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Opgave 9.5.35

Find the volume under the given surface $z = f(x, y)$ and above the rectangle with the given boundaries

$$z = x\sqrt{x^2 + y};\ 0 \le x \le 1,\ 0 \le y \le 1$$

— s. 559

Opstil

$$\begin{aligned} R = 0 \le x \le 1 \\ 0 \le y \le 1 \end{aligned}$$

$$\begin{aligned} V &= \iint_R x\sqrt{x^2 + y}\ dy\ dx \\ &= \int_0^1 \int_0^1 x\sqrt{x^2 + y}\ dy\ dx \\ &= \int_0^1 x \int_0^1 \sqrt{x^2 + y}\ dy\ dx \end{aligned}$$

Løsning af det inderste integral

$$\int_0^1 \sqrt{x^2 + y}\ dy$$

$$\int_0^1 \underbrace{(x^2 + y)^{\frac{1}{2}}}_{u}\ dy$$

Find u

$$\begin{aligned} u &= x^2 + y \\ \frac{du}{dy} &= 1 \\ du &= \ dy \end{aligned}$$

Indsæt tilbage og integrer

$$\int_0^1 u^{\frac{1}{2}}\ du$$

$$\frac{2}{3}u^{\frac{3}{2}}$$

$$\left[\frac{2}{3}(x^2 + y)^{\frac{3}{2}}\right]_0^1$$

$$\frac{2}{3}(x^2 + 1)^{\frac{3}{2}} - \frac{2}{3}x^3$$

Indsæt tilbage i dobbelt integralet

$$V = \int_0^1 x\left(\frac{2}{3}(x^2+1)^{\frac{3}{2}} - \frac{2}{3}x^3\right)\ dx$$
$$V = \int_0^1 \frac{2}{3}x(x^2+1)^{\frac{3}{2}} - \frac{2}{3}x^4\ dx$$
$$V = \int_0^1 \frac{2}{3}x(x^2+1)^{\frac{3}{2}}\ dx - \int_0^1 \frac{2}{3}x^4\ dx$$
$$V = \int_0^1 \frac{2}{3}x(x^2+1)^{\frac{3}{2}}\ dx - \frac{2}{15}$$

Løs integralet ved brug af substitution

$$\int_0^1 \frac{2}{3}x(x^2+1)^{\frac{3}{2}}\ dx$$

Substituer u ind

$$\int_0^1 \frac{2}{3}u^{\frac{3}{2}}\ x\ dx$$

Find u og du

$$u = x^2+1$$
$$\frac{du}{dx} = 2x$$
$$du = 2x\ dx$$
$$\frac{1}{2}\ du = x\ dx$$

Indsæt i integralet

$$\int_0^1 \frac{2}{3}u^{\frac{3}{2}}\frac{1}{2}\ du$$
$$\frac{2}{6}\int_0^1 u^{\frac{3}{2}}\ du$$
$$\frac{2}{6}\left[\frac{2}{5}u^{\frac{5}{2}}\right]_0^1$$
$$\left[\frac{4}{30}u^{\frac{5}{2}}\right]_0^1$$
$$\left[\frac{2}{15}(x^2+1)^{\frac{5}{2}}\right]_0^1$$
$$= \frac{2}{15}2^{\frac{5}{2}} - \frac{2}{15}$$

Indsæt tilbage i integralet

$$V = \frac{2}{15} 2^{\frac{5}{2}} - \frac{2}{15} - \frac{2}{15}$$
$$V = \frac{2}{15}\left(2^{\frac{5}{2}} - 2\right)$$