Transformada de Laplace - Taller N° 12

Alumno: Dennys Alexander Pucha Carrera

Fecha: 31-07-2023

Curso: Ing. en Computación 4to Ciclo "A"

1. Encontrar los siguientes ejercicios, use la definición de Transformada de Laplace, para encontrar $\mathcal{L}\{f(t)\}$

a. $f(t) = \begin{cases} 4, & 0 \leq t < 2 \\ 0, & t \geq 2 \end{cases}$

$$\mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t)\,dt$$

$$= \int_0^2 e^{-st}(4)\,dt + \int_2^{\infty} e^{-st}.(0)\,dt$$

$$= 4\int_0^2 e^{-st}\,dt + 0 \qquad u = -st \quad du = -s\,dt \quad dt = \frac{du}{-s}$$

$$= -\frac{4}{s}\int_0^2 e^{u}\,du = -\frac{4}{s} e^{-st}\Big|_0^2 = \left[\left(-\frac{4}{s}e^{-s(2)}\right) - \left(-\frac{4}{s}e^{-s(0)}\right)\right]$$

$$= \left[-\frac{4}{s}e^{-2s} + \frac{4}{s}\right] = \frac{4-4e^{-2s}}{s} \,//$$

b. $f(t) = \begin{cases} t, & 0 \leq t < 1 \\ 1, & t \geq 1 \end{cases}$

$$\mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t)\,dt$$

Notas

$$= \int_0^1 e^{-st} \cdot (t)\,dt + \int_1^{\alpha} e^{-st} \cdot (1)\,dt$$

$u = t \qquad du = 1\,dt$

$dv = e^{-st}\,dt \qquad v = -\frac{1}{s} e^{-st}$

$$\int dv = \int e^{-st}\,dt$$

$w = -st$

$dw = -s\,dt$

$dt = \frac{dw}{-s}$

$$v = -\frac{1}{-s} \int e^{w}\,dw$$

$$v = -\frac{1}{s} e^{-st} \qquad \int u\,dv = uv - \int v\,du$$

$$= t \cdot e^{-st} - \int -\frac{1}{s} e^{-st}\,dt$$

$$= t \cdot e^{-st} + \frac{1}{s} \int e^{-st}\,dt = t \cdot e^{-st} + \frac{1}{s} \cdot -\frac{1}{s} e^{-st} = t \cdot e^{-st} - \frac{1}{s^2} e^{-st}$$

$$= t \cdot \frac{-1}{s} e^{-st} - \frac{1}{s^2} e^{-st} + \int_1^{\alpha} e^{-st} (1)\,dt = t \cdot e^{-st} - \frac{1}{s^2} e^{-st} - \frac{1}{s} e^{st}$$

$$= \Big|_0^1 t \cdot \frac{-1}{s} e^{-st} - \frac{1}{s^2} e^{st} - \Big|_1^{\alpha} \frac{1}{s} e^{st}$$

$$= \left[\left(1 \cdot \frac{-1}{s} e^{-s(1)} - \frac{1}{s^2} e^{-s(1)}\right) - \left(0 \cdot \frac{-1}{s} e^{-s(0)} - \frac{1}{s^2} e^{-s(0)}\right)\right] - \left[\left(\frac{1}{s} e^{s(\alpha)} - \frac{1}{s} e^{s(1)}\right)\right]$$

$$= \left[\left(\frac{-1}{s} e^{-s} - \frac{1}{s} e^{-s}\right) - \left(0 - \frac{1}{s^2} \cdot (1)\right)\right] - \left[\left(0 - \frac{1}{s} e^{s}\right)\right]$$

$$=\left[\left(\frac{-1}{s}e^{-s}-\frac{1}{s}e^{-s}\right)-\left(-\frac{1}{s^2}\right)\right]-\left[-\frac{1}{s}e^{s}\right]$$

$$=\left[\left(\frac{-1}{s}e^{-s}-\frac{1}{s}e^{-s}\right)+\frac{1}{s^2}\right]+\left[\frac{1}{s}e^{s}\right]$$

$$=\frac{1}{s^2}-\frac{e^{-s}}{s^2}=\frac{1-e^{-s}}{s^2}\ //$$

c. $f(t)=\begin{cases}\operatorname{sen} t, & 0\le t<\pi\\ 0, & t\ge\pi\end{cases}$ $\quad \mathcal{L}\{f(t)\}=\int_0^{\infty}e^{-st}f(t)\,dt$

$$=\int_0^{\pi}e^{-st}\cdot\operatorname{sen}(t)\,dt+\int_{\pi}^{\infty}e^{-st}(0)\,dt$$

$$=\int_0^{\pi}e^{-st}\cdot\operatorname{sen}(t)\,dt+0$$

$u=\operatorname{sen}(t)\quad du=\cos t$

$dv=e^{-st}\quad v=-\frac{1}{s}e^{-st}$

$$=\operatorname{sen}(t)\cdot\frac{-1}{s}e^{-st}-\int-\frac{1}{s}e^{-st}\cos(t)$$

$$\int e^{-st}\cdot\operatorname{sen}(t)\,dt=\operatorname{sen}(t)-\frac{1}{s}e^{-st}+\frac{1}{s}\int e^{-st}\cos(t)$$

$u=\cos t$

$dv=e^{-st}$

$$\int e^{-st}\cdot\operatorname{sen}(t)\,dt=\operatorname{sen}(t)-\frac{1}{s}e^{-st}+\frac{1}{s}$$

$du=-\operatorname{sen} t$

$v=\frac{1}{s}e^{-st}$

$$\int e^{-st}\cdot\operatorname{sen}(t)\,dt=\operatorname{sen}(t)-\frac{1}{s}e^{-st}+\frac{1}{s}\left(\cos t\cdot\frac{1}{s}e^{-st}\right)-\int\frac{1}{s}e^{-st}\cdot-\operatorname{sen}(t)\,dt$$

$$\int e^{-st}\cdot\operatorname{sen}(t)\,dt=\operatorname{sen} t-\frac{1}{s}e^{-st}+\frac{1}{s}\left(\cos t\cdot\frac{1}{s}e^{-st}+\frac{1}{s}\int e^{-st}\cdot\operatorname{sen} t\,dt\right.$$

$$\int e^{-st}\cdot\operatorname{sen}(t)\,dt-\frac{1}{s}\int e^{-st}\cdot\operatorname{sen}(t)\,dt=\operatorname{sen} t-\frac{1}{s}e^{-st}+\frac{1}{s}\left(\cos t\cdot\frac{1}{s}e^{-st}\right)$$

$$\int_0^{\pi} e^{-st}(\operatorname{sen} t)\,dt\left(1+\frac{1}{s^2}\right)=\left(-\frac{\operatorname{sen}\pi}{s}e^{-s(\pi)}-\frac{\cos\pi}{s^2}e^{-s\pi}\right)-\left(-\frac{\operatorname{sen}0}{s}e^{-s(0)}-\right.$$

$$\left.\frac{\cos 0}{s^2}e^{-s(0)}\right)=\int_0^{\pi} e^{-st}(\operatorname{sen} t)\,dt\left(1+\frac{1}{s^2}\right)$$

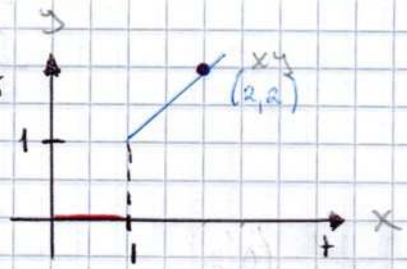
$$=\left(0-\left(-\frac{1}{s^2}e^{-s\pi}\right)\right)-\left(0-\frac{1}{s^2}\right)$$

$$=\frac{\frac{e^{-s\pi}}{s^2}+\frac{1}{s^2}}{1+\frac{1}{s^2}}=\frac{\frac{e^{-s\pi}+1}{s^2}}{\frac{s^2+1}{s^2}}$$

$$=\frac{\cancel{s^2}(e^{-s\pi}+1)}{\cancel{s^2}(s^2+1)}$$

$$=\frac{e^{-s\pi}+1}{s^2+1}\quad //$$

d. f(t)



$$f(t)=\begin{cases} 0, & 0 \le t < 1 \\ t, & t \ge 1 \end{cases}$$

$$= \int_0^1 e^{-st}(0)\,dt - \int_1^{\alpha} e^{-st}(t)\,dt$$

$$= 0 - \int_1^{\alpha} e^{-st}(t)\,dt$$

$u = t \quad du = 1\,dt$

$dv = e^{-st} \quad v = -\frac{1}{s}e^{-st}$

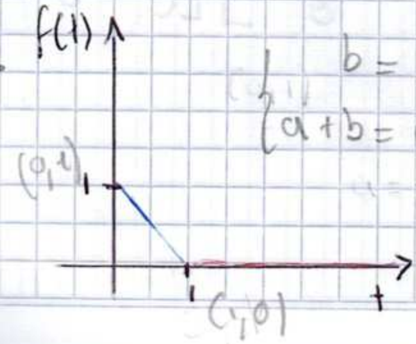
$$= (t)\left(-\frac{1}{s}e^{-st}\right) - \int -\frac{1}{s}e^{-st}\,dt = (t)\left(-\frac{1}{s}e^{-st}\right) + \frac{1}{s}\cdot -\frac{1}{s}\, e^{-st}$$

$$= (t)\left(-\frac{1}{s}e^{-st}\right) - \frac{1}{s^2}e^{-st}\Big|_1^{\alpha} = \left[\alpha \cdot -\frac{1}{s}e^{-s(\alpha)} - \frac{1}{s^2}e^{-s(\alpha)}\right] -$$

$$\left[1 \cdot -\frac{1}{s}e^{-s(1)} - \frac{1}{s^2}e^{-s(1)}\right] = [0-0] - \left[-\frac{1}{s}e^{-s} - \frac{1}{s^2}e^{-s}\right]$$

$$= \frac{1}{s}e^{-s} + \frac{1}{s^2}e^{-s} \quad //$$

e.



$$\begin{cases} b = 1 \\ a + b = 0 \end{cases}, \quad a = -1$$

$$f(t)=\begin{cases} -t+1, & 0 \le t < 1 \\ 0, & t \ge 1 \end{cases}$$

$y = -x + 1$

$$\mathcal{L}\{f(t)\} = \int_0^{\alpha} e^{-st} f(t)\,dt$$

$$= \int_0^1 e^{-st} \cdot t+1 - \int_1^{\alpha} e^{st}(0)\,dt$$

$$= \int_0^1 e^{-st} \cdot (t) + \int_0^1 e^{-st} dt + 0 =$$

$u = t \quad du = 1dt$

$dv = e^{-st} \quad v = -\frac{1}{s} e^{-st}$

$$= (t)(-\frac{1}{s}e^{-st}) - \int_0^1 -\frac{1}{s}e^{-st}dt - \frac{1}{s} e^{-st}$$

$$= (t)\left(-\frac{1}{s}e^{-st}\right) - \left(-\frac{1}{s} - \frac{1}{s}e^{-st}\right) - \frac{1}{s}e^{-st}$$

$$= (t)\cdot\left(-\frac{1}{s}e^{-st}\right) - \frac{1}{s^2}e^{-st} - \frac{1}{s}e^{-st} \Big|_0^1$$

$$= \left[\left((1)\left(-\frac{1}{s}e^{-s(1)}\right) - \frac{1}{s^2}e^{-s(1)} - \frac{1}{s}e^{-s(1)}\right] - \left[(0)\left(-\frac{1}{s}e^{-s(0)}\right)\right.$$

$$\left. - \frac{1}{s}e^{-s(0)} - \frac{1}{s}e^{-s(0)}\right] = \left[\left(-\frac{1}{s}e^{-s}\right) - \frac{1}{s}e^{-s} - \frac{1}{s}e^{-s}\right]\left[\left(0 - \frac{1}{s} - \frac{1}{s}\right)\right]$$

$$= \frac{e^{-s} - 1 + s}{s^2} \quad //$$