ASTR 4260: Problem Set #2

Due: Wednesday, September 27, 2023

Submit in a new subdirectory in the same repository as PS 1.

Problem 1

Write a function that implements a higher-order integrator (either the trapezoidal rule or, preferably, Simpson's rule). Use this integrator to repeat problem 3 from the last problem set, that is, to numerically compute the following integral:

$$\int_0^{\pi/2} \sin(x) dx$$

Again, report the result for a range of step sizes, with the number of steps $N = 10, 10^2, 10^3, 10^4, 10^5$, and again, determine how the fractional error decreases with decreasing step size (increasing N). How does this differ from the method used in the previous problem set? What order is this method?

Problem 2

In problem set 1, you evaluated an analytic function that gave you the ratio of the obscured to unobscured flux of a star during a planetary transit. This was done for a star with a uniform brightness across its disk. However, most stars are actually brighter in their centers and dimmer at their edges – a feature known as limb darkening.[1] This limb darkening can be parameterized with a function $I(r)$, which is the intensity of the sun's surface as a function of radius r (or alternately angle θ – see Fig. 1a of Problem set 1).

This makes the transit calculation more complicated.[2] To complete it, we need to evaluate the ratio of two integrals:

$$F(p, |z|) = \frac{\int_0^1 I(r)\,[1 - \delta(p, r, z)]\, 2r\mathrm{d}r}{\int_0^1 I(r) 2r\mathrm{d}r} \tag{1}$$

where

$$\delta(p, r, z) = \begin{cases} 0 & r \geq z + p \quad \text{or} \quad r \leq z - p, \\ 1 & r + z \leq p \\ \pi^{-1} \arccos[(z^2 - p^2 + r^2)/(2zr)] & \text{otherwise} \end{cases} \tag{2}$$

[1] This is due to the fact that we actually see into the outer layers of a star, rather than a hard surface. When we look at the center (as opposed to the edge), we see deeper into the star, where the gas is hotter and therefore brighter

[2] See Mandel & Agol, 2002, ApJ, 580, L171 for more details.

Use the integrator from the first problem of this problem set to evaluate the function $F(p, |z|)$. Note that it will require two integrals for each evaluation of the function. In particular, I want you to evaluate F using no limb-darkening (i.e. $I(r) = 1$). Do this for the values $p = 0.2$ and $z = 0.9$ and, again, try using different numbers of steps to calculate the integral: $N = 10, 10^2, 10^3, 10^4, 10^5$. How does the fractional error decrease as N increases? Use the analytic formula in the first problem set to check your result.

Note: Since $I(r) = 1$ is basically the same as ignoring $I(r)$, you might be tempted to do just that and ignore it, but resist the temptation. In later problems, we will use this same code with a more interesting limb-darkening function.

Problem 3

Use a Monte-Carlo integration to evaluate the same integrals as in the last problem (again assuming $I(r) = 1$). To do this, generate N random x and y values that are each drawn from a uniform distribution from -1 to 1 (so that you are picking random points inside a box that covers the unit circle). Reject points that lie outside the unit circle (i.e. for which $x^2 + y^2 > 1$). Call the number of accepted points N_1. In addition, count how many of the accepted points lie within the eclipsing planet disk (i.e. for which $(x - z)^2 + y^2 < p^2$) and call that number N_2. Then an estimate for $F(p, z)$ is the ratio of points inside the star's disk that do not lie in the planet's disk to the number of points inside the star's disk (without the planet): $F(p, z) \approx (N_1 - N_2)/N_1$.

Evaluate this for the same p and z values as above. Again repeat for the same range of N values and determine how the error decreases with N. What effective order is this method? (In order to get a clear answer, you may have to repeat the random realizations a number of times and average the results.) Discuss your result in the context of the nature of the problem.

To generate a random number from -1 to 1 in python[3], use:

```
import random
x = random.uniform(-1, 1)
```

Graduates: Problem 4

Evaluate the integral above (using any method) but this time use

$$I(r) = 1 - \sum_{n=1}^{4} c_n (1 - \mu^{n/2})$$

where $\mu = \cos\theta = (1 - r^2)^{1/2}$ (note that $0 \leq r \leq 1$ since r is the normalized radius). The values c_n depend on the particular stellar type. To reproduce Mandel & Agol (2002), Figure

[3]You could also use numpy.random.uniform to generate an array of random values.

5, set $c_i = 1$ for each i from 1 to 4, with all other $c_n = 0$ for $n \neq i$. Evaluate for $p = 0.1$ and many z values from $z = -1.2$ to $z = 1.2$ and overplot all the functions, along with the case $I(r) = 1$.