# Math 542-Modern Algebra II

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**Problem 1:**
(Fri Jan 24) (a) Find an integer $x$ such that $x = 6 \mod 10$ and $x = 15 \mod 21$ and $0 \leq x \leq 210$.
(b) Find the smallest positive integer $y$ such that $y = 6 \mod 10$ and $y = 15 \mod 21$ and $y = 8$ mod 11.

**Solution:**
(a) We will use the proof of the Chinese Remainder Theorem to create a number that is the sum of two integers, $x_1, x_2$, which satisfy: $x_1 \equiv 6 \pmod{10}$ and $x_2 \equiv 15 \pmod{21}$, as well as $x_1 \equiv 0 \pmod{10}$ and $x_2 \equiv 0 \pmod{21}$. This number is hence:

$$x = x_1 + x_2 = 6 \cdot 21 \cdot 1 + 15 \cdot 10 \cdot 19 = 2976 \equiv 36 \pmod{210}$$

This number, $x = 36$ satisfies our system of congruences.

(b) We will use the proof of the Chinese Remainder Theorem to create a number that is the sum of three integers, $y_1, y_2, y_3$ which satisfy: $y_1 \equiv 6 \pmod{10}$, $y_2 \equiv 15 \pmod{21}$ and $y_3 \equiv 8 \pmod{11}$, as well as $y_1 \equiv 0 \pmod{21}$ and $y_1 \equiv 0 \pmod{11}$, $y_2 \equiv 0 \pmod{10}$ and $y_2 \equiv 0 \pmod{21}$ and $y_3 \equiv 0 \pmod{10}$ and $y_3 \equiv 0 \pmod{11}$ repectivley. This number is hence:

$$y = y_1 + y_2 + y_3 = 6 \cdot 231 + 15 \cdot 110 \cdot 17 + 8 \cdot 210 = 31116 \equiv 1086 \pmod{2310}$$

This number, $y = 1086$, satisfies our system of congruences.

**Problem 2:**

(Fri Jan 24) (a) Find integers $i, j$ such that there is no integer $x$ with $x = i \mod 6$ and $x = j \mod 15$. (b) Find all pairs $i, j$ with $i = 0, 1, \ldots 5$ and $j = 0, 1, \ldots, 14$ such that there is an integer $x$ with $x = i \mod 6$ and $x = j \mod 15$.

**Solution:**

First we will display the pairs $i, j$ for which there is no $x$ that satisfy the above system of congruences.

| $i$ | $j$ |
|---|---|
| 0 | 1 |
| 0 | 2 |
| 0 | 4 |
| 0 | 5 |
| 0 | 7 |
| 0 | 8 |
| 0 | 10 |
| 0 | 11 |
| 0 | 13 |
| 0 | 14 |
| 1 | 0 |
| 1 | 2 |
| 1 | 3 |
| 1 | 5 |
| 1 | 6 |
| 1 | 8 |
| 1 | 9 |
| 1 | 11 |
| 1 | 12 |
| 1 | 14 |

| $i$ | $j$ |
|---|---|
| 2 | 0 |
| 2 | 1 |
| 2 | 3 |
| 2 | 4 |
| 2 | 6 |
| 2 | 7 |
| 2 | 9 |
| 2 | 10 |
| 2 | 12 |
| 2 | 13 |
| 3 | 1 |
| 3 | 2 |
| 3 | 4 |
| 3 | 5 |
| 3 | 7 |
| 3 | 8 |
| 3 | 10 |
| 3 | 11 |
| 3 | 13 |
| 3 | 14 |

| $i$ | $j$ |
|---|---|
| 4 | 0 |
| 4 | 2 |
| 4 | 3 |
| 4 | 5 |
| 4 | 6 |
| 4 | 8 |
| 4 | 9 |
| 4 | 11 |
| 4 | 12 |
| 4 | 14 |
| 5 | 0 |
| 5 | 1 |
| 5 | 3 |
| 5 | 4 |
| 5 | 6 |
| 5 | 7 |
| 5 | 9 |
| 5 | 10 |
| 5 | 12 |
| 5 | 13 |

(b) Here we will display the pairs $i, j$ which have a solution $x$, as well as the solution $x$ itself:

| $i$ | $j$ | $x$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 3 | 18 |
| 0 | 6 | 6 |
| 0 | 9 | 24 |
| 0 | 12 | 12 |
| 1 | 1 | 1 |
| 1 | 4 | 19 |
| 1 | 7 | 7 |
| 1 | 10 | 25 |
| 1 | 13 | 13 |
| 2 | 2 | 2 |
| 2 | 5 | 20 |
| 2 | 8 | 8 |
| 2 | 11 | 26 |
| 2 | 14 | 14 |

| $i$ | $j$ | $x$ |
|---|---|---|
| 3 | 0 | 15 |
| 3 | 3 | 3 |
| 3 | 6 | 21 |
| 3 | 9 | 9 |
| 3 | 12 | 27 |
| 4 | 1 | 16 |
| 4 | 4 | 4 |
| 4 | 7 | 22 |
| 4 | 10 | 10 |
| 4 | 13 | 28 |
| 5 | 2 | 17 |
| 5 | 5 | 5 |
| 5 | 8 | 23 |
| 5 | 11 | 11 |
| 5 | 14 | 29 |