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End-to-End Fairness Optimization with Fair Decision Focused Learning

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Abstract. Many real-world systems rely on predictive models to inform algorithmic decisions, where fairness concerns can arise both in the prediction stage and in the resulting decisions. We introduce end-to-end fairness optimization (E2EFO) as a unifying framework that integrates fairness across the entire prediction-to-decision pipeline. We focus on resource allocation problems, where a fair prediction task aims to estimate the allocation impacts without disparity, and then a fair decision task seeks to equitably distribute these impacts. Within this framework, we propose fair decision-focused learning (FDFL), a training approach that jointly optimizes for better prediction fairness and decision regret, which measures the loss in decision fairness due to imperfect predictions. To enable gradient-based training, we develop three methods for computing gradients through the decision optimization problem, including exact closed-form gradients for a tractable class of α-fairness maximizing allocation problems and two gradient approximation methods for general fairness-optimizing decisions. In healthcare resource allocation experiments, FDFL consistently achieves lower decision regret and improves prediction fairness with minimal loss in decision quality. This work underscores the value of addressing fairness holistically in data-driven decision systems and offers a practical framework for improving procedural and outcome fairness in high-stakes applications.

Key words: Equity, Fair Machine Learning, Data-driven decisions

1. Introduction

Data-driven predictions increasingly inform real-world decisions in a wide range of critical domains. In healthcare, forecasts of medical needs guide the allocation of scarce resources such as care management programs for people with complex health needs (Obermeyer et al. 2019). In social services, the predicted risk of child maltreatment is considered in the screening of children for intervention (Chouldechova et al. 2018). In power systems, forecasts of renewable energy supply inform cost-effective scheduling and dispatch of electricity across the power grid (Chen et al. 2021). In finance, loan processing decisions need to account for the expected creditworthiness of loan applicants. These decision-making processes typically consist of two stages: a prediction task, which generates estimates of unknown parameters, and a subsequent decision task that uses these estimates as inputs.

Many real-world applications require fairness in prediction-based decisions. A growing body of literature distinguishes between considering fairness in both how predictions are obtained and how decisions are made (Paulus and Kent 2020, Kuppler et al. 2022, Scantamburlo et al. 2024). Prediction fairness seeks to eliminate discriminative or undesirable disparities in predictions, reflecting technical concerns about statistical properties of the predictor. Decision fairness aims to attain equitable decision outcomes, reflecting moral concerns about the justice of decisions. Kuppler et al. (2022) argue that both aspects should be jointly addressed to design fair and just decision systems, as they characterize independent concepts, that is, fair predictions do not guarantee fair decisions and vice versa.

Consider a simple example that highlights this insufficiency. Suppose a decision maker allocates a fixed amount of beneficial resources to recipients belonging to two groups: an advantaged group A and a disadvantaged group D. Each recipient has a demand for the resource and derives a benefit from receiving it. Practical examples of such settings include distributing public health services to local neighborhoods based on estimated health needs, or allocating financial aid to students based on projected educational return. A predictive model estimates individual benefits, which then guide the resource allocation. We assume that recipients in group A are more likely to have high benefits, and historical data contains systemic bias against group D.

A standard predictive model, trained on historical data to maximize accuracy, tends to be more accurate for group A and underestimates the benefits for members of group D. Based on these standard predictions, the decision maker can seek fairness by adopting a fair decision policy aimed at equalizing the total benefits received by the two groups. To compensate for the underestimation of benefits for group D, a fair policy tends to allocate disproportionately more resources to group D, even when their true benefits may not justify it. This compensatory over-allocation, however, can create reverse discrimination against group A. This outcome demonstrates that fair decisions alone may not correct the negative impacts of biased predictions.

Alternatively, the decision maker can seek fairness with a fair predictor that reduces underestimation for group D, possibly at the expense of introducing more noise into group A's predictions. If the decision maker then applies a standard decision policy aimed solely at maximizing total benefits across all recipients, the allocation will disproportionately favor group A, which contains more recipients with high predicted benefits. As a result, group D suffers from under-allocation, despite the more balanced prediction performances. In this case, addressing prediction fairness alone may be insufficient to ensure fair outcomes in final decisions.

Drawing motivation from these examples, we study a generic decision process that consists of a prediction task and a decision task. The prediction task uses data to forecast uncertain or unknown quantities, which are needed to formulate the optimization model for choosing decisions. The prediction task accounts for *prediction fairness* to mitigate or eliminate unfair bias or discrimination. The decision task seeks *decision fairness* to attain desirable equity performance in the decision outcomes. We refer to this as the End-to-End Fairness Optimization (E2EFO) problem.

A conventional strategy to solve an E2EFO problem addresses the two tasks separately: predictions are made first to attain desirable prediction accuracy and fairness, and then input into a fairness-embedded optimization model, which are known as the predict-then-optimize (PTO) method. This decoupled approach, however, overlooks critical interactions between the two stages. We continue with the previous example to discuss how the misalignment can persist even when fairness is considered in both stages. A fair predictor still incurs prediction errors that may be distributed unevenly. For example, if the fair predictor overestimates the benefit of some group A members and underestimates for some group D members, a fair decision process may still over-assign resources to A and under-assign to D to reduce the benefit gap between groups. Thus, despite intentions to promote fairness in both stages, the resulting allocation remains unfair to one group due to uncoordinated prediction error propagation.

To align prediction fairness with decision fairness in E2EFO, we adopt and extend the framework of Decision-Focused Learning (DFL), a machine learning paradigm that trains prediction models to directly optimize the quality of downstream decisions (Donti et al. 2017, Wilder et al. 2019, Elmachtoub and Grigas 2020). DFL departs from the conventional predict-then-optimize (PTO) approach by embedding the decision objective into the training of the predictor. We build on this foundation to propose Fair Decision-Focused Learning (FDFL), where predictions are generated with joint consideration of prediction and decision fairness. FDFL advances standard DFL in two key respects: (1) we explicitly integrate prediction fairness into the learning objective through regularization, and (2) we extend the framework to accommodate the nonlinear, fairness-oriented objectives common in resource allocation problems.

We summarize our main contributions as follows.

1. We propose end-to-end fairness optimization (E2EFO) as a unifying framework for incorporating fairness into prediction-informed decision-making. Within this general framework, we focus

on prediction fairness, defined in terms of disparities in prediction errors, and decision fairness, represented by the α-fairness of the resulting utility distribution from decisions.

2. We develop fair decision focused learning (FDFL) algorithms to train predictors within the E2EFO framework. FDFL jointly optimizes for low decision regret, which is the gap in the decision fairness objective between prediction-based decisions and true optimal decisions, and high prediction fairness. To enable gradient-based training, we derive exact gradients for a tractable class of decision problems and introduce two general gradient approximation methods for general fairness-maximizing optimization problems lacking a closed-form solution.

3. We demonstrate the performance of FDFL algorithms and fairness-embedded predict-then-optimize (FPTO) through a healthcare resource allocation experiment. Our results show that, when prediction and decision objectives are misaligned, FDFL substantially reduces decision regret compared to a strong FPTO baseline. We further show that integrating prediction fairness into FDFL leads to fairer predictions with minimal impact on decision quality.

The remainder of the paper is organized as follows. Section 2 reviews related works on fairness and decision-focused learning. Section 3 introduces the E2EFO problem and formalizes the FDFL method. Section 4 presents training algorithms for FDFL based on different gradient computation techniques. Section 5 reports experimental results on a healthcare resource allocation problem. Lastly, Section 6 concludes the paper and discusses future research directions.

2. Related Works

We review three lines of literature related to our study: (1) fairness in prediction and optimization models, which offer tools and metrics for defining fairness; (2) outcome-aware fair decision making, which emphasizes the importance of fairness in downstream outcomes and shares our high-level goals; and (3) predict-then-optimize (PTO) and decision-focused learning (DFL), which provide the methodological foundation for our algorithmic framework.

2.1. Fairness in Prediction and Optimization

Fairness has been extensively studied in both machine learning and optimization, though the two communities often pursue different fairness goals. Fair machine learning methods focus on achieving parity across groups or individuals in predictive models to reduce discriminatory biases. Commonly studied fairness criteria include demographic parity (Dwork et al. 2012), equalized odds (Hardt et al. 2016), accuracy parity (Berk et al. 2021), predictive rate parity (Kleinberg et al. 2016) , and individual fairness (Dwork et al. 2012). These goals have been operationalized through a range of techniques, including pre-processing methods that modify input data to eliminate potential bias (e.g., Zemel et al. (2013), Calmon et al. (2017)), in-processing methods that include fairness constraints or regularization during training (e.g., Zafar et al. (2019), Olfat and Aswani (2018), Donini et al. (2018)), and post-processing methods that adjust prediction outputs to attain desirable fairness (e.g., Hardt et al. (2016), Alabdulmohsin (2020)). The survey Mehrabi et al. (2021) provided a comprehensive review of fairness definitions and techniques in machine learning.

Fairness in optimization takes a complementary approach that emphasizes the fairness of decision outcomes, which are typically measured in terms of how utilities, capturing decision-related benefits or costs, are distributed among individuals or groups (Chen and Hooker 2023). Utility-based fairness metrics have been widely studied in operations research and mechanism design. Examples include measures of inequality (e.g., Gini coefficient (Dalton 1920)) that can be minimized to seek equality, Rawlsian max-min fairness (Rawls 1971) that emphasizes prioritizing disadvantaged individuals or groups, and combined metrics that seek to balance both fairness and efficiency (e.g., α-fairness (Mo and Walrand 2000)). Applications of fairness optimization span various domains, such as, facility location (Shehadeh 2023), organ allocation (Bertsimas et al. 2013), food rescue operations (Eisenhandler and Tzur 2019), infrastructure investment prioritization (Baghersad et al. 2025), and crew roster scheduling (Breugem et al. 2022).

2.2. Outcome-Aware Fair Decision Making

Recent works in both machine learning and optimization increasingly emphasize outcome-aware fairness, which seeks fairness in the real-world consequences of predictive and prescriptive decision-making systems. A central motivation is that fair predictions do not guarantee equitable long-term outcomes and impacts. For instance, Liu et al. (2018) showed that fairness in predictive models, which aims to benefit certain protected groups, does not necessarily lead to long-term improvements for the targeted groups. Similarly, Corbett-Davies et al. (2023) found that imposing standard fairness requirements in machine learning models can inadvertently disadvantage the groups that they intend to protect. This perspective motivates frameworks that embed fairness throughout the full decision pipeline. Scantamburlo et al. (2024) proposed a conceptual separation between prediction modelers and decision makers, arguing for a holistic view that aligns their fairness goals in decision making.

An emerging line of research operationalizes outcome-aware fairness by directly learning fair decision policies. Kilbertus et al. (2020) argued that in selective label settings, learning decision policies can provide better utility and fairness performances than learning predictions followed by score-based decision rules. Tang et al. (2023) studied online resource allocation to sequentially arriving individuals, whose allocation outcomes are uncertain but can be learned from individual characteristics. The authors developed allocation policies that aim to optimize long-run outcomes while enforcing capacity and fairness constraints, and provided theoretical performance guarantees in both full-information and sample-based setups. Jia et al. (2024) investigated multi-stage selection problems, and proposed learning interpretable linear selection rules to optimize selection outcomes subject to fairness constraints. The core of their learning approach is using a mixed integer conic optimization problem to obtain asymptotically consistent solutions to an intractable chance-constrained model. Chohlas-Wood et al. (2024) formalized consequentialist algorithmic fairness, which evaluates algorithmic policies or decisions based on their real-world outcomes. Building on the consequentialist principles, the authors provided a policy learning framework that includes the elicitation of stakeholder preferences and the optimization of preference-informed policies.

A persistent gap in the literature is that these outcome-aware frameworks have not simultaneously addressed prediction fairness and decision fairness. Our work fills this gap by linking both aspects of fairness in end-to-end learning.

2.3. Predict-then-Optimize and Decision-Focused Learning

Prediction-based decision making is supported by two main paradigms: predict-then-optimize (PTO), also known as prediction-focused learning, and decision-focused learning (DFL). PTO methods first estimate unknown parameters to maximize estimation accuracy, then plug in these predictions to solve an decision optimization problem. Although PTO aligns with classical stochastic optimization (Bertsimas and Kallus 2020), it can suffer from misalignment between prediction and decision goals. Several works have shown that even accurate predictions may yield poor decisions if the prediction stage does not reflect the desirable optimization structure (Donti et al. 2017, Elmachtoub and Grigas 2020, Wilder et al. 2019, Qi and Shen 2022). Wang et al. (2024) further conceptualized the gaps between good predictions and good decisions due to various factors including treatment effect heterogeneity and feedback loops.

DFL addresses these limitations by integrating prediction and optimization into a unified pipeline. DFL trains the predictive model to directly optimize decision performance, enabling the prediction component to anticipate its impact on final decisions. Literature has studied DFL for a variety of decision tasks, including linear programming (Wilder et al. 2019, Elmachtoub and Grigas 2020, Berthet et al. 2020), quadratic programming (Amos and Kolter 2017, Agrawal et al. 2019), and general nonlinear optimization (Shah et al. 2022). Recent works continue to develop more scalable or general-purpose techniques, such as directional gradients (Huang and Gupta 2024), negative identity backpropagation (Sahoo et al. 2022), surrogate learning via noise contrastive estimation (Mulamba et al. 2020), and landscape surrogates (Zharmagambetov et al. 2023). We refer readers to Mandi et al. (2024) for a comprehensive survey of DFL methods.

Performance differences between DFL and PTO have been explored in theoretical studies. For example, Cameron et al. (2022) linked the performance gap between DFL and PTO to the price of correlation in stochastic optimization, and identified scenarios where PTO leads to optimal decisions and where PTO can perform unboundedly worse than DFL. Elmachtoub et al. (2023) proved that PTO performs better than DFL in terms of regret stochastic dominance, when the predictor model class is well-specified and data is sufficient, and the reverse result applies to the misspecified setting. Ho-Nguyen and Kılınç-Karzan (2022) identified conditions on prediction models that can guarantee high quality decisions, and provided a framework for checking these conditions. Their results are applicable to examine various predictor training methods, including both PTO and DFL, in prediction-based decisions. In addition to theoretical results, applications of DFL have demonstrated performance gains over PTO in practical contexts such as inventory management (Chung et al. 2022, Qi et al. 2023).

As growing research on DFL lays the foundation for integrating prediction and decision, some works have extended DFL to fairness-aware settings. Kotary et al. (2022) included fairness constraints in a ranking decision task, and leveraged linearity of the problem to develop a DFL method to learn fair rankings. Dinh et al. (2024) also studied fair ranking. They defined a fair ranking decision model with the optimization of ordered weighted average functions, and proposed a DFL training algorithm with customized forward and backward propagation computation. Our work extends this literature by presenting a general paradigm for fair end-to-end optimization through fairness-aware DFL algorithms. In contrast to the mentioned papers in fair ranking, we focus on fair resource allocation as the decision problem.

3. Problem Formulation: End-to-End Fairness Optimization

We consider a general resource allocation problem where a decision maker distributes limited resources among stakeholders. The decision maker aims to promote beneficial downstream impacts, such as improved access, health, or opportunity, through these allocations. Stakeholders' outcomes depend on the amount of resources they receive, and these impacts are typically not observable in advance to inform the allocation process. To address this challenge, we study a setting where predicted outcomes guide allocations in a way that balances effectiveness and fairness.

Suppose a decision maker allocates one type of resource to n stakeholders, indexed by $i \in [n] = \{1, \ldots, n\}$. Let $\mathbf{d} \in \mathbb{R}^n_{\geq 0}$ denote allocation decisions, where d_i represents the amount of resources given to stakeholder i. Allocation outcomes depend on how stakeholders are affected by the resources. For $i \in [n]$, we use a parameter r_i to capture the impact of the resource on stakeholder i, aggregating relevant benefits and burdens. The vector $\mathbf{r} \in \mathbb{R}^n$ denotes the impact parameters of all stakeholders, which are unknown at the time of decision-making.

We assume the decision maker has access to feature data $\mathcal{D} = \{\mathbf{x}_1, \ldots, \mathbf{x}_n\}$, where $\mathbf{x}_i$ describes stakeholder i. The **prediction task** uses $\mathcal{D}$ to estimate $\hat{\mathbf{r}} = f_\theta(\mathcal{D})$, parameterized by $\theta \in \Theta$, as a predictive approximation of $\mathbf{r}$. To mitigate potential biases in historical data or structural inequities in feature-to-parameter mapping, we include a prediction fairness criterion $F(\hat{\mathbf{r}})$, which quantifies disparities in the predictions. A fair predictor aims to keep $F(\hat{\mathbf{r}})$ small to reduce bias propagation into predictions.

Given predicted parameters $\hat{\mathbf{r}}$, the decision maker chooses an allocation to promote fairness in stakeholder outcomes. Each stakeholder i derives utility $u_i = U_i(d_i; \hat{r}_i)$, where $U_i : \mathbb{R} \to \mathbb{R}$ is a utility function characterizing i's overall gains from allocation. Thus, a decision vector $\mathbf{d}$ induces a utility distribution $\mathbf{u} = (u_1, \ldots, u_n)$. To evaluate the fairness of an allocation, we introduce a fairness measure $W : \mathbb{R}^n \to \mathbb{R}$ that maps a utility vector to a scalar score: $W(\mathbf{d}; \hat{\mathbf{r}}) = W(U_1(d_1; \hat{r}_1), \ldots, U_n(d_n; \hat{r}_n))$. Higher values of W reflect more equitable outcomes. The **decision task** solves the following optimization problem.

$$\max_{\mathbf{d}} \quad W(\mathbf{d}; \hat{\mathbf{r}}) \text{ s.t.} \quad \mathbf{d} \in \mathcal{S} \tag{1}$$

We assume that the feasible set $\mathcal{S} \subseteq \mathbb{R}^n$ is non-empty, compact, and independent of $\hat{\mathbf{r}}$. Let $\hat{\mathbf{d}} := \mathbf{d}^*(\hat{\mathbf{r}})$ denote the optimal solution to (1) with the predicted parameters. If the true parameters $\mathbf{r}$ were known, the optimal decision $\mathbf{d}^* := \mathbf{d}^*(\mathbf{r})$ could be obtained by solving (1) with $\mathbf{r}$ as input.

We refer to this two-stage prediction-informed decision-making process as the **end-to-end fairness optimization** (E2EFO) framework. The predictive model f_θ trained on $\{\mathcal{D}, \mathbf{r}\}$, which contains historical records of allocations and their observed impacts, is used to estimate stakeholder-specific impact parameters from features. Prediction quality can be evaluated using standard *prediction accuracy* (e.g., mean squared error) and *prediction fairness* $F(\hat{\mathbf{r}})$. While the training process relies on data where $\mathbf{r}$ is observed, in actual deployment, the true parameters $\mathbf{r}$ are not available and predictions $\hat{\mathbf{r}} = f_\theta(\mathcal{D})$ are used in the decision model (1) to obtain $\mathbf{d}^*(\hat{\mathbf{r}})$. Decision performance can be assessed by *decision accuracy* via how close $\mathbf{d}^*(\hat{\mathbf{r}})$ is to $\mathbf{d}^*(\mathbf{r})$ and *decision fairness* via W.

Together, these components define a closed-loop system in which predictions and decisions interact to shape fairness outcomes. Section 3.1 introduces the fairness metrics used in our framework. Section 3.2 presents learning-based approaches to solving the E2EFO problem, then Section 3.3 provides a stylized example to illustrate the performance differences among these approaches.

3.1. Fairness Definitions

The E2EFO framework accommodates a wide range of fairness definitions. In this section, we specify the fairness metrics used in our study. We distinguish between fairness in the prediction task and fairness in the decision task. For consistency, we adopt either group-based fairness or individual-based fairness in both tasks, although mixed perspectives are possible in practice.

Prediction Fairness. The prediction task should ensure that the estimation of stakeholder-specific parameters does not systematically disadvantage particular groups or individuals. We evaluate prediction fairness with accuracy disparity metrics, which quantify variability in prediction errors to capture inequality in the error distribution. Such metrics have been widely applied in fair regression methods (Berk et al. 2017, Agarwal et al. 2019).

In the group-based setting, stakeholders are partitioned into K groups $G_1, \ldots, G_K$, with group G_k contains g_k individuals. The mean squared error (MSE) in the predictions for group k is $MSE_k^{(t)} = \frac{1}{g_k}\sum_{i\in G_k}(r_i - \hat{r}_i)^2$, and $\overline{MSE} = \frac{\sum_{k=1}^K MSE_k}{K}$ is the average error of all groups. We apply the mean absolute deviation (MAD) as the measure of variability, and the group-based accuracy disparity is given by:

$$F(\hat{\mathbf{r}}) = \frac{1}{K}\sum_{k=1}^{K}|MSE_k - \overline{MSE}|. \tag{2}$$

In the individual-based setting, the prediction error for stakeholder i is $e_i = (r_i - \hat{r}_i)^2$, and the mean error is $\overline{e} = \frac{\sum_{i=1}^n e_i}{n}$. The mean absolute deviation (MAD) of all individual errors gives the following individual-based accuracy disparity:

$$F(\hat{\mathbf{r}}) = \frac{1}{n}\sum_{i=1}^{n}|e_i - \overline{e}|. \tag{3}$$

Decision Fairness. The decision task aims to achieve equity in stakeholder outcomes from resource allocation. These outcomes are represented with utility functions, whose definitions may vary across contexts. In our setup, we assume that the parameter r_i captures i's utility gain per unit of resource. Given a predicted parameter $\hat{r}_i$, we define the utility derived from allocation d_i as:

$$U_i(d_i;\hat{r}_i) = \hat{r}_i d_i. \tag{4}$$

Let $\mathbf{u} = (u_1, \ldots, u_n)$ denote the utility distribution generated from decisions $\mathbf{d}$. A fairness measure W aggregates utility values into a scalar indicator of overall fairness. We adopt the widely used α-fairness (e.g., Mo and Walrand (2000), Bertsimas et al. (2011)), a family of measures offering a tunable trade-off between efficiency and equity:

$$W_\alpha(\mathbf{u}) = \begin{cases} \sum_{i=1}^{n} \frac{u_i^{1-\alpha}}{1-\alpha}, & \text{if } \alpha \geq 0 \text{ and } \alpha \neq 1; \\ \sum_{i=1}^{n} \log(u_i), & \text{if } \alpha = 1. \end{cases} \tag{5}$$

When $\alpha = 0$, this measure simplifies to a sum of all utilities, reflecting a pure efficiency goal to maximize the total utility. As α increases, the measure places greater emphasis on equity, with $\alpha \to \infty$ recovering the Rawlsian max-min criterion, which is a pure equity goal to prioritize the worst-off stakeholder. The standard definition (5) applies at the individual level. When using individual-based prediction fairness (3) and $W_\alpha(\mathbf{u})$ as the decision objective, the E2EFO problem targets fair outcomes for individual stakeholders.

To capture group-level fairness, we generalize α-fairness to operate across groups. Let $g_k(\mathbf{u})$ denote an intra-group fairness score for group G_k, defined as:

$$g_k(\mathbf{u}) = \begin{cases} \sum_{i \in G_k} \frac{u_i^{1-\alpha}}{1-\alpha}, & \text{if } 0 \leq \alpha < 1; \\ \sum_{i \in G_k} \log(u_i), & \text{if } \alpha = 1; \\ \frac{\alpha - 1}{\sum_{i \in G_k} u_i^{1-\alpha}}, & \text{if } \alpha > 1. \end{cases} \tag{6}$$

When $\alpha \leq 1$, this function applies the standard α-fairness definition. When $\alpha > 1$, we apply a monotonic transformation to ensure positivity while preserving the relative order of fairness among groups. A higher value of $g_k(\mathbf{u})$ indicates a more equitable utility distribution within group G_k.

We then define group-based α-fairness by aggregating these intra-group fairness scores:

$$W_\alpha^g(\mathbf{u}) = \begin{cases} \sum_{k=1}^{K} \frac{g_k^{1-\alpha}(\mathbf{u})}{1-\alpha}, & \text{if } \alpha \geq 0 \text{ and } \alpha \neq 1; \\ \sum_{k=1}^{K} \log(g_k(\mathbf{u})), & \text{if } \alpha = 1. \end{cases} \tag{7}$$

This two-level formulation ensures that both intra-group fairness (via g_k) and inter-group fairness (via W_α^g) are taken into account. When E2EFO uses group-based prediction fairness (2) and group-based decision fairness objective (7), the framework emphasizes fairness among groups.

3.2. Fair Decision Focused Learning

To solve an E2EFO problem, we first learn a predictor to estimate $\hat{\mathbf{r}}$ then solve for the optimal decisions $\mathbf{d}^*(\hat{\mathbf{r}})$ from (1). The decision task involves a well-structured optimization problem that can often be efficiently handled by standard solvers. In contrast, the prediction task admits greater modeling flexibility, as there are different ways to formulate the training problem depending on the prediction goals.

A common baseline is the predict-then-optimize (PTO) approach, which focuses solely on prediction quality. PTO learns a function f_θ using training data $\{\mathcal{D}, \mathbf{r}\}$ to minimize a standard prediction loss $L(\theta)$, such as the mean squared error. To account for prediction fairness, a fairness penalty $F(f_\theta(\mathcal{D}))$ can be added as a constraint: $\min_{\theta\in\Theta} L(\theta_t)$ s.t. $F(f_\theta(\mathcal{D})) \leq \epsilon$ where ϵ is a predefined tolerance level, or as a regularizer: $\min_{\theta\in\Theta} L(\theta) + \lambda F(f_\theta(\mathcal{D}))$ where $\lambda > 0$ balances prediction accuracy and fairness.

A key limitation of PTO is its potential misalignment with the decision objective. Even highly accurate and fair predictions may lead to low-quality decisions if the decision problem is sensitive to small prediction errors. To address this limitation, we adopt Fair Decision-Focused Learning (FDFL), an end-to-end approach that trains predictive models with explicit consideration of decision performance. FDFL replaces the prediction-error based loss with a regret-based loss, which measures the gap in fairness performance between the optimal decision under true parameters and the prediction-based decision. This loss function has been applied in standard decision-focused learning methods (Tang and Khalil 2023).

$$L_{regret}(\theta) = W(\mathbf{d}^*(\mathbf{r}); \mathbf{r}) - W(\mathbf{d}^*(f_\theta(\mathcal{D})); \mathbf{r}). \tag{8}$$

This regret-based loss becomes zero when the predicted parameters lead to decisions that are as fair as the optimal decision under true parameters. To incorporate prediction fairness, FDFL adds the same fairness disparity measure $F(f_\theta(\mathcal{D}))$, either as a constraint: $\min_{\theta\in\Theta} L_{regret}(\theta)$ s.t. $F(f_\theta(\mathcal{D})) \leq \epsilon$, or as a regularized objective, $\min_{\theta\in\Theta} L_{regret}(\theta) + \lambda F(f_\theta(\mathcal{D}))$ where $\lambda > 0$.

Compared to PTO that emphasizes accurate estimation of $\hat{\mathbf{r}}$, FDFL prioritizes decision quality, specifically generating decisions $\hat{\mathbf{d}}$ that are as fair as $\mathbf{d}^*(\mathbf{r})$. While both approaches can integrate fairness, their training objectives are different.

We next present an example showing how this prediction-decision integration in FDFL can lead to superior performance in end-to-end fairness optimization.

3.3. An Illustrative Example

We present a stylized example to illustrate how different training approaches in the prediction task affect both prediction and decision performances in an E2EFO problem. Suppose a decision maker allocates 20 units of divisible resources to 15 stakeholders belonging to two groups: G_1 has 10 members and G_2 has 5. Each stakeholder is described by a single feature x_i, which is used to predict their impact parameter r_i.

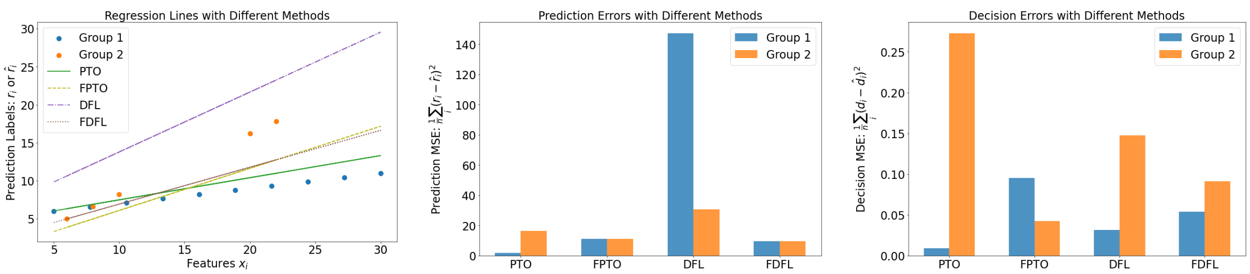
The decision task maximizes group-based α-fairness $W_\alpha^g(\mathbf{u})$ with $\alpha = 0.5$, subject to the constraints $\mathbf{d} \geq 0$ and $\sum_{i=1}^{15} d_i \leq 20$. Prediction fairness is evaluated using group-based accuracy disparity, which simplifies in the two-group setting to $F(\hat{\mathbf{r}}) = |MSE_1 - MSE_2|$, where MSE_k denotes the mean squared error for group G_k.

We compare four methods to learn a linear predictor of $\mathbf{r}$. Two methods follow a predict-then-optimize paradigm: PTO solves a standard linear regression to minimize the MSE between $\mathbf{r}, \hat{\mathbf{r}}$, and fair PTO (FPTO) accounts for prediction fairness by adding accuracy disparity as a regularization term to the prediction error. The other two methods adopt an end-to-end view to emphasize decision quality: DFL fits a linear predictor to minimize the decision regret associated with $\mathbf{d}^*(\hat{\mathbf{r}})$, and FDFL augments DFL with an accuracy disparity regularizer. In both FPTO and FDFL, we set $\lambda = 0.5$ for the prediction fairness term.

Methods	Prediction MSE	Accuracy Disparity	Decision MSE	Decision Regret
PTO	**6.767**	14.606	0.097	0.072
FPTO	10.983	**0.000**	0.077	0.092
DFL	108.341	116.433	0.070	**0.058**
FDFL	9.429	**0.000**	**0.066**	0.064

Table 1: Performance Summary

Figure 1a shows the data points for both groups and the fitted regression lines from all methods. Table 1 summarizes key performance metrics of prediction and decision quality. The results show notable differences among the four methods. PTO achieves the lowest prediction MSE but exhibits disparity in prediction accuracy between groups. FPTO eliminates this disparity at the cost of slightly worse prediction accuracy. In terms of decision quality, both PTO and FPTO are inferior to their end-to-end counterparts, DFL and FDFL. The end-to-end methods attain lower decision MSE and decision regrets, indicating that prediction-based decisions $\hat{\mathbf{d}}$ are closer to the true optimal decisions $\mathbf{d}^*(\mathbf{r})$ both in allocation amounts and in fairness levels. Notably, FDFL improves upon DFL by substantially reducing prediction errors and prediction disparities, while maintaining strong decision performance. Overall, FDFL emerges as the most effective approach, balancing fair and accurate predictions with high-quality decision outcomes.



(a) Data and regression results (b) Prediction errors (c) Decision errors

Figure 1 Regression results, group comparisons of prediction and decision errors

We further compare group-level performances. Figure 1b shows that incorporating prediction fairness helps eliminate accuracy gaps between groups in both the predict-then-optimize and end-to-end paradigms. In Figure 1c, we observe that prediction disparities in PTO are amplified in the decision stage, and the fair predictions in FPTO still lead to disparities in decision errors. These results highlight the misalignment between prediction and decision under PTO and FPTO. In contrast, DFL attains a smaller decision error gap between groups despite its high prediction accuracy disparity, and FDFL achieves both fair predictions and equitable decision outcomes between groups.

4. Fair Decision Focused Learning Algorithm

We next present algorithmic approaches for training predictive models under the FDFL setup. We adopt the training formulation that integrates prediction fairness as a regularization term.

$$\min_{\theta\in\Theta} L_{regret}(\theta) + \lambda F(f_\theta(\mathcal{D})) = \min_{\theta\in\Theta} W(\mathbf{d}^*(\mathbf{r});\mathbf{r}) - W(\mathbf{d}^*(f_\theta(\mathcal{D}));\mathbf{r}) + \lambda F(f_\theta(\mathcal{D})). \tag{9}$$

For notation ease, we refer to the training objective as $\mathcal{L}(\theta)$. To enable gradient-based training methods such as backpropagation, we impose the following assumptions to ensure that $\mathcal{L}(\theta)$ is subdifferentiable with respect to θ.

ASSUMPTION 1. *The prediction model f_θ is differentiable in θ.*

ASSUMPTION 2. *The optimal solution $\mathbf{d}^*(\hat{\mathbf{r}})$ to the parametric decision optimization problem* (1) *is differentiable in the predicted parameters $\hat{\mathbf{r}}$.*

In the regret term, the first component $W(\mathbf{d}^*(\mathbf{r});\mathbf{r})$ is constant with respect to θ, and therefore its gradient vanishes. The second component, $W(\mathbf{d}^*(\hat{\mathbf{r}});\mathbf{r}) = W(\mathbf{d}^*(f_\theta(\mathcal{D}));\mathbf{r})$, depends on θ through the predicted parameters $\hat{\mathbf{r}} = f_\theta(\mathcal{D})$ and the resulting optimal decisions $\mathbf{d}^*(\hat{\mathbf{r}})$. Under Assumptions 1 and 2, we apply the chain rule to compute the regret gradient as:

$$\nabla_\theta L_{\text{regret}}(\theta) = -\frac{\partial W(\mathbf{d}^*(\hat{\mathbf{r}});\mathbf{r})}{\partial \mathbf{d}^*(\hat{\mathbf{r}})} \cdot \frac{\partial \mathbf{d}^*(\hat{\mathbf{r}})}{\partial \hat{\mathbf{r}}} \cdot \frac{\partial \hat{\mathbf{r}}}{\partial \theta}. \tag{10}$$

In this decomposition, the first term is the gradient of the decision objective W with respect to allocation decisions $\mathbf{d}$, which is well defined for our chosen decision fairness measures (5) and (7). The last term is the Jacobian of the predictor f_θ, which is straightforward to compute using automatic differentiation in standard machine learning frameworks.

The middle term, $\frac{\partial \mathbf{d}^*(\hat{\mathbf{r}})}{\partial \hat{\mathbf{r}}}$, requires differentiating through an optimization problem and poses the main technical challenge. Prior works have derived regularity conditions under which this middle term is well defined. For example, Theorem 1 in Amos and Kolter (2017) and Theorem 4.4 in Still (2018) provide sufficient conditions for the differentiability of the optimal solution mapping in parametric convex optimization. In our case, Assumption 2 holds for allocation problems maximizing group-based or individual-based α-fairness over a knapsack, which we explore further in Section 4.1 and numerical experiments in Section 5

We now turn to the prediction fairness regularizer $F(f_\theta(\mathcal{D}))$. Both group-based and individual-based accuracy disparity measures, defined in (2) and (3), are subdifferentiable with respect to $\hat{\mathbf{r}}$. Using the chain rule and Assumption 1, we compute the gradient of the regularizer as:

$$\frac{\partial F(f_\theta(\mathcal{D}))}{\partial \boldsymbol{\theta}} = \frac{\partial F(\hat{\mathbf{r}})}{\partial \hat{\mathbf{r}}} \cdot \frac{\partial \hat{\mathbf{r}}}{\partial \boldsymbol{\theta}}. \tag{11}$$

Since both components of $\mathcal{L}(\theta)$ are subdifferentiable in θ, we apply standard first-order method for optimization. Algorithm 1 summarizes the training procedure to learn f_θ. In each epoch, a forward propagation step evaluates the training loss, then a back propagation step computes a gradient $\nabla_\theta \mathcal{L}$ or a subgradient $\partial_\theta \mathcal{L}$ and updates the predictor parameters accordingly.

The key computational challenge in this algorithm lies in computing the gradient or subgradient in each iteration. Among all the terms involved in the gradient formulas, (10) and (11), $\frac{\partial \mathbf{d}^*(\hat{\mathbf{r}})}{\partial \hat{\mathbf{r}}}$ is the most technically difficult. In the following subsections, we discuss strategies for computing or approximating this term. The other terms, including gradients of W, F, f_θ, are generally straightforward to compute in practice.

Algorithm 1 Gradient-based Training for Fair Decision Focused Learning

Require: Training data $\mathcal{D}$; Target labels $\mathbf{r}$; decision objective $W(\mathbf{d};\mathbf{r})$; prediction fairness F; learning rate η.

1: Initialize predictor parameter $\theta \in \Theta$ for f_θ.
2: **for** each training epoch **do**
3: $\quad \hat{\mathbf{r}} \leftarrow f_\theta(\mathcal{D})$. ▷ Predict parameters in decision model
4: $\quad \mathbf{d}^*(\hat{\mathbf{r}}) \leftarrow \arg\max_{\mathbf{d}\in\mathcal{S}} W(\mathbf{d};\hat{\mathbf{r}})$. ▷ Solve optimal decision based on predicted parameters
5: $\quad \mathcal{L}(\theta) \leftarrow W(\mathbf{d}^*(\hat{\mathbf{r}});\mathbf{r}) - W(\mathbf{d}^*(\mathbf{r});\mathbf{r}) + \lambda F(\hat{\mathbf{r}})$. ▷ Evaluate loss (end of forward propagation)
6: $\quad$ Compute gradient $\nabla_\theta \mathcal{L}$ and update $\theta \leftarrow \theta - \eta\nabla_\theta\mathcal{L}$. ▷ Backward propagation
7: $\quad$ If gradient is not available, compute subgradient $\partial_\theta\mathcal{L}$ and update $\theta \leftarrow \theta - \eta\partial_\theta\mathcal{L}$.
8: **end for**
9: **return** θ

Remark on theoretical consideration. This work focuses on developing and empirically evaluating FDFL algorithms as practical methods for integrating fairness into end-to-end prediction and decision-making. We note that the decision regret loss is generally nonconvex in the predictor parameters. Although the differentiability of the decision regret enables gradient-based training, our training procedure does not offer guarantees of global optimality. Rigorous theoretical analysis of regret-based losses, covering aspects such as convexity, statistical consistency and generalization guarantees, is an important direction for future research. Recent advances in end-to-end learning (e.g., Ho-Nguyen and Kılınç-Karzan (2022), Elmachtoub et al. (2023)) offer promising foundations for advancing theoretical understandings of E2EFO and FDFL.

4.1. Closed-Form Gradients for α-Fairness Optimization

In specific settings, the decision optimization problem admits closed-form solutions, allowing us to derive an exact analytical formula for the gradient term $\frac{\partial \mathbf{d}^*(\hat{\mathbf{r}})}{\partial \hat{\mathbf{r}}}$ and use it in the regret gradient computation (10). This approach avoids the need for implicit differentiation or numerical approximation. Such exact gradient computation is possible for a specific class of decision problem: maximizing α-fairness over a non-negative knapsack, where stakeholder utilities are linear in resource allocations. We explore this setup with both group-based and individual-based fairness objectives.

Suppose a decision maker has a total budget of $Q > 0$ to distribute resources among n stakeholders belonging to K groups. For each unit of resource, the stakeholder i is associated with an allocation cost $c_i > 0$ and a benefit parameter r_i (or its prediction $\hat{r}_i$). The allocation goal is to maximize the

α-fairness of the resulting utility distribution under a budget constraint. Given predicted parameters $\hat{\mathbf{r}}$, the corresponding decision optimization problem is formulated as follows.

$$\begin{aligned} \text{Group-based: } & \max_{\mathbf{d}} \sum_{i=1}^{n} W_{\alpha}^{g}(\mathbf{u}) \\ & \text{s.t. } u_i = \hat{r}_i d_i \ \forall i, \sum_{i=1}^{n} c_i d_i \leq Q, \ \mathbf{d} \geq 0. \end{aligned} \tag{12}$$

$$\begin{aligned} \text{Individual-based: } & \max_{\mathbf{d}} \sum_{i=1}^{n} W_{\alpha}(\mathbf{u}) \\ & \text{s.t. } u_i = \hat{r}_i d_i \ \forall i, \sum_{i=1}^{n} c_i d_i \leq Q, \ \mathbf{d} \geq 0. \end{aligned} \tag{13}$$

We assume all parameters $\hat{r}_i, c_i, Q$ are positive. For $\alpha > 0$, both models maximize a strictly concave function in $\mathbf{d}$ over a compact convex set, ensuring the existence of a unique optimal solution. In the following propositions, we use the Karush-Kuhn-Tucker (KKT) conditions to derive closed-form formulas for the optimal allocation decisions. For notation ease, we state the solution formulas for when true benefit parameters $\mathbf{r}$ is used as input. We then differentiate these formulas with respect to $\mathbf{r}$ to obtain exact analytical gradients. In the training iterations, to compute $\frac{\partial \mathbf{d}^*(\hat{\mathbf{r}})}{\partial \hat{\mathbf{r}}}$, we need to replace $\mathbf{r}$ with $\hat{\mathbf{r}}$ in the gradient formulas derived in Proposition 2 or 4.

In the main text, we discuss the general case where $\alpha \in (0,1) \bigcup (1, \infty)$. Special cases including $\alpha = 0$, $\alpha = 1$, and $\alpha \to \infty$ are treated separately and discussed in Appendix A. The proofs for all propositions are also given in Appendix A.

PROPOSITION 1 **(Closed-Form Decisions, Group-based Fairness)**. *Let* $\alpha \in (0,1) \cup (1,\infty)$, *the optimal solution to* (12) *with parameter* $\mathbf{r}$ *is given by:*

$$d_i^* = \begin{cases} \dfrac{Q c_i^{-\frac{1}{\alpha}} r_i^{\frac{1-\alpha}{\alpha}} S_{k(i)}^{\frac{1}{\alpha-2}}}{\sum_{k=1}^{K} S_k^{1+\frac{1}{\alpha-2}}}, & \textit{if } 0 < \alpha < 1; \\[2ex] \dfrac{Q c_i^{-\frac{1}{\alpha}} r_i^{\frac{1-\alpha}{\alpha}} S_{k(i)}^{\frac{-\alpha+2}{-\alpha^2+2\alpha-2}}}{\sum_{k=1}^{K} S_k^{1+\frac{-\alpha+2}{-\alpha^2+2\alpha-2}}}, & \textit{if } \alpha > 1 \end{cases}, \ \forall i \in [n].$$

where $k(i)$ *is the group of stakeholder i, and for each group* $k \in [K]$, $S_k = \sum_{i \in G_k} \left(c_i^{-1/\alpha} r_i^{1/\alpha} \right)^{1-\alpha}$.

PROPOSITION 2 (**Analytical Gradient Formulas, Group-based Fairness**). *Let* $\mathbf{d}^*$ *denote the optimal solution to* (12) *with parameter* $\mathbf{r}$, *and define* $\beta = \begin{cases} \frac{1}{\alpha-2}, & \text{if } 0<\alpha<1; \\ \frac{-\alpha+2}{-\alpha^2+2\alpha-2}, & \text{if } \alpha>1 \end{cases}$. *For* $k \in [K]$, $S_k = \sum_{i\in G_k}\left(c_i^{-1/\alpha} r_i^{1/\alpha}\right)^{1-\alpha}$. *For* $i \in [n]$, $k(i)$ *denote the group it belongs to.*

The Jacobian $\frac{\partial \mathbf{d}^*(\mathbf{r})}{\partial \mathbf{r}}$ *consists of the following components: for all* $i \in [n]$,

- *Derivative of decision* d_i *with respect to* i*'s own benefit:*

$$\frac{\partial d_i^*}{\partial r_i} = \frac{1-\alpha}{\alpha} Q c_i^{-\frac{1}{\alpha}} r_i^{\frac{1}{\alpha}-2} \frac{S_{k(i)}^{\beta}}{\sum_{k=1}^K S_k^{1+\beta}} + Q c_i^{-\frac{1}{\alpha}} r_i^{\frac{1}{\alpha}-1} \frac{\partial S_{k(i)}^{\beta}}{\partial r_i}\left(\frac{1}{\sum_{k=1}^K S_k^{1+\beta}} - \frac{S_{k(i)}^{\beta}}{(\sum_{k=1}^K S_k^{1+\beta})^2}\right).$$

- *Derivative of decision* d_i *with respect to the benefit of another member in the same group of* i*:*

$$\frac{\partial d_i^*}{\partial r_k} = Q c_i^{-\frac{1}{\alpha}} r_i^{\frac{1}{\alpha}-1} \frac{\partial S_{k(i)}}{\partial r_k}\left(\frac{\beta S_{k(i)}^{\beta-1}}{\sum_{k=1}^K S_k^{1+\beta}} - \frac{(1+\beta) S_{k(i)}^{2\beta}}{(\sum_{k=1}^K S_k^{1+\beta})^2}\right), \ \forall k \neq i, k \in G_{k(i)}.$$

- *Derivative of decision* d_i *with respect to the benefit of a member in a different group:*

$$\frac{\partial d_i^*}{\partial r_j} = -Q c_i^{-\frac{1}{\alpha}} r_i^{\frac{1}{\alpha}-1} S_{k(i)}^{\beta} \frac{\partial S_{k(j)}}{\partial r_j} \frac{(\beta+1) S_{k(j)}^{\beta}}{(\sum_{k=1}^K S_k^{1+\beta})^2}, \ \forall j \notin G_{k(i)}.$$

In these formulas, the gradient of S_k *is given by*

$$\frac{\partial S_k}{\partial r_i} = \frac{1-\alpha}{\alpha} c_i^{1-\frac{1}{\alpha}} r_i^{\frac{1}{\alpha}-2}, \ \forall i \in G_k; \ \frac{\partial S_k}{\partial r_i} = 0, \ \forall i \notin G_k.$$

PROPOSITION 3 (**Closed-Form Decisions, Individual-based Fairness**). *Let* $\alpha \in (0,1) \cup (1,\infty)$, *the optimal solution to* (13) *with parameter* $\mathbf{r}$ *is given by:*

$$d_i^* = \frac{c_i^{-\frac{1}{\alpha}} \cdot r_i^{\frac{1}{\alpha}-1} \cdot Q}{\sum_{j=1}^n c_j^{1-\frac{1}{\alpha}} \cdot r_j^{\frac{1}{\alpha}-1}}, \ \forall i \in [n].$$

PROPOSITION 4 (**Analytical Gradient Formulas, Individual-based Fairness**). *Let* $\mathbf{d}^*$ *denote the optimal solution to* (13) *with parameter* $\mathbf{r}$, *and define* $S = \sum_{i=1}^n c_i^{1-\frac{1}{\alpha}} \cdot r_i^{\frac{1}{\alpha}-1}$.

The Jacobian $\frac{\partial \mathbf{d}^*(\mathbf{r})}{\partial \mathbf{r}}$ *consists of the following components: for all* $i \in [n]$,

$$\frac{\partial d_i^*}{\partial r_i} = \frac{Q\left(-1+\frac{1}{\alpha}\right) c_i^{-\frac{1}{\alpha}} \cdot r_i^{\frac{1}{\alpha}-2}\left(S - c_i^{1-\frac{1}{\alpha}} \cdot r_i^{\frac{1}{\alpha}-1}\right)}{S^2};$$

$$\frac{\partial d_i^*}{\partial r_k} = \frac{-Q\left(-1+\frac{1}{\alpha}\right) c_i^{-\frac{1}{\alpha}} \cdot r_i^{\frac{1}{\alpha}-1} \cdot r_k^{\frac{1}{\alpha}-2} \cdot c_k^{1-\frac{1}{\alpha}}}{S^2}, \ \forall k \neq i.$$

4.2. Gradient Approximation Techniques

When a closed-form solution to the decision optimization model is not available, the gradient component $\frac{\partial \mathbf{d}^*(\hat{\mathbf{r}})}{\partial \hat{\mathbf{r}}}$ cannot be derived analytically. In such cases, we rely on gradient approximation techniques, which are broadly applicable to a wide range of decision models and are compatible with the gradient computation chain used in backpropagation. Even when exact gradients are available, approximate methods may offer computational benefits, as explored in our numerical experiments.

4.2.1. Finite Difference Approximation A simple and general strategy for gradient approximation is the finite difference method. This technique has been applied in DFL with linear decision optimization models, such as the differentiable black-box optimizer (DBB) approach proposed in Pogančić et al. (2020). Given a predicted parameter vector $\hat{\mathbf{r}}$, we consider a perturbation along the gradient of the decision objective with respect to the decision variables: $\mathbf{r}' = \hat{\mathbf{r}} + \epsilon \frac{\partial W(\mathbf{d}^*(\hat{\mathbf{r}}), \mathbf{r})}{\partial \mathbf{d}^*(\hat{\mathbf{r}})}$. Then we can approximate the Jacobian at $\hat{\mathbf{r}}$ as:

$$\frac{\partial \mathbf{d}^*(\hat{\mathbf{r}})}{\partial \hat{\mathbf{r}}} \approx \frac{1}{\epsilon}(\mathbf{d}^*(\mathbf{r}') - \mathbf{d}^*(\hat{\mathbf{r}})). \tag{14}$$

Here, $\epsilon > 0$ controls the size of perturbation. This approach provides non-zero approximate gradients and requires only two calls to the decision solver (for generating $\mathbf{d}^*(\mathbf{r}'), \mathbf{d}^*(\hat{\mathbf{r}})$) per gradient computation. A key limitation of this technique is the lack of theoretical guarantees on approximation quality. Moreover, the approximation may introduce numerical instability if ϵ is chosen poorly. Nonetheless, it provides a useful and flexible baseline for fair decision optimization models where solvers are available but gradients are not analytically tractable.

4.2.2. Folded Optimization based Approximation A more principled, yet still general, technique for computing gradients through optimization models is the Folded Optimization (Fold-Opt) approach proposed by Kotary et al. (2023). Fold-Opt provides a framework for differentiating through a wide range of optimization problems, including both convex and non-convex models, by leveraging the fixed point structure of iterative solution algorithms for these problems.

Let the decision problem be solved using an iterative algorithm that repeatedly applies an update rule U: $\mathbf{d}_{k+1}(\hat{\mathbf{r}}) = U(\mathbf{d}_k(\hat{\mathbf{r}}), \hat{\mathbf{r}})$. The sequence of updates converges the optimal solution, $\mathbf{d}_k(\hat{\mathbf{r}}) \rightarrow \mathbf{d}^*(\hat{\mathbf{r}})$ as $k \rightarrow \infty$. At convergence, the fixed-point condition holds: $\mathbf{d}^*(\hat{\mathbf{r}}) = U(\mathbf{d}^*(\hat{\mathbf{r}}), \hat{\mathbf{r}})$. Using the implicit function theorem, we can differentiate the fixed-point condition and obtain the following linear system defining the Jacobian $\frac{\partial \mathbf{d}^*(\hat{\mathbf{r}})}{\partial \hat{\mathbf{r}}}$:

$$\frac{\partial \mathbf{d}^*(\hat{\mathbf{r}})}{\partial \hat{\mathbf{r}}} = \frac{\partial U(\mathbf{d}^*(\hat{\mathbf{r}}), \hat{\mathbf{r}})}{\partial \mathbf{d}^*(\hat{\mathbf{r}})} \cdot \frac{\partial \mathbf{d}^*(\hat{\mathbf{r}})}{\partial \hat{\mathbf{r}}} + \frac{\partial U(\mathbf{d}^*(\hat{\mathbf{r}}), \hat{\mathbf{r}})}{\partial \hat{\mathbf{r}}}$$

$$\Rightarrow \left(\mathbf{I} - \underbrace{\frac{\partial U(\mathbf{d}^*(\hat{\mathbf{r}}), \hat{\mathbf{r}})}{\partial \mathbf{d}^*(\hat{\mathbf{r}})}}_{\Phi} \right) \frac{\partial \mathbf{d}^*(\hat{\mathbf{r}})}{\partial \hat{\mathbf{r}}} = \underbrace{\frac{\partial U(\mathbf{d}^*(\hat{\mathbf{r}}), \hat{\mathbf{r}})}{\partial \hat{\mathbf{r}}}}_{\Psi}$$

where I is the identity matrix and the Jacobians of U are computed at the fixed point $(\mathbf{d}^*(\hat{\mathbf{r}}), \hat{\mathbf{r}})$.

Although the linear system can be solved directly to obtain $\frac{\partial \mathbf{d}^*(\hat{\mathbf{r}})}{\partial \hat{\mathbf{r}}}$, this approach is often inefficient due to the complexity of explicitly forming and inverting $I - \Phi$. Fold-Opt implements a more efficient alternative method. Instead of computing the decision Jacobian, it directly computes the Jacobian-vector product needed in the gradient chain, $\frac{\partial W(\mathbf{d}^*(\hat{\mathbf{r}});\mathbf{r})}{\partial \mathbf{d}^*(\hat{\mathbf{r}})} \cdot \frac{\partial \mathbf{d}^*(\hat{\mathbf{r}})}{\partial \hat{\mathbf{r}}}$. To obtain this product, Fold-Opt solves for $\mathbf{v} \in \mathbb{R}^n$ from the transposed linear system, $\mathbf{v}^T(I - \Phi) = (\frac{\partial W(\mathbf{d}^*(\hat{\mathbf{r}});\mathbf{r})}{\partial \mathbf{d}^*(\hat{\mathbf{r}})})^T$, then computes $\frac{\partial W(\mathbf{d}^*(\hat{\mathbf{r}});\mathbf{r})}{\partial \mathbf{d}^*(\hat{\mathbf{r}})} \cdot \frac{\partial \mathbf{d}^*(\hat{\mathbf{r}})}{\partial \hat{\mathbf{r}}} = v^T \Psi$. Kotary et al. (2023) showed that this strategy significantly reduces computational and memory overhead.

In our implementation, we apply Fold-Opt to FDFL by unfolding a projected gradient descent solver for the decision problem. The update rule takes the form $U(\mathbf{d}, \hat{\mathbf{r}}) = \mathcal{P}_{\mathcal{S}}(\mathbf{d} + \eta \nabla_{\mathbf{d}} W(\mathbf{d}; \hat{\mathbf{r}}))$, where $\mathcal{P}_{\mathcal{S}}$ denotes the projection onto the feasible set $\mathcal{S}$ and η is the step size. As we have described above, Fold-Opt uses this update rule to approximate the required vector-Jacobian product, supporting efficient backpropagation in each training iteration.

5. Numerical Experiments

We evaluate FDFL algorithms on a healthcare resource allocation problem inspired by Obermeyer et al. (2019), which uncovered racial bias in a commercial medical risk prediction algorithm. The algorithm was designed to generate risk scores that indicate patients' health needs, with higher scores representing more complex needs. These risk scores were used to guide patient enrollment in a care management program. In the health system studied by Obermeyer et al. (2019), patients scoring above the 97th percentile were automatically enrolled, while those above the 55th percentile were referred to their primary care physicians for further review. This study found that, at equal risk scores, black patients were generally sicker than white patients, but were less likely to be enrolled.

We use the synthetic dataset generated in Obermeyer et al. (2019) to replicate real-world medical data without protected information. The dataset contains 48,784 patient records with features about demographic information, comorbidities, past care utilization (e.g., healthcare costs, hospitalizations), and biomarker values (e.g, blood pressure). For our experiments, we select a random sample of $n = 5,000$ patients (11.56% Black) and split them evenly into training and testing sets of 2500 patients each.

We formulate an E2EFO problem with two modifications of the original decision process in Obermeyer et al. (2019). First, instead of predicting medical risk as a proxy for health need, we predict each patient's potential benefit from program enrollment. Second, we replace binary enrollment decisions with continuous resource allocations. Next, we describe the prediction and decision tasks in detail.

Prediction task. The prediction task aims to estimate patients' potential benefits from enrollment. Since the dataset does not contain this information directly, we construct a ground truth benefit score r_i for each patient i by combining two components: (1) potential for health improvement, and (2) cost savings. We treat a patient's number of active chronic illnesses as a proxy for the health improvement potential, denoted h_i. Our rationale is that the care management program can help patients manage their chronic conditions. In the dataset, the original count of chronic illnesses ranges from 0 to 17. We apply min-max normalization to scale h_i to $[0,1]$. Cost savings, denoted s_i, are estimated from avoidable healthcare costs from program enrollment. The dataset contains some patients enrolled in the program, for whom we directly observe their avoidable costs. For others, we impute s_i using nearest-neighbor matching with the observable avoidable costs of enrolled patients. The original avoidable costs range from \$0 to \$642,700. To reduce skewness, we apply the log transformation $\log(1+s_i)$ adopted in Obermeyer et al. (2019) followed by min-max normalization to scale the values to $[0,1]$. The final benefit score is computed as a weighted average, $r_i = 0.5h_i + 0.5s_i$. To ensure numerical stability, we rescale all r_i values to lie in the interval $[2,101]$.

The prediction task trains a predictor f_θ to estimate $\hat{r}_i = f_\theta(\mathbf{x}_i)$ from patient features $\mathbf{x}_i$. Prediction fairness is assessed using either group-based or individual-based accuracy disparity, measured by mean absolute deviation (MAD) of prediction errors. Small values of accuracy disparity indicate comparable predictive performances across groups or individuals.

Decision task. A decision maker allocates program resources to patients based on their predicted benefit scores. The decision d_i represents the amount of resources assigned to patient i. The utility received by patient i is modeled as: $u_i(d_i) = \hat{r}_i d_i$. Thus, patients with higher benefit scores or larger allocation amounts receive higher utilities. The decision maker has a total budget of $Q = 2500$, and each unit of resource given to patient i requires a cost c_i, which we derive from the patient's total healthcare spending (ranging from \$0 to \$527,900) in the dataset. To reduce the influence of extreme values, we cap cost values at the 99th percentile and scale them to the range of $[1, 101]$ using min-max normalization.

The decision task solves the group-based or individual-based fairness optimization problem defined respectively in (12) or (13). The objective is to equitably distribute the allocation benefits across population groups or individuals while satisfying the budget constraint.

5.1. Methods and Implementation

We compare four training methods for learning a predictor in the E2EFO problem.

- **Fairness-embedded Predict-then-Optimize (FPTO):** A baseline method that trains a predictor by minimizing a prediction-fairness-regularized prediction loss: $L(\theta) = \mathrm{MSE}(\hat{\mathbf{r}}, \mathbf{r}) + \lambda \cdot \mathrm{MAD}(\hat{\mathbf{r}}, \mathbf{r})$.
- **FDFL with Closed-Form Gradients (FDFL-CF):** An end-to-end method that trains a predictor to minimize a prediction-fairness-regularized decision regret, as defined in (9). Backpropagation uses analytical gradients derived in Proposition 2 or 4.
- **FDFL with Finite Difference based Approximate Gradients (FDFL-FD):** An end-to-end method using the same training objective as FDFL-CF. Gradients are approximated using the finite difference method described in Section 4.2.1.
- **FDFL with Folded Optimization based Approximate Gradients (FoldOpt):** Another end-to-end method using the same training objective as FDFL-CF. Gradients are approximated by unrolling projected gradient descent steps to compute Jacobian-vector products as described in Section 4.2.2.

For all training methods, we consider two regularization settings: $\lambda = 0$ ignores prediction fairness, and $\lambda = 0.5$ incorporates prediction fairness. When $\lambda = 0$, FPTO reduces to a standard predict-then-optimize approach, and FDFL methods reduce to decision-focused learning that does not seek fair predictions. In all cases, the generated predictions are input into the decision optimization problem to maximize α-fairness. We conduct experiments for two values of α to capture different equity-efficiency tradeoffs in the decision objective: $\alpha = 0.5$ places greater emphasis on pursuing efficiency measured by total utilities, and $\alpha = 2$ places greater emphasis on seeking fairness measured by the smallest utility.

We implement two predictor architectures: a linear regression model and a feed-forward neural network model with two hidden layers (64 units per layer), batch normalization, ReLU activations and dropout. Both models use a Softplus output layer to ensure non-negative benefit predictions.

All models are trained using the Adam optimizer for 50 epochs. We report average results obtained from 50 trials with different random seeds.

We use the *PyEPO* library (Tang and Khalil 2023) to implement FDFL training with closed-form and finite-difference gradients, and the *fold-opt* package (Kotary et al. 2023) to implement FoldOpt based FDFL method. In all experiments, the decision problem is solved using the closed-form solutions derived in Section 4.1.

All methods are evaluated along three dimensions: prediction accuracy measured by the mean squared error (MSE) between true benefits $\mathbf{r}$ and predicted benefits $\hat{\mathbf{r}}$, prediction fairness measured by the mean absolute deviation (MAD) of prediction errors between $\mathbf{r}, \hat{\mathbf{r}}$ (group-based or individual-based), and decision quality evaluated using the normalized decision regret $\frac{L_{\text{regret}}(\hat{\mathbf{r}},\mathbf{r})}{|W(\mathbf{d}^*(\mathbf{r}),\mathbf{r})|}$ (Tang and Khalil 2023). We note that a smaller normalized decision regret indicates better decisions whose fairness is closer to the optimal level under ground truth benefits.

5.2. Results: Group-based Fairness

Table 2 Results: $\alpha = 2.0$**, group-based fairness, Neural Network predictor**

Algorithm	λ	Decision Regret	Prediction MSE	Prediction Fairness	Training Time (seconds)
FPTO	0.0	0.0983 ± 0.0018	35.4627 ± 1.2644	**1.7344** ± 0.1045	1.2996 ± 0.0329
	0.5	0.0968 ± 0.0039	**35.2676** ± 2.0286	1.8322 ± 0.3261	**1.2913** ± 0.0412
FDFL-CF	0.0	**0.0358** ± 0.0005	119.3416 ± 2.8384	32.4739 ± 0.4682	10.1872 ± 0.0213
	0.5	0.0425 ± 0.0001	106.9212 ± 3.0555	21.1977 ± 2.9267	10.2478 ± 0.1541
FDFL-FD	0.0	0.0544 ± 0.0041	188.4053 ± 32.0814	51.0395 ± 6.1237	103.6114 ± 0.4983
	0.5	0.0618 ± 0.0088	190.4567 ± 13.8893	30.2942 ± 0.3262	102.2925 ± 0.7698
FoldOpt	0.0	0.1195 ± 0.0068	366.9125 ± 18.7837	92.0644 ± 6.9585	5.5543 ± 0.0590
	0.5	0.1347 ± 0.0138	328.9056 ± 22.5657	58.8678 ± 12.0956	5.5451 ± 0.0407

Table 3 Results: $\alpha = 0.5$**, group-based fairness, Neural Network predictor**

Algorithm	λ	Decision Regret	Prediction MSE	Prediction Fairness	Training Time (seconds)
FPTO	0.0	**0.0176** ± 0.0004	35.4627 ± 1.2644	**1.7344** ± 0.1045	**1.2987** ± 0.0048
	0.5	0.0179 ± 0.0011	**35.2676** ± 2.0286	1.8322 ± 0.3261	1.3070 ± 0.0671
FDFL-CF	0.0	0.0189 ± 0.0012	207.9338 ± 12.7443	24.5686 ± 3.0886	7.6495 ± 0.0986
	0.5	0.0763 ± 0.0013	220.4741 ± 9.3006	31.9948 ± 4.4498	3.2721 ± 0.0649
FDFL-FD	0.0	0.0223 ± 0.0035	246.7065 ± 41.6353	39.4929 ± 11.4958	78.2144 ± 4.0072
	0.5	0.0810 ± 0.0007	257.7810 ± 7.9979	26.5533 ± 1.2669	30.1630 ± 0.2920
FoldOpt	0.0	0.1325 ± 0.0039	363.7616 ± 15.4504	91.8474 ± 8.1903	5.5577 ± 0.0307
	0.5	0.1325 ± 0.0039	363.7616 ± 15.4504	91.8474 ± 8.1903	5.4976 ± 0.0547

We begin by evaluating the methods under group-based fairness definition, which emphasizes equity between racial groups, specifically between Black and White patients. Tables 2 and 3 respectively report results for fairness-sensitive ($\alpha = 2$) and efficiency-focused ($\alpha = 0.5$) decision objectives. For clarity, we focus the discussion on results obtained from using neural network prediction models. Complete results for the linear regression model are provided in Appendix C.

In the fairness sensitive case with $\alpha = 2$, Table 2 shows that FDFL with closed-form gradients (FDFL-CF) and finite-difference approximate gradients (FDFL-FD) attain superior decision quality over the FPTO baseline. Notably, FDFL-CF achieves the lowest decision regret of **0.0358** when $\lambda = 0$, which is nearly three times lower than the FPTO regret of **0.0983** in the same setting. These results illustrate the misalignment between prediction accuracy and decision quality under $\alpha = 2$ and underscore the value of the end-to-end optimization perspective offered by FDFL. Even though FPTO methods achieve lower prediction MSE, these accurate predictions do not translate into high-quality decisions. We also highlight that, among all FDFL methods, the closed-form gradients enable more effective training than the approximate gradients, as FDFL-CF consistently outperforms FDFL-FD and FoldOpt in decision regret, prediction MSE, and prediction fairness.

In contrast, when $\alpha = 0.5$, prediction and decision performances are better aligned. As shown in Table 3, FPTO methods achieve both the lowest prediction MSE and the lowest decision regret. In this setting, accurate predictions lead to high-quality decisions that closely approximate the true optimal fair decision. Comparing between the two α values, the same FPTO predictor leads to almost five times lower decision regret under $\alpha = 0.5$ than under $\alpha = 2$, illustrating the sensitivity of outcome quality to the decision objective. FDFL methods perform worse in all metrics when $\alpha = 0.5$. Nevertheless, we continue to observe advantages of the closed-form gradients (FDFL-CF) over the approximate gradients (FDFL-FD, FoldOpt) in terms of decision and prediction performances.

We next discuss the prediction performance further. In all settings, FPTO achieves better predictions, with lower prediction MSE and lower prediction disparity, than all FDFL methods. This aligns with intuition that FPTO is able to optimize predictor performance without accounting for impacts on decisions. Adding a prediction fairness regularizer ($\lambda = 0.5$) in FPTO does not substantially reduce prediction disparity, nor does it introduce a clear trade-off with prediction accuracy. This suggests that, in this dataset, the MSE-minimizing predictor already captures most of the structure necessary to reduce disparity, leaving limited room for improvement via fairness regularization alone. The larger standard deviations observed in the $\lambda = 0.5$ setting indicate that fairness–accuracy trade-offs may exist in specific training trails, even if they do not appear in the aggregate results.

For FDFL methods, we observe from Table 2 that, under $\alpha = 2$, including a prediction fairness regularizer ($\lambda = 0.5$) effectively reduces the prediction disparity with minimal increase in decision regret in all three variants. This reflects a favorable trade-off: when end-to-end training (FDFL with $\lambda = 0$) already aligns the predictor with the decision objective, integrating prediction fairness consideration allows for fairer predictions without compromising decision quality. Conversely, when FDFL is less effective in optimizing decisions, as observed under $\alpha = 0.5$ in Table 3, there are no clear gains from integrating prediction fairness.

Finally, we compare training efficiency across methods. As expected, FPTO is the fastest to train due to its reliance on standard learning objectives. Among FDFL methods, both FDFL-CF and FoldOpt are more computationally efficient than FDFL-FD, which incurs higher overhead due to repeated solver calls.

5.3. Results: Individual-based Fairness

Table 4 Results: $\alpha = 2.0$**, individual-based fairness, Neural Network predictor**

Algorithm	λ	Decision Regret	Prediction MSE	Prediction Fairness	Training Time (seconds)
FPTO	0.0	0.0960 ± 0.0012	35.4627 ± 1.2644	45.8432 ± 1.4942	1.2746 ± 0.0313
	0.5	0.1116 ± 0.0020	40.9074 ± 1.5535	47.2346 ± 1.9825	**1.2675** ± 0.0485
FDFL-CF	0.0	**0.0358** ± 0.0005	119.3431 ± 2.8367	159.0209 ± 4.2124	7.7938 ± 0.0406
	0.5	0.0367 ± 0.0001	**14.3596** ± 0.2939	**20.8680** ± 0.4506	8.2520 ± 0.2871
FDFL-FD	0.0	0.0609 ± 0.0102	211.2497 ± 21.2805	290.9811 ± 27.9017	31.8402 ± 0.4804
	0.5	0.0524 ± 0.0043	44.9116 ± 6.9998	58.7328 ± 10.0792	32.0983 ± 0.5131
FoldOpt	0.0	0.0329 ± 0.0029	200.6606 ± 16.1298	266.6893 ± 21.4344	5.7159 ± 0.0395
	0.5	0.0438 ± 0.0035	71.8843 ± 15.1922	83.8750 ± 19.6823	5.4628 ± 0.0453

Table 5 Results: $\alpha = 0.5$**, individual-based fairness, Neural Network predictor**

Algorithm	λ	Decision Regret	Prediction MSE	Prediction Fairness	Training Time (seconds)
FPTO	0.0	0.0363 ± 0.0008	35.4627 ± 1.2644	45.8432 ± 1.4942	**1.2830** ± 0.0166
	0.5	0.0473 ± 0.0007	40.9074 ± 1.5535	47.2346 ± 1.9825	1.3054 ± 0.0555
FDFL-CF	0.0	0.0104 ± 0.0003	119.9672 ± 12.1895	160.2802 ± 17.0537	7.5437 ± 0.4503
	0.5	**0.0099** ± 0.0001	**14.0935** ± 0.5389	**20.6355** ± 0.9980	7.4795 ± 0.2586
FDFL-FD	0.0	0.0185 ± 0.0030	190.9773 ± 34.3295	258.0777 ± 43.9605	31.2052 ± 0.1342
	0.5	0.0152 ± 0.0006	164.9002 ± 1.1193	222.4113 ± 0.1263	31.4317 ± 0.1755
FoldOpt	0.0	0.1247 ± 0.0046	334.9719 ± 12.7858	464.9778 ± 12.9515	5.6179 ± 0.0283
	0.5	0.1247 ± 0.0046	334.9495 ± 12.8280	464.9783 ± 12.9525	5.3891 ± 0.0595

We now turn to the individual-based fairness setting, which aims to achieve equity among all patients. Tables 4 and 5 respectively summarize results for $\alpha = 2$ and $\alpha = 0.5$. We again focus on the results using neural network predictors. Appendix C reports additional results from using linear regression predictors.

We find that the advantages of FDFL approaches are more pronounced in this setup compared to group-based setting. The FPTO results in Tables 4 and 5 reveal persistent misalignment between prediction and decision performances under both α values, creating opportunities for end-to-end training with FDFL.

When $\alpha = 2$, Table 4 shows that all FDFL methods outperform FPTO in terms of decision regret. Specifically, FDFL-CF without prediction fairness achieves the lowest decision regret of **0.0358**. When prediction fairness is included ($\lambda = 0.5$), FDFL-CF demonstrates balanced and superior performance: it not only obtains a low decision regret of **0.0367** but also drastically reduces the prediction MSE to **14.3596** and the prediction Fairness MAD to **20.8680**. These results illustrate the practical value of integrating both prediction and decision fairness in end-to-end training. In addition, Tables 4 shows similar patterns for FDFL-FD and FoldOpt: both methods attain low decision regrets, and adding a prediction fairness regularizer decreases prediction disparity with only a minor trade-off in decision quality. When $\alpha = 0.5$, we observe strong performance for FDFL-CF in Table 5, but approximate gradients appear insufficient for effective end-to-end training in this case. The training time performance is consistent with the group-based fairness case: FPTO methods remain the most computationally efficient; among FDFL methods, closed-form gradients provide the best training efficiency, followed by FoldOpt and then FDFL-FD.

In summary, across both group-based and individual-based fairness settings, FDFL-CF with a prediction fairness regularizer consistently delivers the best balance of decision quality, prediction fairness, and training efficiency. These findings highlight its potential as a practical approach to pursuing end-to-end fairness.

6. Conclusion and Discussion

This study introduces end-to-end fairness optimization (E2EFO) as a unifying framework for fair prediction-informed decision making. We focus on a resource allocation setting, where the prediction task estimates the impact of allocating resources, and the decision task seeks equitable distribution of these impacts. Within E2EFO, we define prediction fairness as the mean absolute deviation of prediction errors, and decision fairness as the α-fairness of allocation utilities. We consider both group-based and individual-based definitions to distinguish fairness goals involving groups or individuals.

To operationalize E2EFO, we propose fair decision-focused learning (FDFL), a learning paradigm that jointly accounts for prediction fairness and decision fairness by minimizing a composite training loss, which combines decision regret and prediction fairness regularization. In contrast to the fair predict-then-optimize (FPTO) approach, which trains a predictor to attain low prediction error and disparity, FDFL directly aligns prediction training with the decision objective. A central technical challenge in training an FDFL predictor with backpropagation lies in computing the (sub)gradient of the prediction-based decision with respect to the predicted parameters, that is, differentiating through the decision optimization model. We provide three approaches to address this challenge: (1) exact closed-form gradients for α-fairness maximization under non-negativity and knapsack constraints, (2) approximate gradients obtained from perturbation-based finite difference method, and (3) approximation based on unrolling an iterative optimization algorithm to solving the decision model.

We evaluate FDFL in a healthcare resource allocation problem using a synthetic dataset generated from real patient records, which include demographic, physiological, and utilization-related features. The task involves allocating enrollment resources of a care management program to patients based on their predicted benefits. We examine both group-based fairness to seek parity between white and black patients, and individual-based fairness to seek equity among all patients. Our results show that FDFL with closed-form gradients and finite-difference approximate gradients consistently achieve lower decision regret than FPTO, especially when prediction and decision objectives are misaligned. Compared with DFL without prediction fairness consideration, FDFL attains notably better prediction fairness with minimal loss in decision quality, demonstrating its ability to balance the fairness goals of different tasks in end-to-end decision making.

We highlight two practical implications of our study. First, E2EFO provides a general and flexible framework for integrating fairness in data-driven decision systems. It is important to distinguish fair predictions and fair decisions for their different practical impacts: while prediction fairness can be interpreted as a type of procedural equity to seek comparable predictive performances, decision fairness emphasizes outcome equity to attain fair impact distribution. Second, FDFL offers a principled and practical solution to achieving end-to-end fairness. Our experiment results show strong empirical performance from both generic gradient approximation techniques and customized gradient computations leveraging problem structure.

Several directions remain for future work. The current problem scope does not address important theoretical questions, such as performance guarantees of FDFL under different fairness definitions, performance comparisons of FDFL versus standard DFL. In addition, our method is developed for specific fairness definitions and the resource allocation problem. Future research could explore E2EFO and FDFL with alternative fairness criteria or in other decision contexts. These new settings may also motivate the development of more scalable FDFL training strategies, such as gradient free methods (Shah et al. 2022, Zharmagambetov et al. 2023).

Appendix A: Proofs for Closed-Form Solutions and Analytical Gradients

A.1. Group-based Fairness

PROPOSITION 5 **(Closed-Form Decisions, Group-based Fairness: Special Cases of α).** *For special cases of $\alpha = 0, 1, \infty$, the optimal solution to* (12) *with parameter* $\mathbf{r}$ *is, respectively, given by:*

- $\alpha = 0$*:* $d_i^* = \begin{cases} \frac{Q}{c_i}, & \text{if } i = \arg\max_j \frac{r_j}{c_j}; \\ 0, & \text{otherwise.} \end{cases}$, $\forall i \in [n]$*, representing the allocation of the entire budget to the stakeholder with the highest reward-to-cost ratio.*
- $\alpha = 1$*:* $d_i^* = \frac{Q}{K|G_{k(i)}|c_i}$, $\forall i \in [n]$*, where $G_{k(i)}$ is the size of the group $G_{k(i)}$ to which i belongs.*
- $\alpha \to \infty$*:* $d_i^* = \frac{Q}{r_i \sum_{j=1}^n \frac{c_j}{r_j}}$, $\forall i \in [n]$*, representing an allocation to equalize the utilities of all stakeholders.*

PROPOSITION 6 **(Analytical Gradient Formulas, Group-based Fairness: Special Cases of α).** *Let* $\mathbf{d}^*$ *denote the optimal solution to* (12) *with parameter* $\mathbf{r}$*. For special cases of $\alpha = 0, 1, \infty$, the Jacobian* $\frac{\partial \mathbf{d}^*(\mathbf{r})}{\partial \mathbf{r}}$ *is, respectively, given by:*

- $\alpha = 0$*:* $\mathbf{d}^*(\mathbf{r})$ *is not differentiable at the maximizer* $i^* = \arg\max_j \frac{r_j}{c_j}$*, and has* $\frac{\partial d_i^*}{\partial r_j} = 0$ *for all $i \neq i^*$ and all j.*
- $\alpha = 1$*:* $\frac{\partial d_i^*}{\partial r_k} = 0$ *for all $i, k \in [n]$ as $\mathbf{d}^*(\mathbf{r})$ does not depend on $\mathbf{r}$.*
- $\alpha \to \infty$*: for all $i \in [n]$,* $\frac{\partial d_i^*}{\partial r_k} = \begin{cases} -\frac{Q}{r_k^2(\sum_{j=1}^n c_j/r_j)} + \frac{Qc_k}{r_k^3(\sum_{j=1}^n c_j/r_j)^2}, & \text{if } i = k; \\ \frac{Qc_k}{r_i r_k^2(\sum_{j=1}^n c_j/r_j)^2}, & \text{if } i \neq k \end{cases}$.

Proof for closed-form decisions:

- **Special case $\alpha = 0$:** The objective function simplifies to the linear utility sum, $W_\alpha^g(\mathbf{u}) = \sum_{k=1}^k \sum_{i \in G_k} r_i d_i$. Maximizing a linear function under a knapsack constraint yields a greedy solution: allocate the full budget to the stakeholder with the highest reward-to-cost ratio, $\frac{r_i}{c_i}$. Therefore, the optimal solution assigns $d_i^* = \frac{Q}{c_i}$ if $\frac{r_i}{c_i} = \max_{j \in [n]}\{\frac{r_j}{c_j}\}$, and $d_i^* = 0$ for all other stakeholders.

This function $\mathbf{d}^*(\mathbf{r})$ is piecewise constant, so its derivative is zero everywhere except at points where the identity of i^* changes, where it is non-differentiable. The subgradient at these points is a set, from which the zero matrix is a valid selection for optimization algorithms.

- **General case $\alpha > 0$ and $\alpha \neq 1$:** When $\alpha > 0$, the objective function is concave in $\mathbf{d}$, and we need a different strategy to conclude the optimal solution. For notation ease, we denote $W_\alpha^g(\mathbf{u})$ with $W(g(\mathbf{d}))$, where $g(\mathbf{d}) = (g_1(\mathbf{d}), \ldots, g_K(\mathbf{d}))$ computes all $g_k(\mathbf{u})$ as defined in (6). The Lagrangian function of (12) is:

$$\mathcal{L}(\mathbf{d}, \lambda, \mu) = -W(g(\mathbf{d})) + \lambda(\sum_{i=1}^n c_i d_i - Q) - \sum_{i=1}^n \mu_i d_i,$$

where $\lambda \in \mathbb{R}_{\geq 0}$ and $\mu \in \mathbb{R}_{\geq 0}^n$ are Lagrangian multipliers for the constraints. The Karush-Kuhn-Tucker (KKT) stationarity condition requires $\frac{\partial \mathcal{L}}{\partial d_i} = 0$ for each $i \in [n]$. For an individual i belonging to a group G_k, we can apply the chain rule to obtain the derivative as:

$$\frac{\partial \mathcal{L}}{\partial d_i} = -\frac{\partial W}{\partial g_k} \cdot \frac{\partial g_k}{\partial d_i} + \lambda c_i - \mu_i.$$

The components in the first term are respectively:

$$\frac{\partial W}{\partial g_k} = g_k(\mathbf{d})^{-\alpha}, \ \frac{\partial g_k}{\partial d_i} = \begin{cases} r_i^{1-\alpha} d_i^{-\alpha}, \ \text{if } 0 < \alpha \leq 1; \\ \left(\frac{\alpha - 1}{\sum_{i \in G_k}(r_i d_i)^{1-\alpha}}\right)^2 r_i^{1-\alpha} d_i^{-\alpha}, \ \text{if } \alpha > 1. \end{cases}$$

The stationarity condition $\frac{\partial \mathcal{L}}{\partial d_i} = 0$ simplifies to $g_k(\mathbf{d})^{-\alpha} \frac{\partial g_k}{\partial d_i} = \lambda c_i - \mu_i$. The complementary slackness condition $\mu_i d_i = 0$ implies that either $\mu_i = 0, d_i > 0$ or $\mu_i > 0, d_i = 0$. We observe that the latter case is impossible because it conflicts with the stationarity condition. Therefore, using $\mu_i = 0, d_i > 0$ for all i, we can obtain the following formula for d_i.

$$d_i = \begin{cases} (\lambda c_i g_k^{\alpha} r_i^{\alpha-1})^{-1/\alpha}, \text{ if } 0 < \alpha \leq 1; \\ \left(\lambda c_i g_k^{\alpha} r_i^{\alpha-1} \left(\frac{\sum_{i \in G_k} (r_i d_i)^{1-\alpha}}{\alpha-1}\right)^2\right)^{-1/\alpha}, \text{ if } \alpha > 1. \end{cases} \tag{15}$$

To further simplify (15), we define D_k, a coefficient that is constant for all members of group k.

$$D_k = \begin{cases} (\lambda g_k^{\alpha})^{-1/\alpha}, \text{ if } 0 < \alpha \leq 1; \\ \left(\lambda g_k^{\alpha} \left(\frac{\sum_{i \in G_k} (r_i d_i)^{1-\alpha}}{\alpha-1}\right)^2\right)^{-1/\alpha}, \text{ if } \alpha > 1. \end{cases} \tag{16}$$

Using D_k, we reformulate d_i as below, with $k(i)$ as the group index of i:

$$d_i = D_{k(i)} c_i^{-\frac{1}{\alpha}} r_i^{\frac{1-\alpha}{\alpha}}. \tag{17}$$

We substitute (17) back into the definition of $g_k(\mathbf{d})$.

$$g_k(\mathbf{d}) = \begin{cases} \frac{1}{1-\alpha} \sum_{i \in G_k} \left(D_k c_i^{-\frac{1}{\alpha}} r_i^{\frac{1}{\alpha}}\right)^{1-\alpha}, & \text{if } 0 < \alpha < 1; \\ (\alpha - 1) \left(\sum_{i \in G_k} (D_k c_i^{-\frac{1}{\alpha}} r_i^{\frac{1}{\alpha}})^{1-\alpha}\right)^{-1}, & \text{if } \alpha > 1. \end{cases} \tag{18}$$

We continue to define another coefficient term S_k to simplify the above g_k formula.

$$S_k = \sum_{i \in G_k} \left(c_i^{-1/\alpha} r_i^{1/\alpha}\right)^{1-\alpha}. \tag{19}$$

Then we reformulate (18) as:

$$g_k(\mathbf{d}) = \begin{cases} \frac{1}{1-\alpha} D_k^{1-\alpha} S_k, & \text{if } 0 < \alpha < 1; \\ (\alpha - 1)(D_k^{1-\alpha} S_k)^{-1}, & \text{if } \alpha > 1. \end{cases} \tag{20}$$

Next, we substitute these formulas back into the definition of D_k in (16). When $\alpha < 1$, the following derivation applies. Note that the final formula is always valid since $-2\alpha + \alpha^2 \neq 0$ for all $0 < \alpha < 1$.

$$\begin{aligned} D_k^{-\alpha} = \lambda \left(\frac{1}{1-\alpha} D_k^{1-\alpha} S_k\right)^{\alpha} &\Rightarrow D_k^{-\alpha-\alpha(1-\alpha)} = \lambda (1-\alpha)^{-\alpha} S_k^{\alpha} \\ &\Rightarrow D_k = \left(\lambda (1-\alpha)^{-\alpha} S_k^{\alpha}\right)^{\frac{1}{-2\alpha+\alpha^2}}. \end{aligned}$$

When $\alpha > 1$, we have the following derivation. Similarly, the final formula is valid because $-\alpha^2 + 2\alpha - 2 \neq 0$ for all $\alpha > 1$.

$$\begin{aligned} D_k^{-\alpha} = \lambda \left(\frac{\alpha-1}{D_k^{1-\alpha} S_k}\right)^{\alpha} \left(\frac{\sum_{i \in G_k} (r_i d_i)^{1-\alpha}}{\alpha - 1}\right)^2 &\Rightarrow D_k^{-\alpha} = \lambda \left(\frac{\alpha-1}{D_k^{1-\alpha} S_k}\right)^{\alpha} \left(\frac{D_k^{1-\alpha} S_k}{\alpha-1}\right)^2 \\ &\Rightarrow D_k^{-\alpha} = \lambda (\alpha-1)^{\alpha-2} (D_k^{1-\alpha} S_k)^{-\alpha+2} \\ &\Rightarrow D_k = \left(\lambda (\alpha-1)^{\alpha-2} S_k^{-\alpha+2}\right)^{\frac{1}{-\alpha^2+2\alpha-2}}. \end{aligned}$$

Lastly, we plug d_i formulas (17) with the above D_k expressions into the budget constraint $\sum_{i=1}^n c_i d_i = Q$. For each group k, recall $S_k = \sum_{i \in G_k} c_i^{\frac{\alpha-1}{\alpha}} r_i^{\frac{1-\alpha}{\alpha}}$, then

$$\sum_{i \in G_k} c_i d_i = D_k \sum_{i \in G_k} c_i^{\frac{\alpha-1}{\alpha}} r_i^{\frac{1-\alpha}{\alpha}} = D_k S_k.$$

Together with the simplified D_k formulas, we can solve for λ, which can be further plugged back into D_k. When $\alpha < 1$, we follow the steps below to conclude $\mathbf{d}$.

$$\begin{aligned}
\sum_{k=1}^{K}\sum_{i\in G_k} c_i d_i = Q \;&\Rightarrow \sum_{k=1}^{K} S_k \left(\lambda(1-\alpha)^{-\alpha} S_k^{\alpha}\right)^{\frac{1}{-2\alpha+\alpha^2}} = Q \\
&\Rightarrow \lambda^{\frac{1}{-2\alpha+\alpha^2}} \sum_{k=1}^{K} S_k \left((1-\alpha)^{-\alpha} S_k^{\alpha}\right)^{\frac{1}{-2\alpha+\alpha^2}} = Q \\
&\Rightarrow D_k = \left(\frac{S_k}{1-\alpha}\right)^{\frac{\alpha}{-2\alpha+\alpha^2}} \frac{Q}{\sum_{k=1}^{K} S_k \left(\frac{S_k}{1-\alpha}\right)^{\frac{\alpha}{-2\alpha+\alpha^2}}} \\
&\Rightarrow d_i = \left(\frac{S_k}{1-\alpha}\right)^{\frac{1}{-2+\alpha}} \frac{Q c_i^{-\frac{1}{\alpha}} r_i^{\frac{1-\alpha}{\alpha}}}{\sum_{k=1}^{K} S_k \left(\frac{S_k}{1-\alpha}\right)^{\frac{1}{-2+\alpha}}}
\end{aligned}$$

When $\alpha > 1$, we derive the following:

$$\begin{aligned}
\sum_{k=1}^{K}\sum_{i\in G_k} c_i d_i = Q \;&\Rightarrow \sum_{k=1}^{K} S_k \left(\lambda(\alpha-1)^{\alpha-2} S_k^{-\alpha+2}\right)^{\frac{1}{-\alpha^2+2\alpha-2}} = Q \\
&\Rightarrow \lambda^{\frac{1}{-\alpha^2+2\alpha-2}} \sum_{k=1}^{K} S_k \left((\alpha-1)^{\alpha-2} S_k^{-\alpha+2}\right)^{\frac{1}{-\alpha^2+2\alpha-2}} = Q \\
&\Rightarrow D_k = \left(\frac{S_k}{\alpha-1}\right)^{\frac{-\alpha+2}{-\alpha^2+2\alpha-2}} \frac{Q}{\sum_{k=1}^{K} S_k \left(\frac{S_k}{\alpha-1}\right)^{\frac{-\alpha+2}{-\alpha^2+2\alpha-2}}} \\
&\Rightarrow d_i = \left(\frac{S_k}{\alpha-1}\right)^{\frac{-\alpha+2}{-\alpha^2+2\alpha-2}} \frac{Q c_i^{-\frac{1}{\alpha}} r_i^{\frac{1-\alpha}{\alpha}}}{\sum_{k=1}^{K} S_k \left(\frac{S_k}{\alpha-1}\right)^{\frac{-\alpha+2}{-\alpha^2+2\alpha-2}}}
\end{aligned}$$

- **Special case** $\alpha = 1$**:** We follow the same strategy as in the general case. Using the KKT conditions, we again derive the expression $d_i = D_{k(i)} c_i^{-\frac{1}{\alpha}} r_i^{\frac{1-\alpha}{\alpha}}$ in (17). Since $\alpha = 1$, this formula simplifies $d_i = \frac{D_k}{c_i}$ and $D_k = \frac{1}{\lambda g_k^{\alpha}}$. To determine $\mathbf{d}$, we only need to solve for D_k. We plug all d_i formulas into the budget constraint to obtain $\sum_{k\in[K]}\sum_{i\in G_k} c_i \frac{D_k}{c_i} = \sum_{k\in[K]} |G_k| D_k = Q$, where $|G_k|$ is the size of group k. The budget constraint is satisfied by setting $D_k = \frac{Q}{K|G_k|}$. Therefore, we conclude the closed-form solution when $\alpha = 1$: $d_i = \frac{Q}{K|G_k|c_i}$, $\forall i \in G_k$.
- **Special case** $\alpha \to \infty$**:** The objective is $\max\min_k g_k(\mathbf{d})$. The utility $g_k(\mathbf{d})$ for large α is a harmonic-mean-like function, which is dominated by the individual with the smallest utility $r_i d_i$ within that group. To maximize the minimum of these group utilities (each of which is determined by its own internal minimum), all individual utilities must be equalized across all groups. Essentially, the optimal solution needs to maximize the minimum utility by attaining an optimum equal utilities for each i. Let $r_i d_i = C$ for some constant C. From this, we have $d_i = C/r_i$. Substituting into the budget constraint gives $\sum_i c_i(C/r_i) = C\sum_i (c_i/r_i) = Q$. Solving for C yields $C = Q/\sum_j (c_j/r_j)$. The closed form solution is $d_i = C/r_i = \frac{Q}{r_i \sum_j (c_j/r_j)}$.

A.2. Individual-Based Fairness

PROPOSITION 7 **(Individual-based Closed-Form Decisions for Special Case** α**).** *For special cases of* $\alpha = 0, 1, \infty$*, the optimal solution to* (13) *with parameter* $\mathbf{r}$ *is, respectively, given by:*

- $\alpha = 0$: $d_i^* = \begin{cases} \frac{Q}{c_i}, & \text{if } i = \arg\max_j \frac{r_j}{c_j}; \\ 0, & \text{otherwise.} \end{cases}$, $\forall i \in [n]$, *representing the allocation of the entire budget to the stakeholder with the highest reward-to-cost ratio.*
- $\alpha = 1$: $d_i^* = \frac{Q}{nc_i}$, $\forall i \in [n]$.
- $\alpha \to \infty$: $d_i^* = \frac{Q}{r_i \sum_{j=1}^n \frac{c_j}{r_j}}$, $\forall i \in [n]$, *representing an allocation to equalize the utilities of all stakeholders.*

PROPOSITION 8 **(Individual-based Analytical Gradient Formulas for Special Case** α**).** *Let* $\mathbf{d}^*$ *denote the optimal solution to* (13) *with parameter* $\mathbf{r}$. *For special cases of* $\alpha = 0, 1, \infty$, *the Jacobian* $\frac{\partial \mathbf{d}^*(\mathbf{r})}{\partial \mathbf{r}}$ *is, respectively, given by:*

- $\alpha = 0$: $\mathbf{d}^*(\mathbf{r})$ *is not differentiable at the maximizer* $i^* = \arg\max_j \frac{r_j}{c_j}$, *and has* $\frac{\partial d_i^*}{\partial r_j} = 0$ *for all* $i \neq i^*$ *and all* j.
- $\alpha = 1$: $\frac{\partial d_i^*}{\partial r_k} = 0$ *for all* $i, k \in [n]$ *as* $\mathbf{d}^*(\mathbf{r})$ *does not depend on* $\mathbf{r}$.
- $\alpha \to \infty$: *for all* $i \in [n]$, $\frac{\partial d_i^*}{\partial r_k} = \begin{cases} -\frac{Q}{r_k^2(\sum_{j=1}^n c_j/r_j)} + \frac{Qc_k}{r_k^3(\sum_{j=1}^n c_j/r_j)^2}, & \text{if } i = k; \\ \frac{Qc_k}{r_i r_k^2(\sum_{j=1}^n c_j/r_j)^2}, & \text{if } i \neq k \end{cases}$.

Proof for closed-form decisions:

- **Special case** $\alpha = 0$**:** The objective simplifies to $\max_{\mathbf{d}} \sum r_i d_i$, which is the same as in the case of $\alpha = 0$ under group-based fairness. The derivation for closed-form decisions and gradients is identical to the proof given in the group-based fairness case.
- **General case** $\alpha > 0$ **and** $\alpha \neq 1$**:** We begin by formulating the Lagrangian for problem (13):

$$\mathcal{L}(\mathbf{d}, \lambda, \boldsymbol{\mu}) = \sum_{i=1}^n \frac{(r_i d_i)^{1-\alpha}}{1-\alpha} - \lambda \left(\sum_{i=1}^n c_i d_i - Q \right) - \sum_{i=1}^n \mu_i(-d_i).$$

For any $\alpha > 0$, the objective function penalizes zero allocations with either an infinite marginal utility (for $0 < \alpha < 1$) or an infinite penalty (for $\alpha \geq 1$), ensuring an interior solution $d_i^* > 0$ given $Q, c_i, r_i > 0$. By complementary slackness, this implies $\mu_i = 0$ for all i. The KKT stationarity condition $\frac{\partial \mathcal{L}}{\partial d_i} = 0$ is:

$$(r_i)^{1-\alpha} d_i^{-\alpha} - \lambda c_i = 0 \implies d_i = (\lambda c_i)^{-1/\alpha} r_i^{\frac{1-\alpha}{\alpha}}.$$

At the optimum, the budget constraint holds with equality. We substitute d_i into the constraint to find λ:

$$\sum_{j=1}^n c_j \left((\lambda c_j)^{-1/\alpha} r_j^{\frac{1-\alpha}{\alpha}} \right) = \lambda^{-1/\alpha} \sum_{j=1}^n c_j^{1-\frac{1}{\alpha}} r_j^{\frac{1-\alpha}{\alpha}} = Q.$$

Solving for the term $\lambda^{-1/\alpha}$ gives:

$$\lambda^{-1/\alpha} = \frac{Q}{\sum_{j=1}^n c_j^{1-\frac{1}{\alpha}} r_j^{\frac{1-\alpha}{\alpha}}}.$$

Substituting this back into the expression for d_i gives the closed form solution formula $d_i = \frac{c_i^{-\frac{1}{\alpha}} \cdot r_i^{\frac{1}{\alpha}-1} \cdot Q}{\sum_{j=1}^n c_j^{1-\frac{1}{\alpha}} \cdot r_j^{\frac{1}{\alpha}-1}}$, $\forall i \in [n]$. The gradient in Proposition 4 follows from direct differentiation.

- **Special case** $\alpha = 1$**:** The objective becomes $\max \sum_{i=1}^n \log(r_i d_i)$. The Lagrangian is $\mathcal{L} = \sum_{i=1}^n \log(r_i d_i) - \lambda(\sum_i c_i d_i - Q)$. The objective's logarithmic nature ensures an interior solution $d_i^* > 0$. The KKT stationarity condition is $\frac{\partial \mathcal{L}}{\partial d_i} = \frac{1}{d_i} - \lambda c_i = 0$, which implies $d_i = 1/(\lambda c_i)$. Substituting this into the budget constraint gives $\sum_i c_i (1/(\lambda c_i)) = \sum_i 1/\lambda = n/\lambda = Q$, yielding $\lambda = n/Q$. Therefore, $d_i^* = Q/(nc_i)$. This expression is independent of $\mathbf{r}$, so its gradient is zero.

Special case $\alpha \to \infty$**:** The objective becomes $\max \min_i \{r_i d_i\}$. To maximize the minimum utility, all utilities must be equal at the optimum. The same derivation for the case of $\alpha \to \infty$ under group-based fairness applies here.

Appendix B: Statistics on Dataset used in Experiments

Table 6 and Table 7 present statistics of the key variables used in our experiments.

Table 6 Descriptive statistics of Dataset: Mean

	Count	Original Number of Chronic Illnesses (h_i)	Original Avoidable Cost (s_i)	Unscaled Benefit Score (r_i)	Unscaled Healthcare Cost (c_i)
White	4422	1.25	2335.91	0.11	7380.42
Black	578	2.03	2791.87	0.15	8785.47

Table 7 Descriptive statistics of Dataset: Standard Deviation

	Count	Original Number of Chronic Illnesses (h_i)	Original Avoidable Cost (s_i)	Unscaled Benefit Score (r_i)	Unscaled Healthcare Cost (c_i)
White	4422	1.79	13792.69	0.15	18653.43
Black	578	2.34	11280.94	0.18	25477.35

Appendix C: Additional Experiment Results

This section presents additional experimental results from using linear regression (LR) and neural network (NN) predictors and different choices of λ for fairness regularization. Tables 8, 9, 10, 11 contain results from using group-based fairness definitions, and Tables 12, 13, 14, 15 contain results from using individual-based fairness definitions. All results report the mean and standard deviation of specific performance metrics over 5 trials using different seeds.

Table 8 Decision regret results: group-based fairness

Algorithm	Predictor	λ	$\alpha = 0.5$	$\alpha = 2.0$
FPTO	LR	0.0	0.1019 ± 0.0002	0.4741 ± 0.0048
		0.05	0.1019 ± 0.0002	0.4740 ± 0.0046
		0.5	0.1015 ± 0.0003	0.4699 ± 0.0050
	NN	0.0	0.0176 ± 0.0004	0.0983 ± 0.0018
		0.05	0.0176 ± 0.0004	0.0979 ± 0.0015
		0.5	0.0179 ± 0.0011	0.0968 ± 0.0039
FDFL-CF	LR	0.0	0.0234 ± 0.0003	0.0737 ± 0.0016
		0.05	0.0915 ± 0.0006	0.0747 ± 0.0013
		0.5	0.0916 ± 0.0005	0.1004 ± 0.0025
	NN	0.0	0.0189 ± 0.0012	0.0358 ± 0.0005
		0.05	0.0735 ± 0.0015	0.0389 ± 0.0004
		0.5	0.0763 ± 0.0013	0.0425 ± 0.0001
FDFL-FD	LR	0.0	0.0444 ± 0.0000	0.1470 ± 0.0022
		0.05	0.0727 ± 0.0002	0.1455 ± 0.0026
		0.5	0.0728 ± 0.0002	0.1600 ± 0.0036
	NN	0.0	0.0223 ± 0.0035	0.0544 ± 0.0041
		0.05	0.0801 ± 0.0007	0.0694 ± 0.0081
		0.5	0.0810 ± 0.0007	0.0618 ± 0.0088
FoldOpt	LR	0.0	0.1278 ± 0.0006	0.1117 ± 0.0119
		0.05	0.1278 ± 0.0006	0.1129 ± 0.0115
		0.5	0.1278 ± 0.0006	0.1129 ± 0.0115
	NN	0.0	0.1325 ± 0.0039	0.1195 ± 0.0068
		0.05	0.1325 ± 0.0039	0.1347 ± 0.0138
		0.5	0.1325 ± 0.0039	0.1347 ± 0.0138

Table 9 Prediction MSE results: group-based fairness

Algorithm	Predictor	λ	$\alpha = 0.5$	$\alpha = 2.0$
FPTO	LR	0.0	208.8934 ± 0.8357	208.8934 ± 0.8357
		0.05	208.8758 ± 0.8303	208.8758 ± 0.8303
		0.5	209.2880 ± 1.1204	209.2880 ± 1.1204
	NN	0.0	35.4627 ± 1.2644	35.4627 ± 1.2644
		0.05	35.3691 ± 1.2873	35.3691 ± 1.2873
		0.5	35.2676 ± 2.0286	35.2676 ± 2.0286
FDFL-CF	LR	0.0	262.6600 ± 2.2327	239.5040 ± 1.5876
		0.05	253.9755 ± 0.9030	236.3998 ± 1.1029
		0.5	253.8507 ± 0.8424	222.0302 ± 0.1822
	NN	0.0	207.9338 ± 12.7443	119.3416 ± 2.8384
		0.05	218.2368 ± 9.4937	112.0740 ± 3.9208
		0.5	220.4741 ± 9.3006	106.9212 ± 3.0555
FDFL-FD	LR	0.0	319.2976 ± 0.8805	296.5613 ± 2.6815
		0.05	300.2613 ± 1.2245	301.8743 ± 1.6218
		0.5	300.2064 ± 1.2518	282.6524 ± 3.9307
	NN	0.0	246.7065 ± 41.6353	188.4053 ± 32.0814
		0.05	257.2754 ± 7.6626	225.3377 ± 1.1809
		0.5	257.7810 ± 7.9979	190.4567 ± 13.8893
FoldOpt	LR	0.0	364.0685 ± 15.3294	362.8136 ± 15.3300
		0.05	364.0685 ± 15.3294	352.4806 ± 19.2327
		0.5	364.0685 ± 15.3294	352.4806 ± 19.2327
	NN	0.0	363.7616 ± 15.4504	366.9125 ± 18.7837
		0.05	363.7616 ± 15.4504	328.9272 ± 22.5315
		0.5	363.7616 ± 15.4504	328.9056 ± 22.5657

Table 10 Prediction fairness (MAD) results: group-based fairness

Algorithm	Predictor	λ	$\alpha = 0.5$	$\alpha = 2.0$
FPTO	LR	0.0	28.6400 ± 0.3371	28.6400 ± 0.3371
		0.05	28.6206 ± 0.3259	28.6206 ± 0.3259
		0.5	28.7634 ± 0.4367	28.7634 ± 0.4367
	NN	0.0	1.7344 ± 0.1045	1.7344 ± 0.1045
		0.05	1.8268 ± 0.0954	1.8268 ± 0.0954
		0.5	1.8322 ± 0.3261	1.8322 ± 0.3261
FDFL-CF	LR	0.0	48.2703 ± 0.6530	58.5934 ± 0.5557
		0.05	37.6696 ± 0.5499	53.0743 ± 0.2425
		0.5	37.7932 ± 0.6163	35.6152 ± 0.4845
	NN	0.0	24.5686 ± 3.0886	32.4739 ± 0.4682
		0.05	30.5865 ± 4.0275	37.4980 ± 2.8442
		0.5	31.9948 ± 4.4498	21.1977 ± 2.9267
FDFL-FD	LR	0.0	63.6933 ± 0.4787	60.5401 ± 0.0245
		0.05	53.0269 ± 0.1726	63.0944 ± 1.6179
		0.5	53.0252 ± 0.2076	50.8770 ± 2.1584
	NN	0.0	39.4929 ± 11.4958	51.0395 ± 6.1237
		0.05	26.3016 ± 1.3442	42.6929 ± 3.8612
		0.5	26.5533 ± 1.2669	30.2942 ± 0.3262
FoldOpt	LR	0.0	91.3055 ± 8.1582	88.9219 ± 9.9793
		0.05	91.3055 ± 8.1582	77.1218 ± 11.1481
		0.5	91.3055 ± 8.1582	77.1218 ± 11.1481
	NN	0.0	91.8474 ± 8.1903	92.0644 ± 6.9585
		0.05	91.8474 ± 8.1903	58.8679 ± 12.0956
		0.5	91.8474 ± 8.1903	58.8678 ± 12.0956

Table 11 Training time in seconds: group-based fairness

Algorithm	Predictor	Lambda	$\alpha = 0.5$	$\alpha = 2.0$
FPTO	LR	0.0	1.8909 ± 0.4243	2.2822 ± 0.0001
		0.05	2.3208 ± 0.0785	2.3862 ± 0.1126
		0.5	2.3467 ± 0.0247	2.5401 ± 0.2606
	NN	0.0	1.2987 ± 0.0048	1.2996 ± 0.0329
		0.05	1.2830 ± 0.0500	1.3325 ± 0.0338
		0.5	1.3070 ± 0.0671	1.2913 ± 0.0412
FDFL-CF	LR	0.0	8.9967 ± 0.0408	9.0340 ± 0.1632
		0.05	3.1345 ± 0.0094	8.9340 ± 0.1853
		0.5	3.0887 ± 0.2231	9.0255 ± 0.0321
	NN	0.0	7.6495 ± 0.0986	10.1872 ± 0.0213
		0.05	3.1456 ± 0.1892	10.4324 ± 0.1570
		0.5	3.2721 ± 0.0649	10.2478 ± 0.1541
FDFL-FD	LR	0.0	85.5750 ± 1.0894	100.3663 ± 1.4406
		0.05	34.8349 ± 0.0017	99.8769 ± 0.8692
		0.5	34.1850 ± 0.3900	99.5620 ± 0.7987
	NN	0.0	78.2144 ± 4.0072	103.6114 ± 0.4983
		0.05	30.2685 ± 0.3014	91.8562 ± 10.4313
		0.5	30.1630 ± 0.2920	102.2925 ± 0.7698
FoldOpt	LR	0.0	5.4392 ± 0.0613	5.4296 ± 0.0474
		0.05	5.3969 ± 0.0515	5.4522 ± 0.0274
		0.5	5.3300 ± 0.0408	5.5180 ± 0.0438
	NN	0.0	5.5577 ± 0.0307	5.5543 ± 0.0590
		0.05	5.5588 ± 0.0120	5.4581 ± 0.0388
		0.5	5.4976 ± 0.0547	5.5451 ± 0.0407

Table 12 Decision regret results: individual-based fairness

Algorithm	Predictor	λ	$\alpha = 0.5$	$\alpha = 2.0$
FPTO	LR	0.0	0.1680 ± 0.0003	0.3847 ± 0.0035
		0.05	0.1686 ± 0.0003	0.3873 ± 0.0038
		0.5	0.1720 ± 0.0008	0.4015 ± 0.0057
	NN	0.0	0.0363 ± 0.0008	0.0960 ± 0.0012
		0.05	0.0374 ± 0.0008	0.0968 ± 0.0016
		0.5	0.0473 ± 0.0007	0.1116 ± 0.0020
FDFL-CF	LR	0.0	0.0263 ± 0.0004	0.0737 ± 0.0016
		0.05	0.0264 ± 0.0004	0.0769 ± 0.0018
		0.5	0.0267 ± 0.0005	0.1266 ± 0.0010
	NN	0.0	0.0104 ± 0.0003	0.0358 ± 0.0005
		0.05	0.0103 ± 0.0003	0.0407 ± 0.0006
		0.5	0.0099 ± 0.0001	0.0367 ± 0.0001
FDFL-FD	LR	0.0	0.0795 ± 0.0011	0.1501 ± 0.0006
		0.05	0.0781 ± 0.0018	0.1493 ± 0.0002
		0.5	0.0786 ± 0.0020	0.1635 ± 0.0032
	NN	0.0	0.0185 ± 0.0030	0.0609 ± 0.0102
		0.05	0.0146 ± 0.0018	0.0521 ± 0.0015
		0.5	0.0152 ± 0.0006	0.0524 ± 0.0043
FoldOpt	LR	0.0	0.1324 ± 0.0079	0.1550 ± 0.0039
		0.05	0.1323 ± 0.0080	0.1557 ± 0.0044
		0.5	0.1322 ± 0.0079	0.1776 ± 0.0038
	NN	0.0	0.1247 ± 0.0046	0.0329 ± 0.0029
		0.05	0.1247 ± 0.0046	0.0306 ± 0.0024
		0.5	0.1247 ± 0.0046	0.0438 ± 0.0035

Table 13 Prediction MSE results: individual-based fairness

Algorithm	Predictor	λ	$\alpha = 0.5$	$\alpha = 2.0$
FPTO	LR	0.0	208.8934 ± 0.8357	208.8934 ± 0.8357
		0.05	208.9190 ± 0.8265	208.9190 ± 0.8265
		0.5	209.2074 ± 0.7166	209.2074 ± 0.7166
	NN	0.0	35.4627 ± 1.2644	35.4627 ± 1.2644
		0.05	35.9598 ± 1.2627	35.9598 ± 1.2627
		0.5	40.9074 ± 1.5535	40.9074 ± 1.5535
FDFL-CF	LR	0.0	249.9550 ± 0.4244	239.5040 ± 1.5876
		0.05	249.5863 ± 0.3867	233.6981 ± 1.7326
		0.5	245.7071 ± 0.9966	189.4951 ± 0.2940
	NN	0.0	119.9672 ± 12.1895	119.3431 ± 2.8367
		0.05	101.2593 ± 17.7797	53.1035 ± 3.3320
		0.5	14.0935 ± 0.5389	14.3596 ± 0.2939
FDFL-FD	LR	0.0	309.4249 ± 3.2944	306.0839 ± 7.2223
		0.05	311.0781 ± 2.8873	292.1106 ± 0.1010
		0.5	309.8823 ± 0.9849	272.6047 ± 0.1271
	NN	0.0	190.9773 ± 34.3295	211.2497 ± 21.2805
		0.05	145.6472 ± 23.0778	192.2392 ± 3.8204
		0.5	164.9002 ± 1.1193	44.9116 ± 6.9998
FoldOpt	LR	0.0	350.3413 ± 20.2815	322.6967 ± 11.9854
		0.05	350.4042 ± 20.1777	318.1791 ± 11.3882
		0.5	350.0677 ± 20.5458	284.8901 ± 11.0219
	NN	0.0	334.9719 ± 12.7858	200.6606 ± 16.1298
		0.05	334.9088 ± 12.7697	161.8870 ± 15.6173
		0.5	334.9495 ± 12.8280	71.8843 ± 15.1922

Table 14 Prediction fairness (MAD) results: individual-based fairness

Algorithm	Predictor	λ	$\alpha = 0.5$	$\alpha = 2.0$
FPTO	LR	0.0	287.6690 ± 0.9304	287.6690 ± 0.9304
		0.05	287.4169 ± 0.9522	287.4169 ± 0.9522
		0.5	286.2050 ± 0.9299	286.2050 ± 0.9299
	NN	0.0	45.8432 ± 1.4942	45.8432 ± 1.4942
		0.05	46.0048 ± 1.4603	46.0048 ± 1.4603
		0.5	47.2346 ± 1.9825	47.2346 ± 1.9825
FDFL-CF	LR	0.0	340.3346 ± 0.4798	327.9019 ± 2.0445
		0.05	339.7767 ± 0.4510	319.8157 ± 2.2505
		0.5	334.2433 ± 1.2528	261.8894 ± 0.5599
	NN	0.0	160.2802 ± 17.0537	159.0209 ± 4.2124
		0.05	134.6005 ± 24.4601	67.3570 ± 4.5287
		0.5	20.6355 ± 0.9980	20.8680 ± 0.4506
FDFL-FD	LR	0.0	426.8138 ± 4.3966	423.6866 ± 8.6738
		0.05	429.0042 ± 4.1919	406.0150 ± 0.0050
		0.5	427.4020 ± 0.8930	380.5660 ± 0.0494
	NN	0.0	258.0777 ± 43.9605	290.9811 ± 27.9017
		0.05	198.6770 ± 30.2688	260.5411 ± 5.5329
		0.5	222.4113 ± 0.1263	58.7328 ± 10.0792
FoldOpt	LR	0.0	483.2485 ± 25.6683	446.0058 ± 13.4441
		0.05	483.4956 ± 25.3096	440.3739 ± 12.7412
		0.5	482.8425 ± 26.0771	399.4644 ± 11.5138
	NN	0.0	464.9778 ± 12.9515	266.6893 ± 21.4344
		0.05	464.9798 ± 12.9548	210.6158 ± 21.8302
		0.5	464.9783 ± 12.9525	83.8750 ± 19.6823

Table 15 Training time in seconds: individual-based fairness

Algorithm	Predictor	λ	$\alpha = 0.5$	$\alpha = 2.0$
FPTO	LR	0.0	2.0203 ± 0.0277	2.2036 ± 0.0784
		0.05	2.1642 ± 0.0942	2.0219 ± 0.0300
		0.5	2.0870 ± 0.1006	2.1175 ± 0.1081
	NN	0.0	1.2830 ± 0.0166	1.2746 ± 0.0313
		0.05	1.2821 ± 0.0379	1.2998 ± 0.0495
		0.5	1.3054 ± 0.0555	1.2675 ± 0.0485
FDFL-CF	LR	0.0	6.9646 ± 0.5505	6.6350 ± 0.1740
		0.05	7.4325 ± 0.2365	6.9338 ± 0.1572
		0.5	7.0560 ± 0.4732	6.7906 ± 0.0618
	NN	0.0	7.5437 ± 0.4503	7.7938 ± 0.0406
		0.05	7.6349 ± 0.6052	7.8034 ± 0.0027
		0.5	7.4795 ± 0.2586	8.2520 ± 0.2871
FDFL-FD	LR	0.0	31.6774 ± 0.0265	31.5571 ± 0.1472
		0.05	31.3788 ± 0.0158	31.8195 ± 0.2763
		0.5	31.2279 ± 0.1334	31.7332 ± 0.0100
	NN	0.0	31.2052 ± 0.1342	31.8402 ± 0.4804
		0.05	31.3667 ± 0.1038	32.0444 ± 0.1237
		0.5	31.4317 ± 0.1755	32.0983 ± 0.5131
FoldOpt	LR	0.0	5.4498 ± 0.0368	5.5284 ± 0.1099
		0.05	5.4593 ± 0.0241	5.5895 ± 0.0793
		0.5	5.3375 ± 0.0690	5.5354 ± 0.0866
	NN	0.0	5.6179 ± 0.0283	5.7159 ± 0.0395
		0.05	5.4080 ± 0.0807	5.5624 ± 0.0191
		0.5	5.3891 ± 0.0595	5.4628 ± 0.0453

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