

Math 542-Modern Algebra II

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Problem:

(Wed Feb 19) For F a finite field call $a \in F$ a generator of F iff every nonzero element of F is a power of a .

- (a) Find a generator of \mathbb{Z}_7 .
- (b) How many generators does \mathbb{Z}_{17} have?
- (c) How many generators does \mathbb{Z}_{31} have?

Solution:

- (a) 3 is a generator.

i	3^i
1	$3^1 = 3 \pmod{7}$
2	$3^2 = 2 \pmod{7}$
3	$3^3 = 6 \pmod{7}$
4	$3^4 = 4 \pmod{7}$
5	$3^5 = 5 \pmod{7}$
6	$3^6 = 1 \pmod{7}$

- (b) The multiplicative group of \mathbb{Z}_{17} is the cyclic group C_{16} . This group has 8 generators.
- (c) The multiplicative group of \mathbb{Z}_{31} is the cyclic group C_{30} . This group has 8 generators.