Opgave 6.2.9

Pollution A lake polluted by bacteria is treated with an anti- bacterial chemical. After t days, the number N of bacteria per milliliter of water is approximated by

$$N(t) = 20\left(\frac{t}{12} - \ln\left(\frac{t}{12}\right)\right) + 30, 1 \leq t \leq 12$$

a. When during this time will the number of bacteria be a minimum?
b. What is the minimum number of bacteria during this time?
c. When during this time will the number of bacteria be a maximum?
d. What is the maximum number of bacteria during this time?

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Opstil funktion og under funktioner

$$N(t) = 20\left(\frac{t}{12} - \ln\left(\frac{t}{20}\right)\right) + 30, 1 \leq t \leq 12$$
$$N(t) = f(g(x) - h(m(x)))$$

$$f(x) = 20x - 30 \Leftrightarrow f'(x) = 20$$
$$g(x) = \frac{t}{12} \Leftrightarrow g'(x) = \frac{1}{12}$$
$$h(x) = \ln(x) \Leftrightarrow h'(x) = x^{-1}$$
$$m(x) = \frac{t}{12} \Leftrightarrow m'(x) = \frac{1}{12}$$

Differentier de forskellige dele

$$(h(m(x)))' = h'(m(x)) \cdot m'(x)$$
$$(h(m(x)))' = \left(\frac{t}{12}\right)^{-1} \cdot \frac{1}{12}$$
$$(h(m(x)))' = \frac{1}{t} = t^{-1}$$

$$(f(g(x) - h(m(x))))' = N'(t) = f'(g(t) - h(m(t))) \cdot (g(t) - h(m(t)))'$$
$$N'(t) = 20 \cdot \left(\frac{1}{12} - t^{-1}\right)$$
$$N'(t) = \frac{5}{3} - \frac{20}{t}$$

Opgave a. When during this time will the number of bacteria be a minimum?

$$N'(t) = 0 = \frac{5}{3} - \frac{20}{t}$$
$$\frac{20}{t} = \frac{5}{3}$$
$$20 = \frac{5}{3} \cdot t$$
$$\frac{20}{\frac{5}{3}} = t$$
$$12 = t$$

Check yder punkterne og ekstrermerne for funktionen

$$N(1) = 81.36$$
$$N(12) = 50$$
$$N(15) = 50.54$$

Der er færrest bakterier efter $t = 12$ dage

Opgave b. What is the minimum number of bacteria during this time?

$$N(12) = 50$$

Der er 50 bakterier efter 12 dage.

Opgave c. When during this time will the number of bacteria be a maximum?

Se Opgave a.
Der vil være flest bakterier efter $t = 1$ dage

Opgave d. What is the maximum number of bacteria during this time?

$$N(1) = 81.36$$

Der er 81.36 bakterier efter 1 dag