End-to-End Fairness Optimization with Fair Decision Focused Learning

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Many real-world applications require algorithmic decisions based on predictions, where fairness must be considered throughout the entire decision-making pipeline. Traditional two-stage approaches that handle fairness separately in prediction and optimization stages can lead to misalignment between prediction fairness and decision fairness. We present Fair Decision Focused Learning (FDFL), an end-to-end framework that optimizes predictions and decisions while incorporating fairness at both stages. Our framework defines utility functions with explicit fairness considerations and uses alpha-fairness measures to evaluate decision outcomes. For linear utility functions, we derive closed-form solutions that enable efficient gradient-based training. For general alpha-fairness measure we leverage gradient-free methods adapted for fairness optimization. Through experiments on healthcare resource allocation, we show FDFL can achieve better fairness outcomes compared to two-stage approaches. Our results show the importance of considering fairness holistically across the prediction-to-decision pipeline and provide a practical framework for applications requiring both accurate and fair algorithmic decisions.

CCS Concepts: • **Do Not Use This Code → Generate the Correct Terms for Your Paper**; *Generate the Correct Terms for Your Paper*; Generate the Correct Terms for Your Paper; Generate the Correct Terms for Your Paper.

Additional Key Words and Phrases: Do, Not, Us, This, Code, Put, the, Correct, Terms, for, Your, Paper

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1 Introduction

Many real-world applications require consideration of fairness throughout the chain of decisions. When public health agencies distribute scarce medical resources to healthcare facilities in different regions, they need to forecast demands then develop allocation plans accordingly. Fairness in forecasts ensure that demands from disadvantaged regions are not overlooked, while fair allocations provide appropriate prioritization among all facilities. Similarly, the planning and distribution of other social resources, such as urban transportation and education funding, encounter fairness challenges from understanding heterogeneous needs and attaining a fair distribution of benefits. Another domain, where a holistic approach to fairness is necessary, is hiring. In the complicated process of sourcing candidates, screening applications, and choosing candidates to interview and hire, bias and unfairness in any step could perpetuate systematic inequalities and undermine organization diversity. These examples highlight the value of end-to-end fairness integration across the entire decision process.

Drawing motivation from the above examples, we study a generic decision process that consists of a prediction task and an optimization task. The prediction task uses data to forecast uncertain or unknown quantities, which are needed to formulate the optimization model for choosing the optimal actions. To describe the process from the other direction,

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the outcomes are determined by the optimal decisions, which depend on the predictions and underlying data. The prediction task should be free of unfair discrimination, and the optimization task should pursue the desirable equity performance in the final outcomes. A conventional strategy addresses these two tasks sequentially: predictions are made first and then input into the optimization model. In two-stage predict-then-optimize (PTO) approaches, fairness can be handled independently in each task, that is, we apply fair prediction followed by fair optimization. There is a large number of fair machine learning algorithms for seeking group-level or individual-level fairness in predictions [28]. In optimization models, fairness measures capturing various equity perspectives can be formulated as objectives or constraints [9].

These separate fairness handling methods overlook the potential interconnection and misalignment of fairness from different decision steps. For example, optimizing fair decisions based on fair predictions may still generate unfair outcomes. Consider a policymaker allocating a new type of healthcare service to local communities, where rural areas tend to have lower demands than urban areas due to a historical lack of healthcare access and awareness. Moreover, such historical bias exist in data and causes demand predictions without any fairness consideration to underestimate some rural areas' demands. In this problem, a fair prediction model will try to increase the demand forecasts for rural areas to reduce disparity. However, these forecast increases would cause some rural areas to be less prioritized than their actual levels, thus perpetuating the systematic inequity in healthcare access between rural and urban areas. To seek fair outcomes, a more desirable alternative is to integrate fair prediction and fair optimization in an end-to-end manner to link prediction fairness directly to decision fairness.

In this paper, we formulate end-to-end fairness optimization (E2EFO) as a paradigm for making data-driven fair decisions. E2EFO aims to optimize fairness in decisions generated based on predictions. We develop fair decision focused learning (FDFL) as an integrated approach to E2EFO. The FDFL framework builds upon standard decision focused learning (DFL), an end-to-end machine learning approach that directly trains a prediction model to optimize downstream decision accuracy [16, 19, 42]. DFL is an alternative to the conventional two-stage PTO approaches, where a prediction model is trained separately from the decision optimization process to optimize the predictive accuracy. In contrast, DFL emphasizes obtaining accurate decisions, measured in terms of the decision objective, despite possible prediction errors. Recent works have revealed limitations of PTO approaches, for example, [16] demonstrated the possible mismatch between prediction and decision goals and argued that prediction-focused methods are prone to model selection bias. [41] further conceptualized the gaps between good predictions and good decisions due to various factors including treatment effect heterogeneity and feedback loops between decisions and predicted outcomes. The integrated structure of DFL methods mitigates these limitations and supports an improved alignment between prediction accuracy and decision accuracy.

Our FDFL framework considers fairness in end-to-end decisions. In FDFL, prediction fairness focuses on eliminating undesirable disparity in forecasts, and is formulated with constraints or regularizers in terms of the predictor model. We note that all fairness formulations in existing in-processing fair supervised machine learning methods [28] are applicable to capture prediction fairness in FDFL. Decision fairness characterizes the equity goal of decision outcomes, and is represented in the objective function of the optimization task. More specifically, the optimization objective is a utility-based fairness measure assessing the desirability of a decision's utility distribution with respect to the underlying fairness perspective. Such fairness measures are broadly applied in operations research and mechanism design [9, 21]. FDFL aims to learn a prediction model to optimize fair decisions. Compared with standard DFL, FDFL aligns prediction and optimization performances in terms of both accuracy and fairness.

We make the following contributions in this paper. First, we formulate the E2EFO problem, and develop the FDFL framework to find optimal fair decisions. Second, we propose gradient-based algorithms for FDFL training. In particular, we utilize the closed-form solution to a specific class of alpha fairness optimization model to design a highly efficient specialized FDFL algorithm for this class of problem. Additionally, we present a perturbation based gradient approximation method to support FDFL with a general fair optimization decision model. Lastly, we demonstrate the benefits of FDFL in comparison with traditional two-stage approaches through a stylized example for theoretical intuition and real data based experiments for empirical insights.

The rest of the paper is organized as follows. Section 2 reviews the related works on DFL versus two-stage PTO, and fairness methods in machine learning and optimization. Section 3 describes the problem formulation to E2EFO and formally presents the FDFL framework. Section **??** discusses training algorithms to implement FDFL. In Section 5, we apply FDFL and two-stage PTO with fairness consideration on a real application motivated medical resource allocation problem. Besides comparing the performances between FDFL and PTO, we also compare among different FDFL algorithms to understand their runtime, training and decision performances. Lasty, Section 6 concludes the paper and discusses future directions to investigate.

2 Related Works

2.1 Predict-then-Optimize and Decision-Focused Learning

Data-driven decision-making frequently encounters scenarios where optimization models depend on parameters that must be estimated from data. Two primary paradigms have emerged to address this challenge: Prediction-Focused Learning (PFL), also known as Predict-then-Optimize (PTO) and Decision-Focused Learning (DFL), also known as End-to-End Learning (E2E). The conventional PTO approach follows a two-stage process, first employing machine learning to predict unknown parameters from relevant features, then using these predictions as deterministic inputs to optimize decisions. While PTO aligns well with classical stochastic optimization [6], it has fundamental limitations arising from its segregated structure. Most critically, the prediction stage operates independently of downstream decision-making, potentially leading to suboptimal decisions even when predictions appear accurate [19, 36, 42]. DFL addresses these limitations by integrating prediction and optimization into a unified framework. By embedding the optimization problem directly into the predictive model's training process, DFL enables the prediction component to anticipate its impact on final decisions. The key challenge in DFL lies in computing gradients through the optimization procedure to train the predictive model.

Three main approaches have emerged to address this challenge: differentiation through optimization, surrogate optimization, and surrogate loss functions. Early DFL methods focused on differentiation through optimization, primarily dealing with linear programming models in the downstream decision task [5, 19, 42]. These approaches have since expanded to handle quadratic programming [1, 3] and general nonlinear optimization [**?**]. Recent innovations include Negative Identity Backpropagation [38] and Perturbation Gradient Loss [23], which provide more efficient ways to compute gradients through optimization procedures. The challenge of non-differentiability in optimization has led to the development of surrogate loss approaches. Smart Predict-then-Optimize loss [19] and Noise Contrastive Estimation [31] provide convex surrogates that upper bound the decision regret.

More recent advances include Learning Optimal Decision Losses (LODL) [**?**], which learns instance-specific surrogate losses but faces computational challenges due to its per-instance training requirement. The Landscape Surrogate (LANCER) model [45] addresses these limitations by learning a global surrogate that can generalize across problem instances, significantly reducing computational overhead while maintaining performance. Additionally, the recently proposed Locally Convex Global Loss Network (LCGLN) offers a novel approach using partial input convex neural networks to guarantee convexity for chosen inputs while maintaining non-convex structure where needed.

Research comparing PTO and DFL has yielded important theoretical insights about their relative performance. Cameron et al. [8] demonstrated that DFL's advantages stem from its ability to adaptively handle stochastic prediction targets, while PTO must make an a priori choice about which statistics of the target distribution to model. They showed that the performance gap between PTO and DFL is closely related to the price of correlation in stochastic optimization, and identified scenarios where PTO can perform unboundedly worse than DFL, particularly when multiple prediction targets are combined to obtain each objective function coefficient. Elmachtoub et al. [20] revealed when the model class is well-specified and data is sufficient, PTO can outperform integrated approaches (such as DFL) in terms of regret stochastic dominance. These theoretical results have useful implications for our FDFL framework. Since fairness objectives often involve multiple interrelated predictions and typically operate under some degree of model mis-specification, e.g., due to historical biases in training data, the theoretical advantages of DFL may be particularly relevant. Additionally, as the tension between prediction accuracy and fairness creates additional complexity not captured in existing theoretical analyses, our work lays foundation for further theoretical study related to end-to-end fairness.

Compared to existing literature, our work introduces two key advancements. First, we study fairness decision objectives that are typically nonlinear, and explore the applicability of state-of-the-art DFL algorithms to handle nonlinearity. Second, while standard DFL addresses a trade-off between prediction accuracy and decision accuracy, our FDFL framework connects prediction and decision quality along the dimension of both accuracy and fairness.

2.2 Fairness in Decision Making

Fairness in decision-making has been extensively studied, with growing interest in integrating fairness into machine learning (ML) and optimization. These fields are characterized by different fairness definitions and goals. While fair ML methods focus on achieving parity in predictions to remove discriminative bias against groups or individuals, fair optimization models emphasize equitable outcomes and impacts as measured by utilities. Among the vast literature, we next review selected works in each direction to give a necessary overview, and highlight papers that are more relevant to end-to-end fair decision-making.

Statistical fairness metrics dominate the field of fair ML. These metrics aim to ensure decisions from ML models, e.g., predictions, are free from discrimination against protected groups. Examples of widely studied measures include demographic parity [17], equalized odds [22], accuracy parity [4], predictive rate parity [24] and individual fairness [17]. Majority of fair ML methods aim to address biases in standard ML models through pre-, in-, or post-processing techniques. Pre-processing methods modify input data to eliminate potential (e.g., [7, 44]). In-processing methods incorporate fairness during model training by including fairness components as constraints or objective regularizers (e.g., [15, 34, 43]). Lastly, post-processing methods adjust model outputs to attain desirable fairness (e.g., [2, 22]). The survey paper [28] provides a comprehensive review of fairness definitions and techniques in machine learning.

Fairness in optimization typically employs utility-based metrics grounded in social welfare theory [9]. This is a primary methodology to make fair decisions in operations research and mechanism design. Utility values capture the benefits or costs people associate from decisions of interest, and utility-based fairness metrics evaluate the desirability of utility distributions. There are three broad categories of metrics capturing different fairness perspectives. First, equality can be viewed as a proxy of fairness. Common metrics reflecting this perspective are inequality metrics, such as Gini index [13], which measure the disparity in utility outcomes. The second category emphasizes fairness for the disadvantaged, and the best known definition is the Rawlsian fairness criteria [37] that seek to prioritize people with lower utilities. The third category reflects a combined view balancing fairness and efficiency, which focus solely on optimizing the overall utilities regardless of individual differences. A popular combined metric is α-fairness [29], which can characterize the full range from Rawlsian fairness to efficiency. Applications of fairness optimization span various domains, such as, fair resource allocation in telecommunication networks [27, 33], balancing fairness and efficiency in assigning projects to university students [10], fair food bank operations [18] and disaster preparation [30] in humanitarian operations.

Along with the significant progress in both directions, recent works call for integrating ML and optimization perspectives to address outcome-centric fairness. One key motivation is the potential insufficiency of relying on fair predictions alone to ensure equitable outcomes and impacts. For example, [26] looked into the delayed impacts of fairness in ML, and demonstrated that inserting fairness, which aims to benefit certain protected groups, in ML model does not guarantee long-term improvements for the targeted groups. [39] proposed a conceptual framework distinguishing the roles of prediction modeler and decision maker in a decision-making system, and argued the importance of a holistic approach for embedding fairness throughout the entire system. Another strong motivation for bridging the two perspectives is that many practical problems naturally require both ML and optimization. The position paper [21] provide a comprehensive discussion of opportunities for combining ML and mechanism design, an important application area of optimization, to address fairness in complex decisions. More concretely, [11, 12] explore the application of a consequentialist approach, where the fairness of a decision algorithm or policy is evaluated based on the produced real-world outcomes. [12] argued that traditional fairness definitions in ML could inadvertently harm the groups they intend to protect, which shares common ground with [26]. Building on the consequentialist principles, [11] presented a practical framework that include the elicitation of stakeholder preferences and the optimization of preference-informed policies. The authors also developed a contextual bandit algorithm to operationalize this framework.

The integration of fair ML and optimization aligns with the goal of end-to-end decision making, which indicates the potentials of DFL techniques in attaining end-to-end fairness. There have been a few recent works that explored the incorporation of fairness into DFL for the task of learning a fair ranking. For example, [25] included fairness requirement with constraints in the ranking decision model, and utilized the linearity of decision objectives to extend the DFL algorithm developed in [19]. In contrast, [14] defined a fair ranking decision model with the optimization of ordered weighted average functions, and proposed a training algorithm with customized forward and backward propagation computation. Our work extends this literature by presenting a general paradigm for fair end-to-end optimization through fair decision focused learning algorithms. Compared with the mentioned prior works, we do not restrict the problem setup and study a general utility-based fairness optimization model as the decision problem.

3 Problem Formulation: End-to-End Fairness Optimization

We consider a decision-making problem with n stakeholders indexed by $i \in [n] = \{1, \dots, n\}$. Let $\mathbf{d} = (d_1, \dots, d_n) \in \mathbb{R}^n$ denote the decision vector, where d_i represents the decision (e.g., resource allocation, selection probability) for stakeholder i. The decisions must satisfy certain constraints represented by a feasible region $\mathcal{S} \subseteq \mathbb{R}^n$. We assume the feasible region $\mathcal{S}$ is non-empty, compact, and convex. Each stakeholder i derives utility from their received decision according to a utility function $U_i : \mathbb{R} \to \mathbb{R}$, which characterizes i's overall gains and costs from decision. Let $\mathbf{u} = (u_1, \dots, u_n)$ denote the utility vector where $u_i = U_i(d_i)$. The fairness of decisions is evaluated using a fairness measure $W : \mathbb{R}^n \to \mathbb{R}$ that maps utility vectors to scalar values. Higher values of W indicate more desirable fairness properties. Formally, given W and $\{U_i\}_{i=1}^n$, the fairness of decisions $\mathbf{d}$ is computed as: $W(\mathbf{d}) = W(U_1(d_1), \ldots, U_n(d_n))$.

The decision maker's objective is to find decisions that maximize fairness while satisfying feasibility constraints:

$$\max_{\mathbf{d}} \quad W(\mathbf{d}) \text{ s.t. } \quad \mathbf{d} \in \mathcal{S} \tag{1}$$

We assume W is concave in $\mathbf{d}$, which is satisfied when W is a concave function and each utility function U_i is concave and non-decreasing in d_i, or when W is a convex function and each utility function U_i is convex and non-increasing in d_i. When this model is fully specified, solving for the optimal decision is straightforward. We consider the case where some parameters of the decision model are not directly available and need to be estimated using data. Let $\mathbf{r}$ denote unknown parameters, and we suppose these parameters only exist in the decision objective. For clarity, we denote the objective function as $W(\mathbf{d}; \mathbf{r})$ to emphasize its dependence on both the decision variables and the unknown parameters. Note that the feasible region $\mathcal{S}$ does not depend on $\mathbf{r}$. In the *full information* case where $\mathbf{r}$ is known, we represent the optimal solution to (1) with $\mathbf{d}^*(\mathbf{r})$, which is the true optimal fair decision. In the *limited information* case, we aim to make a data-driven decision, $\hat{\mathbf{d}}$, that is as close to the true decision as possible. To generate this decision, we can follow a **separate view** to first predict the unknown parameters with $\hat{\mathbf{r}}$ then solve the decision optimization model to obtain $\hat{\mathbf{d}} \coloneqq \mathbf{d}^*(\hat{\mathbf{r}})$, or adopt a **direct view** to predict $\hat{\mathbf{d}}$ without explicit parameter estimation.

Let $\mathcal{D} = \{\mathbf{x}_1, \ldots, \mathbf{x}_n\}$ represent available data that contain features relevant to $\mathbf{r}$ or $\mathbf{d}^*(\mathbf{r})$. Due to potential historical biases in $\mathcal{D}$ and the underlying association with parameters or decisions, estimation of $\hat{\mathbf{r}}$ or $\hat{\mathbf{d}}$ using this dataset might perpetuate or amplify unfairness. Let F represent prediction fairness criteria to incorporate. Besides optimizing the utility-based fairness in decision, we also require prediction models to attain satisfactory performance in terms of F, e.g., if F measures a type of undesirable disparity in $\hat{\mathbf{r}}$, then $F(\hat{\mathbf{r}})$ should be sufficiently low. So far, we have formulated the **end-to-end fairness optimization** (E2EFO) problem. Next, we describe two-stage PTO and FDFL as two distinctive frameworks for solving E2EFO.

A two-stage PTO approach, following the separate view, first estimates unknown parameters and then optimizes decisions. The prediction task uses $\mathcal{D}, \mathbf{r}$ as training data to learn a prediction model f_θ parameterized by $\theta \in \Theta$. Given a feature vector $\mathbf{x}_i$, the model produces parameter estimates $\hat{\mathbf{r}}_i \coloneqq f_\theta(\mathbf{x}_i)$. To train a predictor while accounting for fairness, all fair supervised ML methods, as reviewed in Section 2, are applicable. We focus on in-processing methods that modify standard ML models with fairness components. Let $L(\theta)$ denote the standard loss function, e.g., mean squared error in regression, and $F(f_\theta(\mathbf{x}), \mathcal{D})$ capture disparity or unfairness to reduce. The training problem can be formulated with a fairness constraint, i.e., $\min_{\theta \in \Theta} L(\theta)$ s.t. $F(f_\theta(\mathbf{x}), \mathcal{D}) \le \epsilon$ where ϵ is a predefined tolerance level, or fairness in regularization, i.e., $\min_{\theta \in \Theta} L(\theta) + \lambda F(f_\theta(\mathbf{x}), \mathcal{D})$ where the hyperparameter $\lambda > 0$ regulates the trade-off between prediction accuracy and fairness. After training f_θ, decisions are obtained by solving for $\mathbf{d}^*(\hat{\mathbf{r}})$ from (1).

3.1 Fair Decision Focused Learning

We propose Fair Decision Focused Learning (FDFL) as an end-to-end framework that directly optimizes decision quality while accounting for fairness. FDFL aims to generate fair decisions $\hat{\mathbf{d}}$ that are close to the unknown true optimal fair decisions, $\mathbf{d}^*(\hat{\mathbf{r}})$. Higher decision accuracy provides better fairness performance, as measured by $W(\hat{\mathbf{d}}; \mathbf{r})$. Within FDFL, there are two primary ways to integrate fair prediction as part of fair optimization.

Separate View. FDFL uses the training data $(\mathcal{D}, \mathbf{r})$ to learn a parametric predictor f_θ for estimating $\hat{\mathbf{r}} \coloneqq f_\theta(\mathbf{x})$. The final decisions are determined separately by solving (1) with $\hat{\mathbf{r}}$ plugged in as parameters. The training model needs to reflect the goal of seeking decision accuracy, and there are different ways to define the training loss function. First, a training model can seek a direct maximization of the decision objective, namely, to minimize the training loss $L_{decision}(\theta) = -W(\mathbf{d}^*(f_\theta(\mathbf{x})); \mathbf{r})$. We can also define the training problem to minimize the decision regret, $L_{regret}(\theta) = W(\mathbf{d}^*(f_\theta(\mathbf{x})); \mathbf{r}) - W(\mathbf{d}^*(\hat{\mathbf{r}}); \mathbf{r})$, or minimize the mean square error between decisions, $L_{error}(\theta) = \|\mathbf{d}^*(f_\theta(\mathbf{x})) - \mathbf{d}^*(\hat{\mathbf{r}})\|$. It is easy to observe that all three training losses become zero when decisions are fully accurate, namely $\mathbf{d}^*(f_\theta(\mathbf{x})) = \mathbf{d}^*(\mathbf{r})$. We also remark that standard DFL methods have been proposed using these training loss definitions [40].

Let $L_{DFL}(\mathbf{d}^*(f_\theta(\mathbf{x})))$ denote one of the above loss functions. In addition, we again use $F(f_\theta(\mathbf{x}), \mathcal{D})$ as a prediction fairness criterion representing disparity or unfairness to reduce. Then training under a separate view involves finding optimal θ in the following models:

$$\begin{aligned} \text{Prediction fairness as constraint: } & \max_{\theta\in\Theta} L_{DFL}(\mathbf{d}^*(f_\theta(\mathbf{x}))) \text{ s.t. } F(f_\theta(\mathbf{x}), \mathcal{D}) \le \epsilon; \\ \text{Or prediction fairness in regularization: } & \max_{\theta\in\Theta} L_{DFL}(\mathbf{d}^*(f_\theta(\mathbf{x}))) - \lambda F(f_\theta(\mathbf{x}), \mathcal{D}). \end{aligned} \tag{2}$$

Direct View. Alternatively, following the direct view, FDFL learns from the training data $\mathcal{D}$, $\mathbf{d}^*(\mathbf{r})$ (a solver of (1) is required to generate $\mathbf{d}^*(\mathbf{r})$) to obtain a decision predictor M_θ, from which $\hat{\mathbf{d}} \coloneqq M_\theta(\mathbf{x})$. The previous training loss definitions also apply in this setup. For example, the decision objective loss can be written as $L_{decision}(\theta) = -W(M_\theta(\mathbf{x}); \mathbf{r})$. We use $L_{DFL}(M_\theta(\mathbf{x}))$ to denote a training loss function. Training under a direct view solves the following models.

$$\begin{aligned} \text{Prediction fairness as constraint: } & \max_{\theta\in\Theta} L_{DFL}(M_\theta(\mathbf{x})) \text{ s.t. } F(M_\theta(\mathbf{x}), \mathcal{D}) \le \epsilon; \\ \text{Or prediction fairness in regularization: } & \max_{\theta\in\Theta} L_{DFL}(M_\theta(\mathbf{x})) - \lambda F(M_\theta(\mathbf{x}), \mathcal{D}). \end{aligned} \tag{3}$$

This approach bypasses explicit parameter estimation. The training problem can be viewed as fitting a *policy* or *decision map* that yields the best fair decisions from the input features. As illustrated in standard DFL literature, e.g., [45], M_θ acts as a smooth surrogate function to approximate the optimal decision objective, and is especially useful to support efficient end-to-end learning when the original optimization (1) is expensive to solve or has non-differentiable solutions.

Compared to PTO that emphasizes accuracy of $\hat{\mathbf{r}}$, FDFL prioritizes accuracy of prediction-based decisions $\hat{\mathbf{d}}$ with respect to the true decisions $\mathbf{d}^*(\mathbf{r})$. Although both frameworks could integrate the same prediction and decision fairness components, their different training models lead to different fairness performances. In Section 3.3, we present an example where two-stage PTO is insufficient to attain satisfactory decision fairness. Before this example, we explain further details about defining the decision fairness objective.

3.2 Utility-based Decision Fairness

A decision, such as resource allocation, provides benefits or harms to the decision recipient. The overall impact can be quantified by a utility function. The definition of utility functions varies with the decision contexts to reflect the needed impact assessment. We construct a generic class of utility functions that unify a broad range of utility specifications commonly studied in literature. As discussed in the problem formulation, d_i denotes the decision received by stakeholder i. In addition, we suppose a stakeholder is characterized by three values: $q_i \in [0, 1]$, the level of need or desert for the decision of interest; $g_i \in \mathbb{R}_{\ge 0}$, the utility gain rate for the received decision; $a_i \in \mathbb{R}_{\ge 0}$, the base utility level before receiving any decision. Note that q_i accounts for all factors influencing how strongly the stakeholder requests, desires or needs the decision. For example, in the allocation of medical treatment resources, patients with higher medical risks would have higher desert levels. The other two values, g_i and a_i, quantify a stakeholder's utilities by distinguishing the decision-induced change and the starting position. Variations in these values reflect stakeholders' inherent differences. To continue with the previous example, after receiving the same medical resource, two patients with different health levels may experience different health improvements.

For a fixed decision, it should provide a greater utility for stakeholders who need or deserve the decision more strongly (i.e., a larger q_i), gain utilities more effectively (i.e., a larger g_i) or are better-off to start with (i.e., a larger a_i). Following this intuition, we compute i's utility from decision d_i as:

$$U_i(d_i) = q_i(g_i d_i + a_i)^{\rho}. \tag{4}$$

In this function, $\rho \in \mathbb{R}_+$ regulates the shape and rate of utility changes. When $\rho < 1$, the utility shows diminishing return with respect to decision, that is, utility increases more slowly as the decision amount increases. When $\rho = 1$, the utility function is linear, namely, a constant return rate from decision. When $\rho > 1$, the utility has increasing return. While we focus on a single decision for each stakeholder, we remark that this utility definition can be generalized to represent multiple decision types. For recipient i, let $\mathbf{d}_i = (d_i^1, \ldots, d_i^m)$ denote i's assigned decisions for all m types. Such heterogeneous decisions can characterize the allocation of multiple resources. The need/desert level, utility gain rate and base level for a recipient similarly extend to vectors: $\mathbf{q}_i = (q_i^1, \ldots, q_i^m)$, $\mathbf{g}_i = (g_i^1, \ldots, g_i^m)$, $\mathbf{a}_i = (a_i^1, \ldots, a_i^m)$. The power parameter ρ also can vary across decision types. The general utility definition for recipient i becomes: $U_i(\mathbf{d}_i) = \sum_{j=1}^{m} q_i^j (g_i^j d_i^j + a_i^j)^{\rho_j}$.

Given a utility distribution $\mathbf{u}$ generated from decisions $\mathbf{d}$, a fairness measure W aggregates utility values into a scalar indicator of the desirability of $\mathbf{u}$ with respect to fairness. As reviewed in Section 2, there are many fairness measures that fall into three broad categories: fairness via equality, fairness for the disadvantaged, and combined measures balancing fairness and efficiency. Recall that we focus on W that gives a concave function of $\mathbf{d}$, which requires W to be convex or concave in $\mathbf{u}$ depending on the shape of utility functions.

Here, we highlight one class of concave fairness measures from literature. α-fairness is a widely studied fairness measure in optimization and mechanism design that balances fairness and efficiency. When $\alpha = 0$, the metric characterizes pure efficiency, as it seeks to maximize the total utility without consideration for fairness. The function assigns greater emphasis on fairness as α increases. At the other extreme when α approaches infinity, α-fairness reduces to the Rawlsian maximin fairness criterion, which prioritizes the worst-off utility to be maximized.

$$W_\alpha(\mathbf{u}) = \begin{cases} \frac{1}{1-\alpha} \sum_{i=1}^{n} u_i^{1-\alpha}, & \text{if } \alpha \geq 0 \text{ and } \alpha \neq 1; \\ \sum_{i=1}^{n} \log(u_i), & \text{if } \alpha = 1. \end{cases} \tag{5}$$

The general compatibility with E2EFO does not mean all fairness measures are practical. As we will discuss further in Section **??**, different fairness objectives lead to decision-making models of varying complexity, thus affecting the difficulty of designing FDFL algorithms. In a fairness measure that serves as the decision objective, all unknown parameters are included in $\mathbf{r}$. There may be multiple groups of unknown parameters. For instance, when specifying utility functions of resource allocation decisions, it is possible we do not know stakeholders' exact need levels and utility gain rates, but have access to stakeholder features that can be used to estimate these values.

3.3 An Illustrative Example

We use a simple example to illustrate the prediction and decision performance differences between two-stage versus end-to-end methods. Suppose 50 units of resources need to be distributed among 15 people belonging to two groups.

Group 1, as the majority group, consists of 10 people. Group 2, as the minority group, has 5 people. Each person is characterized by a single feature $x_i \in [0,1]$, a need level q_i and a utility gain rate $g_i \in [0,1]$ for the resource. We generate the data so that Group 2 is the disadvantaged group. To give a concrete context, we can consider the resource as budget to take physical exams, $\{x_i\}$ as people's age, $\{q_i\}$ as overall health risks (older people tend to have higher risks). Due to historical bias, current data tend to under-report the risks of group 2 at higher age and over-report at younger age.

From receiving resource d_i, person i's utility is defined as $u_i = q_i g_i d_i$. The decision maker aims to allocate all resources to maximize a α-fairness objective with $\alpha = 2$ to reflect a strong emphasis on fairness over efficiency subject to the resource capacity constraint. Namely, the decision optimization model can be stated as: $\max_{\mathbf{d}} \frac{1}{1-2} \sum_{i=1}^{n} u_i^{1-2}$ s.t. $u_i = q_i g_i d_i \ \forall i, \sum_{i=1}^{n} d_i \leq Q, \mathbf{d} \geq 0$. At the time of decision, the need levels $\{q_i\}$ is unknown, but can be estimated using the feature $\{x_i\}$. This means $\mathbf{r} \coloneqq \mathbf{q}$ in the example. For the prediction task to generate $\hat{\mathbf{r}}$, we consider a linear regression model for prediction, and quantify accuracy disparity, i.e., the difference between two groups' mean square error (MSE) of $\mathbf{r}, \hat{\mathbf{r}}$, as the prediction unfairness to reduce.

On this example, we compare four models for fitting the linear regression line. Two of the models adopt a two-stage approach to focus on prediction accuracy in regression: the first one (M1) runs a standard linear regression to minimize the MSE between $\mathbf{r}, \hat{\mathbf{r}}$, and the second one (M2) accounts for prediction fairness by adding accuracy disparity as a regularization term to prediction error. The other two models follow an end-to-end view to emphasize decision accuracy: the third model (M3) fits a regression line to minimize the MSE between $\mathbf{d}^*(\mathbf{r}), \mathbf{d}^*(\hat{\mathbf{r}})$ (i.e., L_{error}), and the last model (M4) adds the accuracy disparity regularization to the decision MSE. We note that the last model accounts for fairness in both decision and prediction. In Figure 1, the green and blue points display the features and the true need levels, and the lines show the regression results from all four models. As we expect, the standard linear regression model reaches the best prediction accuracy, and the end-to-end regression model without prediction fairness attains the highest decision accuracy (Table 1). We also highlight that accounting for both types of fairness in an end-to-end manner significantly benefits prediction performance while maintaining decision accuracy. In Figure 2, we observe additional decision differences between two-stage and end-to-end approaches. Both two-stage models carry over some discriminative bias in predictions to decisions, as the decision errors are higher for the disadvantaged group 2. Accounting for prediction fairness in M2 helps reducing the gap between groups, but is insufficient to fully remove the bias. In contrast, through optimizing decision fairness, both end-to-end models attain more accurate decisions for group 2 while getting closer to the true optimal fair decisions.

	M1	M2	M3	M4
$MSE(\mathbf{r}, \hat{\mathbf{r}})$	**1.283**	1.680	70.274	2.710
$MSE(\mathbf{d}^*(\mathbf{r}), \mathbf{d}^*(\hat{\mathbf{r}}))$	0.314	0.201	**0.186**	0.188

Table 1. Results from Four Linear Regression Fitting Models

4 Fair Decision Focused Learning Algorithms

To develop an FDFL algorithm, we must first specify its training formulation. The training model should reflect the goal of optimizing prediction-based decision objectives while maintaining fairness. There are different approaches to achieve this, broadly categorized into gradient-based and gradient-free methods. Gradient-based methods follow a direct approach of minimizing regret loss through gradient descent, where regret is defined as the difference between the full-information optimal objective value and the objective value realized by the prediction-based decision:

$$\min_{\theta} L_{\text{regret}} = \left[W\left(\mathbf{d}^*(\mathbf{r}), \mathbf{r}\right) - W\left(\mathbf{d}^*(\hat{\mathbf{r}}), \mathbf{r}\right) \right] + \lambda F(\cdot) \tag{6}$$

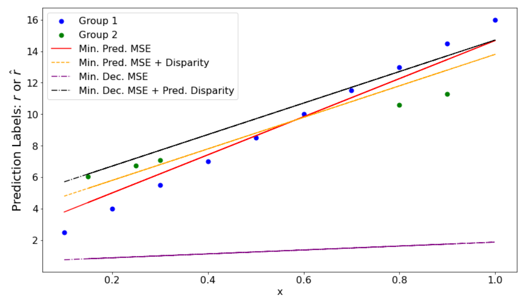

Fig. 1. Data points and Linear Regression Results

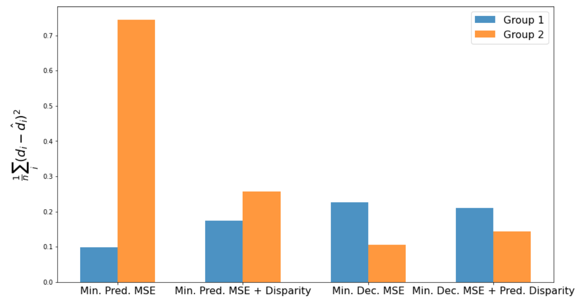

Fig. 2. Errors in Decisions based on Different Regression Models

where $F(\cdot)$ represents the fairness penalty. Note that minimizing regret is equivalent to maximizing $W(\mathbf{d}^*(\hat{\mathbf{r}});\mathbf{r})$ since $W(\mathbf{d}^*(\mathbf{r});\mathbf{r})$ is constant with respect to the prediction model. While regret is the quintessential task loss, other options exist. For example, when ground-truth data includes optimal decisions $\mathbf{d}^*(\mathbf{r})$, we can directly minimize $\|\mathbf{d}^*(\hat{\mathbf{r}}) - \mathbf{d}^*(\mathbf{r})\|^2$.

Alternative approaches that avoid gradient computation have also been developed, including surrogate loss functions, landscape learning, and ranking-based methods. In this work, we focus on gradient-based methods and we provide a detailed discussion of the alternative approaches in Appendix **??**.

4.1 Gradient-Based Methods

The key computational challenge lies in computing gradients through the optimization procedure. Using the chain rule:

$$\frac{\partial L_{Regret}(\hat{\mathbf{r}},\cdot)}{\partial \boldsymbol{\theta}} = \frac{\partial L_{Regret}(\hat{\mathbf{r}},\cdot)}{\partial \mathbf{d}^*(\hat{\mathbf{r}})} \cdot \frac{\partial \mathbf{d}^*(\hat{\mathbf{r}})}{\partial \hat{\mathbf{r}}} \cdot \frac{\partial \hat{\mathbf{r}}}{\partial \boldsymbol{\theta}}$$

The first term is the gradient of the regret loss with respect to the decision variables. The third term is the gradient of the predictions with respect to the model parameters, which can be handled by modern deep learning frameworks. The challenge lies in computing the middle term, which involves differentiating the optimal solution of the optimization problem. The core computational challenge in Algorithm 1 is differentiating through the solver, i.e., computing $\partial\, \mathbf{d}^*(\hat{\mathbf{r}})/\partial\, \hat{\mathbf{r}}$. This term often vanishes or is undefined if $\mathbf{d}^*(\cdot)$ is piecewise constant (typical for linear or discrete problems). Researchers have proposed numerous *surrogate* or *perturbation-based* methods for approximating these gradients as discussed in section 2. We address this using a finite difference approximation inspired by the Differentiable Black-box Optimizer (DBB) approach [35]. Given the current predicted parameters $\hat{\mathbf{r}}$, we introduce a perturbation:

$$\mathbf{r}' = \hat{\mathbf{r}} + \lambda \frac{\partial l(\hat{\mathbf{r}},\cdot)}{\partial \mathbf{d}^*(\hat{\mathbf{r}})} \tag{7}$$

and approximate the gradient as:

$$\frac{\partial \mathbf{d}^*(\hat{\mathbf{r}})}{\partial \hat{\mathbf{r}}} \approx \frac{1}{\lambda}(\mathbf{d}^*(\mathbf{r}') - \mathbf{d}^*(\hat{\mathbf{r}})) \tag{8}$$

where $\lambda > 0$ is a perturbation parameter that controls the approximation accuracy. This approach provides non-zero gradients that can guide optimization while requiring only two solver calls per gradient computation.

For certain special cases, we can avoid such approximations by deriving closed-form expressions for $\frac{\partial \mathbf{d}^*(\hat{\mathbf{r}})}{\partial \hat{\mathbf{r}}}$. We discuss this in Section 4.1.1 for a specific class of α-fairness problems. For completeness, we note that other gradient computation techniques exist in the DFL literature, including surrogate losses [19, 31] and gradient-free methods [45], which we summarize in Appendix **??**.

Algorithm 1 Fair Decision-Focused Learning with Gradient Descent

Require: Training set $\{(\mathbf{x}_i, \mathbf{r}_i)\}$ or $\{(\mathbf{x}_i, \mathbf{d}_i^*(\mathbf{r}_i))\}$;

1: decision objective $W(\mathbf{d}; \hat{\mathbf{r}})$;
2: fairness penalty $F(\theta)$; learning rate η.
3: Initialize predictor parameters θ.
4: **for** each training epoch **do**
5: Predict costs: $\hat{\mathbf{r}} \leftarrow f_\theta(\mathbf{x}_i)$.
6: Solve decision: $\mathbf{d}^*(\hat{\mathbf{r}}) \leftarrow \arg\max_{\mathbf{d}\in\mathcal{S}} W(\mathbf{d}; \hat{\mathbf{r}})$.
7: Evaluate loss: $\mathcal{L}(\theta) = -W\big(\mathbf{d}^*(\hat{\mathbf{r}}); \mathbf{r}\big) + \lambda F(\theta)$.
8: Backprop to get $\nabla_\theta \mathcal{L}$
9: Update $\theta \leftarrow \theta - \eta \nabla_\theta \mathcal{L}$.
10: **end for**
11: **return** Trained parameters θ.

4.1.1 Closed-Form Solutions and Analytical Gradients for Alpha Fairness. We consider the alpha-fairness optimization problem 5 in which each decision variable d_i contributes a linear utility $u_i = r_i g_i d_i$. The goal is to allocate a total budget Q among the n decision variables $\{d_i\}_{i=1}^n$ to maximize the alpha-fairness measure of these linear utilities. Formally, the problem can be stated as

$$\max_{\{d_i\}_{i=1}^n} \sum_{i=1}^n W_\alpha(r_i g_i d_i) \quad \text{subject to} \quad \sum_{i=1}^n c_i d_i \le Q \quad \text{and} \quad d_i \ge 0 \tag{9}$$

Throughout, we assume $\alpha > 0$ and $r_i, g_i, Q > 0$. Under these mild regularity conditions, one can show via Lagrangian duality and optimal conditions that this problem admits a closed-form solution d_i^*. The next proposition states the solution explicitly for the case $\alpha \neq 1$; a similar expression holds for $\alpha = 1$ with the logarithmic objective.

Proposition 4.1 (Closed-Form Solution for Alpha-Fairness Problem). *For the alpha-fairness maximization problem with linear utility $u_i = r_i g_i d_i$, the optimal solution d_i^* is given by:*

$$d_i^* = \frac{c_i^{-\frac{1}{\alpha}} \cdot (r_i g_i)^{\frac{1}{\alpha}-1} \cdot Q}{\sum_{j=1}^n c_j^{-\frac{1}{\alpha}} \cdot (r_j g_j)^{\frac{1}{\alpha}-1}}$$

The closed-form solution enables direct computation of gradients required in the training process. Specifically, we can derive the analytical expression for how the optimal decisions change with respect to changes in the unknown parameters.

Proposition 4.2 (Analytical Gradient of the Optimal Solution). *The partial derivative of the optimal decision d_i^* with respect to the parameter r_k is:*

$$\frac{\partial d_i^*}{\partial r_i} = \frac{Q\left(\left(-1+\frac{1}{\alpha}\right) c_i^{-\frac{1}{\alpha}} \cdot g_i^{\frac{1}{\alpha}-1} \cdot r_i^{-2+\frac{1}{\alpha}} \left(S - c_i^{1-\frac{1}{\alpha}} \cdot (r_i g_i)^{-1+\frac{1}{\alpha}}\right)\right)}{S^2}$$

where $S = \sum_{j=1}^n c_j^{1-\frac{1}{\alpha}} \cdot (r_j g_j)^{-1+\frac{1}{\alpha}}$.

[ADD PROOF in appendix] The closed-form solution in Proposition 4.1 and its analytical gradient (Proposition 4.2) enables fast gradient computation in equation (??).

5 Experiments

5.1 Problem Setting

We evaluate our framework on a healthcare resource allocation problem using a synthetic dataset generated from real-world medical risk prediction data studied in [32]. The dataset contains 48,784 patient records with demographic information, comorbidity indicators, historical healthcare costs, biomarkers, and program enrollment decisions. The original study revealed significant racial bias in commercial risk prediction algorithms: at equivalent risk scores, Black patients were substantially sicker than White patients but less likely to be identified for program enrollment.

In the current system, enrollment decisions follow a two-stage process: first, a commercial algorithm generates risk scores from medical data. Patients scoring above the 97th percentile are automatically enrolled, while those above the 55th percentile are referred to their primary care physician for enrollment consideration based on the risk score and additional contextual information.

We model their utility from program enrollment as: $u_i(d_i) = r_i g_i d_i$, where r_i is the patient's risk score, g_i is an unobserved gain factor representing how much the patient would benefit from enrollment, and d_i is the enrollment decision. The gain factor g_i is estimated using avoidable costs (mapped via nearest neighbor matching for patients lacking this data) and a logistic regression model trained on patient features to generate a propensity score that serves as a continuous proxy for the binary enrollment decision threshold (patients above the 55th percentile of risk scores). The decision optimization maximizes alpha-fair utility:

$$
\begin{aligned}
\max_{\mathbf{d}} \quad & W_\alpha(\mathbf{u}) = \begin{cases} \frac{1}{1-\alpha} \sum_{i=1}^{n} u_i^{1-\alpha}, & \alpha \geq 0, \alpha \neq 1 \\ \sum_{i=1}^{n} \log(u_i), & \alpha = 1 \end{cases} \\
\text{s.t.} \quad & d_i \geq 0, \quad \forall i \\
& \sum_i c_i d_i \leq Q
\end{aligned}
\tag{10}
$$

where c_i represents the cost of enrolling patient i and Q is the total resource budget. While this simplified formulation cannot capture all complexities of real-world medical decision-making, it serves to demonstrate the advantages of FDFL over traditional two-stage approaches in balancing accuracy and fairness.

5.2 Methods and Implementation

We compare two main approaches: traditional two-stage methods and our FDFL framework. For two-stage methods, we start with a basic predict-then-optimize approach and create variants by adding fairness components: fairness in prediction, fairness in optimization, and fairness in both stages. Similarly, we implement FDFL without fairness considerations and enhance it with prediction fairness, decision fairness, and both types of fairness.

Specifically, within FDFL, we utilize two gradient computation strategies:

(1) Differentiable Black-Box (DBB) : This finite difference approach approximates gradients by perturbing the predicted parameters.
(2) Closed-Form Gradient Computation: For specific cases where the optimization problem admits a closed-form solution, we derive exact gradients analytically.

Table 2. Performance Comparison Across Methods

Method	MSE (Mean ± Std)	Regret (Mean ± Std)	Fairness (Mean ± Std)	Time (s)
2-Stage No Prediction Fairness	16.87 ± 1.91	9.71 ± 0.40	1.742 ± 0.291	0.28
2-Stage With Fairness	16.49 ± 1.97	10.03 ± 0.49	1.398 ± 0.288	0.19
DFL No Fairness	23.07 ± 2.35	1.48 ± 0.24	1.329 ± 0.217	16.73
DFL With Fairness (Closed-Form)	23.03 ± 2.41	1.48 ± 0.25	1.304 ± 0.218	17.11
DFL With Fairness (DBB)	44.22 ± 4.44	4.38 ± 4.80	0.047 ± 0.032	38.50
DFL With Fairness (DBB 2)	45.58 ± 3.97	4.87 ± 0.67	0.204 ± 0.051	32.68

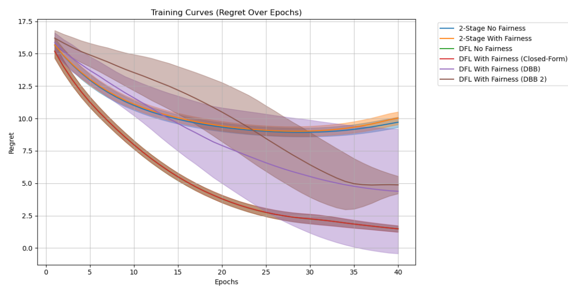

Fig. 3. Enter Caption

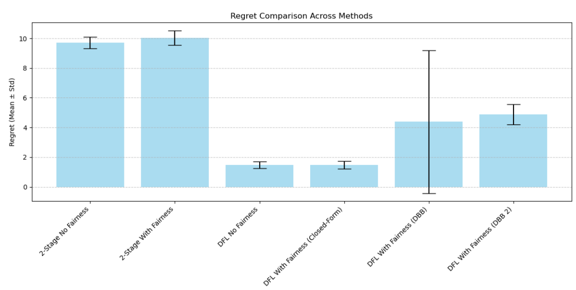

Fig. 4. Enter Caption

All methods use a neural network with two 64-unit hidden layers and ReLU activation. We incorporate prediction fairness through a regularization term with weight λ that penalizes statistical disparity between racial groups, and decision fairness through an alpha-fairness objective function. We evaluate each method through 10 trials with 50/50 train-test splits, using Adam optimizer with adaptive learning rate. For FDFL, we compute gradients using closed-form solutions when possible, or finite difference approximations otherwise. The implementations use PyTorch for neural networks and MOSEK solver for optimization problems, with hyperparameters tuned via grid search.

5.3 Results

We evaluate all methods across four key metrics: prediction accuracy (MSE), decision quality (normalized regret), fairness (statistical disparity), and computational efficiency. Table 1 summarizes the performance of each method averaged over 10 trials.

6 Conclusion

We tackle end-to-end fairness optimization through decision focused learning to provide holistic approaches for fair data-driven decision making.

We view our paper as the beginning of broad research scope of E2EFO and FDFL. There are important open questions in theory, algorithm and practical aspects. On the theory side, there lacks precise understanding of the performance differences between fair two-stage and FDFL approaches with respect to various fairness criteria, as well as between different FDFL algorithms. In algorithm development, we see great needs and potentials for efficient FDFL algorithms that utilize the problem structure of fair prediction and fair optimization. The convergence of algorithm is another critical challenge. Lastly, the applicability of E2EFO and FDFL should be investigated further in practical problems.

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