Opgave 12.3.57

> Sensitivity and Specificity The sensitivity and specificity for breast cancer during a mammography exam are approximately 79.6% and 90.2%, respectively. Source: National Cancer Institute.
>
> **a.** It is estimated that 0.5% of U.S. women under the age of 40 have breast cancer. Find the probability that a woman under 40 who tests positive during a mammography exam actually has breast cancer.
> **b.** Given that a woman under 40 tests negative during a mammography exam, find the probability that she does not have breast cancer.
> **c.** According to the National Cancer Institute, failure to diagnose breast cancer is the most common cause of medical malpractice litigation. Given that a woman under 40 tests negative for breast cancer, find the probability that she does have breast cancer.
>
> — s. 731

Opskriv sandsynligheder

Sandsynlighed for kræft	F	0.5%	0.005
Sandsynlighed for ingen kræft	F'	99.5%	0.995
Sandsynlighed for positiv test	E	-	-
Sensitivity: Sandsynligheden for positiv test givet brystkræft	$P(E\|F)$	79.6%	0.796
Specificity: Sandsynligheden for negativ test givet ingen brystkræft	$P(E'\|F')$	90.2%	0.902

Opgave a.

Vi skal finde sandsynligheden for brystkræft givet positiv test, altså:

$$P(F|E)$$

Til det skal vi bruge Bayes' sætning

$$P(F|E) = \frac{P(F) \cdot P(E|F)}{P(F) \cdot P(E|F) + P(F') \cdot P(E|F')}$$

Først skal vi udrenge $P(E|F')$

$$\begin{aligned} P(E|F') &= 1 - P(E'|F') \\ &= 1 - 0.902 \\ &= 0.098 \end{aligned}$$

$$\begin{aligned} P(F|E) &= \frac{0.005 \cdot 0.796}{0.005 \cdot 0.796 + 0.995 \cdot 0.098} \\ &= 0.039 \\ &= 3.9\% \end{aligned}$$

Sandsynligheden for brystkræft givet positiv test er $\underline{\underline{3.9\%}}$

Opgave b.

Vi skal finde sandsynligheden for ingen brystkræft givet negativ test, altså:

$$P(F'|E')$$

Find $P(E'|F)$

$$\begin{aligned} P(E'|F) &= 1 - P(E|F) \\ &= 1 - 0.796 \\ &= 0.204 \end{aligned}$$

Bayes' sætning

$$\begin{aligned} P(F'|E') &= \frac{P(F') \cdot P(E'|F')}{P(F') \cdot P(E'|F') + P(F) \cdot P(E'|F)} \\ &= \frac{0.995 \cdot 0.902}{0.995 \cdot 0.902 + 0.005 \cdot 0.204} \\ &= 0.999 \\ &= 99.9\% \end{aligned}$$

Sandsynligheden for ingen brystkræft givet negativ test $\underline{\underline{99.9\%}}$

Opgave c.

$$\begin{aligned} P(F|E') &= 1 - P(F'|E') \\ &= 1 - 0.999 \\ &= 0.001 \end{aligned}$$

Sandsynligheden for brystkræft givet negativ test er $\underline{\underline{0.1\%}}$