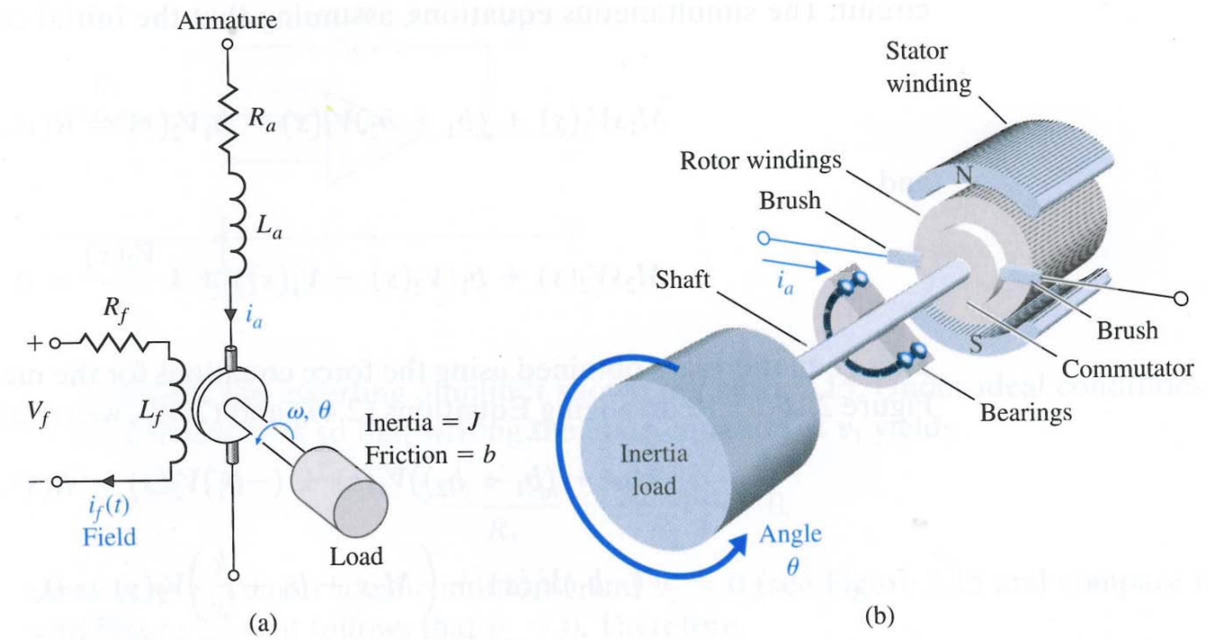
DC Motor

The DC motor is a power actuator device that delivers energy to a load. The DC motor converts direct current (DC) electrical energy into rotational mechanical energy. A major fraction of the torque generated in the rotor (armature) of the motor is available to drive an external load. Because of features such as high torque, speed controllability over a wide range, portability, well-behaved speed-torque characteristics, and adaptability to various types of control methods, DC motors are widely used in numerous control applications, including robotic manipulators, tape transport mechanisms, disk drives, machine tools, and servo-valve actuators.
The transfer function of the DC motor will be developed for a linear approximation to an actual motor, and second-order effects, such as hysteresis and the voltage drop across the brushes, will be neglected. The input voltage may be applied to the field or armature terminals.



The air-gap flux of the motor is proportional to the field current, provided the field is unsaturated, so that

$$\phi = K_f i_f,$$

where i_f is the field current, K_f is the proportional constant, and ϕ is the field flux. The torque derived by the motor is assumed to be related linearly to ϕ and the armature current as follows:

$$T_m(t) = K_1 \phi i_a(t) = K_1 K_f i_f(t) i_a(t),$$

where i_a is the armature current, and K_1 is a constant. It is clear from this equation that, to have a linear system, one current must be maintained constant while the other current becomes the input current. The armature-controlled DC motor uses the armature current as the control variable. The stator field can be established by a field coil and current or a permanent magnet. When a constant field current is established in a field coil, the motor torque in *Laplace transform* notation is

$$T_m(s) = (K_1 K_f I_f) I_a(s) = K_m I_a(s),$$

where K_m is a function of the permeability of the magnetic material.
The armature current is related to the input voltage applied to the armature by

$$V_a(s) = (R_a + L_a s)I_a(s) + V_b(s),$$

where $V_b(s)$ is the back electromotive-force voltage proportional to the motor speed. Therefore, we have

$$V_b(s) = K_b \omega(s),$$

where $\omega(s) = s\theta(s)$ is the transformation of the angular speed.

The motor torque is equal to the torque delivered to the load. This relation may be expressed as

$$T_m(s) = T_L(s) + T_d(s),$$

where $T_L(s)$ is the load torque and $T_d(s)$ is the disturbance torque. The load torque for rotating inertia is written as:

$$T_L(s) = Js^2\theta(s) + bs\theta(s).$$

- Calculate the transfer function $G(s) = \frac{\theta(s)}{V_a(s)}$ using the equations presented above, and letting $T_d(s) = 0$;
- Build a Simulink/Matlab model of the DC motor including a controller (P, PD or PID). ($J = 2, K_m = 10, R_a = 1, L_a = 1, b = 0.5, K_b = 0.1$)