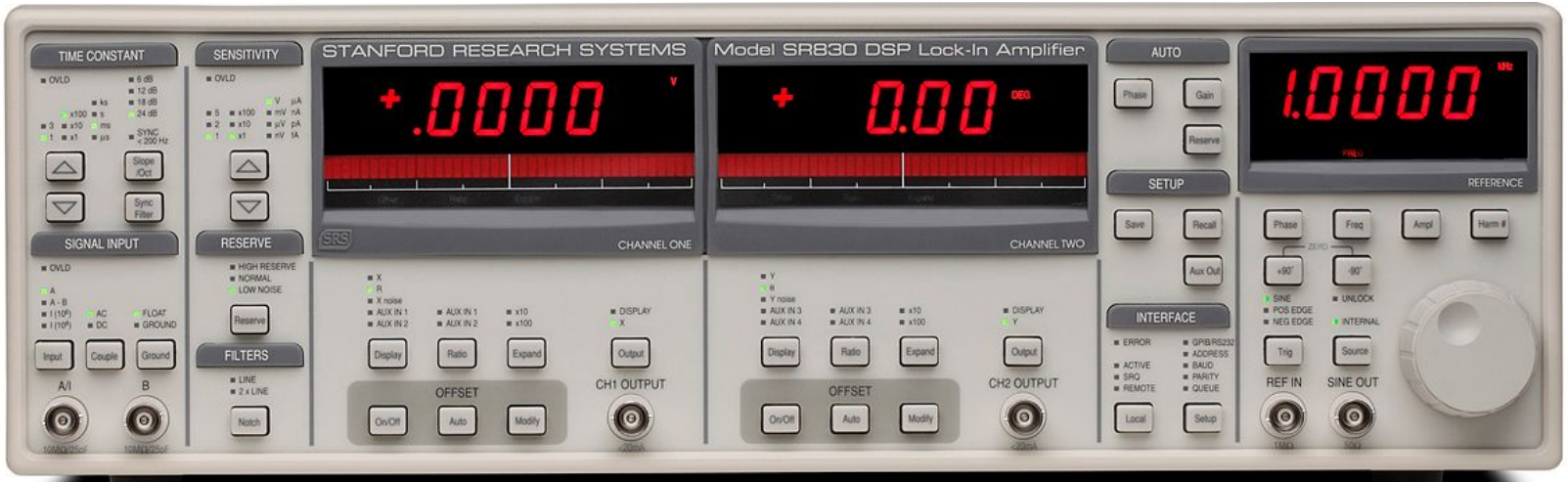

Standard settings and terms

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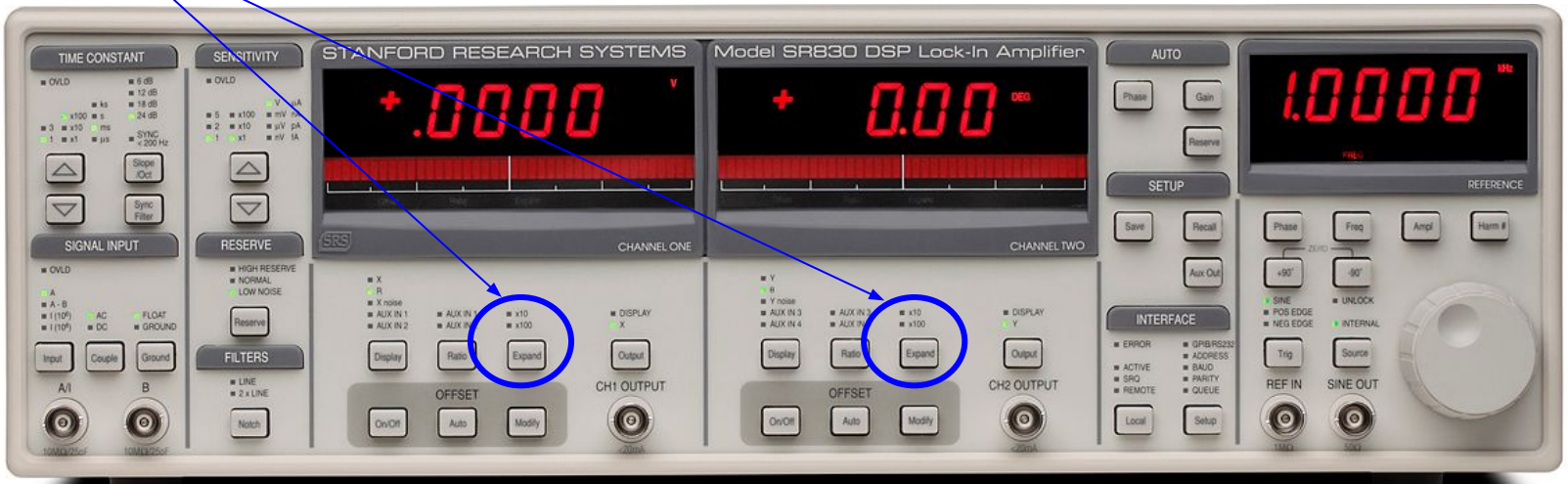
Expand modifies the output scale, adding extra gain
factor of ×10 or ×100
comes *after* Offset

Output = (Signal / Sensitivity - Offset) × Expand × 10 V

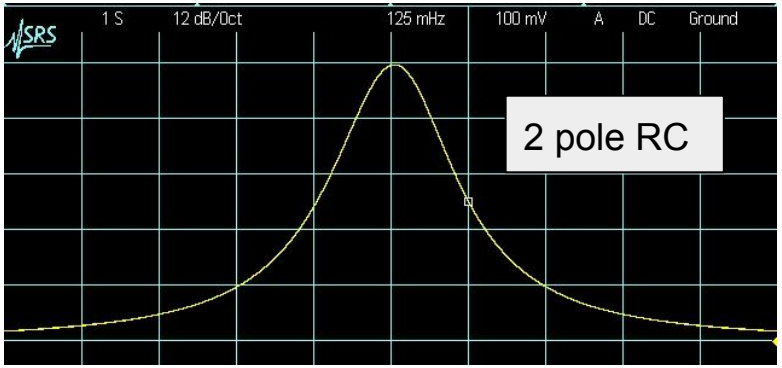


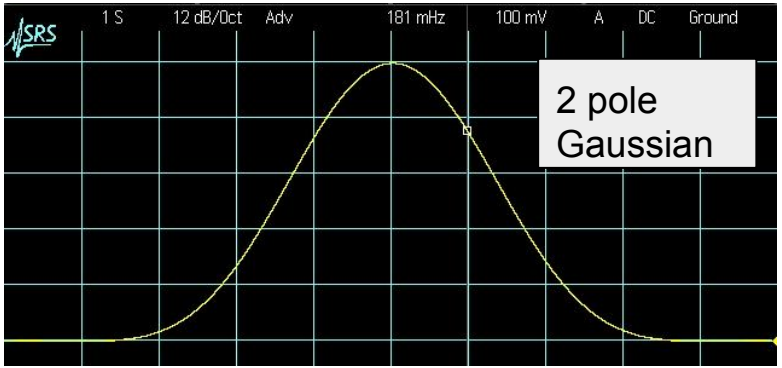
Reference section

- Select Internal or External
- Understand the scope of each setting:
  - Freq: Internal mode
  - Amp: Sine Out only
  - Phase: PSD only
  - Harmonic: PSD only(*)
  - DC level: Sine Out only
- External: Ref In
  - Sine: AC coupled
  - TTL (rising or falling)
  - Watch for Unlock

*: harmonic affects sine out on analog lock-ins

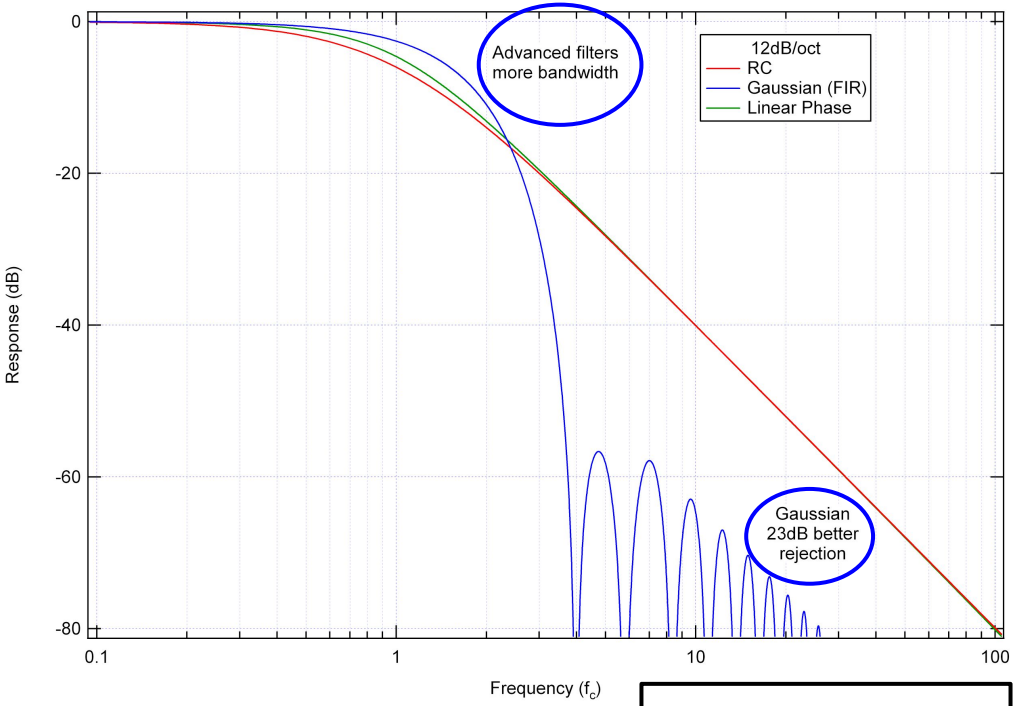
Time constants in the frequency domain





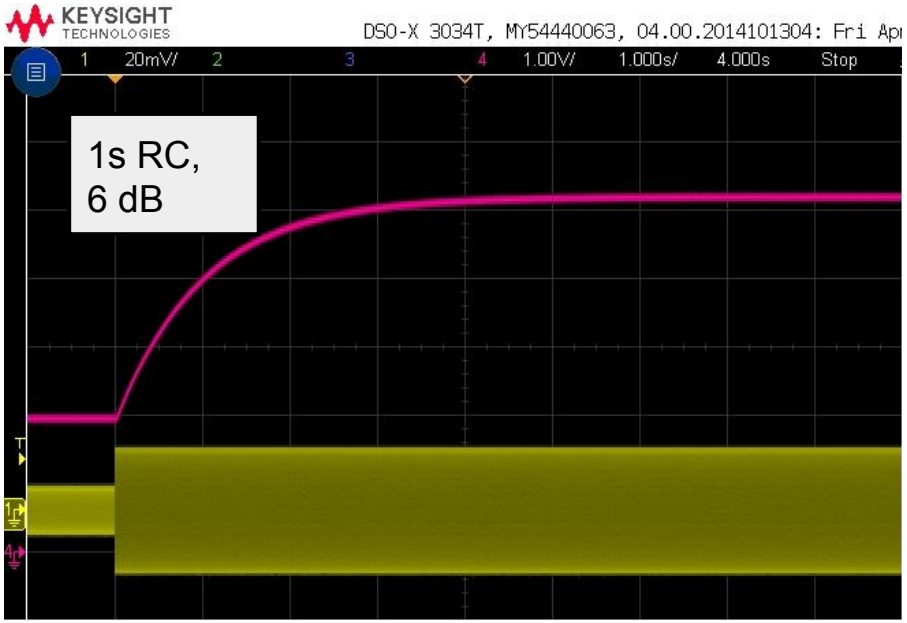
–.80 –.48 –.16 0 +.16 +.48 +.80

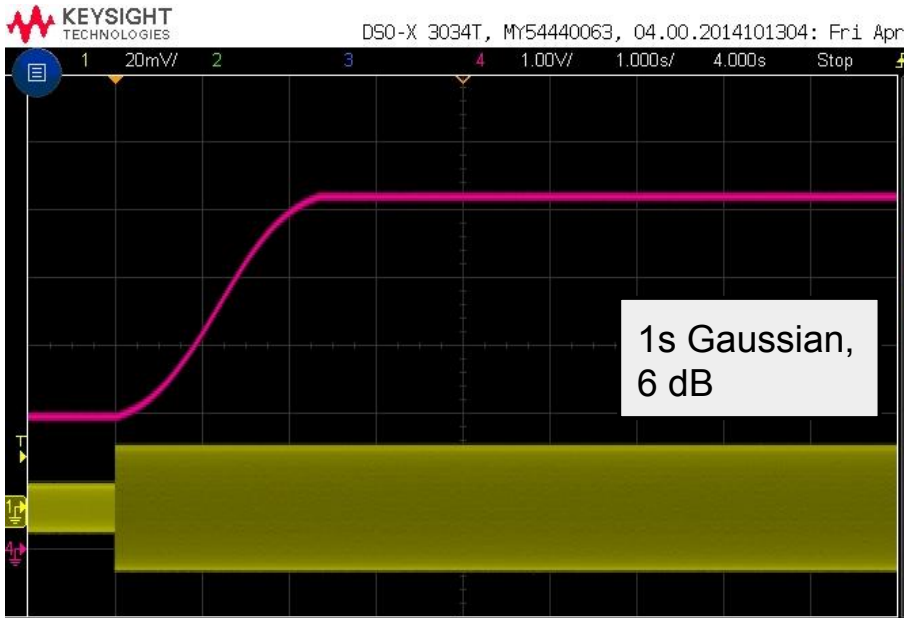
Hz, offset from f_{Ref}



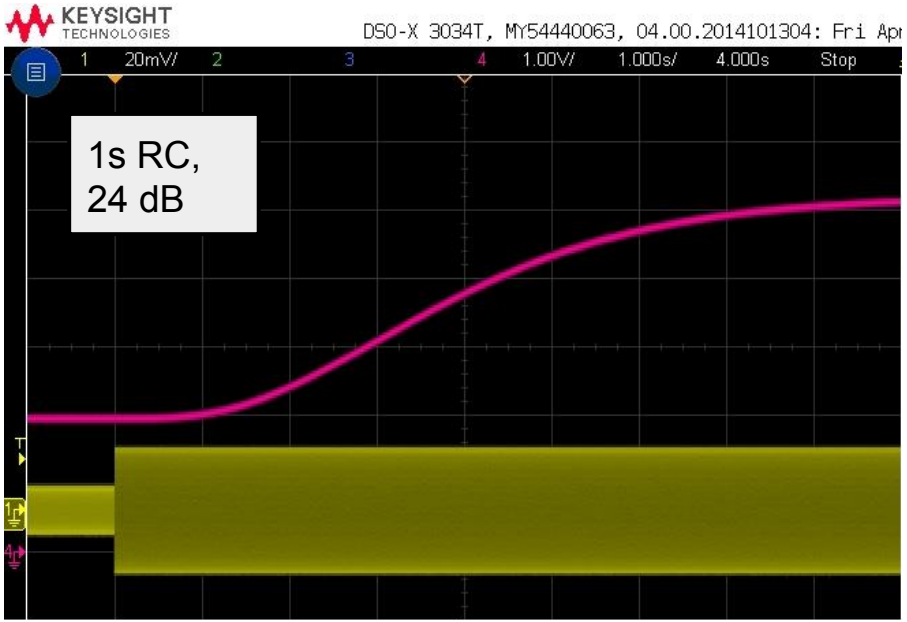
$$f_c = 1/(2\pi\ TC)$$

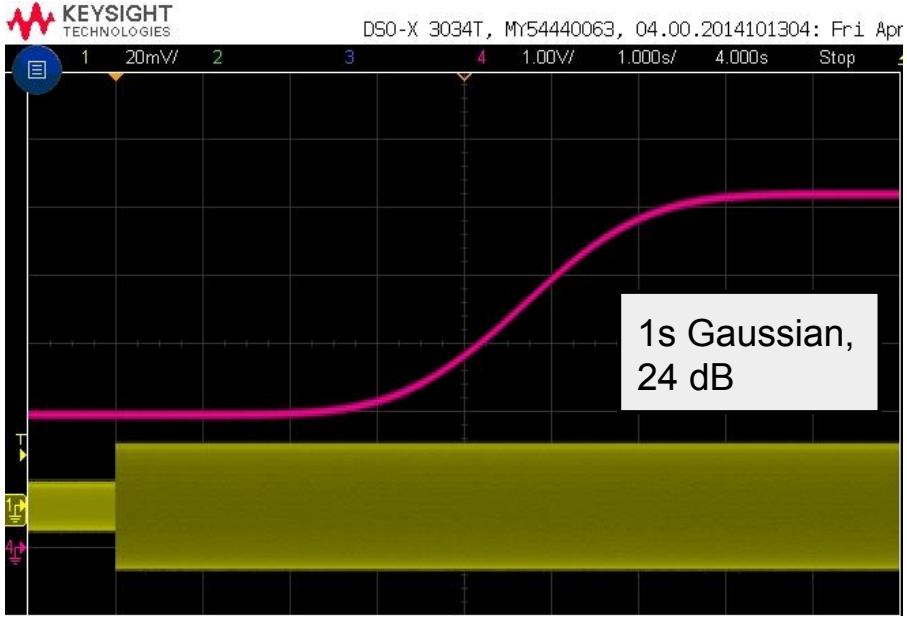
Time constants in the time domain





... more time domain





Ripple from 2f

What to do?

... increase the filter order

... but more lag, and longer ultimate settling time

... use an advanced filter

... some lag, but faster settling; available on newer instruments

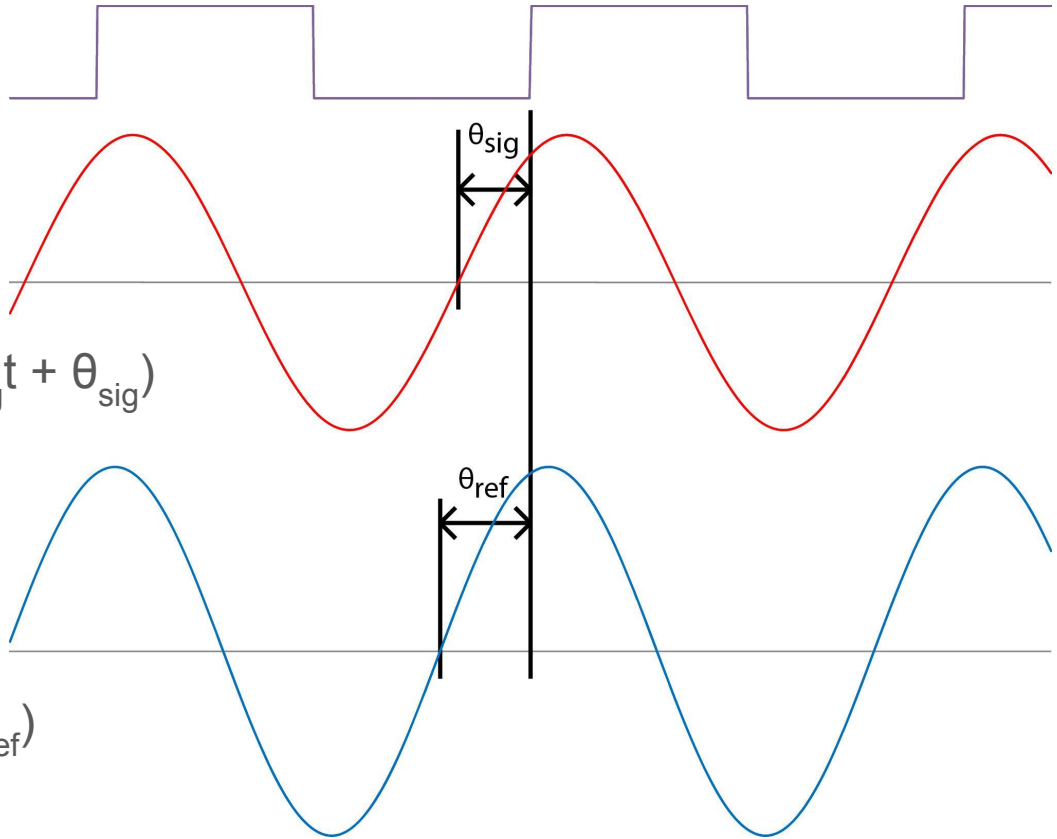
... or try Sync filter (*)

(*): only available at lower frequencies; older instruments had buggy implementation

Phase

$$\theta = \theta_{sig} - \theta_{ref}$$

Ext. Reference



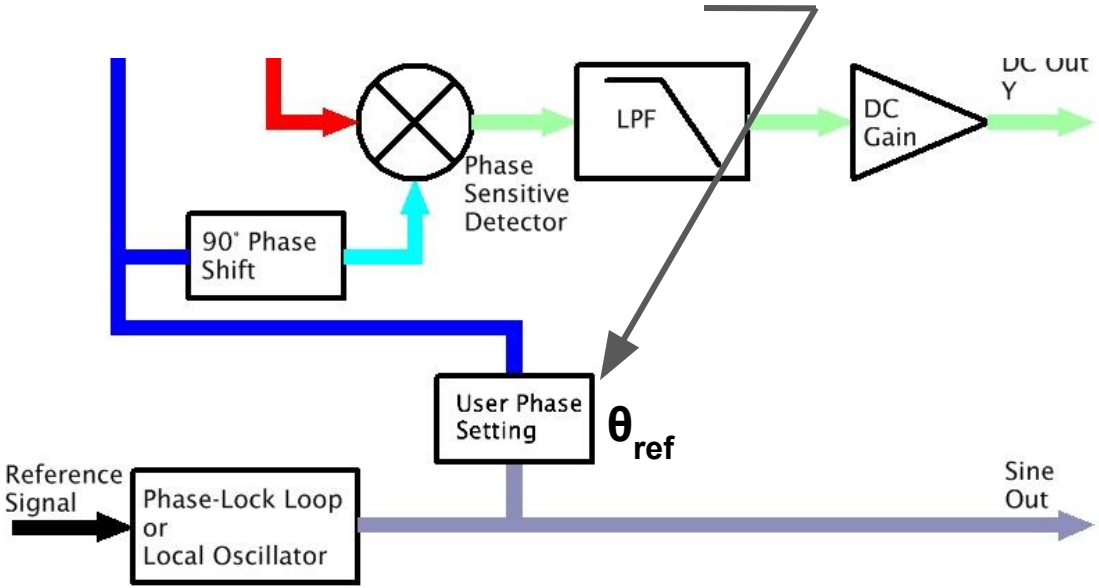
$s(t)=\sqrt{2}V_{sig}\sin(2\pi f_{sig}t + \theta_{sig})$

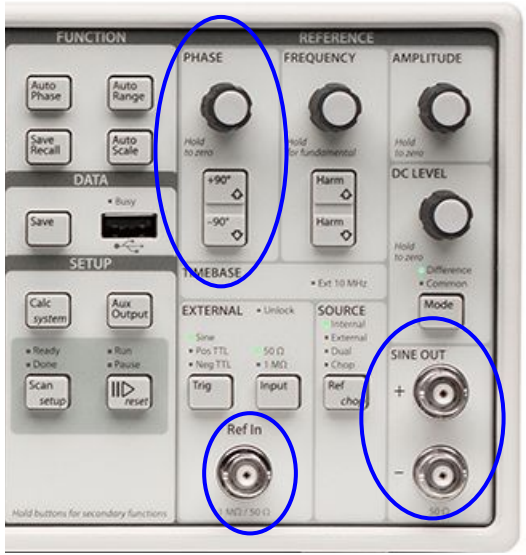
$r_X(t)=\sin(2\pi f_{ref}t + \theta_{ref})$

Phase

Sine Out always tracks Ref In (External reference)

Lock-in PHASE setting is always *here*

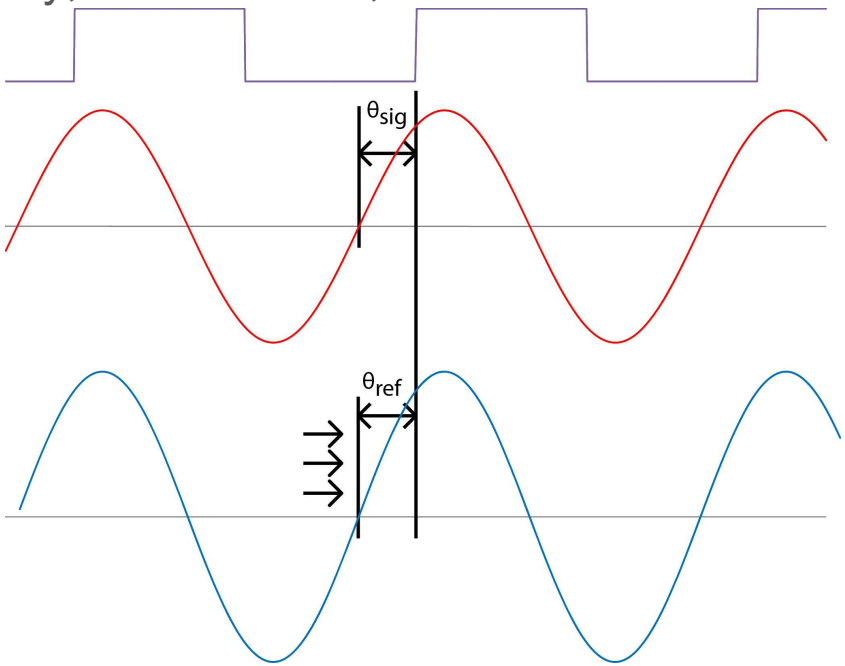




Phase

Auto-Phase sets $\boldsymbol{\theta}_{\mathbf{ref}}$ (phase) to make $\boldsymbol{\theta}_{\mathbf{sig}} = \boldsymbol{\theta}_{\mathbf{ref}}$

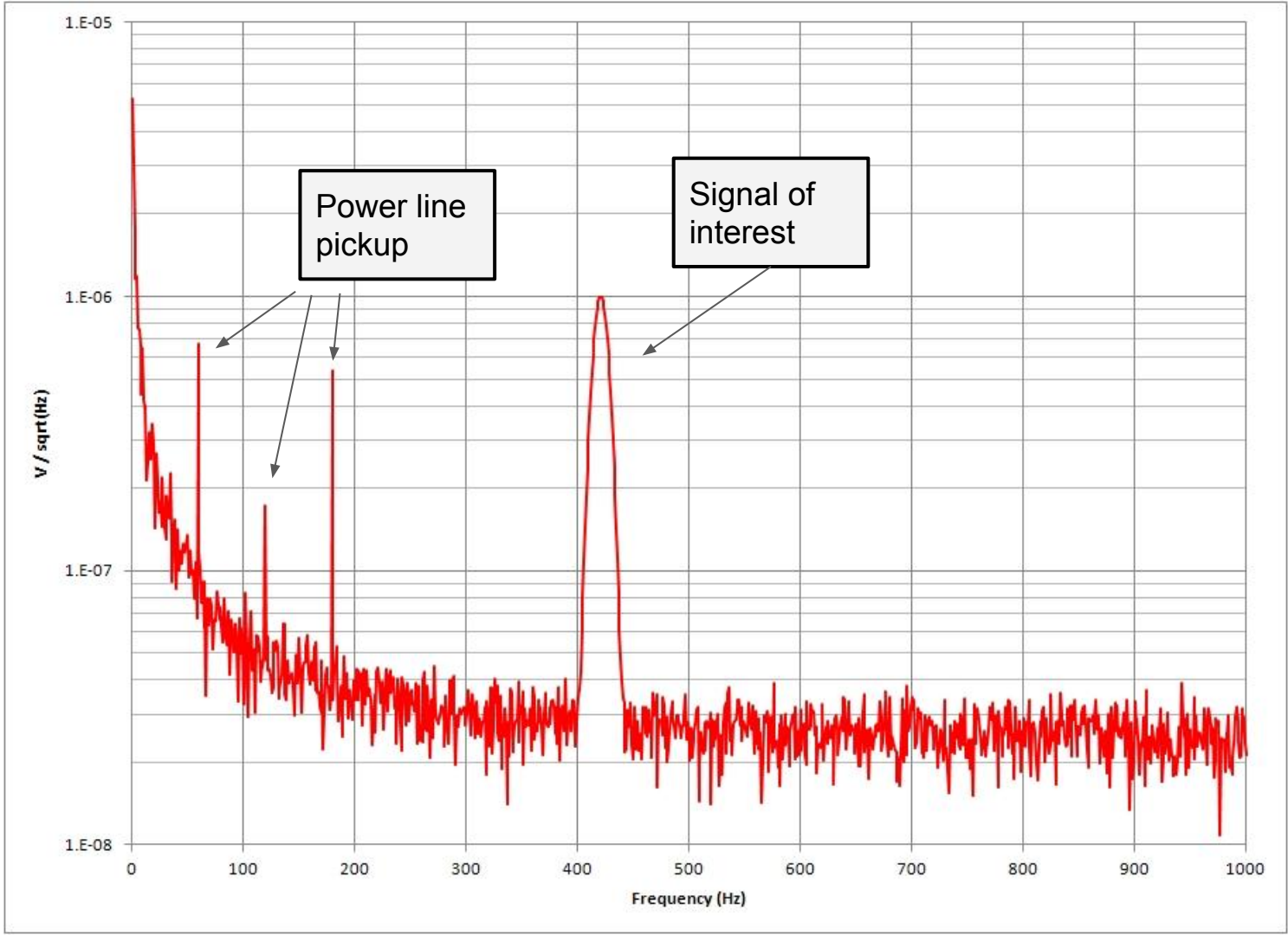
Equivalently, makes X = R, and Y = 0



Lock-in amplifier in the frequency domain

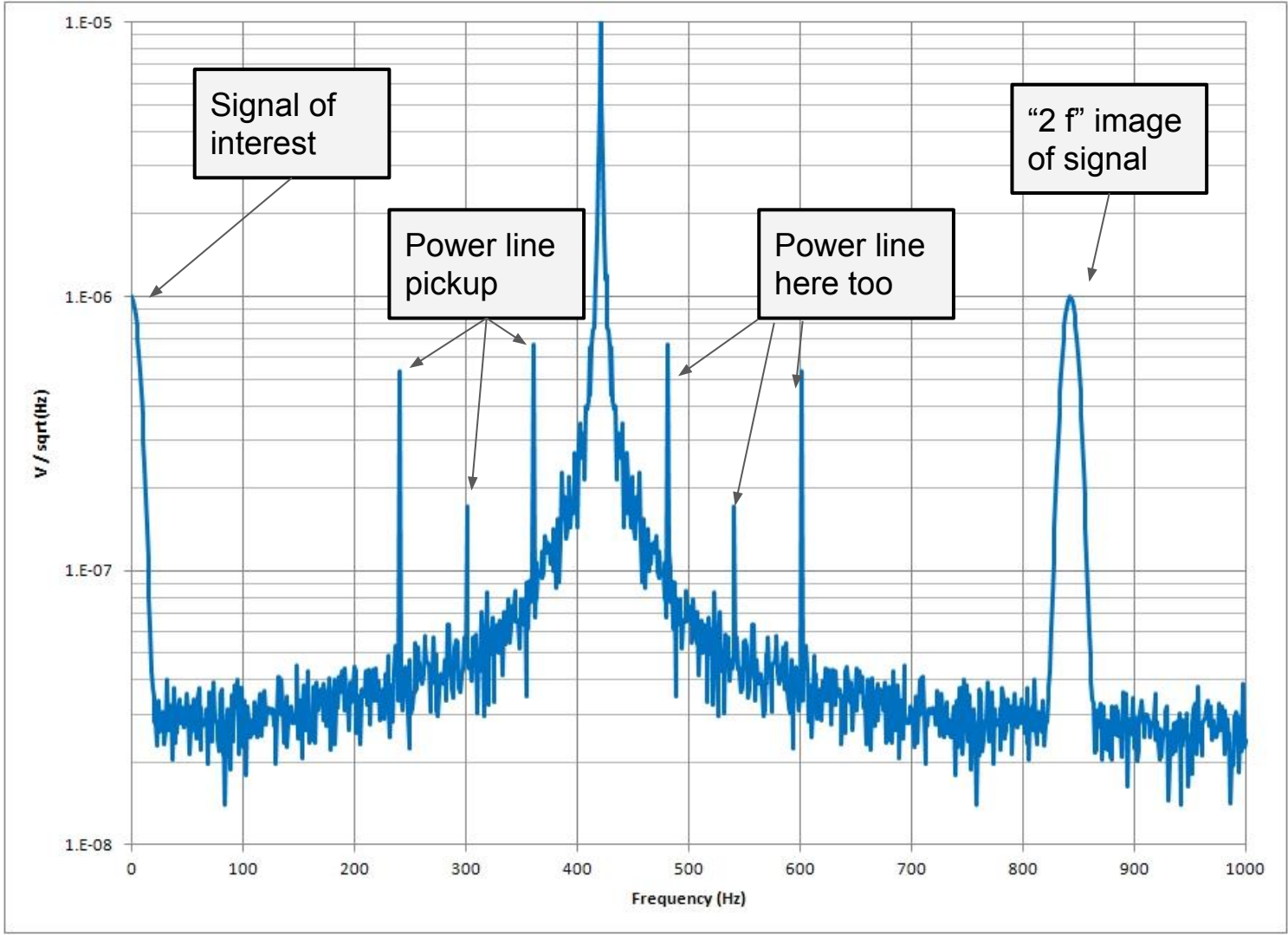
- Looking at the power spectrum amplitude for a toy signal
- Signal at 421 Hz, around 1μV
- “1/f noise” knee around 200 Hz
- Power line pickup at 60 Hz, 120 Hz, and 180 Hz

Input signal

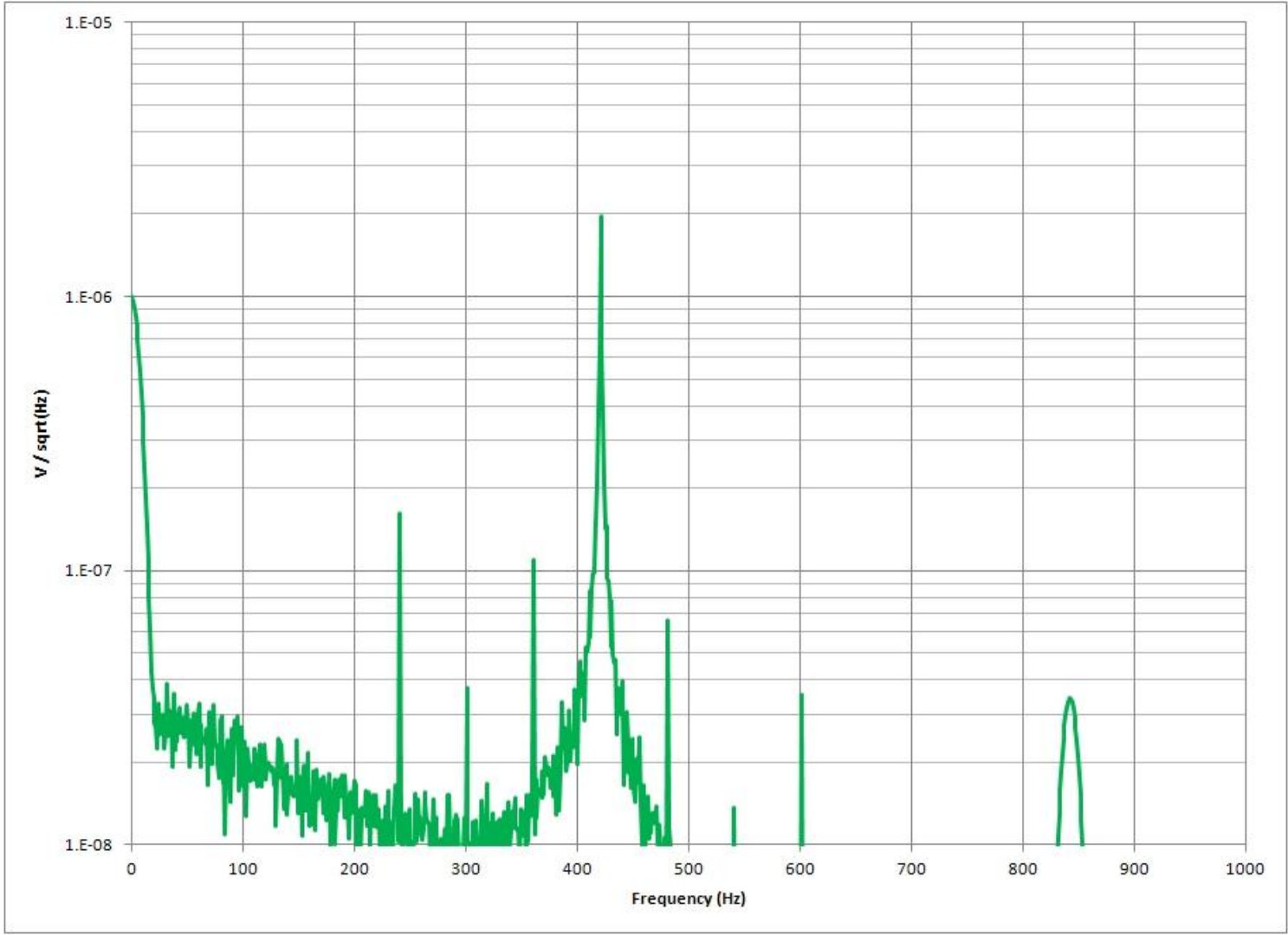


Signal immedately after the PSD

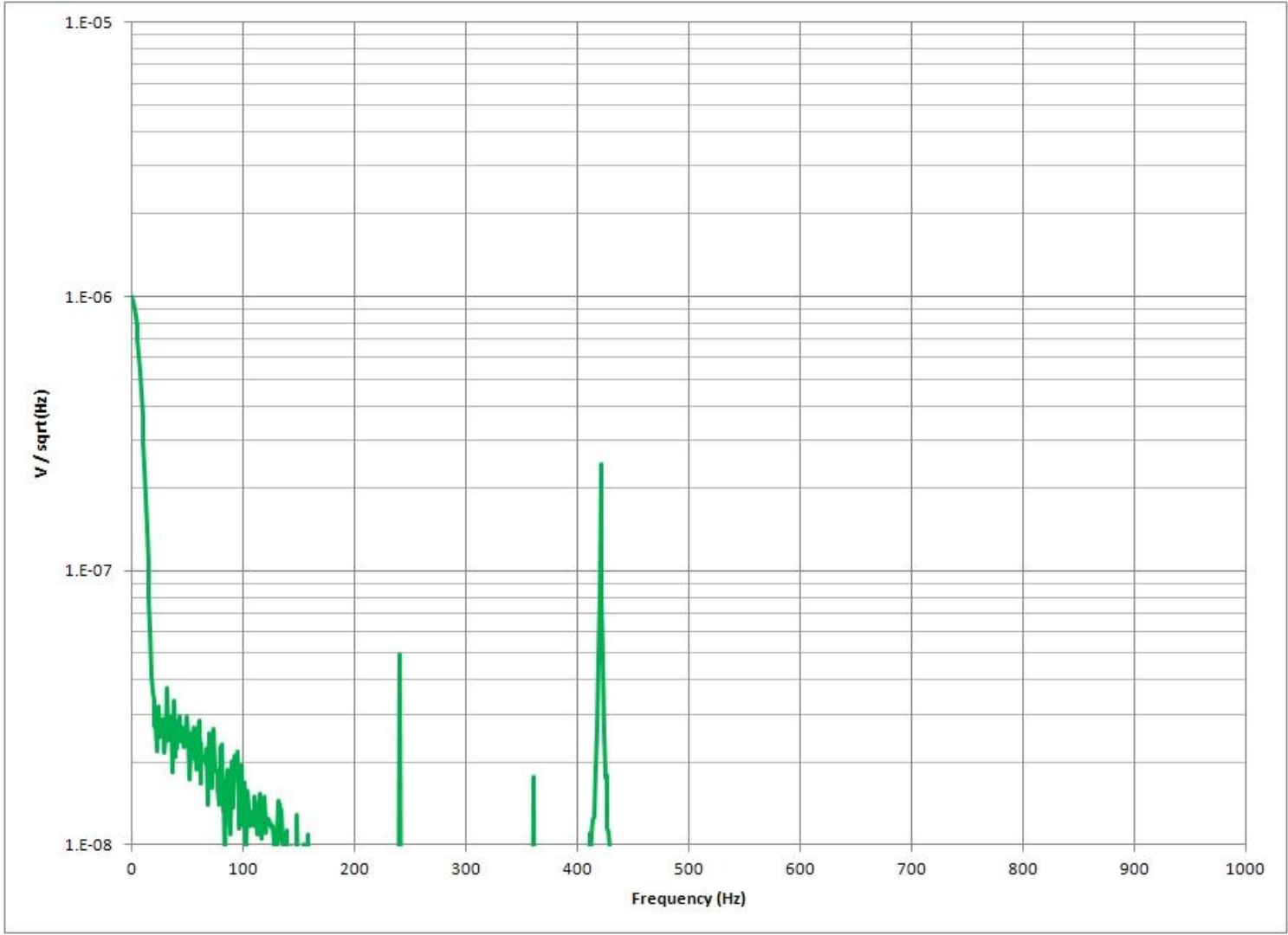
(Lock-in reference at 421 Hz)



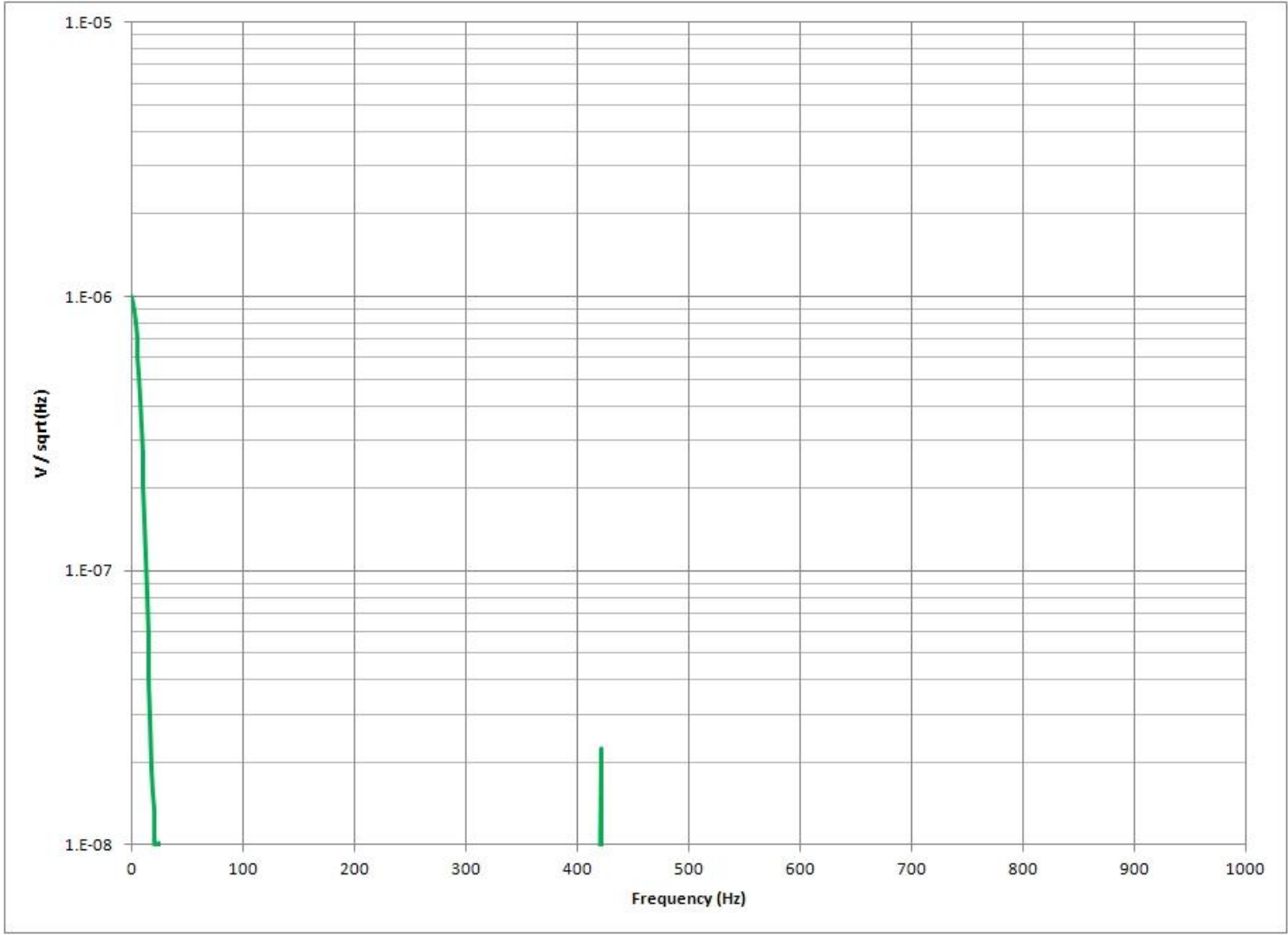
1 ms, 12dB RC-type filter



1 ms, 24dB RC-type filter

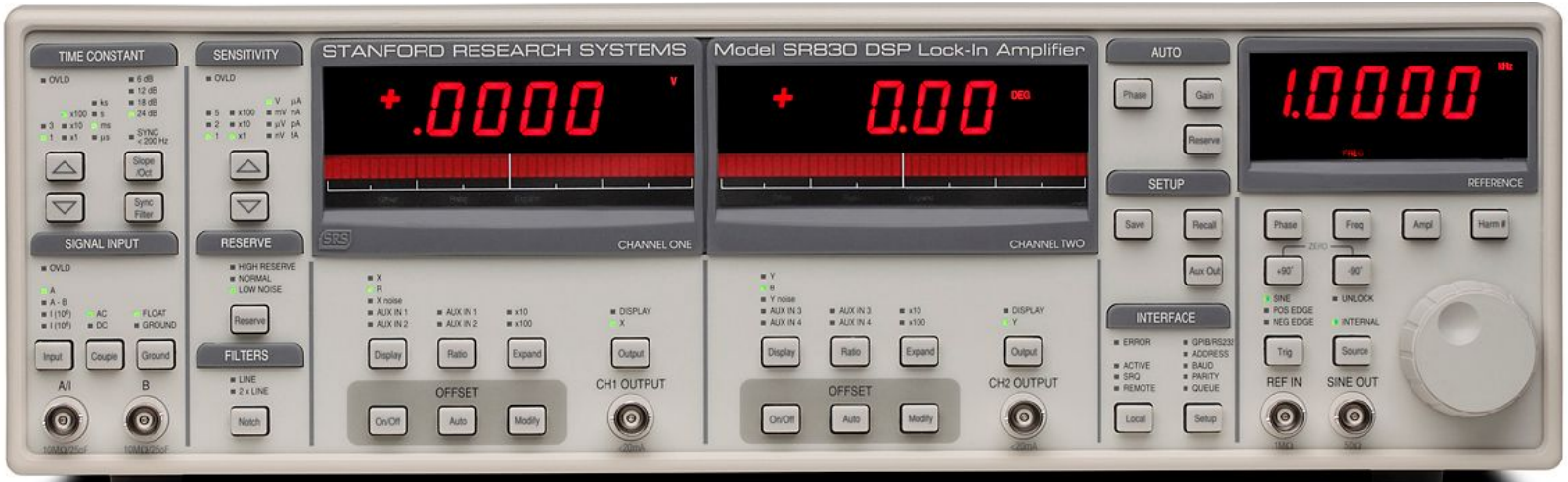


10 ms, 12dB RC-type filter



Common mistakes

- Input mis-configured
- Overload
- Harmonic
- Unlock
- Poor choice of TC



Common mistakes

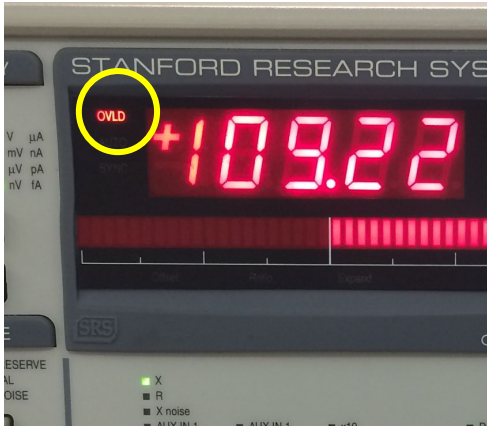
- Input mis-configured
- Overload
- Harmonic
- Unlock
- Poor choice of TC

- A or A-B
- AC or DC Coupling
- Current input ??

Look for “1× f” ripple when signal has DC offset

Common mistakes

- Input mis-configured
- Overload
- Harmonic
- Unlock
- Poor choice of TC



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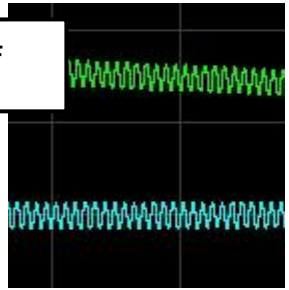
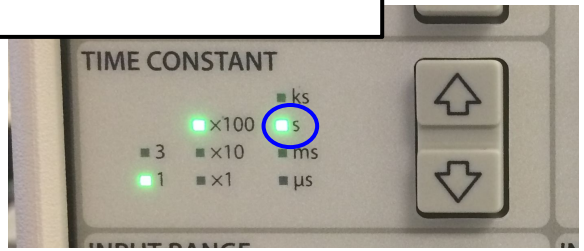
ripple from 2f



100 s instead of 100 ms



Other possible troubles

- Parasitic impedance
- Noisy reference

Other possible troubles

- Parasitic impedance
  - To check, try varying the excitation frequency
  - Capacitive losses will typically scale ∝ f

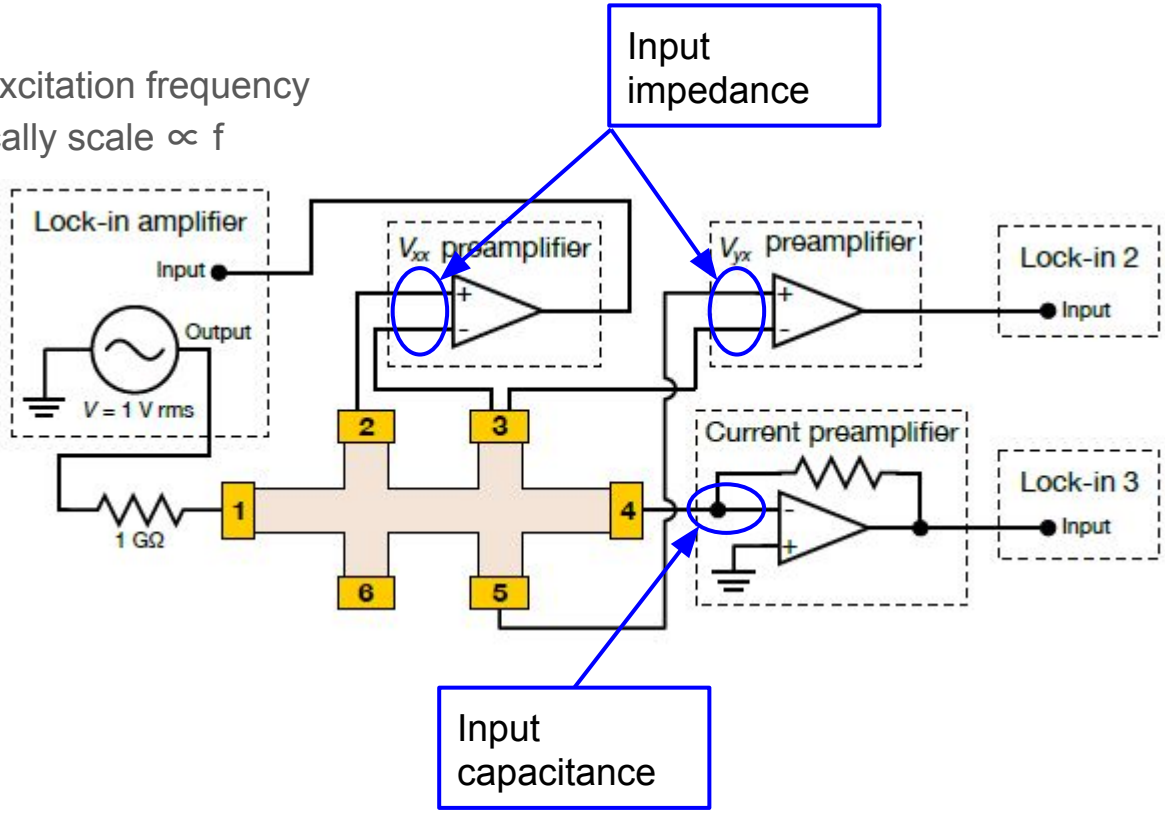


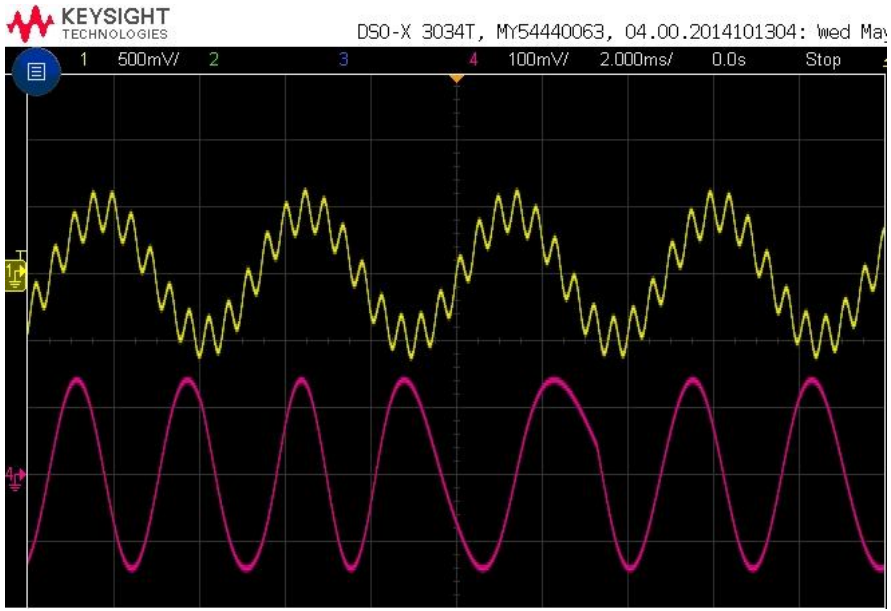
Figure modified from Bestwick et al 2015, arXiv:1412.3189, supplementary info

Other possible troubles

- Noisy or spurious reference

Ref In signal has large pickup.

Sine out is unstable; LIA is *unlocked*

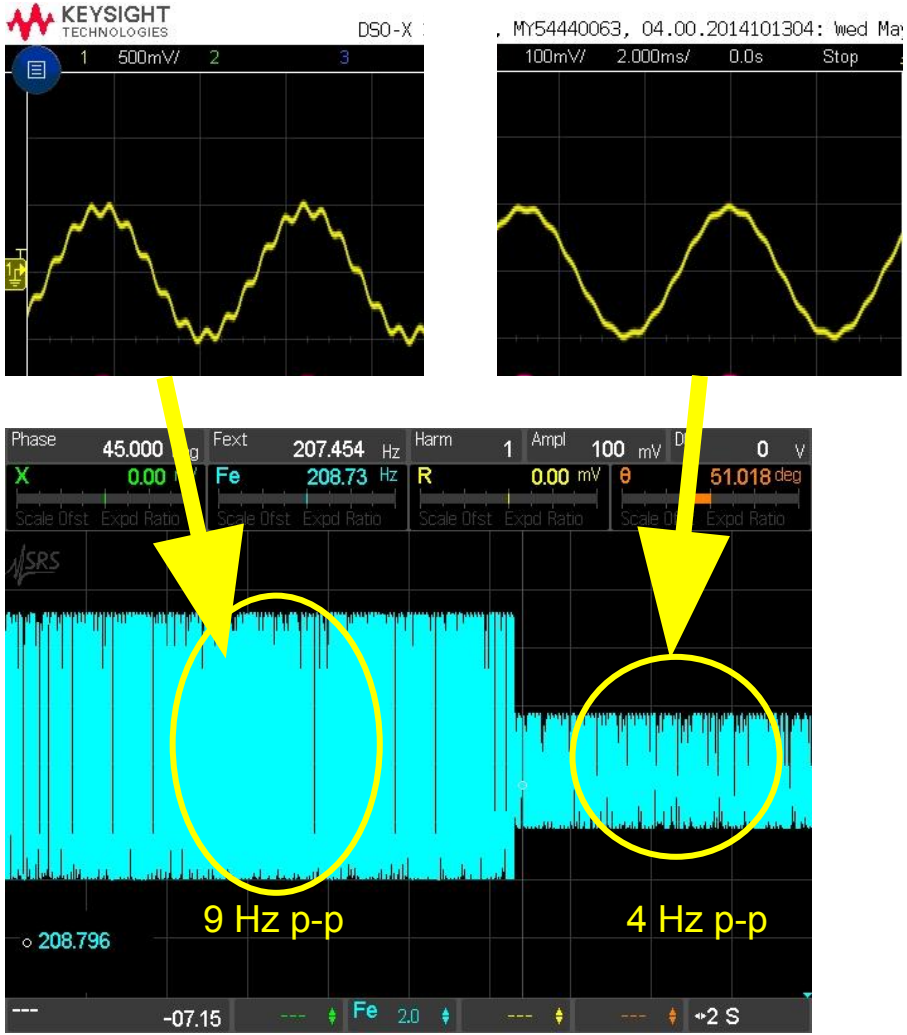


Other possible troubles

- Noisy or spurious reference

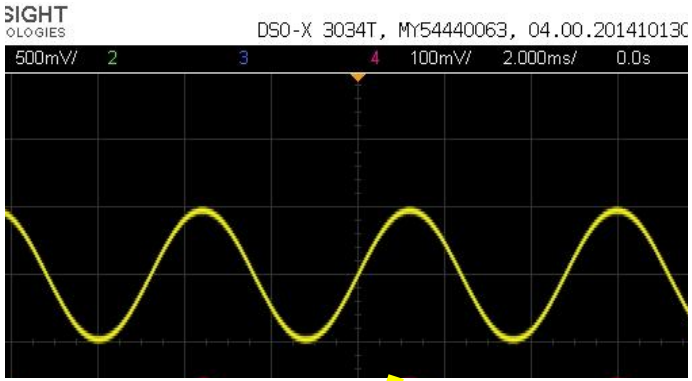
Note: external reference input locks to the positive (negative) zero crossing, not the “fundamental”

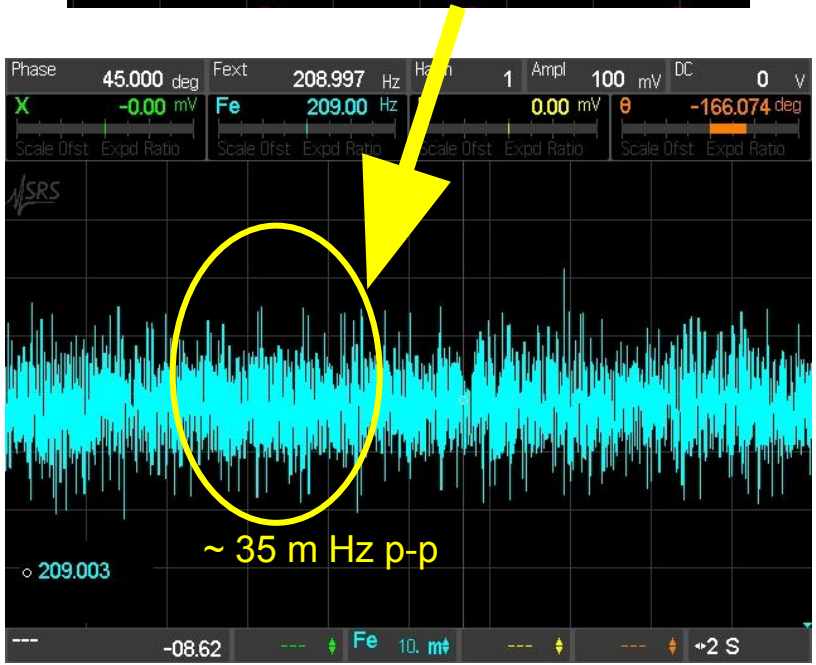
Locked, but lots of noise on reference oscillator!



Other possible troubles

- Noisy or spurious reference





Quiet reference should give a good, stable lock

Specialty applications: Analog Lock-Ins

- (probably) Square Wave PSD mixer
  - ⇒ odd harmonics
- Input filter
  - ⇒ operates on AC, prior to PSD
- Best to read results from BNC output

When to use?

- Eliminate any possible RF interference
- Very low noise front end input stage
- No sampling or quantization artifacts
  - No extra latency — feedback loops

Specialty applications: Analog Lock-Ins

- (probably) Square Wave PSD mixer
  - ⇒ odd harmonics
- Input filter
  - ⇒ operates on AC, prior to PSD
- Best to read results from BNC output
  - ⇒ prevents CPU clock from starting

When to use?

- Eliminate any possible RF interference
- Very low noise front end input stage
- No sampling or quantization artifacts
  - No extra latency — feedback loops

Other special tricks ...

- DC Bias
  - May be handy for some transport measurements
- Differential sine out
- Parameter scans
  - Handy for quick version of experiments
- Double modulation
  - Example follows....

Example of double modulation technique

ARTICLES

Controllable valley splitting in silicon quantum devices

SRIJIT GOSWAMI[1]*, K. A. SLINKER[1]*, MARK FRIESEN[1]*†, L. M. McGUIRE[1], J. L. TRUITT[1], CHARLES TAHAN[2], L. J. KLEIN[1], J. O. CHU[3], P. M. MOONEY[4], D. W. VAN DER WEIDE[5], ROBERT JOYNT[1], S. N. COPPERSMITH[1] AND MARK A. ERIKSSON[1]†

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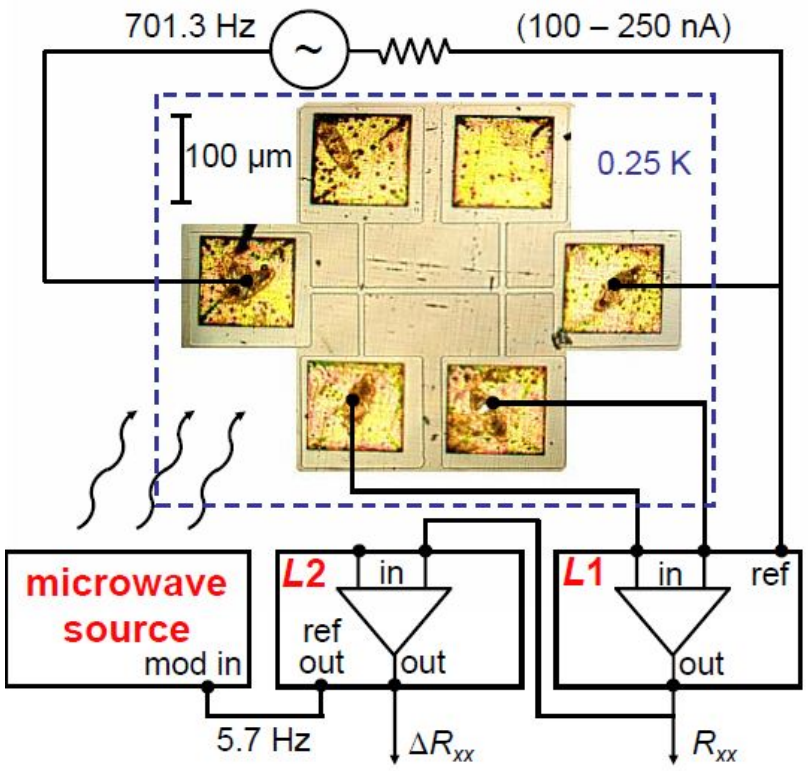
Goswami, Srijit, et al. "Controllable valley splitting in silicon quantum devices." *Nature Physics* 3.1 (2007): 41-45.



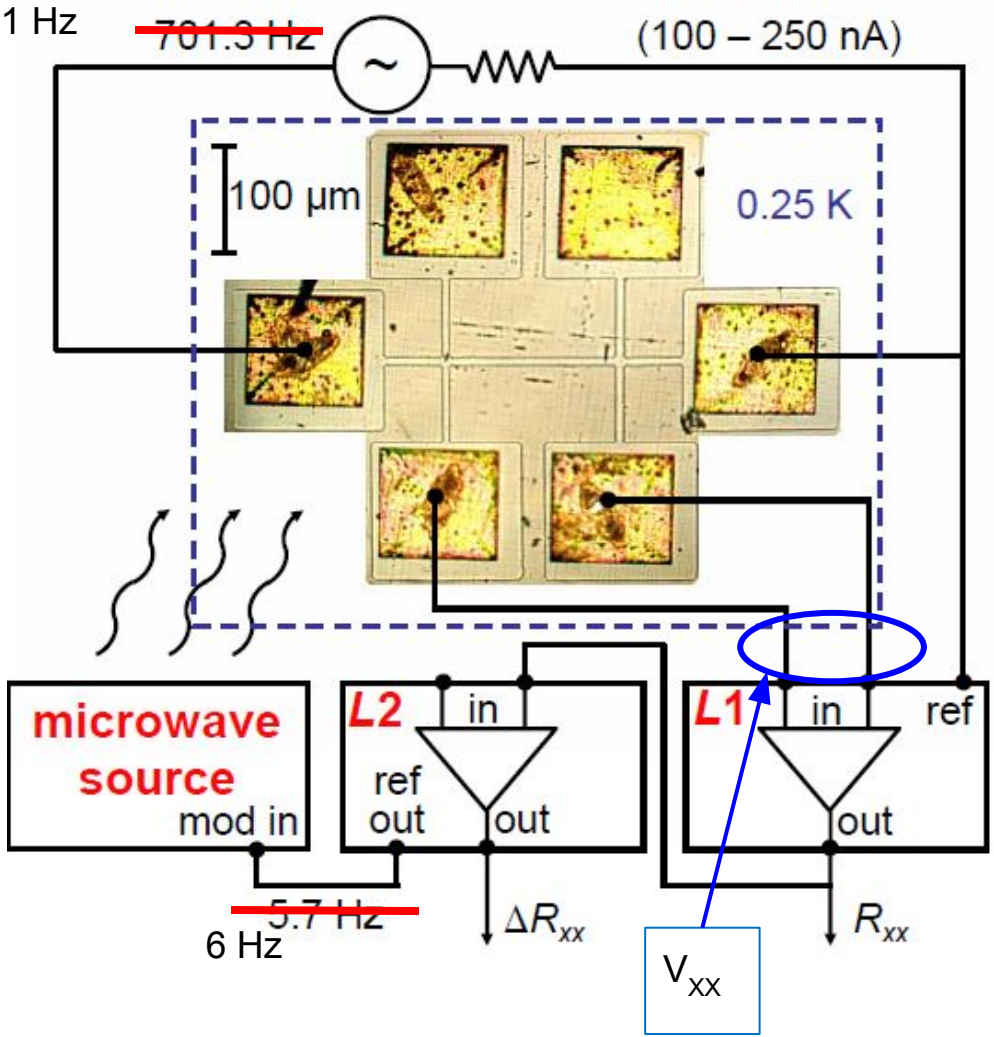
L1 measures R_{xx} at 701.3 Hz

L2 measures ΔR_{xx} from R_{xx} at 5.7 Hz

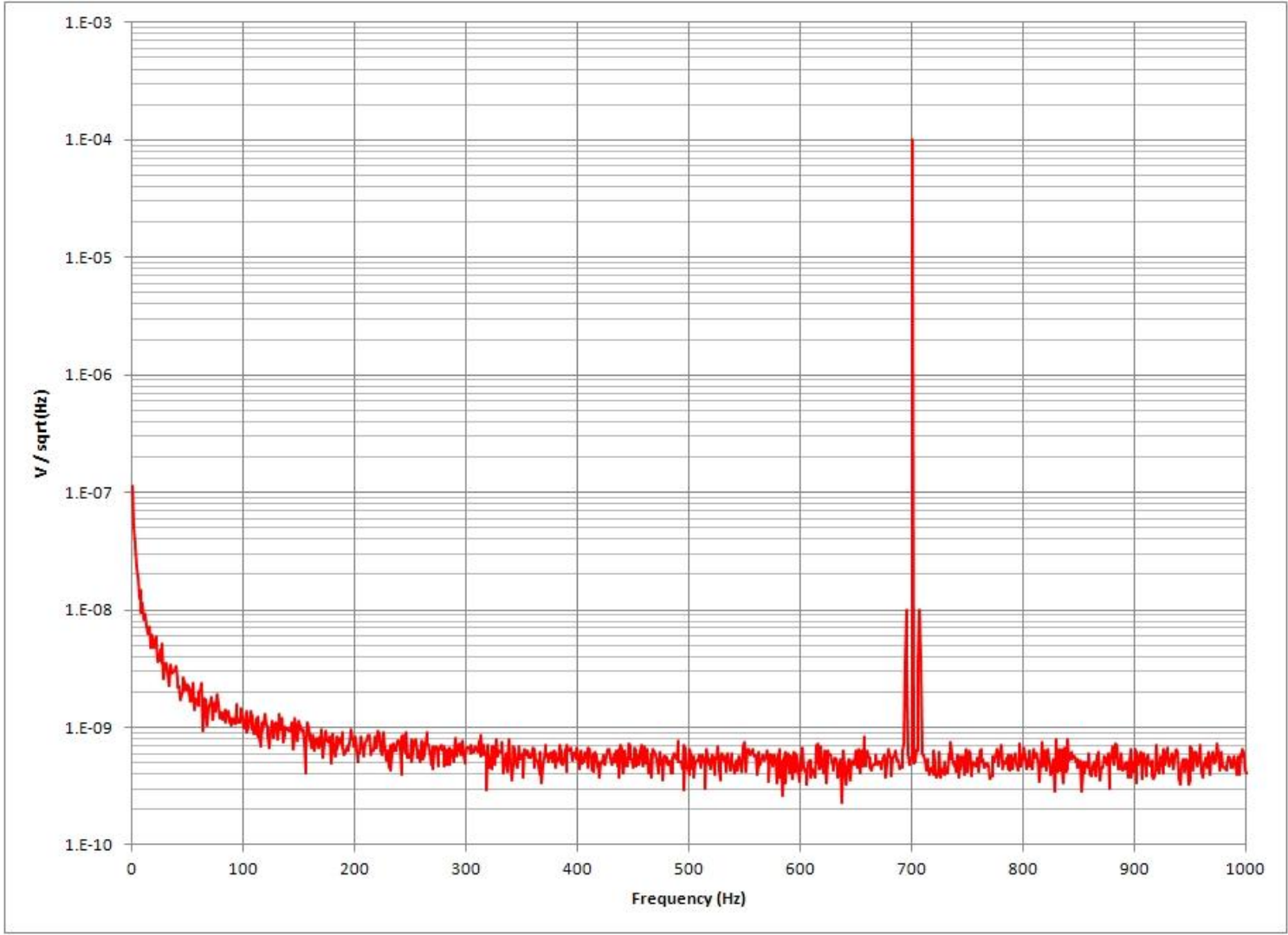
double modulation

Following spectra are fake data generated to demonstrate the frequency content of double-modulated experiments.

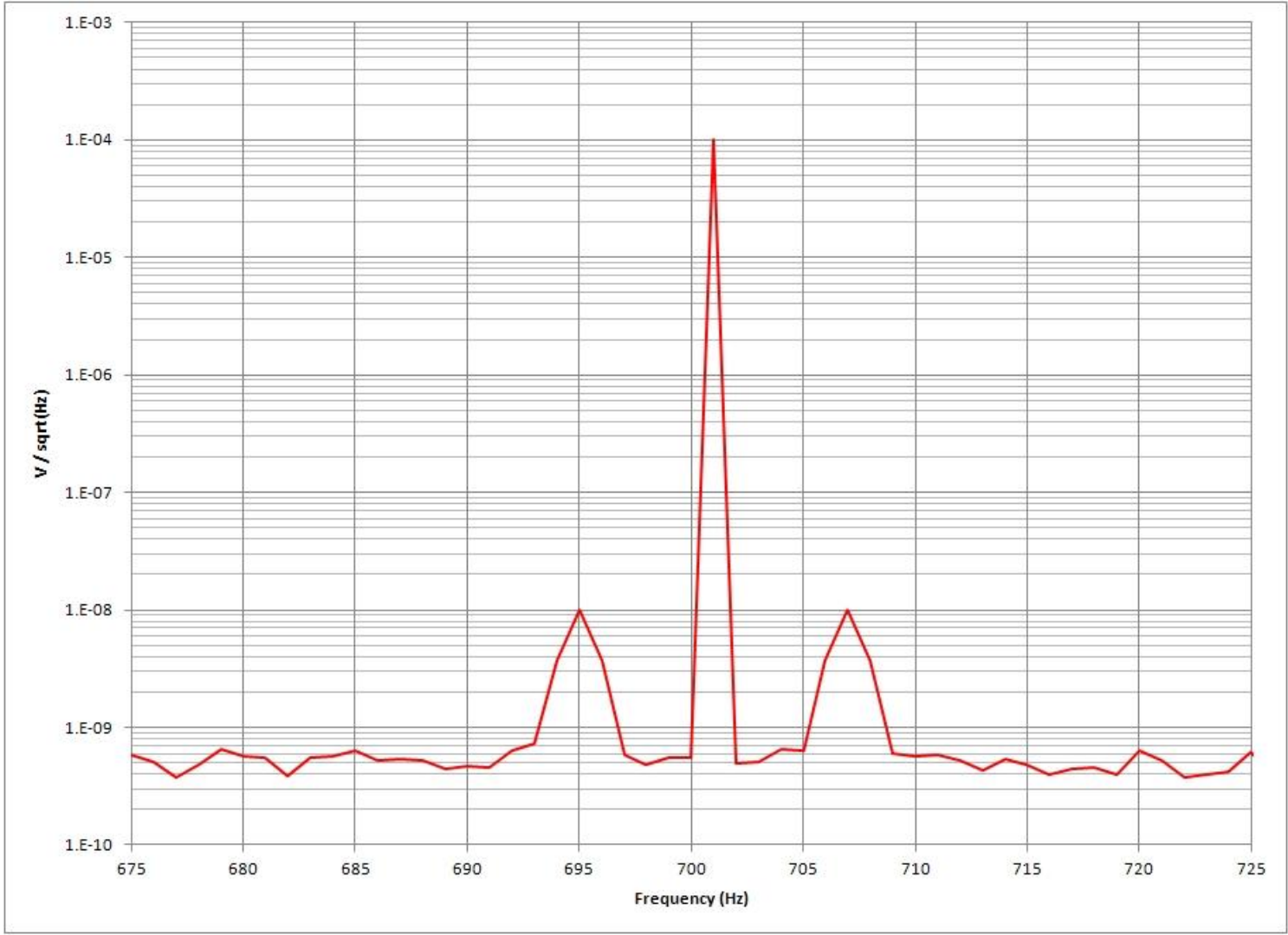
In this example, the Hall resistance R_{XX} is modulated by about 1 part in 10^{-4} by the chopping microwave source.



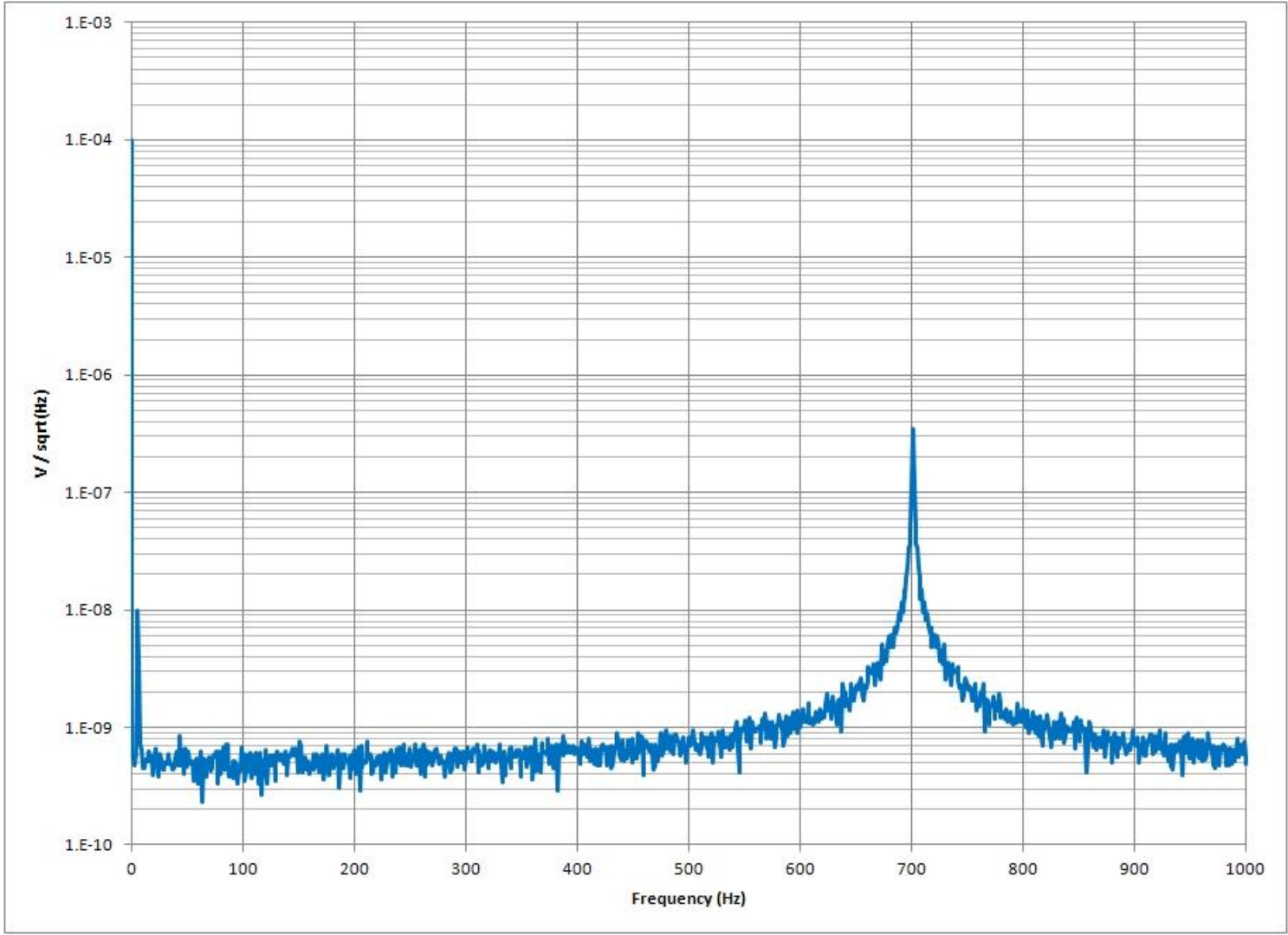
“raw” V_{xx}
(note sidebands)



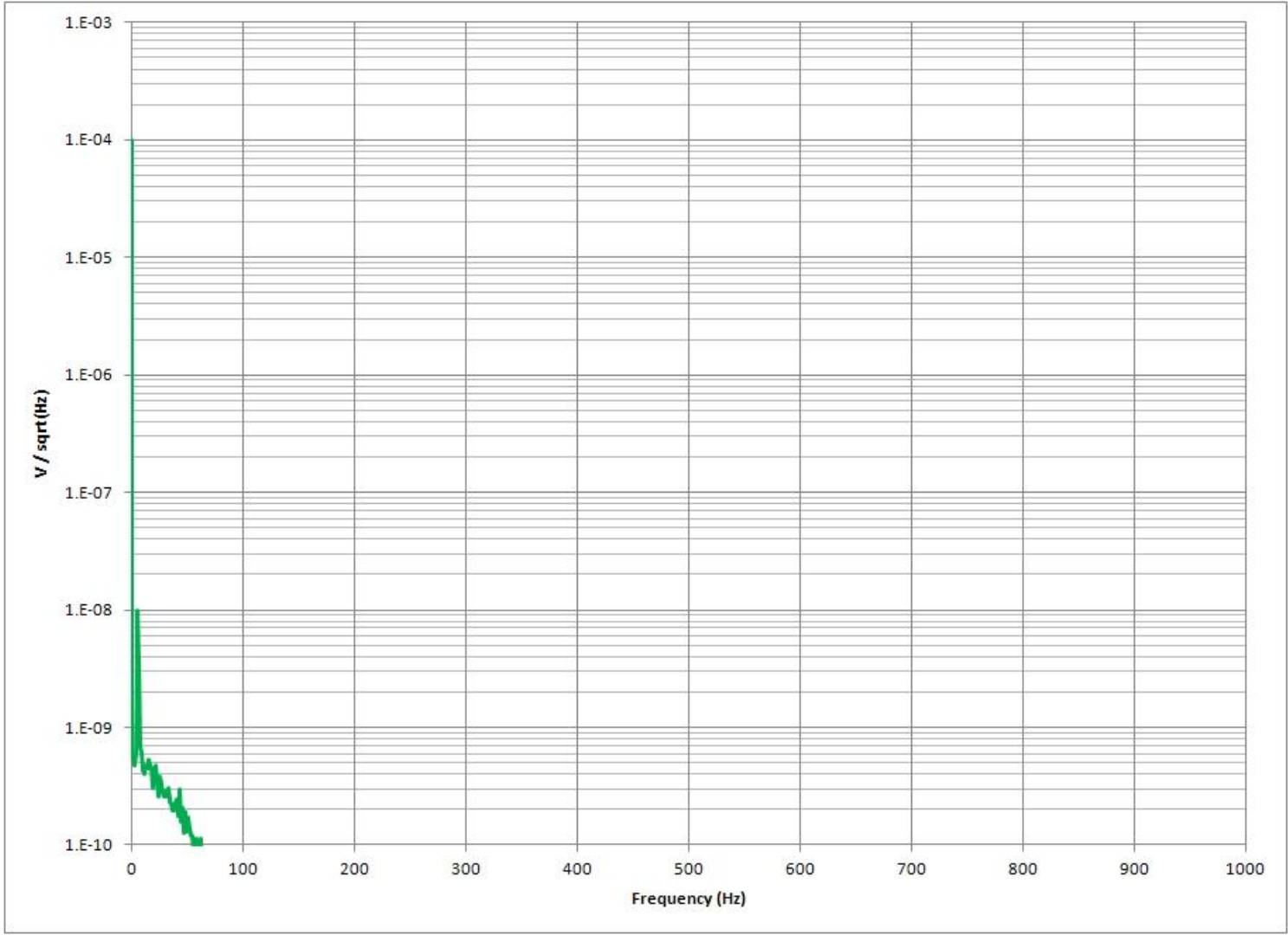
“raw” V_{xx}
(zoomed in)



PSD output for V_{xx}

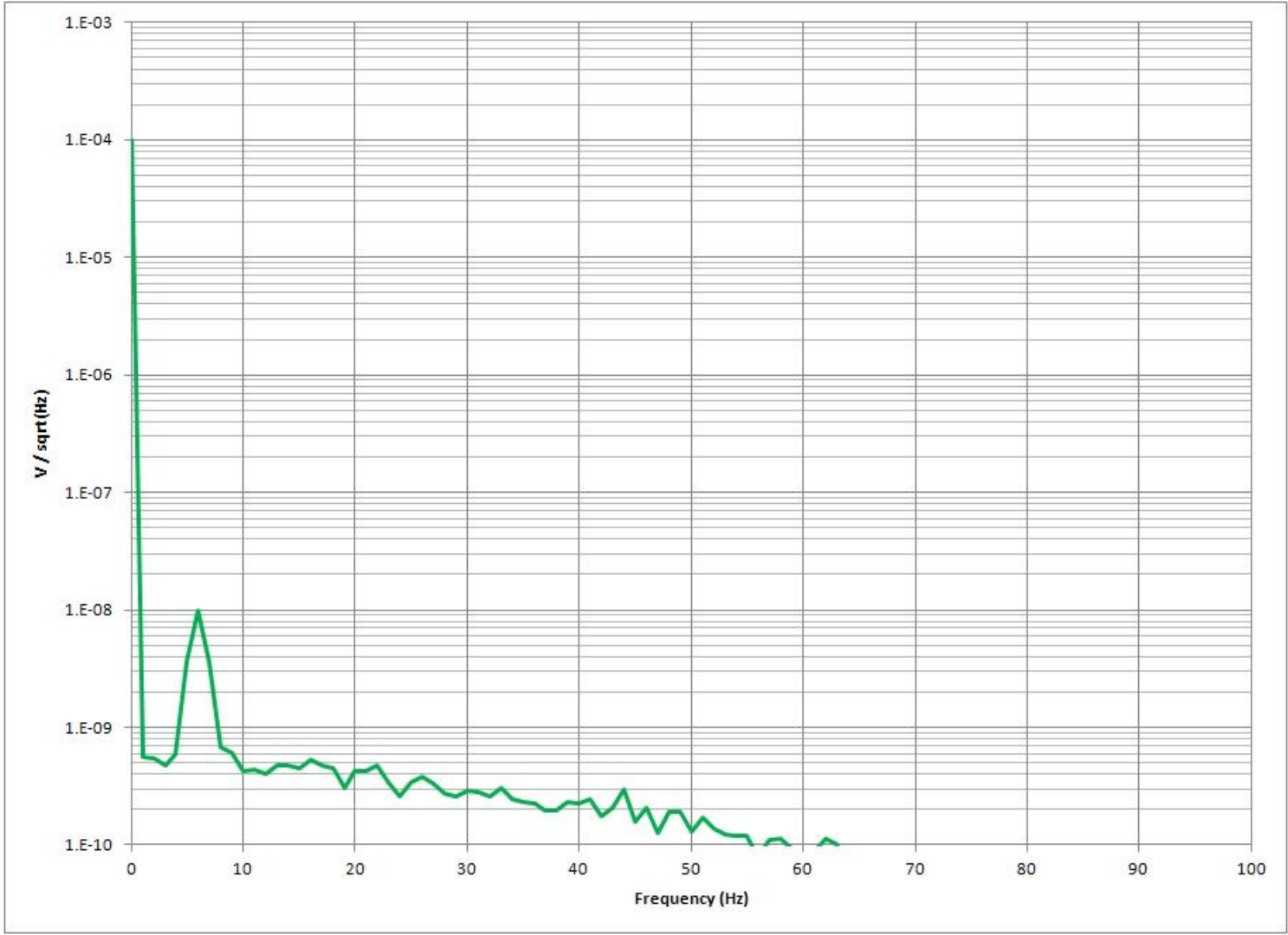


filtered output for V_{xx}
3ms, 24dB “RC”



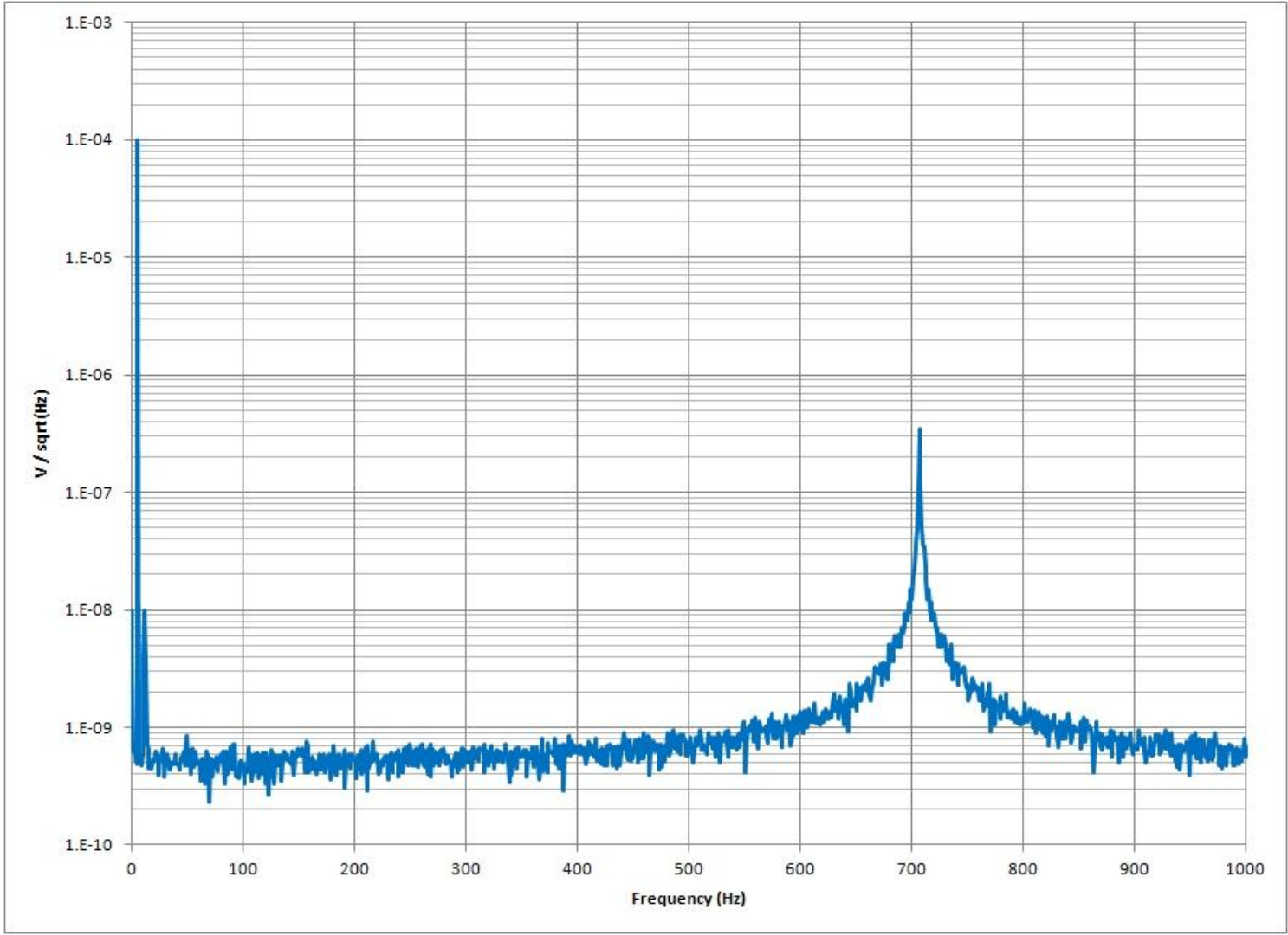
filtered output for V_{XX}
3ms, 24dB “RC”

Detail shows *input* signal to L2; ΔV_{XX} at 6 Hz



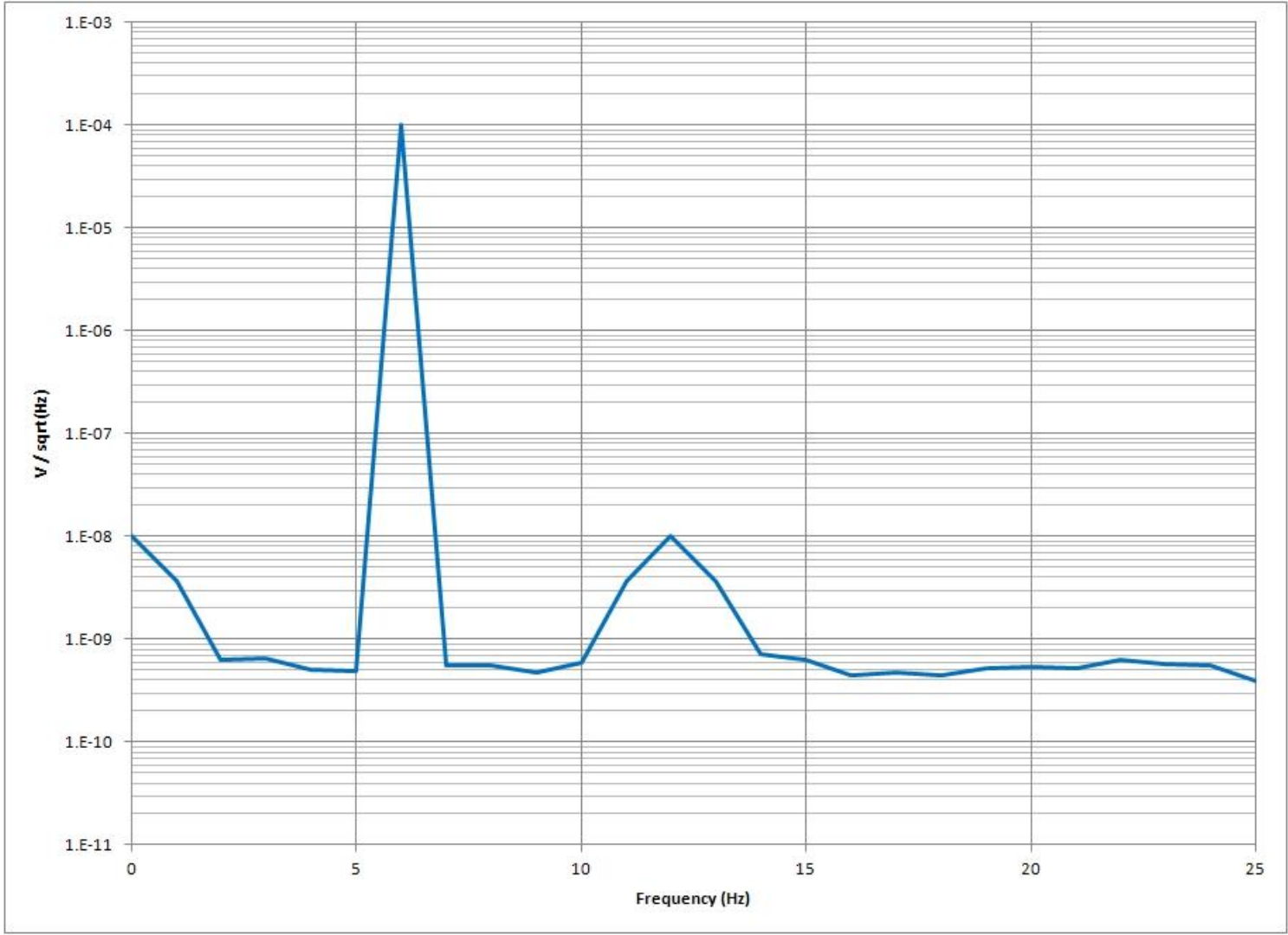
Same measurement can be made with one LIA, in “dual” mode

Reference is set to demodulate *difference* frequency



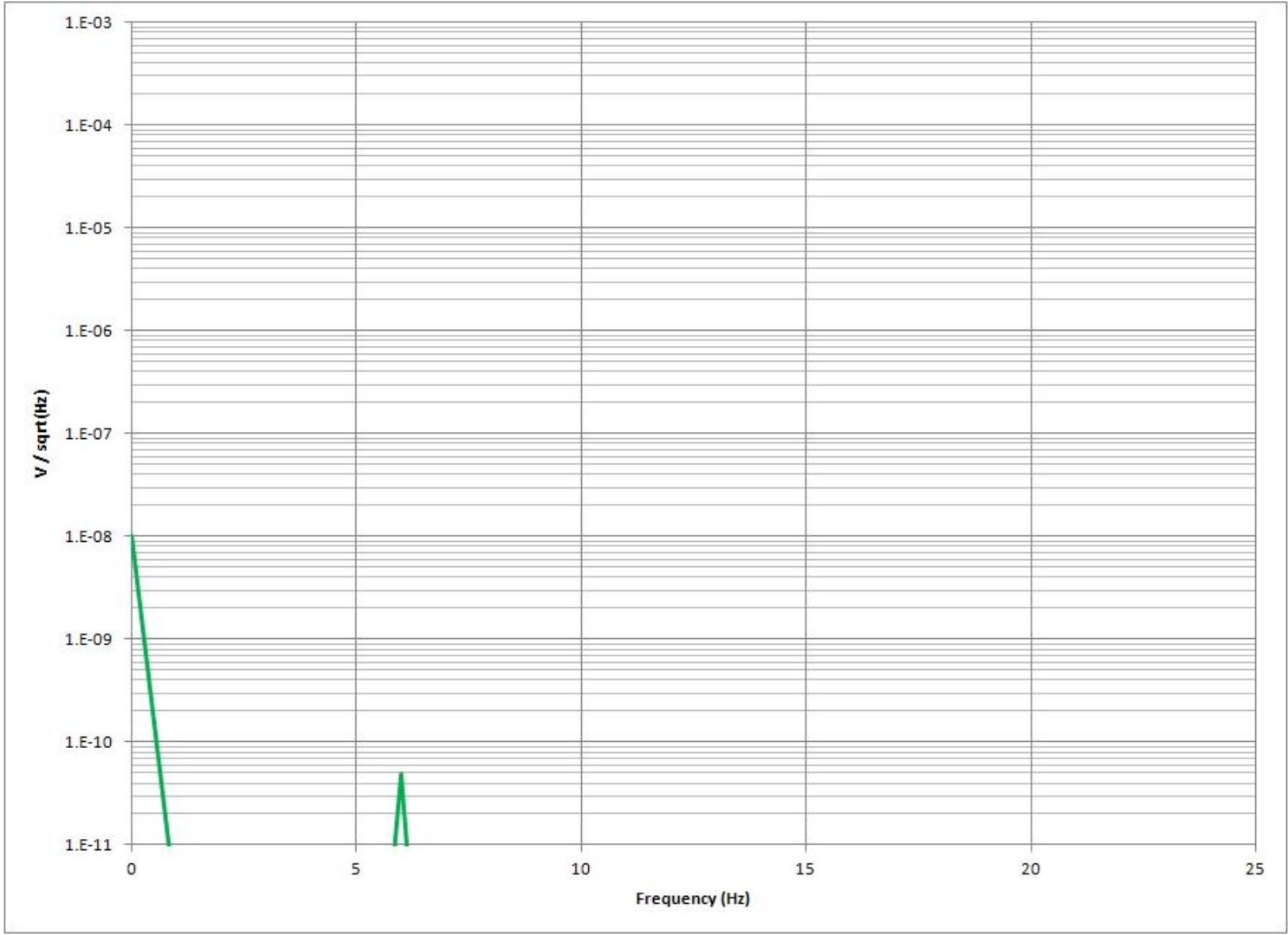
Same measurement can be made with one LIA, in “dual” mode

Zoomed in



Same measurement can be made with one LIA, in “dual” mode

Filtered,
1s, 24 dB “RC”



Handy facts: filter details

Time to settle to within 1% of final value

Filter slope	RC	Gaussian	Linear Phase (≥ 10s)
6 dB/octave	**4.6 TC**	**2.3 TC**	**4.6 TC**
12 dB/octave	**6.6 TC**	**4.0 TC**	**4.4 TC**
18 dB/octave	**8.4 TC**	**5.6 TC**	**4.8 TC**
24 dB/octave	**10.0 TC**	**7.1 TC**	**5.4 TC**

Equivalent Noise Bandwidth (ENBW) in Hz

Filter slope	RC	Gaussian	Linear Phase (≥ 10s)
6 dB/octave	**0.255 / TC**	**0.255 / TC**	**0.255 / TC**
12 dB/octave	**0.126 / TC**	**0.175 / TC**	**0.148 / TC**
18 dB/octave	**0.094 / TC**	**0.150 / TC**	**0.132 / TC**
24 dB/octave	**0.080 / TC**	**0.129 / TC**	**0.124 / TC**