Yolo

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$$\ddot{q}_{M,i} = \frac{1}{J_{M,i}}\left(-f_{M,i}\dot{q}_{M,i} - \frac{1}{n_i}\tau_i + k_{T,i}u_i\right)$$

$$\dot{\boldsymbol{x}} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{f_{M,i}}{J_{M,i}} \end{bmatrix}\boldsymbol{x} + \begin{bmatrix} 0 \\ k_{T,i} \end{bmatrix}u_i + \begin{bmatrix} 0 \\ -\frac{1}{n_i} \end{bmatrix}\tau_i$$

$$\boldsymbol{x} = \begin{bmatrix} q_{M,i} \\ \dot{q}_{M,i} \end{bmatrix}$$

$$\begin{aligned}
&J_{M,i} = J_{eff,i} - \frac{1}{n_i^2}\sup_q D_{ii}(q) = J_{eff,i} - \frac{1}{n_i^2}\widehat{D}(q) \\
&\tau_i = \widehat{D}(q)\ddot{q} + T_L \\
&u_i = n_i\left[K_{p,i}(q_i^r - q_i) + K_{D,i}(\dot{q}_i^r - \dot{q}_i)\right] \\
&q_{M,i} = q_i n_i \\
&\ddot{q}_{M,i} = \frac{1}{J_{M,i}}\left(-f_{M,i}\dot{q}_{M,i} - \frac{1}{n_i}\tau_i + k_{T,i}u_i\right) \\
&f_{eff} = f_M + \frac{f_L}{n_i^2} = f_M, \qquad f_L = 0
\end{aligned}$$

$$n_i\ddot{q}_i = \frac{1}{J_{eff,i} - \frac{1}{n_i^2}\sup_q D_{ii}(q)}\left\{-f_{eff}n_i\dot{q}_i - \frac{1}{n_i}\left(\widehat{D}(q)\ddot{q} + T_L\right) + k_{T,i}n_i\left[K_{p,i}(q_i^r - q_i) + K_{D,i}(\dot{q}_i^r - \dot{q}_i)\right]\right\}$$

Laplace transform

$$q_i\left(\left(n_i J_{eff,i} - \frac{1}{n_i}\sup_q D_{ii}(q) + \frac{1}{n_i}\widehat{D}(q)\right)s^2 + \left(f_{eff}n_i + k_{T,i}n_iK_{D,i}\right)s + k_{T,i}n_iK_{p,i}\right) = -\frac{1}{n_i}T_L + k_{T,i}n_i\left(K_{p,i} + K_{D,i}s\right)q_i^r$$

If $\sup_q D_{ii}(q) = \widehat{D}(q)$

$$q_i\left(s^2 + \frac{\left(f_{eff} + k_{T,i}K_{D,i}\right)}{J_{eff,i}}s + \frac{k_{T,i}K_{p,i}}{J_{eff,i}}\right) = \frac{k_{T,i}\left(K_{p,i} + K_{D,i}s\right)}{J_{eff,i}}q_i^r - \frac{1}{n_i^2 J_{eff,i}}T_L$$

$$q_i = \frac{1}{N_i(s)}\left[F_i(s)q_i^r(s) - G_i(s)T_{L,i}(s)\right]$$

Hvor

$$N_i(s) = s^2 + \frac{\left(f_{eff} + k_{T,i}K_{D,i}\right)}{J_{eff,i}}s + \frac{k_{T,i}K_{p,i}}{J_{eff,i}}$$

$$F_i(s) = \frac{k_{T,i}\left(K_{p,i} + K_{D,i}s\right)}{J_{eff,i}}$$

$$G_i(s) = \frac{1}{n_i^2 J_{eff,i}}$$

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$$s^2 + \frac{\left(f_{eff} + k_{T,i}K_{D,i}\right)}{J_{eff,i}}s + \frac{k_{T,i}K_{p,i}}{J_{eff,i}} = s^2 + 2\omega_{n,i}\zeta_i s + \omega_{n,i}^2$$

$$\omega_{n,i}^2 = \frac{k_{T,i}K_{p,i}}{J_{eff,i}} \quad \Leftrightarrow \quad K_{p,i} = \frac{J_{eff,i}\omega_{n,i}^2}{k_{T,i}}$$

$$2\omega_{n,i}\zeta_i = \frac{\left(f_{eff} + k_{T,i}K_{D,i}\right)}{J_{eff,i}} \quad \Leftrightarrow \quad K_{D,i} = \frac{2\omega_{n,i}\zeta_i J_{eff,i} - f_{eff}}{k_{T,i}} = \frac{2\zeta_i}{\omega_{n,i}}K_{p,i} - \frac{f_{eff}}{K_{T,i}}$$

$$\begin{aligned}
&J_{M,i} = J_{eff,i} - \frac{1}{n_i^2}\sup_q D_{ii}(q) = J_{eff,i} - \frac{1}{n_i^2}\widehat{D}(q) \\
&\tau_i = \widehat{D}(q)\ddot{q} + T_L \\
&u_i = n_i\left[K_{p,i}(q_i^r - q_i) + K_{D,i}(\dot{q}_i^r - \dot{q}_i) + K_{I,i}\int (q_i^r - q_i)\,dt\right] \\
&q_{M,i} = q_i n_i \\
&\ddot{q}_{M,i} = \frac{1}{J_{M,i}}\left(-f_{M,i}\dot{q}_{M,i} - \frac{1}{n_i}\tau_i + k_{T,i}u_i\right) \\
&f_{eff} = f_M + \frac{f_L}{n_i^2} = f_M, \qquad f_L = 0
\end{aligned}$$

$$n_i\ddot{q}_i = \frac{1}{J_{eff,i} - \frac{1}{n_i^2}\sup_q D_{ii}(q)}\left\{-f_{eff}n_i\dot{q}_i - \frac{1}{n_i}\left(\widehat{D}(q)\ddot{q} + T_L\right) + k_{T,i}n_i\left[K_{p,i}(q_i^r - q_i) + K_{D,i}(\dot{q}_i^r - \dot{q}_i) + K_{I,i}\int (q_i^r - q_i\,dt\right]\right\}$$

$$\ddot{q}_i = \frac{1}{J_{eff,i}}\left\{-f_{eff}\dot{q}_i - \frac{1}{{n_i}^2}T_L + k_{T,i}\left[K_{p,i}(q_i^r - q_i) + K_{D,i}(\dot{q}_i^r - \dot{q}_i) + K_{I,i}\int (q_i^r - q_i\,dt\right]\right\}$$

Laplace transform

$$n_iq_is^2\left(J_{eff,i} - \frac{1}{n_i^2}\sup_q D_{ii}(q)\right) = -f_{eff}n_iq_is - \frac{1}{n_i}\left(\widehat{D}(q)q_is^2 + T_L\right) + k_{T,i}n_i\left[K_{p,i}(q_i^r - q_i) + K_{D,i}(q_i^rs - q_is) + K_{I,i}\frac{1}{s}(q_i^r - q_i)\right]$$

$$n_iq_is^2\left(J_{eff,i} - \frac{1}{n_i^2}\sup_q D_{ii}(q)\right) + f_{eff}n_iq_is + \frac{1}{n_i}\widehat{D}(q)q_is^2 = -\frac{1}{n_i}T_L + k_{T,i}n_i\left[K_{p,i}(q_i^r - q_i) + K_{D,i}(q_i^rs - q_is) + K_{I,i}\frac{1}{s}(q_i^r - q_i)\right]$$

$$k_{T,i}n_i\left[K_{p,i}(q_i^r-q_i)+K_{D,i}(q_i^r s-q_i s)+K_{I,i}\frac{1}{s}(q_i^r-q_i)\right]=k_{T,i}n_iK_{P,i}q_i^r-k_{T,i}n_iK_{P,i}q_i+k_{T,i}n_iK_{D,i}q_i^r s-k_{T,i}n_iK_{D,i}q_i s+k_{T,i}n_iK_{I,i}q_i^r\frac{1}{s}-k_{T,i}n_iK_{I,i}q_i\frac{1}{s}$$
$$=k_{T,i}n_i\left(K_{P,i}+K_{D,i}s+K_{I,i}\frac{1}{s}\right)q_i^r-k_{T,i}n_i\left(K_{P,i}+K_{D,i}s+K_{I,i}\frac{1}{s}\right)q_i$$

$$q_i\left\{s^2\left(n_iJ_{eff,i}-\frac{1}{n_i}\sup_q D_{ii}(q)+\frac{1}{n_i}\widehat{D}(q)\right)+s\left(f_{eff}n_i+k_{T,i}n_iK_{D,i}\right)+k_{T,i}n_iK_{P,i}+k_{T,i}n_iK_{I,i}\frac{1}{s}\right\}=-\frac{1}{n_i}T_L+k_{T,i}n_i\left(K_{P,i}+K_{D,i}s+K_{I,i}\frac{1}{s}\right)q_i^r$$

If $\sup_q D_{ii}(q)=\widehat{D}(q)$

$$q_i\left\{s^2+s\frac{\left(f_{eff}+k_{T,i}K_{D,i}\right)}{J_{eff,i}}+\frac{k_{T,i}K_{P,i}}{J_{eff,i}}+\frac{k_{T,i}K_{I,i}}{J_{eff,i}}\frac{1}{s}\right\}=\frac{k_{T,i}}{J_{eff,i}}\left(K_{P,i}+K_{D,i}s+K_{I,i}\frac{1}{s}\right)q_i^r-\frac{1}{n_i^2J_{eff,i}}T_L$$

$$q_i\{J_{eff,i}s^3+s^2\left(f_{eff}+k_{T,i}K_{D,i}\right)+k_{T,i}K_{P,i}s+k_{T,i}K_{I,i}\}=k_{T,i}\left(K_{P,i}s+K_{D,i}s^2+K_{I,i}\right)q_i^r-\frac{s}{n_i^2}T_L$$

$$q_i=\frac{1}{N_i(s)}[F_i(s)q_i{}^r(s)-G_i(s)T_{L,i}(s)]$$

Hvor
$N_i(s)=J_{eff,i}s^3+s^2\left(f_{eff}+k_{T,i}K_{D,i}\right)+k_{T,i}K_{P,i}s+k_{T,i}K_{I,i}$
$F_i(s)=k_{T,i}\left(K_{P,i}s+K_{D,i}s^2+K_{I,i}\right)$
$G_i(s)=\dfrac{s}{n_i^2}$

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$$J_{eff,i}s^3+s^2\left(f_{eff}+k_{T,i}K_{D,i}\right)+k_{T,i}K_{P,i}s+k_{T,i}K_{I,i}=\left(s^2+2\omega_{n,i}\zeta_i s+\omega_{n,i}^2\right)(1+\tau_i' s)=s^2+2\omega_{n,i}\zeta_i s+\omega_{n,i}^2+\tau_i' s^3+2\omega_{n,i}\zeta_i\tau_i' s^2+\omega_{n,i}^2\tau_i' s$$

$$J_{eff,i}s^3+s^2\left(f_{eff}+k_{T,i}K_{D,i}\right)+k_{T,i}K_{P,i}s+k_{T,i}K_{I,i}=\tau_i' s^3+\left(2\omega_{n,i}\zeta_i\tau_i'+1\right)s^2+\left(\omega_{n,i}^2\tau_i'+2\omega_{n,i}\zeta_i\right)s+\omega_{n,i}^2$$

Then we have
$\tau_i'=|J_{eff,i}|$
$2\omega_{n,i}\zeta_i\tau_i'+1=f_{eff}+k_{T,i}K_{D,i} \quad\Leftrightarrow\quad K_{D,i}=\dfrac{2\omega_{n,i}\zeta_i\tau_i'+1-f_{eff}}{k_{T,i}}$
$\omega_{n,i}^2\tau_i'+2\omega_{n,i}\zeta_i=k_{T,i}K_{P,i} \quad\Leftrightarrow\quad K_{P,i}=\dfrac{\omega_{n,i}^2\tau_i'+2\omega_{n,i}\zeta_i}{k_{T,i}}$
$\omega_{n,i}^2=k_{T,i}K_{I,i} \quad\Leftrightarrow\quad K_{I,i}=\dfrac{\omega_{n,i}^2}{k_{T,i}}$

(1) $D(q)\cdot\ddot{q}+v(q,\dot{q})=\tau=[\tau_1,\tau_2,\tau_3,\tau_4]^T$
(2) $u_i\cdot k_{T,i}-\dfrac{1}{n_i}\cdot\tau_i=J_{M,i}\cdot\ddot{q}_{M,i}+f_{M,i}\cdot\dot{q}_{M,i}$

Reduce to
$\widetilde{D}(q)\cdot\ddot{q}+\tilde{v}(q,\dot{q})=u$

Start with (2):
$u_i\cdot k_{T,i}-\dfrac{1}{n_i}\cdot\tau_i=J_{M,i}\cdot\ddot{q}_{M,i}+f_{M,i}\cdot\dot{q}_{M,i} \quad\Leftrightarrow$
$\tau_i=u_i\cdot k_{T,i}n_i-J_{M,i}\cdot n_i\ddot{q}_{M,i}-f_{M,i}\cdot n_i\dot{q}_{M,i} \quad\Leftrightarrow$
$\tau_i=u_i\cdot k_{T,i}n_i-J_{M,i}\cdot n_i{}^2\,\ddot{q}_i-f_{M,i}\cdot n_i^2\dot{q}_i$

Insert (1)
$u_i\cdot k_{T,i}n_i-J_{M,i}\cdot n_i^2\,\ddot{q}_i-f_{M,i}\cdot n_i^2\dot{q}_i=D(q)\cdot\ddot{q}+v(q,\dot{q}) \quad\Leftrightarrow$
$u_i=\dfrac{1}{k_{T,i}n_i}\left[D(q)\cdot\ddot{q}+v(q,\dot{q})+J_{M,i}\cdot n_i{}^2\,\ddot{q}_i+f_{M,i}\cdot n_i^2\dot{q}_i\right] \quad\Leftrightarrow$
$u_i=\dfrac{1}{k_{T,i}n_i}\left[\left(D(q)+J_{M,i}\cdot n_i{}^2\right)\cdot\ddot{q}+v(q,\dot{q})+f_{M,i}\cdot n_i^2\dot{q}_i\right] \quad\Leftrightarrow$

$(*)\ \widetilde{D}(q)=\dfrac{D(q)+J_{M,i}\cdot n_i}{k_{T,i}}$ and $(**)\ \tilde{v}(q,\dot{q})=\dfrac{v(q,\dot{q})+f_{M,i}\cdot n_i\dot{q}_i}{k_{T,i}}$