Opgave 13.2.26

Petal length The length (in centimeters) of a petal on a certain flower is a random variable with probability density function defined by

$$f(x) = \frac{1}{2\sqrt{x}} \quad \text{for } x \in [1, 4]$$

a. Find the expected petal length.
b. Find the standard deviation.
c. Find the probability that a petal selected at random has a length more than 2 standard deviations above the mean.
d. Find the median petal length.

— s. 774

Opgave a.

Find middelværdien

$$\begin{aligned}
\mu &= \int_1^4 x \cdot \left(\frac{1}{2\sqrt{x}}\right)\, dx \\
&= \frac{1}{2}\int_1^4 x \cdot x^{-\frac{1}{2}} \\
&= \frac{1}{2}\int_1^4 x^{\frac{1}{2}} \\
&= \frac{1}{2}\left[\frac{2}{3}x^{\frac{3}{2}}\right]_1^4 \\
&= \frac{1}{3}\left[x^{\frac{3}{2}}\right]_1^4 \\
&= \frac{1}{3} \cdot \left(4^{\frac{3}{2}} - 1^{\frac{3}{2}}\right) \\
&= 2.33
\end{aligned}$$

Den forventede længde er $\underline{\underline{2.33}}$

Opgave b.

$$\begin{aligned}\sigma^2 &= \int_1^4 x^2 \left(\frac{1}{2\sqrt{x}}\right)\ dx - \mu^2 \\ &= \frac{1}{2}\int_1^4 x^2 \cdot x^{-\frac{1}{2}}\ dx - \mu^2 \\ &= \frac{1}{2}\int_1^4 x^{\frac{3}{2}}\ dx - \mu^2 \\ &= \frac{1}{2}\left[\frac{2}{5}x^{\frac{5}{2}}\right]_1^4 - \mu^2 \\ &= \frac{1}{5}\left(4^{\frac{5}{2}} - 1\right) - \mu^2 \\ &= \frac{4^{\frac{5}{2}}}{5} - \frac{1}{5} - (2.33)^2 \\ &= 0.77\end{aligned}$$

$$\begin{aligned}\sigma &= \sqrt{\sigma^2} \\ &= \sqrt{0.77} \\ &= 0.88\end{aligned}$$

Standard afvigelsen er $\underline{\underline{0.88}}$

Opgave c.

Vi skal finde en længden som er 2 standard diviationer over middelværdien:

$$\begin{aligned}u &= \mu + 2\sigma \\ &= 2.33 + (0.88 \cdot 2) \\ &= 4.09\end{aligned}$$

Da længden ligger udenfor vores intervaller for vores tæthedsfunktion, er dette altså umuligt.

Opgave d.

Medianen kan findes ved følgende

$$\begin{aligned}\frac{1}{2} &= \int_1^m \frac{1}{2\sqrt{x}}\ dx \\ \frac{1}{2} &= \frac{1}{2}\int_1^m x^{-\frac{1}{2}}\ dx \\ \frac{1}{2} &= \frac{1}{2}\left[\frac{2}{1}x^{\frac{1}{2}}\right]_1^m \\ \frac{1}{2} &= \sqrt{m} - 1 \\ \sqrt{m} &= \frac{3}{2} \\ m &= 2.25\end{aligned}$$

Medianen er $\underline{\underline{2.25}}$