## **Opgave 9.2.63**

Zooplankton Growth The rate that zooplankton consume phytoplankton has been described by the equation

$$C(x,y) = \frac{k}{a + (x - by)^2}$$

where a, b, and k are constants, and x and y represent the cell size of zooplankton and phytoplankton, respectively. Source: The American Naturalist. Calculate  $C_x(x,y)$  and  $C_y(x,y)$ . What do you notice about the signs of  $C_x(x,y)$  and  $C_y(x,y)$ ? Explain why this makes sense.

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Opskriv funktionen

$$C(x,y) = \frac{k}{a + (x - by)^2}$$
$$= k(a + (x - by)^2)^{-1}$$

Find  $C_x(x,y)$ 

$$\begin{split} C_x(x,y) &= (f(g(x)))' = f'(g(x)) \cdot g'(x) \\ g(x) &= a + (x - by)^2 \\ f'(x) &= -k(g(x))^{-2} \\ g'(x) &= 2(x - by) \\ C_x(x,y) &= -k\big(a + (x - by)^2\big)^{-2} \cdot 2(x - by) \\ C_x(x,y) &= -\frac{2k(x - by)}{(a + (x - by)^2)^2} \end{split}$$

Find  $C_y(x,y)$ 

$$\begin{split} f(u) &= k(u)^{-1} \Leftrightarrow -k(u)^{-2} \\ g(x,y) &= a + (x-by)^2 \Leftrightarrow g_y(x,y) = 2(x-by) \cdot (-b) \\ C_y(x,y) &= -k\big(a + (x-by)^2\big)^{-2} \cdot 2(x-by) \cdot (-b) \\ &= \frac{2kb(x-by)}{\big(a + (x-by)^2\big)^2} \end{split}$$