Problemset 1

Building a Strategic Portfolio

1. Introduction

This project’s goal is to construct and analyze a strategic investment portfolio using a sufficiently large dataset of asset prices.

Our investments fond focus is on the following five stocks: Apple (AAPL), Microsoft (MSFT), Tesla (TSLA) Ørsted (ORSTED.CO) and Visa (V). In addition, the SP500 (^GSPC) is used for comparative purposes.

These assets were selected to ensure a broad diversification across different industries and regions. Apple, Microsoft, and Tesla represent leading U.S. technology and innovation companies with high market capitalizations. Ørsted, a

Danish company in the renewable energy sector, adds a European and sustainable component to the portfolio. Visa represents the financial sector and complements the selection with a stable, globally operating payment company.

1. Data Collection

The historical price data were obtained using Yahoo Finance and its *yfinance* python library, aggregated into monthly intervals. For comparability and improved modeling, logarithmic returns were calculated, defined as:

Figure 1: log-return formula

where defined as the price at time . This transformation linearizes percentage changes and better captures extreme values.

1. Jarque-Bera Test for Normality

To assess whether the monthly stock return follow a normal distribution, the Jarque-Bera (JB) test was used. This statistical test evaluates whether the skewness and kurtosis of the sample distribution significantly deviate from those of a normal distribution. Specifically, the JB test statistic is calculated as:

Figure 2: Jarue-Bera distribution

where is the number of observations, is the skewness and the kurtosis. Under the null hypothesis , the data is normal distributed, skewness is *0* and kurtosis is *3*. The JB test follows a *- distribution* with two degrees of freedom.

Testing for normality is crucial in the context of Mean-Variance Portfolio (MVP) optimization, as the Markowitz framework assumes normally distributed returns. This assumption allows risk and return to be fully described by the mean and variance. Violations of normality — such as skewed distributions or excess kurtosis — may lead to underestimating risk and poor asset allocation.

Based on a significance level of 5% () and the provides p-values from the Jarque-Bera test, we can determine whether to reject or retain the null hypothesis of normality for each stock.

|  |  |  |
| --- | --- | --- |
| **Asset** | **p-value** | **Interpretation** |
| AAPL | 0.45376205 | Normality can be assumed |
| MSFT | 0.90774303 | Normality can be assumed |
| TSLA | 0.44097407 | Normality can be assumed |
| ORSTED.CO | 0.08249381 | Normality can be assumed |
| V | 0.487308 | Normality can be assumed |

For all five assets, the cannot be rejected at the 5% significance level. This means there is **no strong evidence against normal distribution**, and thus the use of **Mean-Variance Optimization** under the assumption of normally distributed returns is statistically justified in this context.

1. Building the Minimum Variance Frontier

The minimum variance frontier represents the sets of all possible portfolios, that offer the lowest level of risk for a given expected return value. It is a fundamental concept in the Modern Portfolio Theory and a crucial step for finding the optimal tangency portfolio.

To construct the frontier, the portfolio variance is minimized subject to different level of expected returns. This is done by combining our assets with different risk and return characteristics while considering their covariances.

The optimization problem is defined as:

under the following conditions:

ensuring that the expected return is accomplished and *100%* of the investment capital is invested.

This results in the follwoing Constrained Mai