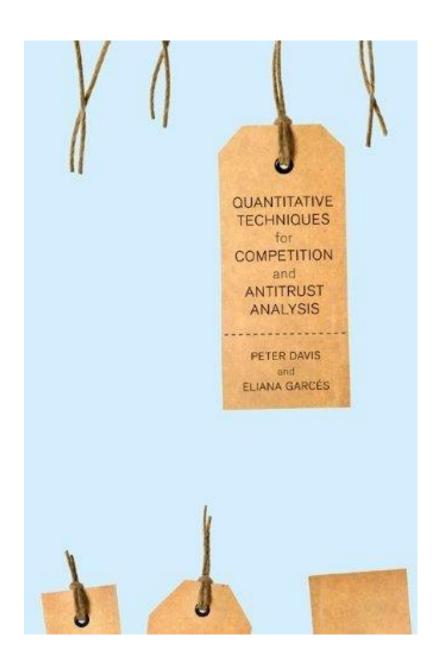


# MASTER OF COMMERCE IN COMPETITION AND ECONOMIC REGULATION

Quantitative Methods and Econometrics for application in Competition and Economic Regulation (QEC9X01)

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Lecture 2: Oligopolistic Competition



"Quantitative Techniques for Competition and Antitrust Analysis" by Peter Davies and Eliana Garces

Chapter: 1

#### **Outline**

- 1. The Cournot Game
  - Quantity competition
- 2. Bertrand's Paradox
  - Price competition
- Solutions to the Bertrand Paradox:
  - Price competition with differentiated products
  - Kreps-Scheinkman → firms first set capacities and then price competition (see Appendix)

#### **Demand:**

Assume that consumers' behaviour is summarized by linear inverse demand function: P(Q) = a - bQ.

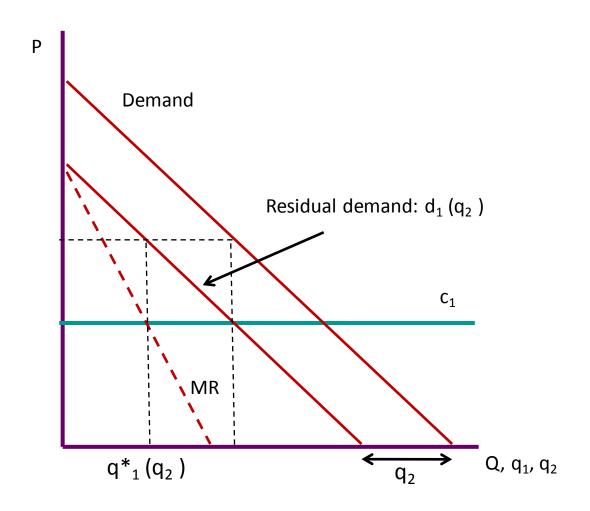
#### **Supply:**

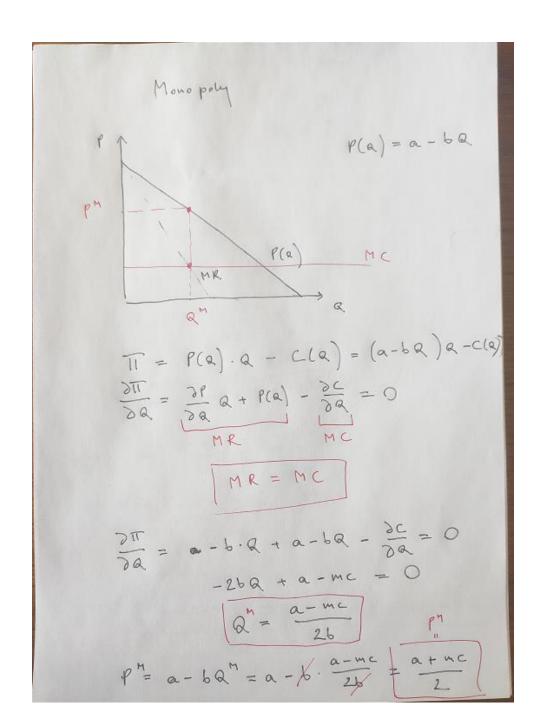
- □ Suppose there are two firms: Firm 1 and Firm 2 which produce homogenous good at a constant marginal cost of c (symmetry) and assume there are no fixed costs. (i.e. their per unit cost is c, cost function C(q) = cq, where a>c
- ☐ The firms are assumed to choose production quantities as strategic variables.

Firms' profits can be written as:

$$\begin{cases}
\Pi_1 = (p(q_1 + q_2^c) - c_1)q_1 \\
\Pi_2 = (p(q_1^c + q_2) - c_1)q_2
\end{cases}$$

- ☐ Firm 1 decides how much to produce without knowing the production of competitor but knowing that his decision will have an effect on the quantity produced by Firm 2.
- Firm 1 forms expectations about quantity supplied by Firm 2.
- □ **Residual demand curve** of Firm 1 is the demand curve which is a portion of total market demand that is not supplied by Firm 2.
- □ To find the quantity which maximizes profits of Firm 1 we need to find the intersection point of marginal cost curve and marginal revenues derived for the residual demand curve.





Profit-maximization by Firm 1:

$$\Pi_{1} = (p(q_{1} + q_{2}^{c}) - c_{1})q_{1}$$

$$\frac{\partial \Pi_{1}}{\partial q_{1}} = p(q_{1} + q_{2}^{c}) - c_{1} + p'(q_{1} + q_{2}^{c})q_{1} = 0$$

$$\frac{\partial \Pi_{1}}{\partial q_{1}} = a - c_{1} - 2bq_{1} - bq_{2}^{c} = 0$$

Optimal production of Firm 1 as a function of expectations about quantity produced by Firm 2 (best-response function of Firm 1):

$$r_1(q_2^c) = q_1 = \frac{a - c_1 - bq_2^c}{2b}$$

If Firm 1 expects that Firm 2 will not produce at all, it chooses monopoly quantity:

$$r_1(0) = \frac{a - c_1}{2b}$$

If Firm 1 were not interested to produce at all, we need to have:

$$a - c_1 - bq_2^c = 0 \Longrightarrow q_2^c = \frac{a - c_1}{b}$$

This implies that price is equal to marginal cost (hence, there are no incentives to enter).

Fim 1 devides to supply 
$$q_1 = 0$$

$$P(a) = a - b (q_1 + q_2)$$

$$= a - b (0 + a - c)$$

$$= a - (a - c) = c$$

Counsot solution
$$q_1 = \frac{a - c - bq_1}{2b}$$

$$q_2 = \frac{a - c - bq_1}{2b}$$

$$q_1 = \frac{a - c - bq_1}{2b}$$

$$2bq_1 = a - c + bq_1$$

$$q_1 = \frac{a - c}{3b}$$

$$q_2 = a - c - b \cdot \frac{a - c}{3b}$$

$$q_2 = \frac{a - c}{3b}$$

$$q_2 = \frac{a - c}{3b}$$

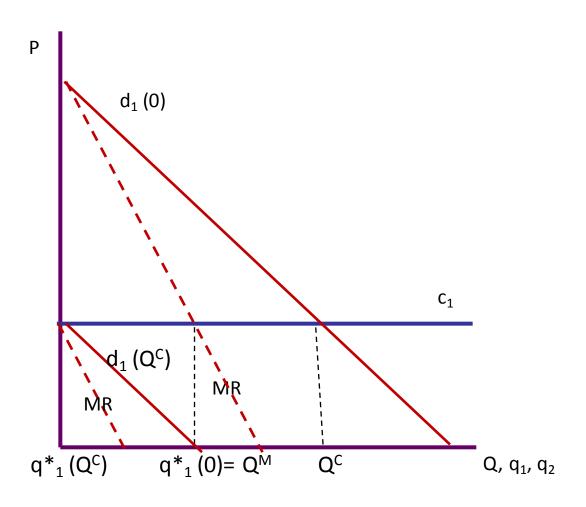
$$q_3 = \frac{a - c}{3b}$$

$$q_4 = \frac{a - c}{3b}$$

$$q_5 = \frac{a - c}{3b}$$

$$q_6 = \frac{a - c}{3b}$$

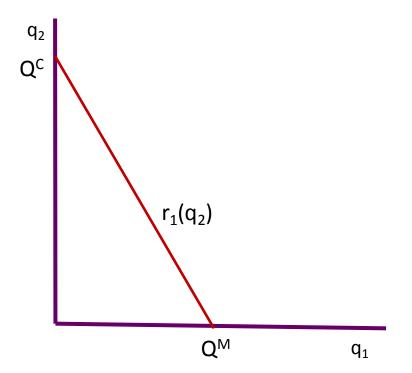
$$q_7 = \frac{a - c}{3b}$$

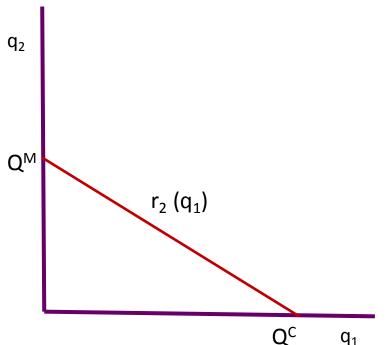


Using similar reasoning we may derive and draw the best-response function for Firm 2.

$$r_1(q_2^c) = q_1 = \frac{a - c - bq_2^c}{2b}$$

$$r_2(q_1^c) = q_2 = \frac{a - c - bq_1^c}{2b}$$





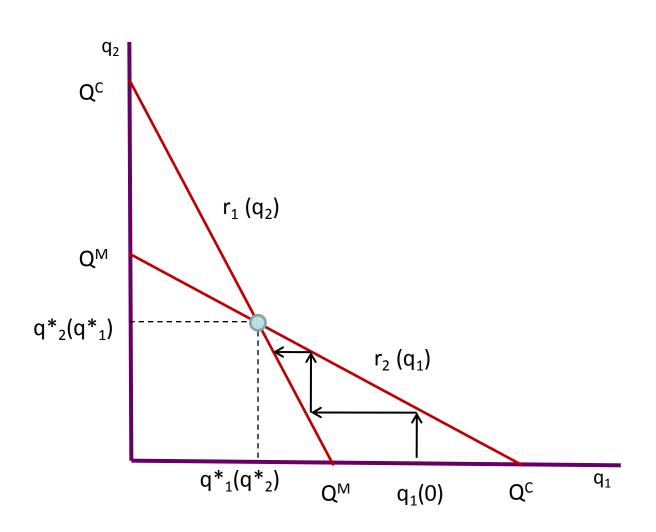
☐ There are many pairs of production quantities which satisfy profit maximization condition of both firms => we need to find the one which represents equilibrium.

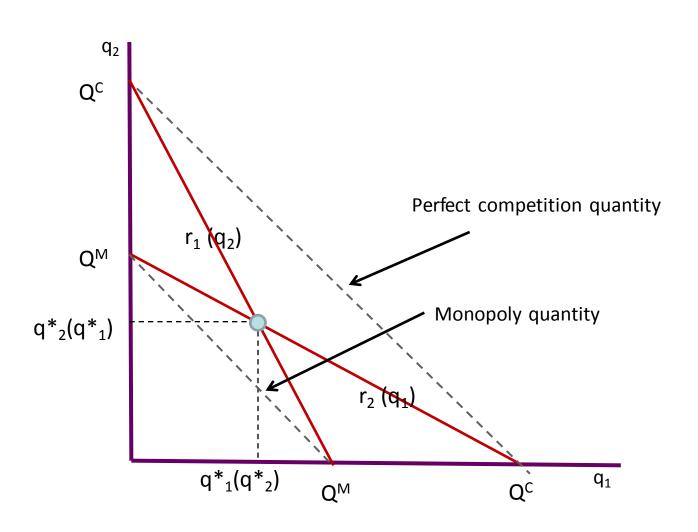
$$(q_1, q_2) = \left(\frac{a - c - bq_2^c}{2b}, \frac{a - c - bq_1^c}{2b}\right)$$

■ Lets assume that when maximizing profits firms form correct expectations about the quantity produced by the competitor:

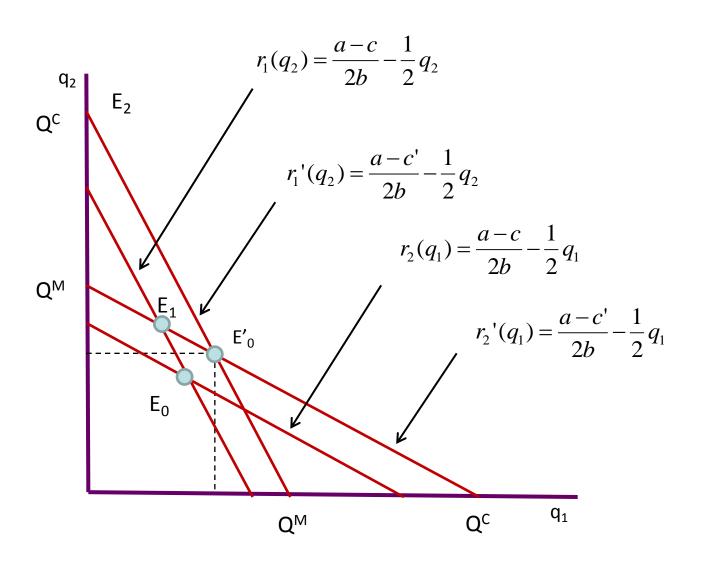
$$q_1^c = q_1 \quad q_2^c = q_2$$

□ Production quantities which satisfy these conditions represent Nash-Cournot equilibrium. The equilibrium is in the intersection of the best-response functions.





- ☐ If Firm 1 manages to achieve a technological advantage which allows it to reduce production cost from c to c', while Firm 2 maintains its production cost at c
  - → the best-response function of Firm 1 is shifted upwards
- $\Box$  The equilibrium changes from point  $E_0$  to point  $E_1$ 
  - → Firm 1 increases and Firm 2 decreases production quantity
- $\Box$  If both firms changed their m marginal costs from c to c', the equilibrium point would move to  $E'_0$ 
  - → both firms increase production quantities



- ☐ Firms are assumed to have different marginal costs.
- □ Let us take two firms i and j and compute the ratios of margins and market shares using their first order conditions:

$$\begin{cases} p - c_i = -\frac{\partial p(Q)}{\partial q_i} q_i \\ p - c_j = -\frac{\partial p(Q)}{\partial q_j} q_j \end{cases} \qquad \frac{\frac{q_i}{(p - c_i)}}{\frac{(p - c_i)}{(p - c_j)}} = \frac{\frac{q_i}{Q}}{\frac{q_j}{Q}} = \frac{s_i}{s_j}$$

- ☐ The ratio of shares increases when the ratio of margins increases
- □ The lower is the marginal cost => the greater is the market share
   => the greater is the margin and profits.

$$c_i > c_j \Longrightarrow s_j > s_i \Longrightarrow \prod_j > \prod_i$$

Asymmetric Count

$$Ti = P(Q)qi - ci(qi)$$

$$\frac{\partial Ti}{\partial q_i} = \frac{\partial P}{\partial q_i}q_i + P(Q) - \frac{\partial ci}{\partial q_i} = 0$$

$$P - mci = -\frac{\partial P}{\partial q_i}q_i$$

$$P - mci = -\frac{\partial$$

#### Cournot duopoly: comparative statics

- Comparative statics predicts how market equilibrium will change in result of changes in various exogenous conditions => comparing the equilibria ex-ante and ex-post
- Example: changes in input costs due to technological progress, exchange rate fluctuations, changes in factors determining demand, etc.

The equilibrium exists when all firms j=1,..., N maximize profits with correctly formed expectations about quantities produced by the other firms:

$$\frac{\partial \Pi_i}{\partial q_i} = \frac{\partial p(Q)}{\partial q_i} q_i + p(Q) - \frac{\partial C(q_i)}{\partial q_i} = 0$$

where 
$$Q = q_i + \sum_{j \neq i}^n q_j^c$$

There is a unique symmetric equilibrium in which all firms produce the same quantity (take two FOCs and subtract sidewise to see this):

$$q_1^* = ... = q_n^* = q^*$$

$$\frac{\partial \Pi_i}{\partial q_i} = a - b \sum_{j=1}^N q_j^* - b q_i^* - c = 0$$

Counset: the core of N symmetric firms

$$Ti = p(Q)qi - c(qi)$$

$$\frac{\partial Ti}{\partial qi} = \frac{\partial p}{\partial qi} = 0$$

$$\frac{\partial qi}{\partial qi} = \frac{\partial p}{\partial qi} = 0$$

$$\frac{\partial qi}{\partial qi} = -bqi + (a-b\sum_{k=1}^{N}q_k) = a-b(q_1+q_2+...q_N)$$

$$\frac{\partial T}{\partial qi} = -2bqi + (a-b\sum_{k=1}^{N}q_k) = \frac{\partial c}{\partial qi} = 0$$

$$= -2bqi + (a-b\sum_{k=1}^{N}q_k) - \frac{\partial c}{\partial qi} = 0$$
another Foc for  $j:$ 

$$-2bj + (a-b\sum_{k=1}^{N}q_k) - \frac{\partial c}{\partial qi} = 0$$

$$= -2bj + (a-b\sum_{k=1}^{N}q_k) - \frac{\partial c}{\partial qi} = 0$$

$$= -2bj + 2bqj + (a-b\sum_{k=1}^{N}q_k) - mai = 0$$

$$= -(a-b\sum_{k=1}^{N}q_k) + mai = 0$$

$$= -2bqi + 2bqj + (a-b\sum_{k=1}^{N}q_k) - mai = 0$$

$$= -2bqi + 2bqj + (a-b\sum_{k=1}^{N}q_k) + mai = 0$$

$$= -2bqi + 2bqj + (a-bqj) + bqi = 0$$

$$= -2bqi + 2bqj + bqi = 0$$

$$= -2bqi + 2bqj + bqi = 0$$

$$= -2bqi + 2bqj + bqi = 0$$

Coumot, the cose of N symmetric firms - 269; + (a - 6 5 9 c) - 2c = 0 9,=92= ... = 9N = 9K -2bg'+(a-b(N+1)g') - mc; = 0 a - b (N+1) 9 " - mc; = 0 9 = a - mc = b (++1) P'= a - b \( \frac{2}{5} = a - b N \q^{\frac{1}{5}} P" = a - 16N . a - mc 16(N+1) \_ a (N+1) - at - N. mc P = 2 - N. mc

$$a-c-b(n+1)q^* = 0$$

$$q^* = \frac{a-c}{b(n+1)}$$

$$p^* = \frac{a+nc}{n+1} \quad \Pi^* = \frac{1}{b} \left( \frac{a-c}{n+1} \right)^2$$

The equilibrium price decreases in the number of firms:

$$\frac{\partial p^*}{\partial n} = \frac{c(n+1) - a - nc}{(n+1)^2} = \frac{-a+c}{(n+1)^2} < 0$$

For infinite number of firms the price is equal to marginal costs.

- ☐ Cournot model illustrates causality between market structure and performance.
- $\Box$  Since firms are symmetric industry profits are equal to  $n\Pi^*$

$$\frac{\partial n\Pi^*}{\partial n} = \frac{(a-c)^2}{b} \left( \frac{(n+1)^2 - 2n(n+1)}{(n+1)^4} \right) = \frac{(a-c)^2}{b} \left( \frac{-n^2 + 1}{(n+1)^4} \right) < 0$$

When the number of firms increases (a change in structure) => industry profits decrease (a change in performance).

Deadweight loss in Cournot oligopoly rapidly decreases with an increase in the number of firms:

$$DL = \frac{1}{2}(P^* - P^C)(Q^C - Q^*) = \frac{1}{2} \left(\frac{a + nc}{n+1} - c\right) \left(\frac{a - c}{b} - \frac{n(a - c)}{b(n+1)}\right)$$

$$DL = \frac{1}{2b} \left( \frac{a-c}{n+1} \right)^2$$

#### **Demand:**

Assume that consumers' behaviour is summarized by an inverse demand function: P(Q) = a - bQ, where a > c.

#### Supply:

- □ Suppose there are two firms: Firm 1 and Firm 2 which produce homogenous good at a constant marginal cost of c (symmetry) and assume there are no fixed costs, i.e., their cost function is C(q) = cq.
- The firms are assumed to simultaneously choose prices!

<u>Proposition</u>: If each firm has a constant marginal cost of production c, then the unique Nash equilibrium is

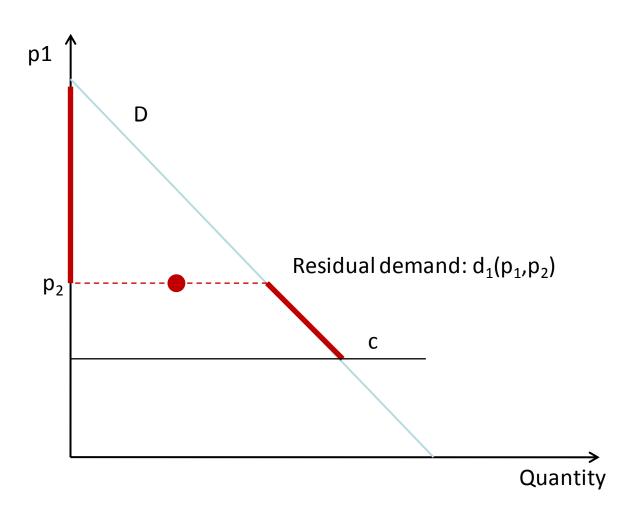
$$p_1^* = p_2^* = c$$

- The good is homogenous => competitors' products are perfect substitutes.
- □ Whichever firm sets the lowest price, it gets all of the demand.
- ☐ If each firm sets the same price, then they evenly split the demand.
- Residual demand function for Firm 1 is defined as follows (it is discontinuous at  $p_1=p_2$ ):

$$d_1(p_1, p_2) = \begin{cases} D(p_1) & p_1 < p_2 \\ D(p_1)/2 & p_1 = p_2 \\ 0 & p_1 > p_2 \end{cases}$$

#### Bertrand competition: residual demand

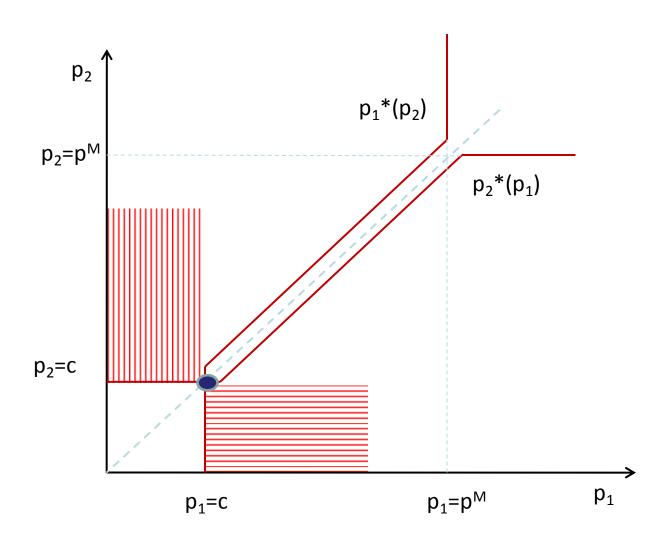
Residual demand of Firm 1 at price p<sub>2</sub>.



- □ Firm 1's optimal price depends on what it conjectures Firm 2's price will be, and vice versa:
- □ If Firm 1 expects Firm 2 to price above monopoly price, then Firm 1's optimal strategy is to price at the monopoly level.
  - => Firm 1 gets all of the demand and makes monopoly profit.
- If Firm 1 expects Firm 2 to price below monopoly price but above marginal cost, then Firm 1 should set a price just below that of Firm 2:  $p_1=p_2-\epsilon$
- □ => Firm 1 gets all the demand and full profit instead of half of it.

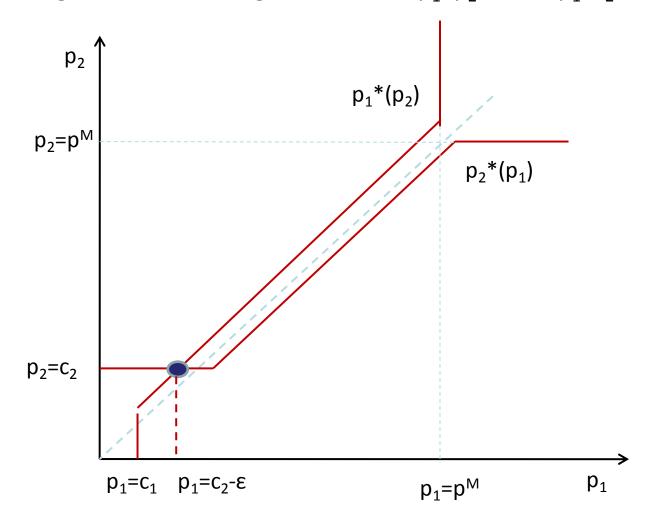
$$\Pi_{1}(p_{1}-\varepsilon,p_{2}) = D(p_{2}-\varepsilon)((p_{2}-\varepsilon)-c) \approx D(p_{2})(p_{2}-c) > \frac{D(p_{2})}{2}(p_{2}-c) > 0$$

- But as long as Firm 1 sets price above marginal cost, then Firm 2 also has an incentive to undercut Firm 1, otherwise it makes zero profits.
- ☐ This price war occurs until each firm prices at marginal cost.
  - => Both firms split the demand and make zero profit.
- ☐ If Firm 1 expects Firm 2 to price below marginal cost, then Firm 1's optimal price would be to price at marginal cost or above.
  - => Firm 2 gets all the demand and makes negative profit, while Firm 1 gets no demand and makes zero profit.



- ☐ In equilibrium both competitors set the same price at marginal cost.
- Why is this the only equilibrium? Because this is the only case where neither firm would profitably deviate.
- Setting price above marginal cost is not an equilibrium strategy.
   the other firm has an incentive to undercut the price and capture the market and make a higher profit.
- Setting price below marginal cost is not an equilibrium strategy.=> you capture all the demand, but you make a negative profit.

When firms differ in marginal costs, only the low cost firm will sell. For instance, if Firm 1 is low cost, it will charge a price just under the marginal cost of the high cost Firm 2: p₁=p₂-ε → p₁=c₂-ε



#### Summary – Bertrand Paradox

- Homogeneous product price competition with identical constant marginal costs
- □ The unique pure strategy Nash equilibrium is for firms to set price = mc and make zero profits!
  - Just two firms are sufficient to ensure p=mc.
  - This is very frustrating for the firms since increasing both their prices would make each of them better off!
- ☐ The result is interpreted as a paradox because we don't expect oligopoly pricing to yield the competitive outcome.
  - Theory seems to run directly counter to the data observed lots of oligopolies who didn't seem to price at mc.

#### Bertrand Paradox 'resolutions'

- ☐ "Resolutions" of the paradox:
  - 1. Differentiated products
  - 2. Repeated interaction
  - 3. Capacity constraints (Bertrand-Edgeworth model)
  - 4. Two stage game: first quantities, then prices (Kreps and Scheinkman, 1983):
    - Capacity chosen in the first stage are Cournot quantities
    - Prices chosen in the second stage with capacity constraints
  - 5. Search costs (Diamond, 1971): if consumers face a (however small) cost of searching for the lowest price, each firm charging the monopoly price is an equilibrium! This is called Diamond paradox.

#### Product differentiation model

Suppose firms face a differentiated product linear demand system:

Demand for good 1:  $q_1 = a_1 - b_{11}p_1 + b_{12}p_2$ 

 $q_2 = a_2 - b_{22}p_2 + b_{21}p_1$ Demand for good 2:

Recall good 2 is a substitute for good 1 if an increase in the price of good 2 increases the demand for good 1

$$\frac{\partial q_1}{\partial p_2} = b_{12} > 0$$

 $\frac{\mathcal{C}q_1}{\partial p_2} = b_{12} > 0$  Eg., Dell and Compaq computers

And good 2 is a complement for good 1 if an increase in the price of good 2 decreases the demand for good 1

$$\frac{\partial q_1}{\partial p_2} = b_{12} < 0$$

Eg., MP3 files and MP3 players

#### **Profit maximization**

□ Profit maximization with constant marginal costs

$$\pi_i(p_i, p_{-i}) = (p_i - c)D_i(p_i, p_{-i})$$

FOC: 
$$\pi_i^i(p_i, p_{-i}) = D_i(p_i, p_{-i}) + (p_i - c)D_i^i(p_i, p_{-i}) = 0$$
$$= (a_i - b_{ii}p_i + b_{ij}p_j) + (p_i - c)(-b_{ii}) = 0$$

$$\Box$$
 So that  $\left(a_i + b_{ii} p_i\right) = (2p_i - c)b_{ii}$ 

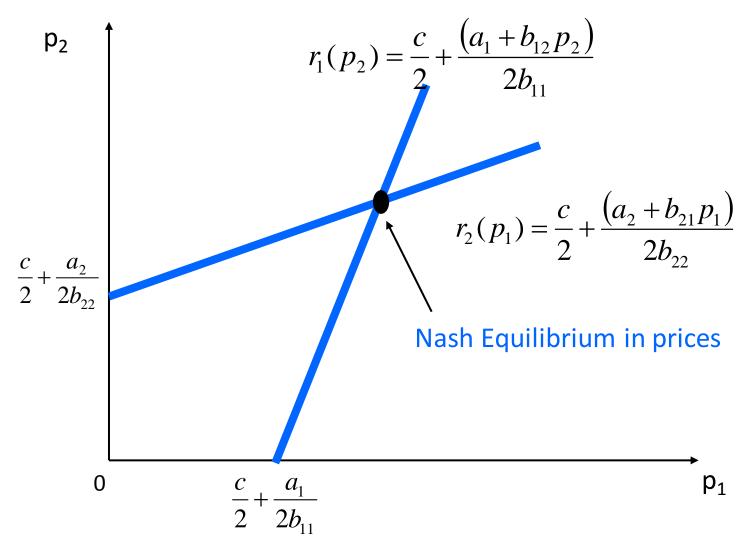
■ And hence:

$$r_i(p_{-i}) = p_i = \frac{c}{2} + \frac{(a_i + b_{ij} p_j)}{2b_{ii}}$$

Slope of reaction function depends  $b_{ij.}$  In particular, whether slopes up or down depends on sign of  $b_{ij}$ .

#### Strategic complements

In differentiated product price game with demand substitutes ( $b_{ij}>0$ ) prices are strategic complements (reaction curves slope up).



# Substitute goods means prices are strategic complements

☐ We showed that the FOC are:

$$\frac{\partial \pi_i \left( p_i, p_{-i} \right)}{\partial p_i} = \left( a_i - b_{ii} p_i + b_{ij} p_j \right) + \left( p_i - c \right) \left( -b_{ii} \right) = 0$$

□ So that the cross derivative is

$$\frac{\partial^{2} \pi_{i} (p_{i}, p_{j})}{\partial p_{i} \partial p_{i}} = b_{ij}$$

■ With linear demands, if products are substitutes (b<sub>ij</sub>>0) then reaction curves in price games will slope upwards. If the goods are complements (b<sub>ij</sub><0) then reaction curves will slope downwards.</p>

### **Take-away Points**

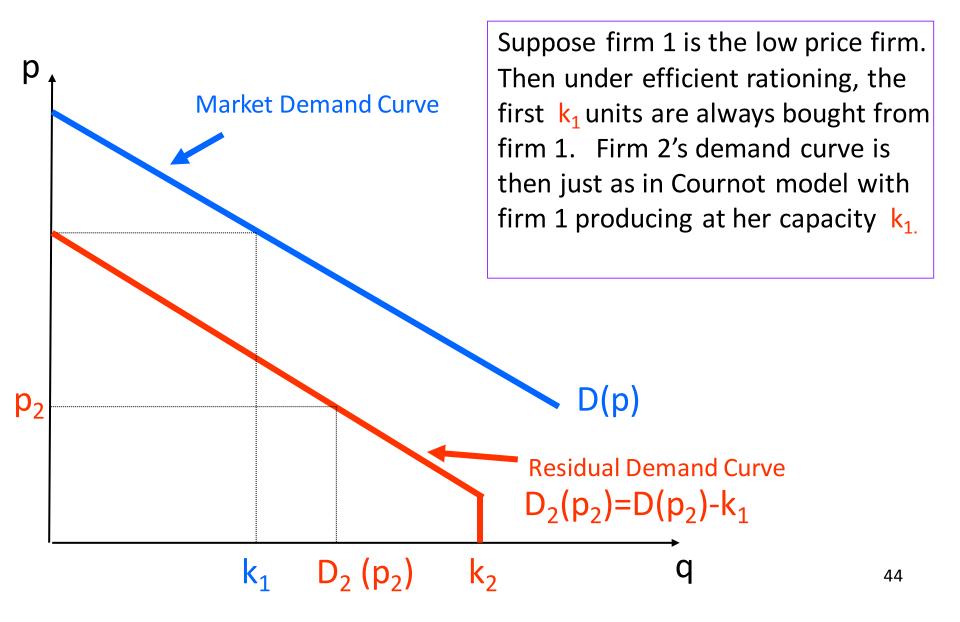
- Competition in quantities and prices have very different Nash equilibria.
- Data usually indicates that oligopolists do not price at marginal cost and so do not often support the homogeneous product version of Bertrand's theory.
- □ Resolutions to the Bertrand paradox include among others:
  - Product differentiation
  - Capacity constraints (see Appendix)

## **Appendix**

#### Price competition with capacity constraints

- Kreps-Scheinkman (Bell Journal, 1983) consider the two stage game:
  - Stage 1: Choose capacities
  - Stage 2: Bertrand price competition given capacities
- □ KS show that provided there is 'efficient rationing' at the second stage (given capacities) the sub-game perfect equilibrium of this two stage game can look a lot like the equilibrium in the one-shot Cournot quantity game.
- □ With capacity constraints, supply can be less than total demand for a given price (at least out of equilibrium) and so we have to know which consumers would get the good. Common assumptions are:
  - Efficient Rationing: The consumers who value the good most are served first by the lowest price firm until capacity exhausted.
  - Proportional (Random) Rationing: each consumer has an equal probability of being served.

### Residual demand with efficient rationing



## Stage 2: Bertrand given capacities (k<sub>1</sub>,k<sub>2</sub>)

Analytically, sales are therefore:

Low price firm gets all demand or sells capacity

$$\min\{D(p_i), k_i\} \qquad \text{if} \qquad p_i < p_j \\ q_i(p_i, p_j; k_i, k_j) = \min\{k_i, \max\{D(p_i) - k_j, 0\}\} \quad \text{if} \quad p_i > p_j \\ \text{High price firm gets} \quad \min\{k_i, (k_i / (k_i + k_j))D(p)\} \quad \text{if} \quad p_i = p_j \\ \text{Residual demand if any and can sell upto capacity}$$

□ At stage 2, firm i chooses its price to solve:

$$r_i(p_j; k_i, k_j) = \underset{p_i}{\operatorname{arg \, max}} \ \pi_i(p_i, p_j; k_i, k_j)$$
$$= \underset{p_i}{\operatorname{arg \, max}} \ (p_i - c)q_i(p_i, p_j; k_i, k_j)$$

#### Case 1: capacities are large

Capacities are not an effective constraint and so sales are:

$$q_{i}(p_{i}, p_{j}; k_{i}, k_{j}) = \begin{cases} D(p_{i}) & \text{if} & p_{i} < p_{j} \\ 0 & \text{if} & p_{i} > p_{j} \\ (k_{i}/(k_{i} + k_{j}))D(p) & \text{if} & p_{i} = p_{j} \end{cases}$$

- If prices are not equal then low price firm gets whole market. If prices are equal, the sharing rule applies.
- ☐ Firm's demand curve looks exactly as it does in the homogeneous product Bertrand game and p=mc is the unique equilibrium outcome to the sub-game.

### Case 2: capacities are small

and 
$$k_i \le \left(\frac{k_i}{k_i + k_j}\right) D(p_i) \Leftrightarrow k_i + k_j \le D(p_i)$$

□ then capacity constraints bind and

$$q_{i}(p_{i}, p_{j}; k_{i}, k_{j}) = \begin{cases} k_{i} & \text{if} & p_{i} < p_{j} \\ \min\{k_{i}, D(p_{i}) - k_{j}\} = k_{i} & \text{if} & p_{i} > p_{j} \\ k_{i} & \text{if} & p_{i} = p_{j} \end{cases}$$

- And equilibrium price will equate industry supply (which is total capacity) to market demand:  $k_i + k_j = D(p^*)$
- Inverting gives the equilibrium price as a function to industry capacity:  $p^* = D^{-1}(k_i + k_j) \equiv P(k_i + k_j)$

#### Stage 1: capacity choices

Substituting in the equilibrium price function

$$r_{i}(k_{j}) = \underset{k_{i}}{\operatorname{arg \, max}} \quad \pi_{i}(p_{i}^{*}, p_{j}^{*}; k_{i}, k_{j})$$

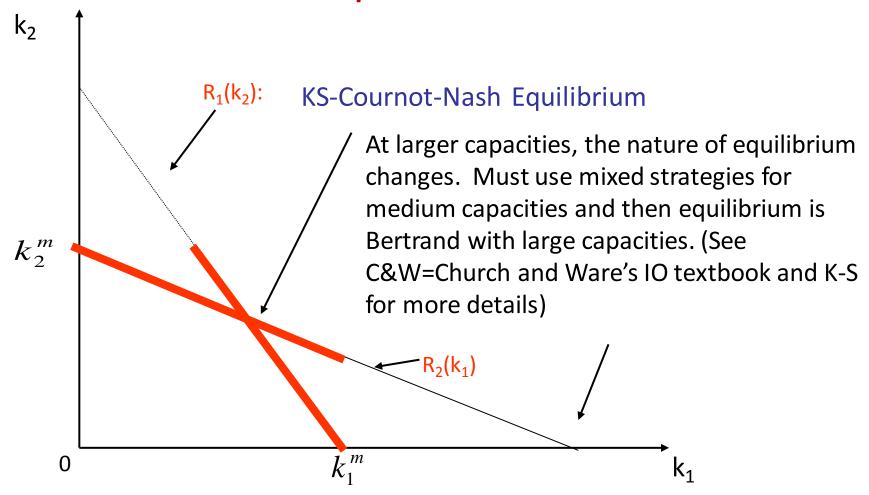
$$= \underset{k_{i}}{\operatorname{arg \, max}} \quad (p_{i}^{*} - c)q_{i}(p_{i}^{*}, p_{j}^{*}; k_{i}, k_{j})$$

$$= \underset{k_{i}}{\operatorname{arg \, max}} \quad (P(k_{i} + k_{j}) - c)k_{i}$$

$$(P(k_{i} + k_{j}) - c)k_{i}$$

□ Which looks exactly like the one-shot Cournot game profit function with choice variable capacity  $k_i$  instead of output,  $q_i$  and with the inverse demand function  $P(k_i + k_j)$ 

# Reaction curve diagram – KS's 'Cournot in capacities'



### Davidson and Denekere (1986)

- □ Show that KS's result is sensitive to the exact rationing rule used. And they argue 'Efficient Rationing' isn't very likely.
- □ Recall, under efficient rationing, the most highly valued units must be bought from the low price firm, e.g., if consumers are randomly distributed between the two firms then this won't happen.

**See:** Deneckere, R., Davidson, C., 1986. "Long-run competition in capacity, short-run competition in price, and the Cournot model", *Rand Journal of Economics* 16, pp.404-415.