Trabajo Autónomo

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Curso: 4 to ciclo "A"

1. Obtenga la ecuación diferencial de la familia dada de curvas

a. $y = C_1 e^{-x}$

$y = c_1 \cdot e^{-x}$

$\frac{dy}{dx} = \frac{d}{dx}(c_1 \cdot e^{-x})$

$\frac{dy}{dx} = -c_1 \cdot e^{-x} = \frac{dy}{dx} = -c_1 \cdot e^{-x}$ //

b. $c_1 y + 2x = 3$

$= \frac{d}{dx}(c_1 y + 2x) = \frac{d}{dx} 3$

$= c_1 \cdot \frac{dy}{dx} + 2 = 0 = \frac{dy}{dx} = \frac{-2}{c_1}$ //

c. $y^2 = c_1 (x+1)$

$\frac{d}{dx} y^2 = \frac{d}{dx}(c_1 (x+1))$

$2y \cdot \frac{dy}{dx} = c_1$

$\frac{dy}{dx} = \frac{c_1}{2y}$ //

2. Resuelva la ecuación por separación de variables

a. $e^x \frac{dy}{dx} = 2x$

$\int dy = 2\int x \cdot \frac{1}{e^x} dx$ $\quad u = x \quad du = dx \quad dv = e^{\frac{1}{x}} \quad v = -e^{-x}$

$y = 2\left(x \cdot e^{-x} - \int -e^{-x} dx\right)$

$y = 2\left(x \cdot e^{-x} + e^{-x} dx\right)$

$y = 2xe^{-x} - 2e^{-x}$

$y = -2xe^{-x} - 2e^{-x} + C$

$y = \frac{-2x - 2}{e^x} + C$

b. $\frac{dy}{dx} + 2xy = 0$

$\frac{dy}{dx} = -2xy$

$dy = (-2xy)dx$

$\int \frac{dy}{y} = \int -2x\,dx$

$\ln y + c = -2\int x\,dx$

$\ln y + c = -\cancel{2}\frac{x^2}{\cancel{2}} + C$

$\ln y + c = -x^2 + c$

$e^{\ln(y)} = e^{-x^2 + c}$

$y = e^{-x^2 + c}$

$y = e^{-x^2} \cdot e^c$

$y = c \cdot e^{-x^2}$

c. $dx + e^{3x}dy = 0$

$$dx = -e^{3x}dy$$

$$\frac{dx}{-e^{3x}} = dy$$

$$dy = \frac{1}{-e^{3x}}dx$$

$$\int dy = -\int e^{-3x}dx$$

$u = -3x \quad du = -3\,dx \quad dx = du/3$

$$y + c = \frac{+1}{3}\int e^{u}\,du$$

$$y + c = +\frac{1}{3}e^{-3x} + C$$

$$y = \frac{1}{3}\cdot e^{-3x} + C$$

3. Ecuaciones Diferenciales Homogéneas. Resuelva la ecuación diferencial dada usando una sustitución apropiada.

a $\frac{dy}{dx} = \frac{y^2 - x^2}{3xy} = 3xy\,dy = y^2 - x^2\,dx = y^2 - x^2dx - 3xy\,dy = 0$

$= ((ux)^2 - x^2)dx - 3x\,(ux)(u\,dx - x\,du) = 0$

$= 3u^2x^2dx + 3u\,x^3du - u^2x^2dx + x^2dx = 0$

$= x^2(2u^2+1)\,dx + 3ux^3du = 0$

$= x^2(2u^2+1)dx = -3u\,x^3du$

$= \frac{x^2\,dx}{x^3} = \frac{-3u\,du}{(2u^2+1)}$

$$\int \frac{1}{x}\,dx = \int \frac{-3u\,du}{3u^2+1}$$

$\ln(x)+c = -3\int \frac{u\,du}{2u^2+1}$ $\quad v = 2u^2+1$

$dv = 4u\,du$

$\ln(x)+c = -\frac{3}{4}\int \frac{dv}{v}$ $\quad \frac{dv}{4} = u\,du$

$\ln(x)+c = -\frac{3}{4}\cdot \ln(v)+c$

$\ln(x)+c = -\frac{3}{4}\ln(2u^2+1)+c$

$\ln(x) = -\frac{3}{4}\ln\left(\frac{2y^2}{x^2}+1\right)+c$ // respuesta

b. $(x-y)dx + x\,dy = 0$

$(x-ux)dx + x(u\,dx + x\,du) = 0$

$x\,dx - \cancel{ux\,dx} + \cancel{ux\,dx} + x^2du = 0$

$x\,dx + x^2du = 0$

$x\,dx = -x^2du$

$-du = \frac{\cancel{x}}{-x^{\cancel{2}}}dx$

$\int -du = \int -\frac{1}{x}dx$ $\quad u = \frac{y}{x}$

$-u + c = \ln(x)$

$-\frac{y}{x} + c = \ln(x)$

$-\frac{y}{x} = \ln(x) - c \quad = -y = x\ln(x) - xc \quad (-1)$

$y = -x\ln(x) + xc$ // respuesta

c. $x\,dx + (y-2x)\,dy = 0$

$$uy(u\,dy + y\,du) + (y - 2uy)\,dy = 0$$

$$u^2y\,dy + uy^2\,du + y\,dy - 2uy\,dy = 0$$

$$uy^2\,du + y(u^2 - 2u + 1)\,dy = 0$$

$$uy\,du + (u-1)^2\,dy = 0$$

$$u = u-1 \quad v = u+1 \quad du = du$$

$$= \int \frac{u\,du}{(u-1)^2} + \frac{dy}{y} = 0 \quad = \int \frac{u+1}{u^2}\,du + \frac{dy}{y} = 0$$

$$= \ln u - u^{-1} + \ln|y| + C$$

$$= \ln(u-1) - \frac{1}{u-1} + \ln(y) = C$$

$$= \ln\left(\frac{x-y}{y}\right) - \left(\frac{x-y}{y}\right)^{-1} = -\ln y + C$$

$$= \ln(x-y) - \ln y - \frac{y}{x-y} = -\ln y + C$$

$$= (x-y)\ln(x-y) - y = C(x-y)$$

$$= \ln(x-y) - (x-y) - y = C.(x-y)$$

// respuesta

4. Ecuaciones diferenciales exactas: Determine si la ecuación dada es exacta. Si es exacta resuélvala

a. $\overset{m}{(\text{sen } y - y \cdot \text{sen } x)}\,dx + \overset{N}{(\cos x + x \cdot \cos y - y)}\,dy = 0$

$m(x,y) = \text{sen } y - y \cdot \text{sen } x$

$N(x,y) = \cos x + x \cdot \cos y - y$

$\dfrac{\partial m}{\partial y} = \dfrac{\partial}{\partial y}\, \text{sen } y - y \cdot \text{sen } x$ $\qquad$ $\dfrac{\partial N}{\partial x} = \dfrac{\partial}{\partial x}\, \cos x + x \cdot \cos y - y$

$= \cos y - \text{sen } x \,//$ $\qquad$ $= -\text{sen } x + \cos y$

$= \cos y - \text{sen } x \,//$ son exactas

$f(x,y) = \int m(x,y) + g(y)$

$= \int (\text{sen } y - y \cdot \text{sen } x)\,dx + g(y)$

$= \text{sen } y \cdot x - y \cdot -\cos x + g(y)$

$= x \cdot \text{sen } y + \cos x \cdot y + g(y)$

$g(y) = -\dfrac{y^2}{2} + C$

$f(x,y) = x \cdot \text{sen } y + \cos x \cdot y - \dfrac{y^2}{2} + C$ // respuesta

b. $(5x + \overset{m}{4y})dx + (4x - \overset{N}{8y^3})dy = 0$

$m(x,y)dx + N(x,y)dy = 0$

$m(x,y) = 5x + 4y$

$N(x,y) = 4x - 8y^3$

$\frac{dm}{dy} = \frac{d}{dy} 5x + 4y$

$= 4$

$\frac{dN}{dx} = 4x - 8y^3$

$= 4$ // son exactas

$f(x,y) = \int m(x,y)dx + g(y)$

$= \int 5x\,dx + \int 4y\,dx + g(y)$

$= 5\frac{x^2}{2} + 4xy + g(y)$

$g(y) = -\int 8y^3$

$g(y) = -8\int y^3$

$g(y) = -\cancel{8}\frac{y^4}{\cancel{4}} = -2y^4 + C$

$f(x,y) = \frac{5x^2}{2} + 4xy - 2y^4 + C$ //respuesta

c. $\overset{m}{(2y^2x-3)}dx + \overset{N}{(2yx^2+4)}dy = 0$

$m(x,y)\,dx + N(x,y)\,dy = 0$

$m(x,y) = 2y^2x-3$

$N(x,y) = 2yx^2+4$

$\frac{\partial m}{\partial y} = \frac{\partial}{\partial y} 2y^2x-3$ $\qquad$ $\frac{\partial N}{\partial x} = \frac{\partial}{\partial x} 2yx^2+4$

$= 4yx$ $\qquad$ $= 4yx$

$f(x,y) = \int m(x,y)\,dx + g(y)$

$= \int 2y^2x - 3 + g(y)$

$= 2y^2\int x - \int 3 + g(y)$

$= \cancel{2}y^2\frac{x^2}{\cancel{2}} - 3x + g(y)$

$= x^2y^2 - 3x + g(y)$

$g(y) = 4\int 1\,dy$

$= 4y + C$

$f(x,y) = x^2y^2 - 3x + 4y = C$ // respuesta