Jérémie Dentan
jeremie.dentan@live.com

Homework # 3
Convex optimization 2022

11/26/22

1 Question 1

This question is a direct consequence of question 2 of homework 2. The dual problem we obtained for (LASSO) is:

$$\min_v \frac{1}{2}||v||_2^2 + y^T v$$

$$\text{s.t. } ||X^T v||_\infty \leq \lambda$$

With the notations of the homework, this is a particular form of (QP) with:

$$Q = 0.5 I_n \ \ ; \ \ p = y \ \ ; \ \ A = \begin{bmatrix} X^T \\ -X^T \end{bmatrix} \in \mathbb{R}^{2d} \ \ ; \ \ d = \begin{bmatrix} \lambda \\ \vdots \\ \lambda \end{bmatrix} \in \mathbb{R}^{2d}$$

2 Question 2

We want to solve (QP). With the notations of the homework, we first write $(a_1, ...a_M)$ the lines of matrix A:

$$A = \begin{bmatrix} a_0^T \\ \vdots \\ a_m^T \end{bmatrix}$$

Then, by definition, the barrier problem is :

$$\min_v t(vtQv + p^T v) - \sum_{i=1}^m \log(b_i - a_i^T v)$$

Let $f_t(v)$ be the objective function of this minimization problem. For more simplicity, let $g_i(v) = b_i - a_i^T v$. The gradient is given by:

$$\nabla f_t(v) = 2tQv + tp + \sum_{i=1}^m \frac{a_i}{g_i(v)}$$

Let $h_i(v) = \frac{a_i}{g_i(v)}$. The the hessian is given by:

$$\nabla^2 f_t(v) = 2tQ + \sum_{i=1}^m \frac{a_i a_i^T}{g_i(v)^2} = 2tQ + \sum_{i=1}^m h_i(v) h_i(v)^T$$

Those computations are then used in our implementation of the Newton method. Please refer to scripts in `/src/centering.py` and `/src/barrier.py`.

3 Question 3

Please refer to our implementation in `/src/mse.py`. Please refer to `README.md` for further detail to run the code.
The figure we obtained is presented in figure 1

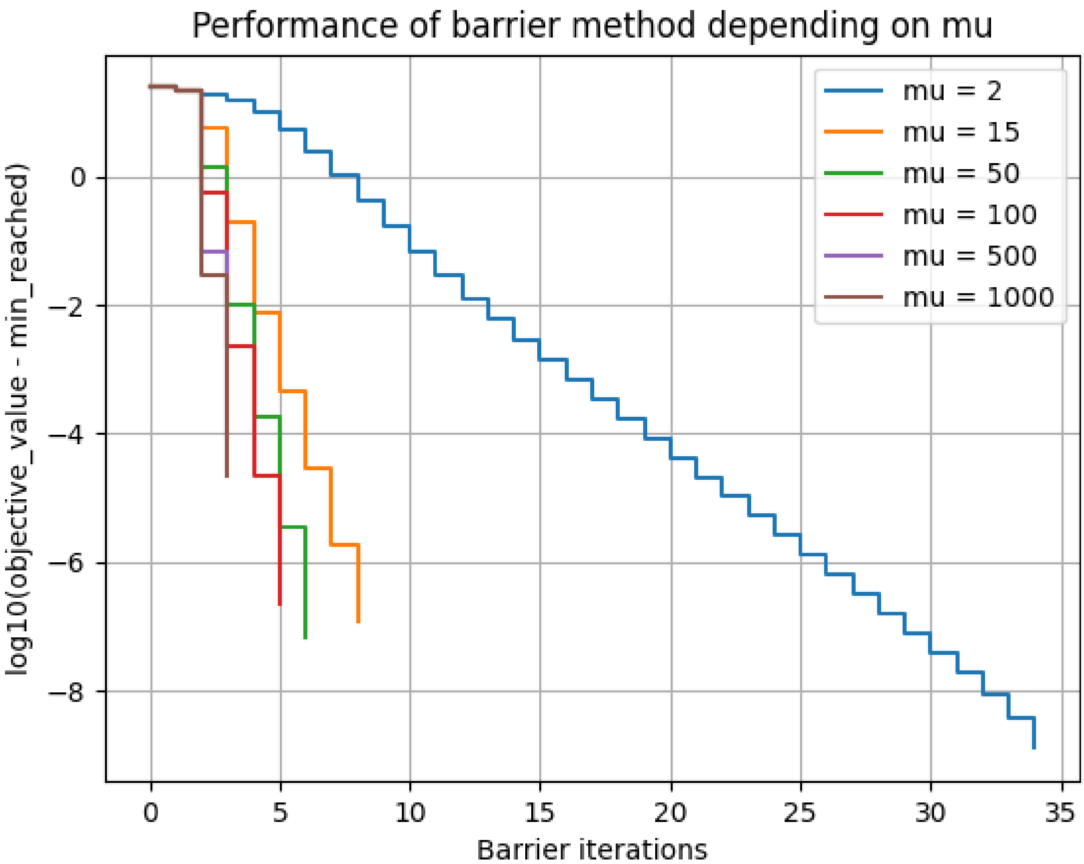

Figure 1: Performances depending on μ