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PROBLEMS AND SOLUTIONS

PROBLEMS

89.3.1. *The Asymptotic Distribution of the Iterated Gauss-Newton Estimators of an ARIMA Process*; proposed by Sastry G. Pantula. Schwert [1] considered the model

$$Y_t = \rho Y_{t-1} + e_t + \beta e_{t-1}, \qquad t = 1, \ldots, n \tag{1}$$

where $\rho = 1$, $|\beta| < 1$, $e_0 = 0$, $Y_0 = 0$, and $\{e_t\}$ is a sequence of i.i.d. $N(0,1)$ random variables. Using a Monte Carlo study, Schwert [1] compared the finite-sample properties of several unit-root test criteria on β ($= -\theta$ in his notation) ranged from -0.8 to 0.8. One of the test statistics he considered is the Said–Dickey [2] t-statistics based on the Gauss–Newton estimators of ρ and β. Said and Dickey [2] obtained the asymptotic distributions of the test statistics based on the *one-step* Gauss–Newton estimators starting from the *initial values* $\rho = 1$ and $\beta = \beta^*$, where β^* is a consistent estimator of β such that $\beta^* - \beta = O_p(n^{-1/4})$. Schwert [1], on the other hand, decided to use the *iterated* Gauss–Newton estimators in his Monte Carlo study, because he felt that "empirical researchers who estimate ARIMA (1,0,1) models and discover an estimated autoregressive parameter close to unity would want to know the reliability of the t test for the unit root when iterative least squares is used." Also, he clearly states that "this is *not* the procedure suggested by Said and Dickey [2]; their results *require* only one Gauss–Newton step from the *unit root.*"

1. Does the t-statistic based on the iterated Gauss–Newton estimators have the same asymptotic distribution as the t-statistic based on the one-step Gauss–Newton estimators starting from the initial values $\rho = 1$ and β^*, where $\beta^* - \beta = O_p(n^{-1/4})$?
2. Is it necessary to start the Gauss–Newton iterations with $\rho = 1$?
3. Do the answers for Questions 1 and 2 extend to ARIMA $(p,1,q)$ processes?

REFERENCES

1. Said, S.E. & D.A. Dickey [2]. Hypothesis testing in ARIMA (p,1,q) models. *Journal of the American Statistical Association* 80 (1985): 369–374.
2. Schwert, G.W. [1]. Tests for unit roots: A Monte Carlo Investigation. *Journal of Business and Economic Statistics* 7 (1989): 147–160.

89.3.2. *Simultaneous Confidence Ellipsoids*, proposed by R.W. Farebrother. Let $\hat{\beta}$ and $\hat{\sigma}^2$ be estimators of the $p \times 1$ matrix β and the positive scalar σ^2, and let $\hat{\beta}$ be normally distributed with mean β and variance $\sigma^2 V$, where V is a known $p \times p$ positive definite matrix. Furthermore, let $m\hat{\sigma}^2/\sigma^2$

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be distributed as χ^2 with m degrees of freedom independently of β. The reader is asked to show that β simultaneously satisfies the constraints

$$(\beta - \hat{\beta})'H(H'VH)^{-1}H'(\beta - \hat{\beta}) \leq c\hat{\sigma}^2$$

for all $p \times q$ matrices H of rank q with probability $1 - \alpha$, where c/p is the upper α critical value of an F distribution with p and m degrees of freedom.

89.3.3. *The Equivalence of the Boothe–MacKinnon and the Hausman Specification Tests in the Context of Panel Data*; proposed by Badi H. Baltagi. Consider the following panel data model:

$$Y_{it} = X_{it}\beta + \mu_i + \nu_{it}, \qquad i = 1,2,\ldots,N,\ t = 1,2,\ldots,T \tag{1}$$

where β is a $K \times 1$ vector of regression parameters. The μ_i's are the individual time-invariant effects which are assumed to be independent of each other with zero mean and variance equal to σ_μ^2. ν_{it} is the remainder disturbance term, assumed to be independent of X_{it} and μ_i, and itself independent and identically distributed with mean zero and variance σ_ν^2. A critical assumption in this model is that $E(\mu_i/X_{it}) = 0$. Failure of this assumption affects the properties of the GLS estimator $\hat{\beta}_{\text{GLS}}$, the between estimator $\hat{\beta}_B$, and the within estimator $\hat{\beta}_W$, differently. Hausman [2] and later Hausman and Taylor [3] suggest testing this hypothesis using three specification tests which are based upon the length of the difference between these estimators. In particular, let

$$\hat{q}_1 = \hat{\beta}_{\text{GLS}} - \hat{\beta}_{\text{W}}, \qquad \hat{q}_2 = \hat{\beta}_{\text{GLS}} - \hat{\beta}_{\text{B}}, \qquad \hat{q}_3 = \hat{\beta}_{\text{W}} - \hat{\beta}_{\text{B}} \tag{2}$$

then Hausman's test can be written as

$$\hat{q}_i' V_i^{-1}\hat{q}_i \to \chi_K^2, \qquad \text{for } i = 1,2,3, \tag{3}$$

where $V_i = \text{cov}(\hat{q}_i)$. Hausman and Taylor [3] proved that these three tests are numerically exactly identical.

Boothe and MacKinnon [1] suggest an alternative specification test for models estimated by GLS. This test is based on the difference between the GLS and OLS estimators, and is demonstrated for a model with a simple form of heteroskedasticity and another model with AR(1) disturbances.

Let $\hat{q}_4 = \hat{\beta}_{\text{GLS}} - \hat{\beta}_{\text{OLS}}$, prove that for the panel data model given in (1), $\hat{q}_4' V_4^{-1}\hat{q}_4$ is numerically exactly identical to the three tests given in (3).

REFERENCES

1. Boothe, P.B. & J.G. MacKinnon. A specification test for models estimated by GLS. *The Review of Economics and Statistics* 68 (1986): 711–714.
2. Hausman, J.A. Specification tests in econometrics. *Econometrica* 46 (1978): 1251–1271.
3. Hausman, J.A. & W.E. Taylor. Panel data and unobservable individual effects. *Econometrica* 49 (1981): 1377–1398.