Introduction to R and econometrics

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Acknowledgement

This tutorial follows that developed by Prof. Sebastian Kranz (Ulm University) on market analysis with econometrics and machine learning:

- https://github.com/skranz/MarketAnalysis

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Objectives

By the end of this class, you should be able to:

- Create summary statistics and plots in R using dplyr & ggplot2
- Understand the basics of linear regression models
- Consider endogeneity, and how to mitigate this problem
- Run a linear model and instrumental variable regression in R

Getting started

Download R, Latex and Github

We are going to use R, Latex and GitHub for reproducible research!
It is free. Please download and install:

- R (econometrics software): https://cran.r-project.org/
- RStudio (works with R and Latex):
  https://rstudio.com/products/rstudio/download/#download
- Latex (I use Texlive) : https://www.tug.org/texlive/
- Sign up for Github (for version control): https://github.com/
  and download Github desktop: https://desktop.github.com/
- You can always use rstudio.cloud if you are struggling with
  installing R, Rstudio and Latex

Additional resources

Additional resources for R can be found at:

- Learning the basics of R: https://r4ds.had.co.nz/
- Free online course on introduction to R:
  https://www.datacamp.com/
- Introduction to econometrics with R:
  https://www.econometrics-with-r.org/
- Learning microeconometrics with R:
  https://www.routledge.com/Learning-Microeconometrics-
  with-R/Adams/p/book/9780367255381
- Merger simulation tool using the antitrust package:
  https://daag.shinyapps.io/antitrust_shiny/

Getting started

Once you have installed everything, and signed up for the necessary:

- Go to https://github.com/ryanhawthorne/IOIntroduction, click on 'fork', then 'code', then 'Open with Github desktop'
- Open RStudio on your computer, click on 'new project', then browse to the folder that you just cloned the git repo into (in Windows, probably: /Documents/GitHub/IOIntroduction)

Trying out RMarkdown

In RStudio, once you have opened your project:

- Click on 'file', 'new file', 'Rmarkdown', 'PDF', and save it ('tutorial1')
- Click on 'knit'
- RMarkdown is a great way to have your report text and econometrics code in the same file

Trying out Git

In RStudio:

- Head over to git on the top right hand side, select files you'd like to commit, click 'commit', add a message (e.g. 'first commit'), and then click 'commit', then 'push'
- Git is a version control system that you can use to roll back to any version of your work
- Commit frequently
- You may want to add a branch, perhaps called 'local', so that you are not working on the 'master' (production) version

You can have a look at your latest commit and push on github.com

Trying out the R console

- Type in '1+1', you should see:

[1] 2

- R uses 'objects', stored by typing: 'clever_stuff <- 3' and then call it with: 'clever_stuff'

[1] 3

- We often use lists in R, type in the following: my_first_list <- c(3,4), and then call it:

[1] 3 4

- When you want to type in strings, use "": my_first_string <- c("this","that"), and call it:

[1] "this" "that"

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Loading your first dataset

To load the data as an object, type this into a new code chunk (hold down ctrl+alt+I) in your Rmarkdown script:

ice_cream_sales <- read.csv("ice cream sales.csv")

First have a look at the data, with: str(ice_cream_sales):

```
## 'data.frame':    200 obs. of  5 variables:
##  $ X: int  1 2 3 4 5 6 7 8 9 10 ...
##  $ t: int  1 2 3 4 5 6 7 8 9 10 ...
##  $ w: num  31.7 15.6 86.1 98.9 55 ...
##  $ p: num  26.8 26.3 26.1 23.1 27.7 ...
##  $ q: num  53 52.3 50.5 44.2 54.3 ...
```

You can always also: View(ice_cream_sales)

Libraries and packages

We are going to use the Tidyverse package in R, including ggplot2 for plotting graphs:

- install.packages("tidyverse") - this installs the R package
- library("tidyverse") - this loads the R package

We will also need the various other packages in "0 starthere.R", so please install and load them.

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We have just bought an ice-cream business!

We are trying to figure out how to price ice cream
We are provided with a nice price and sales history
The data, for each time period t:

- the price p_t for each ice-cream scoop
- the quantity q_t of ice-cream scoops sold
- the wholesale price w_t for ice-cream tubs

Summarise the data

Then summarise the data (in a new, separate chunk), with:

ice_cream_sales %>%
 select(w,p,q) %>%
 summary() %>%
 kable()

Where:

- The '%>%' is known as a 'pipe' - it means 'then'
- 'select' chooses variables we are interested in
- 'summary()' produces summary statistics
- 'kable()' produces nice tables

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Summary result

w	p	q
Min. : 0.3912	Min. :21.76	Min. :42.81
1st Qu.:23.9664	1st Qu.:24.46	1st Qu.:47.98
Median :48.0086	Median :25.35	Median :49.74
Mean :47.3499	Mean :25.38	Mean :49.81
3rd Qu.:70.9175	3rd Qu.:26.32	3rd Qu.:51.76
Max. :99.8161	Max. :28.31	Max. :55.56

Is this relationship between price & quantity correct?

Type in a new code chunk: plot(p $\sim$ q, data = ice_cream_sales)
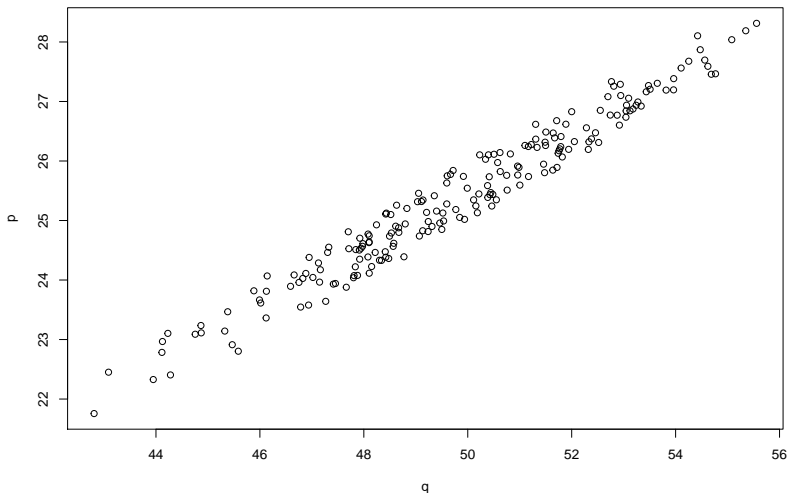
Some explanations for why price might +ve

Demand shocks might generate positive relationship

- Advertising may increase demand
- Better quality increase demand
- Price may also be affected by these demand shocks

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How do we model demand for ice cream?

We want to be able to maximise profits. Lets assume demand takes the following form:

$$q_t = a_t - bp_t$$

Where:

- the market size parameter a_t is: $a_t = a_0 + \varepsilon_t$
- ε_t in turn is a random variable measuring a demand shock
- a_0 and b_0 are exogenous parameters, and $a_0 > 0, b_0 > 0$

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How do we set prices for ice cream?

Our ice cream profit (assuming constant marginal costs, c_t):

$$\pi_t = p_t q_t(q_t) - c_t q_t(p_t) - F$$

Substituting $q_t = a_t - b p_t$ into the formula, profits are:

$$\pi_t = p_t(a_0 + \varepsilon_t) - b p_t^2 - c_t(a_0 + \varepsilon_t) + c_t b p_t$$

Differentiating profits with respect to price, setting equal to zero:

$$\frac{\partial \pi_t}{\partial p_t} = a_0 + \varepsilon_t - 2 b p_t + c_t b = 0$$

Equlibrium prices and quantities

After rearranging, the equilibrium price is:

$$p_t = \frac{a_0 + \varepsilon_t}{2b} + \frac{c_t}{2}$$

Substituting this back into our demand function:

$$q_t = a_0 + \varepsilon_t - b\left(\frac{a_0 + \varepsilon_t}{2b} + \frac{c_t}{2}\right)$$

Rearranging gives us our equilibrium quantity:

$$q_t = \frac{a_0 + \varepsilon_t}{2} - \frac{bc_t}{2}$$

Positive demand shock $(+a_0)$ results in higher price & quantity

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Elasticities in different models of demand

Model	Demand	Transformed demand	Own-price elasticity
Linear	$Q = \alpha + \beta P + \gamma X$	$Q = \alpha + \beta P + \gamma X$	$\varepsilon = \beta * P/Q$
Semi-log	$Q = \exp(\alpha + \beta P + \gamma X)$	$lnQ = \alpha + \beta P + \gamma X$	$\varepsilon = \beta * P$
Log-log	$Q = \exp(\alpha)P^{\beta}X^{\gamma}$	$lnQ = \alpha + \beta lnP + \gamma lnX$	$\varepsilon = \beta$

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Understanding prices and quantities in R (1a)

- We will now consider how demand can be estimated in R

You will learn how to:

- Run a random simulation in R

- Perform a basic linear regression using ordinary least squares

- Start to appreciate the endogeneity problem in demand estimation

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Simple linear regression model

- A simple linear regression model satisfies the following relationship for all observations $t = 1, ..., T$

$$y_t = \beta_0 + \beta_1 x_t + \varepsilon_t$$

- $y = (y_1, ... y_T)$ is the dependent variable.

- $x = (x_1, ... x_T)$ is the explanatory variable.

- $\varepsilon = (\varepsilon_1, ..., \varepsilon_T)$ (epsilon) is a random variable that describes unobserved influences on y, sometimes called *disturbance*. We will typically make some assumptions on the distribution of ε. We will also use the letters u and η (eta) to denote disturbances.

- $\beta = (\beta_0, \beta_1)$ is the vector of true coefficients.

Estimates, predicted values, resideuals

- Let $\hat{\beta}$ be an *estimate* of the true parameter vector β.

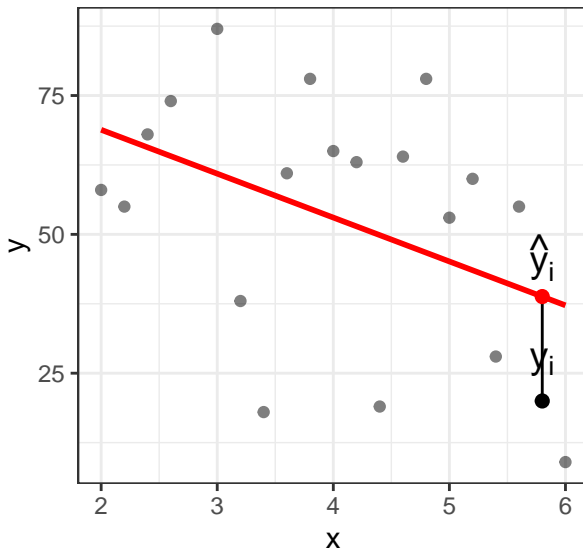- The *predicted values* (also called fitted values) of y are given by

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x$$

- The *residuals* (estimated values of the disturbance) are given by

$$\hat{\varepsilon} = y - \hat{y} = y - \hat{\beta}_0 - \hat{\beta}_1 x$$

  - The residuals $\hat{\varepsilon}$ are close to the true disturbances ε if our estimate $\hat{\beta}$ is close to the true parameters β.

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Considering our ice cream seller

Ordinary least squares minimises this disturbance term

Ordinary Least Squares Estimation (OLS)

- An ordinary least squares (OLS) estimate minimises the sum of squared residuals

$$\hat{\beta} = \arg\min \sum_{t=1}^{T} \hat{\varepsilon}_t^2$$

- For the simple linear regression (one explanatory variable), the OLS estimator $\hat{\beta}_1$ has the following formula

$$\hat{\beta}_1 = \frac{Cov(x_t, y_t)}{Var(x_t)} = cor(x_t, y_t) \frac{sd(y)}{sd(x)}$$

where *cor* denotes an empirical correlation and *sd* an empirical standard deviation for our sample data.

Linear regression model in matrix notation

- One often writes a linear regression model in matrix notation:

$$y = X\beta + \varepsilon$$

  with

$$X = \begin{pmatrix} 1 & x_1 \\ ... & ... \\ 1 & x_T \end{pmatrix} = \begin{pmatrix} \mathbf{1} & x \end{pmatrix}$$

- The OLS estimator $\hat{\beta} = (\hat{\beta}_0, \hat{\beta}_1)$ is then given by

$$\hat{\beta} = (X'X)^{-1}X'y$$

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Estimators and estimates

- Since $y = X\beta + \varepsilon$, the OLS estimator can be rewritten as

$$\hat{\beta} = (X'X)^{-1}X'y$$
$$= (X'X)^{-1}X'(X\beta + \varepsilon)$$
$$= \beta + (X'X)^{-1}X'\varepsilon$$

- This means $\hat{\beta}$ is a linear transformation of the true parameters β and the disturbance ε

- As a function of a random variable ε the OLS *estimator* $\hat{\beta}$ is itself a random variable

- The OLS *estimate* $\hat{\beta}$ is a realisation of the OLS estimator, i.e. the value for particular draws of ε and X.

- To understand what econometrics, one should keep in mind that *an estimator is a random variable*.

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Standard Error of OLS estimator

- In a simple linear regression (one explanatory variable)

$$y = \beta_0 + \beta_1 x + \varepsilon$$

where the ε are independently, identically normal distributed, the standard deviation of the OLS estimator $\hat{\beta}_1$ can be *estimated* by

$$se(\hat{\beta}_1) = \hat{sd}(\hat{\beta}_1) = \frac{1}{\sqrt{T}} \frac{sd(\hat{\varepsilon})}{sd(x)}$$

- We call this estimate of the standard deviation the *standard error* of $\hat{\beta}_1$.

  - Observations: We can estimate β_1 more precisely if we have. . .

    - a larger sample size T

    - more variation in x (higher standard deviation).

Analysis in R: We run a linear regression with `lm` and call `summary`

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Robust Standard Errors

- There is also a matrix formula to compute the standard errors for all $\hat{\beta}$ that can also be used for multiple linear regressions with more than one explanatory variable.

- If the ε are not identically, independently normal distributed, one should use appropriate *robust* standard errors. Most empirical papers in economics use some robust standard errors.

- We don't explain robust standard errors further in this course. Just note that in R a convenient way to use robust standard errors is the function `lm_robust` in the package `estimatr` or the function `felm` in the package `lfe`.

Criteria for estimators: Bias

- **Bias:** Recall that an estimator $\hat{\beta}$ is a random variable since it depends on the realizations of ε. Let $E\hat{\beta}$ be the expected value of $\hat{\beta}$. The bias of $\hat{\beta}$ measures a systematic over- or underestimation of $\hat{\beta}$ compared to β:

$$Bias(\widehat{\beta}) = E\hat{\beta} - \beta.$$

- **Unbiasedness:** An estimator $\hat{\beta}$ is unbiased if its Bias is 0, i.e.

$$E\hat{\beta} = \beta$$

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Criteria for estimators: Standard Deviation

- For two unbiased estimators of β_i, one would typically prefer an estimator with a lower standard deviation $sd(\hat{\beta}_i)$ (or equivalently the one with the lower variance $Var(\hat{\beta}_i)$)

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Criteria for estimators: Mean Squared Error

- **Mean squared error**: The mean squared error of $\hat{\beta}_i$ is given by

$$MSE(\hat{\beta}_i) = E(\hat{\beta}_i - \beta_i)^2$$
$$= Bias(\hat{\beta}_i)^2 + Var(\hat{\beta}_i)$$

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Criteria for estimators: Consistency

- An estimator $\hat{\beta}$ is (strongly) **consistent** if its MSE converges to 0 as the sample size T grows large

$$\lim_{T \to \infty} MSE(\hat{\beta}) = 0.$$

- Estimated parameters $\widehat{\beta}$ converge (in probability) to true β

$$\operatorname*{plim}_{T \to \infty} \widehat{\beta} = \beta$$

- **Consistency: the most important requirement for an estimator**

- If an estimator is inconsistent that is typically because it is biased and the bias does not go away as $T \to \infty$.

Criteria for estimators: Efficiency

- An estimator $\hat{\beta}$ is **efficient** (within a specified class of estimators) if there is no other estimator that has a lower mean squared error.

Assumptions of the simple linear regression model

- We now state a series of assumptions for the simple linear regression model (one explanatory variable).

    - A1: $E(\varepsilon_t|x) = 0$

    - A2: The ε_t are identically and independently distributed.

    - A3: The ε_t are normally distributed

    - A1-A3 are often compactly written as $\varepsilon_t \stackrel{i.i.d.}{\sim} N(0, \sigma^2)$.

    - A4: The explanatory variable x must have positive variance and be deterministic or a stationary random variable. (We don't discuss what stationary means in this course, but you can look it up on Wikipedia.)

- If all assumptions are satisfied, the OLS estimator $\hat{\beta}$ will be consistent, unbiased and efficient.

The main assumption (A1)

- **A1** No matter which values of x we observe, the conditional expected value of ε_t is always zero:

$$E(\varepsilon_t|x) = 0$$

- The important thing is not the 0 on the right. If it were a positive or negative value, we could always redefine the constant β_0 to make it 0.

- The important thing is that the expected value of ε_t does not depend on x. This means knowing x shall give us no information about the expected value of ε_t.

- In our ice cream example with profit maximizing prices this condition is violated. Higher demand shocks lead to higher prices. This means if we observe a high price, we expect that there was a positive demand shock ε_t.

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Exogenous and Endogenous Variables

- We say the explanatory variable x is **exogenous**, if it is uncorrelated with ε

$$\text{cor}(x_t, \varepsilon_t) = 0$$

- We say x is **endogenous** if $\text{cor}(x_t, \varepsilon_t) \neq 0$

- Condition A1 $E(\varepsilon|x) = 0$ can only be satisfied if x is exogenous.

- We will typically just check whether x is exogenous, even though A1 is a stronger condition. A1 is sometimes called *strong exogeniety*. In all examples studied in this course, exogeniety of x implies that also A1 holds.

A2, No auto-correlation and no heteroskedasticity

- **A2** The ε_t are identically and independently distributed.

- Typical violations of A2:

    - auto-correlation: demand shocks may be persistent across periods

    - heteroskedasticity: the variance of ε_t can depend on the explanatory variable (this alone does not yet mean that A1 is violated)

- A2 is moderately important. If violated, the OLS estimator $\hat{\beta}$ is still consistent but not efficient. One must calculate standard errors using an appropriate formula for robust standard errors.

- We don't study violations of A2 in this course.

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A3: Normally distributed disturbances

- **A3** ε_t is normally distributed

- It is nice if A3 holds, but it is not crucial. Even if A3 is violated, the OLS estimate $\hat{\beta}$ is the best unbiased linear estimators of β (Gauss-Markov Theorem). Significance tests would only be asymptotically correct.

- If A1-A3 (and the other assumptions) holds, $\hat{\beta}$ coincides with Maximum Likelihood estimator and is efficient.

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95% Confidence Intervals

- If assumptions A1 holds (no endogeniety problem) then with approximately 95% probability we find an estimate $\hat{\beta}_i$ such that the interval of plus-minus 2 standard errors around $\hat{\beta}_i$ contains the true parameter β_i.

- We call this interval

$$[\hat{\beta}_i - 2 \cdot se(\hat{\beta}_i) \; ; \; \hat{\beta}_i + 2 \cdot se(\hat{\beta}_i)]$$

the approximate **95% confidence interval**.

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"Bias Formula"

- Consider a simple linear regression

$$y = \beta_0 + \beta_1 x + \varepsilon.$$

and assume we would observe ε.

- One can show that

$$\hat{\beta}_1 - \beta_1 = cor(x, \varepsilon) \frac{sd(\varepsilon)}{sd(x)}$$

using the sample correlations and sample standard deviations.

- This expression is an estimator of the bias of $\hat{\beta}_1$. (The actual bias is the expected value of it.)

- Thus essentially the bias has the same sign as the correlation between x and ε.

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Understanding bias and endogeneity in R (1b)

You will learn how to:

- Develop a Monte-Carlo simulation

- Analyse the results

- Consider the endogeneity problem in demand estimation