Math 542-Modern Algebra II

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Problem:
(Mon Feb 17) Suppose for every $x \in G$ that $x^2 = e$. Prove that G is abelian.

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(Mon Feb 17) Suppose $H \subseteq G$ is subgroup of index 2, i.e., $[G : H] = 2$. Prove that it is a normal subgroup of G.

Solution:
(a) First not that for any $x \in G$, we have

$$x^2 = e \Leftrightarrow x = x^{-1}$$

So any element is equal to it's own inverse. Next, let $a, b \in G$. This implies that $ab \in G$ and

$$(ab)^2 = e \Leftrightarrow abab = e \Leftrightarrow ab = b^{-1}a^{-1} \Leftrightarrow ab = ba.$$

Hence, G is abelian, as a, b are arbitrary and all the elements of G commute.

(b) Since the index of H in G is 2, we can create a homomorphism $G \to G/H$ such that an element of $h \in H \subset$ maps to 0 and an element $g \notin H$ but $g \in G$ maps to 1. Hence, $G/H \cong \mathbb{Z}/2\mathbb{Z}$, and since such a homomorphism exists, we can see that H must be normal.