Aflevering uge 3

Opgave 8.2.53

Rate of Growth The area covered by a patch of moss is growing at a rate of

$$A'(t) = \sqrt{t}\ln(t)$$

cm^2 per day, for $t \geq 1$. Find the additional amount of area covered by the moss between 4 and 9 days

$-$ s. 493

Vi starter med at finde den ubestemte integral ved hjælp af delvis integration, derfor skal vi starte med at dele funktion

$$A'(t) = \underbrace{\ln(t)}_{u} \cdot \underbrace{\sqrt{t}\ dt}_{dv}$$

$$u = \ln(t) \Leftrightarrow du = t^{-1}$$

$$dv = t^{\frac{1}{2}} \Leftrightarrow v = \int t^{\frac{1}{2}}\ dt$$

$$= \frac{2}{3}t^{\frac{3}{2}}$$

Opskriv formlen for delvis integration

$$A = u \cdot v - \int v \cdot u'\, dt$$

Indsæt

$$A = \frac{2}{3}t^{\frac{3}{2}}\ln(t) - \int \frac{2}{3}t^{\frac{3}{2}} \cdot t^{-1}\ dt$$

$$= \frac{2}{3}t^{\frac{3}{2}}\ln(t) - \frac{2}{3}\int t^{\frac{3}{2}-1}\ dt$$

$$= \frac{2}{3}t^{\frac{3}{2}}\ln(t) - \frac{2}{3}\int t^{\frac{1}{2}}\ dt$$

$$= \frac{2}{3}t^{\frac{3}{2}}\ln(t) - \frac{2}{3}\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + C$$

$$= \frac{2}{3}t^{\frac{3}{2}}\ln(t) - \frac{4}{9}t^{\frac{3}{2}} + C$$

Herefter skal vi finde det bestemte integral fra 4 til 9

$$A(t)\ |_4^9 = A(9) - A(4)$$

$$= 23.712\ cm^2$$