Adaptive Tracking Algorithm Based on Modified Strong Tracking Filter

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Abstract: **The strong tracking filter (STF) can reduce adaptively estimate bias and thus has ability to track maneuvering target in nonlinear systems. However, STF achieves the perfect performance in maneuvering segment at a cost of the precision in non-maneuvering segment. So based on the Strong Tracking Filter, a new adaptive tracking algorithm(Modified Strong Tracking Filter, MSTF) is derived in this paper, which is also suitable for tracking maneuvering target and has improved the precision of STF in non-maneuvering segment as well as maneuvering segment. The Monte-Carlo simulation results show that the MSTF algorithm has a more excellent performance than STF and can estimate efficiently.**

Index Terms: **extended Kalman filter, Strong Tracking Filter, target tracking**

I. INTRODUCTION

It is well known that the extended Kalman filter(EKF) can be used for the state estimation of a class of nonlinear systems. However, in most cases, the EKF can only give a biased state estimation, which has poor robustness and precision against model mismatching [1] and is not suitable for maneuvering target tracking. For this reason, a nonlinear adaptive extended Kalman filter (Strong Tracking Filter, STF) [2,3] is proposed by Prof. Zhou Donghua in 1991, which is based on the orthogonality principle[4]. By introducing a time-varying suboptimal fading factor $\lambda(k+1)$ into the prediction error covariance of EKF, STF adjusts the corresponding gain matrix on-line, and compels the residual error series to be orthogonal to each other. It is proved that STF has strong robustness against model mismatching, and can be applied to maneuvering target tracking.

Nevertheless, we observed that the tracking precision of STF is inferior to EKF in non-maneuvering segment, i.e. STF obtains a perfect performance in maneuvering segment at a cost of the precision in non-maneuvering segment. Further more, the state estimation is sometimes overregulated because of the influence of the suboptimal fading factor in STF. In order to solve the above problems, a new adaptive tracking algorithm-Modified Strong Tracking Filter(MSTF) is provided in this paper. STF regards the maneuver of the target state as model mismatching and improves the tracking precision by many modifications. Simulation results show that MSTF algorithm has a more excellent performance in maneuvering target tracking than STF and can estimate efficiently.

II. STRONG TRACKING FILTER

Consider a class of stochastic systems represented by the following discrete state-space model:

$$\begin{aligned} X(k+1) &= F(k+1,k)X(k)+w(k) \\ Z(k+1) &= h(k+1,X(k+1))+v(k+1) \end{aligned} \tag{1}$$

where the plant state $X(\cdot)$ is a n-vector, $F(\cdot)$ denotes the state transition matrix, $w(k)$ is a sequence of zero-mean, white, Gaussian process noise with covariance $Q(k)$. The measurement $Z(\cdot)$ is a m-vector, $h(\cdot)$ is a nonlinear function about the state, and $v(k+1)$ is a sequence of zero-mean, white, Gaussian process noise with covariance $R(k+1)$.

The extended Kalman filter to solve the state estimation problem is given by:

$$\hat{X}(k+1|k+1)=\hat{X}(k+1|k)+K(k+1)\gamma(k+1) \tag{2}$$

where the one-step prediction of the state is

$$\hat{X}(k+1|k)=F(k+1,k)X(k,k) \tag{3}$$

the filter gain is given by

$$\begin{aligned} K(k+1|k) &= P(k+1|k)H^T(k+1,\hat{X}(k+1,k)) \\ &\bullet[H(k+1,\hat{X}(k+1,k))P(k+1|k) \\ &\bullet H^T(k+1,\hat{X}(k+1,k))+R(k+1)]^{-1} \end{aligned} \tag{4}$$

the prediction covariance is given by

$$P(k+1|k)=F(k+1,k)P(k|k)F^T(k+1,k)+Q(k) \tag{5}$$

the covariance matrix of the state is given by

$$\begin{aligned} &P(k+1,k+1) \\ &=[I-K(k+1)H(k+1,\hat{X}(k+1,k))]P(k+1,k) \end{aligned} \tag{6}$$

the measurement residual is

$$\gamma(k+1)=Z(k+1)-h(k+1,\hat{X}(k+1,k)) \tag{7}$$

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in (4) there is the Jacobian of the measurement vector evaluated at the a priori estimate:

$$H(k+1,\hat{X}(k+1\mid k)) = \left.\frac{\partial h(k+1, X(k+1))}{\partial X}\right|_{X(k+1)=\hat{X}(k+1,k)} \quad (8)$$

Equations (2)~(8) constitutes the recursive algorithm of the well-known extended Kalman filter.

Based on the extended Kalman filter and orthogonality principle, the Strong Tracking Filter(STF) is deduced by introducing a suboptimal fading factor $\lambda(k+1)$ into the prediction error covariance equation of the EKF as follows:

$$\begin{aligned} &P(k+1\mid k) \\ &= \lambda(k+1)F(k+1,k)P(k\mid k)F^T(k+1,k) + Q(k) \end{aligned} \quad (9)$$

It makes the residual error series orthogonal to each other at each step, so that all the useful information in the residual error series could be extracted. The fading factor $\lambda(k+1)$ can be computed by unconstrained nonlinear programming method. However, it needs complicated optimization process and huge computational efforts, and is not suitable for real-time computation. Therefore, an approximative suboptimal formula is usually used:

$$\lambda(k+1) = \begin{cases} \lambda_0 & \lambda_0 \geq 1 \\ 1 & \lambda_0 < 1 \end{cases} \quad (10)$$

where

$$\lambda_0 = \frac{tr[N(k+1)]}{tr[M(k+1)]} \quad (11)$$

$$\begin{aligned} N(k+1) = V_0(k+1) - \beta R(k+1) - H(k+1,\hat{X}(k+1,k)) \\ \bullet Q(k)H^T(k+1,\hat{X}(k+1,k)) \end{aligned} \quad (12)$$

$$\begin{aligned} M(k+1) = H(k+1,\hat{X}(k+1,k))F(k+1,k)P(k\mid k) \\ \bullet F^T(k+1,k)H^T(k+1,\hat{X}(k+1,k)) \end{aligned} \quad (13)$$

The covariance of the residual $V_0(k+1)$ can be calculated approximately as the following equation.

$$\begin{aligned} V_0(k+1) &= E[\gamma(k+1)\gamma^T(k+1)] \\ &= \begin{cases} \gamma(1)\gamma^T(1), & k=0 \\ \dfrac{[\rho V_0(k) + \gamma(k+1)\gamma^T(k+1)]}{1+\rho}, & k=1 \end{cases} \end{aligned} \quad (14)$$

where $\gamma(1)$ is the initial residual and $0 < \rho \leq 1$ is the preselected forgetting factor, it may be selected according to the real processes. β is a softening factor, which is selected to smooth out the state estimation [4].

III. MODIFIED STRONG TRACKING FILTER

As what is said above, when the maneuver happens, STF performs much better than EKF by introducing the fading factor $\lambda(k+1)$. However, the precision of STF in non-maneuvering segment is not satisfying. It's easy to realize that STF has ignored the effect of the second term in the right part of (9), which is the covariance $Q(k)$. In fact, the maneuver of the target state can be regarded as model mismatching, so it seems quite reasonable to increase the weight of the covariance $Q(k)$ in the recursion of $P(k+1\mid k)$. Thus the expression of $P(k+1\mid k)$ can be defined as

$$\begin{aligned} &P(k+1\mid k) \\ &= \lambda(k+1)\left(F(k+1,k)P(k\mid k)F^T(k+1,k) + Q(k)\right) \end{aligned} \quad (15)$$

So the following task comes to determining the fading factor $\lambda(k+1)$ using orthogonality principle which is provided in [4].

In order to submit to the orthogonality principle, i.e. $E\{r(k+1+j)r^T(k+1)\} = 0$, a time-varying gain matrix $K(k+1)$ should be selected to make

$$P(k+1\mid k)H^T(k+1,\hat{X}(k+1\mid k)) - K(k+1)V_0(k+1) \equiv 0 \quad (16)$$

Hence, using (4) in (16) yields

$$\begin{aligned} &P(k+1\mid k)H^T(k+1,\hat{X}(k+1\mid k)) - P(k+1\mid k) \\ &\bullet H(k+1,\hat{X}(k+1\mid k))[H(k+1,\hat{X}(k+1\mid k))P(k+1\mid k) \\ &\bullet H^T(k+1,\hat{X}(k+1\mid k)) + R(k+1)]^{-1}V_0(k+1) \equiv 0 \end{aligned} \quad (17)$$

i.e.

$$\begin{aligned} &P(k+1\mid k)H^T(k+1,\hat{X}(k+1\mid k)) \\ &\{I - [H(k+1,\hat{X}(k+1\mid k))P(k+1\mid k) \\ &\bullet H^T(k+1,\hat{X}(k+1\mid k)) + R(k+1)]^{-1}V_0(k+1)\} \equiv 0 \end{aligned} \quad (18)$$

While, a sufficient condition to ensure (18) is

$$\begin{aligned} &I - [H(k+1,\hat{X}(k+1\mid k))P(k+1\mid k) \\ &\bullet H^T(k+1,\hat{X}(k+1\mid k)) + R(k+1)]^{-1}V_0(k+1) = 0 \end{aligned} \quad (19)$$

i.e.

$$\begin{aligned} &H(k+1,\hat{X}(k+1\mid k))P(k+1\mid k)H^T(k+1,\hat{X}(k+1\mid k)) \\ &= V_0(k+1) - R(k+1) \end{aligned} \quad (20)$$

So substituting (15) into (20) results in the following

$$\lambda(k+1)H(k+1,\hat{X}(k+1|k))[F(k+1,k)P(k|k)$$
$$\bullet F^T(k+1,k)+Q(k)]H^T(k+1,\hat{X}(k+1|k)) \quad (21)$$
$$=V_0(k+1)-R(k+1)$$

Define

$$N(k+1)\triangleq V_0(k+1)-\beta R(k+1)$$
$$M(k+1)\triangleq H(k+1,\hat{X}(k+1|k))[F(k+1,k)P(k|k)$$
$$\bullet F^T(k+1,k)+Q(k)]H^T(k+1,\hat{X}(k+1|k)) \quad (22)$$

And (21) can be written as

$$\lambda(k+1)M(k+1)=N(k+1) \quad (23)$$

The following purpose is to avoid the overregulating effect effectively and make the state estimation smoother. Therefore, using the characteristic of the square root function, the equations (10) and (11) can be modified as

$$\lambda(k+1)=\begin{cases}\lambda_0 & \lambda_0\geq 1\\ 1 & \lambda_0<1\end{cases} \quad (24)$$

$$\lambda_0=\sqrt{\frac{tr[M(k+1)]}{tr[N(k+1)]}} \quad (25)$$

In this way, the MSTF algorithm which is suitable for maneuvering target tracking is obtained. The improvements that made upon STF is relatively having increased the weight of the covariance $Q(k)$ in the recursion of $P(k+1|k)$ while the target maneuver happens, and given a modified formula for $\lambda(k+1)$. Theoretically, the modifications can improve the tracking precision effectively.

IV. SIMULATION AND ANALYSIS

To illustrate the effectiveness of MSTF, a simulation is performed using EKF, STF and MSTF algorithm.

Consider a 3D maneuvering target model in Cartesian coordinates, and the stochastic system is given as (1) using constant velocity model. Two sensors *A* and *B* are simulated. Assume that sensor *A* is located at the origin, and sensor *B* is located at coordinate (u,v,w), where $u=150$, $v=0$, and $w=0$ nm(nautical mile) in the system plane. We use $(R_A(k),\theta_A(k),\eta_A(k))$ and $(R_B(k),\theta_B(k),\eta_B(k))$ to represent the range, azimuth and elevation measurements of the target measured by sensors *A* and *B*. The coordinate of the target is described as $(x(k),y(k),z(k))$, and the velocity is $(\dot{x}(k),\dot{y}(k),\dot{z}(k))$. Then the state vector is defined as

$$\boldsymbol{X}(k)=\begin{bmatrix}x(k) & \dot{x}(k) & y(k) & \dot{y}(k) & z(k) & \dot{z}(k)\end{bmatrix}^T \quad (26)$$

The measurement vector is

$$\boldsymbol{Z}(k)=\begin{bmatrix}R_A(k) & \theta_A(k) & \eta_A(k) & R_B(k) & \theta_B(k) & \eta_B(k)\end{bmatrix}^T \quad (27)$$

The simulation results are based on 100 Monte-Carlo runs. The sampling interval of two sensors is T=1s. The range errors of the sensors are $\sigma_{\rho A}=\sigma_{\rho B}=50\text{m}$, the azimuth errors are $\sigma_{\theta A}=\sigma_{\theta B}=0.4^\circ$, the elevation errors are $\sigma_{\eta A}=\sigma_{\eta B}=0.1^\circ$.

Suppose the initial state of the target is

$$X(0)$$
$$=[60000\text{m},70\text{m/s},50000\text{m},80\text{m/s},10000\text{m},50\text{m/s}]^T \quad (28)$$

The motion of the target costs 1000s, and the information of the motion is shown as the following table:

the time maneuver happens	t=600s	t=800s
X-Velocity	105m/s	157.5m/s
Y-Velocity	160m/s	320m/s
Z-Velocity	50m/s	50m/s

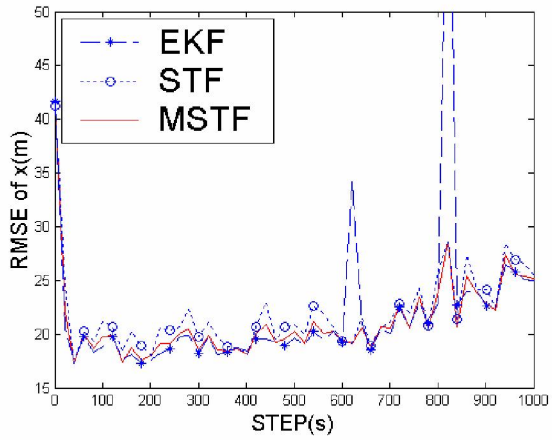
In this simulation, the results is scaled by two statistical indexes, such as the Root Mean Square Error(RMSE) and Accumulative Error(AE). The AE represents the whole accumulative error (a dimensionless parameter) of the filter, which is defined as

$$AE(k)=\frac{1}{M}\sum_{j=1}^{M}\sum_{i=1}^{n}\left|X_i^j(k)-\hat{X}_i^j(k|k)\right| \quad (29)$$

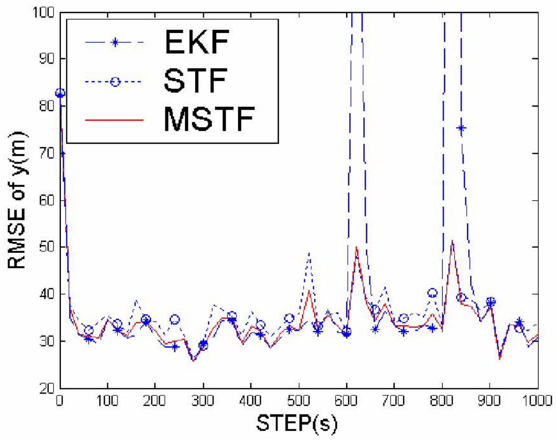
where M is the number of Monte-Carlo simulation(M=1000), and n is the dimension of the state vector(n=6). $X_i^j(k)$ denotes the *i*th element of the state vector at time *k* in the *j*th run, while $\hat{X}_i^j(k|k)$ denotes the corresponding state estimate. So *AE*(*k*) describes the whole accumulative error at time *k*.

The simulation results including RMSE and AE curves are shown Fig. 1. The RMSE curves show that EKF has an efficient precision in non-maneuvering segment, however, the performance is deteriorative when maneuver happens. STF has a good performance when maneuver happens. But actually, the achievement of the good performance in maneuvering segment is at a cost of the precision in non-maneuvering segment. From these figures, it is clear that the tracking errors of MSTF tracking target in non-maneuvering segment are much lower than that of STF, while its tracking precision in maneuvering segment remains at the same level as that of STF, sometimes MSTF performs even better than STF. In other words, MSTF has improved the performance in non-maneuvering segment as well as maneuvering segment. In addition, it can be seen from the AE curves in Fig. (d) that the accumulative error of MSTF is obviously lower than the error of STF and EKF.

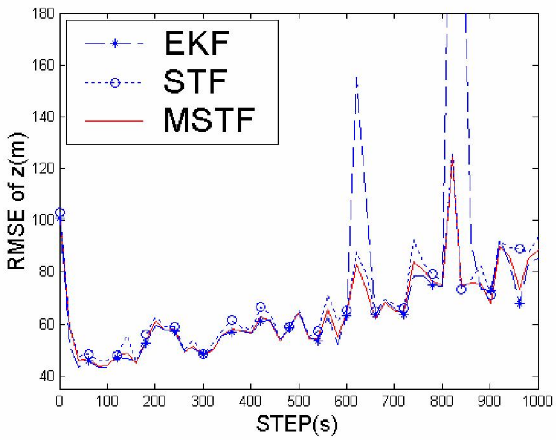
It shows the superiority and efficiency of MSTF from another viewpoint.



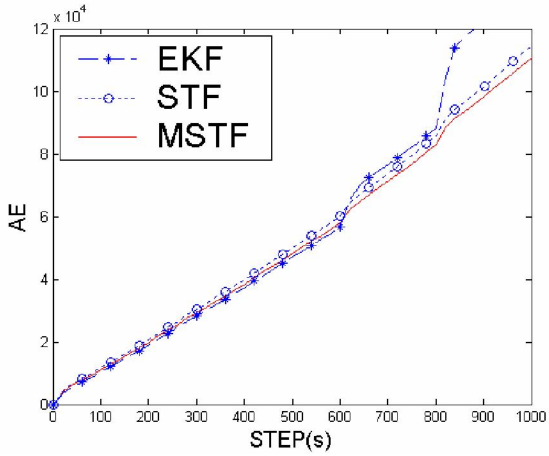
(a)RMSE in X-Direction



(b) RMSE in Y-Direction



(c)RMSE in Z-Direction



(d) Accumulative Error(AE)

Fig.1 the RMSE and AE curves

V. CONCLUSION

In this paper, a new adaptive tracking algorithm-Modified Strong Tracking Filter(MSTF) is presented, which is suitable for tracking maneuvering target. The Monte-Carlo simulation results show that the MSTF algorithm has a more excellent performance in maneuvering target tracking than STF and can estimate efficiently.

REFERENCES

[1] A. H. Jazwinski. Stochastic Process and Filter Theory, Academic Press, 1970.

[2] Zhou Donghua, Xi Yugeng, Zhang Zhongjun. Suboptimal fading extended kalman filtering for nonlinear systems[J]. Decision and Control, 1990, 5(5): 1-6.

[3] Zhou Donghua, Xi Yugeng, Zhang Zhongjun. A suboptimal multiple fading extended kalman filter[J]. Acta Automatica Sinica, 1991, 17(6):689-695.

[4] Zhou Donghua. Fault detection and diagnostics for a class of nonlinear systems: filter approaches. PH. D. Dissertation, Shanghai Jiaotong University, 1990.

[5] Zhou Donghua, Ye Yinzhong. Modern fault diagnostic and fault tolerant control[M]. Beijing: Tsinghua University Press, 2000.

[6] B. Friedland. Treatment of bias in recursive filtering. IEEE Trans. Automatic Control, vol. AC-14, pp.359-367, August 1969.

[7] Bar-shalom,Y., Fortmann,T.E. Tracking and data association. New York, Academic press,1988.

[8] He You, Wang Guohong, Lu Dajin, Peng Yingning. Multisensor information fusion with applications[M]. Beijing:Publishing House of Electronics Industry, 2000.

[9] He You, Xiu Jianjuan, Zhang Jingwei, Guan Xin. Radar data processing with application[M]. Beijing: Publishing House of Electronics Industry, 2006.