解决99%二叉树问题的算法——分治法 Divide & Conquer

主讲人 令狐冲
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分治法 Divide & Conquer

将大规模问题拆分为若干个小规模的**同类型问题**去处理的算法

分治法和二分法（Binary Search）有什么区别?

分治法 Divide & Conquer

二分法会每次丢弃掉一半

分治法分完以后两边都要处理

什么样的数据结构适合分治法？

数组：一个大数组可以拆分为若干个不相交的子数组

归并排序，快速排序，都是基于数组的分治法

二叉树：整棵树的左子树和右子树都是二叉树

二叉树的大部分题都可以使用分治法解决

独孤九剑 —— 破枪式

碰到二叉树的问题，就想想整棵树在该问题上的结果
和左右儿子在该问题上的结果之间的联系是什么

二叉树的高度是多少?

A: O(n)

B: O(logn)

C: O(h)

二叉树的高度是多少?

最坏 O(n) 最好 O(logn)

一般用 O(h) 来表示更合适

二叉树考点剖析

考察形态：二叉树上求值，求路径

代表例题：http://www.lintcode.com/problem/subtree-with-maximum-average/

考点本质：深度优先搜索（Depth First Search）

考察形态：二叉树结构变化

代表例题：http://www.lintcode.com/problem/invert-binary-tree/

考点本质：深度优先搜索（Depth First Search）

考察形态：二叉查找树（Binary Search Tree）

代表例题：http://www.lintcode.com/problem/validate-binary-search-tree/

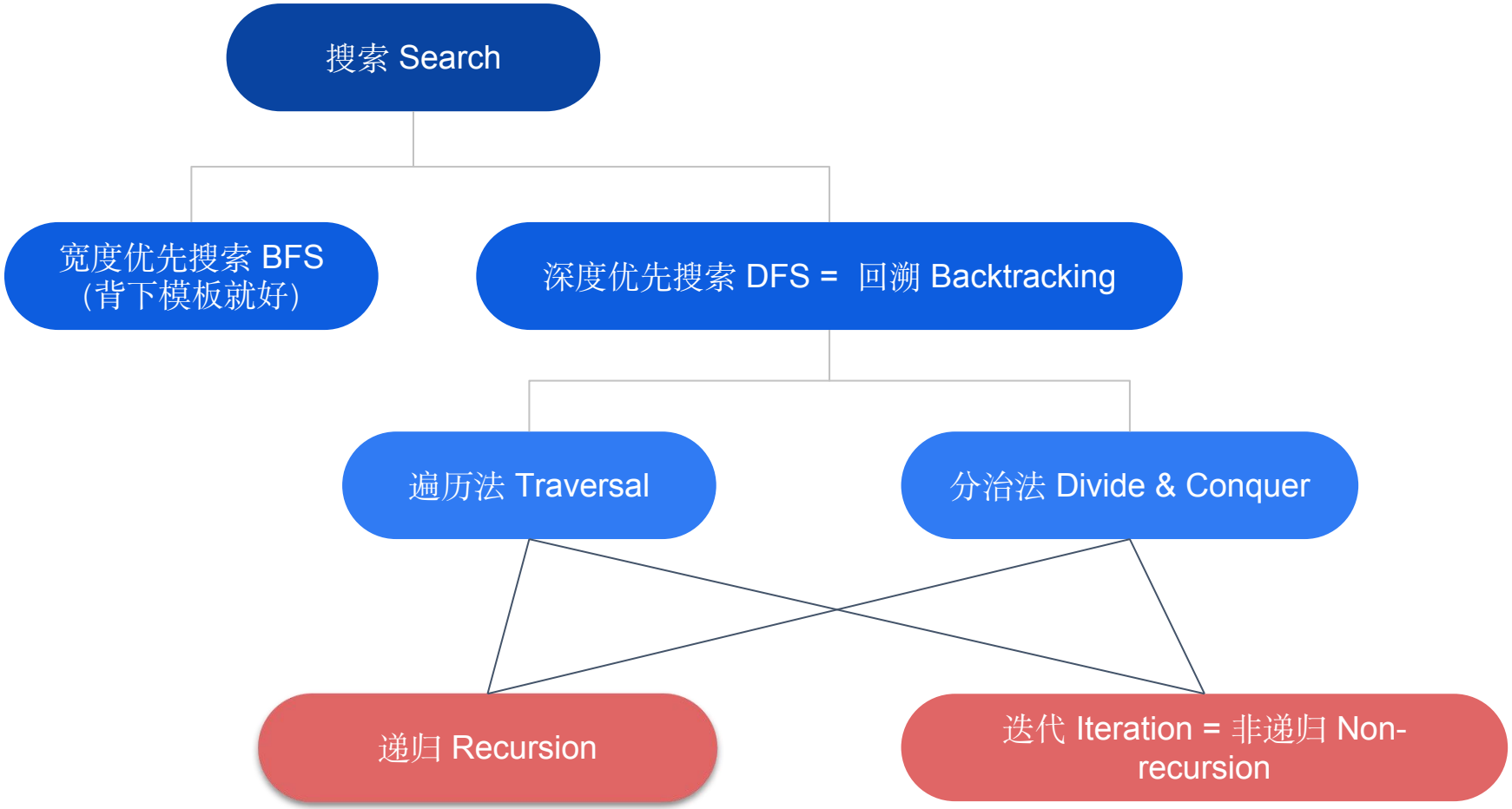
考点本质：深度优先搜索（Depth First Search）

Tree-based Depth First Search

不管二叉树的题型如何变化

考点都是基于树的深度优先搜索

一张图搞明白：递归，DFS，回溯，遍历，分治，迭代



将递归和非递归理解为算法的一种实现方式而不是算法

第一类考察形态

二叉树上求值，求路径

Maximum / Minimum / Average / Sum / Paths

Minimum Subtree

http://www.lintcode.com/problem/minimum-subtree/

http://www.jiuzhang.com/solutions/minimum-subtree/

求和最小的子树

一棵二叉树有多少棵子树

A: O(n)

B: O(2^n)

C: O(n^2)

D: O(n) - 叶子节点个数

代码 - 使用了全局变量的分治法


```java
public class Solution {
    private int minSum;
    private TreeNode minRoot;

    public TreeNode findSubtree(TreeNode root) {
        minSum = Integer.MAX_VALUE;
        minRoot = null;
        getSum(root);
        return minRoot;
    }

    private int getSum(TreeNode root) {
        if (root == null) {
            return 0;
        }

        int sum = getSum(root.left) + getSum(root.right) + root.val;
        if (sum < minSum) {
            minSum = sum;
            minRoot = root;
        }

        return sum;
    }
}
```

```python
class Solution:
    def findSubtree(self, root):
        self.minimum_weight = float('inf')
        self.minimum_subtree_root = None
        self.getTreeSum(root)

        return self.minimum_subtree_root

    # 得到 root 为根的二叉树的所有节点之和
    # 顺便打个擂台求出 minimum subtree
    def getTreeSum(self, root):
        if root is None:
            return 0

        left_weight = self.getTreeSum(root.left)
        right_weight = self.getTreeSum(root.right)
        root_weight = left_weight + right_weight + root.val

        if root_weight < self.minimum_weight:
            self.minimum_weight = root_weight
            self.minimum_subtree_root = root

        return root_weight
```

全局变量的坏处

函数不“纯粹”，容易出 Bug

不利于多线程化，对共享变量加锁带来效率下降

代码 - Python 纯分治的方


```python
class Solution:

    def findSubtree(self, root):
        minimum, subtree, sum_of_root = self.helper(root)
        return subtree

    def helper(self, root):
        if root is None:
            return sys.maxsize, None, 0

        left_minimum, left_subtree, left_sum = self.helper(root.left)
        right_minimum, right_subtree, right_sum = self.helper(root.right)

        sum_of_root = left_sum + right_sum + root.val
        if left_minimum == min(left_minimum, right_minimum, sum_of_root):
            return left_minimum, left_subtree, sum_of_root
        if right_minimum == min(left_minimum, right_minimum, sum_of_root):
            return right_minimum, right_subtree, sum_of_root

        return sum_of_root, root, sum_of_root
```

代码 - Java 纯分治的方法


```java
public ResultType helper(TreeNode node) {
    if (node == null) {
        return new ResultType(null, Integer.MAX_VALUE, 0);
    }

    ResultType leftResult = helper(node.left);
    ResultType rightResult = helper(node.right);

    ResultType result = new ResultType(
        node,
        leftResult.sum + rightResult.sum + node.val,
        leftResult.sum + rightResult.sum + node.val
    );

    if (leftResult.minSum <= result.minSum) {
        result.minSum = leftResult.minSum;
        result.minSubtree = leftResult.minSubtree;
    }

    if (rightResult.minSum <= result.minSum) {
        result.minSum = rightResult.minSum;
        result.minSubtree = rightResult.minSubtree;

    }

    return result;
}
```

```java
public TreeNode findSubtree(TreeNode root) {
    ResultType result = helper(root);
    return result.minSubtree;
}
```

```java
class ResultType {
    public TreeNode minSubtree;
    public int sum, minSum;
    public ResultType(TreeNode minSubtree, int minSum, int sum) {
        this.minSubtree = minSubtree;
        this.minSum = minSum;
        this.sum = sum;
    }
}
```

Lowest Common Ancestor

http://www.lintcode.com/problem/lowest-common-ancestor/

http://www.jiuzhang.com/solutions/lowest-common-ancestor/

with parent pointer vs no parent pointer

follow up: LCA II & III

问法1：如果有父指针

http://www.lintcode.com/problem/lowest-common-ancestor-ii/

http://www.jiuzhang.com/solutions/lowest-common-ancestor-ii/

使用 HashSet 记录从 A 到根的所有点

访问从 B 到根的所有点，第一个出现在 HashSet 中的就是

代码


```java
public class Solution {
    public ParentTreeNode lowestCommonAncestorII(ParentTreeNode root,
                                                 ParentTreeNode A,
                                                 ParentTreeNode B) {
        Set<ParentTreeNode> parentSet = new HashSet<>();
        // 把A的祖先节点都加入到哈希表中
        ParentTreeNode curr = A;
        while (curr != null) {
            parentSet.add(curr);
            curr = curr.parent;
        }
        // 遍历B的祖先节点，第一个在哈希表中出现的即为答案
        curr = B;
        while (curr != null) {
            if (parentSet.contains(curr)) {
                return curr;
            }
            curr = curr.parent;
        }
        return null;
    }
}
```

```python
def lowestCommonAncestorII(self, root, A, B):
    parent_set = set()
    # 把A的祖先节点加入到哈希表中
    curr = A
    while curr is not None:
        parent_set.add(curr)
        curr = curr.parent
    # 遍历B的祖先节点，第一个在哈希表中出现的即为答案
    curr = B
    while curr is not None:
        if curr in parent_set:
            return curr
        curr = curr.parent
    return None
```

问法2：两个节点都在树里

http://www.lintcode.com/problem/lowest-common-ancestor/

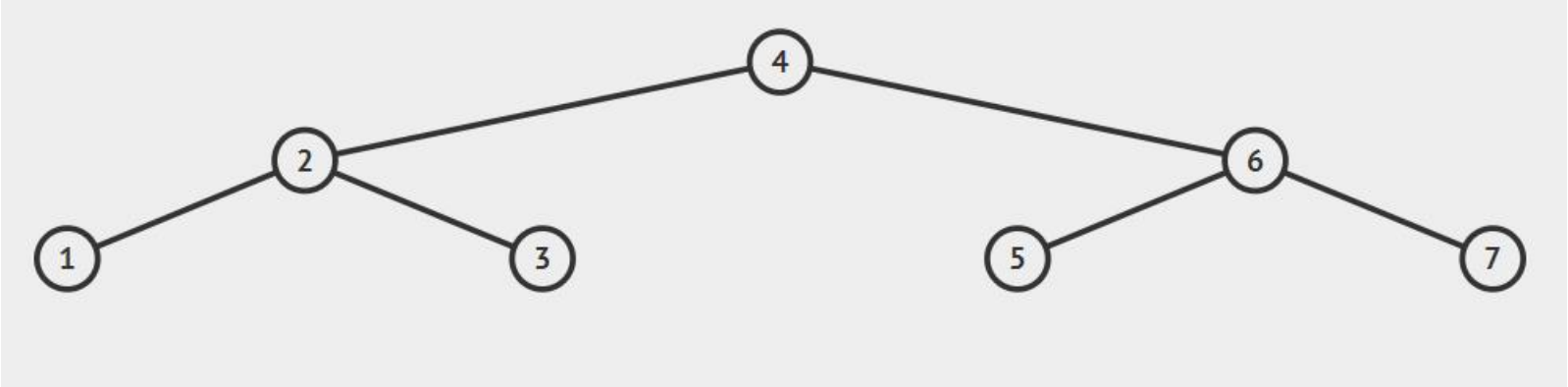
http://www.jiuzhang.com/solutions/lowest-common-ancestor/

给你 root, A, B 三个点的信息

A和B保证都在 root 的下面

方法一：直接遍历的方法

找出两个节点遍历之间深度最小的节点



遍历顺序													
node	4	2	1	2	3	2	4	6	5	6	7	6	4
depth	1	2	3	2	3	2	1	2	3	2	3	2	1

方法2：分治法

定义返回值：

A,B 都存在 -> return LCA(A,B)

只有A --> return A

只有B --> return B

A,B 都不存在 --> return null

代码


```java
public TreeNode lowestCommonAncestor(TreeNode root, TreeNode A, TreeNode B) {
    if (root == null){
        return null;
    }
    if (root == A || root == B){
        return root;
    }

    TreeNode left = lowestCommonAncestor(root.left, A, B);
    TreeNode right = lowestCommonAncestor(root.right, A, B);
    if (left != null && right != null){
        return root;
    }
    if (left != null){
        return left;
    }
    if (right != null){
        return right;
    }
    return null;
}
```

```python
def lowestCommonAncestor(self, root, A, B):
    if root is None:
        return None
    if root == A or root == B:
        return root
    left = self.lowestCommonAncestor(root.left, A, B)
    right = self.lowestCommonAncestor(root.right, A, B)
    if left and right:
        return root
    if left:
        return left
    if right:
        return right
    return None
```

问法3：两个节点不一定都在树里

http://www.lintcode.com/problem/lowest-common-ancestor-iii/

http://www.jiuzhang.com/solutions/lowest-common-ancestor-iii/

root, p, q

但是不保证 root 里一定有 p 和 q

代码

```
public TreeNode lowestCommonAncestor3(TreeNode root,
                                      TreeNode A,
                                      TreeNode B) {
    ResultType rt = helper(root, A, B);
    if (rt.a_exist && rt.b_exist)
        return rt.node;
    else
        return null;
}
```

```
class ResultType {
    public boolean a_exist, b_exist;
    public TreeNode node;
    ResultType(boolean a, boolean b, TreeNode n) {
        a_exist = a;
        b_exist = b;
        node = n;
    }
}
```

```
public ResultType helper(TreeNode root, TreeNode A, TreeNode B) {
    if (root == null)
        return new ResultType(false, false, null);

    ResultType left_rt = helper(root.left, A, B);
    ResultType right_rt = helper(root.right, A, B);

    boolean a_exist = left_rt.a_exist || right_rt.a_exist || root == A;
    boolean b_exist = left_rt.b_exist || right_rt.b_exist || root == B;

    if (root == A || root == B)
        return new ResultType(a_exist, b_exist, root);

    if (left_rt.node != null && right_rt.node != null)
        return new ResultType(a_exist, b_exist, root);
    if (left_rt.node != null)
        return new ResultType(a_exist, b_exist, left_rt.node);
    if (right_rt.node != null)
        return new ResultType(a_exist, b_exist, right_rt.node);

    return new ResultType(a_exist, b_exist, null);
}
```

代码


```python
import copy
class Solution:

    def lowestCommonAncestor3(self, root, A, B):
        a, b, lca = self.helper(root, A, B)
        if a and b:
            return lca
        else:
            return None
```

```python
def helper(self, root, A, B):
    if root is None:
        return False, False, None

    left_a, left_b, left_node = self.helper(root.left, A, B)
    right_a, right_b, right_node = self.helper(root.right, A, B)

    a = left_a or right_a or root == A
    b = left_b or right_b or root == B

    if root == A or root == B:
        return a, b, root

    if left_node is not None and right_node is not None:
        return a, b, root
    if left_node is not None:
        return a, b, left_node
    if right_node is not None:
        return a, b, right_node

    return a, b, None
```

第二类考察形态

二叉树结构变化

Flatten Binary Tree to Linked List

http://www.lintcode.com/problem/flatten-binary-tree-to-linked-list/

http://www.jiuzhang.com/solutions/flatten-binary-tree-to-linked-list/

将二叉树拆成链表

进行前序遍历，将上一个节点的右指针指向当前节点

代码


```java
public class Solution {
    TreeNode prevNode = null;
    public void flatten(TreeNode root) {
        if (root == null) {
            return;
        }

        if (prevNode != null) {
            prevNode.left = null;
            prevNode.right = root;
        }

        prevNode = root;
        TreeNode right = root.right;
        flatten(root.left);
        flatten(right);
    }
}
```

```python
class Solution:
    prev_node = None
    def flatten(self, root):
        if root is None:
            return

        if self.prev_node is not None:
            self.prev_node.left = None
            self.prev_node.right = root

        self.prev_node = root
        right = root.right
        self.flatten(root.left)
        self.flatten(right)
```

尽可能避免使用全局变量

容易写出 BUG

可以把需要修改的变量作为参数传入到函数里

或者是放在 return value 里

代码


```java
public class Solution {
    public void flatten(TreeNode root) {
        flattenAndReturnLastNode(root);
    }

    private TreeNode flattenAndReturnLastNode(TreeNode root) {
        if (root == null) {
            return null;
        }

        TreeNode leftLast = flattenAndReturnLastNode(root.left);
        TreeNode rightLast = flattenAndReturnLastNode(root.right);

        // connect
        if (leftLast != null) {
            leftLast.right = root.right;
            root.right = root.left;
            root.left = null;
        }

        if (rightLast != null) {
            return rightLast;
        }
        if (leftLast != null) {
            return leftLast;
        }
        if (root != null) {
            return root;
        }

        return null;
    }
}
```

```python
class Solution:
    def flatten(self, root):
        self.flatten_and_return_last_node(root)

    def flatten_and_return_last_node(self, root):
        if root is None:
            return None

        left_last = self.flatten_and_return_last_node(root.left)
        right_last = self.flatten_and_return_last_node(root.right)

        # connect
        if left_last is not None:
            left_last.right = root.right
            root.right = root.left
            root.left = None

        return right_last or left_last or root
```

第三类考察形态

二叉搜索树
Binary Search Tree

BST 基本性质

- 从定义出发：
  - 左子树都比根节点小
  - 右子树都不小于根节点

- 从效果出发：
  - 中序遍历 in-order traversal 是“不下降”序列
  - 如图，中序遍历为 1 2 3 4 5

- 性质：
  - 如果一棵二叉树的中序遍历不是“不下降”序列，则一定不是BST
  - 如果一棵二叉树的中序遍历是不下降，也未必是BST
    - 比如下面这棵树就不是 BST，但是它的中序遍历是不下降序列。
    - 1
    - / \
    - 1 1

BST的高度是多少

A: O(n)

B: O(logn)

C: O(h)

BST的高度是多少

同样是最坏 O(n) 最好 O(logn)

用 O(h) 表示更合适

只有 Balanced Binary Tree (平衡二叉树) 才是 O(logn)

红黑树 Red-black Tree

红黑树是一种 Balanced BST

C++/Java中有一个数据结构的实现用的是 Balanced BST

你知道是什么吗？

Java: TreeMap / TreeSet

C++: map / set

Python: 不好意思我没有

有一定要用 TreeMap 来做的面试题么？——没有

关于红黑树，你需要掌握的是

- 知道他是一个 Balanced BST
- 知道他能干嘛
  - O(LogN) 的时间内实现增删查改
  - O(LogN) 的时间内实现找最大找最小
  - O(LogN) 的时间内实现找比某个数小的最大值(upperBound)和比某个数大的最小值(lowerBound)
- 知道他的工程应用价值
  - Java 1.8 中的 HashMap 的实现里同时用到了 TreeMap 和 LinkedList

就面试而言，你不需要掌握他的代码实现，因为写出来200+行，没人能在面试的时候写完

Kth Smallest Element in BST

https://www.lintcode.com/problem/kth-smallest-element-in-a-bst/

https://www.jiuzhang.com/solution/kth-smallest-element-in-a-bst/

时间复杂度如何分析?

代码


```java
public class Solution {
    public int kthSmallest(TreeNode root, int k) {
        Stack<TreeNode> stack = new Stack<>();

        while (root != null) {
            stack.push(root);
            root = root.left;
        }

        for (int i = 0; i < k - 1; i++) {
            TreeNode node = stack.peek();

            if (node.right == null) {
                node = stack.pop();
                while (!stack.isEmpty() && stack.peek().right == node) {
                    node = stack.pop();
                }
            } else {
                node = node.right;
                while (node != null) {
                    stack.push(node);
                    node = node.left;
                }
            }
        }

        return stack.peek().val;
    }
}
```

```python
class Solution:
    def kthSmallest(self, root, k):
        stack = []

        while root != None:
            stack.append(root)
            root = root.left

        for i in range(k - 1):
            node = stack[-1]

            if node.right == None:
                node = stack.pop(-1)
                while len(stack) != 0 and stack[-1].right == node:
                    node = stack.pop(-1)
            else:
                node = node.right
                while node != None:
                    stack.append(node)
                    node = node.left

        return stack[-1].val
```

时间复杂度分析

O(k + h)

当 k 是 1 的时候 ==> O(h)

当 k 是 n 的时候 ==> O(n)

k和h两者取大值

Follow up: 二叉树经常被修改

如何优化 kthSmallest 这个操作?

优化方法

在 TreeNode 中增加一个 counter，代表整个树的节点个数

也可以用一个 HashMap<TreeNode, Integer> 来存储某个节点为代表的子树的节点个数

在增删查改的过程中记录不断更新受影响节点的 counter

在 kthSmallest 的实现中用类似 Quick Select 的算法去找到 kth smallest element

时间复杂度为 O(h)，h 为树的高度。

Strong Hire: 能够答出 Follow Up 的算法，并写出kthSmallest核心代码（不需要写增删查改，45分钟写不完的），bug free or minor bug，不需要提示

Hire / Weak Hire : 能够答出 Follow up 的算法，大致写出 kthSmallest 核心代码，存在一定bug，或者需要提示

No Hire: 答不出 follow up

Strong No: 连第一问的 Inorder traversal 都不会写

Closest Binary Search Tree Value

https://www.lintcode.com/problem/closest-binary-search-tree-value/

http://www.jiuzhang.com/solution/closest-binary-search-tree-value/

如果使用中序遍历，时间复杂度是多少?

如果使用 lowerBound / upperBound 的方法，时间复杂度是多少?

Follow up: 寻找 k 个最接近的值

https://www.lintcode.com/problem/closest-binary-search-tree-value-ii/

https://www.jiuzhang.com/solution/closest-binary-search-tree-value-ii/

如果是用中序遍历得到从小到大的所有值，接下来的问题相当于之前学过的哪个题？

有没有更快的办法？

方法1 暴力做法

先用 inorder traversal 求出中序遍历

找到第一个 >= target 的位置 index

从 index-1 和 index 出发，设置两根指针一左一右，获得最近的 k 个整数

代码


```java
public List<Integer> closestKValues(TreeNode root, double target, int k) {
    List<Integer> values = new ArrayList<>();

    traverse(root, values);

    int i = 0, n = values.size();
    for (; i < n; i++) {
        if (values.get(i) >= target) {
            break;
        }
    }

    if (i >= n) {
        return values.subList(n - k, n);
    }

    int left = i - 1, right = i;
    List<Integer> result = new ArrayList<>();
    for (i = 0; i < k; i++) {
        if (left >= 0 && (right >= n || target - values.get(left) < values.get(right) -
            target)) {
            result.add(values.get(left));
            left--;
        } else {
            result.add(values.get(right));
            right++;
        }
    }

    return result;
}
```

```java
private void traverse(TreeNode root, List<Integer> values) {
    if (root == null) {
        return;
    }

    traverse(root.left, values);
    values.add(root.val);
    traverse(root.right, values);
}
```

代码


```python
def closestKValues(self, root, target, k):
    if root is None or k == 0:
        return []

    nums = self.get_inorder(root)
    left = self.find_lower_index(nums, target)
    right = left + 1
    results = []
    for _ in range(k):
        if self.is_left_closer(nums, left, right, target):
            results.append(nums[left])
            left -= 1
        else:
            results.append(nums[right])
            right += 1
    return results
```

```python
def get_inorder(self, root):
    dummy = TreeNode(0)
    dummy.right = root
    stack = [dummy]
    inorder = []

    while stack:
        node = stack.pop()
        if node.right:
            node = node.right
            while node:
                stack.append(node)
                node = node.left
        if stack:
            inorder.append(stack[-1].val)

    return inorder
```

代码


```python
def find_lower_index(self, nums, target):
    start, end = 0, len(nums) - 1
    while start + 1 < end:
        mid = (start + end) // 2
        if nums[mid] < target:
            start = mid
        else:
            end = mid

    if nums[end] < target:
        return end

    if nums[start] < target:
        return start

    return -1

def is_left_closer(self, nums, left, right, target):
    if left < 0:
        return False
    if right >= len(nums):
        return True
    return target - nums[left] < nums[right] - target
```

方法2 使用两个 Iterator

一个 iterator move forward

另一个iterator move backward

每次 i++ 的时候根据 stack，挪动到 next node

每次i--的时候根据 stack, 挪动到 prev node

代码


```java
public List<Integer> closestKValues(TreeNode root, double target, int k) {
    List<Integer> values = new ArrayList<>();

    if (k == 0 || root == null) {
        return values;
    }

    Stack<TreeNode> lowerStack = getStack(root, target);
    Stack<TreeNode> upperStack = new Stack<>();
    upperStack.addAll(lowerStack);
    if (target < lowerStack.peek().val) {
        moveLower(lowerStack);
    } else {
        moveUpper(upperStack);
    }

    for (int i = 0; i < k; i++) {
        if (lowerStack.isEmpty() ||
                !upperStack.isEmpty() && target - lowerStack.peek().val > upperStack
                        .peek().val - target) {
            values.add(upperStack.peek().val);
            moveUpper(upperStack);
        } else {
            values.add(lowerStack.peek().val);
            moveLower(lowerStack);
        }
    }

    return values;
}
```

```java
private Stack<TreeNode> getStack(TreeNode root, double target) {
    Stack<TreeNode> stack = new Stack<>();

    while (root != null) {
        stack.push(root);

        if (target < root.val) {
            root = root.left;
        } else {
            root = root.right;
        }
    }

    return stack;
}
```

代码


```java
public void moveUpper(Stack<TreeNode> stack) {
    TreeNode node = stack.peek();
    if (node.right == null) {
        node = stack.pop();
        while (!stack.isEmpty() && stack.peek().right == node) {
            node = stack.pop();
        }
        return;
    }

    node = node.right;
    while (node != null) {
        stack.push(node);
        node = node.left;
    }
}
```

```java
public void moveLower(Stack<TreeNode> stack) {
    TreeNode node = stack.peek();
    if (node.left == null) {
        node = stack.pop();
        while (!stack.isEmpty() && stack.peek().left == node) {
            node = stack.pop();
        }
        return;
    }

    node = node.left;
    while (node != null) {
        stack.push(node);
        node = node.right;
    }
}
```

代码


```python
def closestKValues(self, root, target, k):
    if root is None or k == 0:
        return []

    lower_stack = self.get_stack(root, target)
    upper_stack = list(lower_stack)
    if lower_stack[-1].val < target:
        self.move_upper(upper_stack)
    else:
        self.move_lower(lower_stack)

    result = []
    for i in range(k):
        if self.is_lower_closer(lower_stack, upper_stack, target):
            result.append(lower_stack[-1].val)
            self.move_lower(lower_stack)
        else:
            result.append(upper_stack[-1].val)
            self.move_upper(upper_stack)

    return result

def get_stack(self, root, target):
    stack = []
    while root:
        stack.append(root)
        if target < root.val:
            root = root.left
        else:
            root = root.right

    return stack
```

```python
def move_upper(self, stack):
    if stack[-1].right:
        node = stack[-1].right
        while node:
            stack.append(node)
            node = node.left
    else:
        node = stack.pop()
        while stack and stack[-1].right == node:
            node = stack.pop()

def move_lower(self, stack):
    if stack[-1].left:
        node = stack[-1].left
        while node:
            stack.append(node)
            node = node.right
    else:
        node = stack.pop()
        while stack and stack[-1].left == node:
            node = stack.pop()

def is_lower_closer(self, lower_stack, upper_stack, target):
    if not lower_stack:
        return False

    if not upper_stack:
        return True

    return target - lower_stack[-1].val < upper_stack[-1].val - target
```

Closest Binary Search Tree Value 评分标准

Strong Hire

找1个点和找k个点都答出来，且找 k 个点的能用 O(k + logn) 的时间复杂度

Hire

找1个点和找k个点都答出来，且找 k 个点的能用 O(klogn) 的时间复杂度完成，少 bug，无需提示

Weak Hire:

找1个点和找k个点都答出来，且找 k 个点的能分别用 O(klogn) 和 O(n) 的时间复杂度完成，bug 多，需要提示

No Hire

答出1个点，答不出 k 个点非O(n)的算法

Strong No Hire

啥都答不出来

Related Questions

- Search Range in Binary Search Tree
- http://www.lintcode.com/problem/search-range-in-binary-search-tree/
- Insert Node in a Binary Search Tree
- http://www.lintcode.com/problem/insert-node-in-a-binary-search-tree/
- Remove Node in a Binary Search Tree
- http://www.lintcode.com/problem/remove-node-in-binary-search-tree/
- http://www.mathcs.emory.edu/~cheung/Courses/171/Syllabus/9-BinTree/BST-delete.html

在第 7 周的互动课中继续学习如下二叉树的内容

- 第 34 章 后序遍历非递归与 Morris 算法
  - 不作为必须要掌握的知识点，但是学了可以提高 Coding 能力
- 第 35 章 二叉查找树的增删查改
  - 必须掌握增查改，删除操作不作要求