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Fecha: 15-08-2023

Curso: 4to ciclo "A" - Ing. en Computación

1. En los siguientes ejercicios, use la definición de Transformada de Laplace, para encontrar $\mathcal{L}\{F(t)\}$

a. $F(t) = 2t^4$

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{2t^4\}$$
$$= 2\mathcal{L}\{t^4\}$$
$$= 2\left(\frac{4!}{s^{4+1}}\right) = \frac{48}{s^5}$$

b. $F(t) = t^5$

$$\mathcal{L}\{t^5\} = \frac{5!}{s^{5+1}}$$
$$= \frac{120}{s^6}$$

c. $F(t) = 4t - 10$

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} \cdot 4t - \int_0^\infty e^{-st} \cdot 10$$

$$u = 4t, \quad du = 4$$
$$dv = e^{-st} \quad v = -\frac{1}{s}e^{-st}$$

$$= 4t \cdot -\frac{1}{s}e^{-st} - \int -\frac{1}{s}e^{-st} \cdot 4 \quad = 4t - e^{-st} + \frac{4}{s}\int e^{-st} = 4t - e^{-st} + \frac{4}{s^2}e^{-st} - \frac{1}{s}e^{-st}$$

$$4t - \frac{1}{5}e^{-st} + \frac{4}{s^2}e^{-st} - \frac{1}{s}e^{-st} \cdot 10t \Big|_0^{\infty}$$

$$= \left[\left[4(\infty) - \frac{1}{5}e^{-s(\infty)} + \frac{4}{s^2}e^{-s(\infty)} - \frac{1}{s}e^{-s(\infty)} \cdot 10\right] - \left[4(0)\right.\right.$$

$$\left. - \frac{1}{5}e^{-s(0)} + \frac{4}{s^2}e^{-s(0)} - \frac{1}{s}e^{-s(0)} \cdot 10\right]$$

$$= \frac{4}{s^2} - \frac{1}{s} \cdot 10 = \frac{4}{s^2} - \frac{10}{s} \,//$$

d. $f(t) = 7t + 3$

$$\mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} \cdot 7t + 3$$

$$= \int_0^{\infty} e^{-st} \cdot 7t + \int_0^{\infty} e^{-st} \cdot 3 \quad = 7t \cdot \frac{-1}{s}e^{-st} - \int \frac{1}{-s}e^{-st}\,4$$

$u = 7t \quad du = 4$
$dv = e^{-st} \quad v = \frac{1}{-s}e^{-st}$

$$= 7t \cdot \frac{-1}{s^2}e^{-st} + \frac{4}{s^2}e^{-st} + \frac{3}{s}e^{-st} \Big|_0^{\infty}$$

$$= \left(\left[7(\infty) \cdot \frac{-1}{s}e^{-s(\infty)} + \frac{7}{s^2}e^{-s(\infty)} + \frac{3}{s}e^{-s(\infty)}\right] + \left[7(0) \cdot \frac{-1}{s}e^{-s(0)}\right.\right.$$

$$\left. + \frac{7}{s^2}e^{-s(0)} + \frac{3}{s}e^{-s(0)}\right] = 0 + 0 + \frac{7}{s^2} + \frac{3}{s} = \frac{7}{s^2} + \frac{3}{s} \,//$$

e. $4t^2 - 6t - 3$

$$\mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} \cdot 4t^2 - \int_0^{\infty} e^{-st}\,6t - \int_0^{\infty} e^{-st} \cdot 3$$

$u = 4t^2 \quad du = 8t$
$dv = e^{-st} \quad v = -\frac{1}{s}e^{-st}$

$$= 4t^2 \cdot -\frac{1}{s}e^{-st} + \frac{1}{s}\int_0^\infty e^{-st} \cdot 8t \quad = 4t^2 \cdot -\frac{1}{s}e^{-st} + \frac{1}{s}\left(8t \cdot -\frac{1}{s}e^{-st}\right.$$

$u = 8t \quad du = 8$

$dv = e^{-st} \quad v = -\frac{1}{s}e^{-st}$

$$-\int_0^\infty -\frac{1}{s}e^{-st} \cdot 8$$

$$= 4t^2 - \frac{1}{s}e^{-st} + \frac{1}{s}\left(8t - \frac{1}{s}e^{-st}\right) + \frac{8}{s}e^{-st}$$

$$= 4t^2 - \frac{1}{s}e^{-st} + \frac{1}{s}8t - \frac{1}{s^2}e^{-st} + \frac{8}{s}e^{-st} - \int_0^\infty e^{-st} \cdot 6t$$

$u = 6t \quad du = 6$

$dv = e^{-st} \quad v = -\frac{1}{s}e^{-st}$

$$= 4t^2 - \frac{1}{s}e^{-st} + \frac{8}{s}t - \frac{1}{s^2}e^{-st} + \frac{8}{s}e^{-st} - 6t \cdot -\frac{1}{s}e^{-st} - \frac{6}{s}\int_0^\infty e^{\omega}$$

$$= 4t^2 - \frac{1}{s}e^{-st} + \frac{8}{s}t - \frac{1}{s^2}e^{-st} + \frac{8}{s}e^{-st} - 6t \cdot -\frac{1}{s}e^{-st} - \frac{6}{s}e^{-st}$$

$$= 4t^2 - \frac{1}{s}e^{-st} + \frac{8}{s}t - \frac{1}{s^2}e^{-st} + \frac{8}{s}e^{-st} - 6t \cdot -\frac{1}{s}e^{-st} - \frac{6}{s}e^{-st} - \frac{3}{s}e^{-st}$$

$$= \frac{8}{s^3}e^{-st} - \frac{6}{s^2}e^{-st} - \frac{3}{s}e^{-st}\Big|_0^\infty = \left[\frac{8}{s^3}e^{-s(\infty)} - \frac{6}{s^2}e^{-s(\infty)} - \frac{3}{s}e^{-s(\infty)}\right]$$

$$-\left[\frac{8}{s^3}e^{-s(0)} - \frac{6}{s^2}e^{-s(0)} - \frac{3}{s}e^{-s(0)}\right] = \frac{8}{s^3} - \frac{6}{s^2} - \frac{3}{s} \,//$$

a. En los siguientes ejercicios, determinar la transformada inversa dada.

a. $\mathcal{L}^{-1}\left\{\frac{1}{s^3}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s^{2+1}} \cdot \frac{2!}{2!}\right\} = \frac{1}{2!}\mathcal{L}^{-1}\left\{\frac{2!}{s^{2+1}}\right\}$

$= \frac{1}{2}t^2 = \frac{1}{2}t^2$ //

b. $\mathcal{L}^{-1}\left\{\frac{(s+1)^3}{s^4}\right\} = \mathcal{L}^{-1}\left\{\frac{s^3+3s^2+3s+1}{s^4}\right\} = \mathcal{L}^{-1}\left\{\frac{s^3}{s^4}\right\} + \mathcal{L}^{-1}\left\{\frac{3s^2}{s^4}\right\}$

$+ \left\{\frac{3s}{s^4}\right\} + \left\{\frac{1}{s^4}\right\} = \mathcal{L}^{-1}\left\{\frac{s^3}{s^4}\right\} + 3\mathcal{L}^{-1}\left\{\frac{s^2}{s^4}\right\} + 3\mathcal{L}^{-1}\left\{\frac{s}{s^4}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{s^4}\right\}$

$= 1 + 3t + \frac{3t^2}{2} + \frac{1}{6}t^3$ //

c. $\mathcal{L}^{-1}\left\{\frac{1}{s^3} + \frac{1}{s} + \frac{1}{s+2}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s^3}\right\} - \mathcal{L}^{-1}\left\{\frac{1}{s}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{s-2}\right\}$

$= \mathcal{L}^{-1}\left\{\frac{1}{s^{2+1}} \cdot \frac{2!}{2!}\right\} - \mathcal{L}^{-1}\left\{\frac{1}{s}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{s-2}\right\}$

$= \frac{1}{2!}\mathcal{L}^{-1}\left\{\frac{2!}{s^{2+1}}\right\} - \mathcal{L}^{-1}\left\{\frac{1}{s}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{s-2}\right\} = \frac{1}{2}t^2 - 1 + e^{2t}$ //

d. $\mathcal{L}^{-1}\left\{\frac{4}{s} + \frac{6}{s^5} - \frac{1}{s+8}\right\} = \mathcal{L}^{-1}\left\{\frac{4}{s}\right\} + \mathcal{L}^{-1}\left\{\frac{6}{s^5}\right\} - \mathcal{L}^{-1}\left\{\frac{1}{s+8}\right\}$

$= 4t^0 + \frac{6}{24}t^4 - e^{-8t} = 4 + \frac{6t^4}{24} - e^{-8t}$ //

e. $\mathcal{L}^{-1}\left\{\frac{1}{4s+1}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{4(s+\frac{1}{4})}\right\} = \frac{1}{4}\mathcal{L}^{-1}\left\{\frac{1}{s+\frac{1}{4}}\right\}$

$= \mathcal{L}^{-1}\left\{\frac{1}{4s-1}\right\} = \frac{1}{4}e^{-t/4}$ //

f. $\mathcal{L}^{-1}\left\{\frac{5}{s^2+49}\right\} = \left\{\frac{5}{s^2+7^2}\cdot\frac{7}{7}\right\} = \frac{5}{7}\mathcal{L}^{-1}\left\{\frac{7}{s^2+7^2}\right\} = \frac{5}{7}\,\text{sen}\,7t$ //

3. Traslación en el eje "s". Encuentre $F(s)$ o $f(t)$ como se indica

a. $\mathcal{L}\{te^{10t}\}$

$= \mathcal{L}\{e^{at}f(t)\} = F(s-a) = \frac{1}{s^2}\Big|_{s\to s-10} = \frac{1}{(s-10)^2}$ //

b. $\mathcal{L}\{t^3e^{-2t}\}$

$= \mathcal{L}\{e^{at}f(t)\} = F(s-a) = \mathcal{L}\{t^3\}\Big|_{s\to s+2}$

$= \left(\frac{3!}{s^{3+1}}\right)_{s\to s+2} = \frac{6}{s^4}\Big|_{s\to s+2} = \frac{6}{(s+2)^4}$ //

c. $\mathcal{L}\{e^t \cdot \text{sen}\,3t\}$

$= \mathcal{L}^{-1}\{\text{sen}\,3t\}\Big|_{s\to s-1} = \frac{3}{s^2+9} = \frac{3}{(s-1)^2+9}$ //

d. $\mathcal{L}^{-1}\left\{\frac{1}{(s+2)^3}\right.$

$= e^{-2t}\mathcal{L}^{-1}\frac{1}{s^3} = e^{-2t}\mathcal{L}^{-1}\left\{\frac{2!}{s^3}\right\} = \frac{1}{2}e^{-2t}t^2$ //

e $\mathcal{L}^{-1}\left\{\frac{s}{s^2+4s+5}\right\}$

$= \mathcal{L}^{-1}\left\{\frac{s}{(s^2+2\cdot 2s+4)+1}\right\} = \mathcal{L}^{-1}\left\{\frac{s}{(s+2)^2+1}\right\}$

$= \left\{\frac{s+2-2}{(s+2)^2+1}\right. = \mathcal{L}^{-1}\left\{\frac{s+2}{(s+2)^2+1}\right\} + \mathcal{L}^{-1}\left\{\frac{-2}{(s+2)^2+1}\right\} =$

$= e^{-2t}\cdot\cos t - 2e^{-2t}\,\text{sen}\,t$ //

f. $\mathcal{L}^{-1}\left\{\frac{s}{(s+1)^2}\right\} = \mathcal{L}^{-1}\left\{\frac{s+1-1}{(s+1)^2}\right\} = \mathcal{L}^{-1}\left\{\frac{(s+1)}{(s+1)^2}\right\} - \mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2}\right\}$

$= \mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} - \mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s}\right\}_{s\to s+1} - \mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\}_{s\to s+1}$

$= e^{-t} - te^{-t}$ //

4. Traslación en el eje "t". Encuentre $F(s)$ o $f(t)$, como se indica.

a. $\mathcal{L}\{e^{2-t}\, u(t-2)\}$

$= \mathcal{L}\{e^{-(t-2)}\, u(t-2)\}$

$= \frac{1}{s+1} e^{-2s} = \frac{e^{-2s}}{s+1}$

b. $\mathcal{L}\{(3t+1)\, u(t-1)\}$

$\mathcal{L}\{3(t-1)+4)\, u(t-1)\}$

$= \mathcal{L}\{3(t-1)u(t-1)\} +$

$\mathcal{L}\{4u(t-1)\} = \frac{3}{s^2}e^{-s} + \frac{4}{s}e^{-s}$ //

c. $\mathcal{L}\{\cos 2t\; u(t-\pi)\}$

$= e^{-\pi s}\,\mathcal{L}\{\cos(2t+2\pi)\} = \mathcal{L}\{\cos(2(t-\pi))\, u(t-\pi)\}$

$= \frac{s}{s^2+4} e^{-\pi s}$ //

d. $\mathcal{L}\{\operatorname{sen} t\, u(t-\pi/2)\}$

$= \mathcal{L}\{\cos(t-\frac{\pi}{2})\, u(t-\frac{\pi}{2})\}$

$= \frac{s}{s^2+1} e^{-\pi/s}$ //

e. $\mathcal{L}^{-1}\left\{\frac{e^{-2s}}{s^3}\right\}$

$= \mathcal{L}^{-1}\left\{\frac{2!}{2!\,s^3}\, e^{-2s}\right\}$

$= \frac{1}{2}(t-2)^2\, u(t-2)$ //

f) $\mathcal{L}^{-1}\left\{\frac{e^{-\pi s}}{s^2+1}\right\} = \mathcal{L}^{-1}\left\{e^{-\pi s}\left(\frac{1}{s^2+1}\right)\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\}\Big|_{t\to t-\pi}$

$= \operatorname{sen}(t-\pi) - u(t-\pi) = -\operatorname{sen} t\, u(t-\pi)$ //

g) $\mathcal{L}^{-1}\left\{\frac{e^{-s}}{s(s+1)}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s(s+1)}e^{-s}\right\} = \frac{1}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1}$

$1 = A(s+1) + Bs = A(-1+1) + B(-1) = -1 \quad B = -1$

$1 = A(0+1) + B(0) = 1 \quad A = 1$

$= \left\{\frac{1}{s} - \frac{1}{s+1}\right\}e^{-s} = \mathcal{L}^{-1}\left\{\frac{1}{s} - \frac{1}{s+1}\right\} = 1 - e^{-t} = \mathcal{L}^{-1}\left\{\left(\frac{1}{s} - \frac{1}{s+1}\right)e^{-s}\right\}$

$= \left(1 - e^{-(t-1)}\right)u(t-1)$ //

Transformada de una derivada. Use la transformada de Laplace para resolver el problema con valores iniciales

a. $\frac{dy}{dt} - y = 1,\ y(0) = 0$

$\mathcal{L}\left\{\frac{dy}{dt}\right\} - \mathcal{L}\{y\} = \mathcal{L}\{1\} = s\mathcal{L}\{y\} - y(0) - \mathcal{L}(y) = \mathcal{L}\{1\}$

$s(y(s)) - y(s) = \frac{1}{s},\quad y(s)(s-1) = \frac{1}{s} = \frac{1}{s(s-1)}$

$\left[\frac{1}{s(s-1)} = \frac{A}{s} + \frac{B}{s-1}\right] \quad \begin{cases} 1 = A(1-1) + B(1) \\ 1 = A(0-1) + B(0) \end{cases} \quad \begin{matrix} B = 1 \\ A = -1 \end{matrix}$

$y(s) = \frac{1}{s(s-1)} = \frac{A}{s} + \frac{B}{s-1} = y(t) = \mathcal{L}^{-1}\left\{\frac{1}{s}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\}$

$y = -1 + e^{t}$ //

b. $2\frac{dy}{dt}+y=0,\ y(0)=-3$

$2\mathcal{L}\{y'\}+\mathcal{L}\{y\}=\mathcal{L}\{0\}$

$2[s\mathcal{L}\{y\}-y(0)]+\mathcal{L}\{y\}=0 \quad = 2s\mathcal{L}\{y\}-2y(0)+\mathcal{L}\{y\}=0$

$\mathcal{L}\{y\}[2s+1]=-6$

$\mathcal{L}\{y\}=\frac{-6}{2s+1}=\mathcal{L}^{-1}\left\{\frac{-3}{s+\frac{1}{2}}\right\}=-3e^{-1/2t}$

c. $y'+6y=e^{4t},\ y(0)=2.$

$\mathcal{L}\{y'\}+6\mathcal{L}\{y\}=\mathcal{L}\{e^{4t}\} = s\,y(s)-2=\mathcal{L}\{e^{4t}\}=\frac{1}{s-4}$

$s\,y(s)-2+6y(s)=\frac{1}{s-4},\ s\,y(s)+6y(s)=\frac{1}{s-4}+2$

$y(s)=\frac{1}{(s-4)(s+6)}+\frac{2}{(s+6)}=\frac{1+2s-8}{(s-4)(s+6)}=\frac{2s-7}{(s-4)(s+6)}$

$\left[\frac{2s-7}{(s-4)(s+6)}=\frac{A}{(s-4)}+\frac{B}{(s+6)}\right]$

$2s-7=A(s+6)+B(s-4)$

$2(-6)-7=A(-6+6)+B(-6-4)$

$-12-7=-10B \rightarrow -19=-10B \cdot B=19/10$

$2(4)-7=A(4+6)+B(4-4)$

$8-7=10A \qquad A=1/10$

$$g(s)\ \frac{2s-7}{(s-4)(s+6)} = \frac{A}{s-4} + \frac{B}{(s+6)} = \frac{1}{10}\left(\frac{1}{s-4}\right) + \frac{19}{10}\left(\frac{1}{s+6}\right)$$

$$g(t) = \frac{1}{10}\mathcal{L}^{-1}\left\{\frac{1}{s-4}\right\} + \frac{19}{10}\mathcal{L}^{-1}\left\{\frac{1}{s+6}\right\} = \frac{1}{10}e^{4t} + \frac{19}{10}e^{-6t}$$

6. Derivada de una transformada: Evaluar cada una de las transformadas de laplace.

a. $\mathcal{L}\{te^{-10t}\}$

$$=(-1)^1\frac{d}{ds}\left(\frac{1}{s+10}\right) = \frac{-d}{ds}(s+10)^{-1} = -(-1)(s+10)^{-2}(1)$$

$$= (s+10)^{-2} = \mathcal{L}\{te^{-10t}\} = \frac{1}{(s+10)^2}$$

b. $\mathcal{L}\{t \cdot \cos 2t\} = \frac{-d}{ds}\mathcal{L}\{\cos 2t\}$

$$= \frac{d}{ds}\left(\frac{-s}{s^2+2^2}\right)$$

$$= \frac{2s^2-(s^2+4)}{(s^2-4)^2} = \frac{2s^2-s^2-4}{(s^2+4)^2} = \frac{s^2-4}{(s^2+4)^2}$$

c. $\mathcal{L}\{t^2 \operatorname{sen} h\, t\} = \mathcal{L}\left\{t^2\left(\frac{e^t-e^{-t}}{2}\right)\right]$

$$= \frac{1}{2}\mathcal{L}[t^2e^t] - \frac{1}{2}\mathcal{L}[t^2e^{-t}]$$

$$= \frac{1}{2}\frac{2!}{s^3}\Big|_{s\to s-1} - \frac{1}{2}\frac{2!}{s^3}\Big|_{s\to s+1}$$

$$= \frac{1}{(s-1)^3} - \frac{1}{(s+1)^3}$$ //