ASTR 4260: Problem Set #5

Due: Wednesday, October 25, 2023

Problem 1

Write an N-body code for integrating solar system dynamics. Use the leapfrog or RK4 method to integrate the orbits of N bodies under their mutual self-gravity. (*Graduates:* please implement both methods with a way to choose between them.) You can assume N is small enough to evaluate the gravitational accelerations by direct summation (you may do this in either 2 or 3 dimensions). Note that leapfrog requires evaluation of new positions, accelerations from those positions, and new velocities, in that order.

Test the method on a system with $N = 2$ where the mass of the central object (e.g. the sun) is much, much larger than the satellite (e.g. the earth). Use a circular orbit ($v_c = \sqrt{GM/r}$). Plot the x and y positions of the objects over time to visualize the orbits. Check to see how much the radius of the satellite changes after five orbital periods. Also monitor the total energy (kinetic plus potential) of the system. If the radius and energy change too much (i.e. more than a few tenths of a percent per orbit), you will need to decrease the time step of your integration or improve the accuracy of your integrator.

Graduates: Comparison of positions and total energies for leapfrog and RK4 results will be enlightening. Which is more accurate over short timescales? Over long timescales?

Problem 2

Some extrasolar planetary systems observed have Jupiter-mass planets in very elliptical orbits. Set up a hypothetical solar system composed of three bodies: (1) a one solar mass central star, (2) an earth-mass planet with a circular orbit at 1 A.U. (astronomical unit), and (3) a jupiter-mass planet with a semi-major axis $a = 4$ A.U. (similar to, but slightly lower than Jupiter's) and an eccentricity of $e = 0.6$.[1] Integrate the system for as many years (i.e. earth orbits) as you can, but at least one hundred. What happens? Plot the orbits and check the evolution of the total energy to confirm your results.

[1] It may be helpful to know, as can be confirmed with an elementary dynamics textbook, that when the jupiter-mass planet is at pericenter (closest approach), $r = (1-e)a$, its velocity $v_{\mathrm{tan}} = \sqrt{(GM(1+e)/(a(1-e))}$ is entirely tangential.