Computational Statistics (2008) 23:41–62
DOI 10.1007/s00180-007-0067-7

ORIGINAL PAPER

Simultaneous confidence bands for the integrated hazard function

Anna Dudek · Maciej Goćwin · Jacek Leśkow

Accepted: 20 September 2006 / Published online: 4 July 2007

Abstract The construction of the simultaneous confidence bands for the integrated hazard function is considered. The Nelson–Aalen estimator is used. The simultaneous confidence bands based on bootstrap methods are presented. Four methods of construction of such confidence bands are proposed. The weird and conditional bootstrap methods are used for resampling. Simulations are made to compare the actual coverage probability of the bootstrap and the asymptotic simultaneous confidence bands. It is shown that the equal-tailed bootstrap confidence band has the coverage probability closest to the nominal one. We also present application of our confidence bands to the data regarding survival after heart transplant.

1 Introduction and summary

In biomedical settings, the multiplicative intensity model introduced by Aalen has many applications. This is a model for point processes observed on a fixed time interval for which the stochastic intensity is decomposed into deterministic function $\alpha(t)$ and stochastic process $Y(t)$. The $\alpha(t)$ function may be considered as an individual force of transition at time t and $Y(t)$ as a number at risk just before time t.

This research was partly supported by AGH grant No. 10.420.03.

A. Dudek (✉) · M. Goćwin · J. Leśkow
Department of Applied Mathematics, AGH University of Science and Technology,
Al. Mickiewicza 30, 30-059 Cracow, Poland
e-mail: aedudek@uci.agh.edu.pl

M. Goćwin
e-mail: gocwin@uci.agh.edu.pl

J. Leśkow
e-mail: leskow@wsb-nlu.edu.pl

In broad terms what makes survival data special is the presence of censored data. To analyze such data by the multiplicative intensity model a general assumption of independent censoring is required, which means that at any time t the survival experience in the future is not statistically altered by censoring and survival experience in the past. The censoring mechanism is modelled by Y process and has not any influence on the α function.

In the survival analysis the most interesting is to estimate the survivor function and the integrated hazard function. In this paper we consider the latter, which is estimated by the Nelson–Aalen estimator. An interpretation of this estimator is difficult without construction of some confidence intervals. From our perspective, the pointwise intervals are not totaly satisfactory while one wants to construct confidence region for the whole curve simultaneously for all points.

The construction of the simultaneous confidence bands is difficult since we need the uniform consistency property. However, such confidence bands are badly needed in practical applications. For example, in the works related with ours like in the papers of Cowling et al. (1996) and Snethlage (1999) but also in the time series analysis (Leśkow and Wronka 2004) and the nonparametric regression (Loader 1993).

The formula for the asymptotic confidence interval for the Nelson–Aalen estimator is known, however, it is very complicated and does not work well for small samples (see Andersen et al. 1993). An alternative approach is through the use of bootstrap methods. This idea was first introduced by Efron and Tibshirani (1993) and later developed in many papers (also in cited above). Bootstrapping of the point processes is not yet fully explored. Some results are presented in Braun and Kulperger (1998, 2003). The Poisson process context is treated in the paper Cowling et al. (1996), however these methods cannot be easily adapted to the multiplicative intensity model.

The aim of our work is the construction of the bootstrap simultaneous confidence bands for the Nelson–Aalen estimator. We want to compare constructed bootstrap regions with the asymptotic ones. We make simulations to check if the actual coverage probability is close to a nominal one. In our calculations we use the weird and the conditional bootstrap method.

We show that for the small samples the weird bootstrap models have much better coverage probabilities. Not only the actual coverage probabilities of the bootstrap simultaneous confidence bands are very close to nominal ones but also the left- and right-tail error probabilities are almost equal.

Our paper is organized in the following way. Section 2 contains a short survey of basic results related to the Nelson–Aalen estimator and the bootstrap for point processes. Section 3 is dedicated to construction of simultaneous confidence bands for the estimator considered. A practical example related to heart transplant study is included in Sect. 4, while Sect. 5 contains additional numerical results. Conclusions and open questions are presented in Sect. 6.

2 Problem formulation

In our paper we construct the bootstrap simultaneous confidence bands for the integrated intensity function. We use the weird and conditional bootstrap introduced in

Andersen et al. (1993) and Davison and Hinkley (1999). We compare our results with those presented in Andersen et al. (1993) and Bie et al. (1987). Application of bootstrap is well motivated in the small sample case and when censoring mechanism is quite complex. Moreover, the standard asymptotic theory provides confidence intervals that are quite difficult to apply. To construct bootstrap simultaneous confidence bands we applied one of the methods proposed in Cowling et al. (1996).

While defining our problem we follow (Andersen et al. 1993, p 176). We consider a continuous-time interval $\mathcal{T}$ which may be of the form $[0, \tau]$ or $[0, \tau)$ for a given terminal time τ, $0 < \tau \leq \infty$. Let $(\Omega, \mathcal{F})$ be a measurable space equipped with a filtration $(\mathcal{F}_t, t \in \mathcal{T})$. We define on $(\Omega, \mathcal{F})$ a counting process $\mathbf{N} = (N(t), t \in \mathcal{T})$ adapted to the filtration such that its stochastic intensity function λ is of the form $\lambda(t) = \alpha(t)Y(t)$, where α is nonnegative deterministic function and Y is a predictable process. For example, we can consider an initial group Y_0 of patients with cancer after some medical treatment. Although the patients enter the study at different calendar times, we observe only their time since operation. In this case $\alpha(t)$ is the individual intensity of death and $Y(t)$ is the number at risk at the moment of time t e.g., number of patients who lived till time t. For a practical example see Sect. 4.

The only assumption we have to make about α is its integrability,

$$\int_0^t \alpha(s)ds < \infty \quad \text{for all } t \in \mathcal{T}.$$

We consider the Nelson–Aalen estimator $\widehat{A}$ for

$$A(t) = \int_0^t \alpha(s)ds$$

which is of the form

$$\widehat{A}(t) = \sum_{j:T_j \leq t} \frac{1}{Y(T_j)},$$

where T_j are jump times.

We define an estimator for the mean squared error function as

$$\widehat{\sigma}^2(t) = \sum_{j:T_j \leq t} \frac{1}{Y^2(T_j)}.$$

Under the suitable assumptions the Nelson–Aalen estimator is uniformly consistent on compact intervals (see Andersen et al. 1993, p 190), which means:

$$\sup_{s \in [0,t]} |\widehat{A}^{(n)}(s) - A(s)| \xrightarrow{p} 0 \quad \text{as } n \to \infty \quad \text{for } t \in \mathcal{T}.$$

The asymptotic distribution of the Nelson–Aalen estimator can be obtained from Rebolledo's martingale central limit theorem (for details see Andersen et al. 1993, page 190). It should be pointed out that the problem of constructing simultaneous confidence bands requires a version of the functional central limit theorem for the cumulative intensity function. Such results can be found in (Andersen et al. 1993, p 263), however, the limiting distribution is quite difficult to apply in practice. Moreover, it is still unknown what form of the functional central limit theorem can be established for α alone. (See also Sect. 6 for additional remarks regarding this problem).

The results above can be used to construct pointwise confidence intervals and simultaneous confidence bands for $A(t)$ (Andersen et al. 1993). Unfortunately, formulae for the asymptotic distributions are very complicated. That is why we want to apply bootstrap methods to construct simultaneous confidence bands. Bootstrapping of counting processes is not easy because such processes are not based on i.i.d. samples. The problem is complex and, thus, the methods for the general case are not known. There are some results for the Poisson processes (see Cowling et al. 1996), however in this case one may get similar results without simulations (see Snethlage 1999). Some methods of bootstrapping point processes are also presented in Braun and Kulperger (1998, 2003).

One of the methods we apply in our paper is the weird bootstrap method. The idea is based on the fact that the asymptotic distribution of $a_n(\widehat{A} - A)$ has independent increments and $Var(d\widehat{A}(t)|\mathcal{F}_{t-}) = dA(t)(1-dA(t))/Y(t)$. The bootstraped version N^* of a point process N is generated by random sampling the number of jumps at original failure times from the binomial distribution with parameters $(Y(t), \Delta\widehat{A}(t))$. The conditional distribution of $\widehat{A}^* - \widehat{A}$, given N and Y, estimates the distribution of $\widehat{A} - A$.

For the proof of consistency of this method see (Andersen et al. 1993, p 220).

The word *weird* is not accidental. In every time point $t \in \mathcal{T}$ every individual at risk from the set $Y(t)$ has the same probability of a failure. However, the event at the time t does not exert any influence on any other time moment $s \in \mathcal{T}$.

The second sampling scheme that we consider is the conditional bootstrap method presented in Davison and Hinkley (1999). The idea of this method is first to generate independently failure time for every individual in the group from previously estimated failure distribution. The next step is to generate censoring times taking into account if original observation was censored. Finally we take minimum of these values.

The problem of bootstrapping point processes is not completely solved and quite challenging. Some partial solution are discussed in Cowling et al. (1996), Davison and Hinkley (1999), Braun and Kulperger (1998, 2003). In the next section we use this method of bootstrapping to construct the simultaneous confidence bands.

3 Simultaneous confidence bands

The Nelson–Aalen point estimator is difficult to interpret without some idea of its accuracy. Resolving this problem requires constructing confidence intervals or confidence bands. These bands are also quite interesting because of their hypothesis testing interpretation. We can think of confidence bands as a one-sample test statistics with a null hypothesis $A = A_0$ which is rejected at significance level θ if A_0 is not completely contained in the band. In this case pointwise confidence intervals are not satisfactory. That is why we introduce simultaneous confidence bands.

Definition 3.1 Confidence region

Let $\mathcal{B}$ denote a connected, nonempty, random subset of the rectangle $[0, \tau] \times [0, \infty)$, such that $\mathcal{B} \cap \{(x, y) : 0 \leq y < \infty\}$ is nonempty for each $x \in [0, \tau]$. We call $\mathcal{B}$ a confidence region for A over the set $S \in [0, \tau]$ with a coverage probability $(1 - \theta)$ if $P\{(x, A(x)) \in \mathcal{B}\ for\ all\ x \in S)\} = 1 - \theta$.

In our paper S is always an interval.

Simultaneous confidence bands may be constructed in many different ways. The authors of the book (Andersen et al. 1993, p 209) proposed two types of such bands: EP-band (equal precision band) and HW-band (Hall–Wellner band). These confidence bands are based on the asymptotic distribution of the Nelson–Aalen estimator on compact intervals which can be derived from the martingale central limit theorem.

Both EP- and HW-band for A on $[t_1, t_2]$ are of the form

$$\widehat{A}(s) \pm a_n^{-1} K_{q,\theta}(c_1, c_2) \left(1 + a_n^2 \widehat{\sigma}^2(s)\right) \Big/ q \left(\frac{a_n^2 \widehat{\sigma}^2(s)}{1 + a_n^2 \widehat{\sigma}^2(s)} \right)$$

with $K_{q,\theta}(c_1, c_2)$ being the upper percentile of the distribution of

$$\sup_{x \in [c_1, c_2]} |q(x) W^0(x)|,$$

where W^0 denotes the standard Brownian bridge.

The constants c_1 and c_2 can be approximated by

$$\widehat{c}_i = \frac{a_n^2 \widehat{\sigma}^2(t_i)}{1 + a_n^2 \widehat{\sigma}^2(t_i)},$$

where $a_n = \sqrt{n}$ is a normalizing factor and n is the number of individuals at study.

For EP-band q is chosen as $q_1(x) = \{x(1 - x)\}^{-1/2}$ which yields the confidence bands proportional to the pointwise ones. For HW-band q is chosen as $q_2(x) = 1$.

In both cases θ percentile of the asymptotic distribution are difficult to obtain. These bands also perform badly even with the sample size of 100–200 (Bie et al. 1987). Because of this reason one may consider some transformations to improve the approximation to the asymptotic distribution (Andersen et al. 1993, p 211).

To avoid such problems we consider bootstrap simultaneous confidence bands. The authors of the paper Cowling et al. (1996) proposed a few different methods of constructing these bands. In our calculations we use the weird bootstrap and the conditional bootstrap methods.

Below we present four bootstrap confidence bands.

1. The simplest bootstrap confidence band that we consider is the percentile bootstrap confidence region. We approximate

$$V(x) = \widehat{A}(x) - A(x), \quad x \in \mathcal{T}$$

by its bootstrap version

$$V^*(x) = \widehat{A}^*(x) - \widehat{A}(x), \quad x \in \mathcal{T}.$$

We define the region as

$$\mathcal{B}_1 = \left\{(x, y) : x \in S,\ \max\left[0, \widehat{A}(x) - t_1\right] \leq y \leq \widehat{A}(x) + t_1\right\},$$

where t_1 is chosen so that

$$P\left\{|V^*(x)| \leq t_1,\ \text{all } x \in S|N, Y\right\} = 1 - \theta.$$

This region is symmetric and has the constant width.

2. The square-root transformation is approximately variance stabilizing (for more details see Hall 1992). Therefore, instead of using the V statistic one may consider the statistic U defined by

$$U(x) = \sqrt{\widehat{A}(x)} - E\sqrt{\widehat{A}(x)}, \quad x \in \mathcal{T}$$

and its bootstrap version

$$U^*(x) = \sqrt{\widehat{A}^*(x)} - \sqrt{\widehat{A}(x)}, \quad x \in \mathcal{T}.$$

The region is of the form

$$\mathcal{B}_2 = \left\{(x, y) : x \in S,\ \left(\sqrt{\widehat{A}(x)} - t_2\right)^2 \leq y \leq \left(\sqrt{\widehat{A}(x)} + t_2\right)^2\right\},$$

where t_2 is chosen such that

$$P\{|U^*(x)| \leq t_2,\ \text{all } x \in S|N, Y\} = 1 - \theta.$$

In order to improve the second-order features the studentized statistics should be taken (see Hall 1992).

Our construction of bootstrap-t confidence regions for A is based on the bootstrap approximation

$$T^*(x) = \frac{\widehat{A}^*(x) - \widehat{A}(x)}{\widehat{\sigma}^*(x)}, \quad x \in \mathcal{T}$$

of

$$T(x) = \frac{\widehat{A}(x) - A(x)}{\widehat{\sigma}(x)}, \quad x \in \mathcal{T}.$$

For details see Andersen et al. (1993).

Below we present two bootstrap confidence bands which are constructed on the basis of T statistic:

3. Confidence region is defined by

$$\mathcal{B}_3 = \left\{(x, y) : x \in S,\ \max\left[0, \widehat{A}(x) - t_3\widehat{\sigma}(x)\right] \le y \le \widehat{A}(x) + t_3\widehat{\sigma}(x)\right\},$$

where t_3 is chosen such that

$$P\left\{|T^*(x)| \le t_3,\ \text{all } x \in S|N, Y\right\} = 1 - \theta.$$

The main feature of this region is that at the point x its width is proportional to $\widehat{\sigma}(x)$.

4. In many applications populations cannot be modelled via symmetric distributions. The only reasonable choice is a strongly skewed distribution. In all of the previous presented intervals, skewness was not taken into consideration. This has a quite negative impact on the coverage probability. To adjust for skewness of the distribution one could construct a region which the left- and right-tail error probabilities are equal. This kind of the region is of the form

$$\mathcal{B}_4 = \left\{(x, y) : x \in S,\ \max\left[0, \widehat{A}(x) - t_5\widehat{\sigma}(x)\right] \le y \le \widehat{A}(x) - t_4\widehat{\sigma}(x)\right\},$$

where t_4 and t_5 are chosen such that

$$P\left\{t_4 \le T^*(x) \le t_5,\ \text{all } x \in S|N, Y\right\} = 1 - \theta$$

and

$$P\left\{T^*(x) \le t_4,\ \text{all } x \in S|N, Y\right\} = P\left\{T^*(x) \ge t_5,\ \text{all } x \in S|N, Y\right\}.$$

In the next section we present an example of applying such bands.

4 Practical example

We take under the consideration the group of 64 patients after heart transplant. The data we use are taken from (Kalbfleisch and Prentice 2002, Appendix A, pp 387–389). In our approach, the risk is defined as the rejection of the transplant so the time between the operation and the rejection is considered. A total of 35 observations are censored. The censoring was present if patients were alive at the end of the study or lost to follow-up. The 95% confidence bands simultaneous with respect to the time argument were constructed in the time bandwidth between day 20 and day 1,200 of the observation. The construction of such confidence interval was based on Nelson–Aalen estimator. Figure 1 presents the Nelson–Aalen estimator together with HW and

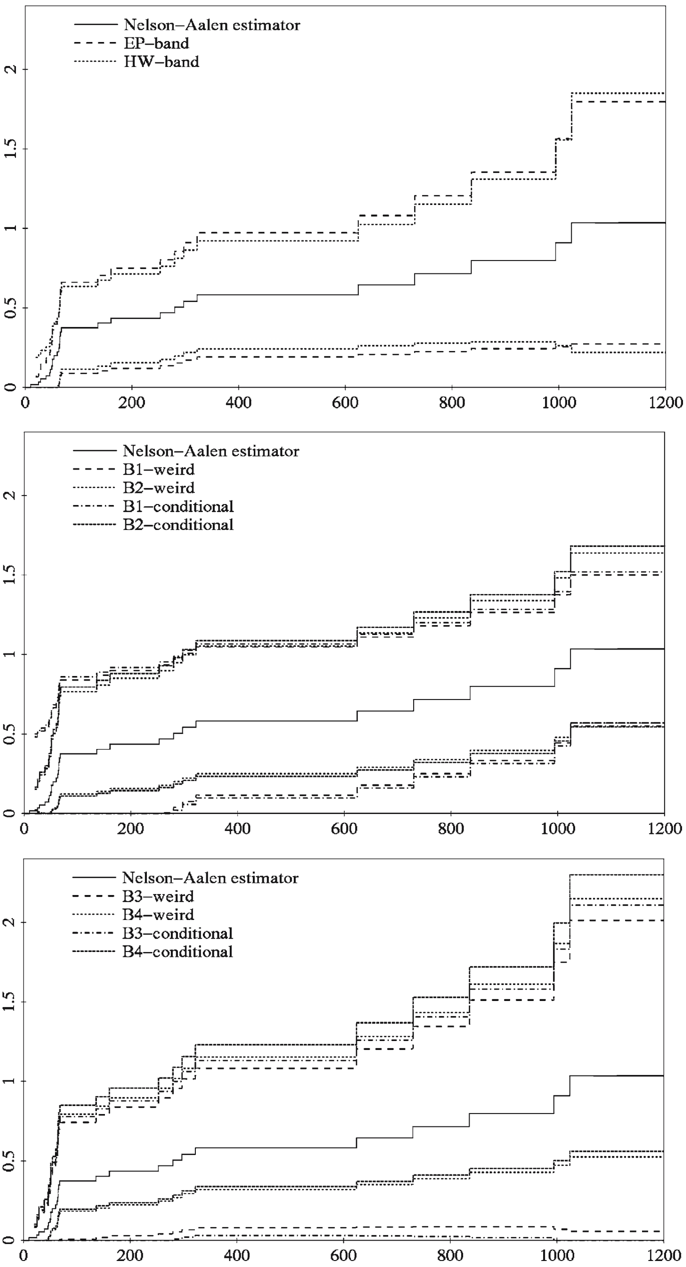

Fig. 1 Nelson–Aalen estimator with asymptotic bands (*top panel*) and bootstrap bands (*middle* and *bottom panels*)

EP bands (top panel), $\mathcal{B}1$ and $\mathcal{B}2$ (middle panel), $\mathcal{B}3$ and $\mathcal{B}4$ bootstrap simultaneous confidence bands (bottom panel). Note that $\mathcal{B}1$, $\mathcal{B}3$, EP and HW bands are symmetric. Only $\mathcal{B}2$ and $\mathcal{B}4$ are not symmetric. The upper bounds of $\mathcal{B}3$ are close to $\mathcal{B}4$ but are located slightly below. The lower bounds of $\mathcal{B}3$ are noticeably too low. This suggests that $\mathcal{B}3$ is too wide. Upper bounds of $\mathcal{B}1$ and $\mathcal{B}2$ are very close to each other and still lower than $\mathcal{B}4$. $\mathcal{B}1$, which has a constant width, is definitely too wide in the first part of the interval. Its upper bound is much lower then the others at the end of the considered time interval. HW and EP bands are close to each other but EP is significantly broader during the most part of the time interval. Moreover, $\mathcal{B}4$ is shifted upwards compared with the asymptotic simultaneous confidence bands.

Now we will verify the actual coverage probability for the considered bands.

5 Numerical results

Our aim is to compare the coverage probability for asymptotic and bootstrap simultaneous confidence bands. Our simulations are based on the multiplicative model for the intensity function $\lambda(t) = Y(t)\alpha(t)$. We concentrate on a few typical examples of the α function. We consider two schemes of generating process Y. In both cases we first choose the beginning value Y_0 (the number of individuals at risk). In the first method (later denoted by **C**) for every individual the time of failure and the time of censoring are sampled from nonhomogeneous Poisson process with the intensity $\alpha(t)$. Then, we take the minimum of these two values and we get the point process under consideration. Because both numbers are generated from the same distribution we get censoring about 50%.

In the second method (denoted later by **P**) we first generate jumps of process Y by sampling from exponential distribution with the mean value 0.3. The generated exponential times indicate when Y—the number of individuals at risk—decreases by 1. Then, we generate a nonhomogeneous Poisson process with the intensity $\alpha(t)Y(t)$.

For the latter method we cannot apply the conditional bootstrap, because for one individual there can be more than one failure and we do not have any information regarding censoring.

For our study we chose four functions:

$$\begin{aligned}
\alpha_1(t) &= 2,\\
\alpha_2(t) &= \frac{7}{6} + 10(t-0.5)^2,\\
\alpha_3(t) &= 2 + 8(t-0.5)^3,\\
\alpha_4(t) &= \frac{17}{6} - 10(t-0.5)^2.
\end{aligned}$$

Curves of such kind can be applied in biomedicine, insurance and demography. For example, the U-shaped functions may reflect behaviour of the intensity of death and the inverted U-shaped functions may describe the intensity of birth. These shapes are reflected in the equation of α_2 and α_4 functions.

We make simulations for the interval $S = [0.2; 0.8]$, the number of bootstrap resamples $B = 500$ and initial number at risk Y_0 : 15, 25, 35, 50, 75. In Tables 1, 2, 3, 4, 5, 6, 7, 8 and 9 we show the actual coverage probability (denoted later by ACP),

Table 1 Actual coverage probabilities for the model **C** when $\theta = 1\%$

Y_0	Method	Function											
		α_1			α_2			α_3			α_4		
15	HW	0.0	8.8	91.2	0.1	8.7	91.3	0.0	9.1	90.9	0.0	12.8	87.2
	EP	0.0	11.1	88.9	0.0	9.2	90.8	0.0	8.7	91.3	0.0	12.5	87.5
	$\mathcal{B}1$ cond	4.0	17.6	78.4	4.3	14.2	81.5	4.0	16.4	79.6	3.6	21.9	74.6
	$\mathcal{B}2$ cond	2.9	5.7	91.4	3.2	4.2	92.6	3.0	6.0	91.1	3.1	10.3	86.7
	$\mathcal{B}3$ cond	0.0	3.4	96.6	0.0	3.0	97.0	0.0	4.5	95.5	0.0	7.2	92.8
	$\mathcal{B}4$ cond	3.5	2.9	93.6	5.3	2.5	92.2	2.7	4.3	93.0	2.5	6.7	90.8
	$\mathcal{B}1$ weird	9.4	16.0	74.6	10.1	13.2	76.7	9.6	15.5	75.0	9.5	22.6	68.0
	$\mathcal{B}2$ weird	2.6	3.9	93.6	3.4	3.0	93.6	3.4	4.6	92.1	5.0	8.3	87.0
	$\mathcal{B}3$ weird	0.0	3.6	96.4	0.0	3.3	96.7	0.0	4.8	95.2	0.0	7.4	92.6
	$\mathcal{B}4$ weird	0.8	3.2	96.0	1.1	3.0	95.8	0.7	4.5	94.7	0.7	6.9	92.4
25	HW	0.0	4.8	95.2	0.0	5.0	95.0	0.0	4.4	95.6	0.0	5.9	94.1
	EP	0.0	6.1	93.9	0.0	5.5	94.5	0.0	5.5	94.5	0.0	8.5	91.5
	$\mathcal{B}1$ cond	3.3	10.9	85.9	2.8	9.6	87.6	2.9	9.8	87.4	3.0	12.8	84.3
	$\mathcal{B}2$ cond	3.0	4.2	92.8	2.9	3.8	93.3	2.8	3.4	93.8	2.7	5.7	91.7
	$\mathcal{B}3$ cond	0.0	1.0	99.0	0.0	1.2	98.8	0.0	0.9	99.1	0.0	1.6	98.4
	$\mathcal{B}4$ cond	2.5	0.6	96.8	3.9	0.7	95.3	2.0	0.4	97.5	1.8	1.4	96.8
	$\mathcal{B}1$ weird	9.0	10.2	80.9	7.8	9.1	83.1	8.1	8.9	82.9	8.5	13.3	78.2
	$\mathcal{B}2$ weird	3.6	3.5	92.9	3.8	3.3	92.9	3.6	2.7	93.8	3.4	5.0	91.6
	$\mathcal{B}3$ weird	0.0	1.0	99.0	0.0	1.2	98.8	0.0	0.8	99.2	0.0	1.6	98.4
	$\mathcal{B}4$ weird	0.7	0.7	98.7	0.9	0.7	98.4	0.6	0.4	98.9	0.5	1.2	98.3
35	HW	0.0	3.3	96.7	0.0	3.5	96.5	0.0	3.0	97.0	0.0	3.6	96.4
	EP	0.0	4.5	95.5	0.0	3.9	96.1	0.0	4.2	95.8	0.0	5.6	94.4
	$\mathcal{B}1$ cond	2.1	8.1	89.8	2.1	7.0	91.0	1.9	7.7	90.4	3.0	8.8	88.2
	$\mathcal{B}2$ cond	2.2	3.4	94.4	2.3	3.1	94.6	2.1	2.9	95.5	2.8	4.4	92.8
	$\mathcal{B}3$ cond	0.0	0.7	99.3	0.0	1.0	99.0	0.0	0.5	98.0	0.0	0.9	99.1
	$\mathcal{B}4$ cond	2.2	0.5	97.3	3.0	0.5	96.5	1.7	0.3	98.0	1.7	0.6	97.8
	$\mathcal{B}1$ weird	7.1	7.6	85.3	5.5	6.6	87.9	5.9	7.1	86.9	8.3	9.6	82.2
	$\mathcal{B}2$ weird	3.7	3.1	93.1	3.5	3.0	93.4	3.5	2.5	94.0	4.0	4.2	91.8
	$\mathcal{B}3$ weird	0.0	0.7	99.3	0.0	0.8	99.2	0.0	0.5	99.5	0.0	0.7	99.3
	$\mathcal{B}4$ weird	0.7	0.4	98.9	0.7	0.5	98.8	0.6	0.2	99.2	0.6	0.4	99.0
50	HW	0.0	2.5	97.5	0.0	2.6	97.4	0.0	2.4	97.8	0.0	2.7	97.3
	EP	0.0	3.2	96.8	0.0	2.9	97.1	0.0	3.3	96.7	0.0	3.9	96.1
	$\mathcal{B}1$ cond	1.9	6.0	92.8	1.3	5.1	93.6	1.5	5.4	93.1	2.5	7.4	90.1
	$\mathcal{B}2$ cond	2.2	2.6	95.2	1.6	2.4	96.0	1.8	2.2	96.0	2.6	4.0	93.3
	$\mathcal{B}3$ cond	0.0	0.5	99.5	0.0	0.6	99.4	0.0	0.5	99.5	0.0	0.7	99.3
	$\mathcal{B}4$ cond	2.1	0.3	97.7	2.5	0.3	97.2	1.8	0.3	98.0	1.3	0.4	98.3
	$\mathcal{B}1$ weird	5.5	5.9	88.6	3.0	4.9	92.1	3.5	5.3	91.2	7.4	7.7	84.9
	$\mathcal{B}2$ weird	4.0	2.7	93.3	2.6	2.5	94.9	3.0	2.4	94.6	4.7	3.9	91.4
	$\mathcal{B}3$ weird	0.0	0.5	99.5	0.0	0.7	99.3	0.0	0.3	99.7	0.0	0.6	99.4

Table 1 continued

Y_0	Method	Function											
		α_1			α_2			α_3			α_4		
	$\mathcal{B}4$ weird	0.7	0.3	99.1	0.8	0.3	98.8	0.7	0.1	99.2	0.7	0.3	99.0
75	HW	0.0	2.0	98.0	0.0	1.7	98.3	0.0	1.9	98.1	0.0	1.8	98.2
	EP	0.0	2.6	97.3	0.0	2.2	97.8	0.0	2.5	97.5	0.0	2.8	97.2
	$\mathcal{B}1$ cond	0.9	4.5	94.6	0.6	3.7	95.8	0.8	3.7	95.5	1.4	6.0	82.7
	$\mathcal{B}2$ cond	1.2	2.2	96.7	0.8	1.8	97.4	1.0	1.8	97.1	1.5	3.0	85.5
	$\mathcal{B}3$ cond	0.0	0.5	99.5	0.0	0.5	99.5	0.0	0.4	99.6	0.0	0.4	99.6
	$\mathcal{B}4$ cond	1.8	0.3	97.9	2.3	0.3	97.3	1.6	0.2	98.2	1.3	0.2	98.5
	$\mathcal{B}1$ weird	2.1	4.4	93.5	0.9	3.5	95.6	1.3	3.8	95.0	4.4	5.3	90.4
	$\mathcal{B}2$ weird	2.2	2.4	95.4	1.3	1.9	96.8	1.6	2.1	96.3	3.4	2.9	93.7
	$\mathcal{B}3$ weird	0.0	0.5	99.5	0.0	0.6	99.4	0.0	0.4	99.6	0.0	0.3	99.7
	$\mathcal{B}4$ weird	0.7	0.2	99.1	0.8	0.3	99.0	0.7	0.2	99.2	0.6	0.2	99.2

Column I and II: left- and right-tail error probabilities
Column III: actual coverage probability when nominal is 99%

when the nominal coverage probability is 0.9, 0.95, 0.99 and the number of iterations is equal to 10,000. For every Y_0, α_i function ($i = 1 \ldots 4$) and method of construction of the confidence region, the first and the second number in these tables are the left- and right-tail error probabilities and the third is the actual coverage probability (all probabilities are measured in percentage).

First we discuss results for the nominal coverage probability $1 - \theta = 99\%$ which are presented in Tables 1 and 2. As we expected, $\mathcal{B}1$ behaves hopelessly independently on bootstrapping method and model chosen. This happens because this kind of region is of the constant width and its construction does not guarantee the variance stabilization. It is therefore of no interest to write about it in more details.

For $Y_0 = 15$ (Tables 1, 2) all considered confidence bands are too narrow. The asymptotic confidence bands EP and HW have ACP about 10% lower than the nominal one. Maybe, the behavior of $\mathcal{B}3$ $cond$, $\mathcal{B}3$ $weird$ and $\mathcal{B}4$ $weird$ is not completely satisfactory, but their ACPs are much better than asymptotic bands (but still about 5% lower than nominal ones) and both kinds of $\mathcal{B}2$. It is worth noticing that $\mathcal{B}2$ $cond$ and $\mathcal{B}2$ $weird$ behave also slightly better then the asymptotic bands.

For $Y_0 = 25$ three above mentioned bootstrap confidence regions ($\mathcal{B}3$ $cond$, $\mathcal{B}3$ $weird$ and $\mathcal{B}4$ $weird$) perform very well. Their coverage probabilities do not differ from nominal ones more then 0.5%. EP and HW are still too narrow and have ACP about 4–5% too low.

For $Y_0 \geq 35$ $\mathcal{B}4$ $weird$ is definitely the best choice. $\mathcal{B}3$ $cond$ and $\mathcal{B}3$ $weird$, which seemed to be quite good for low values of Y_0, are constantly too wide in this case. HW-band is as good as $\mathcal{B}4$ $weird$ not before $Y_0 = 75$, which is still not true for EP. $\mathcal{B}2$ $cond$ and $\mathcal{B}2$ $weird$ are constantly too narrow for all values of Y_0.

In the case of $\theta = 5\%$ (Tables 3, 4) final conclusions are similar.

Table 2 Actual coverage probabilities for the model **P** when $\theta = 1\%$

Y_0	Method	Function											
		α_1			α_2			α_3			α_4		
15	HW	0.0	7.7	92.3	0.0	5.8	94.2	0.0	9.4	90.6	0.0	11.5	88.5
	EP	0.0	8.5	91.5	0.0	6.1	93.9	0.0	8.7	91.3	0.0	10.9	89.1
	$\mathcal{B}1$ weird	4.0	11.4	84.6	3.8	8.8	87.4	4.4	13.5	82.1	4.0	15.6	80.5
	$\mathcal{B}2$ weird	2.1	3.3	94.6	2.3	2.4	95.4	2.0	3.8	94.2	2.1	6.2	91.7
	$\mathcal{B}3$ weird	0.0	2.7	97.3	0.0	1.8	98.3	0.0	3.8	96.2	0.0	6.0	94.0
	$\mathcal{B}4$ weird	0.9	1.9	97.2	0.9	1.1	98.1	0.9	3.2	95.9	1.1	5.3	93.6
25	HW	0.0	3.2	96.8	0.0	3.6	96.3	0.0	4.4	95.5	0.0	4.4	95.6
	EP	0.0	3.8	96.2	0.0	3.8	96.2	0.0	5.9	94.1	0.0	5.8	94.2
	$\mathcal{B}1$ weird	3.3	5.3	91.4	3.0	5.6	91.4	3.1	6.8	90.1	3.3	6.6	90.1
	$\mathcal{B}2$ weird	1.9	1.8	96.3	2.3	2.1	95.6	2.1	1.9	96.0	1.9	2.1	96.0
	$\mathcal{B}3$ weird	0.0	0.9	99.1	0.0	1.2	98.8	0.0	1.4	98.6	0.0	1.4	98.6
	$\mathcal{B}4$ weird	1.0	0.4	98.5	0.9	0.5	98.6	0.8	1.0	98.2	0.9	0.8	98.3
35	HW	0.0	2.5	97.5	0.1	3.1	96.8	0.0	2.7	97.3	0.0	2.4	97.6
	EP	0.0	3.3	96.7	0.1	3.4	96.5	0.0	3.7	96.3	0.0	3.5	96.5
	$\mathcal{B}1$ weird	1.4	3.6	95.0	2.4	3.9	93.8	2.1	4.1	93.8	2.2	3.6	94.2
	$\mathcal{B}2$ weird	1.1	1.6	97.3	2.3	1.7	96.0	1.9	1.5	96.5	1.7	1.1	97.2
	$\mathcal{B}3$ weird	0.0	0.5	99.5	0.1	1.0	98.9	0.0	0.9	99.1	0.0	0.6	99.4
	$\mathcal{B}4$ weird	1.0	0.1	98.9	0.8	0.4	98.8	0.8	0.4	98.8	0.9	0.4	98.7
50	HW	0.1	1.7	98.2	0.1	1.5	98.4	0.1	1.7	98.2	0.0	1.6	98.4
	EP	0.1	2.3	97.6	0.1	1.8	98.1	0.1	2.5	97.5	0.0	2.4	97.6
	$\mathcal{B}1$ weird	1.3	2.7	96.0	1.1	2.0	97.0	1.1	2.4	96.4	1.1	2.4	96.5
	$\mathcal{B}2$ weird	1.6	1.3	97.1	1.3	0.9	97.8	1.1	0.9	98.0	1.1	0.8	98.1
	$\mathcal{B}3$ weird	0.1	0.8	99.1	0.1	0.8	99.1	0.0	0.4	99.5	0.0	0.4	99.5
	$\mathcal{B}4$ weird	0.8	0.4	98.8	0.9	0.4	98.7	0.8	0.2	99.0	0.7	0.2	99.1
75	HW	0.1	1.2	98.8	0.1	1.1	98.8	0.0	1.3	98.6	0.1	1.3	98.6
	EP	0.1	1.4	98.5	0.1	1.2	98.7	0.0	1.6	98.4	0.1	1.9	98.0
	$\mathcal{B}1$ weird	0.5	1.4	98.1	0.6	1.4	98.0	0.5	1.4	98.1	0.5	1.3	98.2
	$\mathcal{B}2$ weird	0.7	0.8	98.5	0.8	0.8	98.5	0.5	0.8	98.7	0.5	0.7	98.8
	$\mathcal{B}3$ weird	0.1	0.7	99.3	0.1	0.8	99.0	0.0	0.6	99.4	0.0	0.5	99.5
	$\mathcal{B}4$ weird	0.8	0.4	98.8	0.9	0.4	98.8	0.8	0.2	99.0	0.9	0.1	99.0

Column I and II: left- and right-tail error probabilities
Column III: actual coverage probability when nominal is 99%

For $Y_0 = 15$ $\mathcal{B}3$ *cond* and $\mathcal{B}3$ *weird* are even better than HW and EP but still have ACP about 3–5% too low. $\mathcal{B}4$ *weird* is only a little worse.

For all values of Y_0 $\mathcal{B}2$ *cond* and $\mathcal{B}2$ *weird* perform badly in the model **C**. Their ACP is about 10% (for higher values of Y_0) to 15% ($Y_0 = 15$) lower then the nominal one. In the model **P** $\mathcal{B}3$ *weird* and $\mathcal{B}4$ *weird* perform significantly better than $\mathcal{B}2$ *weird*, although the latter performs here better then in the model **C**. In both models

Table 3 Actual coverage probabilities for the model **C** when $\theta = 5\%$

Y_0	Method	Function											
		α_1			α_2			α_3			α_4		
15	HW	0.1	12.4	87.6	0.2	13.4	86.4	0.1	12.7	87.3	0.0	15.8	84.2
	EP	0.0	15.0	85.0	0.0	14.3	85.7	0.0	16.0	84.0	0.0	15.8	84.2
	$\mathcal{B}1$ cond	10.0	24.0	66.0	9.6	21.3	69.1	9.7	22.8	67.7	10.8	28.6	61.1
	$\mathcal{B}2$ cond	8.0	12.1	80.0	8.4	10.7	80.9	7.8	11.7	80.6	8.2	16.0	76.0
	$\mathcal{B}3$ cond	0.0	8.3	91.7	0.0	7.6	92.4	0.0	7.9	92.1	0.0	10.6	89.4
	$\mathcal{B}4$ cond	8.5	5.3	86.3	10.7	5.0	84.3	7.7	5.9	86.4	7.3	8.4	84.4
	$\mathcal{B}1$ weird	18.2	22.9	59.0	16.8	20.1	63.1	17.4	22.2	60.7	19.2	29.0	52.5
	$\mathcal{B}2$ weird	9.1	10.9	80.0	10.1	9.8	80.1	8.4	10.3	81.4	8.5	14.6	77.1
	$\mathcal{B}3$ weird	0.0	9.8	90.1	0.0	8.9	91.1	0.0	9.2	90.8	0.0	11.6	88.4
	$\mathcal{B}4$ weird	4.0	5.8	90.2	4.5	5.8	89.7	3.8	6.6	89.6	3.9	9.0	87.1
25	HW	0.0	8.3	91.7	0.1	9.1	90.8	0.0	7.0	93.0	0.0	8.6	91.4
	EP	0.0	10.6	89.4	0.0	9.8	90.2	0.0	9.8	90.2	0.0	11.5	88.5
	$\mathcal{B}1$ cond	8.2	17.6	74.2	7.3	15.3	77.4	8.0	15.6	76.5	8.9	19.9	71.3
	$\mathcal{B}2$ cond	7.8	9.3	82.8	7.2	8.6	84.2	7.7	7.8	84.5	8.4	10.7	81.0
	$\mathcal{B}3$ cond	0.0	4.2	95.8	0.0	4.5	95.4	0.0	3.8	96.2	0.0	5.3	94.7
	$\mathcal{B}4$ cond	7.6	2.2	90.2	9.0	2.4	88.6	7.0	2.3	90.8	6.4	2.6	91.1
	$\mathcal{B}1$ weird	14.9	17.2	67.9	11.8	14.8	73.4	13.5	15.0	71.5	16.9	20.0	63.2
	$\mathcal{B}2$ weird	10.2	9.2	80.7	9.6	8.4	82.0	10.0	7.4	82.6	10.5	10.5	79.0
	$\mathcal{B}3$ weird	0.0	4.7	95.3	0.1	4.9	95.0	0.0	4.1	95.9	0.0	5.5	94.5
	$\mathcal{B}4$ weird	3.6	2.4	94.0	3.8	2.5	93.8	3.5	2.2	94.3	3.3	2.4	94.3
35	HW	0.1	7.0	93.0	0.1	6.6	93.3	0.0	5.8	94.2	0.0	6.4	93.6
	EP	0.0	8.7	91.3	0.0	7.2	92.8	0.0	8.1	91.9	0.0	8.8	91.2
	$\mathcal{B}1$ cond	6.8	14.3	79.0	5.2	12.2	82.6	5.4	12.7	82.0	7.9	15.5	76.6
	$\mathcal{B}2$ cond	7.0	8.2	84.8	6.0	6.6	87.4	6.2	6.7	87.2	7.8	8.8	83.4
	$\mathcal{B}3$ cond	0.0	3.5	96.5	0.0	3.4	96.6	0.0	2.7	97.3	0.0	3.5	96.5
	$\mathcal{B}4$ cond	6.8	1.8	91.5	8.8	1.7	89.5	6.6	1.3	92.1	5.7	3.5	92.6
	$\mathcal{B}1$ weird	12.4	14.1	73.5	8.0	11.9	80.1	8.6	12.4	79.0	14.5	15.8	69.7
	$\mathcal{B}2$ weird	10.4	8.2	81.4	8.0	6.7	85.3	8.3	6.6	85.1	10.9	8.8	80.3
	$\mathcal{B}3$ weird	0.0	3.8	96.3	0.0	3.8	96.1	0.0	2.9	97.1	0.0	3.2	96.8
	$\mathcal{B}4$ weird	3.2	1.8	95.1	3.7	1.9	94.3	3.3	1.1	95.6	3.1	1.7	95.2
50	HW	0.1	5.3	94.5	0.1	6.0	93.9	0.1	5.1	94.8	0.1	5.3	94.6
	EP	0.0	6.4	93.6	0.0	6.6	93.4	0.0	6.7	93.3	0.0	7.5	92.5
	$\mathcal{B}1$ cond	5.2	10.8	84.0	3.7	9.6	86.7	4.3	10.1	85.7	6.1	12.7	81.2
	$\mathcal{B}2$ cond	5.9	6.1	88.0	4.7	5.7	89.6	5.1	5.5	89.4	6.6	7.5	85.9
	$\mathcal{B}3$ cond	0.0	2.7	97.3	0.0	3.2	96.8	0.0	2.2	97.8	0.0	2.8	97.2
	$\mathcal{B}4$ cond	7.0	1.4	91.6	8.1	1.6	90.3	6.1	1.0	93.0	5.2	1.6	93.2
	$\mathcal{B}1$ weird	8.0	11.0	81.0	4.9	9.5	85.6	6.0	10.3	83.7	11.4	12.9	75.9
	$\mathcal{B}2$ weird	8.3	6.9	84.9	6.1	6.1	87.8	6.9	5.7	87.3	10.2	7.6	82.2
	$\mathcal{B}3$ weird	0.0	3.1	96.9	0.1	3.6	96.3	0.0	2.3	97.7	0.0	2.8	97.2

Table 3 continued

Y_0	Method	Function											
		α_1			α_2			α_3			α_4		
	$\mathcal{B}4$ weird	3.1	1.4	95.5	3.2	1.7	95.1	3.2	1.0	95.8	3.1	1.4	95.5
75	HW	0.2	4.7	95.2	0.1	4.8	95.1	0.1	4.2	95.7	0.1	5.0	94.9
	EP	0.1	5.4	94.5	0.1	4.9	95.0	0.0	5.2	94.8	0.1	6.8	93.1
	$\mathcal{B}1$ cond	2.8	8.7	88.5	1.8	7.6	90.5	2.5	7.7	89.8	3.8	11.3	84.9
	$\mathcal{B}2$ cond	3.8	5.2	91.0	2.9	4.3	92.8	3.4	4.3	92.4	4.7	6.9	88.4
	$\mathcal{B}3$ cond	0.0	2.8	97.2	0.1	2.9	97.1	0.0	2.0	98.0	0.0	2.8	97.2
	$\mathcal{B}4$ cond	6.3	1.4	92.4	7.9	1.4	90.7	5.7	0.9	93.4	4.7	1.4	93.9
	$\mathcal{B}1$ weird	4.4	9.2	86.4	2.6	7.6	89.7	3.6	8.3	88.0	6.5	10.9	82.7
	$\mathcal{B}2$ weird	5.5	6.0	88.5	4.0	4.7	91.3	4.9	4.6	90.4	7.0	7.1	85.9
	$\mathcal{B}3$ weird	0.0	3.1	96.9	0.0	3.5	96.5	0.0	2.3	97.7	0.0	3.1	96.9
	$\mathcal{B}4$ weird	3.0	1.6	95.4	3.1	1.6	95.3	3.2	0.9	95.9	2.9	1.5	95.7

Column I and II: left- and right-tail error probabilities
Column III: actual coverage probability when nominal is 95%

the equal-tailed confidence region $\mathcal{B}4$ $weird$ achieves the best ACP for $Y_0 \geq 35$. For $Y_0 = 25$ the difference is not higher than 2%.

Finally, for $Y_0 \geq 35$ $\mathcal{B}3$ $weird$ and $\mathcal{B}3$ $cond$ have ACP 1–3% too high. The asymptotic bands results are acceptable for $Y_0 \geq 50$.

For $\theta = 10\%$ (Tables 5, 6) the major difference with the previously discussed cases is the quick growth of ACP simultaneously for HW, EP and for both $\mathcal{B}3$ bands. In both models these regions get already too wide for $Y_0 = 35$ (ACP is up to 2.5% too high). As previously $\mathcal{B}4$ $weird$ is very stable with ACP very close to the nominal for $Y_0 \geq 35$ and for $Y_0 = 25$ only about 1–2% too low.

Summarizing, in all considered cases the ACP of EP is farther from the nominal one then the ACP of HW. Unfortunately, for $Y_0 = 75$ HW is still too narrow in the case $\theta = 1\%$ and already too wide for $\theta = 10\%$. On the other hand for the two lowest considered values of Y_0 (15 and 25) ACP is 5–10% less then it should be. This happens because these are asymptotic bands and in our case the number of jumps of the point processes is not big enough to apply the asymptotic distribution.

The conditional bootstrap does not seem to be good enough in this model. The confidence intervals generated by $\mathcal{B}1$ and $\mathcal{B}2$ have always ACP that is too low. $\mathcal{B}3$ performs well for small Y_0 but when the number of jumps rises it remains consistently too wide. Its actual coverage probability is about 3% too high for $Y_0 = 50$ and 75. On the other hand, the equal-tailed bootstrap confidence band $\mathcal{B}4$ is too narrow for every value of Y_0.

The weird bootstrap method is a much better choice. Also in this case $\mathcal{B}3$ is always too wide (as in previous case). $\mathcal{B}4$ behaves well in all considered situations. Its actual coverage probability is always close to the nominal, even in the case of small beginning number at risk equal to 25 (when the asymptotic bands fail). Our simulations also show that the left-side failure probability for the EP- and HW-band is significantly

Table 4 Actual coverage probabilities for the model **P** when $\theta = 5\%$

Y_0	Method	Function											
		α_1			α_2			α_3			α_4		
15	HW	0.2	11.8	88.0	0.3	10.7	89.0	0.1	13.4	86.5	0.1	15.1	84.8
	EP	0.2	12.8	87.0	0.3	11.3	88.4	0.1	16.4	83.5	0.1	14.3	85.5
	$\mathcal{B}1$ weird	7.0	17.4	75.7	6.5	15.2	78.3	7.2	19.5	73.4	7.0	20.8	72.4
	$\mathcal{B}2$ weird	5.7	8.1	86.2	5.5	7.5	87.0	5.3	9.2	85.6	4.8	10.7	84.5
	$\mathcal{B}3$ weird	0.2	8.3	91.5	0.4	7.0	92.6	0.1	9.6	90.3	0.1	10.1	89.9
	$\mathcal{B}4$ weird	4.1	5.1	90.8	3.6	3.7	92.6	3.9	6.0	90.1	4.5	7.6	87.9
25	HW	0.3	7.3	92.5	0.4	6.8	92.8	0.3	8.3	91.4	0.4	7.9	91.7
	EP	0.3	8.3	91.4	0.4	7.1	92.5	0.3	9.7	90.0	0.4	9.3	90.3
	$\mathcal{B}1$ weird	5.4	10.6	84.1	5.0	9.6	85.5	5.6	12.0	82.5	5.4	11.4	83.2
	$\mathcal{B}2$ weird	5.0	5.3	89.7	5.2	5.4	89.4	5.1	5.6	89.3	4.4	5.2	90.4
	$\mathcal{B}3$ weird	0.2	4.6	95.1	0.5	5.2	94.4	0.2	5.2	94.6	0.2	5.6	94.2
	$\mathcal{B}4$ weird	3.3	2.5	94.3	3.8	2.8	93.5	3.6	2.7	93.8	3.7	3.9	92.4
35	HW	0.5	5.4	94.0	0.6	5.5	93.9	0.4	6.0	93.7	0.5	5.3	94.2
	EP	0.5	6.1	93.3	0.6	5.8	93.6	0.4	7.6	92.0	0.5	7.9	91.5
	$\mathcal{B}1$ weird	3.9	7.4	88.7	3.5	7.2	89.3	4.7	7.9	87.4	4.5	7.1	88.4
	$\mathcal{B}2$ weird	4.3	4.2	91.5	4.1	4.4	91.5	5.0	4.2	90.9	4.5	3.7	91.9
	$\mathcal{B}3$ weird	0.5	3.7	95.8	0.6	4.8	94.6	0.3	3.8	95.9	0.3	3.5	96.2
	$\mathcal{B}4$ weird	3.6	1.9	94.5	3.1	2.5	94.3	3.7	2.1	94.3	3.0	1.9	95.1
50	HW	0.4	4.6	95.0	0.7	4.3	95.0	0.2	4.8	95.0	0.5	4.5	95.0
	EP	0.5	5.0	94.5	1.0	4.6	94.4	0.1	5.7	94.3	0.5	5.6	93.9
	$\mathcal{B}1$ weird	2.4	5.8	91.9	2.6	5.3	92.1	0.0	3.1	96.9	2.9	5.4	91.8
	$\mathcal{B}2$ weird	2.5	3.4	94.1	2.8	3.5	93.7	3.0	1.3	95.6	3.2	3.2	93.7
	$\mathcal{B}3$ weird	0.5	3.9	95.6	0.9	4.5	94.6	0.0	3.1	96.9	0.3	2.9	96.8
	$\mathcal{B}4$ weird	2.3	2.0	95.7	3.1	2.7	94.2	3.0	1.3	95.6	3.3	1.3	95.4
75	HW	1.0	3.5	95.5	1.0	3.5	95.5	0.9	3.6	95.5	0.9	3.3	95.8
	EP	0.9	3.9	95.1	1.1	3.6	95.2	0.9	4.4	94.7	0.9	4.3	94.8
	$\mathcal{B}1$ weird	2.6	3.6	93.7	2.2	4.0	93.8	2.3	4.0	93.7	2.3	3.5	94.2
	$\mathcal{B}2$ weird	2.7	2.6	94.7	2.5	2.7	94.8	2.6	2.7	94.7	2.8	2.4	94.8
	$\mathcal{B}3$ weird	0.9	3.4	95.7	1.2	3.5	95.4	0.8	3.2	96.0	0.8	2.8	96.4
	$\mathcal{B}4$ weird	3.2	1.7	95.1	3.0	1.9	95.0	3.2	1.6	95.2	3.4	1.2	95.5

Column I and II: left- and right-tail error probabilities
Column III: actual coverage probability when nominal is 95%

too small. Its value is below 1%. This means that our functions $\alpha_i(t)$ almost never cross the lower bound of the confidence region, i.e. the lower bound goes too far away from the estimator. The advantage of the $\mathcal{B}4$ region is the equal tailed feature. The left- and right-hand side probabilities of the lack of coverage are almost equal.

Table 5 Actual coverage probabilities for the model **C** when $\theta = 10\%$

Y_0	Method	Function											
		α_1			α_2			α_3			α_4		
15	HW	0.3	15.3	84.5	0.6	17.1	82.3	0.2	15.0	84.8	0.1	18.5	81.5
	EP	0.1	18.2	81.8	0.0	17.4	82.5	0.0	19.7	80.2	0.0	18.6	81.3
	$\mathcal{B}1$ cond	15.2	28.3	56.9	14.6	25.0	60.5	14.5	28.2	57.7	16.3	34.1	50.8
	$\mathcal{B}2$ cond	12.6	17.3	70.5	13.2	15.4	71.6	11.9	16.3	62.1	13.1	22.0	65.6
	$\mathcal{B}3$ cond	0.1	13.5	86.5	0.1	12.3	87.6	0.1	13.0	86.9	0.1	14.7	85.2
	$\mathcal{B}4$ cond	14.7	8.4	77.3	16.7	7.5	76.0	13.6	7.7	78.9	13.2	10.6	76.7
	$\mathcal{B}1$ weird	22.3	27.6	50.3	21.2	24.2	54.8	21.7	27.1	51.5	24.5	34.3	42.8
	$\mathcal{B}2$ weird	15.0	16.7	68.6	15.7	14.9	69.5	13.4	15.6	71.2	14.8	21.2	64.7
	$\mathcal{B}3$ weird	0.3	16.0	83.8	0.3	14.7	85.0	0.2	15.5	84.3	0.1	16.0	84.0
	$\mathcal{B}4$ weird	8.0	10.0	82.1	9.1	8.9	82.2	8.0	8.8	83.3	8.4	11.4	80.5
25	HW	0.2	11.2	88.7	0.4	12.2	87.4	0.1	9.7	91.2	0.1	12.2	87.7
	EP	0.0	13.4	86.6	0.1	13.0	86.9	0.0	13.3	86.7	0.0	15.0	85.0
	$\mathcal{B}1$ cond	12.9	21.2	66.2	11.7	19.8	68.6	11.9	19.7	68.5	14.8	24.3	61.2
	$\mathcal{B}2$ cond	12.7	13.0	74.5	11.5	12.9	75.7	11.6	11.4	77.1	13.5	15.3	71.6
	$\mathcal{B}3$ cond	0.1	7.7	92.2	0.2	8.7	91.0	0.0	7.1	92.9	0.0	9.6	90.4
	$\mathcal{B}4$ cond	13.2	4.3	82.7	15.0	4.8	80.2	11.2	3.9	84.9	10.9	5.7	83.6
	$\mathcal{B}1$ weird	19.0	21.1	60.1	15.9	19.6	64.5	16.7	19.3	64.0	21.9	24.5	53.9
	$\mathcal{B}2$ weird	15.8	13.4	71.1	14.5	13.0	72.7	14.3	11.3	74.5	16.2	15.3	68.8
	$\mathcal{B}3$ weird	0.2	9.4	90.5	0.4	10.2	89.5	0.0	8.0	92.0	0.1	10.6	89.4
	$\mathcal{B}4$ weird	7.3	4.7	88.1	7.6	5.2	87.2	6.9	4.2	88.9	7.0	5.6	87.5
35	HW	0.3	9.1	90.5	0.3	10.1	89.7	0.2	8.4	91.4	0.2	9.2	90.5
	EP	0.1	10.8	89.0	0.1	10.7	89.2	0.1	10.5	89.4	0.1	12.2	87.8
	$\mathcal{B}1$ cond	11.3	17.8	71.1	9.0	15.9	75.1	9.4	16.1	74.6	12.1	19.8	68.3
	$\mathcal{B}2$ cond	11.2	11.0	77.8	9.7	10.4	79.9	10.0	9.6	80.5	12.0	13.0	75.2
	$\mathcal{B}3$ cond	0.1	6.4	93.5	0.2	7.2	92.6	0.1	5.7	94.3	0.0	6.9	93.1
	$\mathcal{B}4$ cond	12.0	3.4	84.7	14.4	3.7	81.9	11.5	2.7	85.8	10.3	3.5	86.3
	$\mathcal{B}1$ weird	15.9	18.0	66.2	11.4	15.9	72.8	13.1	16.2	70.8	18.5	20.3	61.4
	$\mathcal{B}2$ weird	14.8	11.7	73.6	12.0	10.9	77.2	13.2	9.9	77.0	15.7	13.4	71.1
	$\mathcal{B}3$ weird	0.2	7.4	92.4	0.3	8.6	91.2	0.1	6.3	93.7	0.1	7.3	92.6
	$\mathcal{B}4$ weird	6.6	3.5	90.0	6.9	4.3	88.8	7.0	2.9	90.2	6.7	3.3	90.1
50	HW	0.4	8.3	91.3	0.5	7.7	91.9	0.3	7.3	92.4	0.4	7.5	92.1
	EP	0.1	9.6	90.2	0.3	8.6	91.2	0.1	8.9	90.9	0.1	10.1	89.8
	$\mathcal{B}1$ cond	8.1	15.6	76.6	6.9	12.8	80.3	7.2	13.4	79.4	10.3	16.7	73.1
	$\mathcal{B}2$ cond	9.0	9.5	81.6	7.9	8.4	83.7	8.2	8.1	83.8	11.0	10.9	78.2
	$\mathcal{B}3$ cond	0.1	5.6	94.3	0.3	5.8	93.9	0.1	5.1	94.8	0.1	5.6	94.3
	$\mathcal{B}4$ cond	11.9	2.6	85.6	14.4	2.7	82.9	10.4	2.1	87.5	9.4	2.7	88.0
	$\mathcal{B}1$ weird	11.0	16.0	73.1	8.5	13.3	78.3	9.1	13.7	77.3	14.6	17.1	68.4
	$\mathcal{B}2$ weird	11.9	10.7	77.5	9.7	9.0	81.3	10.2	8.8	81.0	14.5	11.4	74.2
	$\mathcal{B}3$ weird	0.1	6.9	92.5	0.4	7.3	92.4	0.1	5.8	94.1	0.1	6.0	94.0

Table 5 continued

Y_0	Method	Function											
		α_1			α_2			α_3			α_4		
	$\mathcal{B}4$ weird	6.8	3.0	90.2	7.0	3.1	89.9	6.4	2.4	91.3	6.4	2.5	91.1
75	HW	0.8	6.9	92.3	0.7	7.1	92.2	0.6	7.3	92.2	0.6	7.6	91.8
	EP	0.3	7.8	91.9	0.6	7.8	92.0	0.4	8.2	91.4	0.1	9.5	90.3
	$\mathcal{B}1$ cond	6.0	11.5	82.5	5.0	10.8	84.3	5.3	10.6	84.1	8.0	14.4	77.7
	$\mathcal{B}2$ cond	7.0	7.7	85.3	6.0	7.2	86.8	6.5	6.9	86.5	9.0	9.9	81.2
	$\mathcal{B}3$ cond	0.3	5.0	94.8	0.3	5.7	94.0	0.0	5.0	94.8	0.1	5.8	94.1
	$\mathcal{B}4$ cond	11.3	2.4	86.4	13.4	2.7	83.9	10.7	2.3	87.0	9.1	2.6	88.3
	$\mathcal{B}1$ weird	8.5	12.5	79.0	6.5	11.3	82.2	7.3	11.4	81.4	10.8	14.2	75.1
	$\mathcal{B}2$ weird	9.7	8.6	81.7	7.9	8.0	84.1	8.6	7.7	83.7	11.9	10.2	77.9
	$\mathcal{B}3$ weird	0.3	6.5	93.2	0.5	7.3	92.3	0.3	6.2	93.6	0.1	6.7	93.2
	$\mathcal{B}4$ weird	6.2	3.1	90.8	6.5	3.5	90.0	6.9	2.8	90.3	6.3	2.8	91.0

Column I and II: left- and right-tail error probabilities
Column III: actual coverage probability when nominal is 90%

We checked empirically that weird bootstrap $\mathcal{B}4$ is the optimal choice. Independently on the beginning number at risk it has a coverage probability close to the nominal one and, what is very important, it provides almost equally divided failure probability. At the first sight $\mathcal{B}3$ seems to be a good choice but as the sample size grows it gets too wide.

The case $Y_0 = 15$ is problematic. None of the methods give satisfactory results. This happens because of the very small number of jumps in the process with such low beginning number of individuals at risk. It stands to reason not to consider this case any longer and we omit it in the remaining part of this paper.

Now we compare our results with those presented in Bie et al. (1987). The authors there proposed arcsine- and logarithmic-transform of the Nelson–Aalen estimator. They considered the modifications of EP- and HW-band which use these transformations.

Such constructed simultaneous confidence bands perform badly in model **C**. However, their left- and right-hand side failure probability are close to each other, they have significantly too large actual coverage probability.

Using simulation methods presented before we compare the behavior of these bands to the bootstrap band $\mathcal{B}4$ $weird$. The results are presented in Tables 7, 8 and 9. AHW and AEP denote the arcsine-transform of the HW- and EP-band, respectively. The logarithmic-transform bands are denoted by LHW and LEP.

The actual coverage probability of the transformed bands for $\theta = 10\%$ is even 4–5% too high for every value of Y_0. In the case of $\theta = 5\%$ and $\theta = 1\%$ the actual coverage probability is two times too small. For $Y_0 \geq 35$ $\mathcal{B}4$ $weird$ is almost optimal. For $Y_0 = 25$ and $\theta = 10\%$ $\mathcal{B}4$ $weird$ has ACP about 2% lower then nominal one and about 1% for other considered values of θ.

Table 6 Actual coverage probabilities for the model **P** when $\theta = 10\%$

Y_0	Method	Function											
		α_1			α_2			α_3			α_4		
15	HW	0.8	15.5	83.7	1.0	14.1	84.9	0.5	16.1	83.4	0.6	17.3	82.1
	EP	0.8	17.6	81.6	1.1	14.3	84.7	0.3	19.1	80.6	0.8	16.9	82.2
	$\mathcal{B}1$ weird	10.6	21.4	68.3	9.2	19.8	71.2	9.8	22.4	68.0	10.3	24.8	66.5
	$\mathcal{B}2$ weird	9.3	13.4	77.4	9.2	12.4	78.4	8.9	12.2	78.9	8.4	13.4	78.2
	$\mathcal{B}3$ weird	1.0	13.1	86.0	1.6	12.7	85.7	0.5	15.8	83.7	0.6	13.7	85.7
	$\mathcal{B}4$ weird	7.8	8.4	83.9	7.3	7.5	85.2	7.9	8.7	83.6	8.6	9.8	81.9
25	HW	0.7	10.6	88.7	1.6	9.8	88.7	1.0	10.9	88.1	1.0	11.1	87.9
	EP	0.5	12.0	87.5	1.6	10.2	88.3	1.1	11.7	87.2	1.0	14.0	84.9
	$\mathcal{B}1$ weird	9.0	14.0	77.0	7.7	13.3	79.1	8.6	14.9	76.7	9.3	15.0	76.0
	$\mathcal{B}2$ weird	8.3	9.2	82.5	8.2	8.8	83.2	8.3	8.7	83.0	8.3	8.8	83.0
	$\mathcal{B}3$ weird	0.8	9.4	89.8	2.0	9.7	88.3	1.0	9.2	89.8	0.9	8.7	90.4
	$\mathcal{B}4$ weird	7.6	5.3	87.1	6.8	5.3	87.9	7.0	4.7	88.3	7.4	5.3	87.4
35	HW	1.5	6.1	92.5	1.8	8.0	90.2	1.4	8.7	89.9	1.4	8.4	90.3
	EP	1.4	6.9	91.8	1.9	8.6	89.4	1.3	10.3	88.4	1.4	10.0	88.7
	$\mathcal{B}1$ weird	7.7	9.7	82.7	6.5	10.0	83.6	7.1	11.2	81.9	6.9	10.9	82.4
	$\mathcal{B}2$ weird	6.6	5.4	88.1	6.6	7.1	86.3	7.4	7.4	85.2	7.1	6.7	86.3
	$\mathcal{B}3$ weird	1.6	5.9	92.6	2.3	8.8	88.9	1.3	8.2	90.5	1.2	7.4	91.4
	$\mathcal{B}4$ weird	6.5	3.1	90.4	6.0	4.7	89.3	6.8	3.7	89.4	6.8	3.4	89.7
50	HW	1.6	6.8	91.6	2.4	6.9	90.7	1.9	7.4	90.7	1.7	6.7	91.6
	EP	1.6	8.0	90.4	2.5	7.3	90.2	1.8	8.5	89.8	1.7	8.3	90.0
	$\mathcal{B}1$ weird	5.0	8.2	87.0	6.1	7.9	86.0	5.9	8.5	85.6	5.7	7.6	86.7
	$\mathcal{B}2$ weird	4.9	6.3	88.8	6.4	6.1	87.5	6.0	6.3	87.7	5.7	6.0	88.4
	$\mathcal{B}3$ weird	1.9	7.3	90.8	3.0	7.6	89.5	1.8	7.3	90.9	1.8	6.8	91.4
	$\mathcal{B}4$ weird	5.5	4.0	90.6	6.8	4.4	88.9	6.5	3.6	90.0	6.2	2.6	91.2
75	HW	2.8	6.5	90.7	2.3	6.3	91.4	2.5	6.3	91.2	2.1	6.6	91.3
	EP	2.9	7.1	90.0	2.4	6.5	91.1	2.4	6.9	90.7	2.0	7.6	90.5
	$\mathcal{B}1$ weird	4.9	7.2	88.0	4.5	6.8	88.8	5.1	6.8	88.2	5.0	6.7	88.3
	$\mathcal{B}2$ weird	5.3	5.5	89.2	4.8	5.5	89.6	5.1	5.2	89.7	5.0	5.6	89.5
	$\mathcal{B}3$ weird	3.1	7.1	89.8	2.8	6.8	90.4	2.5	6.7	90.9	2.0	7.0	91.0
	$\mathcal{B}4$ weird	6.0	4.1	90.0	5.4	4.4	90.2	6.0	3.6	90.5	6.0	3.6	90.3

Column I and II: left- and right-tail error probabilities
Column III: actual coverage probability when nominal is 90%

We have also made simulations for the second model **P**. Since the results are very similar to those for model **C**, Tables with coverage probabilities are omitted. For $\theta = 1\%$ and every value of Y_0 all considered bands have ACP close to the nominal one. To be precise, ACP of transformed bands is a little too low for $Y_0 = 25$ and a little to high for $Y_0 = 75$. In the case $\theta = 5\%$ transformed bands are too wide for Y_0

Table 7 Actual coverage probabilities for the model **C** when $\theta = 1\%$

Y_0	Method	Function											
		α_1			α_2			α_3			α_4		
25	AHW	0.1	1.3	98.6	0.2	1.6	98.3	0.2	0.9	98.8	0.1	2.0	97.8
	AEP	0.2	1.2	98.6	0.2	1.4	98.5	0.3	0.8	98.9	0.2	1.8	98.1
	LHW	0.4	0.5	99.1	0.4	0.7	98.9	0.5	0.4	99.1	0.7	1.3	98.0
	LEP	0.3	0.4	99.3	0.3	0.5	99.3	0.5	0.2	99.3	0.6	1.0	98.4
	$\mathcal{B}4$ weird	0.7	0.7	98.7	0.9	0.7	98.4	0.6	0.4	98.9	0.5	1.2	98.3
35	AHW	0.2	0.9	98.9	0.1	0.9	99.0	0.1	0.6	99.3	0.1	0.8	99.1
	AEP	0.2	0.9	98.8	0.2	0.9	98.9	0.2	0.6	99.1	0.2	0.8	99.0
	LHW	0.4	0.4	99.1	0.3	0.4	99.3	0.4	0.2	99.4	0.6	0.5	98.9
	LEP	0.4	0.4	99.3	0.3	0.3	99.4	0.4	0.2	99.4	0.5	0.3	99.2
	$\mathcal{B}4$ weird	0.7	0.4	98.9	0.7	0.5	98.8	0.6	0.2	99.2	0.6	0.4	99.0
50	AHW	0.2	0.5	99.3	0.1	0.5	99.3	0.2	0.4	99.4	0.1	0.4	99.4
	AEP	0.3	0.6	99.1	0.3	0.6	99.1	0.3	0.5	99.2	0.2	0.6	99.2
	LHW	0.5	0.2	99.3	0.4	0.3	99.3	0.4	0.1	99.5	0.7	0.2	99.2
	LEP	0.4	0.2	99.4	0.4	0.2	99.4	0.4	0.1	99.5	0.5	0.2	99.3
	$\mathcal{B}4$ weird	0.7	0.3	99.1	0.8	0.3	98.8	0.7	0.1	99.2	0.7	0.3	99.0
75	AHW	0.2	0.3	99.5	0.2	0.4	98.4	0.2	0.3	99.5	0.2	0.3	99.5
	AEP	0.3	0.4	99.3	0.2	0.5	99.4	0.3	0.4	99.3	0.3	0.4	99.2
	LHW	0.5	0.1	99.4	0.3	0.2	99.5	0.4	0.1	99.5	0.7	0.1	99.2
	LEP	0.4	0.2	99.5	0.3	0.2	99.5	0.4	0.1	99.4	0.6	0.1	99.2
	$\mathcal{B}4$ weird	0.7	0.2	99.1	0.8	0.3	99.0	0.7	0.2	99.2	0.6	0.2	99.2

Column I and II: left- and right-tail error probabilities
Column III: actual coverage probability when nominal is 99%

equal to 50 and 75. The similar situation one may observe for $\theta = 10\%$ when ACP of the transformed bands is up to 2% too large.

Our simulations show that for the Nelson–Aalen model the weird bootstrap is better then the conditional one. In models **C** and **P** independently of the shape of the intensity function, we recommend $\mathcal{B}4$ $weird$ as a best choice.

6 Conclusions

In many applications, the hazard function is much more interesting and relevant to estimate than the integrated hazard function, but it is also more challenging to estimate. There are several approaches to that problem, the histogram based sieve estimator considered in Leśkow and Różański (1989) and Leśkow (1988) being one of them. Unfortunately, functional central limit theorem for such estimator is still unknown. Without such result, construction of the simultaneous confidence bands is quite difficult.

In our paper we showed that even for samples as small as 25 the bootstrap simultaneous confidence bands behave better than the asymptotic ones. They also have better

Table 8 Actual coverage probabilities for the model **C** when $\theta = 5\%$

Y_0	Method	Function											
		α_1			α_2			α_3			α_4		
25	AHW	0.9	3.0	96.2	1.0	3.1	95.9	0.8	2.8	96.4	0.8	3.5	95.7
	AEP	1.2	3.4	95.4	1.2	2.0	95.7	1.1	3.1	95.9	1.0	3.3	95.7
	LHW	1.5	1.9	96.4	1.5	1.6	96.5	1.9	1.6	96.5	2.3	2.4	95.4
	LEP	1.4	1.7	96.6	1.4	2.7	96.9	1.9	1.3	96.8	2.0	2.8	96.8
	$\mathcal{B}4$ weird	3.6	2.4	94.0	3.8	2.5	93.8	3.5	2.2	94.3	3.3	2.4	94.3
35	AHW	0.9	2.2	96.9	1.0	2.5	96.5	1.0	1.9	97.0	1.0	2.1	96.9
	AEP	1.2	2.8	96.0	1.3	2.9	95.8	1.4	2.1	96.4	1.2	2.8	96.0
	LHW	1.7	1.3	97.0	1.6	1.6	96.8	1.6	1.0	97.5	2.5	1.5	96.0
	LEP	1.8	1.3	96.9	1.7	1.4	96.9	1.9	0.7	97.3	2.2	1.5	96.3
	$\mathcal{B}4$ weird	3.2	1.8	95.1	3.7	1.9	94.3	3.3	1.1	95.6	3.1	1.7	95.2
50	AHW	0.9	1.5	97.6	1.0	1.8	97.2	1.0	1.4	97.6	1.3	1.4	97.3
	AEP	1.1	2.0	96.9	1.3	2.1	96.6	1.2	1.7	97.1	1.4	1.7	97.0
	LHW	1.9	0.8	97.3	1.4	0.9	97.6	2.1	0.6	97.3	2.6	0.7	96.7
	LEP	1.6	0.9	97.5	1.4	1.1	97.5	1.8	1.0	97.6	2.2	0.9	97.0
	$\mathcal{B}4$ weird	3.1	1.4	95.5	3.2	1.7	95.1	3.2	1.0	95.8	3.1	1.4	95.5
75	AHW	1.3	1.4	97.3	1.2	1.5	97.3	1.1	1.3	97.6	1.2	1.4	97.4
	AEP	1.3	1.8	96.9	1.4	1.5	97.0	1.2	1.7	97.1	1.2	1.9	97.0
	LHW	2.0	0.7	97.3	1.9	0.8	97.4	2.1	0.6	97.3	2.4	0.7	96.9
	LEP	1.9	0.9	97.3	1.8	1.0	97.2	1.9	0.7	97.5	2.0	0.9	97.1
	$\mathcal{B}4$ weird	3.0	1.6	95.4	3.1	1.6	95.3	3.2	0.9	95.9	2.9	1.5	95.7

Column I and II: left- and right-tail error probabilities
Column III: actual coverage probability when nominal is 95%

actual coverage probability independently of the shape of the considered intensity function. The considerable influence of the constructing bootstrap confidence regions on obtained ACPs is shown. An advantage of the equal-tailed type confidence region is the balance of the left- and right-tail error probability. One has to bear in mind that the integrated hazard function is always nondecreasing, however, the lower confidence bound for considered methods may happen to be decreasing. It may be interesting to construct regions taking into consideration the known features of the estimated function (for example monotonicity, unimodality).

The other curious problem is bootstrapping of the point process. We consider two methods (the weird and conditional bootstrap). In the paper Cowling et al. (1996) other methods are proposed but only for Poisson processes. A method for obtaining bootstrap replicates for the one-dimensional point process is presented in Braun and Kulperger (1998) and its multi-dimensional version is also proposed. Because of deficient coverage properties in some cases, Braun and Kulperger proposed in Braun and Kulperger (2003) a technique for one-dimensional point process which

Table 9 Actual coverage probabilities for the model **C** when $\theta = 10\%$

Y_0	Method	Function											
		α_1			α_2			α_3			α_4		
25	AHW	1.8	4.7	93.6	2.1	4.9	93.0	2.1	4.6	93.3	1.9	4.9	93.3
	AEP	2.8	5.5	91.7	2.9	5.2	91.9	3.0	4.9	92.2	2.4	5.0	92.7
	LHW	3.4	3.1	93.6	2.9	3.3	93.9	3.1	3.0	94.0	4.2	3.7	92.3
	LEP	3.6	3.0	93.4	3.2	2.8	94.0	3.8	2.2	94.0	3.8	3.6	92.8
	$\mathcal{B}4$ weird	7.3	4.7	88.1	7.6	5.2	87.2	6.9	4.2	88.9	7.0	5.6	87.5
35	AHW	1.9	3.3	94.8	2.0	3.8	94.2	2.1	3.4	94.6	2.2	3.4	94.4
	AEP	2.7	4.4	92.9	2.9	4.4	92.7	3.0	3.9	93.1	2.5	4.4	93.1
	LHW	3.2	2.2	94.7	2.8	2.2	95.1	3.2	2.1	94.9	4.1	2.4	93.5
	LEP	3.3	2.3	94.5	3.1	2.3	94.7	4.0	1.8	94.2	3.6	2.5	94.0
	$\mathcal{B}4$ weird	6.6	3.5	90.0	6.9	4.3	88.8	7.0	2.9	90.2	6.7	3.3	90.1
50	AHW	1.9	3.2	94.9	2.0	3.4	94.6	2.3	3.4	94.3	2.5	2.8	94.7
	AEP	2.6	3.9	93.5	2.7	3.9	93.4	2.8	3.8	93.4	2.7	3.9	93.4
	LHW	3.3	2.0	94.7	2.8	1.9	95.3	3.3	1.9	94.8	4.2	1.6	94.2
	LEP	3.2	2.5	94.3	3.0	2.2	94.8	3.6	2.0	94.4	3.8	1.9	94.4
	$\mathcal{B}4$ weird	6.8	3.0	90.2	7.0	3.1	89.9	6.4	2.4	91.3	6.4	2.5	91.1
75	AHW	2.6	2.9	94.6	2.5	3.5	94.0	2.7	3.0	94.3	2.8	2.7	94.5
	AEP	3.1	3.0	93.9	3.1	3.6	93.3	3.3	3.2	93.4	2.9	3.3	93.9
	LHW	3.9	1.4	94.6	3.4	1.9	94.7	3.7	1.7	95.6	4.6	1.5	93.9
	LEP	3.8	1.8	94.4	3.7	2.4	93.9	4.3	2.0	93.8	3.9	1.8	94.3
	$\mathcal{B}4$ weird	6.2	3.1	90.8	6.5	3.5	90.0	6.9	2.8	90.3	6.3	2.8	91.0

Column I and II: left- and right-tail error probabilities
Column III: actual coverage probability when nominal is 90%

uses the idea of re-colouring presented in Davison and Hinkley (1999). It remains an open question if these methods can be applied in a general case.

References

Andersen PK, Borgan O, Gill RD, Keiding N (1993) Statistical models based on counting processes. Springer, Heidelberg
Bie O, Borgan O, Liestøl K (1987) Confidence intervals and confidence bands for the cumulative hazard rate function and their small sample properties. Scand J Statist 14:221–233
Braun WJ, Kulperger RJ (1998) A bootstrap for point processes. J Stat Comput Simul 60:129–155
Braun WJ, Kulperger RJ (2003) Re-colouring the intensity-based bootstrap for point processes. Commun Stat Simul Comput 32(2):475–488
Cowling A, Hall P, Phillips MJ (1996) Bootstrap confidence regions for the intensity of a Poisson point process. Am Stat Assoc 91(436):1516–1524
Davison AC, Hinkley DV (1999) Bootstrap methods and their applications. Cambridge University Press, London
Efron B, Tibshirani RJ (1993) An introduction to the bootstrap. Chapman&Hall/CRC, New York
Hall P (1992) The bootstrap and Edgeworth expantion. Springer, New York
Kalbfleisch JD, Prentice RL (2002) The statistical analysis of failure time data. Willey, New York

Leśkow J (1988) Histogram maximum likelihood estimator of a periodic function in the multiplicative intensity model. Stat Decis 6:79–88

Leśkow J, Różański R (1989) Histogram maximum likelihood estimator in the multiplicative intensity model. Stoch Process Appl 31:151–159

Leśkow J, Wronka C (2004) Bootstrap resampling in the analysis of time series. Springer, Heidelberg, pp 267–274

Loader C (1993) Nonparemetric regression, confidence bands and bias correction. Comput Sci Stat. In: Proceedings of the 25th symposium on the interface, pp 131–136

Snethlage M (1999) Is bootstrap really helpful in point process statistics? Metrika 49:245–255