Boundary Behaviour of Differential Equation for AdS/CFT Superconductivity

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The differential equation of interest

$$\begin{aligned} (-2z^3 + z^2\phi^2 + 2)\psi + (z^7 + z^4 - 2z)\psi' + (z^8 - 2z^5 + z^2)\psi'' = 0 \\ (z^5 - z^2)\phi'' + 2\phi\psi^2 = 0 \end{aligned} \tag{1}$$

or

$$\begin{aligned} (-2z^3 + z^2\phi^2 + 2)\psi + z\left((z^3-1)^2 + 3(z^3-1)\right)\psi' + z^2(z^3-1)^2\psi'' = 0 \\ z^2(z^3-1)\phi'' + 2\phi\psi^2 = 0 \end{aligned} \tag{2}$$

The expansions of the solution at $z = 0$ given boundary conditions μ, ρ, ψ_2 and that $\psi'(0) = 0$ is

$$\begin{aligned} \psi(z) &= \psi_2 z^2 - 4\mu^2\psi_2\frac{z^4}{4!} + 20\psi_2(2+\mu\rho)\frac{z^5}{5!} + \mathcal{O}(z^6) \\ \phi(z) &= \mu - \rho z + 4\mu\psi_2^2\frac{z^4}{4!} - 12\rho\psi_2^2\frac{z^5}{5!} + \mathcal{O}(z^6) \end{aligned} \tag{3}$$

It has numerically been found that starting at finite z and integrating down towards $z = 0$ with starting values such that $\psi'(0) = 0$ and ρ is big gives $\rho/\psi_2 = \sqrt{2}$.