

Opgave 6.1.50

Satisfaction Suppose some substance (such as a preferred food) gives satisfaction to an individual, but the substance requires effort to obtain, so that after a while the individual is no longer interested in expending more effort to obtain the substance. A mathematical model of this situation is given by

$$S = a \ln kx - bx$$

, where S is the amount of satisfaction, x is the amount of the substance, and a , b , and k are constants. Source: Mathematical Biology of Social Behavior. Find the amount of the substance that gives the maximum amount of satisfaction.

— s. 372

$$S = f(g(x)) - h(x)$$

Opstil de forskellige funktioner

$$\begin{aligned} f(x) &= a \ln(x) \Leftrightarrow f'(x) = \frac{a}{x} \\ g(x) &= kx \Leftrightarrow g'(x) = k \\ h(x) &= bx \Leftrightarrow h'(x) = b \end{aligned}$$

Brug kædereglene til at differentiere

$$\begin{aligned} (f(g(x)))' &= f'(g(x)) \cdot g'(x) \\ &= \frac{a}{kx} \cdot k \\ &= \frac{a}{x} \end{aligned}$$

Opstil den afledte funktion af S

$$S' = \frac{a}{x} - b$$

Sæt S' lig 0

$$\begin{aligned} S' = 0 &= \frac{a}{x} - b \\ b &= \frac{a}{x} \\ bx &= a \\ x &= \frac{a}{b} \end{aligned}$$