Math 542-Modern Algebra II

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Problem:
(Mon Feb 24) Let R be a commutative ring with 1. Let I be a maximal ideal in R. Suppose $ab = 0$. Prove that $a \in I$ or $b \in I$.

Problem:
(Mon Feb 24) Consider $p(x) = x^3 + x + 1$ as a polynomial in $\mathbb{Z}_2[x]$. Suppose p has a root α is in some field extension. Construct the multiplication table for $\mathbb{Z}_2[\alpha] =^{def} \{a + b\alpha + c\alpha^2 | a, b, c \in \mathbb{Z}_2\}$

Solution:
(a)

0	0	1	α	$1+\alpha$	$\alpha+\alpha^2$	$1+\alpha+\alpha^2$
0	0	0	0	0	0	0
1	0	1	α	$1+\alpha$	$\alpha+\alpha^2$	$1+\alpha+\alpha^2$

(b) The multiplicative group of $\mathbb{Z}_{17}$ is the cyclic group C_{16}. This group has 8 generators.

(c) The multiplicative group of $\mathbb{Z}_{31}$ is the cyclic group C_{30}. This group has 8 generators.