Periodically kicked turbulence

Detlef Lohse*
Department of Applied Physics and J. M. Burgers Centre for Fluid Dynamics, University of Twente, 7500 AE Enschede, Netherlands
(Received 23 May 2000)

Periodically kicked turbulence is theoretically analyzed within a mean-field theory. For large enough kicking strength A and kicking frequency f the Reynolds number grows exponentially and then runs into some saturation. The saturation level Re^{sat} can be calculated analytically; different regimes can be observed. For large enough Re we find $\mathrm{Re}^{sat} \propto Af$, but intermittency can modify this scaling law. We suggest an experimental realization of periodically kicked turbulence to study the different regimes we theoretically predict and thus to better understand the effect of forcing on fully developed turbulence.

PACS number(s): 47.27.$-$i

Periodically driven flow is ubiquitous. Faraday's experiment [1] is an idealized version thereof; more relevant examples are the earth's atmosphere, driven by periodical heating of the sun, or the blood flow in veins, driven by the beating heart. Another example is the gas flow inside a sonoluminescing bubble that is periodically kicked by the collapsing bubble wall [2]. Another example of periodically kicked turbulence is the numerical realization of homogeneous shear flow [3] where periodical remeshing is necessary.

In this paper we set up a mean-field theory for periodically kicked flow, based on the mean-field theory for decaying turbulence [4], which was able to describe the experimentally measured energy decay in turbulent liquid-helium flow with fixed external length scale [5]. The goal of this paper is to theoretically understand the different flow regimes, which are to be expected, to explore the effect of intermittency corrections on these regimes, and to ultimately initiate experiments.

Another motivation for the paper is to study the effect of a specified type of forcing on turbulence. In most theoretical studies on turbulence, a Gaussian random noise, acting on the largest length scales, is assumed. Only recently experimentalists started to systematically vary the type of forcing [6,7]. This paper is a further step towards the analysis of a more specific type of forcing.

To define the model, we have to (i) calculate the energy input during the kick and (ii) know how the energy is dissipated in the time Δt between successive kicks.

(i) Kick: As an illustration, consider plane shear flow in the 1 direction; the flow is sheared in the 3 direction. The width of the channel is L, the velocity of the upper plate is U, the lower one is at rest. The average energy dissipation rate can be calculated to be (see, e.g., Ref. [8])

$$\epsilon = -\frac{U}{L}\left(\langle u_3 u_1\rangle_A - \nu \partial_3 \langle u_1\rangle_A\right), \tag{1}$$

where $\langle\rangle_A$ denotes the average over the x-y plane and ν is the viscosity. For laminar flow the first term in the bracket is zero and the second one is $-\nu U/L$. In turbulent flow in the middle of the channel the second term on the right-hand side will hardly contribute for large enough Reynolds numbers. The first term represents the total turbulent flow energy E, order of magnitude wise. Therefore, in general,

$$\dot{E}(t) = -\epsilon(t) \sim \frac{U}{L}E(t) + \frac{U^2}{L^2}\nu. \tag{2}$$

Imagine now a short intense kick of time $\Delta t_{kick} \ll \Delta t$ on the flow by rapidly moving the upper plate with U. After this kick the initial energy E_0 increases according to Eq. (2),

$$E_1 = E_0 + \left(E_0 + \frac{U\nu}{L}\right)\frac{U}{L}\Delta t_{kick}, \tag{3}$$

where we have assumed $\Delta t_{kick} \ll L/U$. Assume isotropic turbulence in the flow center and define a Reynolds number[1]

$$\mathrm{Re}(t) = \frac{L u_{1,rms}(t)}{\nu} = \sqrt{\frac{2}{3}}\frac{L\sqrt{E(t)}}{\nu}. \tag{4}$$

Then Eq. (3) translates to

$$\mathrm{Re}_1 = \mathrm{Re}_0\sqrt{1 + 2A + \frac{\mathrm{Re}_{lam}^2}{\mathrm{Re}_0^2}} \tag{5}$$

with the dimensionless kicking strength $A = \frac{1}{2}\Delta t_{kick} U/L \ll 1$ and the "laminar" Reynolds number $\mathrm{Re}_{lam} = \frac{2}{3}\Delta t_{kick} U^2/\nu$. We choose this name because for very small Re_0 we have $\mathrm{Re}_1 = \mathrm{Re}_{lam}$. For very large $\mathrm{Re}_0 \gg \mathrm{Re}_{lam}$ we have $\mathrm{Re}_1 = (1+A)\mathrm{Re}_0$. Shear flow and the "derivation" of Eq. (5) are only thought of as a motivation; there will be many other experimental situations where the energy input roughly corresponds to a law of type (5).

(ii) Decay: In Ref. [4] we calculated how the turbulent activity decays within a time t for flow with fixed external length scale L. The calculation was based on Effinger and Grossmann's variable range mean-field theory of turbulence [9] in which viscous subrange and inertial subrange can be

*Email address: lohse@tn.utwente.nl

[1] Note that this is not the standard definition and gives lower values for the laminar-turbulent transition than what one is used to.

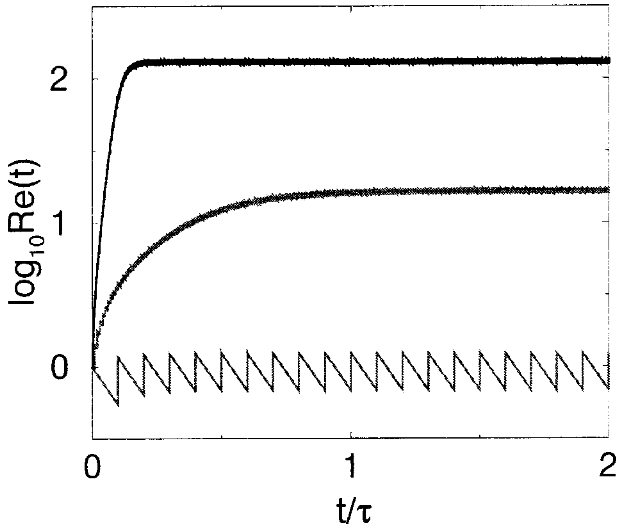

FIG. 1. Time series $\mathrm{Re}(t)$ for fixed $A=0.1$ and three kicking frequencies $f=10/\tau$, $f=100/\tau$, and $f=500/\tau$, bottom to top. The corresponding (upper) saturation levels are $\mathrm{Re}_u^{sat}=1.3$, $\mathrm{Re}_u^{sat}=17.3$, and $\mathrm{Re}_u^{sat}=136$, respectively.

treated equally well. The result of Ref. [4] is that for given initial Reynolds number Re_i, the time dependence of $\mathrm{Re}(t)$ [defined as in Eq. (4)] is determined by the inverse function of

$$\frac{t(\mathrm{Re})}{\tau}=\frac{3}{c_{\epsilon,\infty}}[F(\mathrm{Re})-F(\mathrm{Re}_i)], \tag{6}$$

where $\tau=L^2/\nu$ and $F(\mathrm{Re})$ is given by

$$F(\mathrm{Re})=\frac{1}{2\,\mathrm{Re}^2}\{-\gamma+\sqrt{\gamma^2+\mathrm{Re}^2}\}+\frac{1}{2\gamma}\log\left\{\frac{\gamma+\sqrt{\gamma^2+\mathrm{Re}^2}}{\mathrm{Re}}\right\}. \tag{7}$$

$c_{\epsilon,\infty}=(6/b)^{3/2}$ (in this theory) is the dimensionless energy dissipation in the large Reynolds number limit, $\gamma=9/c_{\epsilon,\infty}$, and b is the Kolmogorov constant [10], which is the only free parameter in the theory of Ref. [4]. From experiment [11] $b=6.0$, a value that we take in all calculations here. Consequently, $c_{\epsilon,\infty}=1.0$ and $\gamma=9.0$. Rather than the Reynolds number [Eq. (4)] one could also give the Taylor-Reynolds number Re_λ [4],

$$\mathrm{Re}_\lambda=\sqrt{\frac{15\,\mathrm{Re}^2}{c_{\epsilon,\infty}(\gamma+\sqrt{\gamma^2+\mathrm{Re}^2})}}. \tag{8}$$

Equations (5)–(7) with $t=\Delta t$ (the time between successive kicks) fully define the present model. The two main physical parameters in the model are the kicking strength A and the kicking frequency $f=1/\Delta t$. The third physical parameter is Re_{lam}, the minimal Reynolds number after a kick. We pick $\mathrm{Re}_{lam}=1$ throughout.

Figure 1 shows $\mathrm{Re}(t)$ for fixed A and three different kicking frequencies f for the initial Reynolds number $\mathrm{Re}_0=\mathrm{Re}_{lam}=1$. During each cycle there is a kick $\mathrm{Re}_0\to\mathrm{Re}_1$ according to Eq. (5) and a subsequent decay according to Eq. (6). Overall, there is growth up to some saturation level $\mathrm{Re}^{sat}(A,f)$, achieved after $t_{sat}(A,f)$. In this saturation state,

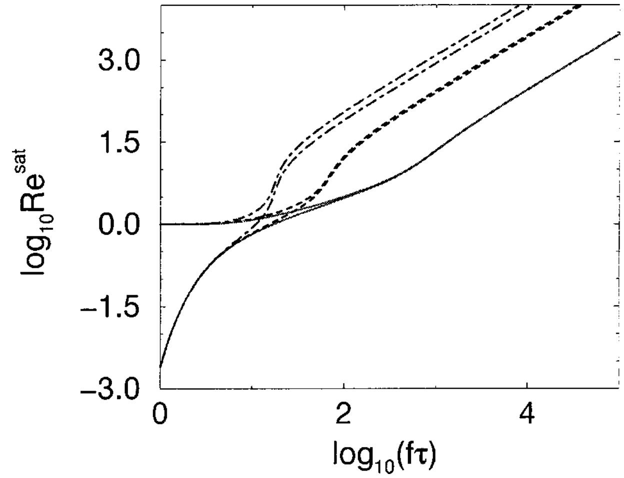

FIG. 2. Saturation level Re_l^{sat} (lower curve of pair) and Re_u^{sat} (upper curve of pair) as a function of f for three different kicking strengths $A=0.01$ (solid), $A=0.1$ (dashed), and $A=0.5$ (dashed-dotted), bottom to top. Below $\mathrm{Re}_u^{sat}\sim\gamma=9.0$ the excited state is laminar, above $\mathrm{Re}_u^{sat}\sim 9$ it is turbulent.

energy input and loss through decay in Δt balance, and the degree of excitation fluctuates between a lower saturation level Re_l^{sat} and an upper saturation level

$$\mathrm{Re}_u^{sat}=\mathrm{Re}_l^{sat}\sqrt{1+2A+(\mathrm{Re}_{lam}/\mathrm{Re}_l^{sat})^2}. \tag{9}$$

The (lower) level of saturation $\mathrm{Re}_l^{sat}(A,f)$ is given through the implicit equation

$$\frac{1}{\tau f}=\frac{3}{c_{\epsilon,\infty}}\left[F(\mathrm{Re}_l^{sat})-F\left(\mathrm{Re}_l^{sat}\sqrt{1+2A+\left(\frac{\mathrm{Re}_{lam}}{\mathrm{Re}_l^{sat}}\right)^2}\right)\right]. \tag{10}$$

For large Re_u^{sat}, $\mathrm{Re}_l^{sat}\gg\gamma$ one has the explicit result

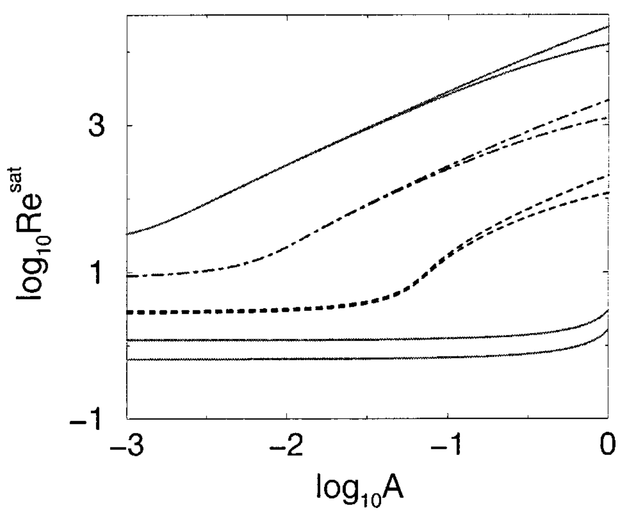

FIG. 3. Re_u^{sat} and Re_l^{sat} for $f=10$, 10^2, 10^3, 10^4, bottom to top. The increasing difference between Re_u^{sat} and Re_l^{sat} at the right edge of the figure, has its origin in the breakdown of the requirement $A\ll 1$.

$$\mathrm{Re}_u^{sat} = \frac{3\tau f}{c_{\epsilon,\infty}}(\sqrt{1+2A}-1) \approx \frac{3\tau}{c_{\epsilon,\infty}} A f. \tag{11}$$

In Fig. 2 we show a log-log plot of $\mathrm{Re}_{l,u}^{sat}$ as a function of f for different kicking strength A. Two regimes are seen: (i) For small $\mathrm{Re}_{l,u}^{sat} \lesssim \gamma$ a laminar regime in which after the decay the kick always brings back the level of excitation to the laminar value $\mathrm{Re}_{lam}=1$. (ii) For large $\mathrm{Re}_{l,u}^{sat} \gtrsim \gamma$ we have a turbulent scaling regime with $\mathrm{Re}_u^{sat} \propto f$. The transition from the laminar to the turbulence regime takes place around

$$f_{trans} = \frac{3}{\tau A}. \tag{12}$$

Similarly, these two regimes are also seen in Fig. 3 where we plotted $\mathrm{Re}_{l,u}^{sat}$ vs A. In the turbulent regime for large $\mathrm{Re}_{l,u}^{sat}$, it is $\mathrm{Re}_{l,u}^{sat} \propto A$.

We now come to the important question of how intermittency effects [12] change the exponents calculated within this mean-field theory. In Ref. [13] intermittency effects have been included into the mean-field theory of Ref. [4] on a phenomenological basis. One possibility for their effect is that the dimensionless energy dissipation rate c_ϵ becomes slightly Reynolds number dependent even in the large Reynolds number limit [13], $c_\epsilon \propto \mathrm{Re}^{-\kappa}$, with $\kappa = (9/8)\delta\zeta_2/(1+3\delta\zeta_2/8)$. Here, $\delta\zeta_2 \approx 0.03$ is the experimentally found deviation from the classical scaling exponent $\zeta_2 = 2/3$ of the second-order velocity structure function. The consequences of this small ($\kappa \approx 0.03$) scaling correction can straightforwardly be embodied in the present mean-field approach to periodically kicked turbulence. The result is that in the turbulent regime the saturation level now obeys

$$\mathrm{Re}_{l,u}^{sat} \propto (Af)^{1/(1-\kappa)} \tag{13}$$

rather than Eq. (11). Equation (13) may offer a new and independent way to experimentally determine intermittency exponents.

Another effect related to intermittency is the following: We expect the total energy to build up again and again over time scales larger than Δt and then to suddenly drop because an energy pulse is traveling downscale. Such behavior has been observed in numerical simulations of periodically remeshed homogeneous shear flow [3] and in simulations of periodically kicked shell models of turbulence [results to be published]. As based on a mean-field theory the model of this paper is only applicable to the mean energy and not to these fluctuations.

We now come back to an experimental realization. It will be easier to perform experiments in a closed system rather than in a channel flow. A particular suited experimental setup for periodically kicked turbulence would be the flow in a cylinder between two counter-rotating disks [6,14]. Also, Rayleigh-Benard convection may be well suited. Here, as an example, we take Taylor-Couette flow [15]: If the radii of the inner and outer cylinder are similar, the energy input will still roughly follow Eq. (5). Take water as a fluid that has $\nu = 10^{-6}\ \mathrm{m^2/s}$ and take $L = 1$ cm, then $\tau = 100$ s. Realistically achievable kick strengths would be $A = \Delta t_{kick} U/(2L) \sim (0.1\ \mathrm{s})(0.1\ \mathrm{m/s})/(2\times 1\ \mathrm{cm}) = 0.5$, e.g., the whole range $A<1$ in which the theory is applicable. For this value the scaling regime sets in at $f_{trans} = 0.06$ Hz. Roughly two decades of scaling are necessary to explore intermittency corrections and to distinguish between Eqs. (11) and (13). That is, one has to go up to frequencies of around 6 Hz, which should be achievable. At these relatively large kicking frequencies, measurements can only reveal *instantaneous* values. To get statements on the averaged quantities dealt with in this paper, ensemble averaging is necessary. This is best done by repeatedly probing the flow at some fixed phase after the respective kick. Averaging over the results at phase 0^+ will give Re_u^{sat}, and averaging over the results at phase Δt^- will give Re_l^{sat}, etc.

We suggest to perform a periodically kicked turbulence experiment and to measure Re^{sat} as a function of both A and f. To our knowledge, it would be one of the first ones with some active control on the type of forcing. Immediate questions to ask are: Does the level of saturation for large Re indeed only depend on the product Af as suggested by Eqs. (11) and (13) or do boundary effects carry on into the strongly turbulent central regime and cause a more subtle relation? If so, an application of the so commonly used volume forcing for turbulence becomes more questionable. Do the (scaling) relations between the quantities introduced in this paper, e.g., $\mathrm{Re}_{l,u}^{sat}(f,A)$ offer a new way to measure intermittency effects? What modifications arise if forcing and decay do not decouple as assumed in this simple model?

The author thanks L. Biferale, M. Brenner, B. Eckhardt, and F. Toschi for discussions. This work is part of the research program of the Stichting voor Fundamenteel Onderzoek der Materie (FOM), which is financially supported by the Nederlandse Organisatie voor Wetenschappelijk Onderzoek (NWO). This research was also supported in part by the European Union under Contract No. HPRN-CT-2000-00162 and by the National Science Foundation under Grant No. PHY94-07194. We also thank the Institute of Theoretical Physics in Santa Barabara for its hospitality.

[1] M. Faraday, Philos. Trans. R. Soc. London **52**, 319 (1831).
[2] L. A. Crum, Phys. Today **47**, 22 (1994); B. P. Barber, R. A. Hiller, R. Löfstedt, S. J. Putterman, and K. R. Weninger, Phys. Rep. **281**, 65 (1997).
[3] A. Pumir, Phys. Fluids **8**, 3112 (1996).
[4] D. Lohse, Phys. Rev. Lett. **73**, 3223 (1994).
[5] M. Smith, R. J. Donelly, N. Goldenfeld, and W. F. Vinen, Phys. Rev. Lett. **71**, 2583 (1993).
[6] R. Labbe, J. F. Pinton, and S. Fauve, J. Phys. II **6**, 1099 (1996).
[7] R. Camussi, S. Ciliberto, and C. Baudet, Phys. Rev. E **56**, 6181 (1997).
[8] Th. Gebhardt, S. Grossmann, M. Holthaus, and M. Löhden, Phys. Rev. E **51**, 360 (1995).
[9] H. Effinger and S. Grossmann, Z. Phys. B: Condens. Matter

66, 289 (1987).
[10] A. S. Monin and A. M. Yaglom, *Statistical Fluid Mechanics* (MIT Press, Cambridge, MA, 1975).
[11] K. R. Sreenivasan, Phys. Fluids **7**, 2778 (1995).
[12] U. Frisch, *Turbulence* (Cambridge University Press, Cambridge, 1995).
[13] S. Grossmann, Phys. Rev. E **51**, 6275 (1995); G. Stolovitzky and K. R. Sreenivasan, *ibid.* **52**, 3242 (1995).
[14] G. Zocchi, P. Tabeling, J. Maurer, and H. Willaime, Phys. Rev. E **50**, 3693 (1994); for a review on this type of turbulence see P. J. Zandbergen and D. Dijkstra, Annu. Rev. Fluid Mech. **26**, 137 (1987).
[15] G. S. Lewis and H. L. Swinney, Phys. Rev. E **59**, 5457 (1999).