TPE → Taller N9 y N10

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Curso: 4 to ciclo "A"

Asignatura: Ecuaciones Diferenciales

Ecuaciones Diferenciales Lineales no Homogéneas. Método de variación de parámetros

- Resuelva cada ecuación diferencial mediante variación de parámetros

* $y'' + y = \operatorname{sen} x$

$m^2 + 1 = 0$

$m_1 = +i$ $\qquad y_1 = e^{\alpha x} \cdot \cos(x)$ $\qquad y_c = e^{\alpha x} \cos(x) + e^{\alpha x} \operatorname{sen}(x)$

$m_2 = -i$ $\qquad y_2 = e^{\alpha x} \cdot \operatorname{sen}(x)$

$y_1 = \cos(x)$

$y_2 = \operatorname{sen}(x)$

$$\begin{cases} u' \cos(x) + v' \operatorname{sen}(x) = 0 & \operatorname{sen}(x) \\ -u' \operatorname{sen}(x) + v' \cos(x) = \operatorname{sen}(x) & \cos(x) \end{cases}$$

$$\cancel{u' \operatorname{sen}(x) \cos(x)} + v' \operatorname{sen}^2(x) = 0$$

$$\cancel{-u' \operatorname{sen}(x) \cos(x)} + v' \cos^2(x) = \operatorname{sen}(x) \cdot \cos(x)$$

$$v' \operatorname{sen}^2(x) + v' \cos^2(x) = \operatorname{sen}(x) \cdot \cos(x)$$

$$v'(\sin^2(x)+\cos^2(x)) = \sin(x)\cdot\cos(x)$$

$$v'(x) = \sin(x)\cdot\cos(x)$$

$$\int v' = \int \sin(x)\cdot\cos(x)\,dx$$

$t = \sin(x)$ $\quad dx = \frac{1}{\cos(x)}$

$dt = \cos(x)\,dx$

$$v = \int \sin(x)\cdot\cancel{\cos(x)}\,\frac{1}{\cancel{\cos(x)}}$$

$$v = \int t$$

$$v = \frac{t^2}{2} = \frac{\sin^2(x)}{2}$$

$$u' = \frac{-v' \cdot y_2}{y_1}$$

$$u' = \frac{-\sin(x)\cdot\cancel{\cos(x)}\cdot\sin(x)}{\cancel{\cos(x)}}$$

$$u' = -\sin^2(x)$$

$$\int u' = \int \frac{1-\cos(2x)}{2} = \frac{1}{2}\int 1-\cos(2x)\,dx$$

$$u = \frac{1}{2}\left(\int 1\,dx - \int \cos(2x)\,dx\right)$$ $\quad t = 2x$

$dt = 2\,dx \quad dx = \frac{1}{2}$

$$= \frac{1}{2}\left(x - \frac{\sin(2x)}{2}\right)$$

$$= \frac{1}{2}x^2 - \frac{\sin 2x}{4} + C =$$

$$y_p = \frac{2\,sen\,x + \cos 2x - 2x}{4}\cos(x) + \frac{sen^2 x}{2} \cdot sen(x)$$

$$y = C_1 \cdot \cos x + C_2 \cdot sen(x) + \frac{sen\,2x - 2x}{4} \cdot \cos(x) + \frac{sen(x)}{2}\,\cancel{sen\,x}$$

$$y_F\ \frac{1}{2}\ sen\,x(\cos^2 x + sen^2 x) - \frac{1}{2}x \cdot \cos x$$

$$y_p = \frac{1}{2}\ sen\ x - \frac{1}{2}\ x \cdot \cos(x)$$

$$y = y_c + y_p \,//$$

$$y = C_1 \cos(x) + C_2 \cdot sen(x) + \frac{1}{2}\ sen(x) - \frac{1}{2}x\cos(x)\,//$$

$*\ y'' + y = \cos^2 x$

$$(m^2 + 1) = 0$$

$m_1 = +i$ $\quad y_1 = e^x \cdot \cos(x) = \cos(x)$

$m_2 = -i$ $\quad y_2 = e^x \cdot sen(x) = sen(x)$

$$y_c = e^x \cdot \cos(x) + e^x \cdot sen(x)$$

$$\begin{cases} u'\cos(x) + v'sen(x) = 0 \\ -u'\,sen(x) + v'\cos(x) = \cos^2(x) \end{cases} \quad \begin{matrix} sen(x) \\ \cos(x) \end{matrix}$$

$$\begin{cases} \cancel{u'\,sen(x)\cos(x)} + v'\,sen^2(x) = 0 \\ \cancel{-u'\,sen(x)\cos(x)} + v'\cos^2(x) = \cos^3(x) \end{cases}$$

$$v'(sen^2 x + \cos^2 x) = \cos^3(x)$$

$$v'(1) = \cos^3(x)$$

$$\int v' = \int \cos^3(x) \qquad v' = \cos^3(x)$$

$$v' = \int \cos^2(x) \cdot \cos(x)$$

$$v = \int \cos(x) \cdot (1 - \sin^2(x))$$

$u = \sin(x)$

$\frac{du}{dx} = \cos(x)$

$dx = \frac{1}{\cos(x)} du$

$$v = \int \cancel{\cos(x)} \cdot (1 - \sin^2(x)) \cdot \frac{1}{\cancel{\cos(x)}}$$

$$v = \int (1 - u^2) du$$

$$v = \int 1 du - \int u^2 du = \sin(x) - \frac{\sin^3(x)}{3}$$

$$u' \cos(x) + v' \operatorname{sen}(x) = 0$$

$$u' = \frac{-\cancel{\cos^3(x)} \cdot \operatorname{sen}(x)}{\cancel{\cos(x)}}$$

$$u' = -\cos^2(x) \cdot \operatorname{sen}(x)$$

$$\int u' = \int -\cos^2(x) \cdot \operatorname{sen}(x)$$

$t = \cos(x) \quad dt = -\operatorname{sen}(x) dx \quad dx = \frac{1}{-\operatorname{sen}(x)}$

$$u = \int -t^2 \cdot \cancel{\operatorname{sen}(x)} \cdot \frac{1}{-\cancel{\operatorname{sen}(x)}}$$

$$u = \int t^2 = \frac{t^3}{3} = \frac{\cos^3(x)}{3} + C$$

$$y_p = \operatorname{sen}(x) - \frac{\sin^3(x)}{3} + \frac{\cos^3(x)}{3}$$

$$y = e^x\cos(x) + e^x\operatorname{sen}(x) + \sin(x) - \frac{\sin^3(x)}{3} + \frac{\cos(x)}{3}$$

* $y'' - 4y = e^{2x}/x$

$(m^2-2)(m+2) = 0$

$m_1 = 2$
$m_2 = -2$

$$u_1 = -\int \frac{y_2 f(x)\,dx}{\omega}$$

$$u_2 = \int \frac{y_1 f(x)\,dx}{\omega}$$

$y_1 = e^{2x} \quad y_2 = e^{-2x}$

$y_c = C_1e^{2x} + C_2e^{-2x}$

$$\omega = \begin{vmatrix} e^{2x} & e^{-2x} \\ 2e^{2x} & -2e^{-2x} \end{vmatrix} = -4$$

$$u_1 = -\int \frac{e^{-2x}\cdot\frac{e^{2x}}{x}\,dx}{-4} = \frac{1}{4}\ln(x) \qquad u_2 = \int \frac{e^{2x}\cdot\frac{e^{2x}}{x}\,dx}{-4} = -\int \frac{e^{4x}}{4x}\,dx$$

$$u_2 = -\frac{1}{4}\int \frac{e^{4x}}{x}\,dx = \int_{x_0}^{x} \frac{e^{4t}}{t}\,dt$$

$$y_p = \frac{1}{4}\ln(x)\cdot e^{2x} - \frac{1}{4}\int_{x_0}^{x} \frac{e^{4t}}{t}\,dt \cdot e^{-2x} \;//$$

$$y = c_1e^{2x} + C_2e^{-2x} + \frac{1}{4}\ln(x)\cdot e^{2x} - \frac{1}{4}\int_{x_0}^{x} \frac{e^{4t}}{t}\,dt\cdot e^{-2x} \;//$$

* $y'' + 3y' + 2y = 1/(1+e^x)$

$m^2 + 3m + 2 = 0$

$(m+1)(m+2) = 0$

$m_1 = -1 \qquad y_1 = C_1e^{-x} \qquad y_c = C_1e^{-x} + C_2e^{-2x}$
$m_2 = -2$
$y_2 = C_2e^{-2x}$

$y_1 = e^{-x}$ $\quad y_2 = e^{-2x}$ $\quad g(x) = \frac{1}{1+e^x}$

$w(y_1, y_2) = \begin{vmatrix} e^{-x} & e^{-2x} \\ -e^{-x} & -2e^{-2x} \end{vmatrix} = -2e^{-3x} + e^{-3x} = -e^{-3x}$

$w_1 = \begin{vmatrix} 0 & e^{-2x} \\ \frac{1}{1+e^x} & -2e^{-2x} \end{vmatrix} = -\frac{e^{-2x}}{1+e^x}$ $\qquad u_1 = \int \frac{w_1}{w} dx = \int \frac{1}{e^{-x}(1+e^x)} dx$

$= \ln|1+e^x|$

$w_2 = \begin{vmatrix} e^{-x} & 0 \\ -e^{-x} & \frac{1}{1+e^x} \end{vmatrix} = \frac{e^{-x}}{1+e^x}$ $\qquad u_2 = \int \frac{1}{e^{-2x}(1+e^x)} dx = -\int \frac{e^x e^x}{1+e^x} dx$

$p = 1 + e^x$

$p - 1 = e^x$

$dp = e^x dx$

$u_2 = -\int \frac{p-1}{p} dp = -\int 1 - \frac{1}{p} = -(1+e^x) + \ln(1+e^x)$

$u_2 = -1 - e^x + \ln(1+e^x)$

$y_p = u_1 y_1 + u_2 y_2$ $\qquad y_p = (\ln(1+e^x))(e^{-x}) + (-1 - e^x + \ln(1+e^x))(e^{-2x})$

$y_p = e^{-x}\ln(1+e^x) - e^{-2x} - e^{-2x} - e^{-x} + e^{-2x}\ln(1+e^x)$

$y_p = e^{-x}\ln(1+e^x) + e^{-2x}\ln(1+e^x)$

$y = y_c + y_p$

$y = C_1 e^{-x} + C_2 e^{-2x} + e^{-x}\ln(1+e^x) + e^{-2x}\ln(1+e^x)$ //

↑

solución general

* $y'' + 3y' + 2y = \operatorname{sen} e^x$

$m^2 + 3m + 2 = 0$

$(m+2)(m+1) = 0$

$m_1 = -2 \qquad m_2 = -1$

$y_1 = e^{-2x} \qquad y_2 = e^{-x}$

$y_c = C_1 e^{-2x} + C_2 e^{-x}$

$y_p = u_1 y_1 + u_2 y_2$

$u_1' = \frac{w_1}{w} \qquad u_2' = \frac{w_2}{w}$

$u_1' = \frac{-e^{-x} \operatorname{sen} e^x}{e^{-3x}} \qquad u_2' = \frac{e^{-2x} \operatorname{sen} e^x}{e^{-3x}}$

$u_1' = -e^{2x} \operatorname{sen} e^x \qquad u_2' = e^x \operatorname{sen} e^x$

$u_1 = -\int e^x (e^x \operatorname{sen} e^x)\, dx$

$u = e^x \qquad dv = e^x \operatorname{sen} e^x\, dx$

$du = e^x dx \qquad v = -\cos e^x$

$\int u\, dv = uv - \int v\, du$

$u_1 = e^x \cos e^x - \int e^x \cos e^x\, dx$

$u_1 = e^x \cos e^x - \int \cos t\, dt$

$u_1 = e^x \cos e^x - \operatorname{sen} e^x$

$y_1 = e^{-2x} \qquad y_2 = e^{-x}$

$w = \begin{vmatrix} e^{-2x} & e^{-x} \\ -2e^{-2x} & -e^{-x} \end{vmatrix} = (e^{-2x})(-e^{-x}) - (-2e^{-2x})(e^{-x})$

$w = -e^{-3x} + 2e^{-3x} \qquad w = e^{-3x}$

$w_1 = \begin{vmatrix} 0 & e^{-x} \\ \operatorname{sen} e^x & e^{-x} \end{vmatrix} = -e^{-x} \operatorname{sen} e^x$

$w_1 = -e^{-x} \cdot \operatorname{sen} e^x$

$w_2 = \begin{vmatrix} e^{-2x} & 0 \\ -2e^{-2x} & \operatorname{sen} e^x \end{vmatrix} = e^{-2x} \operatorname{sen} e^x$

$w_2 = e^{-2x} \operatorname{sen} e^x$

$u_2 = \int e^x \operatorname{sen} e^x\, dx$

$t = e^x \qquad dt = e^x dx$

$u_2 = \int \operatorname{sen} t\, dt$

$u_2 = -\cos t = -\cos e^x$

$u_2 = -\cos e^x$

$y_p = (e^x \cos e^x - \operatorname{sen} e^x) e^{-2x} + (\cos e^x)(e^{-x})$

$y_p = e^{-x} \cos e^x - e^{-2x} \operatorname{sen} e^x - e^{-x} \cos e^x$

$y_p = -e^{-2x} \operatorname{sen} e^x$ //

EDLNH: método de coeficientes indeterminados

- Resuelva la ecuación diferencial dada x el método de coeficientes constantes

1. $y'' - 9y = 54$

$y'' - 9y = 0$ $\qquad g(x) = 54$

$m^2 - 9 = 0$ $\qquad = 0 \Rightarrow D$

$(m+3)(m-3) = 0$

$m_1 = -3$ $\quad y_1 = e^{-3x}$ $\qquad y_c = C_1 e^{-3x} + C_2 e^{3x}$

$m_2 = 3$ $\quad y_2 = e^{3x}$

$D(D^2 - 9) = D(54)$ $\qquad y = C_1 e^{0x} + C_2 e^{-3x} + C_3 e^{3x}$

$m(m+3)(m-3) = 0$ $\qquad y = C_1 + C_2 e^{-3x} + C_2 e^{3x}$

$m_1 = 0$ $\quad y_1 = e^{0x}$

$m_2 = -3$ $\quad y_2 = e^{-3x}$

$m_3 = 3$ $\quad y_3 = e^{3x}$

$y'' - 9y = 54$

$y_p = A$ $\qquad 0 - 9A = 54$

$y' = 0$

$y'' = 0$ $\qquad A = 54 / -9 = -6$

$y = y_c + y_p$

$y = C_1 e^{-3x} + C_2 e^{3x} - 6$ //

2. $y'' + y' = 3$

$m^2 + m = 0$ $\quad y_1 = e^{0x}$ $\quad y_p = 3x$

$m(m+1) = 0$

$m_1 = 0$ $\quad y_2 = e^{x}$ $\quad y = C_1 e^{0x} + C_2 e^{-x} + 3x$

$m_2 = -1$

$g(x) = 3$

$y' = 0 \quad => 0$

$D(D^2 + D) = D(3)$

$m(m^2 + m) = 0$

$m(m^2 + m) = 0$

$m_1 = 0$
$m_2 = -1$

$y_p = C_1 e^{0x} + \underbrace{C_2 x e^{0x}}_{y_p} + C_3 e^{-x}$

$y = Ax$

$y' = A$

$y'' = 0$

$y_p = 3^x$

$y'' + y = 3$

$A = 3$ //

$y = C_1 x + C_2 + C_3 e^{-x}$ //

3. $y'' + 4y' + 4y = 2x + 6$

$y'' + 4y' + 4y = 0$

$(m^2 + 4m + 4) = 0$

$(m+2)(m+2) = 0$

$m_1 = -2 \qquad y_1 = e^{-2x}$
$m_2 = -2 \qquad y_2 = e^{-2x}$

$y_c = C_1 e^{-2x} + C_2 x e^{-2x}$

$y_p = A + Bx \rightarrow y_p \ 1 + \frac{1}{2}x$

$y' = B$

$y'' = 0$

$g(x) = 2x + 6$

$y' = 2$

$y'' = 0$

$D^2(D^2 - 4D + 4) = D^2(2x + 6)$

$m^2(m+2)(m+2) = 0$

$m_1 = 0$
$m_2 = 0$
$m_3 = -2$
$m_4 = -2$

$y = C_1 + C_2 x + C_3 e^{-2x} + C_4 x e^{-2x}$

$y'' + 4y' + 4y = 2x + 6$

$4B + 4A + 4Bx = 2x + 6$

$4B + 4A = 6$

$4B = 2$
$B = 4/2$

$B = \frac{1}{2}$ //

$4(\frac{1}{2}) + 4A = 6$

$4A = 6 - 2$

$A = \frac{4}{4}$

$A = 1$

$y_p = 1 + \frac{1}{2}x$

$y = C_1 e^{-2x} + C_2 x e^{-2} + 1 + \frac{1}{2}x$

4. $y''' + y'' = 8x^2$

$m^3 + m^2 = 0$

$m^3(m+1) = 0$

$m_1 = 0$

$m_2 = -1$

$y_1 = e^{0x}$

$y^2 = xe^{0x}$

$y^3 = e^{-x}$

$g(x) = 8x^2$

$y' = 16x$

$y'' = 16$

$y''' = 0$

$D^3(D^3 + D^2) = D^2(8x^2) = 0$

$m^3(m^3 + m^2) = 0$

$m^3(m^2(m+1)) = 0$

$m_1 = 0$	$m_4 = 0$
$m_2 = 0$	$m_5 = 0$
$m_3 = 0$	$m_6 = -1$

$y = C_1 e^{0x} + C_2 x e^{0x} + C_3 e^{-x}$

$y = C_1 + C_2 x + C_3 x^2 + C_4 x^3 + C_5 x^4 + C_6 e^{-x}$

$y''' + y'' = 8x^2$

$6B + 24Cx + 2A + 6Bx + 12Cx^2 = 8x^2$

$12Cx^2 + 6Bx + 24Cx + 2A + 6B = 8x^2$

$$\begin{cases} 12C = 8 \\ 6B + 24C = 0 \\ 2A + 6B = 0 \end{cases}$$

$12C = 8$

$C = 8/12$

$C = 2/3$

$6B + 24\left(\frac{2}{3}\right) = 0$

$6B = -16$

$B = -8/3$

$2A + 6(-8/3) = 0$

$2A = 16$

$A = 16/2 = 8$

$y_p = 8x^2 - \frac{8}{3}x^3 + \frac{2}{3}x^4$

$y = C_1 + C_2x + C_3e^{-x} + 8x^2 - \frac{8}{3}x^3 + \frac{2}{3}x^4$ //

5. $y'' - y' - 12y = e^{4x}$

$(m^2 - m - 12) = 0$

$(m-4)(m-3) = 0$

$m_1 = 4 \qquad y_1 = e^{4x}$

$m_2 = -3 \qquad y_2 = e^{-3x}$

$y_c = C_1e^{4x} + C_2e^{-3x}$

$g(x) = e^{4x} \Rightarrow D(-4)$

$(D-4)(D^2 - D - 12) = 0$

$(m-4)(m-4)(m+3) = 0$

$m_1 = 4$

$m_2 = 4$

$m_3 = -3$

$y_1 = e^{4x}, \; y_2 = xe^{4x}, \; y_3 = e^{-3x}$

$y_p = Axe^{4x}$

$y_p' = Ae^{4x} + 4Ae^{4x} + 16Axe^{4x}$

$y'' = 8Ae^{4x} + 16Ae^{4x}$

$8Ae^{4x} + 16Axe^{4x} - Ae^{4x} - 4Axe^{4x} - 12Axe^{4x} = e^{4x}$

$7Ae^{4x} = e^{4x} \qquad \div e^{4x}$

$A = \frac{1}{7}$

$y_p = \frac{1}{7}xe^{-4x}$

$y = y_c + y_p$

$y = C_1e^{4x} + C_2e^{-3x} + \frac{1}{7}xe^{4x}$

6. $y'' - 2y' - 3y = 4e^x - 9$

$y'' - 2y' - 3y = 0$

$(m-3)(m+1) = 0$

$m_1 = 3$
$m_2 = -1$

$y_1 = e^{3x}$ $y_2 = e^{-x}$

$g(x) = 4e^x - 9$

$D(D-1)(D^2 - 2D - 3) = D(D-1)(4e^x - 1)$

$(m-3)(m+1) = 0$

$m_1 = 0$
$m_2 = 1$
$m_3 = 3$
$m_4 = -1$

$y_1 = e^{0x}, y_2 = e^x, y_3 = e^{3x}, y_4 = e^{-x}$

$y = C_1 + C_2 e^x + C_3 e^{3x} + C_4 e^{-x}$

$y_p = A + Be^x$

$y' = Be^x$

$y'' = Be^x$

$Be^x - 2Be^x - 3A - 3Be^x = 4e^x - 9$

$\begin{cases} -4B = 4 \\ -3A = 9 \end{cases}$

$-4B = 4$ $A = -9/-3$

$B = -1$ $A = 3$

$y_p = 3 - e^x$

$y = y_c + y_p$

$y = C_1 e^{3x} + C_2 e^{-x} + 3 - e^x$

7. $y'' - 2y' + 5y = e \cdot \operatorname{sen} x$

$y'' - 2y' + 5y = 0$

$m^2 - 2m + 5 = 0$

$m = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$m = \dfrac{2 \pm \sqrt{4 - 20}}{2}$

$m_1 = 1 + 2i$

$m_2 = 1 - 2i$

$y_1 = e^x \cos 2x$

$y_2 = e^x \operatorname{sen} 2x$

$(D^2-2D-5)\,y = e^x \operatorname{sen} x$

$[D^2-2D-5)y = e^x \operatorname{sen} x\,]\,(D^2-2D+2)$

$(D^2-2D+2)\,(D^2-2D+5)=0$

$y_p = D^2-2D+2$

$m = \dfrac{-2 \pm \sqrt{(-2)^2-4(1)(2)}}{2(1)}$

$m = \dfrac{2 \pm \sqrt{4-8}}{2}$

$y_c = C_1 e^x \cos 2x + C_2 e^x \operatorname{Sen} 2x$

$m_1 = 1+i$

$m_2 = 1-i$

$y = C_1 e^x \cos 2x + C_2 e^x \operatorname{Sen} 2x + C_3 e^x \cos x + C_4 e^x \operatorname{sen} x$

$y_p = A e^x \cos + B e^x \operatorname{Sen} x$

$= (A\cos x + B \operatorname{sen} x)\, e^x$

$y' = (A\cos x + B\operatorname{sen} x)\, e^x + (-A\operatorname{sen} x + B\cos x)\,e^x$

$y' = ((A+B)\cos x + (B-A\operatorname{sen} x)\, e^x$

$y'' = (2B\cos - 2A\operatorname{sen} x)\, e^x$

$(2B\cos x - 2A\operatorname{sen} x)\, e^x + [-2A-2B)\cos x + (-2B+2A)\operatorname{sen} x]\, e^x + 5A\cos x$
$5B\operatorname{sen})\, e^x = e^x \operatorname{sen} x$

$3B\operatorname{sen} x^{e^x} + 3A\cos x\, e^x = e^x \operatorname{sen} x$

$\begin{cases} 3A = 0 \\ 3B = 1 \end{cases}$ $\quad B = 1/3$
$\quad A = 0$

$y_p = \dfrac{1}{3} e^x \operatorname{Sen} x$ $\qquad y = C_1 e^x \cos 2x + C_2 e^x \operatorname{Sen} 2x + \dfrac{1}{3} e^x \operatorname{sen} x$ //