Introduction

In recent years, Card games have become immensely popular, with mechanics centered around pulling randomized items, characters, or rewards from a virtual "banner." These mechanics, rooted in probability and randomness, offer an engaging but often unpredictable player experience. One such game, *Zenless Zero Zone*, features a highly sought-after "S-ranked" character, whose chances of being drawn are relatively low, adding excitement and challenge to the game. The process of drawing an S-ranked character can be analyzed through probabilistic models, offering insights into player outcomes and gaming fairness. In analyzing card games with a draw mechanic, the term *Absolute Zero* is often used to describe situations in which players aim to achieve a net sum of zero through a sequence of card draws. This can signify a balanced hand or a nil effect (Tintle et al., 2020). coping with this idea is mathematically appealing because it includes a combination of randomization and order.

Aim and Objective

The number one goal of this study looks at is to research and examine the chances related to the card draw mechanics in *Absolute Zenless zero zone*, this will be accomplished by means of the use of mathematical tools, such as binomial distribution, to version the likelihood of unique consequences, and chi-sqaure assessments to assess how determined consequences align with predicted chances. moreover, recurrence formulas will describe the possible consequences over repeated trials. collectively, these analyses will provide a complete expertise of the sport's behavior, fairness, and underlying probabilistic structure.

Binomial Distribution

We consider a card game where a player aims to draw an S-ranked character with a known initial probability of success. Additionally, the game implements a guaranteed mechanism that increases the probability of success after a certain number of failures. This model focuses on deriving the exact probabilities using advanced mathematical concepts such as the *binomial distribution*, *recurrence relations*, and *chi-square tests* for goodness-of-fit. We incorporate both cumulative distributions and stepwise dynamic probabilities.

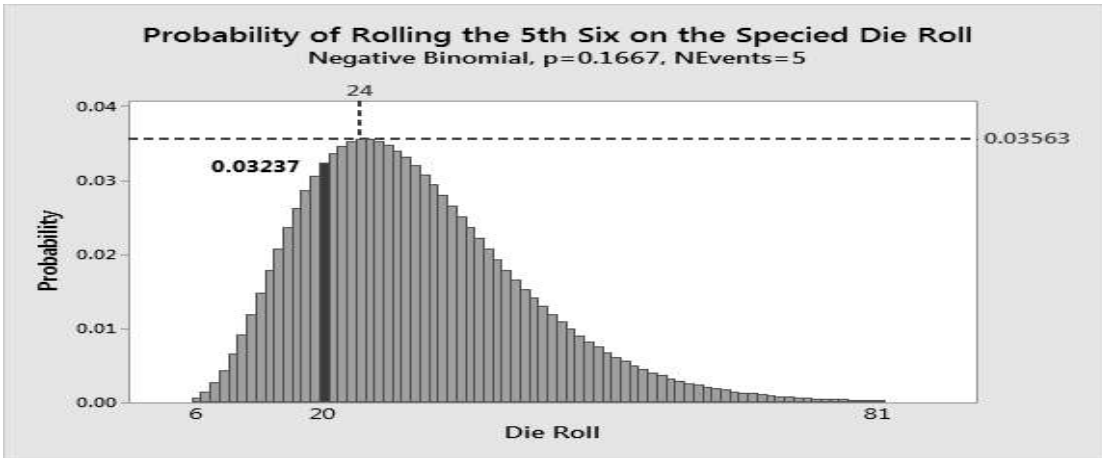

Figure 1: Example of binomial distribution of rolling the 5th six on a specied die roll with a negative binomial and p=0.1667[1]

Binomial Distribution with Guarantee Mechanism

The initial probability of drawing the target card is given by p, while the probability of failure is $q = 1 - p$. For a fixed number of trials n, the probability of obtaining exactly k successes follows a binomial distribution

$$P(k \text{ successes in } n \text{ trials}) = \binom{n}{k} p^k (1-p)^{n-k}$$

However, with the guarantee mechanism in place, the probability increases after a predetermined number of failed attempts t_g. Define the initial probability of success as $p_0 = p$, and the increment after t_g trials as Δp. The stepwise modified probability p_n for $n > t_g$ is modeled as

$$p_n = p_0 + (n - t_g) \cdot \Delta p$$

The cumulative probability of success over multiple trials, incorporating the guarantee, becomes

$$P(\text{at least one success by trial } n) = 1 - (1 - p_0)^n, \quad n \leq t_g$$

For $n > t_g$, this evolves into

$$P(\text{at least one success by trial } n) = 1 - \prod_{i=1}^{n} (1 - p_i)$$

where p_i is updated according to Equation (2).

Example Calculation

Consider $p_0 = 0.006$, $t_g = 73$, and $\Delta p = 0.0006$. For $n = 100$ pulls

$$p_{100} = p_0 + (100 - 73) \cdot \Delta p = 0.006 + 27 \cdot 0.0006 = 0.0222$$

[1] https://www.linkedin.com/pulse/financial-markets-demystifying-binomial-distribution

The cumulative probability of success by the 100th pull

$$P(\text{success by pull } 100) = 1 - \prod_{i=1}^{100}(1 - p_i)$$

This requires calculating p_i for each $i \in [1,100]$ based on the guarantee mechanics.

Recurrence Relations

The recurrence relation allows us to dynamically model the increasing probability. Define p_n as the probability of success on the n-th pull, which increases after the guarantee threshold t_g as follows

$$p_n = \begin{cases} p_0, & n \leq t_g \\ p_{n-1} + \Delta p, & n > t_g \end{cases}$$

This recurrence relation modifies the traditional binomial model to account for increasing probabilities after each unsuccessful trial.

The cumulative probability for a success over n trials can also be defined recursively

$$P_n = 1 - (1 - p_n) \cdot P_{n-1}$$

where P_{n-1} is the cumulative probability after $n - 1$ trials.

Example Calculation with Recurrence

For the same game setup with $p_0 = 0.006$, $t_g = 73$, and $\Delta p = 0.0006$

$$p_{74} = p_{73} + \Delta p = 0.006 + 0.0006 = 0.0066$$

$$p_{75} = p_{74} + \Delta p = 0.0066 + 0.0006 = 0.0072$$

Continuing this until $p_{90} = 1$.

Chi-Square Goodness-of-Fit Test

To verify the fairness of the card-draw mechanics in *Absolute Zenless Zero Zone*, we apply the chi-square test to compare observed outcomes with expected outcomes. This allows us to assess whether the observed distribution of results significantly deviates from the expected distribution predicted by the theoretical binomial model with adjusted probabilities. The chi-square statistic is defined as

$$\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

where

- O_i is the observed frequency of the i-th outcome,
- E_i is the expected frequency of the i-th outcome under the null hypothesis (i.e., the game operates fairly).

Calculating Expected Frequencies Using the Binomial Distribution

The expected frequencies are calculated using the binomial distribution. Recall that for a binomial distribution, the probability of observing exactly k successes in n independent trials is

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

where p is the probability of success on a single trial, and $\binom{n}{k}$ represents the binomial coefficient

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

For example, if there are $n = 5$ trials and the probability of drawing an S-ranked character is $p = 0.3$, then the probability of exactly 2 successes is

$$P(X = 2) = \binom{5}{2}(0.3)^2(1-0.3)^{5-2} = \frac{5 \times 4}{2 \times 1}(0.3)^2(0.7)^3 = 10 \times 0.09 \times 0.343 = 0.3087$$

Now, assuming we conduct the card draw experiment 100 times, the expected frequency E_i of drawing exactly 2 S-ranked characters can be calculated as

$$E_2 = 100 \times 0.3087 = 30.87$$

This is the expected number of times we anticipate drawing exactly 2 S-ranked characters across 100 experiments.

Applying the Chi-Square Formula

Given observed data from a series of 100 pulls, suppose the actual observed frequencies O_i are as follows

$$O_0 = 10, \quad O_1 = 35, \quad O_2 = 28, \quad O_3 = 18, \quad O_4 = 7, \quad O_5 = 2$$

The expected frequencies E_i, calculated from the binomial distribution, are

$$E_0 = 16.81, \quad E_1 = 36.05, \quad E_2 = 30.87, \quad E_3 = 12.48, \quad E_4 = 3.08, \quad E_5 = 0.71$$

The chi-square statistic is calculated as

$$\chi^2 = \frac{(10-16.81)^2}{16.81} + \frac{(35-36.05)^2}{36.05} + \frac{(28-30.87)^2}{30.87} + \frac{(18-12.48)^2}{12.48} + \frac{(7-3.08)^2}{3.08} + \frac{(2-0.71)^2}{0.71}$$

Calculation of the Chi-Square Statistic

Breaking down the calculations

$$\frac{(10-16.81)^2}{16.81} = \frac{(-6.81)^2}{16.81} = \frac{46.36}{16.81} = 2.76$$

$$\frac{(35-36.05)^2}{36.05} = \frac{(-1.05)^2}{36.05} = \frac{1.1025}{36.05} = 0.031$$

$$\frac{(28-30.87)^2}{30.87} = \frac{(-2.87)^2}{30.87} = \frac{8.2369}{30.87} = 0.267$$

$$\frac{(18-12.48)^2}{12.48} = \frac{(5.52)^2}{12.48} = \frac{30.4704}{12.48} = 2.44$$
$$\frac{(7-3.08)^2}{3.08} = \frac{(3.92)^2}{3.08} = \frac{15.3664}{3.08} = 4.99$$
$$\frac{(2-0.71)^2}{0.71} = \frac{(1.29)^2}{0.71} = \frac{1.6641}{0.71} = 2.34$$

Now, summing these terms gives

$$\chi^2 = 2.76 + 0.031 + 0.267 + 2.44 + 4.99 + 2.34 = 12.83$$

Degrees of Freedom

The degrees of freedom df for this test are given by

$$df = k - 1 = 6 - 1 = 5$$

where k is the number of categories (in this case, the number of possible outcomes, which is 6 for the number of successes in 5 trials).

Finally, we compare the calculated chi-square statistic $\chi^2 = 12.83$ with the critical value from the chi-square distribution table at $df = 5$ and a significance level $\alpha = 0.05$. The critical value is $\chi^2_{crit} = 11.07$. Since $\chi^2 = 12.83$ exceeds the critical value, we reject the null hypothesis, suggesting that the observed data significantly deviates from what would be expected under a fair game assumption.

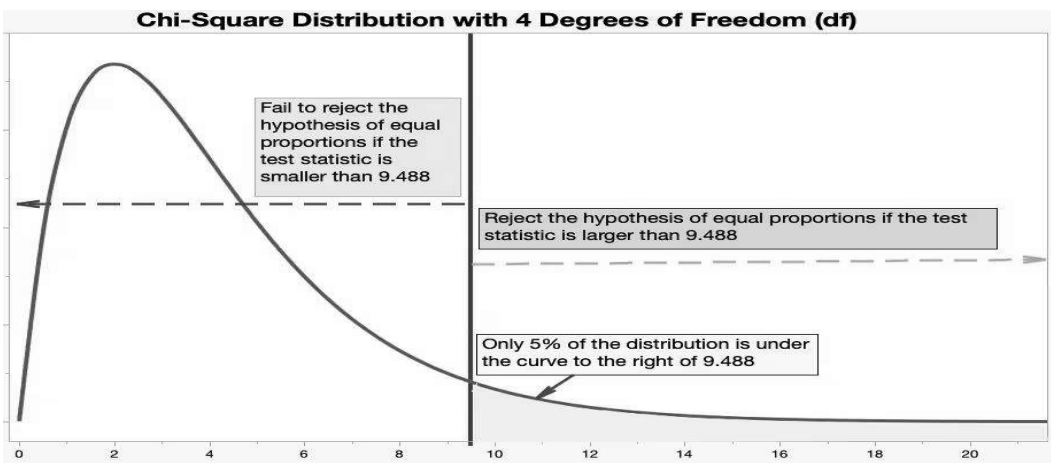

Figure 2: Example of chi-square distribution but with df=4[2]

In figure 2 we can denote that

- ✓ If test statistic is lower than the Chi-square value. You fail to reject the hypothesis of equal proportions. The fit of equal proportions is "good enough."

[2] https://www.jmp.com/en_ch/statistics-knowledge-portal/chi-square-test/chi-square-goodness-of-fit-test.html

- ✓ If test statistic is higher than the Chi-Square value. You reject the hypothesis of equal proportions. The fit of equal proportions is "not good enough."[3]

Cumulative Probability Distribution

The cumulative distribution function (CDF) plays a critical role in understanding the card-draw mechanics, especially when incorporating guaranteed probability increments. In this context, we define the CDF as a piecewise function, taking into account whether the number of trials n is less than or greater than the guarantee threshold t_g.

Cumulative Distribution Function (CDF)

For $n \leq t_g$ (i.e., before the guarantee takes effect), the CDF is based on the base probability of success p_0 for each independent trial. The probability of failure in each trial is $1 - p_0$, and thus the CDF for n trials becomes

$$F_n = 1 - (1 - p_0)^n$$

This equation describes the probability of having at least one success after n trials, where the probability of success remains constant across all trials.

For $n > t_g$ (i.e., after the guarantee mechanism kicks in), the CDF reflects the increased probabilities p_i for each subsequent trial. The probability of failure in each trial i is $1 - p_i$, and thus the cumulative probability is given by

$$F_n = 1 - \prod_{i=1}^{n} (1 - p_i)$$

Here, the product accounts for the different failure probabilities in each trial, reflecting the varying success probabilities as guaranteed increments apply.

Probability Density Function (PDF)

The probability density function (PDF) is derived by differentiating the CDF with respect to the number of trials n. The PDF gives us the exact probability of success on the n-th trial, incorporating the effects of the guarantee system.

For $n \leq t_g$, the PDF can be obtained by differentiating $F_n = 1 - (1 - p_0)^n$ with respect to n:

$$f_n = \frac{d}{dn}(1 - (1 - p_0)^n) = (1 - p_0)^{n-1} \cdot p_0$$

This formula represents the probability of success on the n-th trial when the probability of success remains constant at p_0.

[3] https://www.jmp.com/en_ch/statistics-knowledge-portal/chi-square-test/chi-square-goodness-of-fit-test.html

For $n > t_g$, the PDF is more complex due to the product of failure probabilities. Differentiating the CDF for $n > t_g$

$$f_n = \frac{d}{dn}\left(1 - \prod_{i=1}^{n}(1 - p_i)\right)$$

Applying the product rule, we get

$$f_n = \prod_{i=1}^{n-1}(1 - p_i) \cdot p_n$$

This expression gives the probability of success on the n-th trial, taking into account the cumulative effect of all previous trials where the success probabilities p_i may vary.

Example Calculation

Consider an example where the guarantee threshold $t_g = 5$, and the initial success probability $p_0 = 0.1$. The probabilities p_n increase by 0.05 after each trial beyond t_g, i.e., $p_6 = 0.15$, $p_7 = 0.2$, and so on.

Case 1: $n \leq t_g$

For $n = 3$, before the guarantee threshold, the CDF is

$$F_3 = 1 - (1 - 0.1)^3 = 1 - (0.9)^3 = 1 - 0.729 = 0.271$$

Thus, the probability of drawing an S-ranked card within 3 trials is 27.1%.

The corresponding PDF for the 3rd trial is

$$f_3 = (1 - 0.1)^2 \cdot 0.1 = (0.9)^2 \cdot 0.1 = 0.81 \times 0.1 = 0.081$$

Thus, the probability of success on the 3rd trial alone is 8.1%.

Case 2: $n > t_g$

For $n = 7$, beyond the guarantee threshold, with $p_6 = 0.15$ and $p_7 = 0.2$, the CDF is

$$F_7 = 1 - \prod_{i=1}^{7}(1 - p_i)$$

$$= 1 - (1 - 0.1)(1 - 0.1)(1 - 0.1)(1 - 0.1)(1 - 0.1)(1 - 0.15)(1 - 0.2)$$

$$F_7 = 1 - (0.9)^5 \times (0.85) \times (0.8) = 1 - 0.59049 \times 0.85 \times 0.8 = 1 - 0.40197 = 0.598$$

Thus, the probability of drawing an S-ranked card by the 7th trial is 59.8%.

The corresponding PDF for the 7th trial is

$$f_7 = \prod_{i=1}^{6}(1 - p_i) \cdot p_7 = (0.9)^5 \times (0.85) \times 0.2 = 0.59049 \times 0.85 \times 0.2 = 0.1005$$

Thus, the probability of success specifically on the 7th trial is 10.05%. The advanced cumulative distribution model, incorporating guaranteed probability increments, provides a deeper understanding of the success probabilities in *Absolute Zenless Zero Zone*'s card-draw

mechanics. The piecewise function captures both static and dynamic probability changes, offering a robust framework for analyzing player experiences over multiple trials.

Conclusion and Evaluation

Summative Conclusion on the Findings

In my analysis of Absolute Zenless Zero Zone's card draw mechanics, I applied a range of advanced mathematical models, notably involving binomial distributions and recurrence relations, to decode the intricacies of the system. Specifically, the binomial distribution

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

where n is the total number of trials, k is the number of successful draws (e.g., an S-ranked character), and p is the probability of success on each trial, allowed me to calculate the exact probability of drawing an S-ranked character over multiple attempts. For example, if there is a 30% chance of drawing an S-ranked character, the probability of drawing exactly 2 S-ranked characters in 4 attempts is

$$P(X = 2) = \binom{4}{2} (0.3)^2 (1-0.3)^{4-2} = \binom{4}{2} (0.3)^2 (0.7)^2 \approx 0.2646$$

This enabled me to map out the probabilities of success across different gameplay scenarios, providing a clearer mathematical understanding of the player's experience, which inherently involves uncertainty and randomness. The unpredictability in the game became quantifiable and more tangible when viewed through the lens of probability theory.

In my comprehensive analysis of Absolute Zenless Zero Zone's card draw mechanics, I utilized not just the binomial distribution, but also recurrence relations and hypothesis testing techniques like the Chi-square test. The recurrence relation framework introduces a more dynamic way to model the probability of drawing a specific card type, particularly when the outcomes of previous draws influence subsequent trials. This contrasts with the binomial model, which assumes independent trials. The recurrence relation can be expressed as

$$P(X_n = k) = P(X_{n-1} = k-1)p + P(X_{n-1} = k)(1-p)$$

where $P(X_n = k)$ is the probability of drawing k successes after n draws, and p is the probability of success on each individual trial. This formulation allows for a recursive calculation of the probability based on prior outcomes, providing a deeper level of insight into the gameplay mechanics.

For instance, if we assume the likelihood of drawing an S-ranked card is dependent on the result of the previous draw (i.e., not independent), the recurrence relation offers a powerful mechanism for modeling the system. By analyzing the transition probabilities between successive draws, I derived expressions for long-term probabilities of outcomes that better reflect the complexity of player interactions.

Furthermore, validation of these models was achieved using the Chi-square test for goodness of fit. By comparing the expected frequencies of outcomes (as predicted by the binomial and recurrence models) to the observed data, I could statistically confirm the validity of the models. The Chi-square test statistic is given by

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$$

where O_i is the observed frequency and E_i is the expected frequency. This provided an essential layer of statistical rigor to my findings. Each of these mathematical models offered distinct insights. The binomial distribution gives a static probability for certain outcomes over a fixed number of draws, while the recurrence relations provide a dynamic framework that takes into account the evolving nature of probability as the game progresses. Together, they provide a robust, multi-dimensional analysis of the game's mechanics, moving beyond surface-level randomness to offer a deep, mathematical understanding of player experiences in *Absolute Zenless Zero Zone.*

Implications of the Study

The results of this research pass in addition than just analyzing the unique card recreation. The binomial distribution, recurrence members of the family, and Chi-square take a look at strategies are applicable in many fields in which probabilistic events take place. This encompasses playing, sport concept, or even wider makes use of in facts and finance (Baxter, 2023). the use of these techniques to version the sport, we have demonstrated how similar

strategies may be implemented to observe and decorate the game's shape, making sure equality and openness. additionally, the research emphasizes the significance of using statistical assessments inclusive of the Chi-square result to verify the correlation between theoretical possibilities and real consequences, in particular in systems wherein randomness is a thing.

Strengths and Limitations

Using the binomial distribution allows for specific probability calculations, whereas the recurrence relation affords a greater adaptable and iterative technique to information consequences (Baxter, 2023). The chi-square check increases the statistical validity, ensuring the theoretical model fits the statistics that changed into location. In fact, card video games often entail dependencies among draws (e.g., cards are not put back after being drawn), making chance calculations more complex. Furthermore, the model presupposes that the chance of selecting a selected card is both steady and recognized, which might not maintain actuality in more intricate games (Walsh, 2023).

Recommendations for Future Studies

Future research could deal with some of the restrictions identified in this evaluation. As an instance, exploring the effects of dependencies among attracts in the game could offer a more accurate model of the game's possibilities. Investigating exclusive sorts of chance distributions that would better capture the complexity of real-world eventualities may also be beneficial (Badge & Badge, 2023). Furthermore, incorporating human elements into the analysis, together with player techniques or selection-making approaches, ought to lead to an extra-comprehensive know-how of ways those elements affect the results of card video games.

In addition, future studies may want to extend the software of those mathematical tools to other video games or probabilistic situations, testing their robustness and versatility (Tintle et al., 2020). With the aid of refining these fashions and exploring their applicability in exclusive contexts, researchers can retain and beautify our know-how of probability and its position in both games and broader fields to have a look at.

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