Artificial Neural Networks: Lecture 8

Reinforcement Learning and SARSA

Wulfram Gerstner

EPFL, Lausanne, Switzerland

Objectives for today:

- Reinforcement Learning is learning by rewards
- Agents and actions
- Exploration vs Exploitation
- Bellman equation
- SARSA algorithm

Announcements:

1. No class on Friday ‘ascension’ (May 11)
2. Classes in May start at 10h30 (and not 10:15)
3. Exam: Monday July 2, 4pm

Exam (counts 67%)

- paper and pencil
- no textbook, no slides, no calculator, no notes.
- similar to exercises and quizzes

Miniprojects (count 33%).

- miniproject validated after ‘fraud detection interview’

Reading for this week:

Sutton and Barto, Reinforcement Learning
(MIT Press, 2nd edition 2018, also online)

Chapters: 1.1-1.4; 2.1-2.6; 3.1-3.5; 6.4

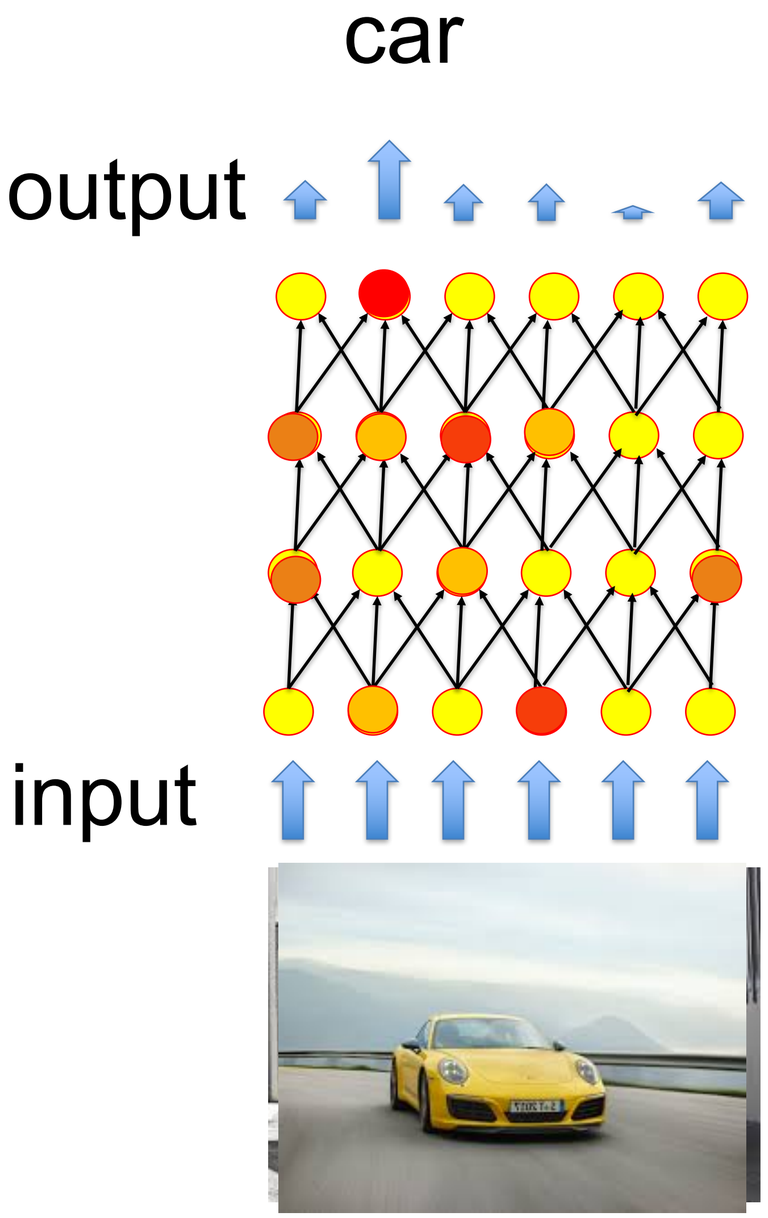
Background reading:

Silver et al. 2017, Archive

Mastering Chess and Shogi by Self-Play with a General Reinforcement Learning Algorithm

Review: Artificial Neural Networks for classification

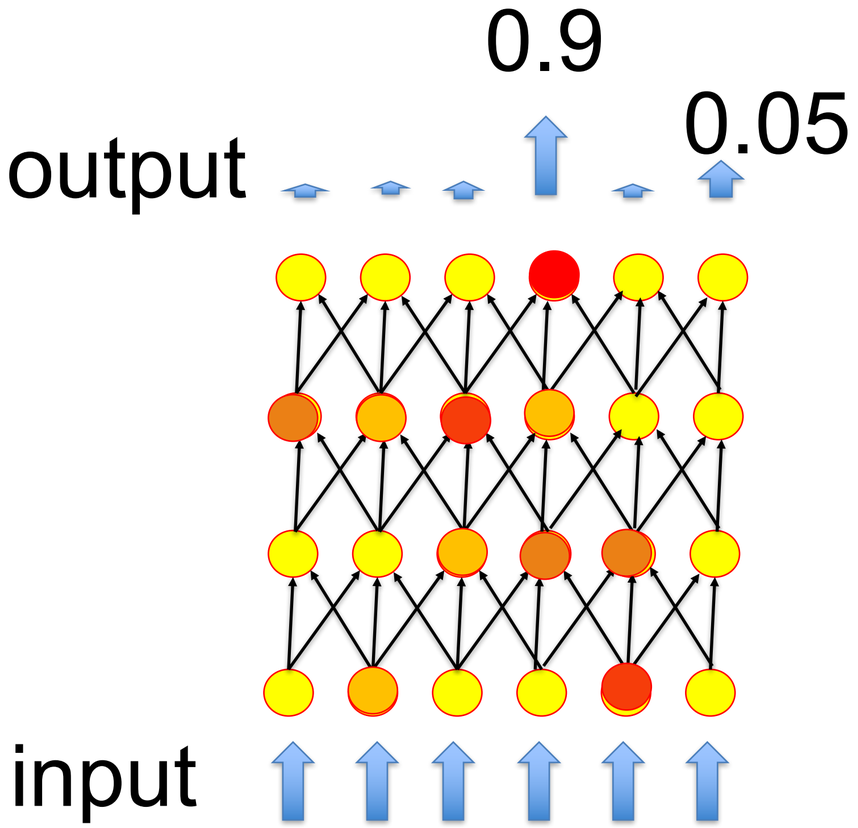
feedforward network



review: Artificial Neural Networks for classification

Prerequisite for learning:
labeled data base

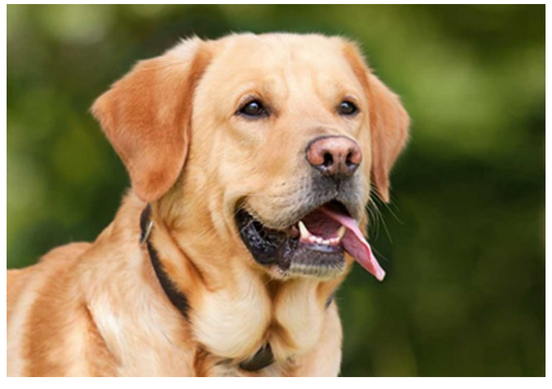
$\{ \quad (\boldsymbol{x}^{\mu}, \boldsymbol{t}^{\mu}) \quad , \qquad 1 \leq \mu \leq P \quad \};$



Aim of learning:
Adjust connections such that output $\boldsymbol{y}^{\mu}$ is correct

$$\boldsymbol{y}^{\mu} = \boldsymbol{t}^{\mu}$$

(for each static input image, $\boldsymbol{x}^{\mu}$)



review: Artificial Neural Networks for classification

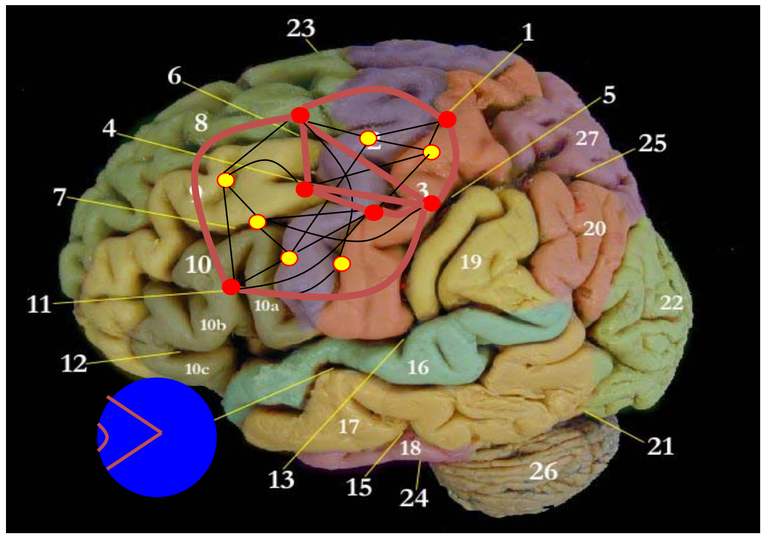
Prerequisite for learning:

labeled data base

$$\{ \quad (\boldsymbol{x}^{\mu}, \boldsymbol{t}^{\mu}) \quad , \qquad 1 \leq \mu \leq P \quad \};$$

Question: Is this realistic?

1. Artificial Neural Networks for action learning



Where is the supervisor?
Where is the labeled data?

Replaced by:
'Value of action'
- 'goodie' for dog
- 'success'
- 'compliment'

BUT:
Reward is rare:
'sparse feedback' after a long action sequence

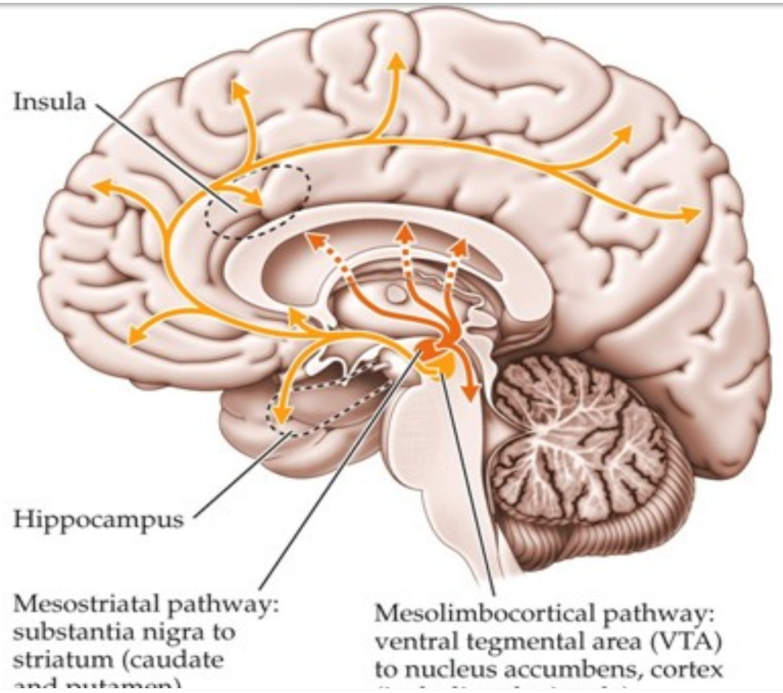
Reward information is available in the brain

Neuromodulator **dopamine**:
Signals reward minus
expected reward

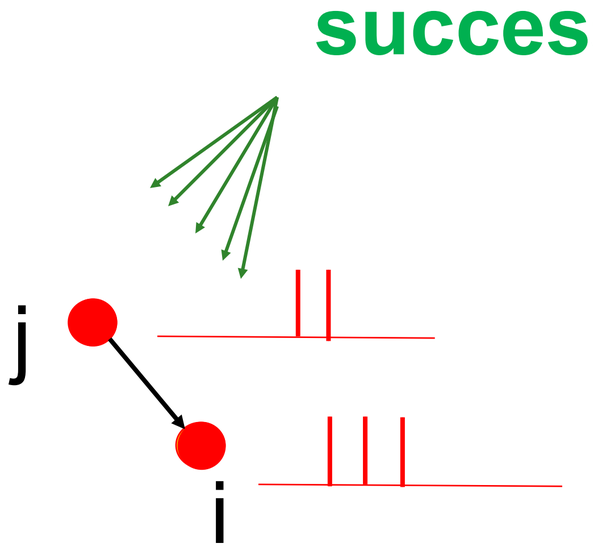
Schultz et al., 1997,
Waelti et al., 2001
Schultz, 2002

‘success signal’

Dopamine



Review: Modeling – the role of reward



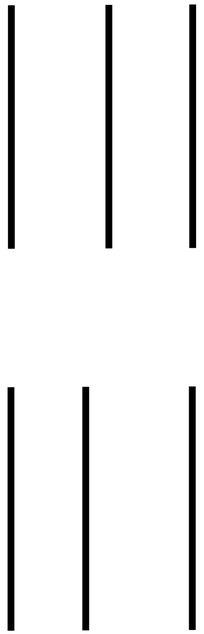
Three factors for changing a connection
- activity of neuron j
- activity of neuron i
- success

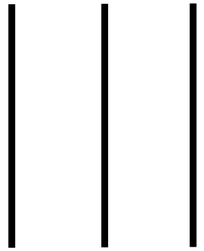
Barto 1985, Schultz et al. 1997; Waelti et al., 2001; Reynolds and Wickens 2002; Lisman et al. 2011

Reinforcement learning = learning based on reward

1. Examples of reinforcment learning

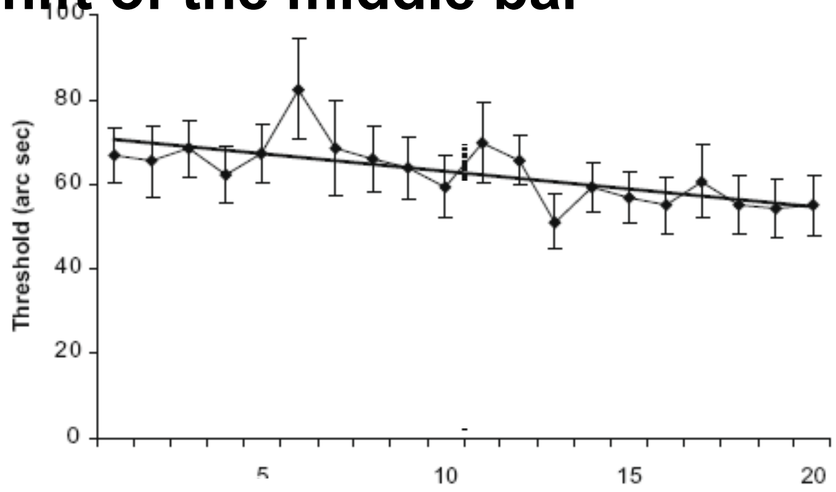
Middle bar: shifted left or shifted right?





Observers get better at seeing the shift of the middle bar

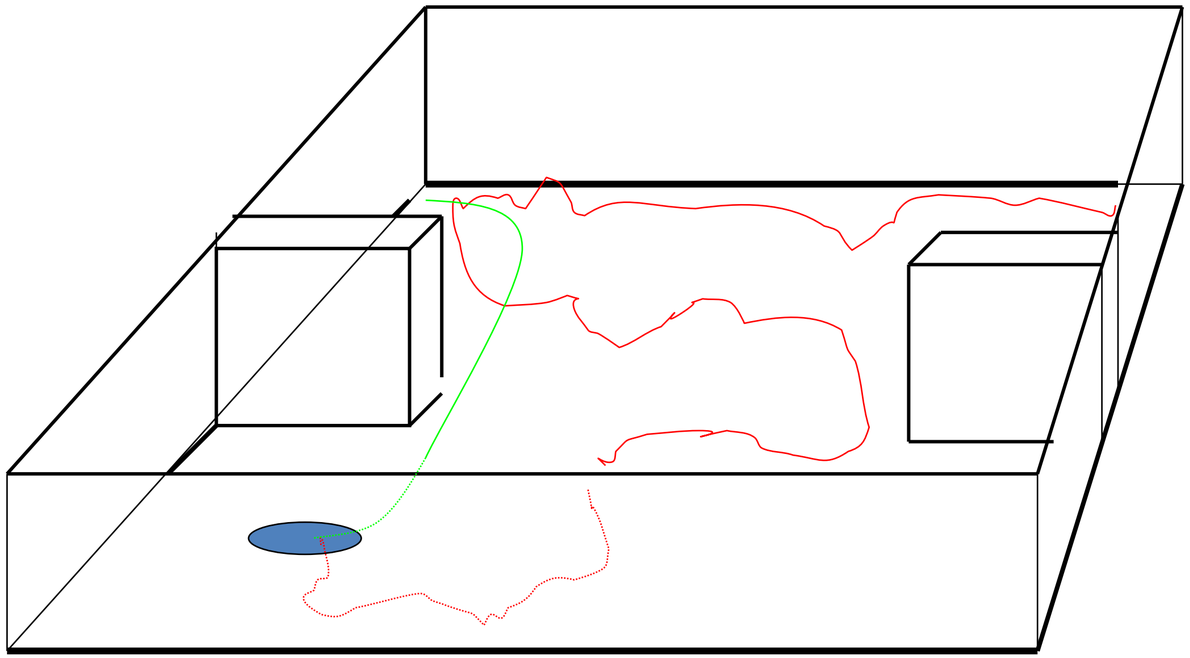
Min. shift



Feedback:
tone for wrong response

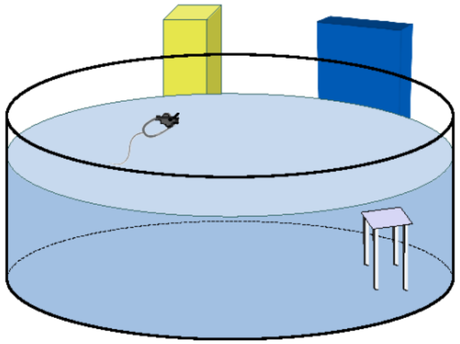
Tartaglia,Aberg,Herzog 2009

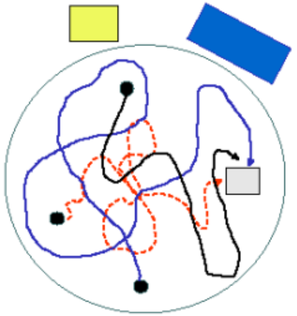
1. Examples of reinforcement learning: animal conditioning

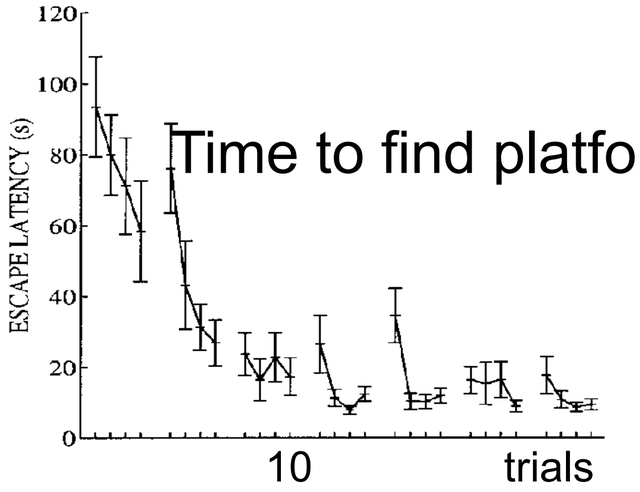


1. Examples of reinforcement learning: animal conditioning

Morris Water Maze







Time to find platform

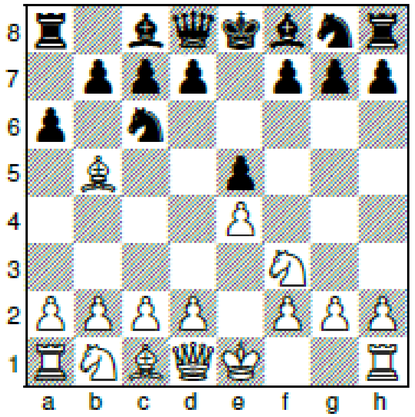
Rats learn to find
the hidden platform

(Because they like to
get out of the cold water)

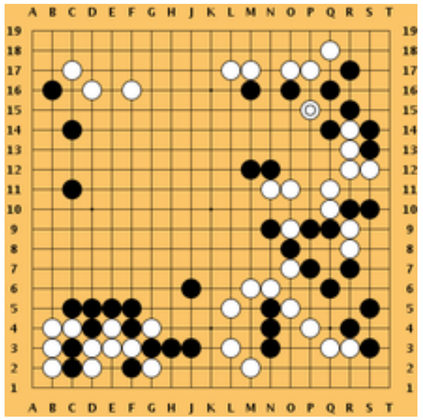
Foster, Morris, Dayan 2000

1. Deep reinforcement learning

Chess



Go



Artificial neural network (*AlphaZero*) discovers different strategies by playing against itself.

In Go, it beats Lee Sedol

1. Deep reinforcement learning

Network for choosing action

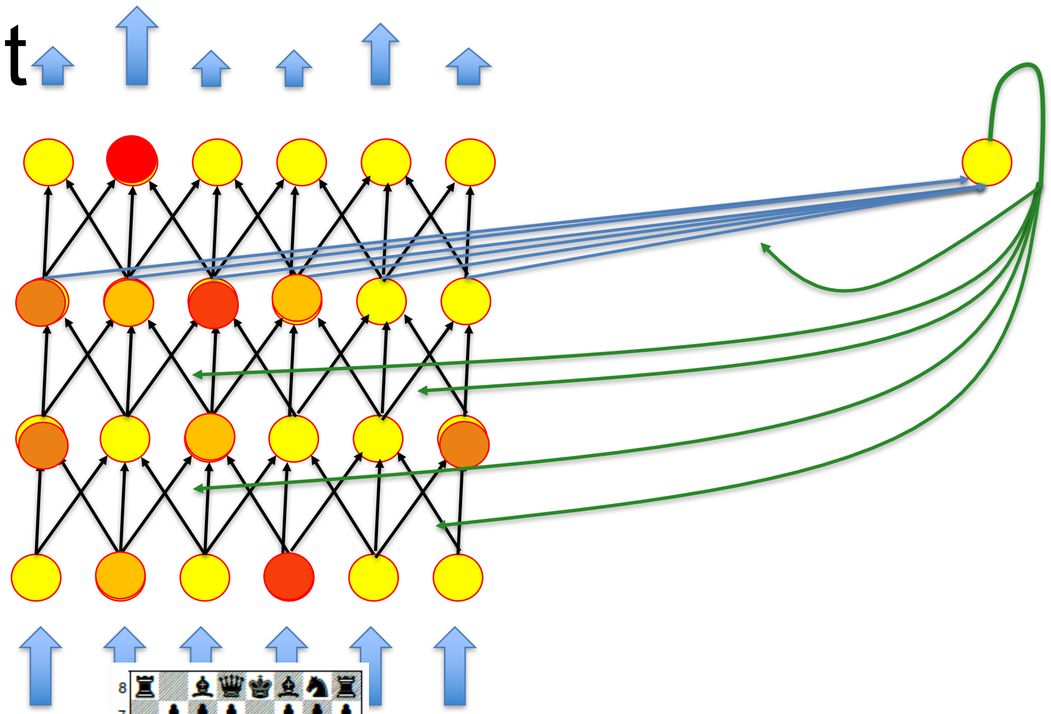
action:

Advance king

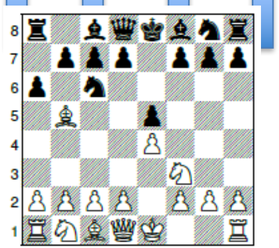
2e output for **value** of action:

probability to win

output



input



learning:

- change connections

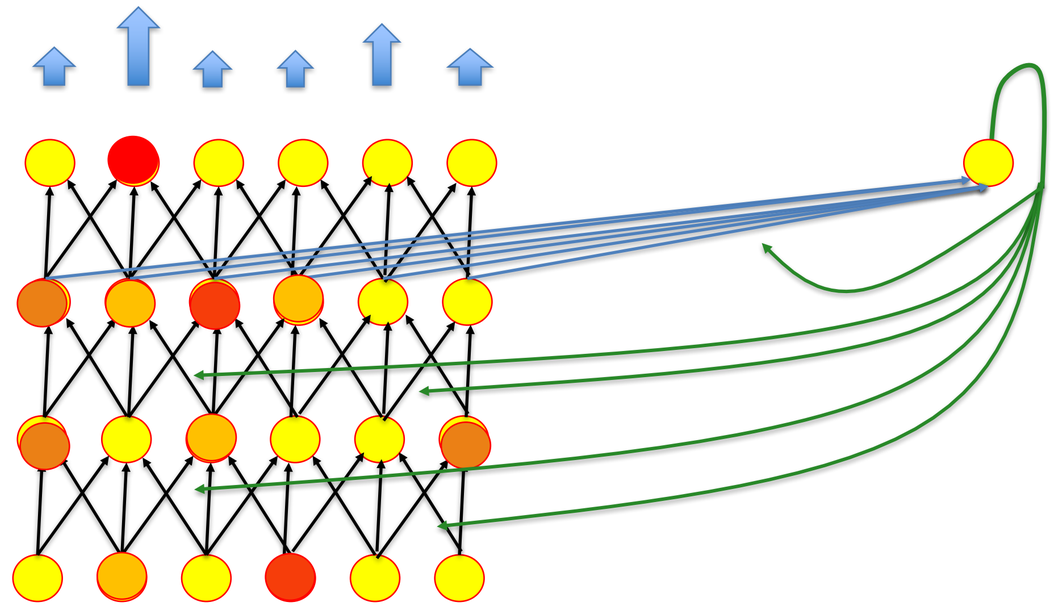
aim:

- Predict value of position
- Choose next action to win

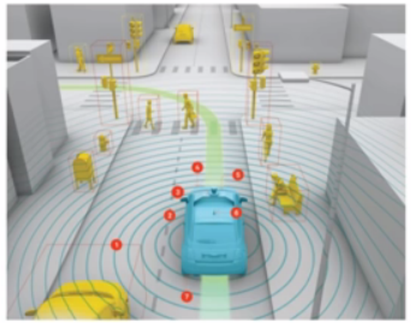
1. Deep Reinforcement Learning: Self-driving cars

https://selfdrivingcars.mit.edu/ *Lex Friedman, MIT*

advance and accerate



Value: security, duration of travel

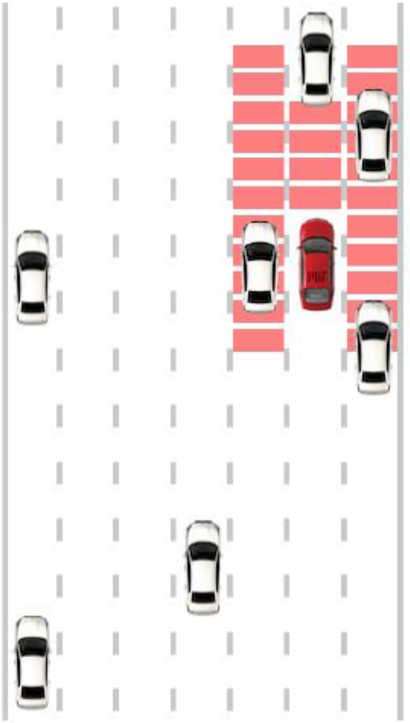


External

1. Radar
2. Visible-light camera
3. LIDAR
4. Infrared camera
5. Stereo vision
6. GPS/IMU
7. CAN
8. Audio

Internal

1. Visible-light camera
2. Infrared camera
3. Audio

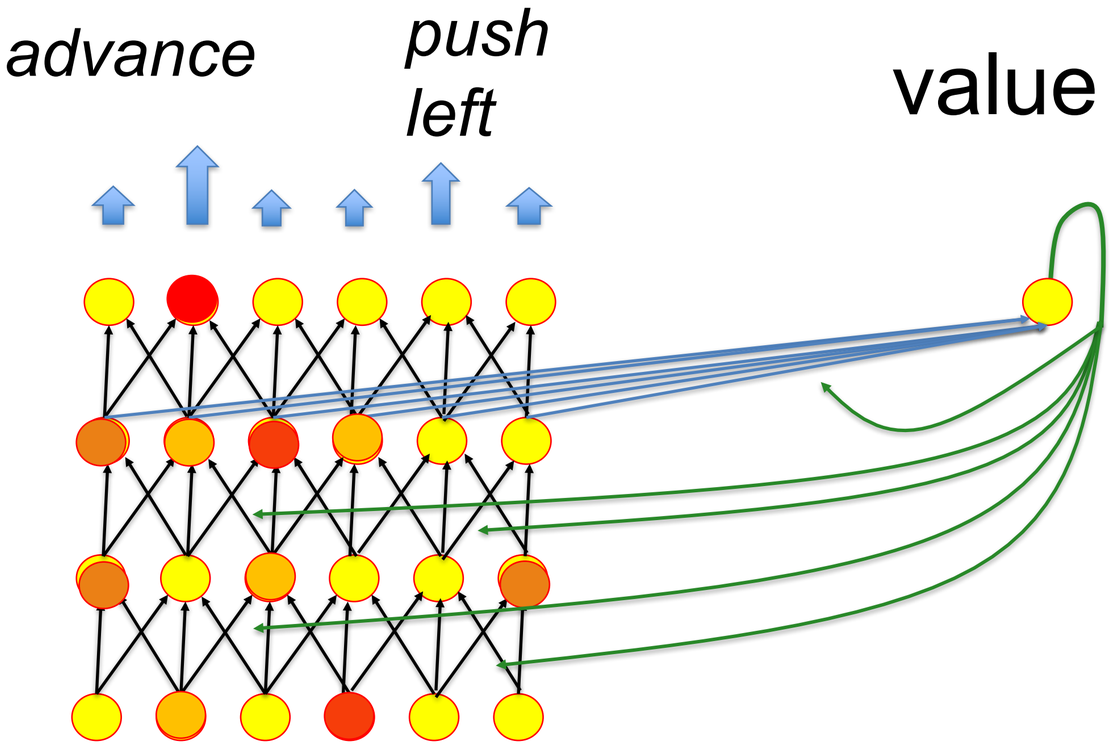


Road Overlay:

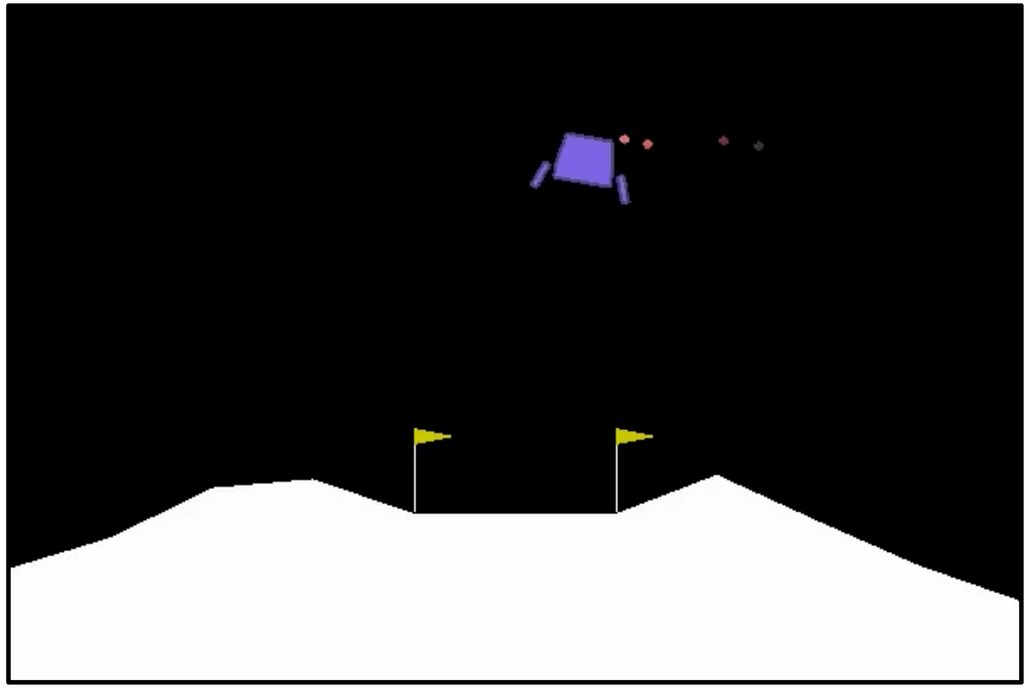
Safety System

1. Deep Reinforcement Learning: Lunar Lander (miniproject)

actions



Aim: land between poles



Quiz: Rewards in Reinforcement Learning

[ ] Reinforcement learning is based on rewards
[ ] Reinforcement learning aims at optimal action choices
[ ] In chess, the player gets an external reward after every move
[ ] In table tennis, the player gets a reward when he makes a point
[ ] A dog can learn to do tricks by giving rewards at appropriate moments

Artificial Neural Networks: Lecture 8

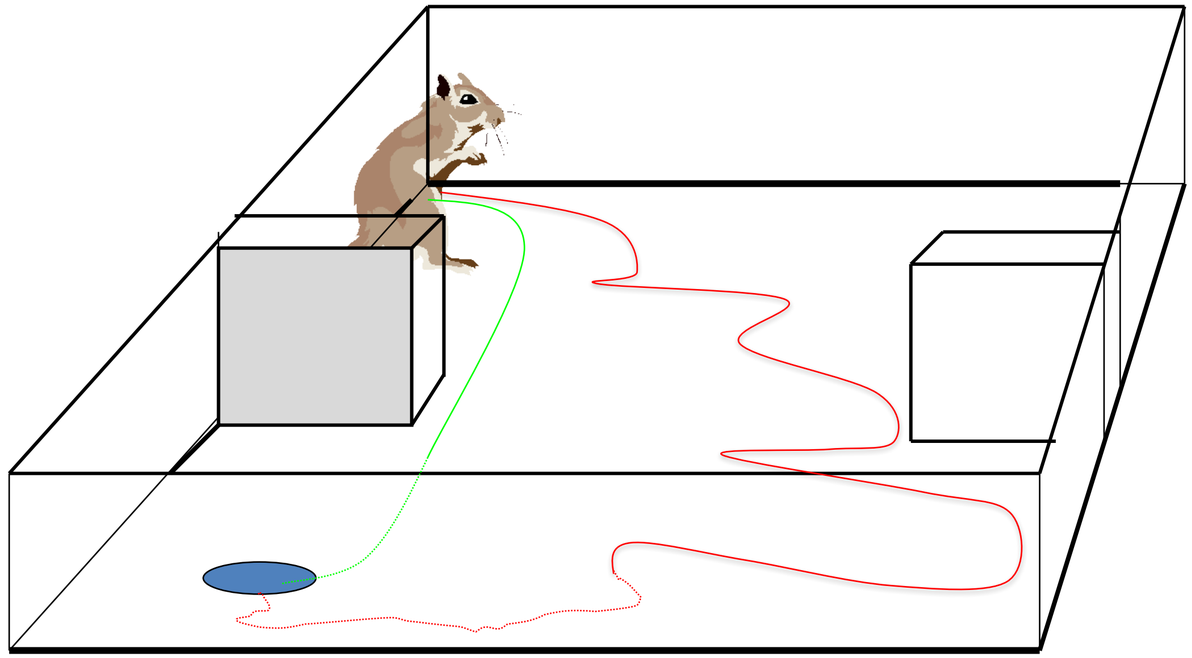
Reinforcement Learning and SARSA

1. Learning by Reward: Reinforcement Learning
2. Elements of Reinforcement Learning

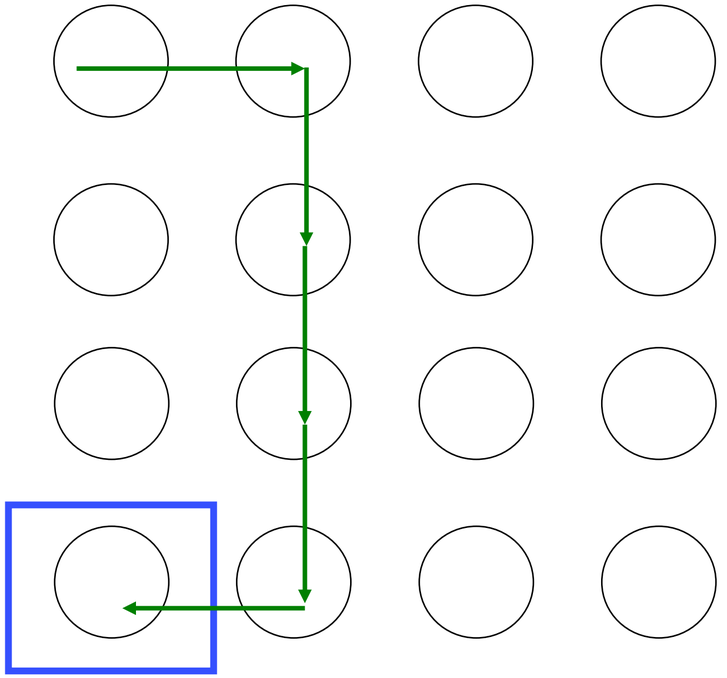
2. Elements of Reinforcement Learning:

-states
-actions
-rewards

2. Elements of Reinforcement Learning:

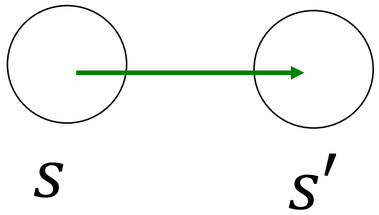


- discrete states
- discrete actions
- sparse rewards



2. Elements of Reinforcement Learning:

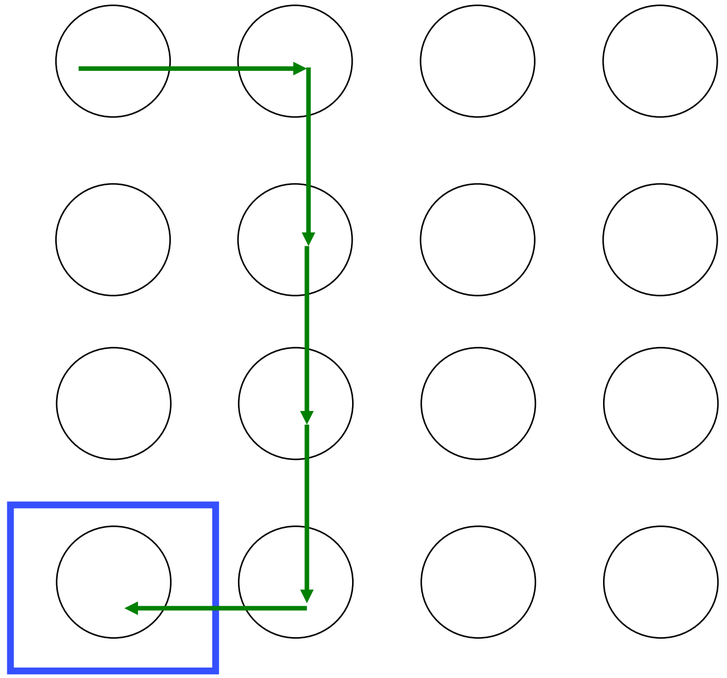
- discrete states:
  - old state s
  - new state s'



- current state: s_t
- discrete actions: a
- Mean rewards for transitions:

$$R^{a}_{s \to s'}$$

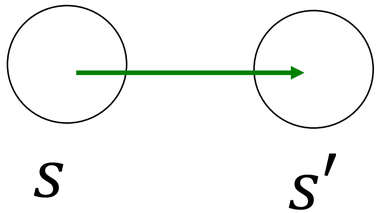
- current reward: r_t



often most transitions have zero reward

2. States Reinforcement Learning:

- discrete states:
  - old state s
  - new state s'
- current state: s_t



state = current configuration/well-defined situation
= generalized ‘location’ of actor in environment

2. Reinforcement Learning: Example Acrobot

3 actions: a1= no torque,
a2 = torque +1 at elbow,
a3 = torque -1 at elbow

States?

→ discretize!

reward if tip above line

Suppose 5 states per dimension,
How many states in total?

[ ] 5
[ ] 25
[ ] 125
[ ] 625

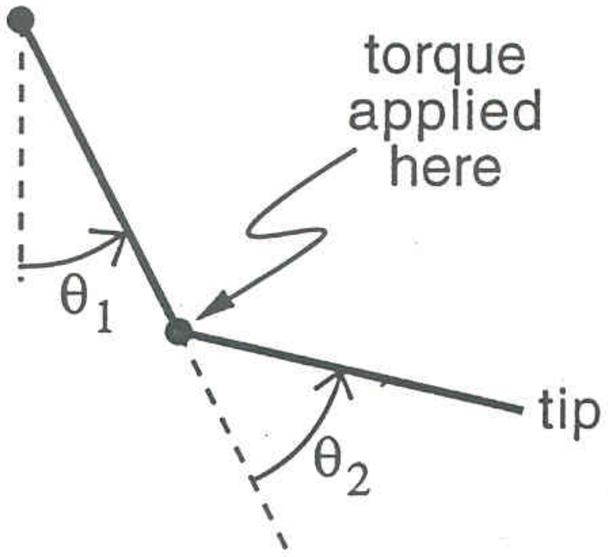


Figure 11.4 The acrobot.

From Book:
Sutton and Barto

2. Reinforcement Learning: Example Acrobot

1st episode: long sequence of random actions
400th episode: short sequence of 'smart' actions

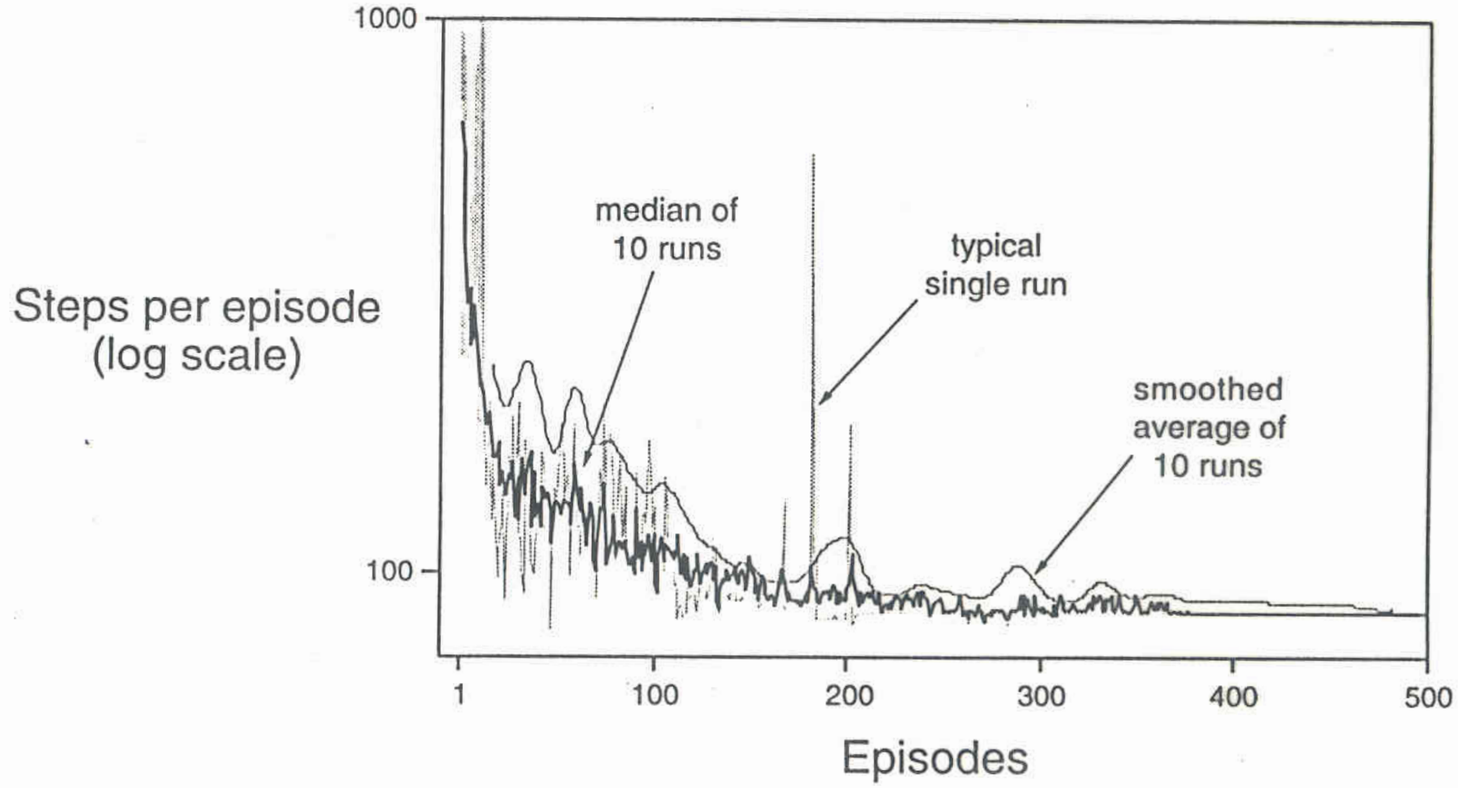


Figure 11.6 Learning curves for Sarsa(λ) on the acrobot task.

From Book:
Sutton and Barto

2. Reinforcement Learning: Example Acrobot

after 400 episodes

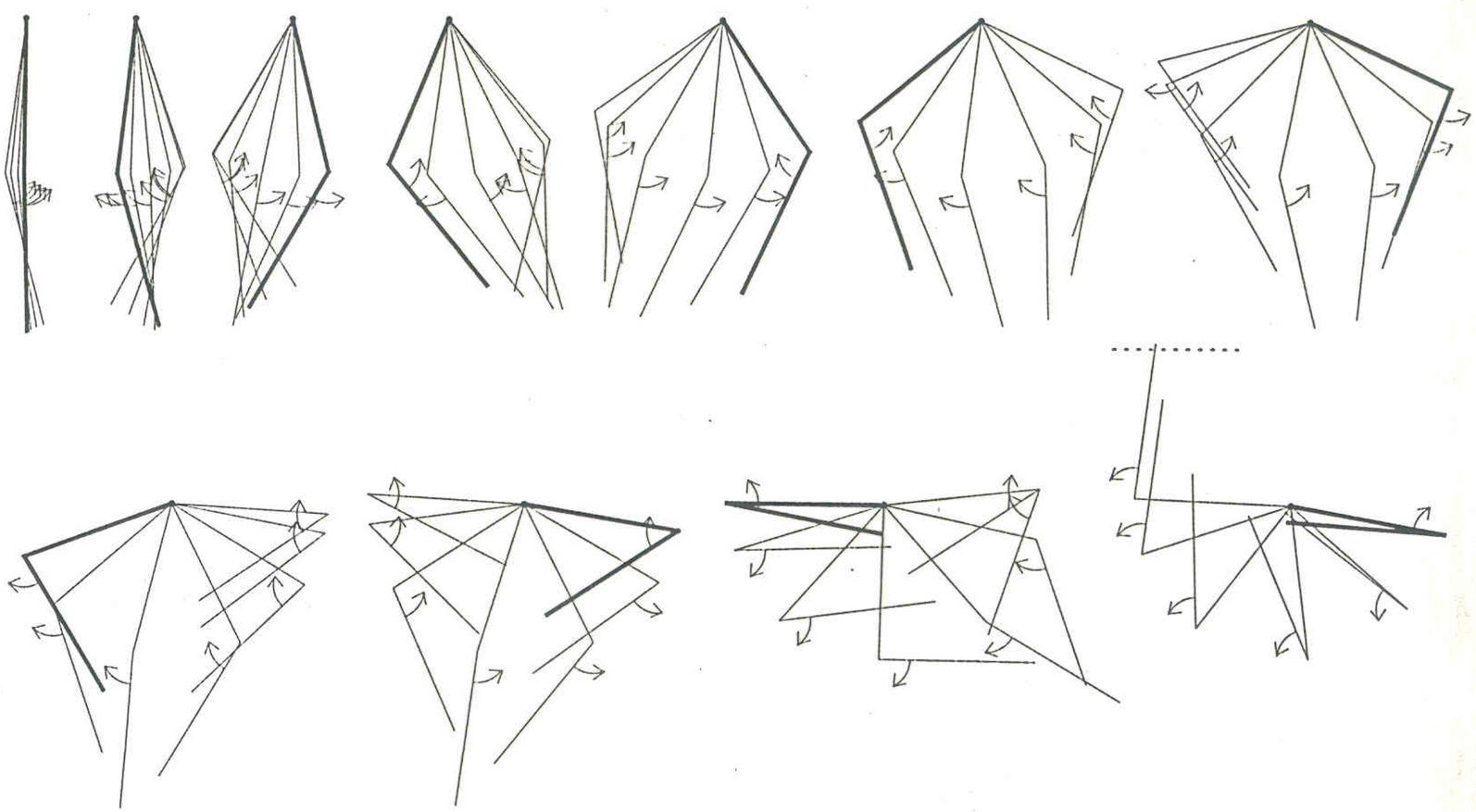


Figure 11.7 A typical learned behavior of the acrobot. Each group is a series of consecutive positions, the thicker line being the first. The arrow indicates the torque applied at the second joint.

From Book:
Sutton and Barto

2. Reinforcement Learning: Example backgammon

Case Studies

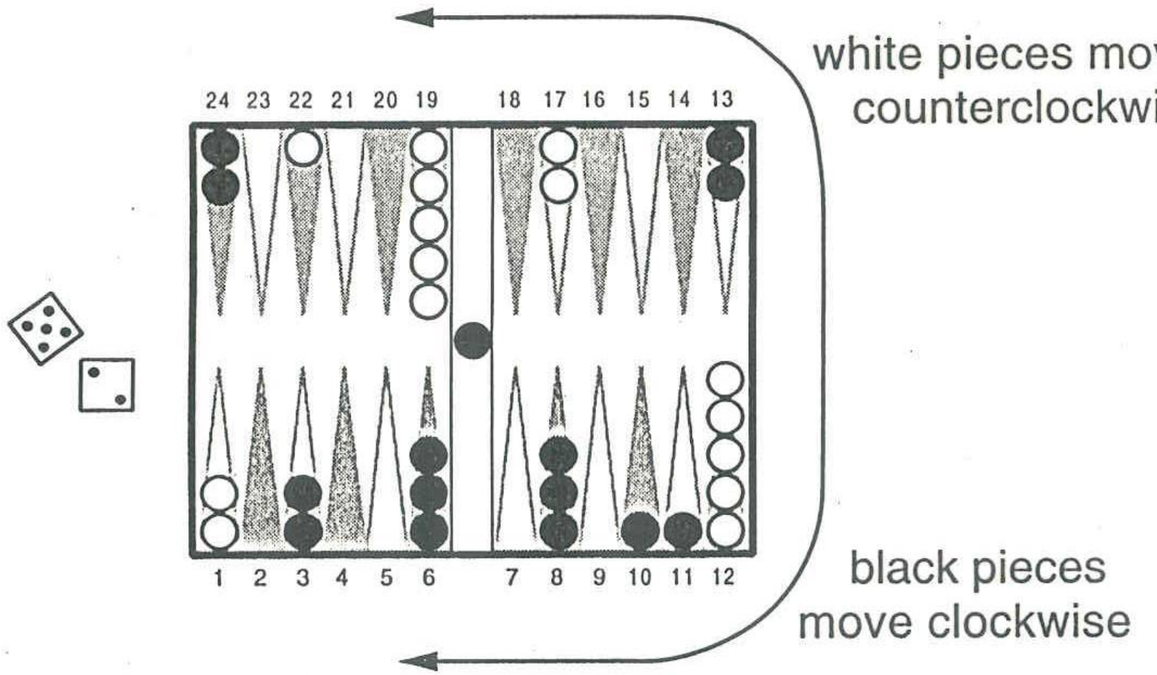


Figure 11.1 A backgammon position.

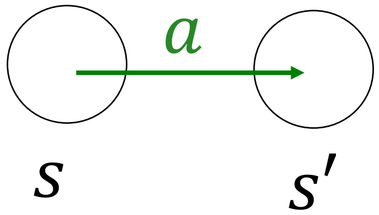
From Book:
Sutton and Barto

Game position = discrete states!

Suppose 2 pieces per player,
How many states in total?
[ ] 100<n<500
[ ] 500<n<5000
[ ] 5 000<n<50 000
[ ] n>50 000

2. Elements of Reinforcement Learning: Summary

There can be MANY states
Often need to discretize first
(→ next week we try to model in continuum)

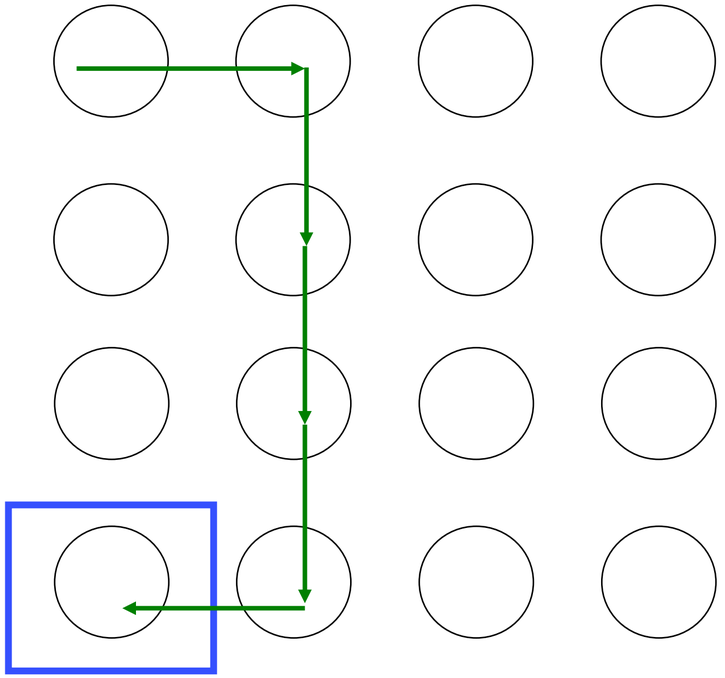


- discrete actions: a

- Mean reward for transition:

$$R^{a}_{s\to s'} = E\left(r\middle|s,a,s'\right)$$

- current actual reward: r_t



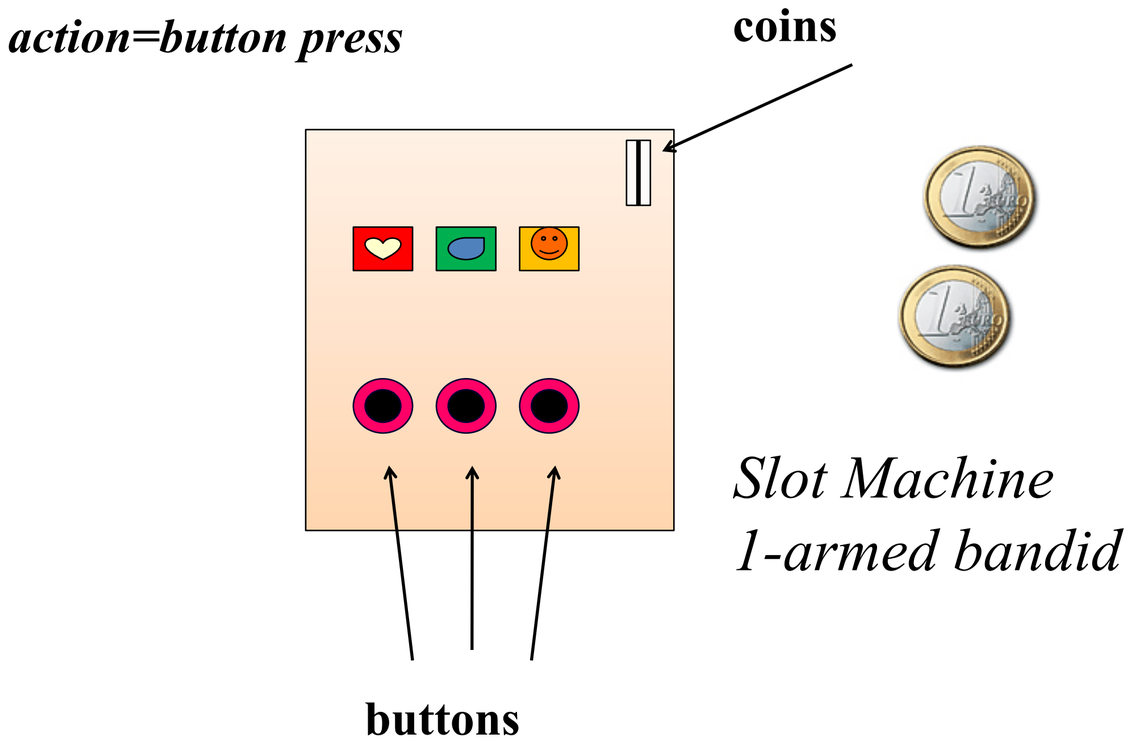
often most transitions have zero reward

Artificial Neural Networks: Lecture 8

Reinforcement Learning and SARSA

1. Learning by Reward: Reinforcement Learning
2. Elements of Reinforcement Learning
3. One-step horizon (bandit problems)

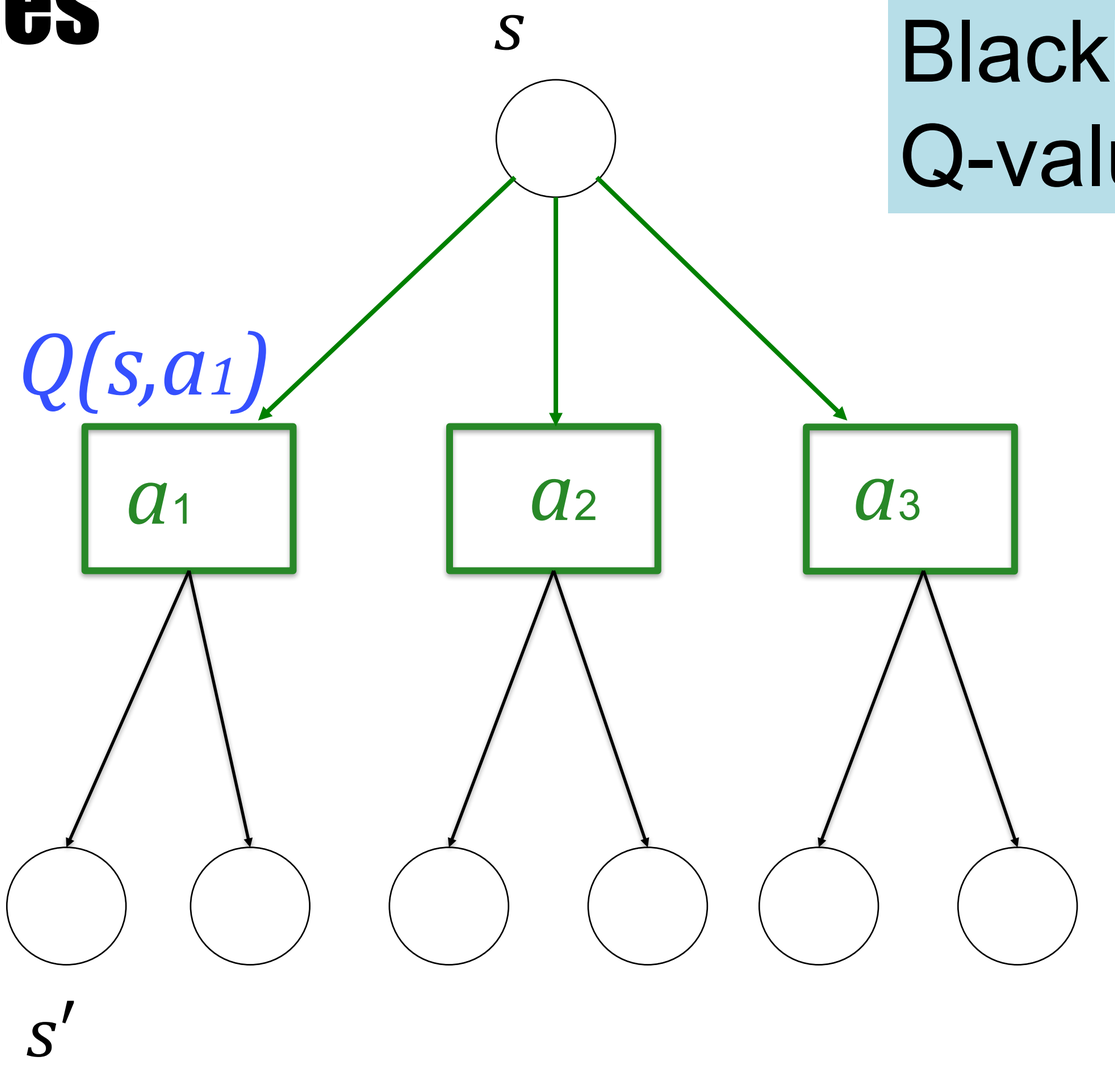
2. One-step horizon games (bandit)



Slot Machine
1-armed bandid

2. One-step horizon games

Q-value: $Q(s,a)$
Expected reward for action a starting from s



Blackboard1:
Q-values

2. One-step horizon games

Blackboard1:
Q-values

2. One-step horizon games: Q-value

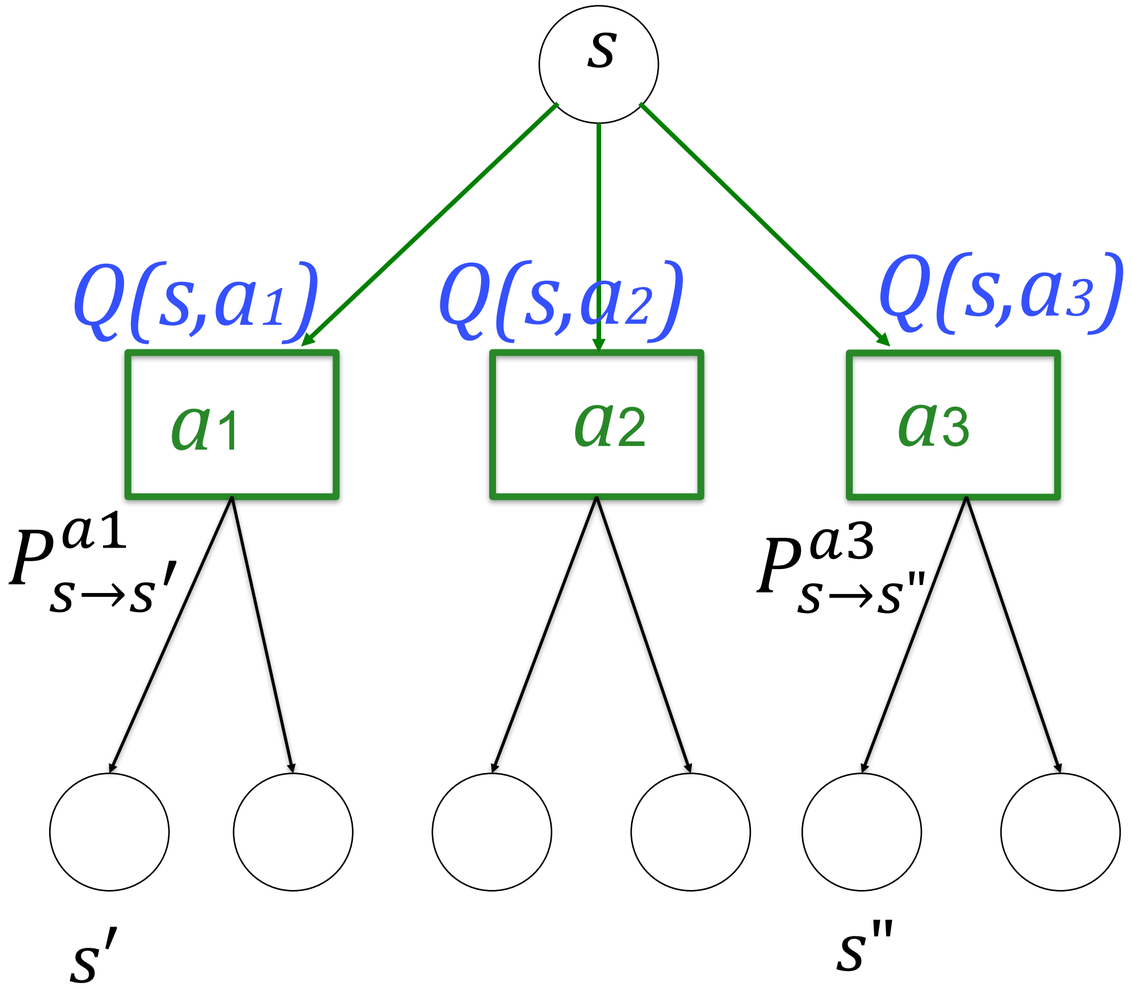
Q-value $Q(s,a)$

Expected reward for action a starting from s

$$Q(s,a) = \sum_{s'} P^{a}_{s\to s'} R^{a}_{s\to s'}$$

Reminder:

$$R^{a}_{s\to s'} = E\left(r \middle| s, a, s'\right)$$



Now we know the Q-values: which action should you choose?

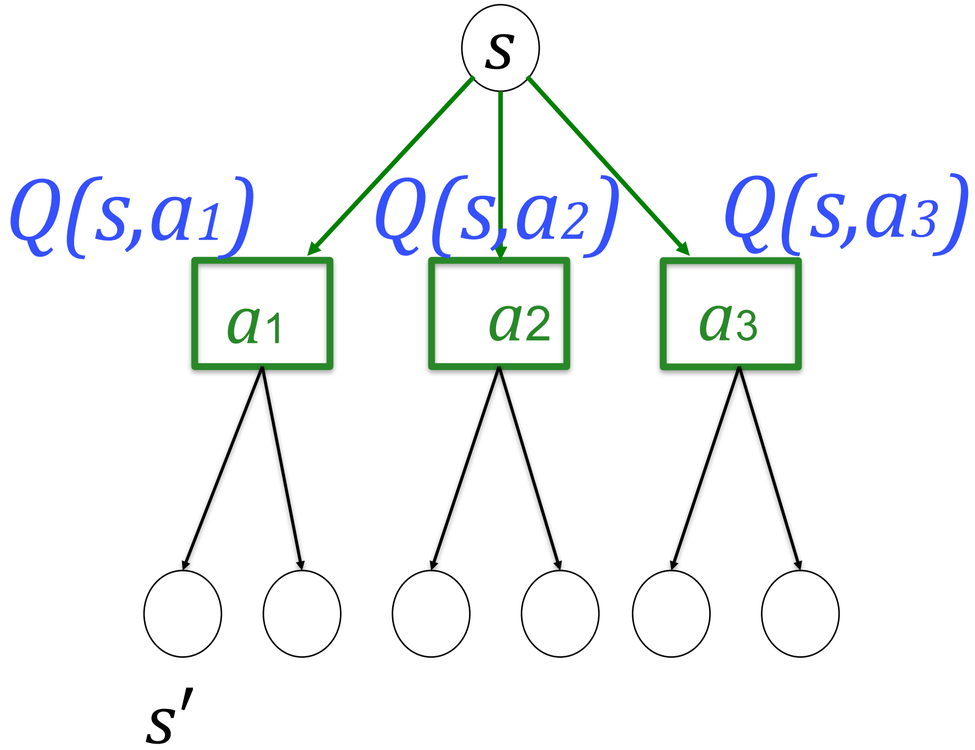
2. Optimal policy (greedy)

Suppose all Q-values are known:

take action a with*

$$Q(s,a^*) > Q(s,a_j)$$

other actions



optimal action:

$$a^* = \text{argmax}_a\,[Q(s,a)]$$

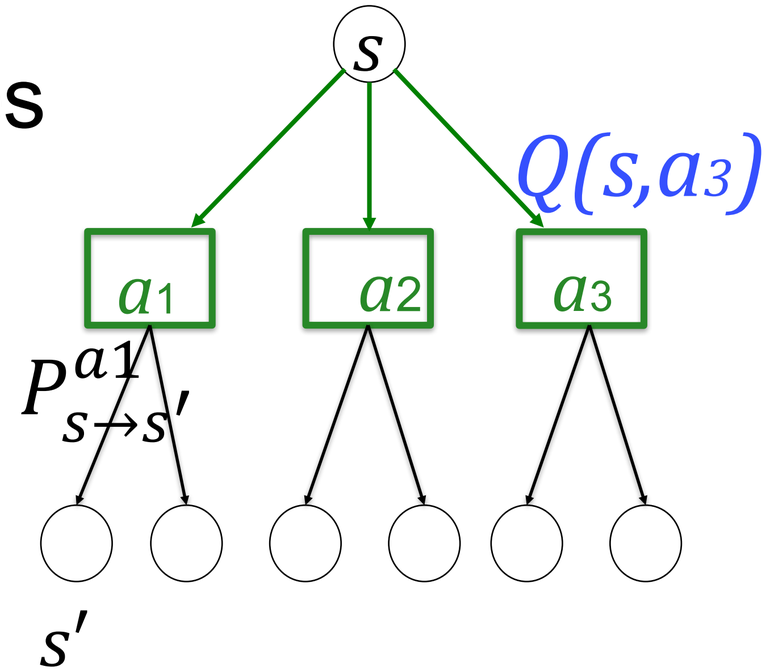
Optimal policy is also called ‘greedy policy’

2. One-step horizon games

Q-value = expected reward for state-action pair

If Q-value is known, choice of action is simple
→ take action with highest Q-value

BUT: we normally do not know the Q-values
→ estimate by trial and error

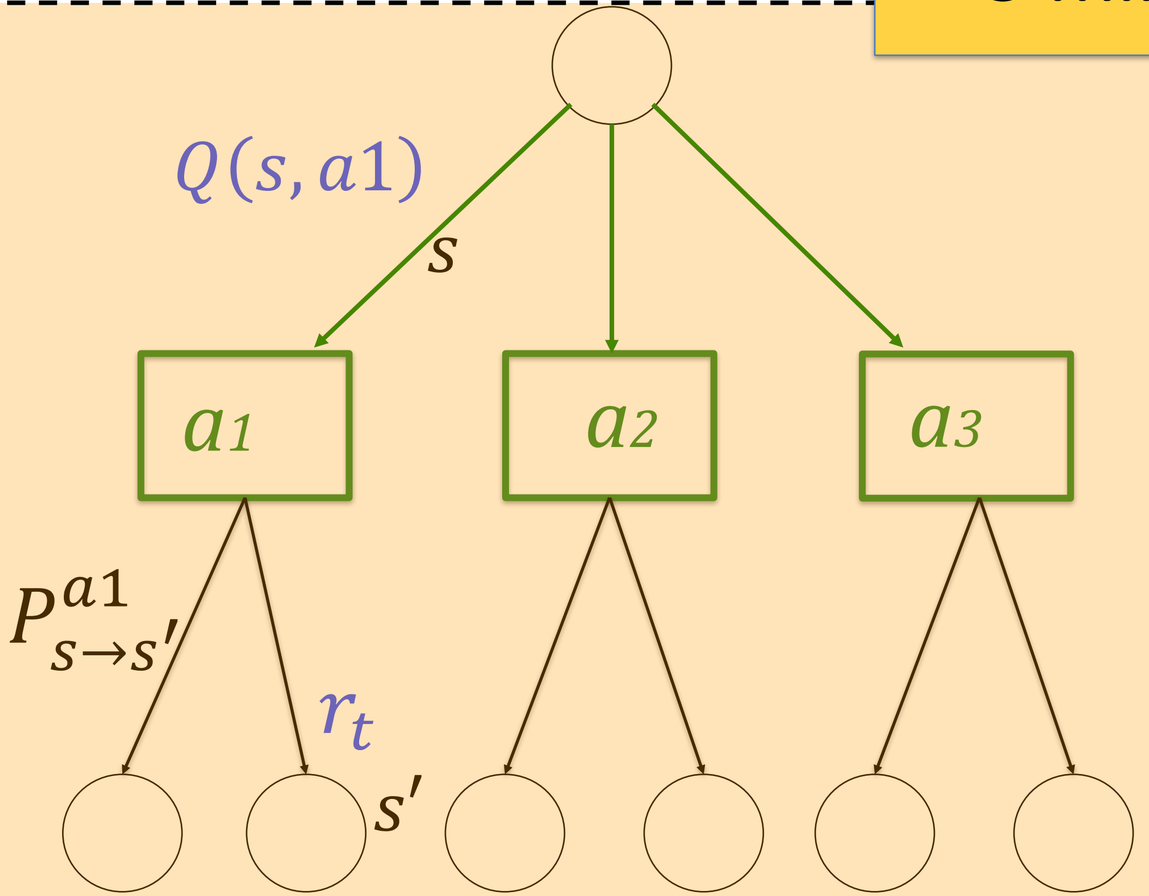


Exercise 1 now (in class)

5 min, now!

Expected reward

$$Q(s,a) = \sum_{s'} P^{a}_{s \to s'} R^{a}_{s \to s'}$$



Show that empirical averaging over k trials gives an update rule

$$\Delta Q(s,a) = \eta\,[r_t - Q(s,a)]$$

Blackboard2: Exercise 1

2. One-step horizon: summary

Q-value = expected reward for state-action pair

If Q-value is known, choice of action is simple

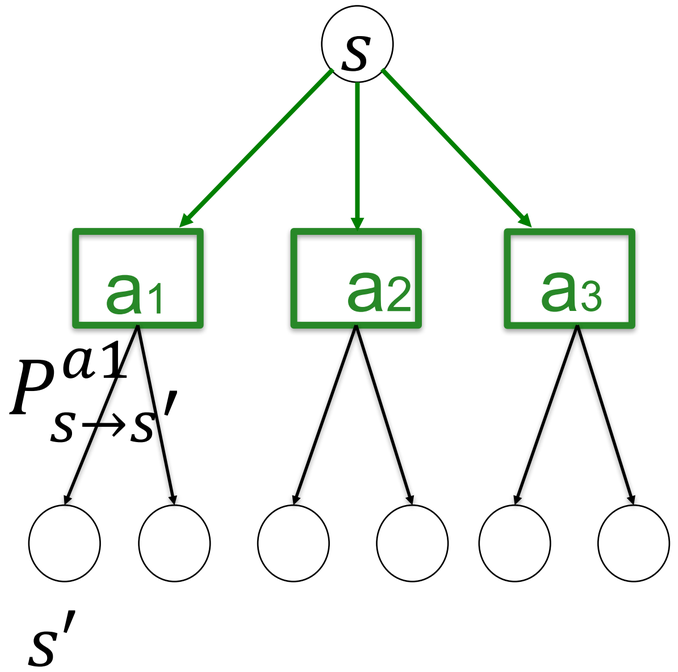
→ take action with highest Q-value

If Q-value not known:

→ estimate by trial and error

→ update with rule

$$\Delta Q(s,a) = \eta\,[r_t - Q(s,a)]$$



Let learning rate η decrease over time

Artificial Neural Networks: Lecture 8

Reinforcement Learning and SARSA

1. Learning by Reward: Reinforcement Learning
2. Elements of Reinforcement Learning
3. Exploration vs Exploitation

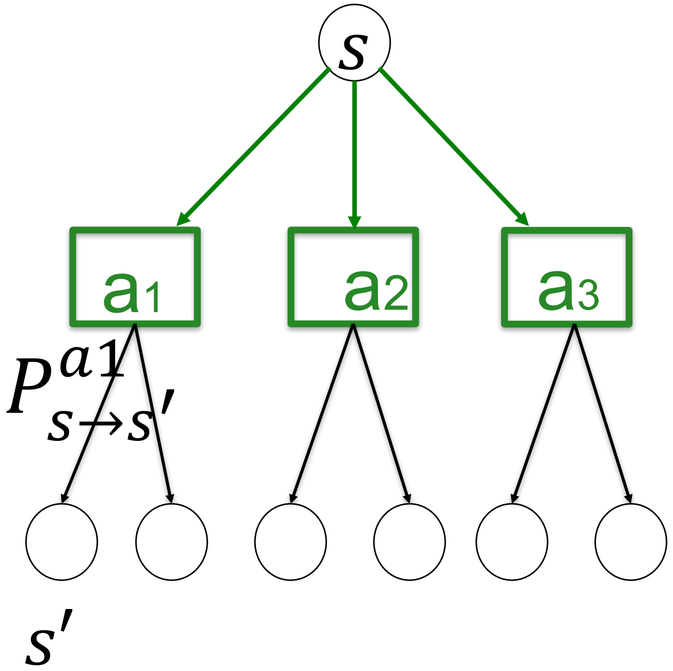
3. Exploration – Exploitation dilemma

Ideal: take action with maximal $Q(s,a)$

Problem:

correct Q values not known

(since reward probabilities and branching probabilities are not known)



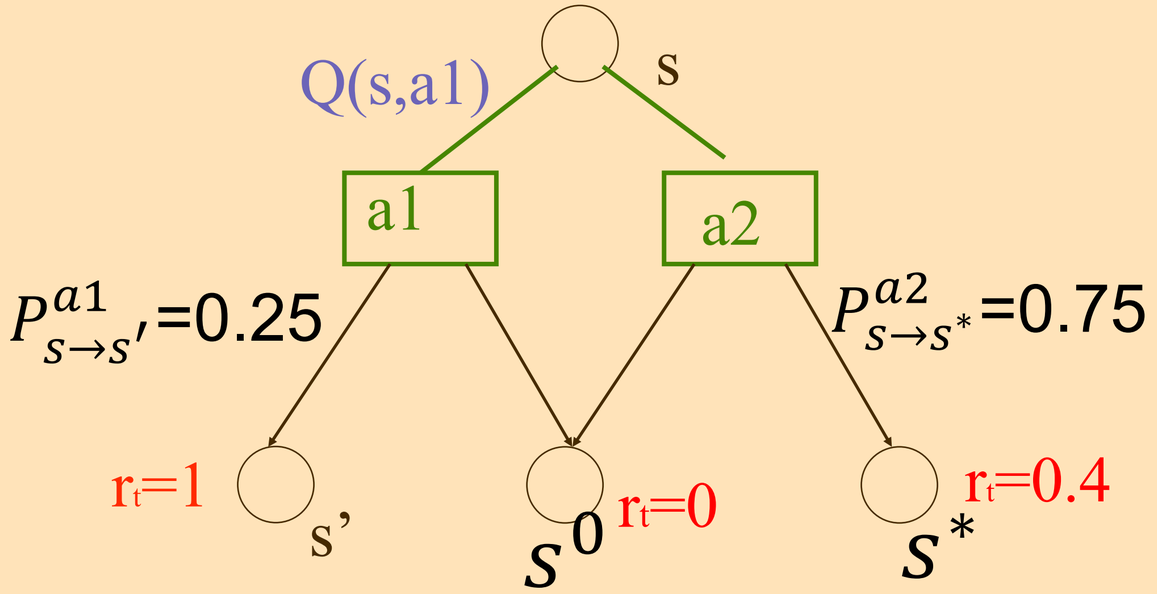
Exploration versus exploitation

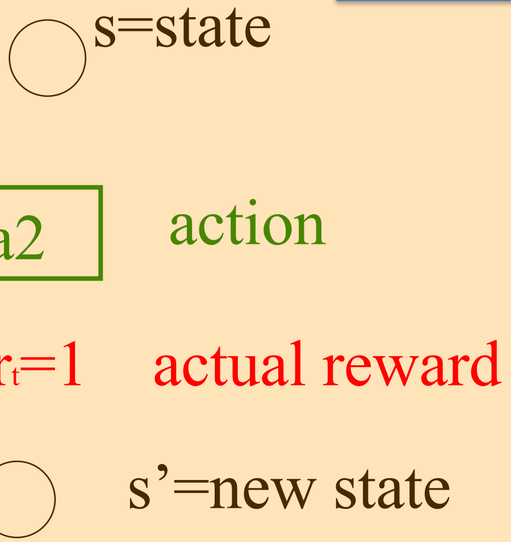
Explore so as to estimate reward probababities

Take action which looks optimal, so as to maximize reward

Exercise 2.1 - 2.3 now: Exploration-Exploitation

5 min, now!





2.1 Assume that you initialize all Q values with zero; set η =0.2

Trial 1: you choose action a1, you get r_t=1

Trial 2: you choose action a2, you get r_t=0.4

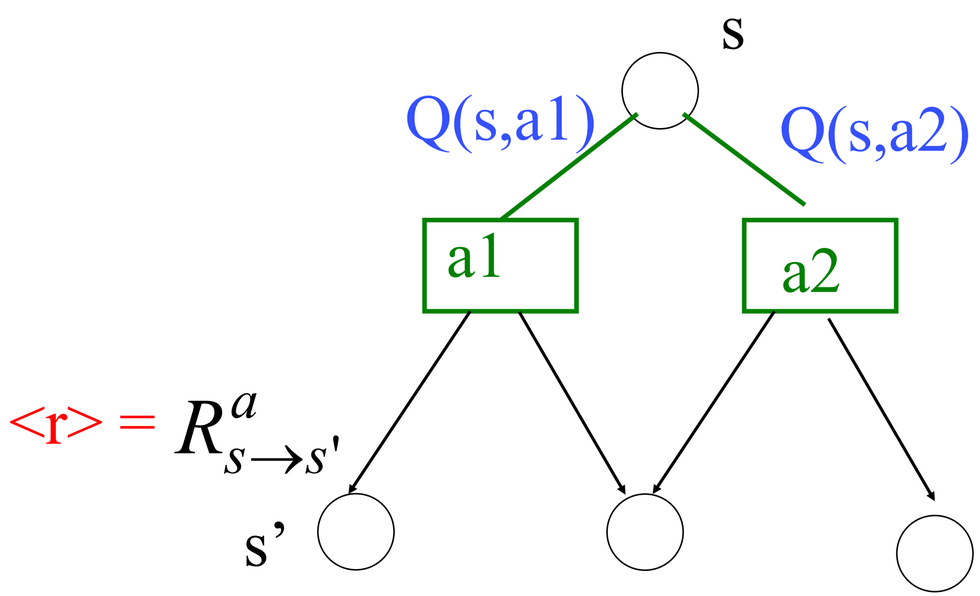
$$\Delta Q(s,a) = \eta\,[r_t - Q(s,a)]$$

2.2 Trial 3 – 5: continue 'greedy' (assume that you do not get rewards)

2.3 Calculate for both actions the expected reward $Q(s,a)=\sum_{s'} P^{a}_{s\to s'} R^{a}_{s\to s'}$

Blackboard3: Exercise 2.1

3. Exploration and Exploitation



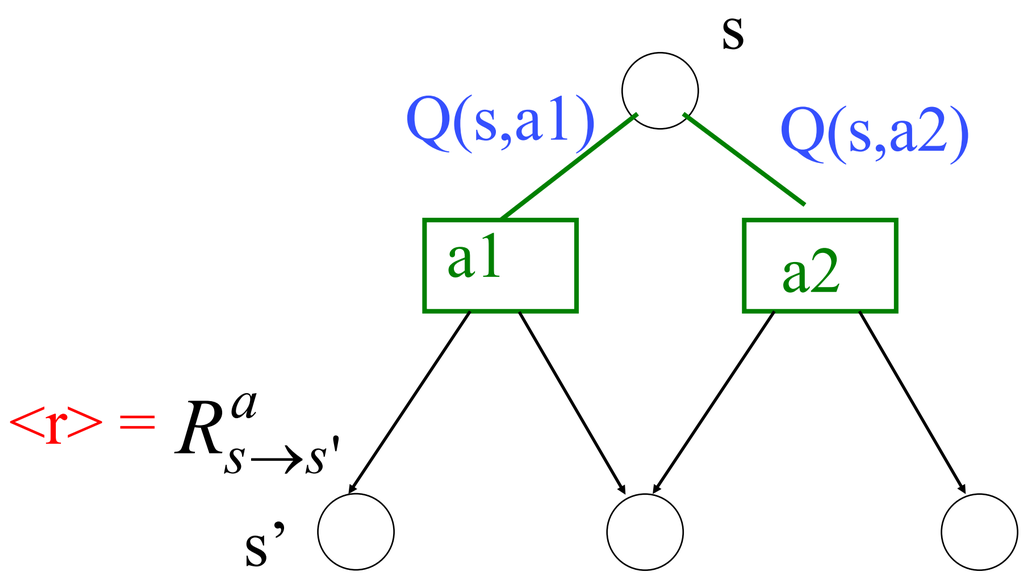
Problem: correct Q values not known

greedy strategy:
- take action a* which looks best

$$Q(s,a^*) > Q(s,a_j)$$

ATTENTION:
with 'greedy' you may get
stuck with a sub-optimal strategy

3. Exploration and Exploitation: practical approach



Problem: correct Q values not known

greedy strategy:
- take action a* which looks best

$$Q(s,a^*) > Q(s,a_j)$$

$\mathcal{E}$greedy strategy:
- take action a* which looks best with prob $P = 1 - \varepsilon$

Softmax strategy: take action a' with prob

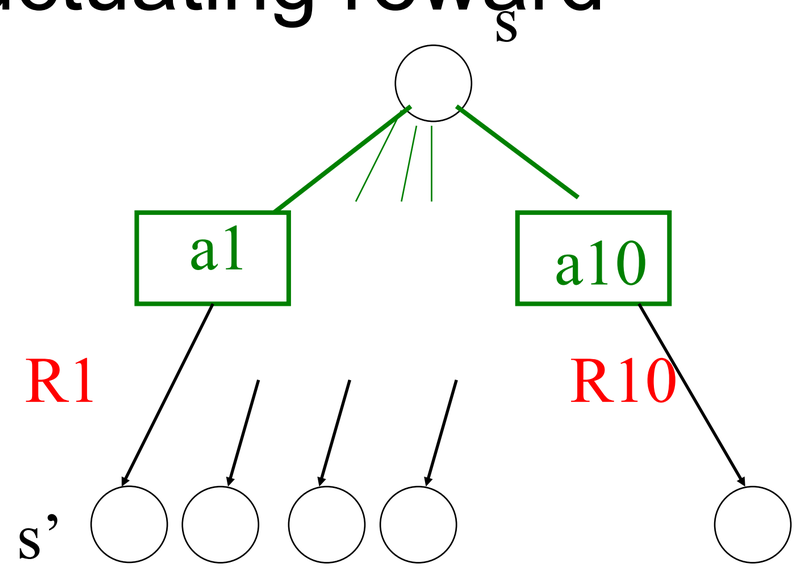
$$P(a') = \frac{\exp[\beta Q(a')]}{\sum_a \exp[\beta Q(a)]}$$

Optimistic greedy:
initialize with Q values that are too big

$$\Delta Q(s,a) = \eta\,[r_t - Q(s,a)]$$

3. Exploration and Exploitation: practical approach

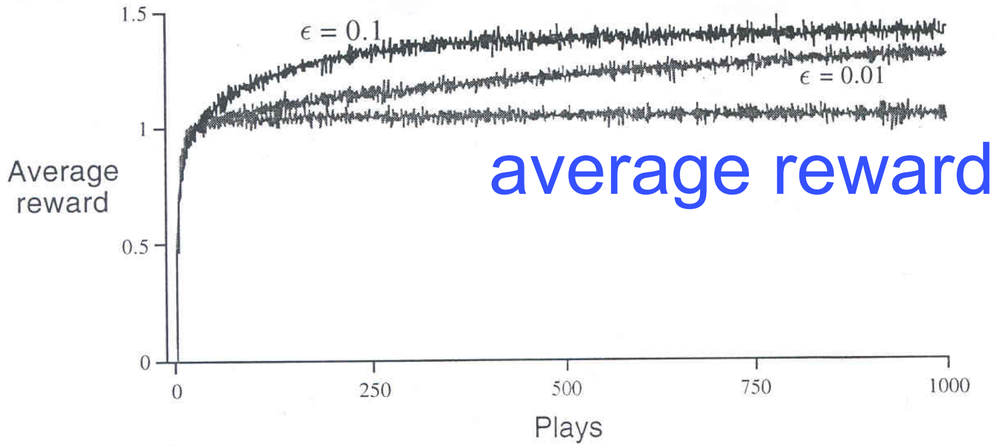
Example: 10-armed bandit with fluctuating reward



in each action, actual rewards fluctuate around a mean

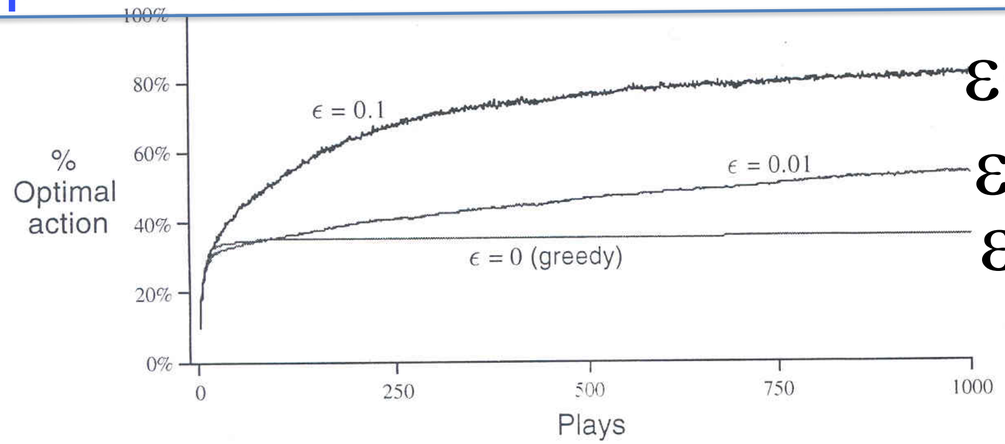
Rk= $R_{s \to s'}^{ak}$

Epsilon-greedy: simulation



average reward

Optimal action



ε=0.1

ε=0.01

ε=0

Figure 2.1 Average performance of ϵ-greedy action-value methods on the 10-armed testbed. These data are averages over 2000 tasks. All methods used sample averages as their action-value estimates.

book: Sutton and Barto

3. Exploration and Exploitation: practical approach

Epsilon-greedy, combined with iterative update of Q-values

A simple bandit algorithm

Initialize, for $a = 1$ to k:

$Q(a) \leftarrow 0$

$N(a) \leftarrow 0$

Repeat forever:

$$A \leftarrow \begin{cases} \arg\max_a Q(a) & \text{with probability } 1-\varepsilon \quad \text{(breaking ties randomly)} \\ \text{a random action} & \text{with probability } \varepsilon \end{cases}$$

$R \leftarrow bandit(A)$

$N(A) \leftarrow N(A) + 1$

$Q(A) \leftarrow Q(A) + \frac{1}{N(A)}\left[R - Q(A)\right]$

Sutton and Barto, ch. 2

3. Quiz: Exploration – Exploitation dilemma

We use a heuristic exploration method and update with **eta=0.2**

[ ] Using an epsilon-greedy method with epsilon = 0.1
means that, even after convergence,
in 10 percent of cases a suboptimal action is chosen.

[ ] If the rewards in the system are between 0 and 1 and Q-values are initialized with Q=2, then each action is played at least 5 times before exploitation starts.

[ ] With a greedy policy the agent uses the best possible action

3. Quiz: Exploration – Exploitation dilemma

We use a heuristic exploration method and update with eta=0.2.
All Q values are initialized with the same value Q=0.1
Rewards in the system are r =0.5 for action 1
and r=1.0 for action 2

[ ] if we use softmax with beta = 0.1, then action 2 is taken about twice as often as action 1.
[ ] if we use softmax with beta = 10, then action 2 is, after a few steps, chosen nearly always

Softmax strategy: take action a'
with prob $$P(a') = \frac{\exp[\beta Q(a')]}{\sum_{a} \exp[\beta Q(a)]}$$

Break now!

Take some time to fill in the online student feedback for ALL YOUR CLASSES

Artificial Neural Networks: Lecture 8

Reinforcement Learning and SARSA

1. Learning by Reward: Reinforcement Learning
2. Elements of Reinforcement Learning
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4. Bellman equation

4. Multistep horizon

Policy $\pi(s,a)$

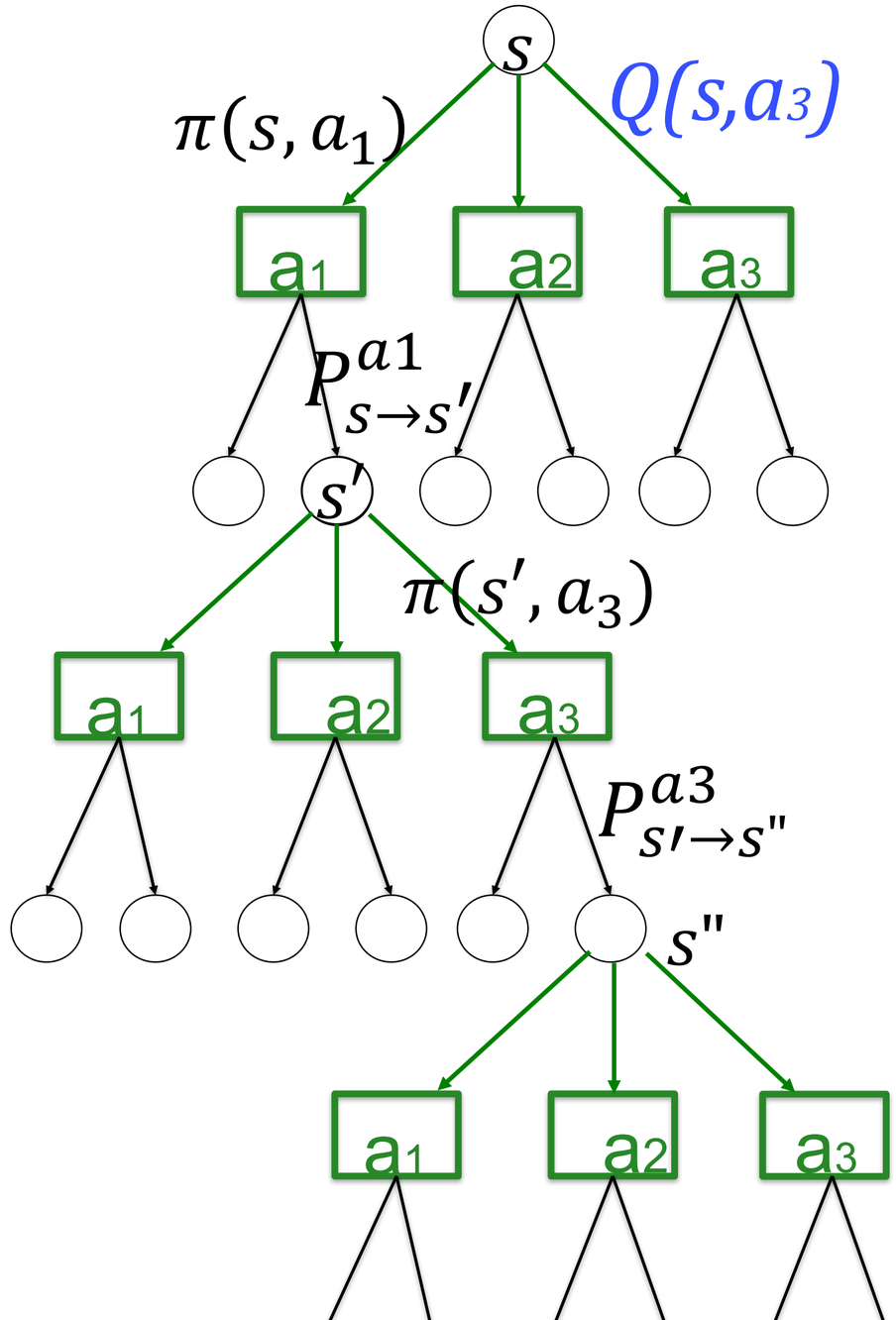
probability to choose action a in state s

$1=\sum_{a'}\pi(s,a')$

Examples of policy:
-epsilon-greedy
-softmax

Stochasticity $P^{a}_{s\to s'}$

probability to end in state s' taking action a in state s

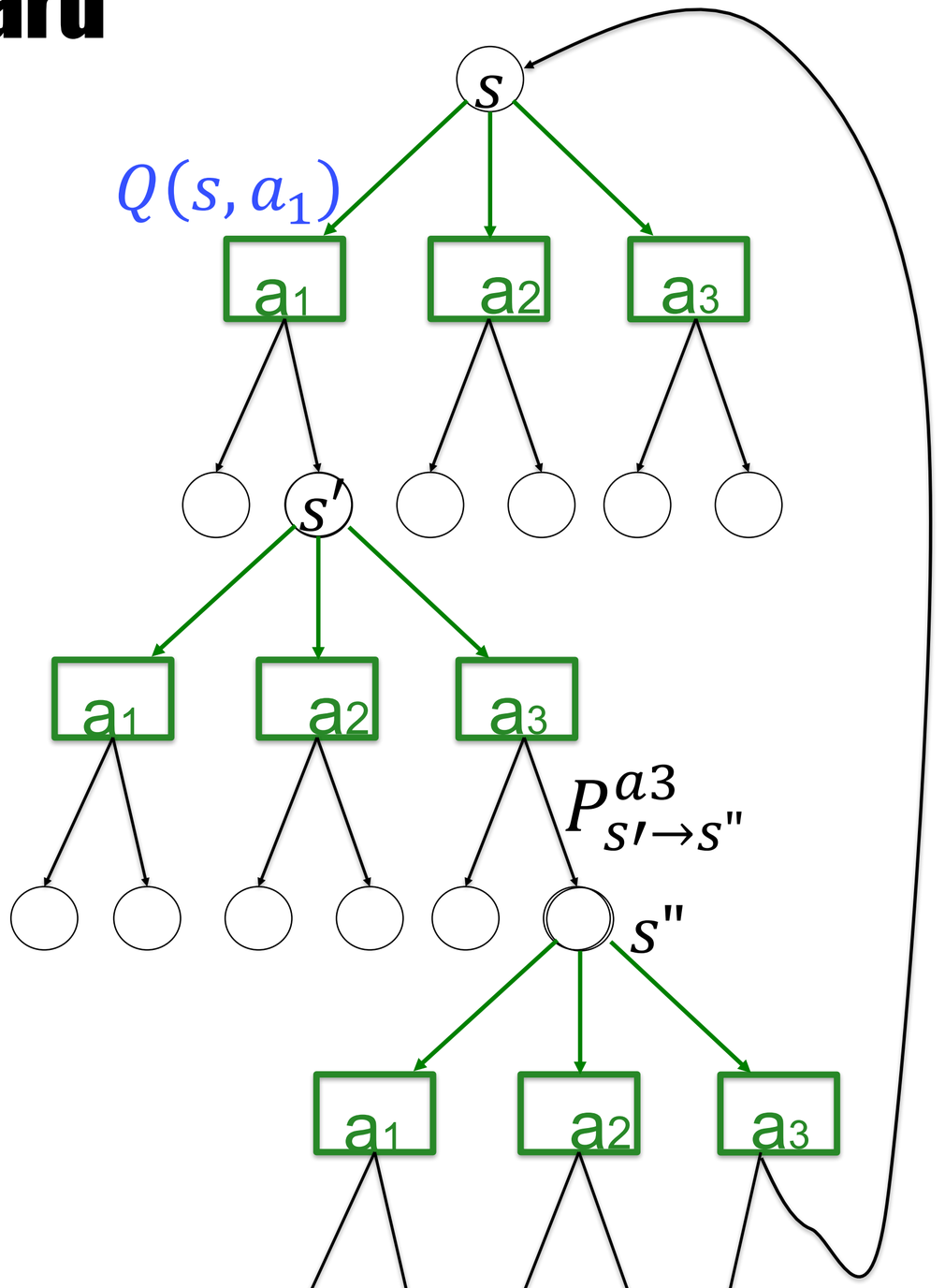


4. Total expected (discounted) reward

Starting in state s with action a

$Q(s,a)$ =

$$\langle r_t + \gamma\, r_{t+1} + \gamma^2 r_{t+2} + \gamma^3\, r_{t+3} + \cdots \rangle$$



Discount factor: γ <1

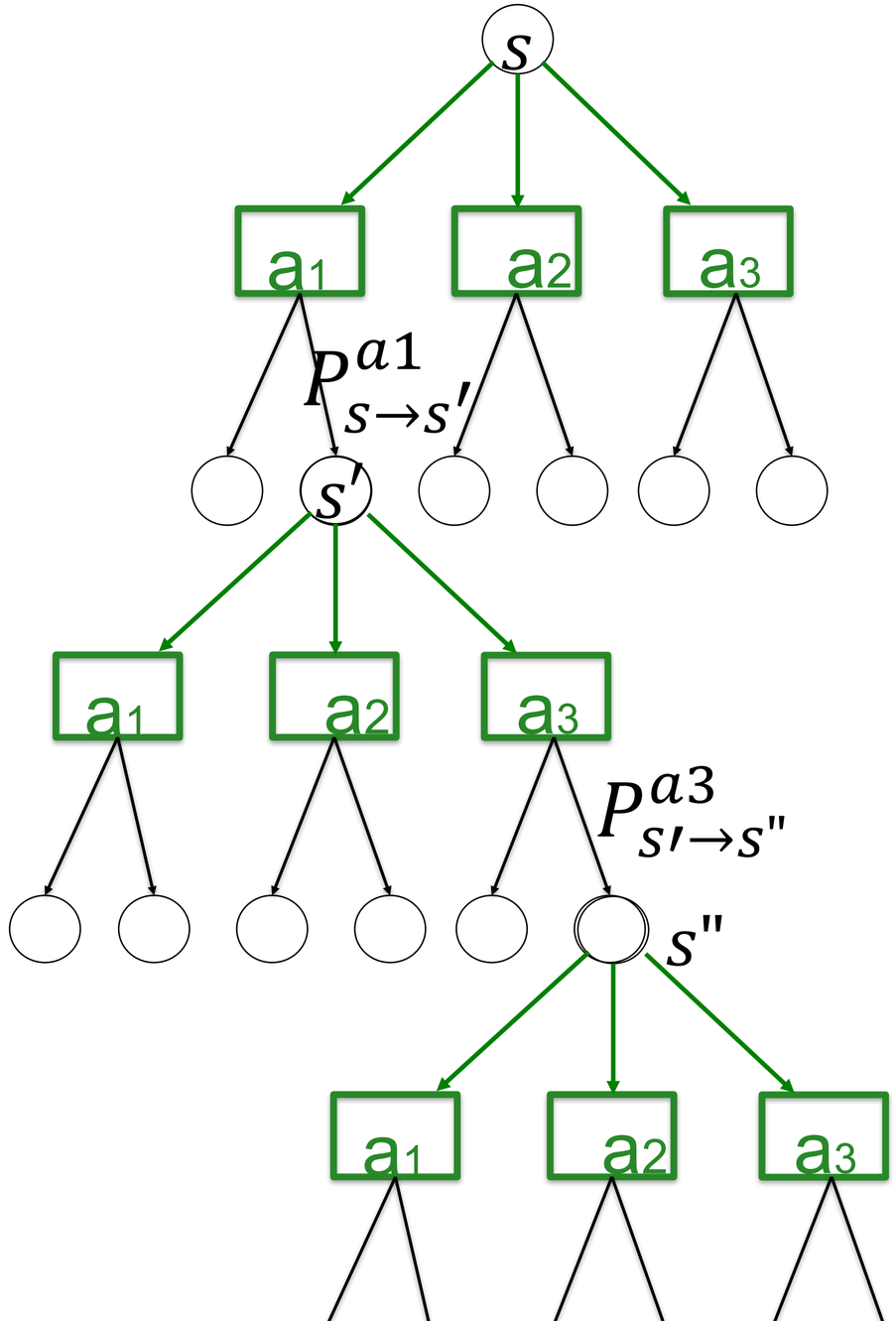
-important for recurrent networks!

-avoids blow-up of summation

-gives less weight to reward in **far** future

4. Bellman equation

Blackboard4:
Bellman eq.



4. Bellman equation with policy π

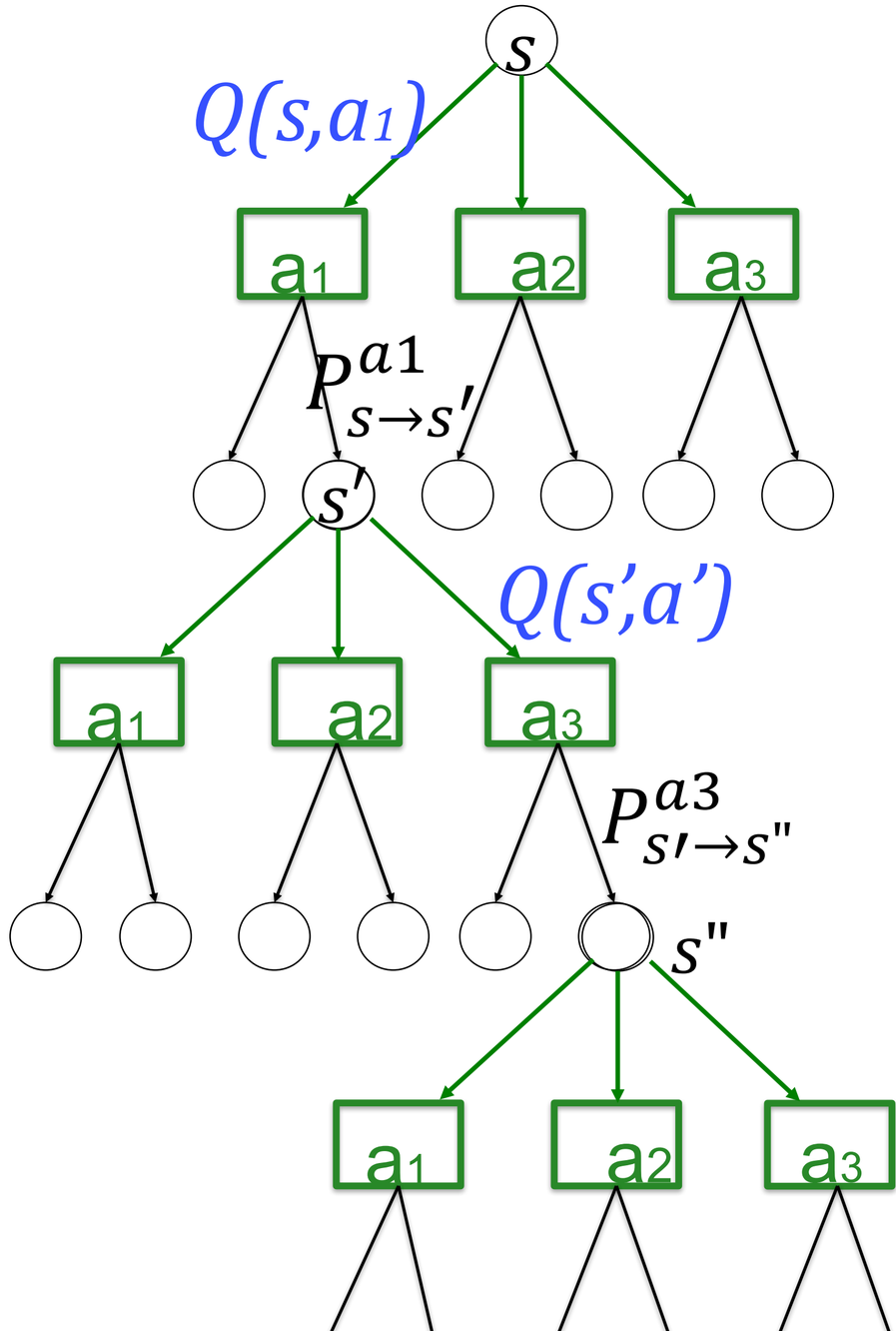
$$Q(s,a) = \sum_{s'} P^{a}_{s\to s'} \left[R^{a}_{s\to s'} + \gamma \sum_{a'} \pi(s',a') Q(s',a') \right]$$

Bellman equation = value consistency of neighboring states

Remark:

Sometimes Bellman equation is written for greedy policy: $\pi(s,a) = \delta_{a,a*}$

with action $a^* = \underset{a'}{\operatorname{argmax}}\, Q(s,a')$



4. Bellman equation (for optimal actions)

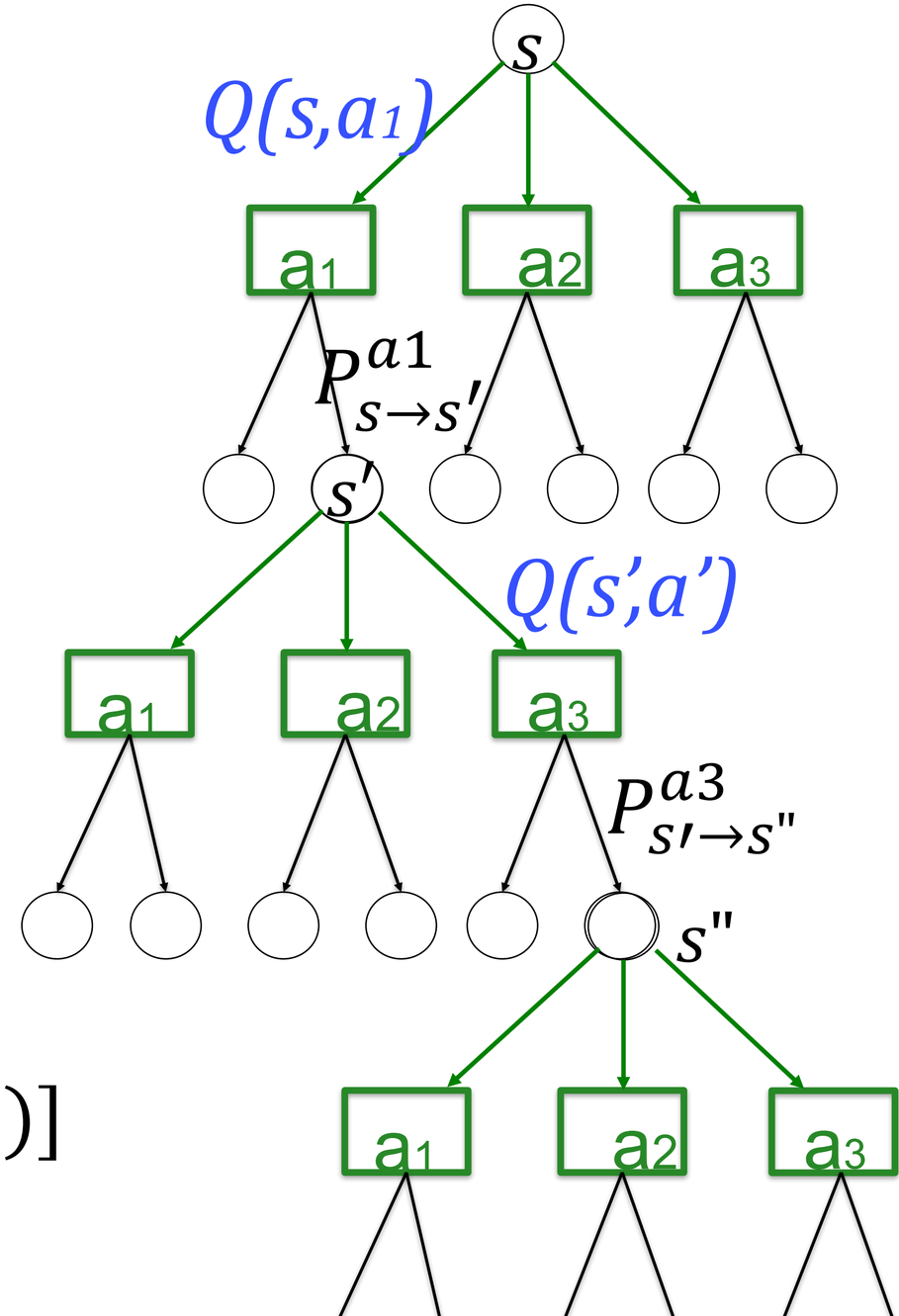
$$Q(s,a) = \sum_{s'} P^{a}_{s\to s'} \left[R^{a}_{s\to s'} + \gamma \sum_{a'} \pi(s',a') Q(s',a') \right]$$

for greedy policy:

$$\pi(s,a) = \delta_{a,a*}$$

with action $a^* = \underset{a'}{\operatorname{argmax}}\, Q(s,a')$

$$Q(s,a) = \sum_{s'} P^{a}_{s'\to s''} \; [\, R^{a}_{s'\to s''} + \gamma max_{a'} Q(s'a')]$$

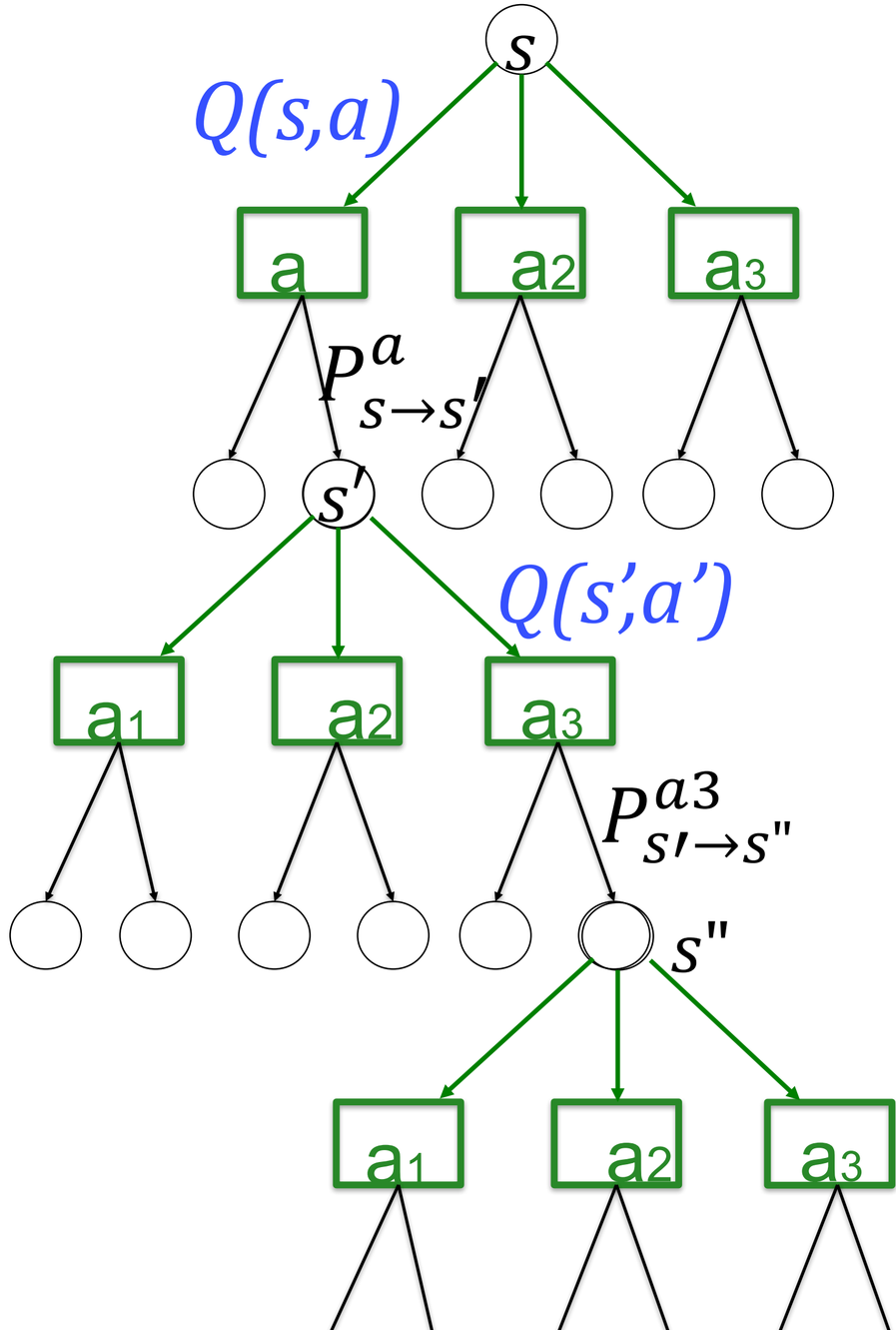


4. Quiz: Bellman equation with policy π

$$Q(s,a) = \sum_{s'} P^{a}_{s \to s'} \left[R^{a}_{s \to s'} + \gamma \sum_{a'} \pi(s',a') Q(s',a') \right]$$

[ ] The Bellman equation is linear in the variables $Q(s'a')$

[ ] The set of variables $Q(s',a')$ that solve the Bellman equation is unique and does not depend on the policy



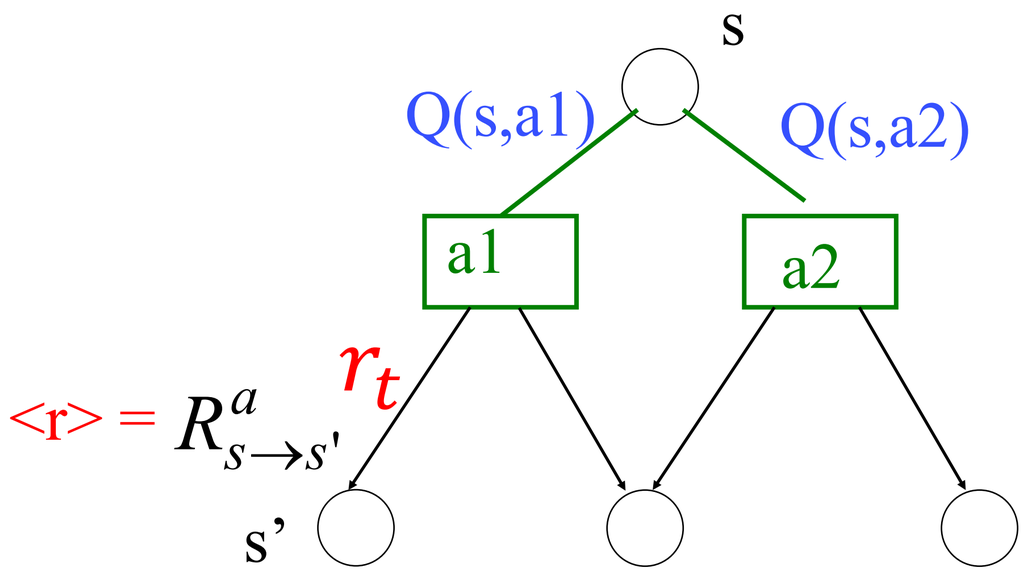
Artificial Neural Networks: Lecture 8

Reinforcement Learning and SARSA

1. Learning by Reward: Reinforcement Learning
2. Elements of Reinforcement Learning
3. Exploration vs Exploitation
4. Bellman equation
5. SARSA algorithm

3. Iterative update of Q-values

Problem: Q-values not given



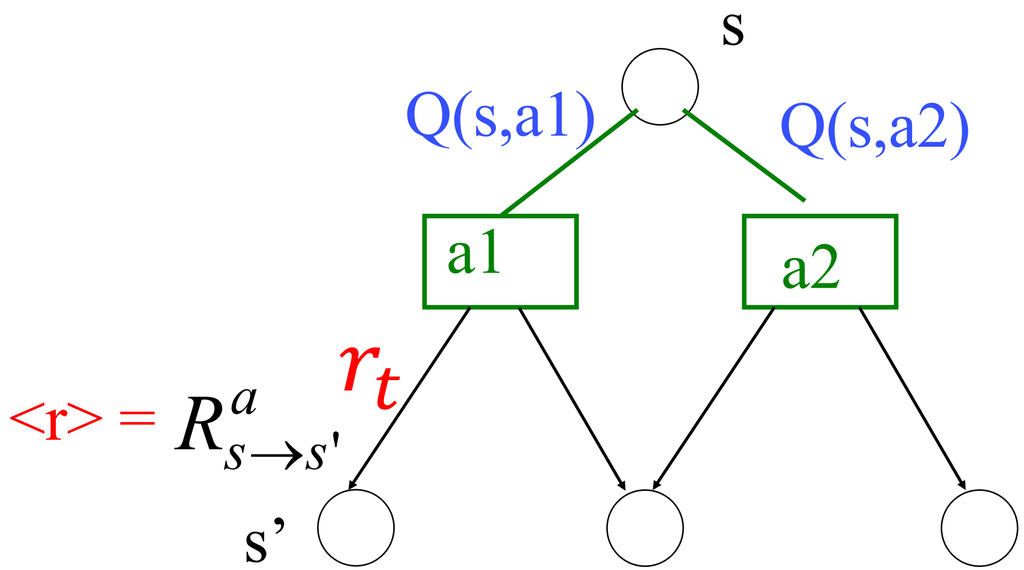
Solution: iterative update

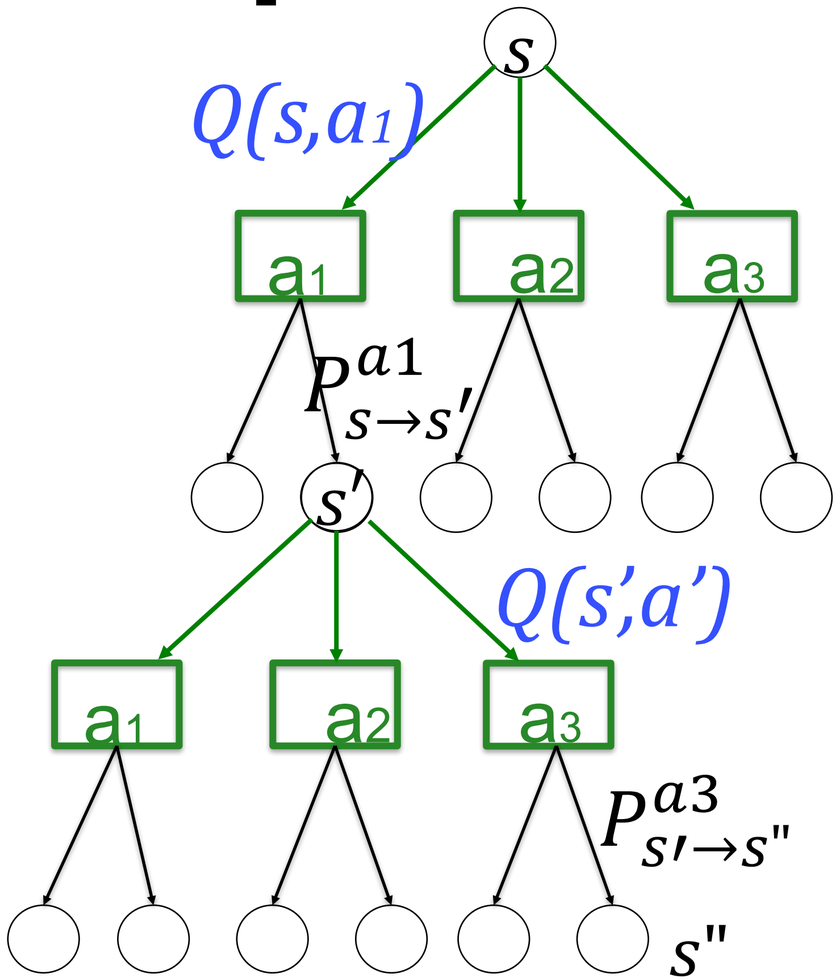
$$\Delta Q(s,a) = \eta\,[r_t - Q(s,a)]$$

while playing with policy $\pi(s,a)$

5. Iterative update of Q-values for multistep environments

Problem: Q-values not given





Solution: iterative update

$$\Delta Q(s,a) = \eta\,[r_t - Q(s,a)]$$

while playing with policy $\pi(s,a)$

$$\Delta Q(s,a) = \;?$$

Blackboard5:
SARSA update

5. Iterative update of Q-values for multistep environments

Bellman equation:

$$Q(s,a) = \sum_{s'} P^{a}_{s \to s'} \left[R^{a}_{s \to s'} + \gamma \sum_{a'} \pi(s',a') Q(s',a') \right]$$

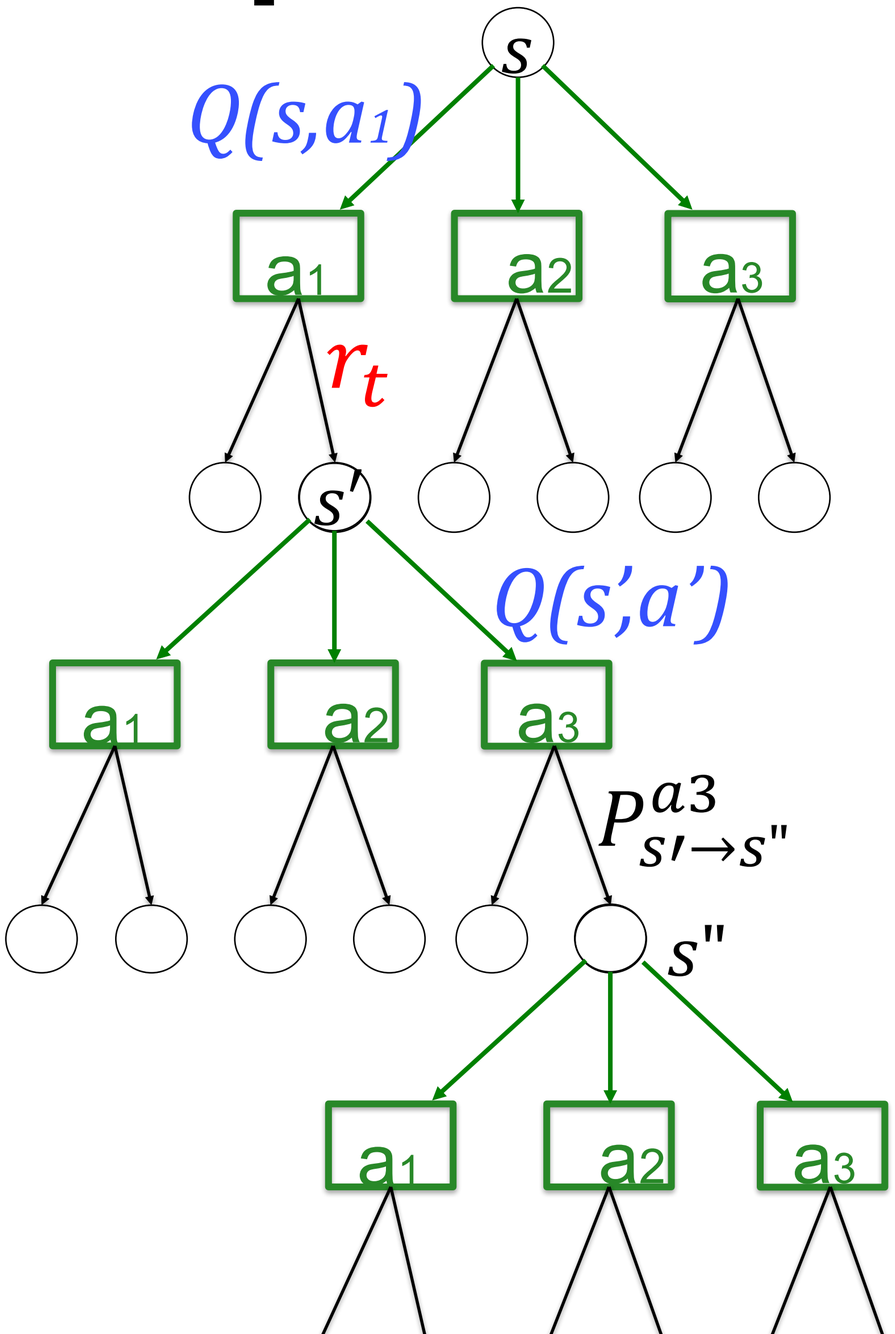
Problem:

- Q-values not given
- branching probabilities not given
- reward probabilities not given

Solution: iterative update

$$\Delta Q(s,a) = \eta \, [r_t + \gamma Q(s',a') - Q(s,a)]$$

while playing with policy $\pi(s,a)$



5. SARSA vs. Bellman equation

$$Q(s,a) = \sum_{s'} P^{a}_{s \to s'} \left[R^{a}_{s \to s'} + \gamma \sum_{a'} \pi(s',a') Q(s',a') \right]$$

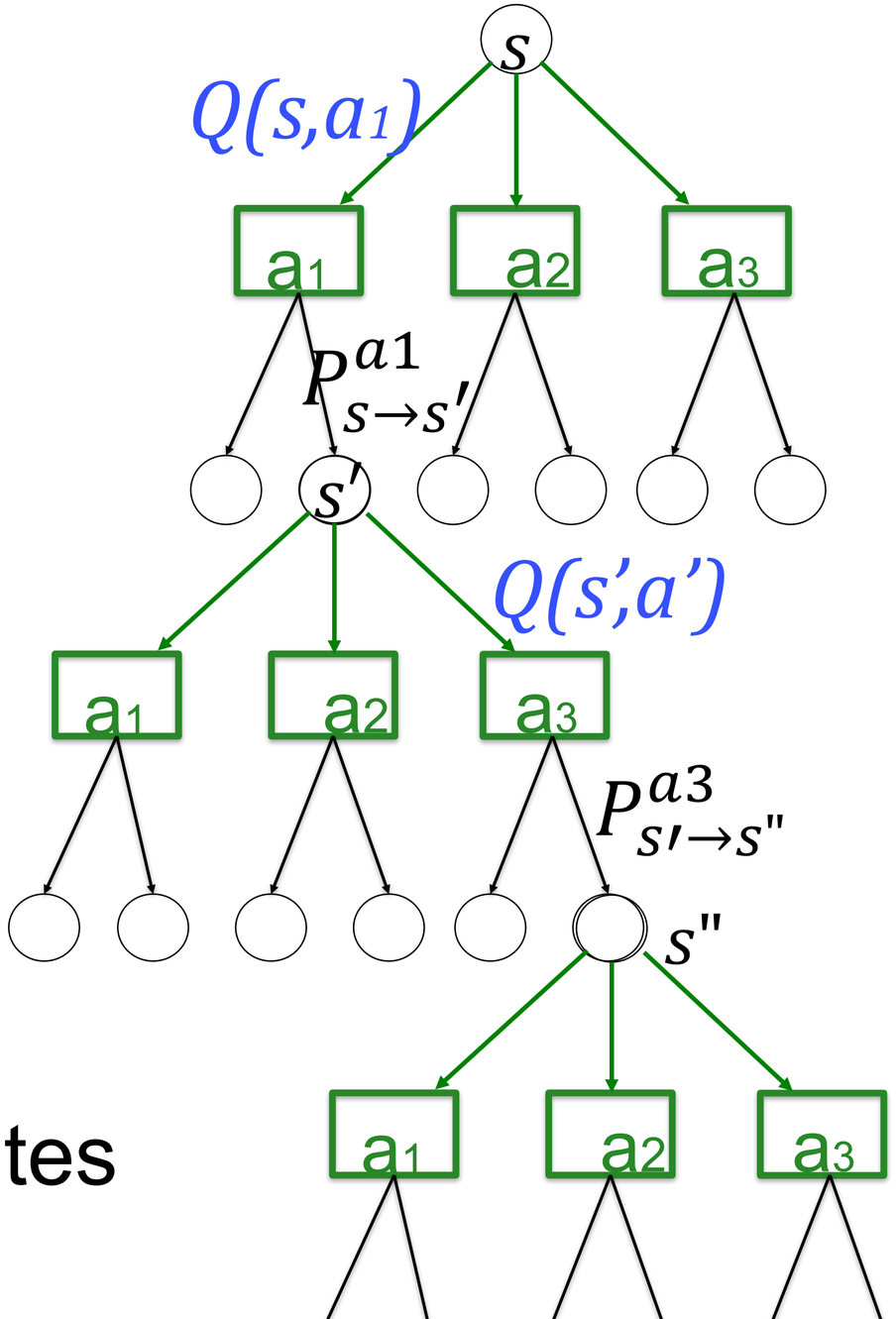
Bellman equation

= consistency of Q-values
across neighboring states

SARSA update rule

$$\Delta Q(s,a) = \eta [r_t + \gamma Q(s',a') - Q(s,a)]$$

= make Q-values of neighboring states
more consistent



5. SARSA algorithm

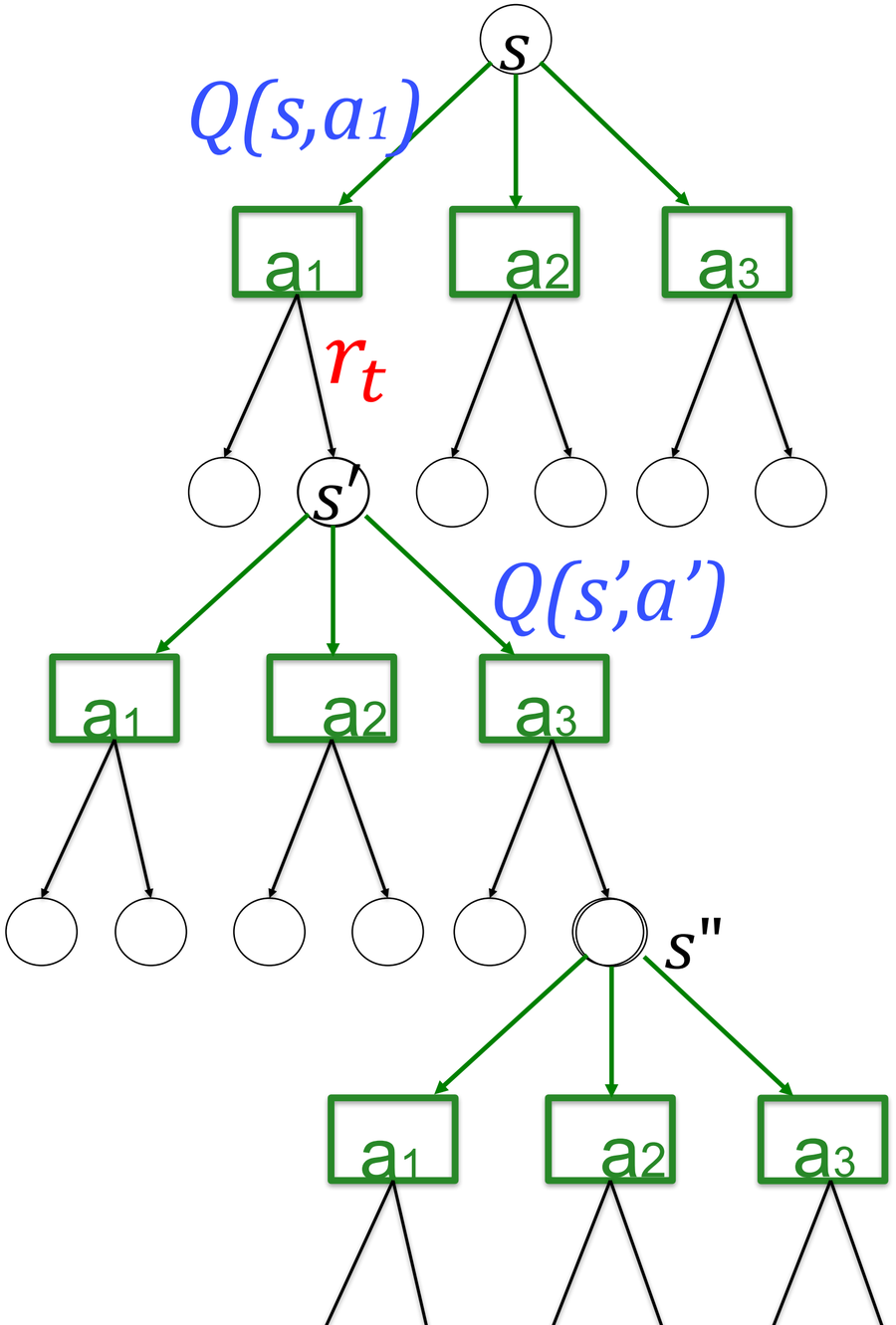
Initialise Q values
Start from initial state **s**

1) being in state **s**
   choose action **a**
   [according to policy $\pi(s,a)$]
2) Observe reward **r**
   and next state **s'**
3) Choose action **a'** in state s'
   [according to policy $\pi(s,a)$]
4) Update with SARSA update rule

$$\Delta Q(s,a) = [r_t + \gamma Q(s',a') - Q(s,a)]$$

5) set: s ← s'; a ← a'
6) Goto 1)

Stop when all Q-values have converged



5. Convergence in expectation of SARSA: theorem

Assumption:

The SARSA algo has been applied for a very long time, using updates

$$\Delta Q(s,a) = \eta\,[r_t + \gamma Q(s',a') - Q(s,a)]$$

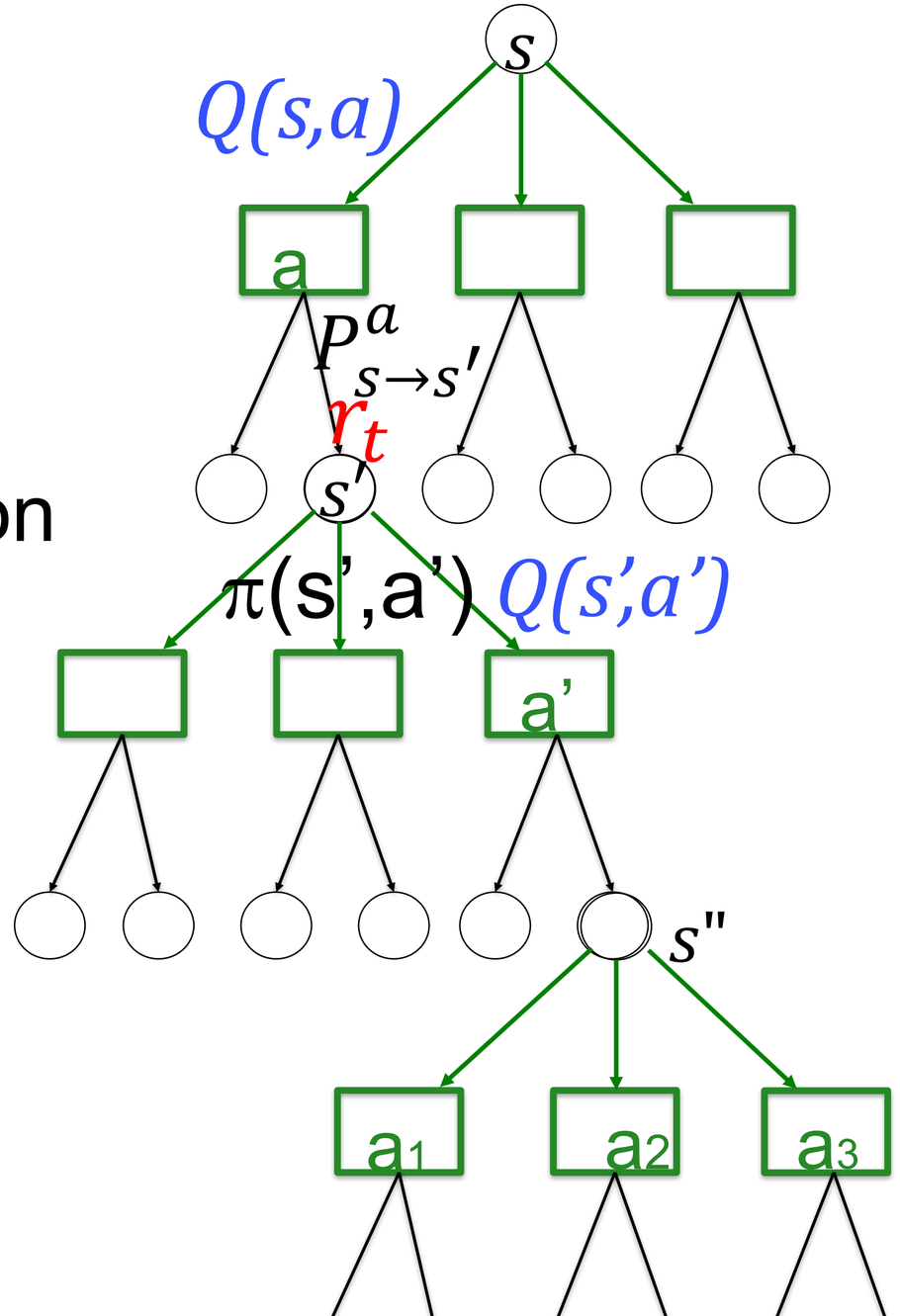
IF all Q-values have converged in expectation

$$\langle \Delta Q(s,a) \rangle = 0$$

THEN

The set of Q-values solves the Bellman eq.

$$Q(s,a) = \sum_{s'} P^{a}_{s\to s'} \left[R^{a}_{s\to s'} + \gamma \sum_{a'} \pi(s',a') Q(s',a') \right]$$



5. Convergence in expectation of SARSA: theorem

Blackboard4:
SARSA convergence

Look at graph to take expectations:

- if algo in state s, all remaining expectations are “given s”
- if algo on a branch (s,a), all remaining exp. are “given s and a”

5. SARSA algorithm

We have initialized SARSA and played for n>2 steps.
Is the following true?

[ ] in SARSA, updates are applied after each move.

[ ] in SARSA, the agent updates the Q-value $Q(s(t),a(t))$ related to the **current** state $s(t)$

[ ] in SARSA, the agent updates the Q-value $Q(s(t-1),a(t-1))$ related to the **previous** state, when it is in state $s(t)$

[ ] in SARSA, the agent moves in the environment using the policy $\pi(s,a)$

[ ] SARSA is an on-policy algorithm (see next week)

Artificial Neural Networks: Lecture 8

Reinforcement Learning and SARSA

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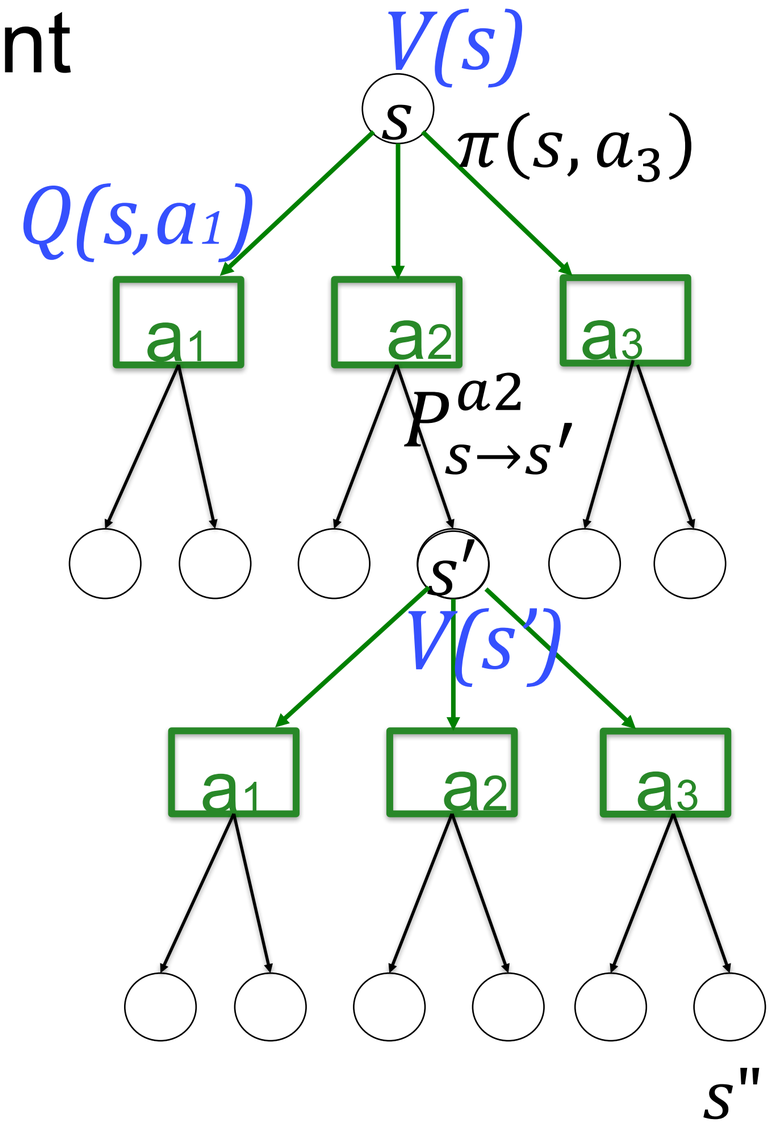
3. Exploration vs Exploitation
4. Bellman equation
5. SARSA algorithm
6. Q-values versus V-values

6. State-values V

Value $V(s)$ of a state s
= total (discounted) expected reward the agent gets starting from state s

$$V(s) = \sum_a \pi(s,a) Q(s,a)$$



Bellman equation for *V(s)*

$$V(s) = \sum_a \pi(s,a) \sum_{s'} P^a_{s \to s'} [\, R^a_{s \to s'} + \gamma V(s')]$$

Reinforcement Learning and SARSA

Objectives:

- **Reinforcement Learning is learning by rewards**
  → world is full of rewards (but not full of labels)
- **Agents and actions**
  → agent learns by interacting with the environment
  → state s, action a, reward r
- **Exploration vs Exploitation**
  → optimal actions are easy if we know reward probabilities
  → since we don't know the probabilities we need to explore
- **Bellman equation**
  → self-consistency condition for Q-values
- **SARSA algorithm:** state-action-reward-state-action
  → update while exploring environment with current policy

Exercise (at home)

- **Update of Q values in SARSA**

$$\Delta Q(s,a) = \eta\ [r-(Q(s,a)-Q(s',a'))]$$

- **policy for action choice:**

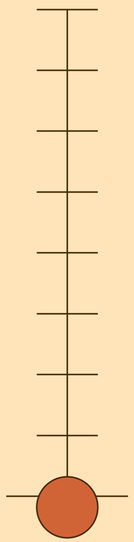
  Pick most often action

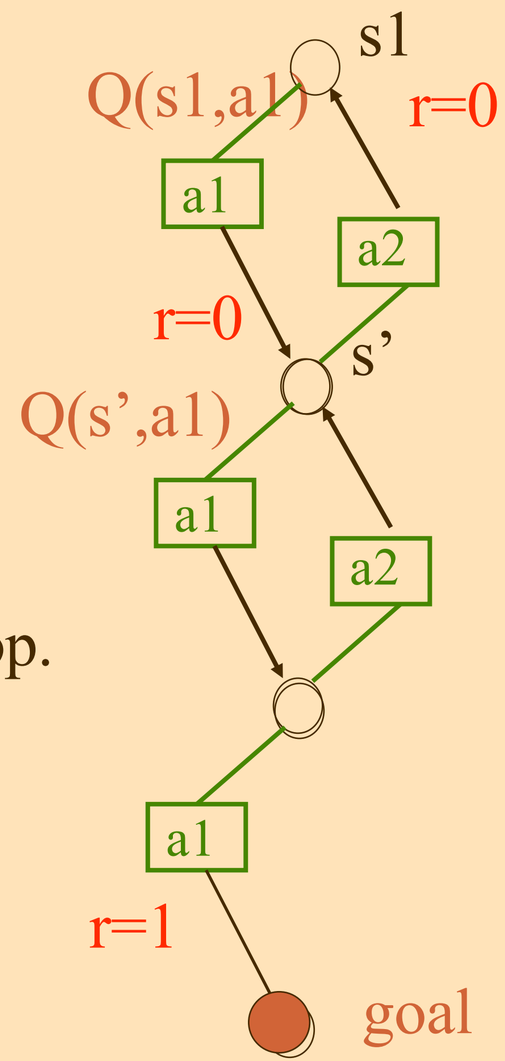
$$a_t^* = \arg\max_a Q_a(s,a)$$

Consider a linear sequence of states. Reward only at goal. Actions are up or down.

a) Initialise Q values at 0. Start at top. How do Q values develop?

b) Q values after 3 complete trials?





Reading for this week:

Sutton and Barto, Reinforcement Learning
(MIT Press, 2nd edition 2018, also online)

Chapters: 1.1-1.4; 2.1-2.6; 3.1-3.5; 6.4

The END

Motivational background reading:

Silver et al. 2017, Archive
Mastering Chess and Shogi by Self-Play with a General Reinforcement Learning Algorithm

Goodfellow et al., Ch. 1 of
Deep Learning